

Started on	Saturday, 17 February 2024, 12:16 PM
State	Finished
Completed on	Saturday, 17 February 2024, 12:35 PM
Time taken	18 mins 23 secs
Marks	7.00/7.00
Grade	10.00 out of 10.00 (100%)

Question 1

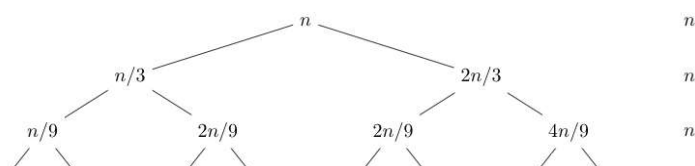
Correct

Mark 1.00 out of 1.00

Recursion Tree is one way to analyze recursive functions. Consider a function with following time complexity.

$$T(n) = T(n/3) + T(2n/3) + n$$

Following figure shows the first 3 levels of the recursion tree.



What is/are the number(s) which can not be appear in the next (4th) level in this recursion tree?

- ☐ a. $8n/27$
- ☒ b. $16n/27$ ✓
- ☐ c. $2n/27$
- ☐ d. $n/27$

The correct answer is: $16n/27$

Question 2

Correct

Mark 1.00 out of 1.00

Which is not a method for analyzing time complexity of recurrences?

- ☐ a. Substitution Method
- ☐ b. Recurrence Tree Method
- ☒ c. Amortized Method ✓
- ☐ d. Master Method

The correct answer is: Amortized Method

Question 3

Correct

Mark 1.00 out of 1.00

Given a set 'S' of n integers and another integer x, an algorithm should determine whether or not there exists two elements in S whose sum is exactly x. A possible algorithm for this task is described below.

- 1) Sort the elements in S using any efficient sorting algorithm.
- 2) Remove the last element from S. Let y be the value of the removed element.
- 3) If S is non-empty, look whether an element z exist in S where $z=x-y$
- 4) If S contains such an element z, then stop, since we have found y and z such that $x=y+z$; otherwise repeat Step 2.
- 5) If S is empty, then no two elements in S sum to x.

Select the correct statement(s) regarding above approach.

- ☒ a. Time complexity of this algorithm is $\Theta(\text{nlg } n)$. ✓
- ☐ b. Best time complexity to do Step 3 is $\Theta(n)$.
- ☒ c. Step 1 can be achieved through merge sort with $\Theta(\text{nlg } n)$ time complexity. ✓
- ☐ d. There are algorithms which can solve this task with better time complexity than above described algorithm

The correct answers are: Step 1 can be achieved through merge sort with $\Theta(\text{nlg } n)$ time complexity., Time complexity of this algorithm is $\Theta(\text{nlg } n)$.

Question 4

Correct

Mark 1.00 out of 1.00

Select the asymptotic upper and lower bounds for $T(n)$ in the following recurrence. Assume that $T(n)$ is constant for $n \leq 3$. Make your bounds as tight as possible.

$$T(n) = T(n-2) + \log(n)$$

Select one:

- ☐ a. $T(n) = \Theta(n)$
- ☐ b. $T(n) = \Theta(\log(n))$
- ☐ c. $T(n) = \Theta(n^2)$
- ☒ d. $T(n) = \Theta(n \log(n))$ ✓

Your answer is correct.

The correct answer is:

$$T(n) = \Theta(n \log(n))$$

Question 5

Correct

Mark 1.00 out of 1.00

The solution to the recurrence $T(n) = 3T(n/3) + O(\lg n)$ is $T(n) = \Theta(n \lg n)$.

Select one:

- ☐ True
- ☒ False ✓

False.

Case 3 of the master theorem applies:

$$f(n) = O(n^{\log_3(\text{base } 3)}) = O(n) \text{ for } f(n) = O(\lg n), \text{ hence, } T(n) = O(n).$$

The correct answer is 'False'.

Question 6

Correct

Mark 1.00 out of 1.00

For the following recurrence, select the correct expression for runtime $T(n)$ if the recurrence can be solved using Master Theorem, Otherwise, indicate that the Master Theorem does not apply.

$$T(n) = 16T(n/4) + n$$

- ☐ a. $T(n) = \Theta(n^2 \log(n))$
- ☐ b. Master Theorem does not apply.
- ☒ c. $T(n) = \Theta(n^2)$ ✓
- ☐ d. $T(n) = n \log(n)$

The correct answer is: $T(n) = \Theta(n^2)$

Question 7

Correct

Mark 1.00 out of 1.00

Solve the following Recursive Algorithm:

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ 2T(\frac{n}{2}) + F'(n) & \text{if } n > 1 \end{cases}$$

Note: $F'(n)$ function is in the order of $O(n)$

- ☐ a. $T(n) = O(n)$
- ☒ b. $T(n) = O(n \log(n))$ ✓
- ☐ c. $T(n) = O(n^2)$
- ☐ d. $T(n) = O(\log(n))$

The correct answer is: $T(n) = O(n \log(n))$