I. GW150914 SIGNAL ANALYSIS [BY AKHILA RAMAN]

• Let the 4096 second H1/L1 signal be represented by $h(t)$ and its Fourier Transform given by $h(f)$. Its Discrete Time version is given by $h[n]$ whose Discrete Fourier Transform, implemented as Fast Fourier Transform(FFT), is given by $H[k]$ as follows, where $k = 0, 1, ...N_1 - 1$, $N_1 = 4096 * 4096$ and sampling frequency $Fs = 4096$.

$$H[k] = \sum_{n=0}^{N_1-1} h[n]e^{-i\frac{2\pi}{N_1}kn} \tag{1}$$

We can divide $h[n]$ into 2000 **1-second blocks** [Note 1] , centered around the GW150914 signal at $tevent$ = 1126259462.422 and take the FFT of each block as follows, where block $B = 1, ..2000$ and $N = 4096$, $N_0 = N*1000+\frac{N}{2}$ and sampling frequency $Fs = 4096$ Hz, and $k = 0, 1, ...N - 1$

$$H[B, k] = \sum_{n=0}^{N-1} h[n + 2048 * N + (B - 1) * N - N_0]e^{-i\frac{2\pi}{N}kn} \tag{2}$$

The signal power in each **1 second block** [Note 1] is given by

$$P[B] = \frac{1}{N}\sum_{k=0}^{N-1} |H[B, k]|^2 = \sum_{n=0}^{N-1} |h[n + 2048 * N + (B - 1) * N - N_0]|^2 \tag{3}$$

and we can write $P[B] = P_0 + P_1 + P_2$ where P_0 is the power in $60 * n$ Hz tones, P_1 is the power in the frequency range 50-300Hz (where GW150914 signal has most of its frequency components) and P_2 is the power **outside** the frequency range 50-300Hz. For a sampling frequency of $Fs = 4096$ Hz, and $N = 4096$, frequency index k is in steps of 1 Hz. The $60 * n$ Hz power line harmonics are $60, 120, 180$ Hz tones.

$$\begin{aligned} P[B] &= P_0 + P_1 + P_2 \\ P_0 &= \frac{1}{N}[|H[B, 60]|^2 + |H[B, 120]|^2 + |H[B, 180]|^2] \\ P_1 &= \frac{1}{N}\sum_{k=50}^{300} |H[B, k]|^2 \\ P_2 &= \frac{1}{N}[\sum_{k=0}^{49} |H[B, k]|^2 + \sum_{k=301}^{N-1} |H[B, k]|^2] \end{aligned} \tag{4}$$

For the case of L1 signal of duration 4096 seconds, we take the central 2000 blocks of duration 2000 seconds,we note that

• Power in $60 * n$ Hz tones, P_0, varies between $P_{0_{min}} = 0.9e - 40$ and $P_{0_{max}} = 1.56966424875e - 40$. [Click here for plot.]

• Power in the frequency range 50-300Hz, **excluding** $60 * n$ Hz tones, for blocks **outside** the central block $B = 1001$ corresponding to $tevent = 1126259462.422$, has an average value of $P_{avg} = 9.4448302344708711e - 41$.

This P_{avg} corresponds to power in **non**-electromagnetic(EM) components.

• Let us assume that for the central block 1001, we have a coincident EM signal with frequency components in the range 50-300Hz, similar to GW150914 signal in this plot. [Click here for plot.].

EM component power in 50-300Hz region , is given by $P_{EM} = P_1[1001] - P_{avg} = 1.40413267663e - 40$. [Note 2]

We can see that $P_{EM} = 1.40413267663e - 40$ is **comparable** to the maximum power in in $60 * n$ Hz tones, $P_{0_{max}} = 1.56966424875e - 40$, making it difficult to distinguish. **Magnetometers** track the **amplitude** of the magnetic field, given that EM power $P = EH = E\frac{B}{\mu}$. They track the amplitude corresponding to the variation in $60 * n$ Hz tones, from $P_{0_{min}} = 0.9e - 40$ to $P_{0_{max}} = 1.56966424875e - 40$ and hence **may not distinguish** the amplitude corresponding to EM component power in 50-300Hz region , given by $P_{EM} = 1.40413267663e - 40$, occurring in the block 1001.[**Note 3**]

• There is an **One to One correspondence** between

[1] Power in EM components P_0 and P_{EM} observed in the discrete time samples in the HDF5 file containing H1 and L1 samples for 4096 seconds, and

[2] Power in EM components observed in the analog electrical signal present at the input of the Analog to Digital Converter(ADC), and also

[3] Power in EM components at whichever point it is picked up, whether it is picked up by the magnets in the mirror suspension or picked up by the analog electrical board containing the pre-amplifier or the 7 KHz anti-aliasing filter.

[4] **Magnetometers** track the variation in EM power P_0 of $60 * n$ Hz tones. If EM power P_{EM} appears in central block, comparable in power to P_0, it should track this P_{EM} as well and show power comparable to P_0 [**Note 4**] .

II. NOTES

• [**Note 1**] We have computed power in terms of 1 second blocks of samples.

[1] If we choose the block size < 1 second, power variation in P_0 is likely to be **more** than $P_{0_{min}} = 0.9e - 40$ and $P_{0_{max}} = 1.56966424875e - 40$.

[2] If we choose the block size > 1 second, we are likely to miss out the short-term power of duration 0.2 seconds, as in GW150914 signal.

• [**Note 2**] It is assumed that the power of EM component in Block 1001 P_{EM} is independent of P_{avg} which is the average power of **non**-electromagnetic(EM) components in all blocks except central Block 1001. Hence we can write $P_{EM} + P_{avg} = P_1[1001]$.

For example, we can generate 2 independent sets of gaussian noise samples of length 4096, given by $w_1[n]$ and $w_2[n]$, whose individual power is given by $P_{w_1} = \frac{1}{N}\sum_{n=0}^{N-1}|w_1[n]|^2 = \frac{1}{N^2}\sum_{k=0}^{N-1}|W_1[k]|^2$ and $P_{w_2} = \frac{1}{N}\sum_{n=0}^{N-1}|w_2[n]|^2 = \frac{1}{N^2}\sum_{k=0}^{N-1}|W_2[k]|^2$ and if we add the 2 sets to get $w[n] = w_1[n] + w_2[n]$, the combined power is given by $P_w = P_{w_1} + P_{w_2}$.

We can represent the signal corresponding to P_{avg} [**non**-electromagnetic(EM) components] as a noise signal $w_{non-EM}[n]$. If we add $w_{non-EM}[n]$ to EM component signal $h_{EM}[n]$, which has most of frequency components in the region 50-300 Hz, and is independent of $w_{non-EM}[n]$, we get $w_4[n] = w_{non-EM}[n] + h_{EM}[n]$ and the combined power is given by $P_{w_4} = P_{w_{non-EM}} + P_{h_{EM}} = P_{avg} + P_{EM}$.

• [**Note 3**] If there are **extra EM** components, **other than** $60 * n$ Hz tones, in the frequency range 0-2048 Hz, in the sensitivity plot [shown here], present during nominally quiet times, then the power of these extra EM components given by $P_{extraEM}$ is added to the power P_0 in the $60 * n$ Hz tones and is present in the baseline EM power $P_{baselineEM} = P_0 + P_{extraEM}$, whose variation is tracked by magnetometers, during quiet times. In this case $P_{EM} = 1.40413267663e - 40$ will be **less than** the maximum power in in $P_{baselineEM}$, making it **difficult** for magnetometers to distinguish. For example, in the sensitivity plot [shown here], there is a dominant interference $10 - 15$ Hz, which could be an EM signal, coming from in-vacuum electro-mechanics, which is picked up by the magnets near the mirror suspension. There may be other unknown EM components in the sensitivity plot [shown here].

[Note 4] Correspondence between electric field E and magnetic field B.

• We know that $E = B * c$ where c is the speed of light in free space [Eq. 457].

There is a one-to-one correspondence between the readings of electric field meter which measures E and magnetometer which measures B.

• **Example 1:** Let us assume the raw H1/L1 signal has **only** 60 Hz power line interference, $h(t) = h_1(t) = A_1 \cos(\omega_1 t) = A_1 \cos(2\pi f_1 t)$ where $f_1 = 60$ Hz.

At the input of the Analog to Digital converter(ADC) present in the analog board with the 7 Khz anti-aliasing filter, there is a voltage signal corresponding to $h_1(t)$, given that this analog board is operating with $\pm 10V$ voltage supply.

If we assume a transverse plane EM wave propagating in direction z and that it arrives at the analog board, electric field meter and magnetometer at the same time, there is a corresponding Electric Field $E_1(t) = A_1 \cos(\omega_1 t) * K_1$ corresponding to the voltage signal $h_1(t)$, incident at the Electric Field Meter placed near this analog board, given that electric field is expressed in Volts/meter $\frac{V}{m}$, K_1 is a proportional constant for a given distance between source of the EM signal and the analog board.

Electric Field Meter will show an average reading of E_1 and Magnetometer placed near this board will show an average reading of $B_1 = \frac{E_1}{c}$.

Now, if we add a 180 Hz power-line interference to $h(t)$, we have $h_2(t) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)$. Corresponding electric field is given by $E_2(t) = [A_1 \cos(\omega_1 t) * K_1 + A_2 \cos(\omega_2 t) * K_1]$.[see **Note 5**]

Electric Field Meter will show an average reading of E_2 and Magnetometer placed near this board will show an average reading of $B_2 = \frac{E_2}{c}$.

• **Example 2:** In the above Example 1, if we change $h_1(t) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t) + A_3 \cos(\omega_3 t)$ where the signal has $60, 120, 180$ Hz power-line EM components, and if we change $h_2(t) = \int_{\omega_A}^{\omega_B} A(\omega) \cos(\omega t) d\omega$ where $\omega_A = 2\pi * 50, \omega_B = 2\pi * 300$, $A(\omega)$ is the magnitude of **EM-only** components in 50-300 Hz range, the arguments still hold.

Electric Field Meter will show an average reading of E_1, E_2, corresponding to the signals $h_1(t), h_2(t)$, and Magnetometer placed near this board will show an average reading of $B_1 = \frac{E_1}{c}, B_2 = \frac{E_2}{c}$.

[Note 5] $60 * n$ Hz tones and magnetic coupling B.

If we look at the amplitudes of 60*n Hz harmonics in L1 signal,[[Click here for plot]] and magnetic coupling [[Click here for plot]] at 60 Hz and 180 Hz, we see that

• 60 Hz amplitude = **6e-22** ; 180 Hz amplitude = **3e-22**
• Magnetic coupling at 60 Hz= **1e-10** ; coupling at 180 Hz = **2e-12**
Coupling ratio at 60 Hz to 180 hz= 1e-10/2e-12 = **50**

This means, observed 180 Hz amplitude of **3e-22** corresponds to source amplitude of 3e-22*50 = **1.5e-20** [compared to 60 Hz source amplitude] which is **25 times higher** than 60 Hz amplitude of **6e-22** !

180 Hz harmonic source amplitude which is 25 times higher than 60 Hz fundamental's source amplitude **cannot** be the case. This suggests that
• a) the frequency response $h(f)$ is more likely to be flat in the frequency range 60 Hz - 180 Hz, and perhaps beyond 180 Hz as well.
• b) there is some difference between the way 60*n Hz EM signals are picked up Vs magnetic coupling calibration method using oscillating magnetic fields.

Example cases

There is a one-to-one correspondence between the amplitude of power grid harmonics in this plot [[Click here for plot]] , and the voltage level of the signal in time domain and the reading shown by an electric field meter and magnetometer at this location .

• For example, let us take **only 60 Hz** component. Let us represent it by $h_1(t) = A_1\cos(2\pi f_1 t)$; f1=60 Hz , A_1 is the amplitude. There is a corresponding voltage amplitude V_1 in this analog electrical board "AEB", which is proportional to A_1, given that this board with 7 Khz anti-aliasing filter and 16-bit Analog to Digital Converter(ADC) has a 10 V power supply. $V_1(t) = V_1 * \cos(2\pi f_1 * t)$. [V_1 proportional to A_1]

Let us assume this 60 Hz EM signal arrives at the same time at the board AEB, electric field meter(E-meter) and magnetometer. There is a corresponding electric field E_1 at E-meter, which is proportional to V_1 and A_1, given E-field is in Volts/m. Given that E=B*c,[Eq. 457] where "c" is speed of light in free space, the magnetometer also should show a corresponding reading $B_1 = \frac{E_1}{c}$.

Next, if we **add the 180 Hz harmonic**, the signal is given by
$h_2(t) = A_1\cos(2\pi f_1 t) + A_2\cos(2\pi f_2 t)$. $f_2 = 180\text{Hz}$

Power of this signal is proportional to $(A_1^2 + A_2^2)$ and corresponding amplitude is proportional to $\sqrt{(A_1^2 + A_2^2)}$. There is a corresponding voltage amplitude V_2 proportional to $\sqrt{(A_1^2 + A_2^2)}$.

• **Case 1**: if the frequency response is **flat** between 60 Hz and 180 Hz:

E-meter should show a reading E_2 proportional to $\sqrt{(A_1^2 + A_2^2)}$ and magnetometer should show a corresponding reading $B_2 = \frac{E_2}{c}$.

as an example, if $A_1 = A_2$, reading $E_2 = E_1 * \sqrt{(2)}$ and $B_2 = B_1 * \sqrt{(2)}$.

If an external EM signal arrives coincident with $f_3 = 90$ Hz, $A_3 = A_1 = A_2$, $E_3 = E_1 * \sqrt{(3)}$ and $B_3 = B_1 * \sqrt{(3)}$.

If 60 Hz and 180 Hz power tones vary in power [[Click here for plot]] , [HDF5 data files from LOSC website] [[Click here for website]] , B_3 and B_2 will be barely distinguishable.

• **Case 2**: if there is a **huge drop** in EM coupling between 60 Hz and 180 Hz by a factor of 50: [this case is ruled out in previous section. we will consider it anyways].

E-meter should show a reading E_2 proportional to $\sqrt{(A_1^2 + (50 * A_2)^2)}$ and magnetometer should show a corresponding reading $B_2 = \frac{E_2}{c}$.

as an example, if $A_1 = A_2$, reading $E_2 = E_1 * \sqrt{(1^2 + 50^2)} = E_1 * 50.01$ and $B_2 = \frac{E_2}{c}$.

If an external EM signal arrives coincident with $f_3 = 90$ Hz, where magnetic coupling = 2e-11,[[Click here for plot]] 5 times lower than 60 Hz coupling,

$h_3(t) = A_1\cos(2\pi f_1 t) + A_2\cos(2\pi f_2 t) + A_3\cos(2\pi f_3 t)$

E-meter should show a reading E_3 proportional to $\sqrt{(A_1^2 + (50 * A_2)^2 + (5 * A_3)^2)}$ and magnetometer should show a corresponding reading $B_3 = \frac{E_3}{c}$.

as an example, if $A_1 = A_2 = A_3$, reading $E_3 = E_1 * \sqrt{(1^2 + 50^2 + 5^2)} = E_1 * 50.259$ and $B_3 = \frac{E_3}{c}$.

we can see that $E_2 = E_1 * 50.01$ and $E_3 = E_1 * 50.259$ are **barely** distinguishable, if A_3 is comparable to A_1 and A_2. Similarly, B_2 and B_3 are **barely** distinguishable.

If external EM signals arrive coincident, whose power is comparable to these power grid harmonics, magnetometers may have a difficult time distinguishing it.