I. GW150914 SIGNAL ANALYSIS [BY AKHILA RAMAN]

• Let the 4096 second H1/L1 signal be represented by $h(t)$ and its Fourier Transform given by $h(f)$. Its Discrete Time version is given by $h[n]$ whose Discrete Fourier Transform, implemented as Fast Fourier Transform(FFT), is given by $H[k]$ as follows, where $k = 0, 1, ...N_1 - 1$, $N_1 = 4096 * 4096$ and sampling frequency $Fs = 4096$.

$$H[k] = \sum_{n=0}^{N_1-1} h[n]e^{-i\frac{2\pi}{N_1}kn} \tag{1}$$

We can divide $h[n]$ into 2000 **1-second blocks**, centered around the GW150914 signal at $tevent = 1126259462.422$ and take the FFT of each block as follows, where block $B = 1, ..2000$ and $N = 4096$, $N_0 = N * 1000 + \frac{N}{2}$ and sampling frequency $Fs = 4096$ Hz, and $k = 0, 1, ...N - 1$

$$H[B, k] = \sum_{n=0}^{N-1} h[n + 2048 * N + (B - 1) * N - N_0]e^{-i\frac{2\pi}{N}kn} \tag{2}$$

The signal power in each **1 second block** is given by

$$P[B] = \frac{1}{N}\sum_{k=0}^{N-1} |H[B, k]|^2 = \sum_{n=0}^{N-1} |h[n + 2048 * N + (B - 1) * N - N_0]|^2 \tag{3}$$

and we can write $P[B] = P_0 + P_1 + P_2$ where P_0 is the power in $60 * n$ Hz tones, P_1 is the power in the frequency range 50-300Hz (where GW150914 signal has most of its frequency components) and P_2 is the power **outside** the frequency range 50-300Hz. For a sampling frequency of $Fs = 4096$ Hz, and $N = 4096$, frequency index k is in steps of 1 Hz. The $60 * n$ Hz power line harmonics are $60, 120, 180$ Hz tones.

$$\begin{aligned} P[B] &= P_0 + P_1 + P_2 \\ P_0 &= \frac{1}{N}[|H[B, 60]|^2 + |H[B, 120]|^2 + |H[B, 180]|^2] \\ P_1 &= \frac{1}{N}\sum_{k=50}^{300} |H[B, k]|^2 \\ P_2 &= \frac{1}{N}[\sum_{k=0}^{49} |H[B, k]|^2 + \sum_{k=301}^{N-1} |H[B, k]|^2] \end{aligned} \tag{4}$$

For the case of L1 signal of duration 4096 seconds, we take the central 2000 blocks of duration 2000 seconds, we note that

• Power in $60 * n$ Hz tones, P_0, varies between $P_{0_{min}} = 0.9e - 40$ and $P_{0_{max}} = 1.56966424875e - 40$. [Click here for plot.]

• Power in the frequency range 50-300Hz, **excluding** $60 * n$ Hz tones, for blocks **outside** the central block $B = 1001$ corresponding to $tevent = 1126259462.422$, has an average value of $P_{avg} = 9.4448302344708711e - 41$.

This P_{avg} corresponds to power in **non**-electromagnetic(EM) components.

• Let us assume that for the central block 1001, we have a coincident EM signal with frequency components in the range 50-300Hz, similar to GW150914 signal in this plot. [Click here for plot.].

EM component power in 50-300Hz region , is given by $P_{EM} = P_1[1001] - P_{avg} = 1.40413267663e - 40$.

We can see that $P_{EM} = 1.40413267663e - 40$ is **comparable** to the maximum power in in $60 * n$ Hz tones, $P_{0_{max}} = 1.56966424875e - 40$, making it difficult to distinguish. **Magnetometers** track the **amplitude** of the magnetic field, given that EM power $P = EH = E\frac{B}{\mu}$. They track the amplitude corresponding to the variation in $60 * n$ Hz tones, from $P_{0_{min}} = 0.9e - 40$ to $P_{0_{max}} = 1.56966424875e - 40$ and hence **may not distinguish** the amplitude corresponding to EM component power in 50-300Hz region , given by $P_{EM} = 1.40413267663e - 40$, occurring in the block 1001.

• There is an **One to One correspondence** between

[1] Power in EM components P_0 and P_{EM} observed in the discrete time samples in the HDF5 file containing H1 and L1 samples for 4096 seconds, and

[2] Power in EM components observed in the analog electrical signal present at the input of the Analog to Digital Converter(ADC), and also

[3] Power in EM components at whichever point it is picked up, whether it is picked up by the magnets in the mirror suspension or picked up by the analog electrical board containing the pre-amplifier or the 7 KHz anti-aliasing filter.

• We have computed power in terms of 1 second blocks of samples.

[1] If we choose the block size < 1 second, power variation in P_0 is likely to be **more** than $P_{0_{min}} = 0.9e - 40$ and $P_{0_{max}} = 1.56966424875e - 40$.

[2] If we choose the block size > 1 second, we are likley to miss out the short-term power of duration 0.2 seconds, as in GW150914 signal.