On the Electro-Magnetic(EM) correlations in LIGO signals

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Abstract. This article analyzes the data for the three gravitational wave (GW) events detected in both Hanford(H1) and Livingston(L1) detectors by the LIGO[1] collaboration. It is shown that the leading candidate for non-GW source for these assumed GW events, is in fact electro-magnetic(EM) interference from terrestrial and non-terrestrial sources. We know that LIGO detector's GW channel does in fact pick up strong 60*n Hz EM interference from power lines and it will be shown that GW channel has a relatively flat frequency response for 60*n Hz EM interference.

The only way to rule out EM interference is by looking at the magnetometers. Let us consider the case where EM interference is the cause of the assumed GW signal. It is shown that GW170814, GW170817, GW151226 and GW170104 are very weak signals whose amplitude does not rise significantly during the GW event, and they are indistinguishable from non-stationary detector noise. In the case of these weak GW events, where assumed GW event does not rise above the background noise, magnetometers also will not show an EM signal rising well above the background noise.

It will be shown for GW151226 and GW170104, that time domain cross-correlation of magnetometer signal during GW event, with the template, does not show peaks in cross-correlation, if the frequency response/transfer function $H(f)$ of magnetometers differ from that of the GW channel, which is likely to be the case. There is no reason to suppose that magnetometers and GW channel have the exact same transfer function $H(f)$. It will be shown that GW channel has various paths for EM interference pickup, with varying transfer functions. Hence it is suggested that GW151226 and GW170104 be questioned as candidates for GW signals. The implications of these results are discussed for GW170814 and GW170817. All the results in this paper are demonstrated using modified versions of LIGO's Python scripts[16].[2]

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[1]The Laser Interferometer Gravitational-Wave Observatory
[2]The specific Python script used to generate Fig.1 to Fig.4 in this manuscript, is mentioned in [16].

Contents

1   Introduction

In the LIGO system, GW channel has ever-present 60*n Hz electro magnetic(EM) interference(figure) and broadband EM interference. It will be shown that EM interference enters GW channel through several paths, suspension magnets or electrical board or elsewhere. Each such path in the GW channel may have a different transfer function (frequency response).

It is possible that an EM interference signal, such as lightning, may be picked up at both sites H1 and L1 and may be mistaken for a GW signal. The only way we can rule out EM interference in the GW channel, is by cross-correlating the magnetometer signal with the template, during GW event duration. If the transfer function experienced by the EM signal in the magnetometer channel is different from the transfer function in the GW channel, it is possible that the magnetometer channel may not produce Cross Correlation Function(CCF) peaks when correlated with the template and hence this EM signal may be mistaken for GW signals.

The first GW signal observed was GW150914 [1] which was a relatively strong signal whose amplitude, after whitening and filtering[1], rose significantly, well over detector noise level, during the 0.2 second GW event duration. In comparison, the second signal[2] GW151226, the third signal[3] GW170104, the fourth signal[4] GW170814 and the fifth signal[5] GW170817 were very weak signals, which look like noise after whitening and filtering and whose signal amplitude does not rise above the detector noise level during the GW event of duration of less than 1 second (Fig.7, Fig.8, Fig.9, Fig.10, Fig.11)).

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[1]Fourth order Butterworth bandpass filters used in frequency range 20-300Hz for GW150914, 43-800Hz for GW170104 and GW151226.

If we check the magnetometers for any spikes during assumed GW events, we may not see a spike, as in above weak signals, if EM interference is the source of observed GW events. This raises the important question of whether GW151226 and GW170104 could have been caused by EM signals from other sources. Because we are more often likely to observe EM signals which may be buried in background noise and whose amplitude does not rise during assumed GW event, it is of paramount importance that we should not classify non-GW signals such as EM, as GW events. We must insist on high standards before classifying an observed time series as a GW signal.

The organization of this paper is as follows. In Section 2, it is shown that the GW channel has multiple paths for EM pickup, with varying transfer functions and also that 60*n Hz EM interference is picked up with a **relatively** flat frequency response. In Sections 4, it will be shown for GW151226 and GW170104, that even small variations in the transfer functions of GW channel vs magnetometer channel, can destroy cross-correlation peaks in one of those channels, when correlated with the template. Section 5 discusses the implications of this analysis for GW150914. Section 6 gives the reasons why weak signals GW170814 and GW170817, observed in 3 detectors, should be questioned.

2 GW channel coupling for EM signals

In this section, it is assumed that magnetometers have a perfectly flat coupling(frequency response) in the range 0-2048 Hz. It will be shown that the EM interference is picked up by multiple paths in the GW channel with varying transfer functions.

LIGO's EM coupling calibration plot (Figure 2, Page 13 in [6], reproduced here) shows a sharp drop by a factor of 50, from 60 Hz to 180Hz. It is shown here that this sharp drop is **not** applicable for 60*n Hz EM interference and may not be applicable for any other EM interference.

We can show that 60*n Hz EM interference is picked up in 60-180 Hz range, as relatively flat frequency response(coupling) in the GW channel. If we look at the detector noise frequency response here, we can see that 60 Hz, 120 Hz, 180 Hz amplitudes are relatively flat. There is a **contradiction** between, the 50 times drop from 1e-10 at 60 Hz to 2e-12 at 180 Hz, observed in EM coupling calibration plot (Figure 2, Page 13 in [6], reproduced here) and amplitudes of 60*n Hz harmonics observed in the L1 signal here .

If we look at the amplitudes of 60*n Hz harmonics in the L1 signal, 60 Hz amplitude = 6e-22 and 180 Hz amplitude = 3e-22. We can see that the EM coupling(frequency response) is relatively flat from 60 Hz to 180Hz, with a drop only by a factor of 2.

We do not see the 50 times drop in EM coupling from 60 Hz to 180 Hz [drop from 1e-10 at 60 Hz to 2e-12 at 180Hz], as implied in the EM coupling calibration plot (Figure 2, Page 13 in [6], reproduced here). If there were 50 times drop, then 180 Hz harmonic will have to be 25 times higher than the 60 Hz fundamental amplitude, which cannot be the case (It is shown in Section 8.1 that the harmonic amplitudes are always lesser than the amplitude of the fundamental frequency).

It is clear that 60*n Hz EM interference is picked up by a path, which is **different** from the path used in EM coupling calibration and hence LIGO's EM coupling calibration plot (Figure 2, Page 13 in [5], reproduced here) may not be applicable to this EM pickup. If 60*n Hz EM interference is more likely picked up by the magnets in the mirror suspension, or a different path, this means that 60*n Hz EM signals travel by air/vacuum from power lines or power outlets to the magnets, where they are picked up and have a relatively flat coupling from 60 Hz -180 Hz.

This means that, external EM signals, such as lightning and EM interference from astrophysical objects, also can be picked up by the same path which picked up 60*n Hz EM interference (may be magnets in the mirror suspension) and have relatively flat coupling, with small variations.

In this section, we have shown that the GW channel has multiple paths for EM pickup, with varying transfer functions and also that 60*n Hz EM interference is picked up with a **relatively** flat frequency response. In the next section, for the purpose of convenience, we will **interchange** the transfer functions for magnetometers and the GW channel and assume that the GW channel has a perfectly flat frequency response $H_{gw}(f) = 1$ and that the magnetometer has a **relatively** flat frequency response with some **small** variation.

3 Difference in Isolation for EM signals in GW channel and magnetometers
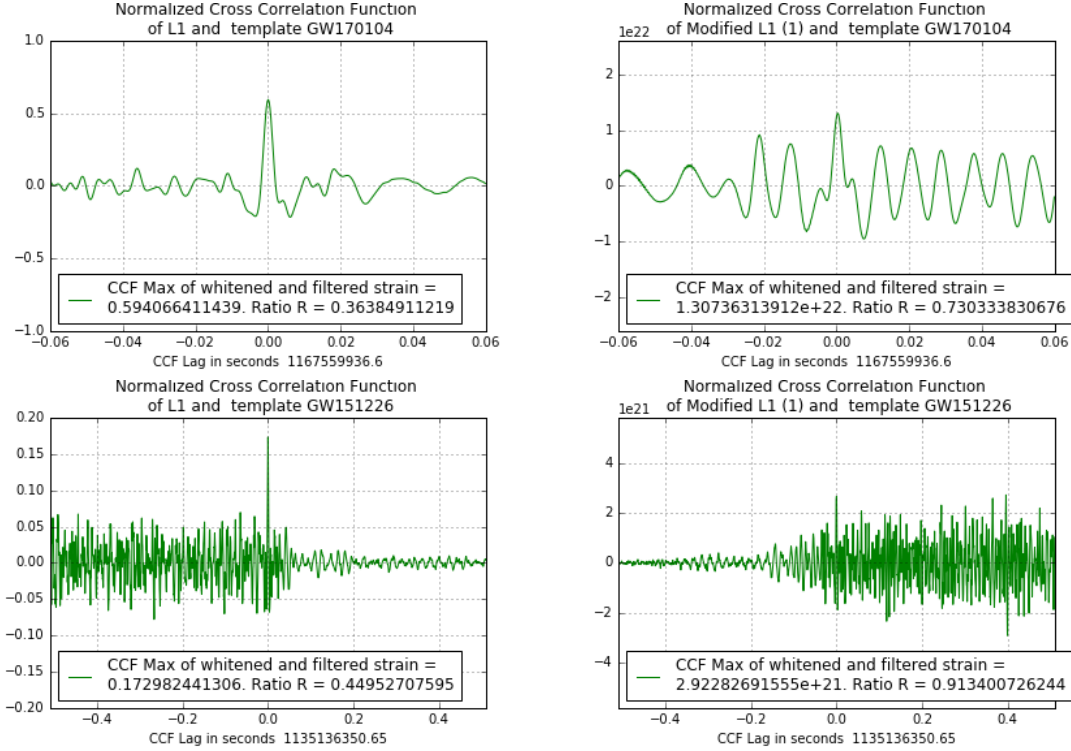

Figure 1. CCF Plots. Top panels: GW170104. Bottom panels: GW151226. Left: GW channel. L1 vs template. Right: Magnetometer channel: (ideal template + noise + larger 60 Hz interference) vs template

Let us assume an EM signal x(t) is incident at the 2 sites H1 and L1, for GW151226 and GW170104. It is picked up as $x(t) = h(t) + w_{em}(t) + w_{non-em}(t)$ in the GW channel. In the magnetometer channel, it will be picked up as $y(t) = h(t) + w_{em}(t) * K_0$ where $h(t)$ is the template, $w_{em}(t)$ is the detector noise due to EM and $w_{non-em}(t)$ is the detector noise due to non-EM sources.

We know that the GW channel has EM isolation of > 40 dB to minimize EM interference (Section 8.2) (Fig.1 and 2), while magnetometers do not, because magnetometers are meant to pick up and measure EM interference. Hence the parameter K_0 in the above expression.

The only way to rule out this EM interference as the source of the observed GW candidate signal, is by doing cross-correlation function(CCF) of the received signal with the template, in both the GW channel (CCF of x(t) vs template) and the magnetometer channel (CCF of y(t) vs template). The two CCFs can look very different in Fig.1, GW channel on left shows a peaky CCF ($R < \frac{1}{e}$), while magnetometer channel on right does not have a peaky CCF ($R > \frac{1}{e}$)

We can see that CCF does not show clear peaks in magnetometer channel, for GW151226 and GW170104. So, we may not see any CCF peaks in magnetometer channel on right panel here and may mistake this EM interference, as a GW signal. Hence it is suggested that **GW151226** and **GW170104** be questioned.

The reason for the absence of CCF peaks in the magnetometer channel is due to the difference in EM isolation factor K_0 which causes differences in **amplitude distortion and phase distortion** of the template, in the 2 channels, due to the whitening procedure used in LIGO python scripts. The whitening procedure is done by taking the Fast Fourier Transform(FFT) of the received signal $x(t)$ and dividing by the magnitude of the fourier transform $X_w(f) = \frac{X(f)}{|X(f)|}$.

This is illustrated in a simplified example, with $w_{em}(t) = A\cos(2\pi f_0 t)$ where $f_0 = 60 + \delta$ where δ is chosen to be in between the FFT frequency bins, so that they will have an effect on the adjacent frequency bins. This is a reasonable example given that 60 Hz EM interference is likely to have a frequency spread around 60 Hz.

$$
\begin{aligned}
x(t) &= h(t) + A\cos(2\pi f_0 t) = h(t) + w_{em}(t) \\
X(f) &= H(f) + W_{em}(f) \\
X_w(f) &= \frac{X(f)}{|X(f)|} = \frac{(H(f) + W_{em}(f))}{|(H(f) + W_{em}(f))|} \\
y(t) &= h(t) + K_0 * w_{em}(t) \\
Y_w(f) &= \frac{(H(f) + K_0 * W_{em}(f))}{|(H(f) + K_0 * W_{em}(f))|}
\end{aligned}
\tag{3.1}
$$

In the continuous time Fourier Transform, we know that $W_{em}(f) = \frac{A}{2}(\delta(f - f_0) + \delta(f + f_0))$. In discrete-time, we have $W_{em}[k] = \sum_{n=0}^{N-1} w_{em}[n] e^{-i2\pi n\frac{k}{N}}$ and given that the chosen frequency f_0 is in between the frequency bins, it will spread into adjacent bins.

We know that the process of whitening the template given by $\frac{H(f)}{|H(f)|}$ causes amplitude distortion in time domain. We can show that the parameter K_0 in the above expressions also causes a difference in phase distortion in the 2 channels.

$$
\begin{aligned}
X(f) = H(f) + W_{em}(f) &= |H(f)|e^{i\theta_h(f)} + |W_{em}(f)|e^{i\theta_w(f)} \\
= [|H(f)|\cos\theta_h(f) + |W_{em}(f)|\cos\theta_w(f)] &+ i[|H(f)|\sin\theta_h(f) + |W_{em}(f)|\sin\theta_w(f)] \\
X_w(f) = \frac{X(f)}{|X(f)|} &= e^{i\tan^{-1}\frac{[|H(f)|\sin\theta_h(f) + |W_{em}(f)|\sin\theta_w(f)]}{[|H(f)|\cos\theta_h(f) + |W_{em}(f)|\cos\theta_w(f)]}} \\
Y_w(f) = \frac{Y(f)}{|Y(f)|} &= e^{i\tan^{-1}\frac{[|H(f)|\sin\theta_h(f) + K_0[W_{em}(f)|\sin\theta_w(f)]}{[|H(f)|\cos\theta_h(f) + K_0[W_{em}(f)|\cos\theta_w(f)]}}
\end{aligned}
\tag{3.2}
$$

We can see that the parameter K_0 in the above expressions also causes a difference in phase distortion in the 2 channels.

4 Different Transfer Functions for EM signals in GW channel and magnetometers

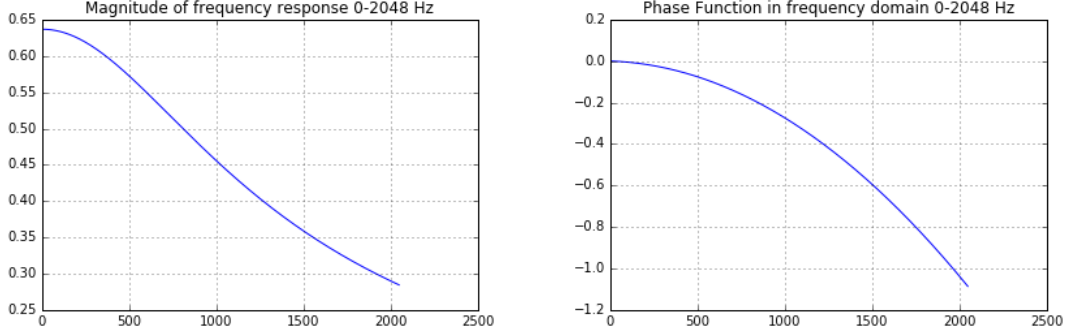### 4.1 Nonlinear Phase Transfer Functions for EM signals in magnetometers

Figure 2. GW151226: Example Transfer function plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged). Transfer Function 1: Nonlinear Phase + single pole magnitude. One sided, causal, practically realizable.
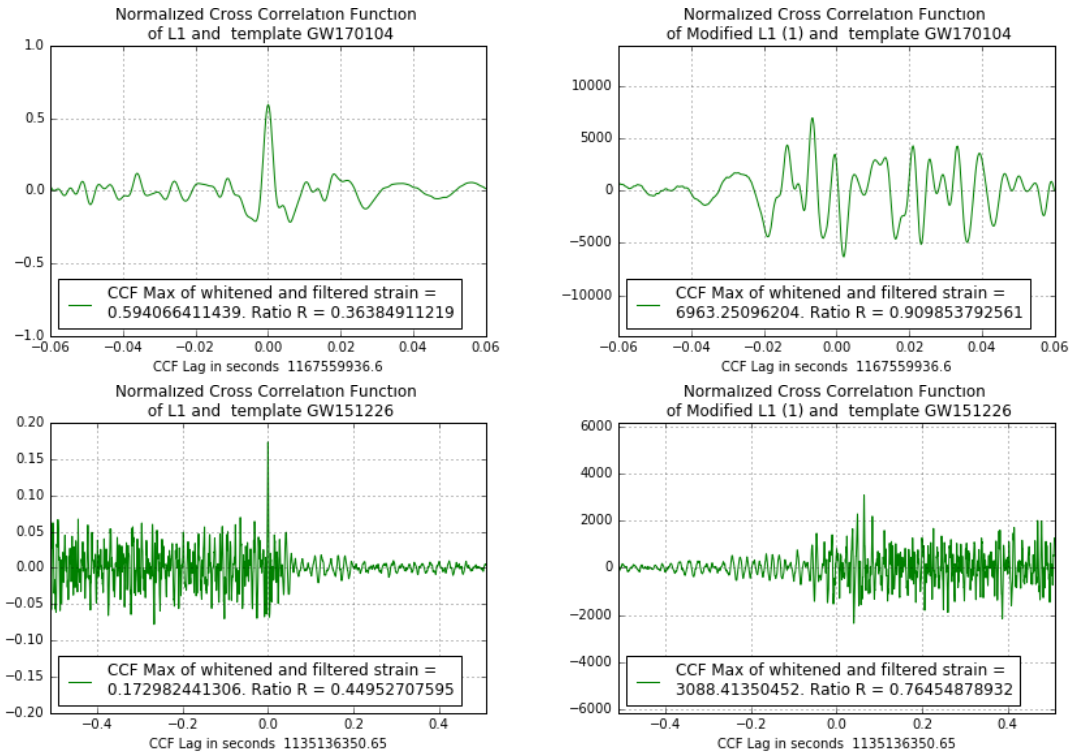

Figure 3. Top panels: GW170104. Bottom panels: GW151226. Left: GW channel. L1 vs template. Right: Magnetometer channel: L1 passed through Transfer Function 1 vs template.

Let us assume an EM signal x(t) is incident at the 2 sites H1 and L1, for GW151226 and GW170104. It is picked up as x(t) in the GW channel. In the magnetometer channel, it will be picked up as $y(t) = x(t) * w(t)$ where * denotes convolution in time domain. In the frequency domain,

$Y(f) = X(f)W(f)$, where $W(f)$ is given by the transfer function in Fig. 2 with nonlinear phase and single pole magnitude transfer function.

The only way to rule out this EM interference as the source of the observed GW candidate signal, is by doing cross-correlation function(CCF) of the received signal with the template, in both the GW channel (CCF of x(t) vs template) and the magnetometer channel (CCF of y(t) vs template).

The two CCFs can look very different in Fig.3, GW channel on left shows a peaky CCF ($R < \frac{1}{e}$), while magnetometer channel on right does not have a peaky CCF ($R > \frac{1}{e}$). [2]

We can see that CCF does not show clear peaks in magnetometer channel, for GW151226 and GW170104. So, we may not see any CCF peaks in magnetometer channel on right panel here and may mistake this EM interference, as a GW signal. Hence it is suggested that **GW151226** and **GW170104** be questioned.

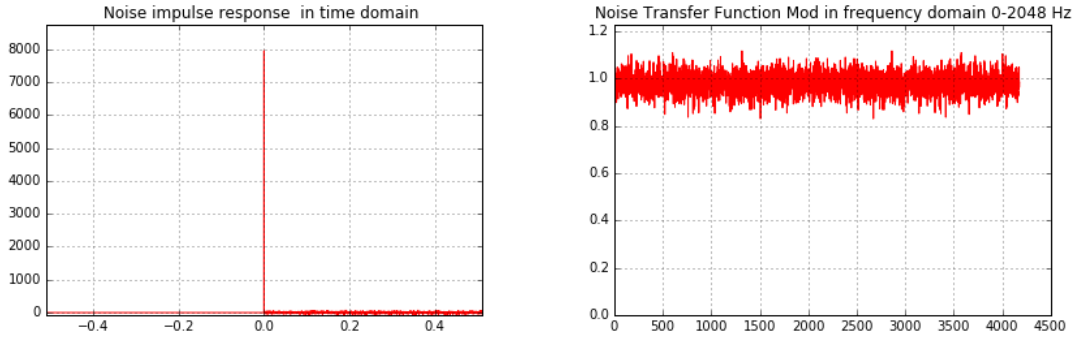## 4.2 Varying Magnitude Transfer Functions for EM signals in magnetometers

Figure 4. GW151226: Example Transfer function plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged.). Transfer Function 2, impulse response $h_1(t)$. One sided, causal, practically realizable.

Let us assume an EM signal x(t) is incident at the 2 sites H1 and L1, for GW151226 and GW170104. It is picked up as x(t) in the GW channel. In the magnetometer channel, it will be picked up as $y(t) = x(t) * (\delta(t) + w(t))$ where * denotes convolution in time domain. In the frequency domain, $Y(f) = X(f)(1 + W(f))$, where $w(t)$ is AWGN with a small standard deviation and $\delta(t)$ is dirac delta function.

The only way to rule out this EM interference as the source of the observed GW candidate signal, is by doing cross-correlation function(CCF) of the received signal with the template, in both the GW channel (CCF of x(t) vs template) and the magnetometer channel (CCF of y(t) vs template).

Causal, one-sided and practically realizable, time domain impulse response $h_1(t)$ and frequency response $H_1(f) = 1 + W(f)$, where $W(f)$ is additive white gaussian noise(AWGN) with a small standard deviation, shown in the upper panels of Fig. 4. The two CCFs can look very different in Fig.5, GW channel on left shows a peaky CCF ($R < \frac{1}{e}$), while magnetometer channel on right does not have a peaky CCF ($R > \frac{1}{e}$).

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[2]By peaky CCF, we mean that the ratio, R, of the the absolute value of CCF for any lag greater than the decorrelation time of the template, to the absolute maximum value of CCF, should be less than a certain threshold. Decorrelation time of the template τ_0 is defined as the time taken for the autocorrelation of the template to fall to $\frac{1}{e} = 0.36$ of the maximum value at zero lag [12]. We will use the ratio R_3, which is the ratio of the absolute value of CCF at any lag greater than $\tau_0 * 3$ to the absolute maximum value of CCF, and test whether $R_3 < \frac{1}{e}$. Lag greater than $\tau_0 * 3$ is taken to allow for some cushion.
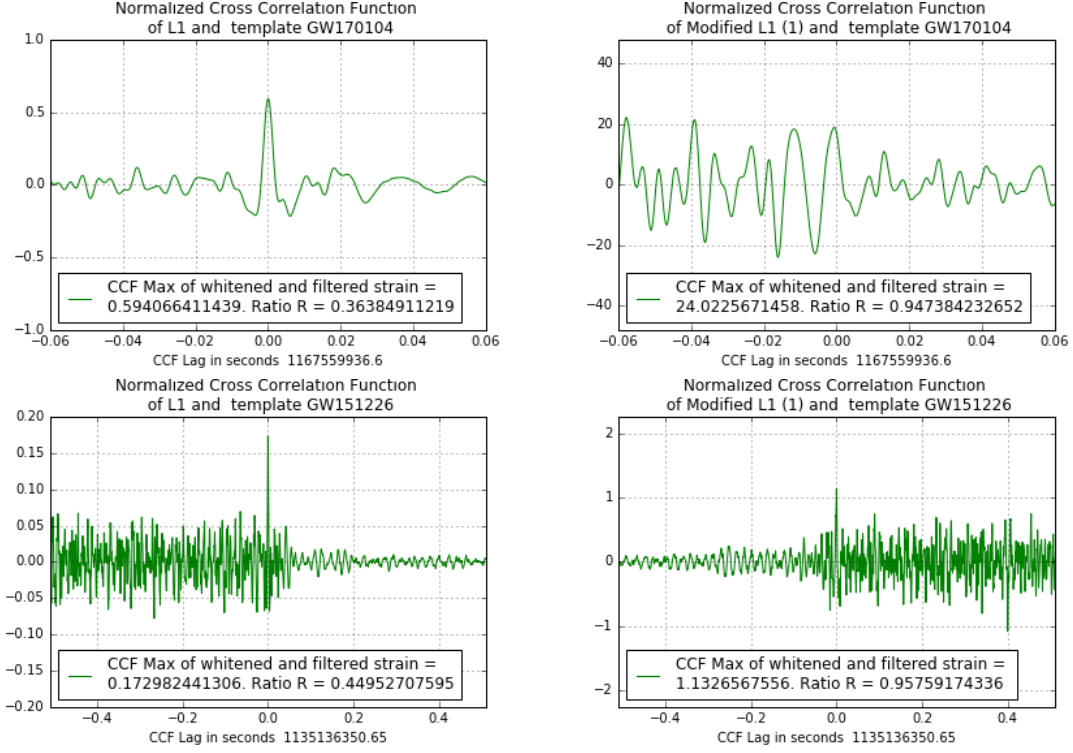
Figure 5. Top panels: GW170104. Bottom panels: GW151226. Left: GW channel. L1 vs template. Right: Magnetometer channel: L1 passed through Noise Transfer Function 2 (impulse response $h_1(t)$) vs template.

We can see that CCF does not show clear peaks in magnetometer channel, for GW151226 and GW170104. So, we may not see any CCF peaks in magnetometer channel on right panel here and may mistake this EM interference, as a GW signal. Hence it is suggested that **GW151226** and **GW170104** be questioned.
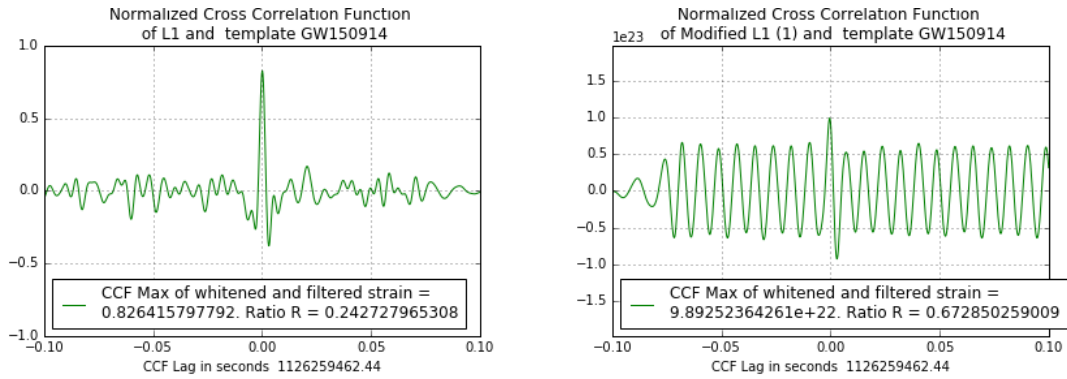
5   Implications for GW150914

Figure 6. CCF Plots. GW150914. Left: GW channel. L1 vs template. Right: Magnetometer channel: (ideal template + noise + larger 60 Hz interference) vs template.

Let us assume an EM signal x(t) is incident at the 2 sites H1 and L1, for GW150914. It is picked up as $x(t) = h(t) + w_{em}(t) + w_{non-em}(t)$ in the GW channel. In the magnetometer channel, it will be picked up as $y(t) = h(t) + w_{em}(t) * K_0$ where $h(t)$ is the template, $w_{em}(t)$ is the detector noise due to EM and $w_{non-em}(t)$ is the detector noise due to non-EM sources.

We know that the GW channel has EM isolation of > 40 dB to minimize EM interference (Section 8.2) (Fig.1 and 2), while magnetometers do not, because magnetometers are meant to pick up and measure EM interference. Hence the parameter K_0 in the above expression.

The two CCFs can look very different in Fig.6, GW channel on left shows a peaky CCF ($R < \frac{1}{e}$), while magnetometer channel on right does not have a peaky CCF ($R > \frac{1}{e}$).

We can see that CCF does not show clear peaks in the magnetometer channel, for GW150914. So, we may not see any CCF peaks in the magnetometer channel on the right panel here and may mistake this EM interference, as a GW signal.

We know that a strong lightning (EM signal) occurred around the same time as GW150914 observation, in Burkina Faso. We also know that excess power was observed in many magnetometer channels. It is stated that "Fluxgate magnetometers indicate that magnetic disturbances at the LIGO detectors produced by coincident lightning strikes were at least 3 orders of magnitude too small to account for the amplitude of GW150914" [Section 6.1 - 6.3 in Page 24-26 in the LIGO noise characterization paper here].

It has been shown in Section 2, that the GW channel has multiple paths for EM pickup and also that 60*n Hz EM interference is picked up with a **relatively** flat frequency response and that LIGO's EM coupling calibration plot (Figure 2, Page 13 in [6], reproduced here) may not be applicable to 60*n Hz EM interference and likely for other external EM signals. Hence the magnetometer excess power "**3 orders of magnitude too small**" argument to rule out Burkina Faso lightning, as a possible candidate for GW150914, is **not** supported.

But GW150914 is a strong signal and if EM interference is the source of the observed GW signal, it is possible that it is seen rising visually in the magnetometer channel, well above the background noise. Unless LIGO team releases magnetometer and Mains Voltage Monitor data (few dozen channels), we cannot be sure that GW150914 was **not** caused by EM interference.

6 Implications for GW170814 and GW170817

GW170814 and GW170817 were observed at three detectors H1, L1 and V1. Their templates have not been published as of date. The analysis in above sections are applicable for GW170814 and GW170817 as well and EM interference could explain these GW signals, irrespective of the exact templates used.

The Gamma Ray Burst(GRB) associated with GW170817 was observed with telescopes. It is well known that astrophysical objects such as stars, emit EM signals in a wide frequency range, all the way down to KHz. Our Sun is known to emit EM signals down to 30 KHz (NASA document). Given that GW170817 was observed as a weak signal, whose amplitude does not rise during GW event, if the astrophysical object which emitted the GRB, also emitted EM signal in the frequency range of 0-2048 Hz, magnetometers will not show any signal spike during the GW event. Hence low frequency EM signals from the astrophysical object which emitted the GRB, is a good candidate for GW170817.

Hence it is suggested that **GW170814** and **GW170817** be questioned.

7 Concluding remarks

In Section 2, it is shown that the GW channel has multiple paths for EM pickup, with varying transfer functions and also that 60*n Hz EM interference is picked up with a **relatively** flat frequency response. Section 4 gives the reasons why weak signals GW151226 and GW170104 should be questioned. Section 5 discusses the implications of this analysis for GW150914. Section 6 gives the reasons why weak signals GW170814 and GW170817 should be questioned.

Reiterating the point made earlier, because LIGO detectors more often likely to pickup EM signals which look like noise and whose amplitude does not rise during assumed GW event, it is of paramount importance that we should not classify EM interference as GW events. We need **high** standards to classify an observed time series as a GW signal.

The only way to avoid wrong classification of EM interference as GW signals, is as follows. LIGO detectors have very strong impulsive interference in 60Hz and harmonics and also in 300Hz-2000Hz range. First, it is very important to clean up the impulsive interference and rest of the EM interference, in the detectors. Secondly, we should insist on signals which rise well above the background noise, in time domain, in the GW channel. When we do observe such a strong signal in the GW channel, we can then look at the magnetometer channel in time domain, for any similar strong signal in time domain, which rises well above the background noise and if it does, we must reject it as a candidate for GW signal, given that CCF may not show clear peaks due to possible difference in transfer function between the GW channel and the magnetometer channel.

Acknowledgments

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[12] Kenneth Y. Jo, Satellite Communications Network Design and Analysis, Page 415.

[13] LOSC LIGO GW150914 Tutorial

[14] LOSC LIGO GW151226 Tutorial matched filter equations are implemented in the section "matched filtering to find the signal"

[15] LIGO Tutorials

[16] Zip Files of Modified LIGO Tutorials which demonstrate the results in this paper.

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Fig.1,2,3,4: LOSC Event tutorial Normalized CCF xfer firls mc.py

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Matched Filter SNR results are demonstrated with 32 second block of data, used in the original LIGO tutorial scripts. Results hold for 512 second block of data also.

8 Appendix A

8.1 60 Hz Harmonic amplitudes

Let us consider the case of 60 Hz EM interference from power lines and represent it as $x_1(t) = A\cos(2\pi f_0 t)$, where $f_0 = 60$ Hz is the fundamental frequency and A is the amplitude. Second harmonic 120 Hz $x_2(t)$ and third harmonic 180 Hz $x_3(t)$ are generated by nonlinearities in the hardware, which can be obtained by taking the square and cube of the fundamental frequency, as follows.

$$x_1(t) = A\cos(2\pi f_0 t)$$
$$x_2(t) = x_1^2(t) = A^2\cos^2(2\pi f_0 t) = \frac{A^2}{2}[1 + \cos(2\pi(2*f_0)t)]$$
$$x_3(t) = x_1^3(t) = A^3\cos^3(2\pi f_0 t) = \frac{A^3}{2}\cos(2\pi f_0 t)[1 + \cos(2\pi(2*f_0)t)] \quad (8.1)$$
$$x_3(t) = \frac{A^3}{2}[\cos(2\pi f_0 t) + \frac{1}{2}(\cos(2\pi(3*f_0)t) + \cos(2\pi f_0 t))]$$

We can see that the harmonic amplitudes are always lesser than the amplitude of the fundamental frequency.

8.2 EM isolation in GW channel

It is shown that LIGO detector has 44 dB more attenuation for 60 Hz EM interference in the GW channel, compared to magnetometer.

The frequency domain plots are in Fig 1 and 2 in Page 10,13 in Noise Characterization paper (Fig.1 and 2)

1. Top panel in Fig.2 shows magnetometer picking up 60 Hz with amplitude 1e-8 Tesla, compared to background noise of 2e-12.

So, $Ratio_1 = 1e-8/2e-12 = 5000$. (Blue line is magnetometer plots during normal operation, no test tones, no GW)

2. GW channel detector noise frequency response plot in Fig.1 shows 60 Hz picked up, with amplitude 6e-22, compared to background noise of 2e-23. So, $Ratio_2 = 6e-22/2e-23 = 30$.

1 Tesla = 1 Volt $\frac{sec}{meter^2}$ (wikipedia).

$20\log 10(\frac{Ratio_1}{Ratio_2}) = 20*log10(\frac{5000}{30}) = 44.43$ dB.
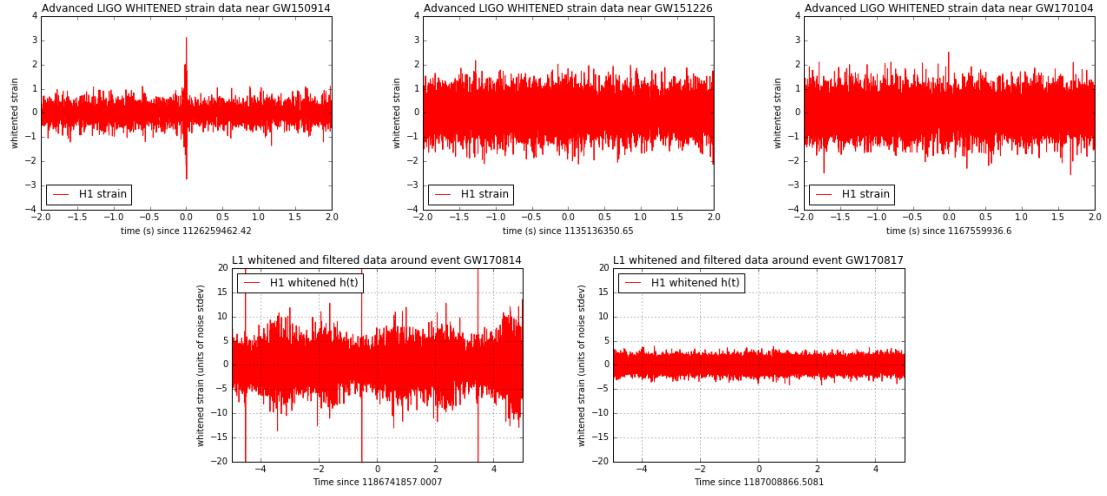
8.3 GW waveforms

Figure 7. Plots of H1 whitened and filtered strain in GW150914, GW151226, GW170104, GW170814 and GW170817.
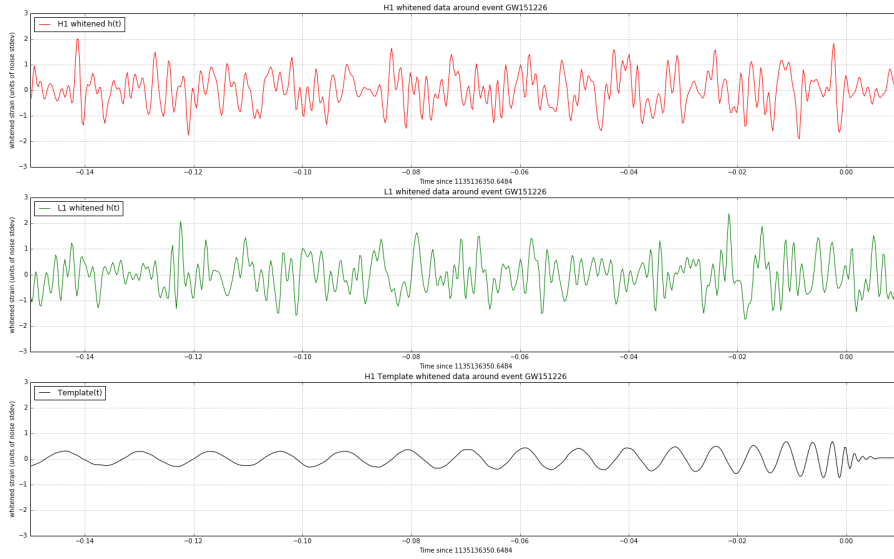

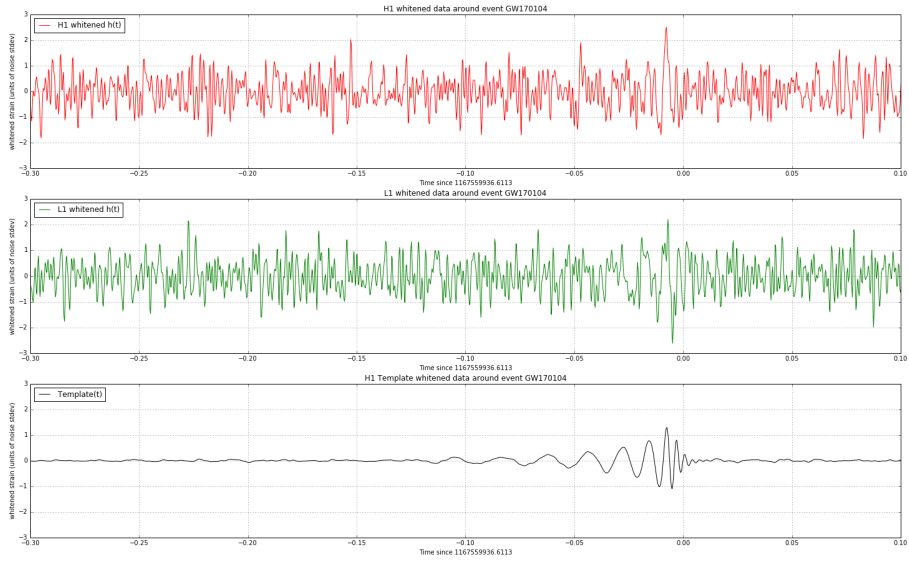**Figure 8**. GW151226 whitened and filtered H1 and L1 strain, and the template

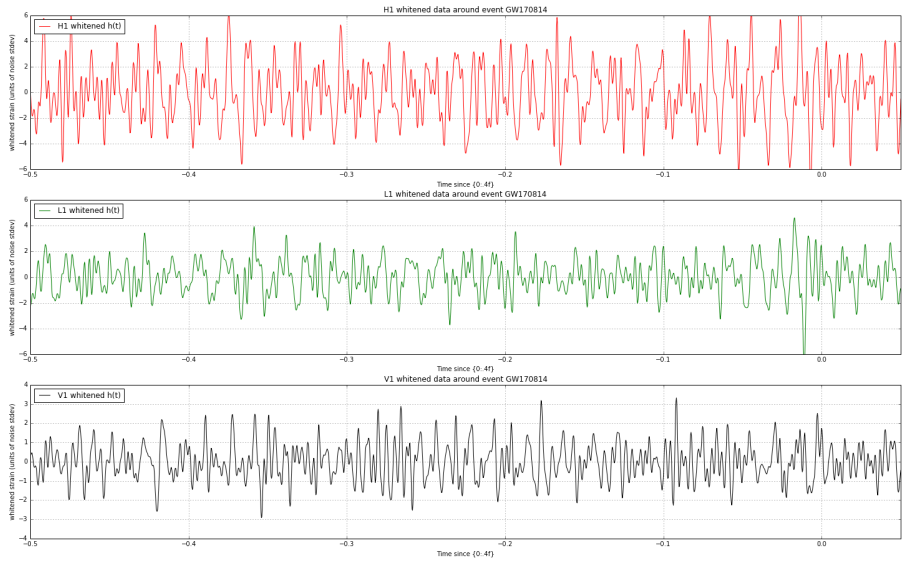**Figure 9**. GW170104 whitened and filtered H1 and L1 strain, and the template

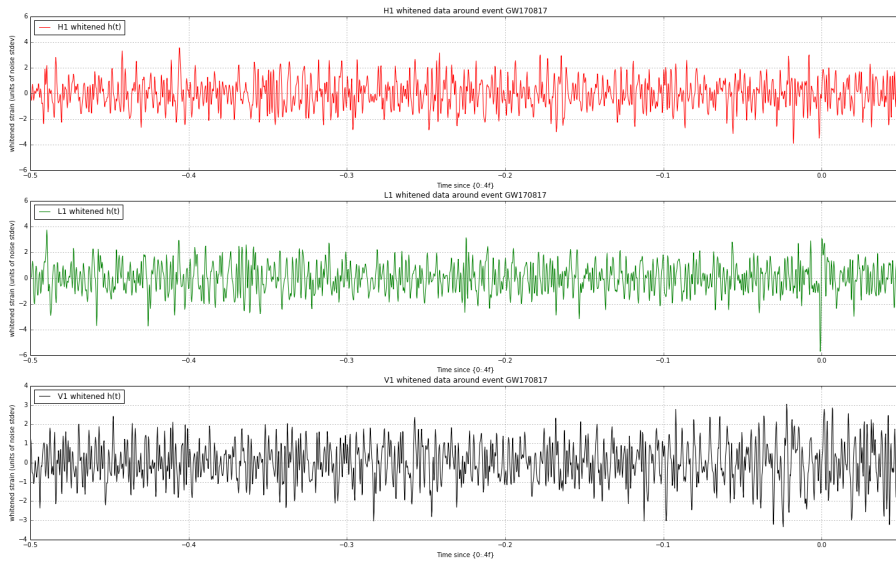

Figure 10. GW170814 whitened and filtered H1, L1 , V1 strain

Figure 11. GW170817 whitened and filtered H1, L1 , V1 strain