

On signal detection and interference rejection in LIGO signals

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Abstract. This article analyzes from the viewpoint of signal detection and interference rejection, the data for the first three gravitational wave (GW) events detected in both Hanford (H1) and Livingston (L1) detectors by the LIGO collaboration. It is shown that we cannot rule out the possibility of electromagnetic (EM) interference from terrestrial and non-terrestrial sources, as a candidate for these assumed GW events, given that LIGO detector's GW channel does in fact pick up strong 60* π Hz EM interference from power lines.

The only way to rule out EM interference is by looking at the magnetometers. Let us consider the case where EM interference is the cause of the assumed GW signal. It will be shown that the time domain cross-correlation of magnetometer signal during GW event with the template, using a matched filter, does not show peaks in cross-correlation, while the GW channel shows cross-correlation peaks. Hence EM interference can be mistaken for GW signals. The reason for the absence of cross-correlation peaks in the magnetometer channel is due to the difference in the nature of the background noise in the GW channel and the magnetometer channel. In the GW channel, detector noise is non-stationary, non-gaussian and non-white noise and in the magnetometer channel, the background noise is approximately white noise.

Hence it is suggested that GW150914, GW151226 and GW170104 be further studied independently and all the relevant data for magnetometers and relevant EM channels be published. The implications of these results are discussed for GW170814 and GW170817. All the results in this paper are demonstrated using modified versions of LIGO's Python scripts[16].

Contents

1	Introduction	1
1.1	GW signals buried in the detector noise	2
2	GW channel frequency response for EM interference signals	3
3	Signal Detection with Non-white Detector Noise in GW channel versus White Noise in magnetometers	3
4	Signal Detection using matched filter with different frequency response in GW channel and magnetometers	5
4.1	Nonlinear Phase frequency response	5
4.2	Varying Magnitude frequency response	6
5	Implications for GW150914	8
6	Implications for GW170814 and GW170817	8
7	Concluding remarks	9
8	Appendix A	10
8.1	60 Hz Harmonic amplitudes	10

1 Introduction

In the [LIGO](#)¹ system, [gravitational wave](#) (GW) channel has ever-present 60*n Hz electromagnetic (EM) interference ([figure](#)) and broadband EM interference. It will be shown that EM interference enters GW channel through several paths and that each of these paths in the GW channel can have a unique frequency response (transfer function).

It is possible that an EM interference signal, such as lightning, may be picked up at both sites H1 and L1 and may be mistaken for a GW signal, if it resembles 1 out of 250,000 GW templates. Lightning has [low frequency](#) components, in the 0-2048 Hz region of interest. EM signals also travel at the speed of light, like the GW signal, and can arrive within the 10 msec window at the 2 detectors.

It is shown in the following subsection that, even the strongest GW event GW150914, is buried in the detector noise and is smaller than detector noise by a factor of 400 in the GW channel. Similarly, if EMI is the source of the assumed GW events, they are also buried in the noise in the magnetometer channel. Hence the magnetometers may not show any spikes (excess power) during assumed GW events.

The only way to rule out EM interference in the GW channel is to cross-correlate the magnetometer signal with the template during GW event duration. If the transfer function experienced by the EM signal in the magnetometer channel is different from the transfer function in the GW channel, it is possible that the magnetometer channel may not produce cross correlation function (CCF) peaks when correlated with the template and hence this EM signal can be mistaken for GW signals. There is no reason to suppose that magnetometers and GW channel have the exact same transfer function

¹The Laser Interferometer Gravitational-Wave Observatory

$H(f)$. It will be shown that GW channel has various paths for EM interference pickup, with varying transfer functions.

Lightning can enter the GW channel through electrical power points and [wires](#), in which case it is necessary to look at the mains voltage monitors to rule out EM interference. The analysis in this paper can be applied to mains voltage monitors as well.

Lightning occurs on average 44 times a second over the entire Earth. There may be many other EMI sources like 30-300 Hz [range](#) and 300-3000 Hz frequency [range](#) used in military applications. The only way we detect EMI is by magnetometers, but if these EMI sources are weak, they may be buried in magnetometer noise, similar to these GW events.

This manuscript **does not** claim that EMI is the source of the assumed GW events. It points out that EMI **cannot be ruled out** as a candidate for the GW events observed so far.

1.1 GW signals buried in the detector noise

The first GW signal observed in the LIGO detector was GW150914 [1] and it was the strongest of the 6 GW signals observed so far. As pointed out in the introduction section [here](#), we see from Fig.1 that the maximum amplitude of the recovered template (1.2×10^{-21}) shown in the right panel, after LIGO whitening procedure and filtered in 43-300 Hz range, is 400 times smaller than the amplitude of the raw strain in the left panel (0.5×10^{-18}). Nearly 85 percent of the template energy is in the 43-300 Hz range. We also see that the recovered template for GW150914 in the right panel is hidden beneath the maximum amplitude of the filtered strain (3×10^{-21}) in the middle panel.

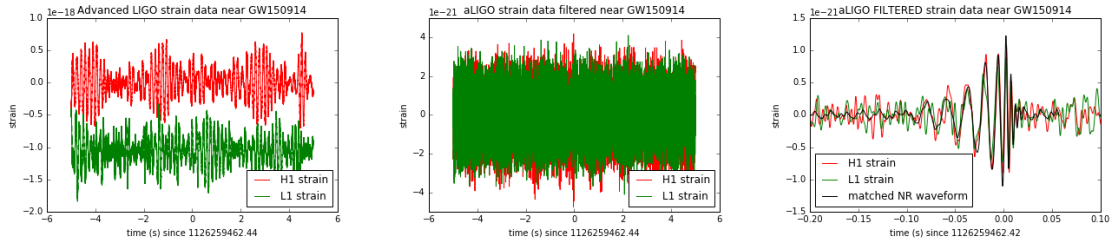


Figure 1. GW150914 H1(Red) and L1(Green) strains: Left Panel: Raw strain. Middle Panel: Raw strain filtered in 43-300 Hz range. Right Panel: LIGO whitened and filtered strains

It has been pointed out in the introduction section [here](#) that 5 out of 6 GW signals observed so far were very weak signals, and they look like noise after whitening and filtering and their signal amplitude does not rise above the detector noise level during the GW event of duration.

If we check the magnetometers for any spikes (excess power) during assumed GW events, to rule out EM interference in the GW channel, we may not see a spike, as in above weak signals, if EM interference is the source of observed GW events. This raises the important question of whether GW151226 and GW170104 could have been caused by EM signals from other sources. Because it is more likely to encounter EM signals which can be buried in background noise and whose amplitude does not rise during assumed GW event, it is of paramount importance that we should not classify non-GW (e.g. electromagnetic) signals, as GW events. We must insist on high standards before classifying an observed time series as a GW signal.

The organization of this paper is as follows. In Section 2, it is shown that the GW channel has multiple paths for EM pickup, with varying transfer functions and also that 60* π Hz EM interference is picked up with a relatively flat frequency response. In Sections 3, it will be shown for GW150914, GW151226 and GW170104, that even small variations in the transfer functions of GW channel and magnetometer channels can destroy cross-correlation peaks in one of those channels, when correlated

with the template. Section 4 discusses the implications of this analysis for GW150914. Section 6 gives the reasons why weak signals GW170814 and GW170817, observed in the three detectors, should be further studied.

2 GW channel frequency response for EM interference signals

We know that a strong lightning (EM signal) occurred around the same time as GW150914 observation, in Burkina Faso. We also know that excess power was observed in many magnetometer channels. It is stated that "Fluxgate magnetometers indicate that magnetic disturbances at the LIGO detectors produced by coincident lightning strikes were **at least 3 orders of magnitude too small** to account for the amplitude of GW150914" (Section 6.1 - 6.3 in Page 24-26 in the LIGO noise characterization [paper](#)).

This "**at least 3 orders of magnitude too small**" defense against Burkino Faso lightning is related to the bottom panel in LIGO's EM coupling calibration [plot](#) (Figure 2, Page 13 in [6]), shows a sharp drop by a factor of 50, from 60 Hz to 180 Hz ($50 \times 50 = 2500$, which is 3 orders of magnitude). It is shown here that this sharp drop in coupling is **not** applicable for 60*n Hz EM interference and may not be applicable for any other EM interference like lightning. In this section, it will be shown that the EM interference is picked up by a different path with relatively flat frequency response.

If we look at the detector noise frequency [response](#) and examine the amplitudes of 60*n Hz harmonics in the L1 signal, we can see that 60 Hz amplitude = $6e-22$ and 180 Hz amplitude = $3e-22$. We can see that the EM coupling's frequency response is **relatively flat** from 60 Hz to 180 Hz, with a drop only by a factor of 2. We know that the harmonic amplitudes are always lesser than the amplitude of the fundamental frequency (Section 8.1) at the power line source. 180 Hz amplitude can be at most be equal to 60 Hz amplitude, at the source. If this 60*n Hz interference were picked up by the path described by LIGO's EM coupling calibration plot, 180 Hz harmonic would be 50 times less than 60 Hz amplitude. But we see only a drop by a factor of 2 in 180 Hz amplitude.

It is clear that 60*n Hz EM interference is picked up by a path which is **different** from the path used in EM coupling calibration [plot](#) and hence LIGO's EM coupling calibration plot may not be applicable to this 60*n Hz EM pickup.

This means that, external EM signals, such as lightning and also EM interference from astrophysical objects (see Section 6), also can be picked up by the same path which picked up 60*n Hz EM interference and have relatively flat frequency response, with small variations.

The simulations in subsequent sections assume that external EMI like lightning is picked up in the GW channel by the same path of pickup as 60*n Hz EMI, with flat frequency response in 50-200 Hz range. This path is different from the path of EMI pickup for the EM coupling calibration experiment and hence "**at least 3 orders of magnitude too small**" defense is not valid for these cases.

3 Signal Detection with Non-white Detector Noise in GW channel versus White Noise in magnetometers

Let us assume an EM signal $x(t)$ is incident at the 2 sites H1 and L1, for GW150914. It is picked up as $x(t) = h(t) + w_d(t)$ in the GW channel. In the magnetometer channel, it will be picked up as $y(t) = h(t) + w_m(t)$ where $h(t)$ is the template ², $w_d(t)$ is the detector noise which is non-stationary, non-gaussian and [non-white](#) noise and $w_m(t)$ is the magnetometer noise due to thermal

²The template $h(t)$ is scaled so that $x(t) = h(t) + w_d(t)$ produces the same Signal to Noise ratio in LIGO's matched filter, as the event GW150914.

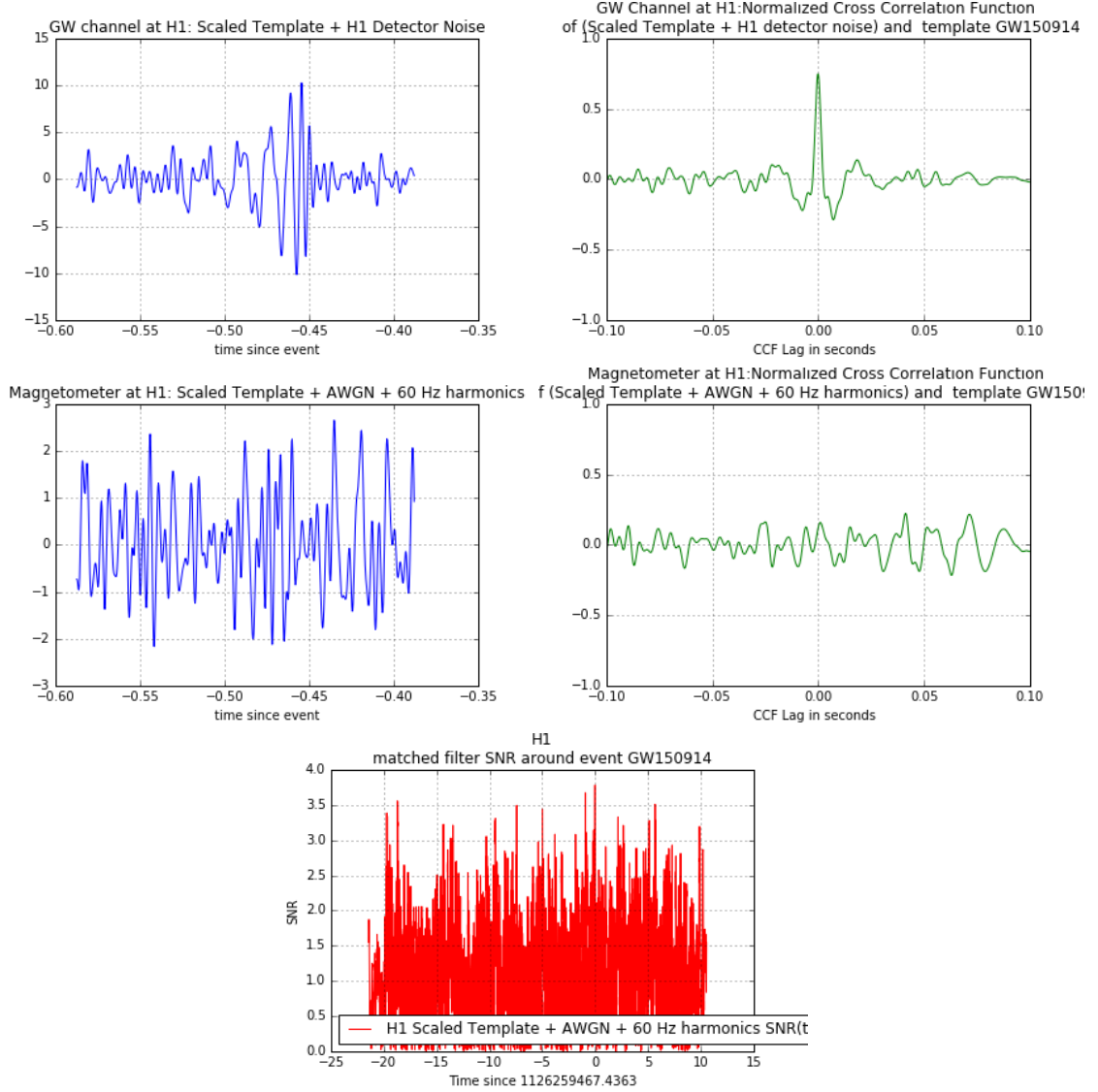


Figure 2. GW150914: Top panels: GW channel, (template+detector noise) whitened and filtered. Top Left: time domain. Top right: Normalized CCF with template. Middle panels: Magnetometer channel, (template+ white noise) whitened and filtered. Middle Left: time domain. Middle right: Normalized CCF with template. Lower panel: LIGO matched filter SNR for Magnetometer channel, (template+ white noise) whitened and filtered.

and broadband EM sources, which is approximately white noise, combined with 60*n Hz impulses (top plot in blue has white noise and 60*n Hz impulses). The white noise in the magnetometer channel has the same noise power equivalent to the detector noise power in the GW channel, in the 43-300 Hz frequency range, where most of the template energy is concentrated.

We can see that the white noise power in the 43-300 Hz frequency range in the magnetometer channel is expected to be greater than or equal to the detector noise power in the 43-300 Hz frequency range in the GW channel for the following reason. Magnetometer channel has thermal noise and picks up 60*n Hz EM harmonics and broadband EM noise by definition, without any attenuation or EM isolation. On the other hand, GW channel has thermal noise and 60*n Hz EM harmonics in the

43-300 Hz frequency range, but it has EM isolation [mechanism](#) and noise reduction hardware.

This experiment is done with the following change. The standard deviation of white noise in the magnetometer channel is scaled down by a factor of 200, compared to the standard deviation of detector noise in the GW channel. This is equivalent to scaling the template amplitude in the magnetometer channel by a factor of 200. This comes very close to addressing ”**at least 3 orders of magnitude too small**” argument, in case lightning was picked up by the path of EMI pickup in EM coupling calibration experiment detailed in the previous section.

The only way to rule out this EM interference as the source of the observed GW candidate signal is by computing cross-correlation of the received signal with the template in both the GW channel (CCF of $x(t)$ versus template) and the magnetometer channel (CCF of $y(t)$ versus template). The two CCFs can look very different in Fig.2, GW channel on the top right panel shows a peaky CCF³, while magnetometer channel on the right panel in the second row does not have a peaky CCF.

We can see that CCF does not show clear peaks in the magnetometer channel, for GW150914. Similar results are observed for GW151226 in Fig.5 and GW170104 in Fig.6. So, we may not see any CCF peaks in the magnetometer channel and may mistake this EM interference, as a GW signal. Hence it is suggested that **GW150914**, **GW151226** and **GW170104** be further studied independently, given that EMI **cannot be ruled out** as a candidate for the GW events.

The reason for the absence of CCF peaks in the magnetometer channel is due to the difference in the nature of the background noise in the GW channel and the magnetometer channel. In the GW channel, detector noise is non-stationary, non-gaussian and non-white noise and in the magnetometer channel, the background noise is approximately white noise.

4 Signal Detection using matched filter with different frequency response in GW channel and magnetometers

4.1 Nonlinear Phase frequency response

In this section, for the purpose of convenience, we will **interchange** the transfer functions for magnetometers and the GW channel and assume that the GW channel has a perfectly flat frequency response $H_{gw}(f) = 1$ and that the magnetometer has a **relatively** flat frequency response with non-linear phase.

Let us assume an EM signal $x(t)$ is incident at the 2 sites H1 and L1, for GW150914. It is picked up as $x(t)$ in the GW channel. In the magnetometer channel, it will be picked up as $y(t) = x(t) * w(t)$ where $*$ denotes convolution in time domain. In the frequency domain, $Y(f) = X(f)W(f)$, where $W(f)$ is given by the transfer function in the upper row in Fig. 3 with nonlinear phase response $W(f) = e^{-i\theta(f)}$ where $\theta(f) = 2\pi f t_0 + 2\pi f^2 t_1$.

The only way to rule out this EM interference as the source of the observed GW candidate signal is by computing cross-correlation function of the received signal with the template in both the GW channel (CCF of $x(t)$ versus template) and the magnetometer channel (CCF of $y(t)$ versus template) using matched filter.

³By peaky CCF, we mean that the ratio, R , of the the absolute value of CCF for any lag greater than the decorrelation time of the template, to the absolute maximum value of CCF, should be less than a certain threshold. Decorrelation time of the template τ_0 is defined as the time taken for the autocorrelation of the template to fall to $\frac{1}{e} = 0.36$ of the maximum value at zero lag [12] We will use the ratio R_3 , which is the ratio of the absolute value of CCF at any lag greater than $\tau_0 * 3$ to the absolute maximum value of CCF, and test whether $R_3 < \frac{1}{e}$. Lag greater than $\tau_0 * 3$ is taken to allow for some cushion.

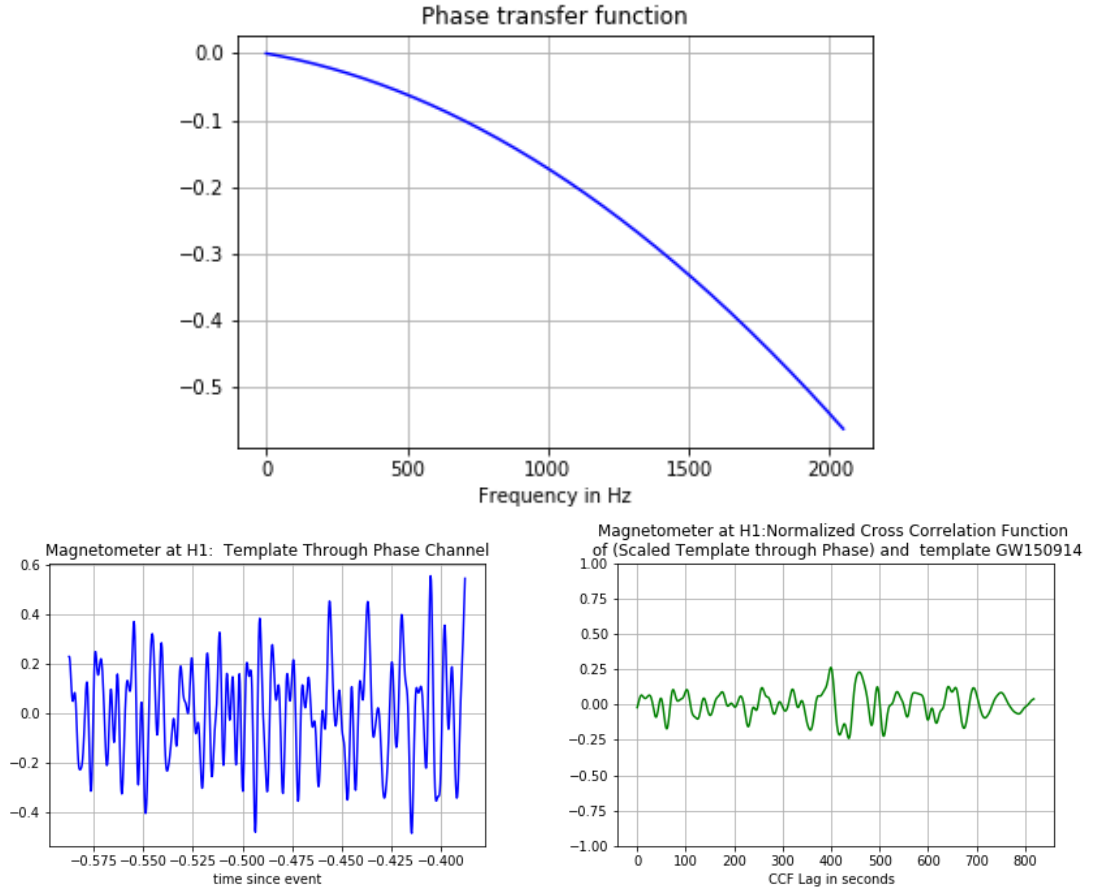


Figure 3. GW150914: Top: System 1: Example Phase frequency response plots of Magnetometer Channel (GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: template passed through System 1. Right: Normalized CCF of (template passed through System 1) versus template.

The two CCFs can look very different. In Fig.2, GW channel on the upper right panel shows a peaky CCF, while magnetometer channel on the lower right panel in Fig.3 does not have a peaky CCF.

We can see that CCF does not show clear peaks in magnetometer channel, for GW150914 and similar results are observed for GW151226 in Fig.7 and GW170104 in Fig.8. So, we may not see any CCF peaks in magnetometer channel on right panel and may mistake this EM interference, as a GW signal. Hence it is suggested that **GW150914**, **GW151226** and **GW170104** be further studied independently, given that EMI **cannot be ruled out** as a candidate for the GW events.

4.2 Varying Magnitude frequency response

In this section, for the purpose of convenience, we will **interchange** the transfer functions for magnetometers and the GW channel and assume that the GW channel has a perfectly flat frequency response $H_{gw}(f) = 1$ and that the magnetometer has a **relatively** flat frequency response with some **small** variations in magnitude, as opposed to phase, in the previous subsection.

Let us assume an EM signal $x(t)$ is incident at the 2 sites H1 and L1, for GW150914. It is picked up as $x(t)$ in the GW channel. In the magnetometer channel, it will be picked up as $y(t) =$

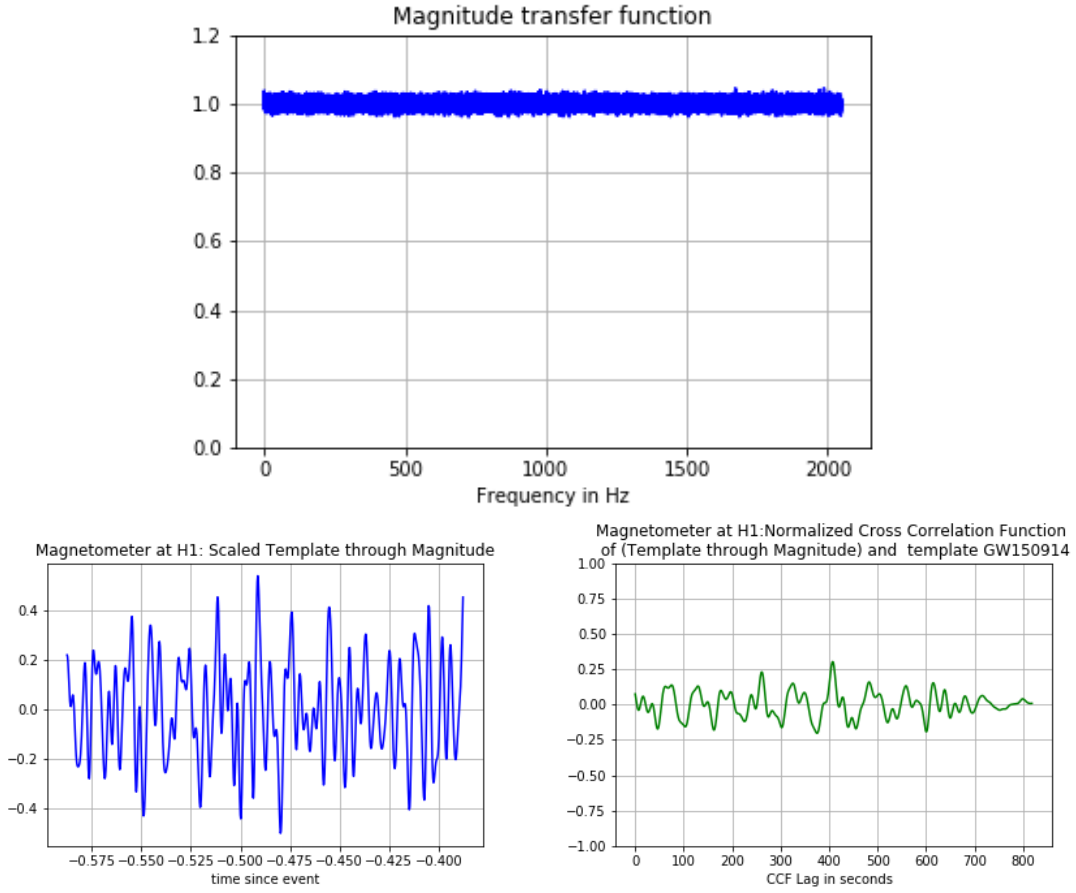


Figure 4. GW150914: Top: System 2: Example Magnitude Frequency response plots of Magnetometer Channel (GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: template passed through System 2. Right: Normalized CCF of (template passed through System 2) versus template.

$x(t) * (\delta(t) + w(t))$ where $*$ denotes convolution in time domain. In the frequency domain, $Y(f) = X(f)(1 + W(f))$, where $w(t)$ is Additive White Gaussian Noise (AWGN) with a small standard deviation and $\delta(t)$ is the dirac delta function. $W(f)$ is the fourier transform of $w(t)$ and $(1 + W(f))$ is given by the transfer function in the upper row in Fig. 4 with unity magnitude frequency response with small variations.

The only way to rule out this EM interference as the source of the observed GW candidate signal is by computing cross-correlation function of the received signal with the template in both the GW channel (CCF of $x(t)$ versus template) and the magnetometer channel (CCF of $y(t)$ versus template) using matched filter.

The two CCFs can look very different. In Fig.2, GW channel on the upper right panel shows a peaky CCF, while magnetometer channel on the lower right panel in Fig.4 does not have a peaky CCF.

We can see that CCF does not show clear peaks in the magnetometer channel, for GW150914 and similar results are observed for GW151226 in Fig.9 and GW170104 in Fig.10. So, we may not see any CCF peaks in the magnetometer channel on the right panel and may mistake this EM interference, as a GW signal. Hence it is suggested that **GW150914** , **GW151226** and **GW170104**

be further studied independently, given that EMI **cannot be ruled out** as a candidate for the GW events.

5 Implications for GW150914

We know that a strong lightning (EM signal) occurred around the same time as GW150914 observation in Burkina Faso. We also know that excess power was observed in many magnetometer channels. It is stated that "Fluxgate magnetometers indicate that magnetic disturbances at the LIGO detectors produced by coincident lightning strikes were at least 3 orders of magnitude too small to account for the amplitude of GW150914" [Section 6.1 - 6.3 in Page 24-26 in the LIGO noise characterization [paper](#)].

It has been shown in Section 2, that the GW channel has multiple paths for EM pickup and also that 60 Hz EM interference is picked up with a **relatively** flat frequency response and that LIGO's EM coupling calibration plot (Figure 2, Page 13 in [6]) may not be applicable to 60 Hz EM interference and likely for other external EM signals. Hence the magnetometer excess power "**3 orders of magnitude too small**" argument to rule out Burkina Faso lightning, as a possible candidate for GW150914, is **not** supported.

We have also shown in Section 3 that CCF does not show clear peaks in the magnetometer channel, for GW150914. So, we may not see any CCF peaks in the magnetometer channel and may mistake this EM interference, as a GW signal. The reason for the absence of CCF peaks in the magnetometer channel is due to the difference in the nature of the background noise in the GW channel and the magnetometer channel. In the GW channel, detector noise is non-stationary, non-gaussian and non-white noise and in the magnetometer channel, the background noise is approximately white noise.

Hence it is suggested that GW150914 be further studied independently, given that EMI **cannot be ruled out** as a candidate for the GW event.

6 Implications for GW170814 and GW170817

GW170814 and GW170817 were observed at three detectors H1, L1 and V1. Their templates have not been published as of date. The analysis in above sections are applicable for GW170814 and GW170817 as well and EM interference could explain these GW signals, irrespective of the exact templates used.

As pointed out in Section 6.1 [here](#), the Gamma Ray Burst (GRB) associated with GW170817 was observed with telescopes. It is well known that astrophysical objects such as stars, emit EM signals in a wide frequency range, all the way down to kHz. Our Sun is known to emit EM signals down to 30 kHz (NASA [document](#)). Given that [GW170817](#) was observed as a weak signal, whose amplitude does not rise during GW event, if the astrophysical object which emitted the GRB, also emitted EM signal in the frequency range of 0-2048 Hz, magnetometers will not show any excess power during the GW event. Hence low frequency EM signals from the astrophysical object which emitted the GRB, is a good candidate for GW170817.

Hence it is suggested that **GW170814** and **GW170817** be further studied independently, given that EMI **cannot be ruled out** as a candidate for the GW events.

7 Concluding remarks

In Section 2, it is shown that the GW channel has multiple paths for EM pickup, with varying transfer functions and also that 60 Hz EM interference is picked up with a **relatively** flat frequency response. Section 3 and Section 4 give the reasons why GW150914, GW151226 and GW170104 should be studied further. Section 5 discusses the implications of this analysis for GW150914. Section 6 gives the reasons why weak signals GW170814 and GW170817 should be further studied.

Reiterating the point made earlier, because LIGO detectors are more likely to pickup EM signals which are buried in the detector noise, it is of great importance that we should not classify EM interference as GW events, given the need for **high** standards.

The only way to avoid wrong classification of EM interference as GW signals, is as follows. LIGO detectors have very strong impulsive interference in 60 Hz and harmonics and also in 300-2000 Hz range. First, it is very important to clean up the impulsive interference and rest of the EM interference, in the detectors. Secondly, we should insist on signals which rise well above the background noise, in time domain, in the GW channel. When we do observe such a strong signal in the GW channel, we can then look at the magnetometer channel in time domain, for any similar strong signal in time domain, which rises well above the background noise and if it does, we must reject it as a candidate for GW signal, given that CCF may not show clear peaks due to possible difference in transfer function between the GW channel and the magnetometer channel.

LIGO team should release magnetometer and Mains Voltage Monitor data (few dozen channels) for all the 6 GW signals, and also publish the templates for GW170608, GW170814 and GW170817, to enable outside researchers to analyze them further.

This manuscript **does not** claim that EMI is the source of the assumed GW events. It points out that EMI **cannot be ruled out** as a candidate for the GW events observed so far. GW signals should be subjected to further **independent** studies and LIGO should provide all the information required to facilitate such studies and publish the data for all the EM channels such as magnetometers and mains voltage monitors.

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References

- [1] Abbott, B. P., Abbott, R., Abbott, T. D., et al., *Observation of Gravitational Waves from a Binary Black Hole Merger*, Physical Review Letters, 116, 061102 (2016) [Online version of paper](#).
- [2] Abbott, B. P., Abbott, R., Abbott, T. D., et al., *GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence*, Physical Review Letters, 116, 241103(2016) [Online version of paper](#).
- [3] Abbott, B. P., Abbott, R., Abbott, T. D., et al. *GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2*, Physical Review Letters, 118, 221101 (2017) [Online version of paper](#).
- [4] Abbott, B. P., Abbott, R., Abbott, T. D., et al. *GW170814: A Three-Detector Observation of Gravitational Waves from a Binary Black Hole Coalescence*, Physical Review Letters, 119, 141101 (2017) [Online version of paper](#).

- [5] Abbott, B. P., Abbott, R., Abbott, T. D., et al. *GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral*, Physical Review Letters, 119, 161101 (2017) [Online version of paper](#).
- [6] Abbott, B. P., Abbott, R., Abbott, T. D., et al. *Characterization of transient noise in Advanced LIGO relevant to gravitational wave signal GW150914*. ([Online version of paper](#).)
- [7] Abbott, B. P., Abbott, R., Abbott, T. D., et al., *GW150914: First Results from the Search for Binary Black Hole Coalescence with Advanced LIGO*, Phys. Rev. D 93, 122003 (2016) [see pp.4-9] [Arxiv copy of paper](#). see pp.8-13.
- [8] Creswell, J., Hausegger, S., Jackson, A. D., Liu, H., Naselsky, P., *On the time lags of the LIGO signals*, 2017, arxiv:1706.04191 [Arxiv copy of paper](#).
- [9] Naselsky, P., Jackson, A. D., & Liu, H., *Understanding the LIGO GW150914 event*, Journal of Cosmology and Astroparticle Physics, 8, 029 (2016)
- [10] Liu, H., & Jackson, A. D., *Possible associated signal with GW150914 in the LIGO data*, Journal of Cosmology and Astroparticle Physics, 10, 014 (2016)
- [11] Garnier, J., Papanicolaou, G., *Passive Sensor Imaging Using Cross Correlations of Noisy Signals in a Scattering Medium*, SIAM J. Imaging Sci., 2(2), 396437. (2009). [Online version of paper](#)
- [12] Kenneth Y. Jo, *Satellite Communications Network Design and Analysis*, Page 415.
- [13] [LOSC LIGO GW150914 Tutorial](#)
- [14] [LOSC LIGO GW151226 Tutorial](#) matched filter equations are implemented in the section "matched filtering to find the signal"
- [15] [LIGO Tutorials](#)
- [16] [Zip Files of Modified LIGO Tutorials which demonstrate the results in this paper](#).

8 Appendix A

8.1 60 Hz Harmonic amplitudes

Let us consider the case of 60 Hz EM interference from power lines and represent it as $x_1(t) = A \cos(2\pi f_0 t)$, where $f_0 = 60$ Hz is the fundamental frequency and A is the amplitude. Second harmonic 120 Hz $x_2(t)$ and third harmonic 180 Hz $x_3(t)$ are generated by nonlinearities in the hardware, which can be obtained by taking the square and cube of the fundamental frequency, as follows.

$$\begin{aligned}
 x_1(t) &= A \cos(2\pi f_0 t) \\
 x_2(t) &= x_1^2(t) = A^2 \cos^2(2\pi f_0 t) = \frac{A^2}{2} [1 + \cos(2\pi(2 * f_0)t)] \\
 x_3(t) &= x_1^3(t) = A^3 \cos^3(2\pi f_0 t) = \frac{A^3}{2} \cos(2\pi f_0 t) [1 + \cos(2\pi(2 * f_0)t)] \\
 x_3(t) &= \frac{A^3}{2} [\cos(2\pi f_0 t) + \frac{1}{2} (\cos(2\pi(3 * f_0)t) + \cos(2\pi f_0 t))]
 \end{aligned} \tag{8.1}$$

We can see that the harmonic amplitudes are always lesser than the amplitude of the fundamental frequency.

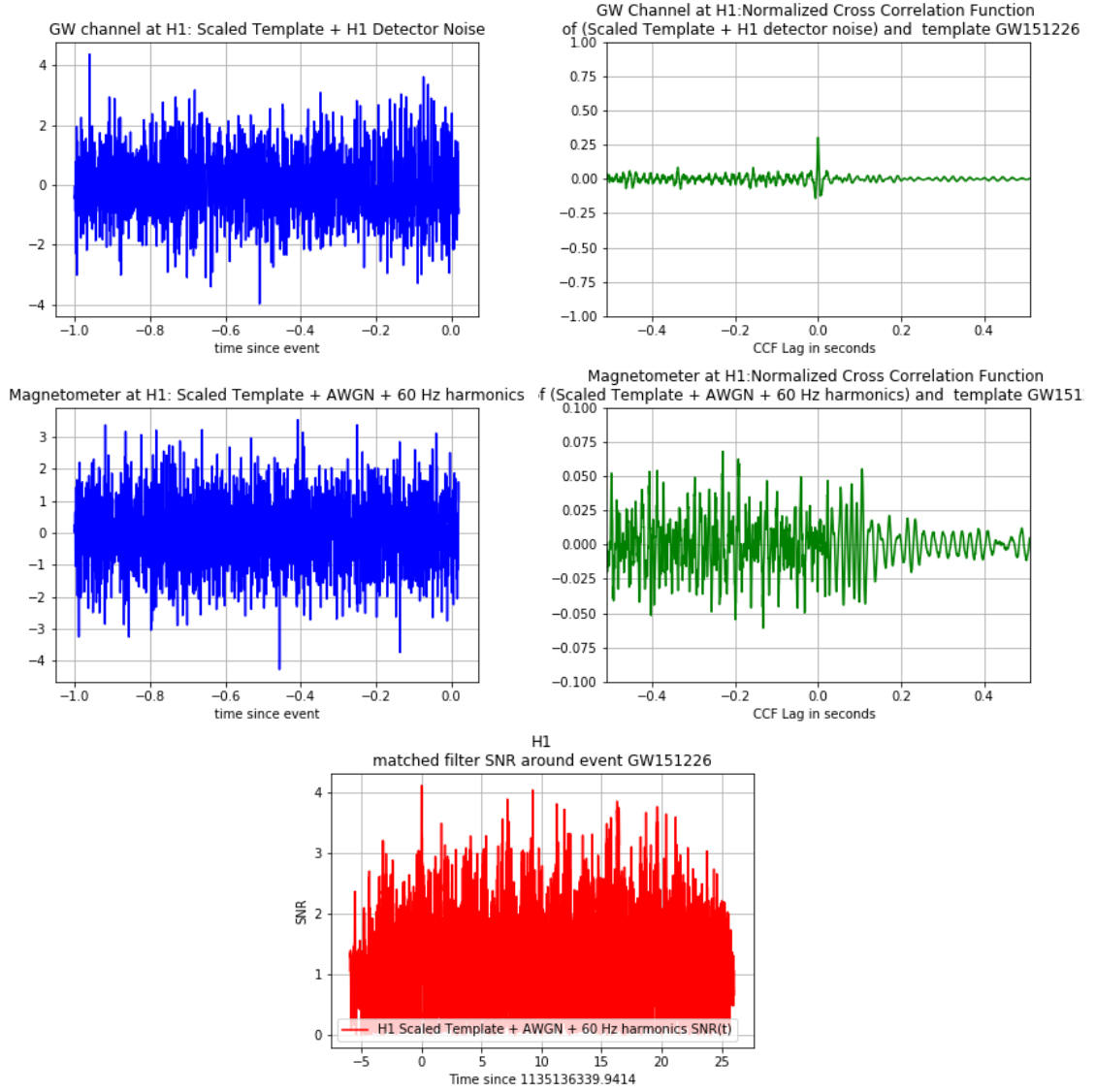


Figure 5. GW151226: Top panels: GW channel, (template+detector noise) whitened and filtered. Top Left: time domain. Top right: Normalized CCF with template. Middle panels: Magnetometer channel, (template+ white noise) whitened and filtered. Middle Left: time domain. Middle right: Normalized CCF with template. Lower panel: LIGO matched filter SNR for Magnetometer channel, (template+ white noise) whitened and filtered.

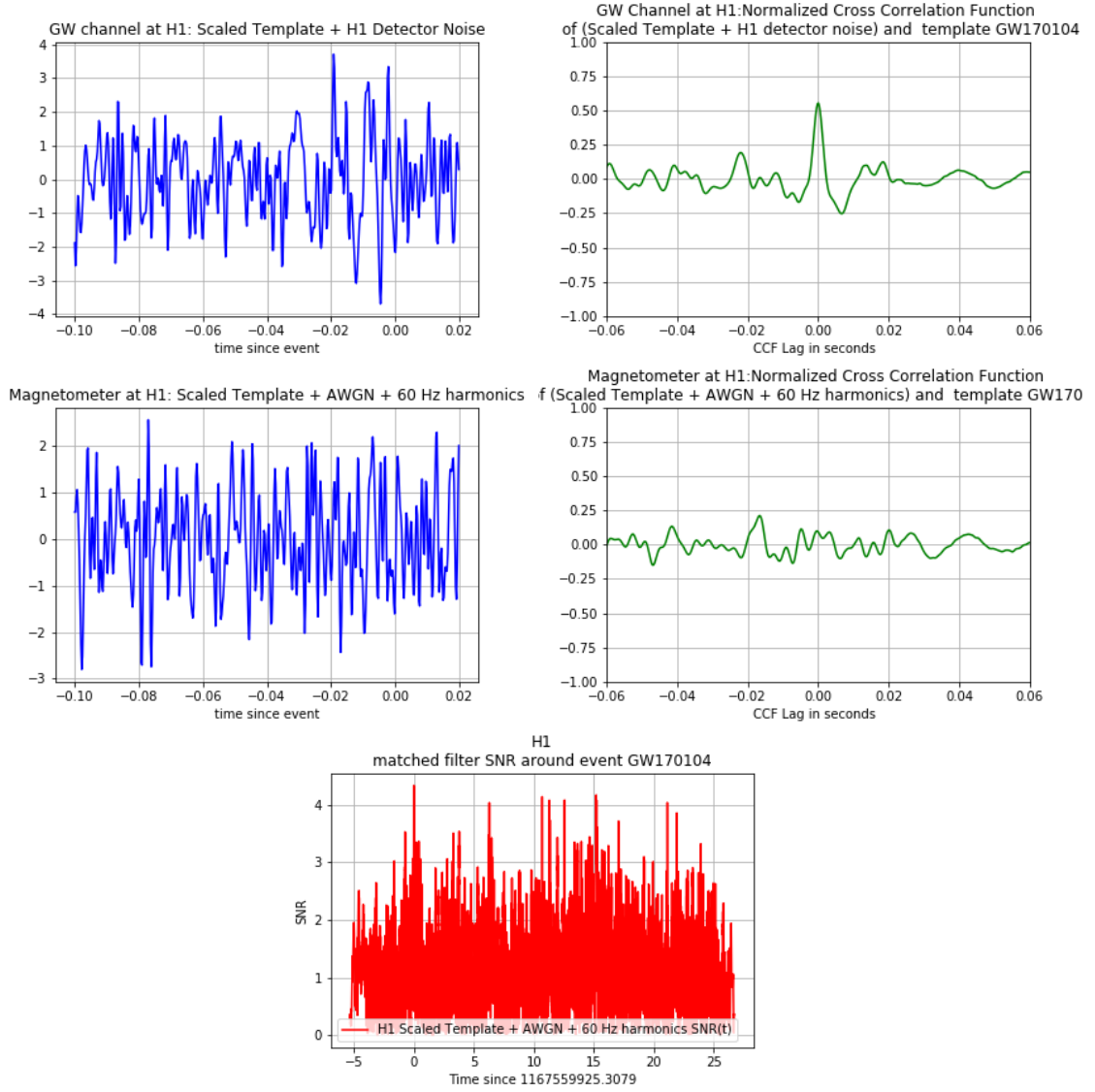


Figure 6. GW170104: Top panels: GW channel, (template+detector noise) whitened and filtered. Top Left: time domain. Top right: Normalized CCF with template. Middle panels: Magnetometer channel, (template+ white noise) whitened and filtered. Middle Left: time domain. Middle right: Normalized CCF with template. Lower panel: LIGO matched filter SNR for Magnetometer channel, (template+ white noise) whitened and filtered.

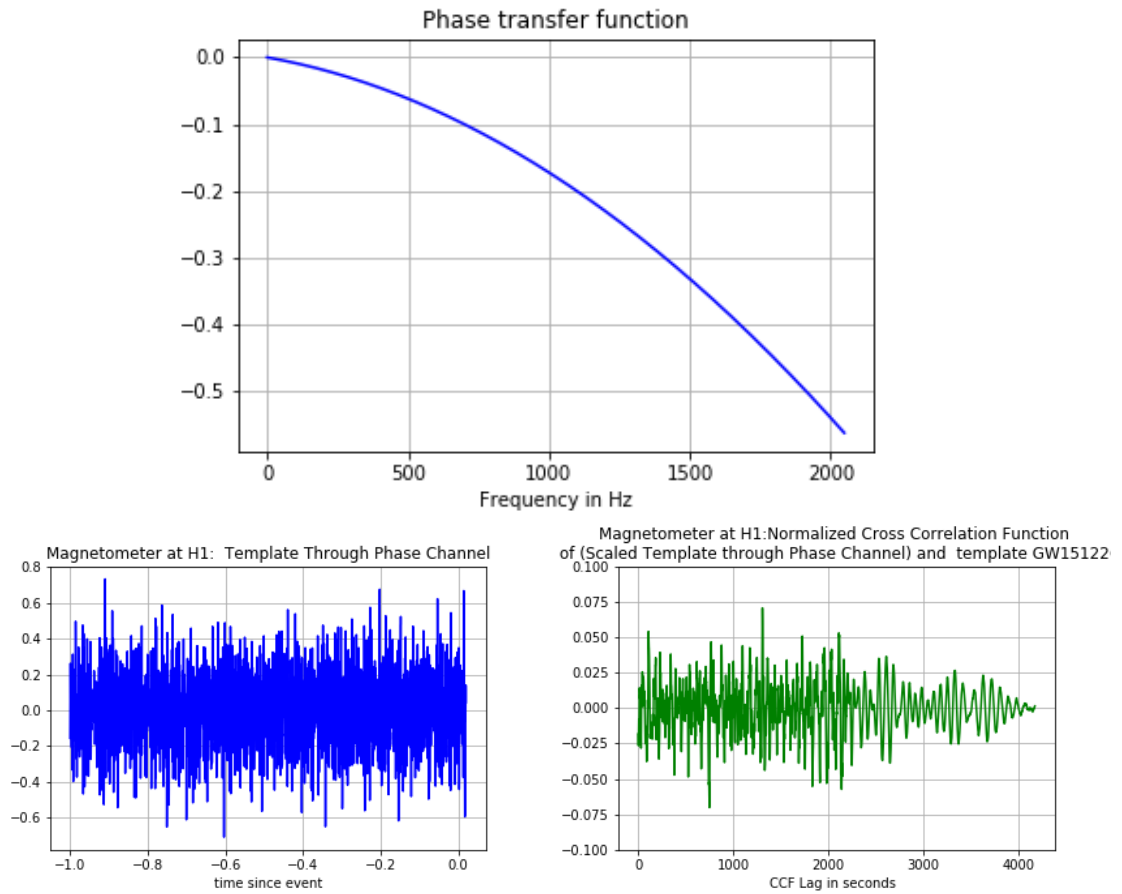


Figure 7. GW151226: Top: System 1: Example Phase frequency response plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: template passed through System 1. Right: Normalized CCF of (template passed through System 1) versus template.

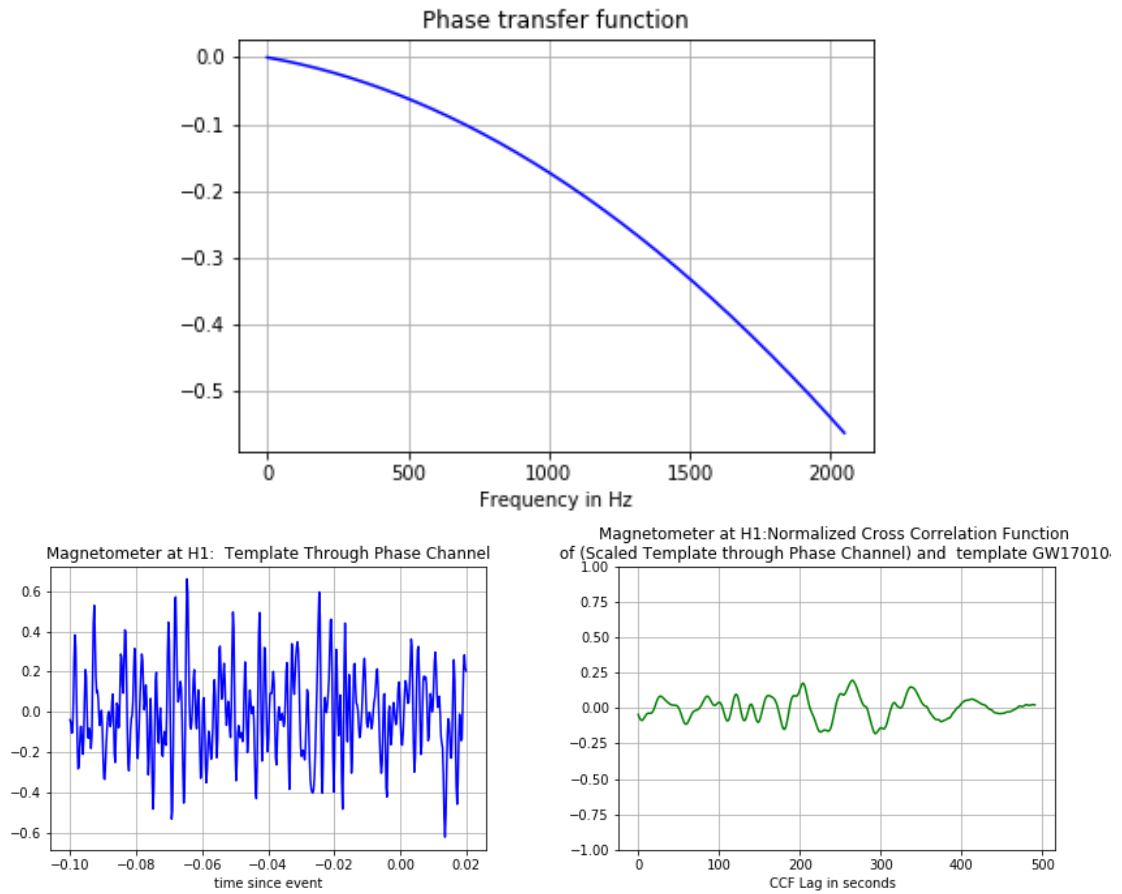


Figure 8. GW170104: Top: System 1: Example Phase frequency response plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: template passed through System 1. Right: Normalized CCF of (template passed through System 1) versus template.

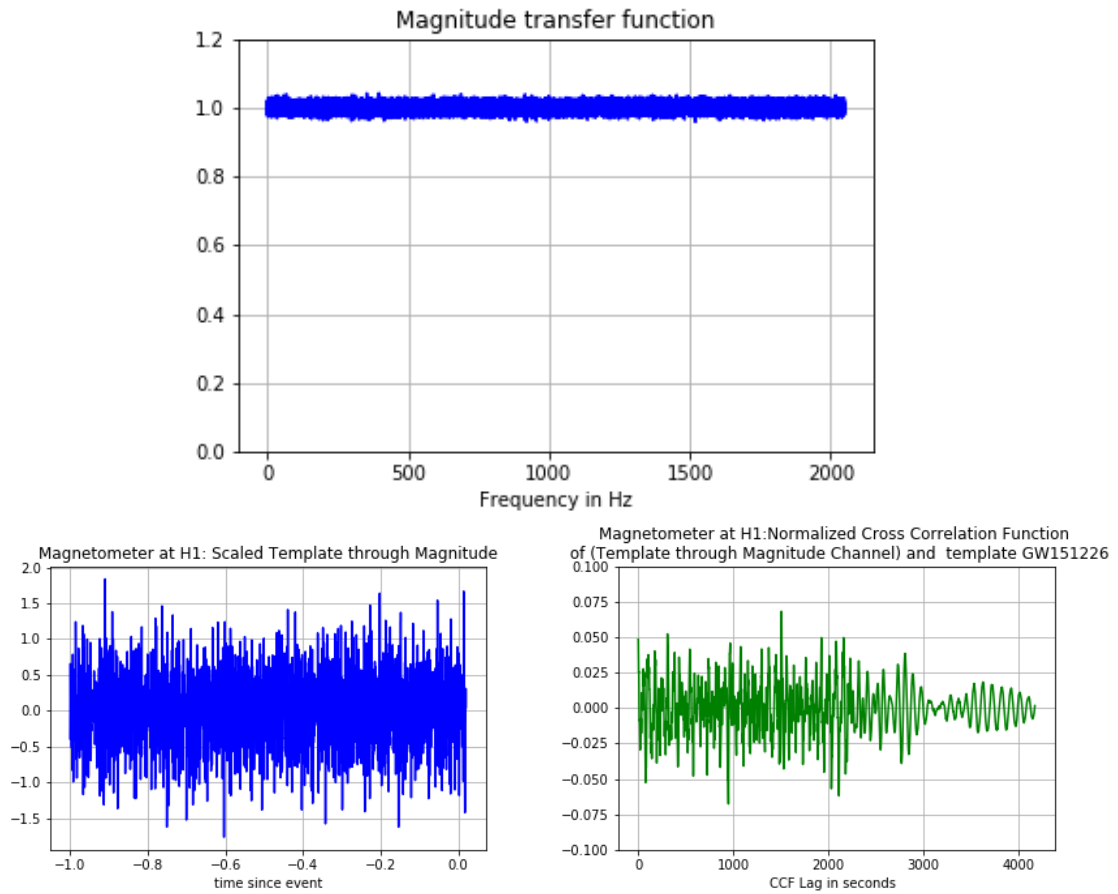


Figure 9. GW151226: Top: System 2: Example Magnitude Frequency response plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: template passed through System 2. Right: Normalized CCF of (template passed through System 1) versus template.

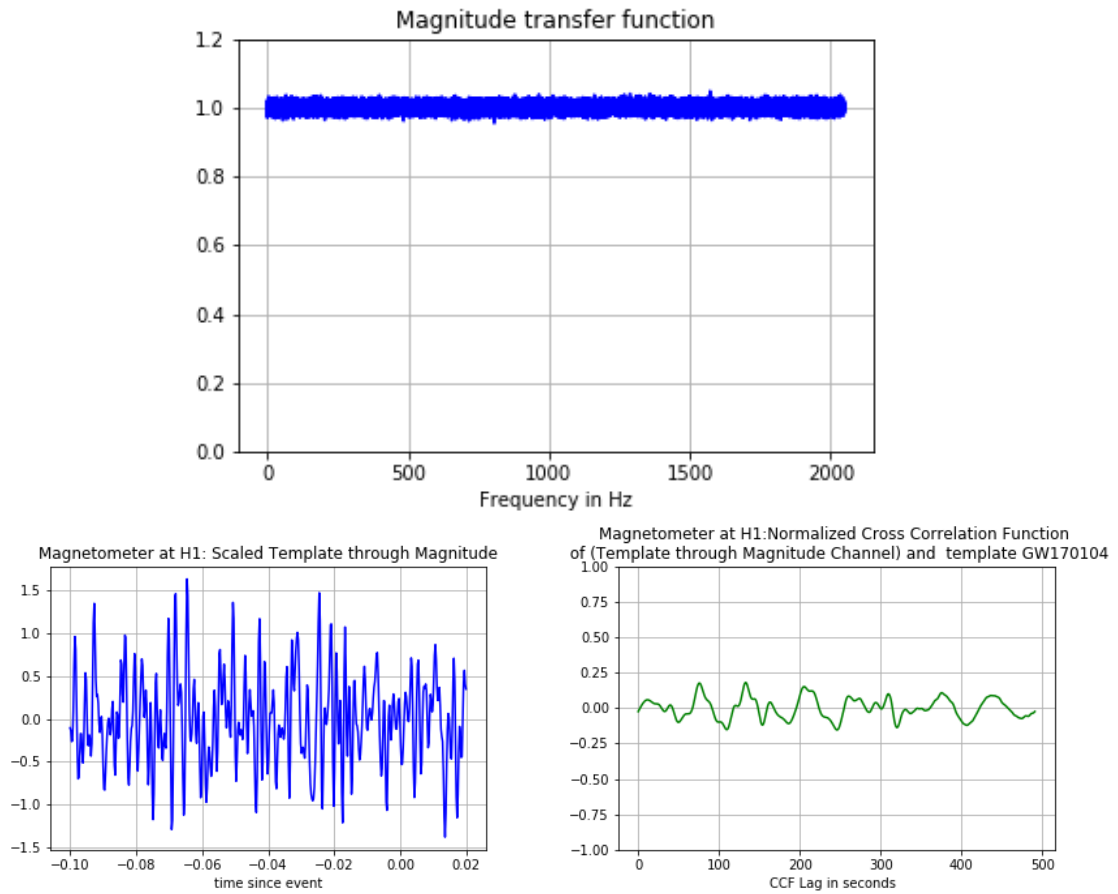


Figure 10. GW170104: Top: System 2: Example Magnitude Frequency response plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: template passed through System 2. Right: Normalized CCF of (template passed through System 1) versus template.