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On the Electro-Magnetic(EM) correlations in LIGO signals

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Abstract. This article analyzes the data for the first three gravitational wave (GW) events detected in both Hanford(H1) and Livingston(L1) detectors by the LIGO[1] collaboration. It is shown that the leading candidate for non-GW source for these assumed GW events, is in fact electro-magnetic(EM) interference from terrestrial and non-terrestrial sources. We know that LIGO detector's GW channel does in fact pick up strong 60*n Hz EM interference from power lines and it will be shown that GW channel has a relatively flat frequency response for 60*n Hz EM interference.

The only way to rule out EM interference is by looking at the magnetometers. Let us consider the case where EM interference is the cause of the assumed GW signal. It is shown that GW150914, GW151226 and GW170104 are very weak signals which are buried in the raw detector noise by a factor of $\frac{1}{400}$. In the case of these weak GW events, where assumed GW event does not rise above the raw detector noise, magnetometers also will not show an EM signal rising well above the background noise. It will be shown that the time domain cross-correlation of magnetometer signal during GW event, with the template, does not show peaks in cross-correlation, while the GW channel shows cross-correlation peaks. The reason for the absence of CCF peaks in the magnetometer channel is due to the difference in the nature of the background noise in the GW channel and the magnetometer channel. In the GW channel, detector noise is non-stationary, non-gaussian and non-white noise and in the magnetometer channel, the background noise is approximately white noise.

It will be shown for GW150914, GW151226 and GW170104, that time domain cross-correlation of magnetometer signal during GW event, with the template, does not show peaks in cross-correlation, if the frequency response/transfer function $H(f)$ of magnetometers differ from that of the GW channel, which is likely to be the case. There is no reason to suppose that magnetometers and GW channel have the exact same transfer function $H(f)$. It will be shown that GW channel has various paths for EM interference pickup, with varying transfer functions.

Hence it is suggested that GW150914, GW151226 and GW170104 be further studied as candidates for GW signals. The implications of these results are discussed for GW170814 and GW170817. All the results in this paper are demonstrated using modified versions of LIGO's Python scripts[16].[2]

[1]The Laser Interferometer Gravitational-Wave Observatory

[2]The specific Python script used to generate Fig.1 to Fig.4 in this manuscript, is mentioned in [16].

Contents

1 Introduction

In the LIGO system, GW channel has ever-present 60*n Hz electro magnetic(EM) interference(figure) and broadband EM interference. It will be shown that EM interference enters GW channel through several paths, suspension magnets or electrical board or elsewhere. Each such path in the GW channel may have a different transfer function (frequency response).

It is possible that an EM interference signal, such as lightning, may be picked up at both sites H1 and L1 and may be mistaken for a GW signal, if it resembles 1 out of 250,000 GW templates. Lightning has low frequency components as well, in the 0-2048 Hz region of interest. EM signals also travel at the speed of light, like the GW signal, and arrive within the 10 msec window at the 2 detectors.

It is shown in the following subsection that, even the strongest GW event GW150914, is buried in the detector noise by a factor of $\frac{1}{400}$ in the GW channel. Similarly, if EMI is the source of the assumed GW events, they are also buried in the noise in the magnetometer channel. Hence the magnetomemeters may not show any spikes (excess power) during assumed GW events.

The only way we can rule out EM interference in the GW channel, is by cross-correlating the magnetometer signal with the template, during GW event duration. If the transfer function experienced by the EM signal in the magnetometer channel is different from the transfer function in the GW channel, it is possible that the magnetometer channel may not produce Cross Correlation Function(CCF) peaks when correlated with the template and hence this EM signal may be mistaken for GW signals.

Lightning may enter the GW channel through electrical power points and wires, in which case, we need to look at the mains voltage monitors, to rule out EM interference. The analysis in this paper can be applied to mains voltage monitors as well.

This manuscript **does not** claim that EMI is the source of the assumed GW events. It points out that EMI **cannot be ruled out** as a candidate for the GW events observed so far.

1.1 GW signals buried in the detector noise

The first gravitational wave GW signal observed in the LIGO detector was GW150914 [1] which was the strongest of the 6 GW signals observed so far. But, we can see from Fig.1 that, the maximum amplitude of the recovered template ($1.2 * 10^{-21}$) shown in the right panel, after LIGO whitening procedure and filtered in 43-300Hz range, is $\frac{1}{400}$th of the amplitude of the raw strain in the left panel ($0.5 * 10^{-18}$). 85 percent of the template energy is in the 43-300Hz range. We can also see that, the recovered template for GW150914 in the right panel, is submerged below the maximum amplitude of the filtered strain ($3 * 10^{-21}$) in the middle panel.

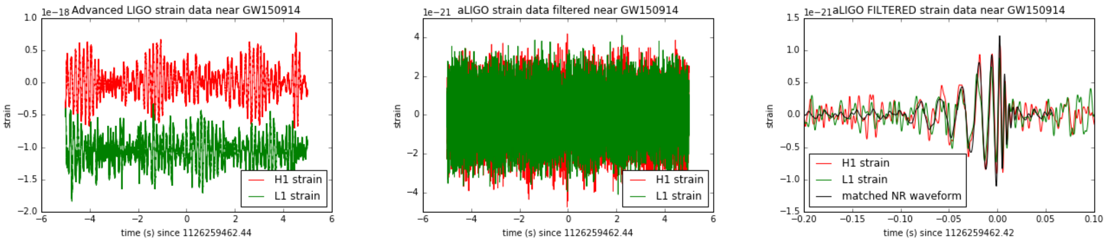

Figure 1. GW150914 H1 and L1 strains: Left Panel: Raw strain. Middle Panel: Raw strain filtered in 43-300Hz range. Right Panel: LIGO whitened and filtered strains

The first GW signal observed was GW150914 [1] which was a relatively strong signal whose amplitude, after whitening and filtering[1], rose significantly, well over detector noise level, during the 0.2 second GW event duration. In comparison, the second signal[2] GW151226, the third signal[3] GW170104, the fourth signal[4] GW170814 and the fifth signal[5] GW170817 were very weak signals, which look like noise after whitening and filtering and whose signal amplitude does not rise above the detector noise level during the GW event of duration of less than 1 second (Fig.5, Fig.6, Fig.7, Fig.8, Fig.9)).

If we check the magnetometers for any spikes during assumed GW events, we may not see a spike, as in above weak signals, if EM interference is the source of observed GW events. This raises the important question of whether GW151226 and GW170104 could have been caused by EM signals from other sources. Because we are more often likely to observe EM signals which may be buried in background noise and whose amplitude does not rise during assumed GW event, it is of paramount importance that we should not classify non-GW signals such as EM, as GW events. We must insist on high standards before classifying an observed time series as a GW signal.

The organization of this paper is as follows. In Section 2, it is shown that the GW channel has multiple paths for EM pickup, with varying transfer functions and also that 60*n Hz EM interference is picked up with a **relatively** flat frequency response. In Sections 3, it will be shown for GW151226 and GW170104, that even small variations in the transfer functions of GW channel vs magnetometer channel, can destroy cross-correlation peaks in one of those channels, when correlated with the

[1]Fourth order Butterworth bandpass filters used in frequency range 43-300Hz for GW150914, 43-800Hz for GW170104 and GW151226.

template. Section 5 discusses the implications of this analysis for GW150914. Section 6 gives the reasons why weak signals GW170814 and GW170817, observed in the three detectors, should be further studied.

2 GW channel coupling for EM signals

We know that a strong lightning (EM signal) occurred around the same time as GW150914 observation, in Burkina Faso. We also know that excess power was observed in many magnetometer channels. It is stated that ”Fluxgate magnetometers indicate that magnetic disturbances at the LIGO detectors produced by coincident lightning strikes were **at least 3 orders of magnitude too small** to account for the amplitude of GW150914” [Section 6.1 - 6.3 in Page 24-26 in the LIGO noise characterization paper here].

This **”at least 3 orders of magnitude too small”** defense against Burkino Faso lightning is related to the bottom panel in LIGO’s EM coupling calibration plot (Figure 2, Page 13 in [6]), shows a sharp drop by a factor of 50, from 60 Hz to 180Hz ($50*50 = 2500$, which is 3 orders of magnitude). It is shown here that this sharp drop in coupling is **not** applicable for 60*n Hz EM interference and may not be applicable for any other EM interference like lightning. In this section, it will be shown that the EM interference is picked up by multiple paths in the GW channel with varying transfer functions.

If we look at the detector noise frequency response here, we can see that the amplitudes of 60*n Hz harmonics in the L1 signal, 60 Hz amplitude = 6e-22 and 180 Hz amplitude = 3e-22. We can see that the EM coupling(frequency response) is **relatively flat** from 60 Hz to 180Hz, with a drop only by a factor of 2. We know that the harmonic amplitudes are always lesser than the amplitude of the fundamental frequency (Section 8.1) at the power line source. 180 Hz amplitude can be at most be equal to 60 Hz amplitude, at the source. If this 60*n Hz interference were picked up by the path described by LIGO’s EM coupling calibration plot, 180Hz harmonic would be 50 times less than 60 Hz amplitude. But we see only a drop by a factor of 2 in 180 Hz amplitude.

It is clear that 60*n Hz EM interference is picked up by a path, which is **different** from the path used in EM coupling calibration and hence LIGO’s EM coupling calibration plot may not be applicable to this EM pickup.

This means that, external EM signals, such as lightning and EM interference from astrophysical objects, also can be picked up by the same path which picked up 60*n Hz EM interference and have relatively flat coupling, with small variations.

3 Non-white Detector Noise in GW channel Vs White Noise in magnetometers for EM signals

Let us assume an EM signal x(t) is incident at the 2 sites H1 and L1, for GW150914. It is picked up as $x(t) = h(t) + w_d(t)$ in the GW channel. In the magnetometer channel, it will be picked up as $y(t) = h(t) + w_m(t)$ where $h(t)$ is the template [2], $w_d(t)$ is the detector noise which is non-stationary, non-gaussian and non-white noise (here) and $w_m(t)$ is the magnetometer noise due to thermal and EM sources, which is approximately white noise (top plot in blue has white noise and 60*n Hz impulses). The white noise in the magnetometer channel has the same noise power equivalent to the detector noise power in the GW channel, in the 43-300Hz frequency range, where most of the template energy is concentrated.

[2] The template $h(t)$ is scaled so that $x(t) = h(t) + w_d(t)$ produces the same Signal to Noise ratio in LIGO’s mathced filter, as the event GW150914.

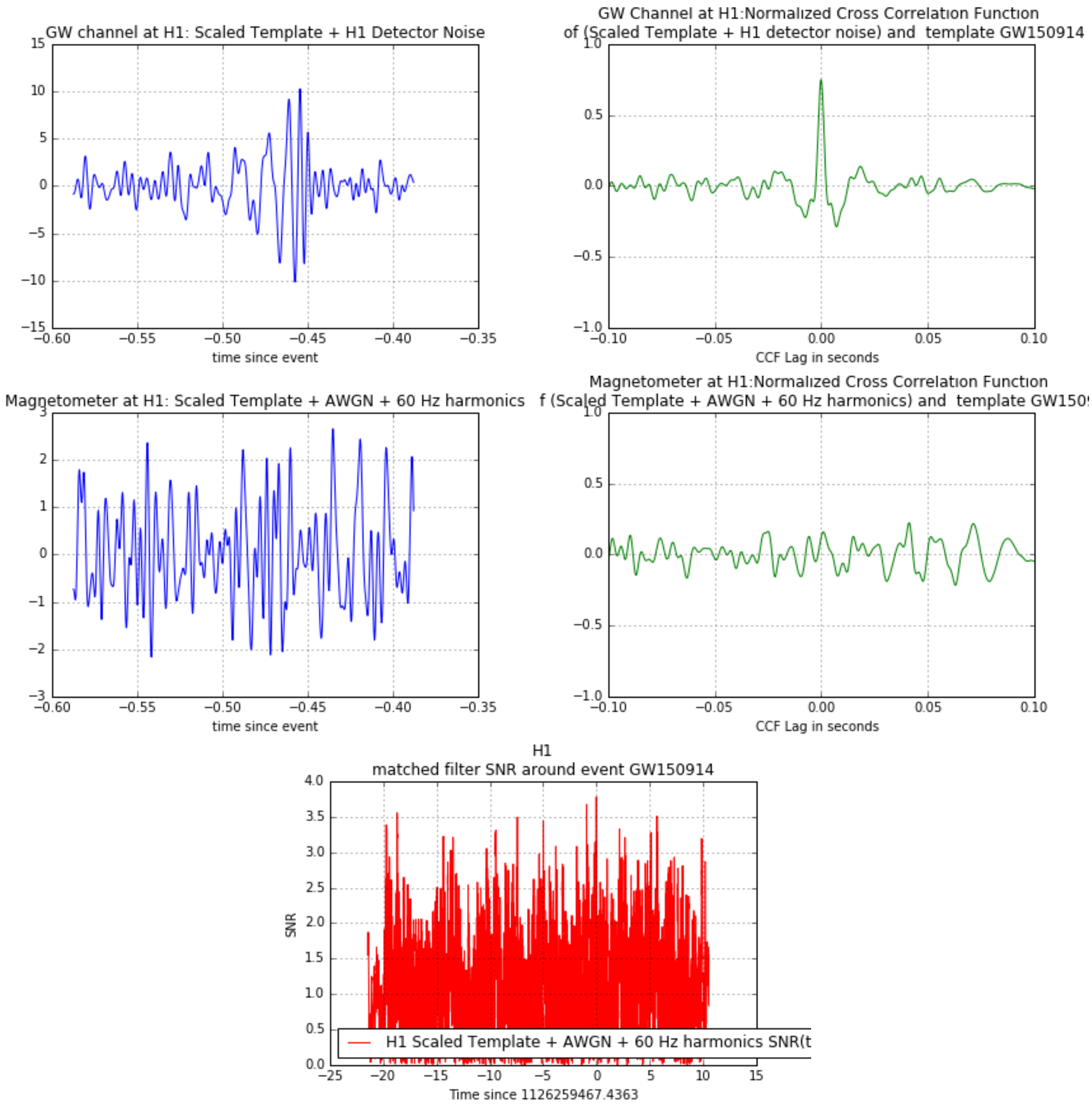

Figure 2. GW150914: Top panels: GW channel, (Template+detector noise) whitened and filtered. Top Left: time domain. Top right: Normalized CCF with template. Middle panels: Magnetometer channel, (Template+ white noise) whitened and filtered. Middle Left: time domain. Middle right: Normalized CCF with template. Lower panel: LIGO matched filter SNR for Magnetometer channel, (Template+ white noise) whitened and filtered.

We can see that the white noise power in 43-300Hz frequency range in the magnetometer channel is expected to be greater than or equal to the detector noise power in 43-300Hz frequency range in the GW channel for the following reason. Magnetometer channel has thermal noise and picks up 60*n Hz EM harmonics and broadband EM noise by definition, without any attenuation or EM isolation. On the other hand, GW channel has thermal noise and 60*n Hz EM harmonics in the 43-300Hz frequency range, but it has EM isolation mechanism and noise reduction hardware.

The only way to rule out this EM interference as the source of the observed GW candidate signal, is by doing cross-correlation function(CCF) of the received signal with the template, in both the GW channel (CCF of x(t) vs template) and the magnetometer channel (CCF of y(t) vs template).

The two CCFs can look very different in Fig.2, GW channel on the top right panel shows a peaky CCF ($R < \frac{1}{e}$) [3], while magnetometer channel on the right panel in the second row does not have a peaky CCF ($R > \frac{1}{e}$)

We can see that CCF does not show clear peaks in the magnetometer channel, for GW150914. Similar results are observed for GW151226 in Fig.10 and GW170104 in Fig.11. So, we may not see any CCF peaks in the magnetometer channel and may mistake this EM interference, as a GW signal. Hence it is suggested that **GW150914** , **GW151226** and **GW170104** be further studied.

The reason for the absence of CCF peaks in the magnetometer channel is due to the difference in the nature of the background noise in the GW channel and the magnetometer channel. In the GW channel, detector noise is non-stationary, non-gaussian and non-white noise and in the magnetometer channel, the background noise is approximately white noise.

EM isolation in the GW channel

We know that the GW channel has EM isolation of > 40 dB to minimize EM interference (Section 8.2) (Fig.1 and 2), while magnetometers do not, because magnetometers are meant to pick up and measure EM interference. If we account for this in the calculations in the above section, it can be shown that this additional difference between the GW channel and magnetometer channel will result in the absence of CCF peaks in the magnetometer channel.

4 Different Transfer Functions for EM signals in GW channel and magnetometers

4.1 Nonlinear Phase Transfer Functions for EM signals in magnetometers

In this section, for the purpose of convenience, we will **interchange** the transfer functions for magnetometers and the GW channel and assume that the GW channel has a perfectly flat frequency response $H_{gw}(f) = 1$ and that the magnetometer has a **relatively** flat frequency response with non-linear phase.

Let us assume an EM signal x(t) is incident at the 2 sites H1 and L1, for GW150914. It is picked up as x(t) in the GW channel. In the magnetometer channel, it will be picked up as $y(t) = x(t) * w(t)$ where * denotes convolution in time domain. In the frequency domain, $Y(f) = X(f)W(f)$, where $W(f)$ is given by the transfer function in the upper row in Fig. 3 with nonlinear phase frequency response.

The only way to rule out this EM interference as the source of the observed GW candidate signal, is by doing cross-correlation function(CCF) of the received signal with the template, in both the GW channel (CCF of x(t) vs template) and the magnetometer channel (CCF of y(t) vs template).

The two CCFs can look very different. In Fig.2, GW channel on the upper right panel shows a peaky CCF ($R < \frac{1}{e}$), while magnetometer channel on the lower right panel in Fig.3 does not have a peaky CCF ($R > \frac{1}{e}$).

We can see that CCF does not show clear peaks in magnetometer channel, for GW150914 and similar results are observed for GW151226 in Fig.12 and GW170104 in Fig.13. So, we may not see any CCF peaks in magnetometer channel on right panel and may mistake this EM interference, as a GW signal. Hence it is suggested that **GW150914** , **GW151226** and **GW170104** be further studied.

[3]By peaky CCF, we mean that the ratio, R, of the the absolute value of CCF for any lag greater than the decorrelation time of the template, to the absolute maximum value of CCF, should be less than a certain threshold. Decorrelation time of the template τ_0 is defined as the time taken for the autocorrelation of the template to fall to $\frac{1}{e} = 0.36$ of the maximum value at zero lag [12] We will use the ratio R_3, which is the ratio of the absolute value of CCF at any lag greater than $\tau_0 * 3$ to the absolute maximum value of CCF, and test whether $R_3 < \frac{1}{e}$. Lag greater than $\tau_0 * 3$ is taken to allow for some cushion.

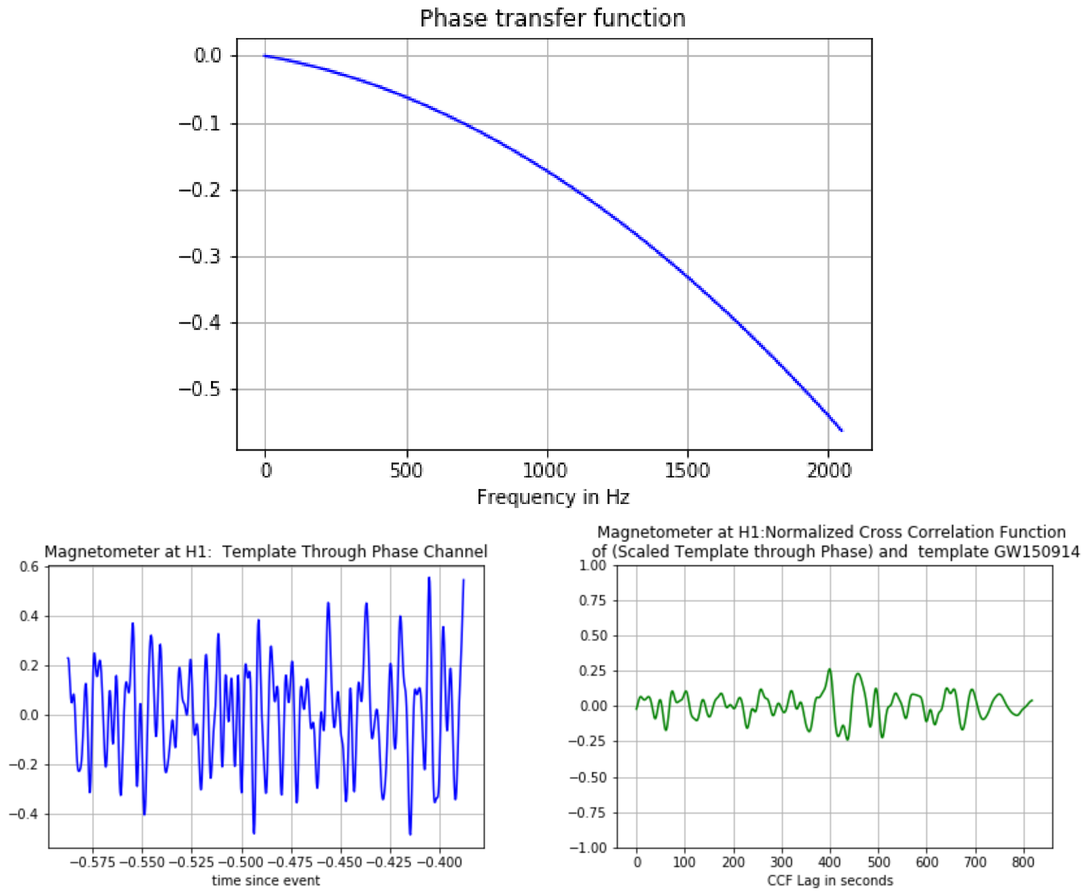

Figure 3. GW150914: Top: System 1: Example Phase frequency response plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: Template passed through System 1. Right: Normalzied CCF of (Template passed through System 1) vs Template.

4.2 Varying Magnitude Transfer Functions for EM signals in magnetometers

In this section, for the purpose of convenience, we will **interchange** the transfer functions for magnetometers and the GW channel and assume that the GW channel has a perfectly flat frequency response $H_{gw}(f) = 1$ and that the magnetometer has a **relatively** flat frequency response with some **small** variation.

Let us assume an EM signal x(t) is incident at the 2 sites H1 and L1, for GW150914. It is picked up as x(t) in the GW channel. In the magnetometer channel, it will be picked up as $y(t) = x(t) * (\delta(t) + w(t))$ where * denotes convolution in time domain. In the frequency domain, $Y(f) = X(f)(1 + W(f))$, where $w(t)$ is Additive White Gaussian Noise(AWGN) with a small standard deviation and $\delta(t)$ is the dirac delta function and $W(f)$ is given by the transfer function in the upper row in Fig. 4 with unity magnitude frequency response with small variations.

The only way to rule out this EM interference as the source of the observed GW candidate signal, is by doing cross-correlation function(CCF) of the received signal with the template, in both the GW channel (CCF of x(t) vs template) and the magnetometer channel (CCF of y(t) vs template).

The two CCFs can look very different. In Fig.2, GW channel on the upper right panel shows a peaky CCF ($R < \frac{1}{e}$), while magnetometer channel on the lower right panel in Fig.4 does not have a

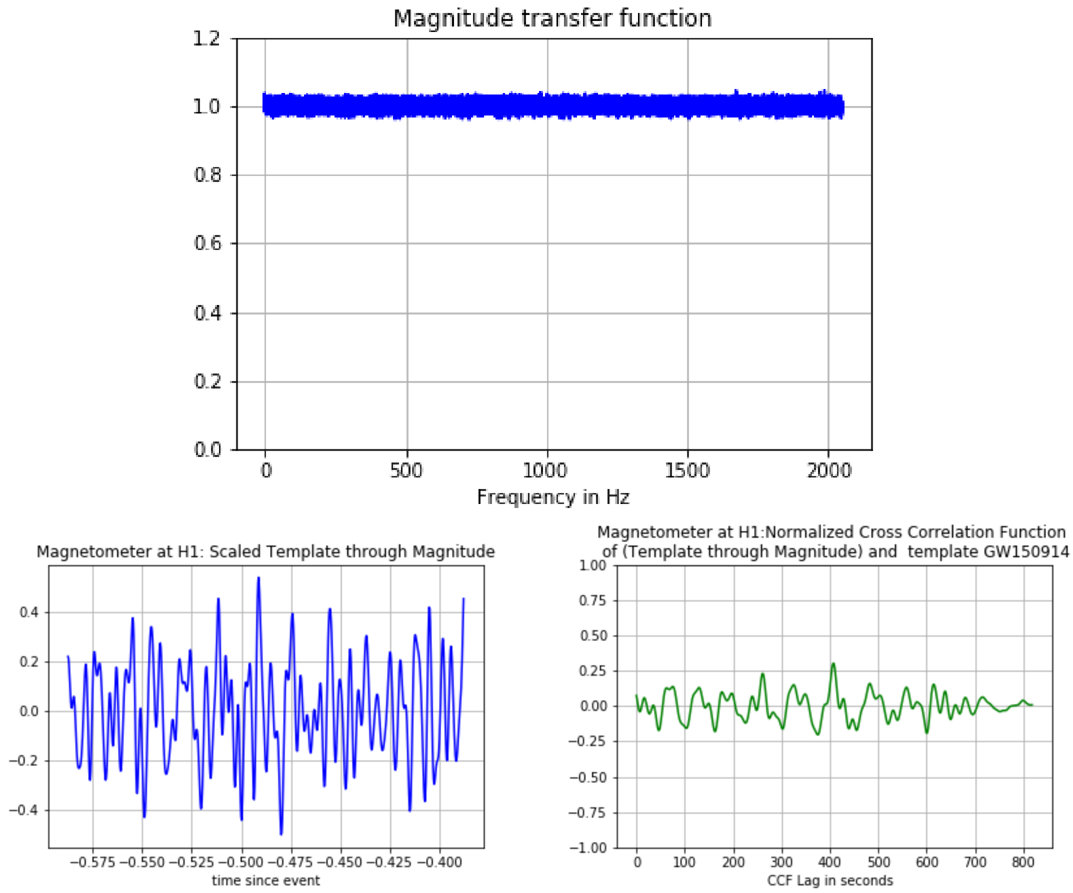

Figure 4. GW150914: Top: System 2: Example Magnitude Frequency response plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: Template passed through System 2. Right: Normalzied CCF of (Template passed through System 2) vs Template.

peaky CCF ($R > \frac{1}{e}$).

We can see that CCF does not show clear peaks in the magnetometer channel, for GW150914 and similar results are observed for GW151226 in Fig.14 and GW170104 in Fig.15. So, we may not see any CCF peaks in the magnetometer channel on the right panel and may mistake this EM interference, as a GW signal. Hence it is suggested that **GW150914** , **GW151226** and **GW170104** be further studied.

5 Implications for GW150914

We know that a strong lightning (EM signal) occurred around the same time as GW150914 observation, in Burkina Faso. We also know that excess power was observed in many magnetometer channels. It is stated that "Fluxgate magnetometers indicate that magnetic disturbances at the LIGO detectors produced by coincident lightning strikes were at least 3 orders of magnitude too small to account for the amplitude of GW150914" [Section 6.1 - 6.3 in Page 24-26 in the LIGO noise characterization paper here].

It has been shown in Section 2, that the GW channel has multiple paths for EM pickup and also that 60*n Hz EM interference is picked up with a **relatively** flat frequency response and that

LIGO's EM coupling calibration plot (Figure 2, Page 13 in [6]) may not be applicable to 60*n Hz EM interference and likely for other external EM signals. Hence the magnetometer excess power "**3 orders of magnitude too small**" argument to rule out Burkina Faso lightning, as a possible candidate for GW150914, is **not** supported.

We have also shown in Section 3 that CCF does not show clear peaks in the magnetometer channel, for GW150914. So, we may not see any CCF peaks in the magnetometer channel and may mistake this EM interference, as a GW signal. The reason for the absence of CCF peaks in the magnetometer channel is due to the difference in the nature of the background noise in the GW channel and the magnetometer channel. In the GW channel, detector noise is non-stationary, non-gaussian and non-white noise and in the magnetometer channel, the background noise is approximately white noise.

6 Implications for GW170814 and GW170817

GW170814 and GW170817 were observed at three detectors H1, L1 and V1. Their templates have not been published as of date. The analysis in above sections are applicable for GW170814 and GW170817 as well and EM interference could explain these GW signals, irrespective of the exact templates used.

The Gamma Ray Burst(GRB) associated with GW170817 was observed with telescopes. It is well known that astrophysical objects such as stars, emit EM signals in a wide frequency range, all the way down to KHz. Our Sun is known to emit EM signals down to 30 KHz (NASA document). Given that GW170817 was observed as a weak signal, whose amplitude does not rise during GW event, if the astrophysical object which emitted the GRB, also emitted EM signal in the frequency range of 0-2048 Hz, magnetometers will not show any signal spike during the GW event. Hence low frequency EM signals from the astrophysical object which emitted the GRB, is a good candidate for GW170817.

Hence it is suggested that **GW170814** and **GW170817** be further studied.

7 Concluding remarks

In Section 2, it is shown that the GW channel has multiple paths for EM pickup, with varying transfer functions and also that 60*n Hz EM interference is picked up with a **relatively** flat frequency response. Section 3 gives the reasons why GW150914, GW151226 and GW170104 should be studied further. Section 5 discusses the implications of this analysis for GW150914. Section 6 gives the reasons why weak signals GW170814 and GW170817 should be further studied.

Reiterating the point made earlier, because LIGO detectors more often likely to pickup EM signals which are buried in the detector noise, it is of paramount importance that we should not classify EM interference as GW events. We need **high** standards to classify an observed time series as a GW signal.

The only way to avoid wrong classification of EM interference as GW signals, is as follows. LIGO detectors have very strong impulsive interference in 60Hz and harmonics and also in 300Hz-2000Hz range. First, it is very important to clean up the impulsive interference and rest of the EM interference, in the detectors. Secondly, we should insist on signals which rise well above the background noise, in time domain, in the GW channel. When we do observe such a strong signal in the GW channel, we can then look at the magnetometer channel in time domain, for any similar strong signal in time domain, which rises well above the background noise and if it does, we must reject it as a candidate for GW signal, given that CCF may not show clear peaks due to possible difference in transfer function between the GW channel and the magnetometer channel.

LIGO team should release magnetometer and Mains Voltage Monitor data (few dozen channels) for all the 6 GW signals, and also publish the templates for GW170608, GW170814 and GW170817, to enable outside researchers to analyze them further.

This manuscript **does not** claim that EMI is the source of the assumed GW events. It points out that EMI **cannot be ruled out** as a candidate for the GW events observed so far. GW signals should be subject to further **independent** studies and LIGO should provide all the information required to facilitate such studies.

Acknowledgments

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[12] Kenneth Y. Jo, Satellite Communications Network Design and Analysis, Page 415.

[13] LOSC LIGO GW150914 Tutorial

[14] LOSC LIGO GW151226 Tutorial matched filter equations are implemented in the section ”matched filtering to find the signal”

[15] LIGO Tutorials

[16] Zip Files of Modified LIGO Tutorials which demonstrate the results in this paper.
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Matched Filter SNR results are demonstrated with 32 second block of data, used in the original LIGO tutorial scripts. Results hold for 512 second block of data also.

8 Appendix A

8.1 60 Hz Harmonic amplitudes

Let us consider the case of 60 Hz EM interference from power lines and represent it as $x_1(t) = A\cos(2\pi f_0 t)$, where $f_0 = 60$ Hz is the fundamental frequency and A is the amplitude. Second harmonic 120 Hz $x_2(t)$ and third harmonic 180 Hz $x_3(t)$ are generated by nonlinearities in the hardware, which can be obtained by taking the square and cube of the fundamental frequency, as follows.

$$
\begin{aligned}
x_1(t) &= A\cos(2\pi f_0 t) \\
x_2(t) = x_1^2(t) = A^2\cos^2(2\pi f_0 t) &= \frac{A^2}{2}[1+\cos(2\pi(2*f_0)t)] \\
x_3(t) = x_1^3(t) = A^3\cos^3(2\pi f_0 t) &= \frac{A^3}{2}\cos(2\pi f_0 t)[1+\cos(2\pi(2*f_0)t)] \\
x_3(t) &= \frac{A^3}{2}[\cos(2\pi f_0 t) + \frac{1}{2}(\cos(2\pi(3*f_0)t) + \cos(2\pi f_0 t))]
\end{aligned}
\tag{8.1}
$$

We can see that the harmonic amplitudes are always lesser than the amplitude of the fundamental frequency.

8.2 EM isolation in GW channel

It is shown that LIGO detector has at least 40 dB more attenuation for 60 Hz EM interference in the GW channel, compared to magnetometer.

The frequency domain plots are in Fig 1 and 2 in Page 10,13 in Noise Characterization paper (Fig.1 and 2)

1. Top panel in Fig.2 shows magnetometer picking up 60 Hz with amplitude 1e-8 Tesla, compared to the background noise of 2.1e-12.

So, $Ratio_{(M)} = 1e-8/2.1e-12 = 4761$. (Blue line is magnetometer plots during normal operation, no test tones, no GW)

2. GW channel detector noise frequency response plot in Fig.1 shows 60 Hz picked up, with amplitude 6.1e-22, compared to the background noise of 2e-23. So, $Ratio_{(GW)} = 6.1e-22/2e-23 = 30.5$.

1 Tesla = 1 Volt $\frac{sec}{meter^2}$ (wikipedia).

$20\log 10(\frac{Ratio_{(M)}}{Ratio_{(GW)}}) = 20 * log10(\frac{4761}{30.5}) = 43.86$ dB.

8.3 GW waveforms

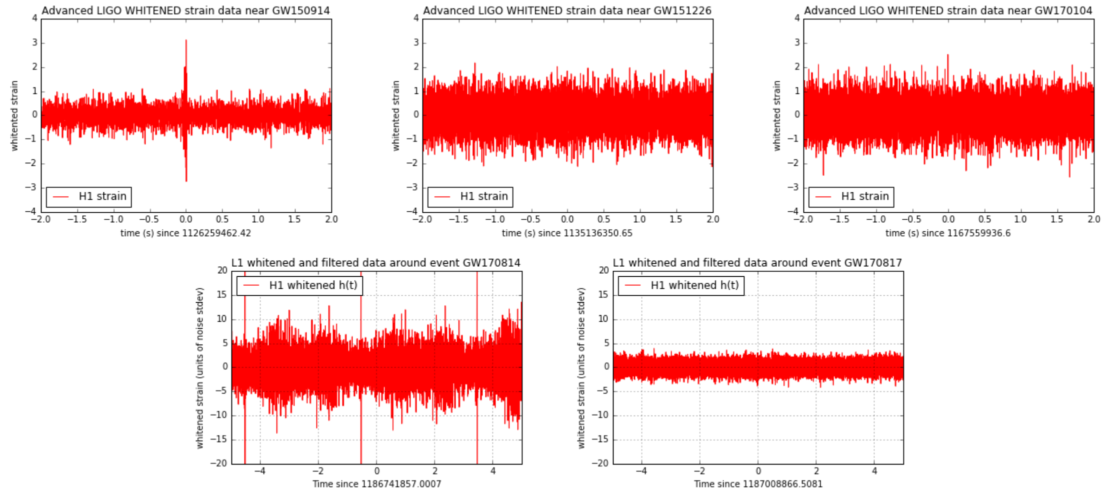

Figure 5. Plots of H1 whitened and filtered strain in GW150914, GW151226, GW170104, GW170814 and GW170817.

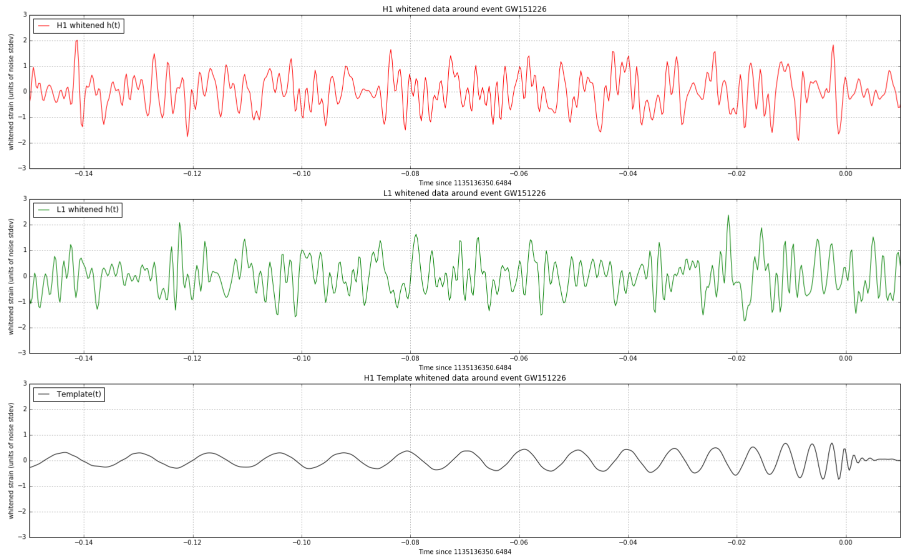

Figure 6. GW151226 whitened and filtered H1 and L1 strain, and the template

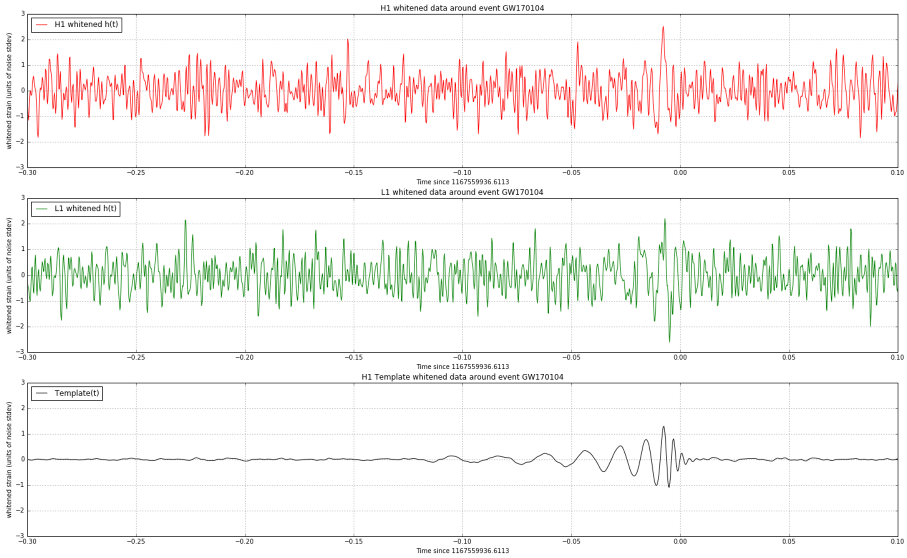

Figure 7. GW170104 whitened and filtered H1 and L1 strain, and the template

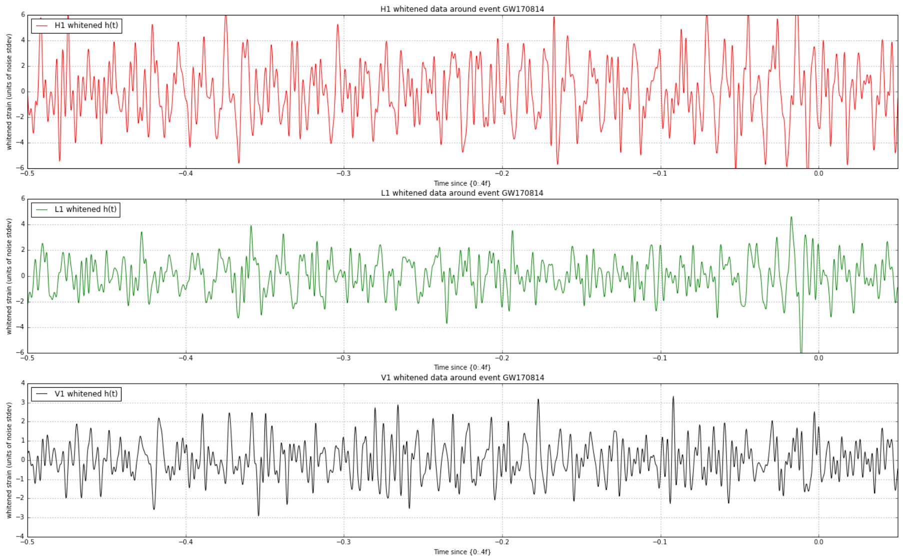

Figure 8. GW170814 whitened and filtered H1, L1 , V1 strain

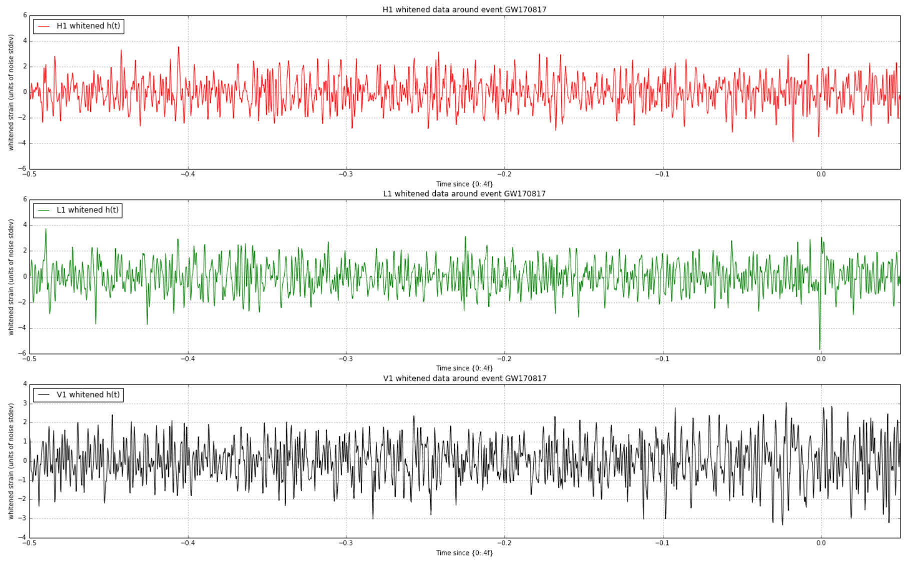

Figure 9. GW170817 whitened and filtered H1, L1 , V1 strain

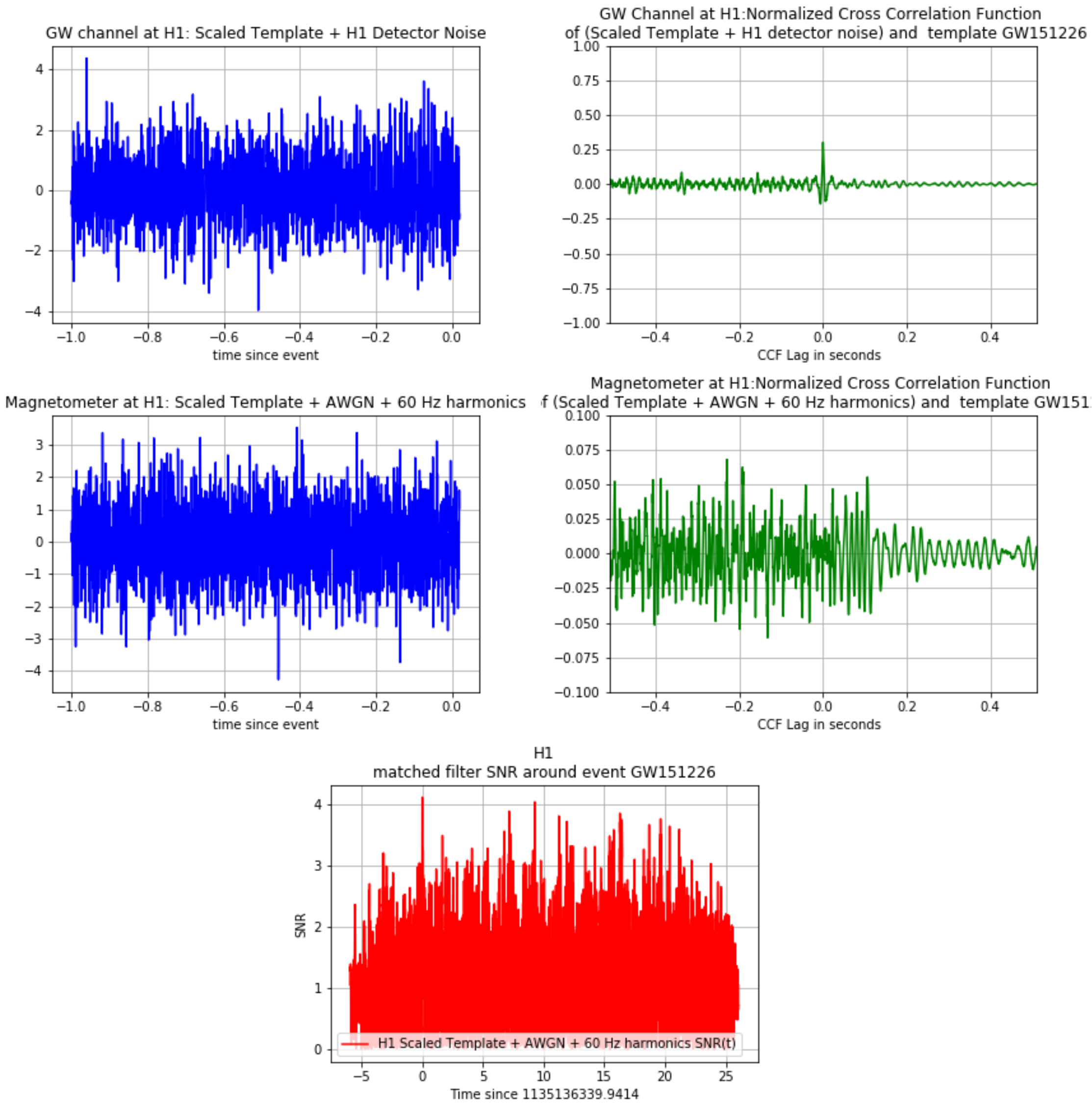

Figure 10. GW151226: Top panels: GW channel, (Template+detector noise) whitened and filtered. Top Left: time domain. Top right: Normalized CCF with template. Middle panels: Magnetometer channel, (Template+ white noise) whitened and filtered. Middle Left: time domain. Middle right: Normalized CCF with template. Lower panel: LIGO matched filter SNR for Magnetometer channel, (Template+ white noise) whitened and filtered.

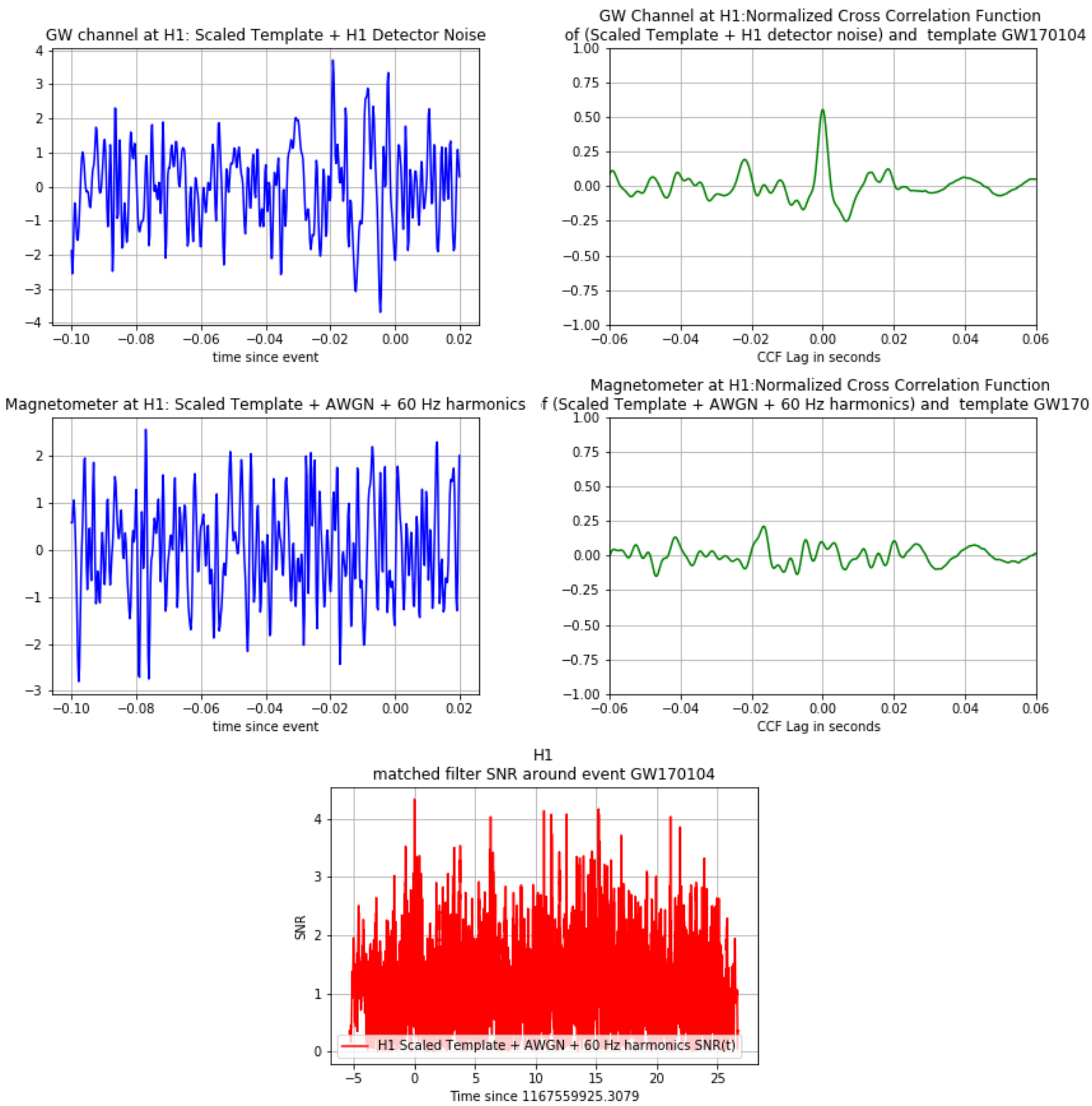

Figure 11. GW170104: Top panels: GW channel, (Template+detector noise) whitened and filtered. Top Left: time domain. Top right: Normalized CCF with template. Middle panels: Magnetometer channel, (Template+ white noise) whitened and filtered. Middle Left: time domain. Middle right: Normalized CCF with template. Lower panel: LIGO matched filter SNR for Magnetometer channel, (Template+ white noise) whitened and filtered.

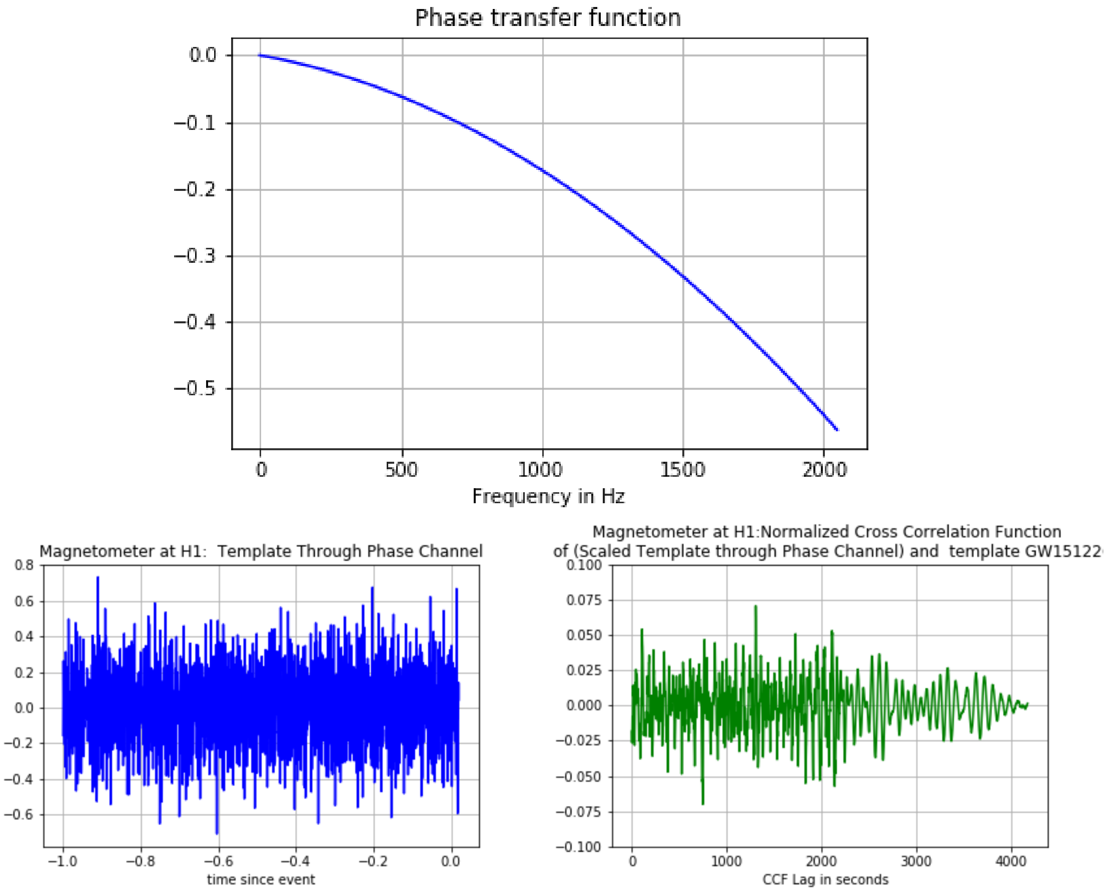

Figure 12. GW151226: Top: System 1: Example Phase frequency response plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: Template passed through System 1. Right: Normalzied CCF of (Template passed through System 1) vs Template.

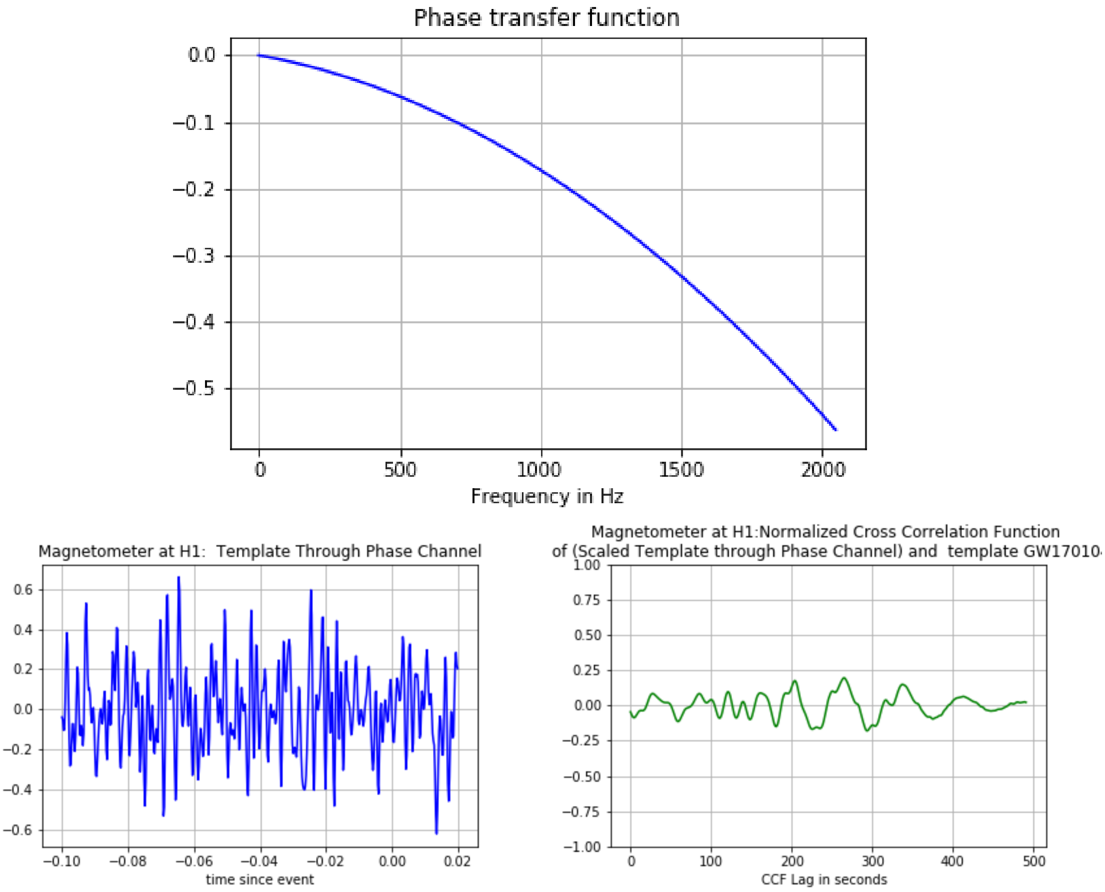

Figure 13. GW170104: Top: System 1: Example Phase frequency response plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: Template passed through System 1. Right: Normalzied CCF of (Template passed through System 1) vs Template.

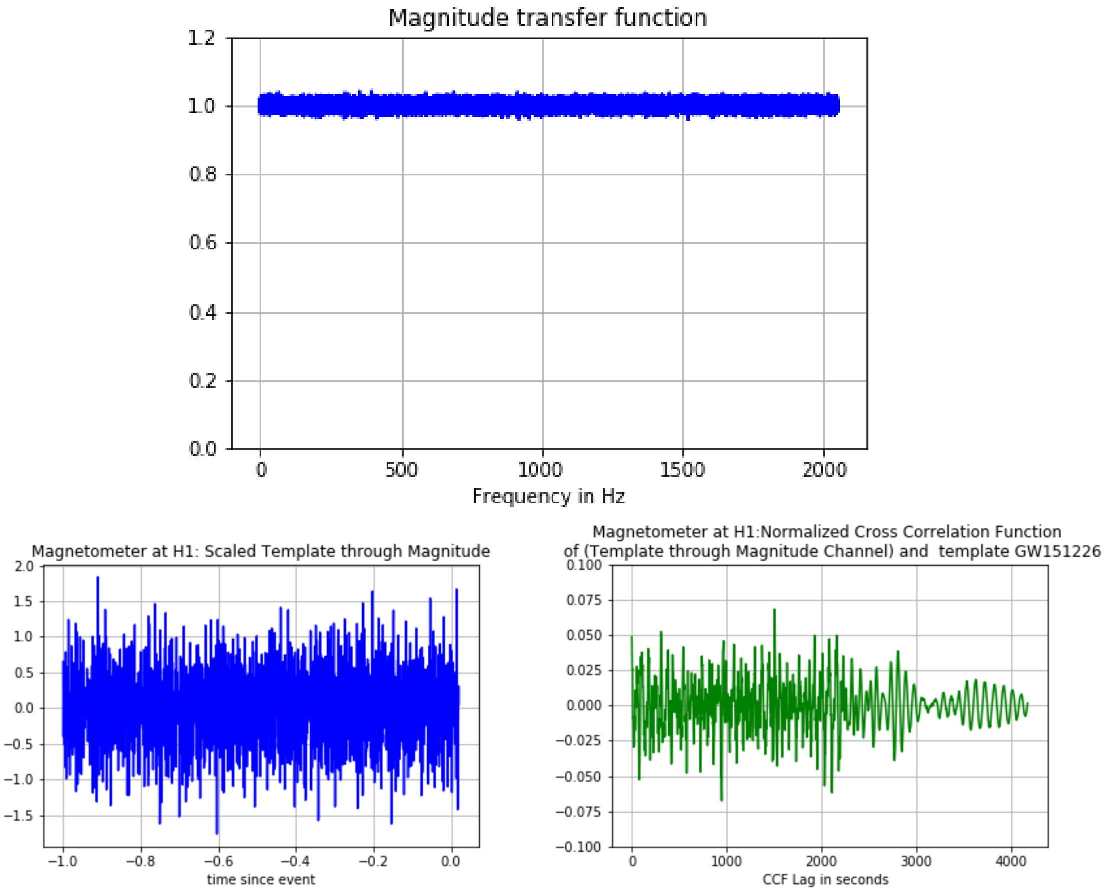

Figure 14. GW151226: Top: System 2: Example Magnitude Frequency response plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: Template passed through System 2. Right: Normalzied CCF of (Template passed through System 1) vs Template.

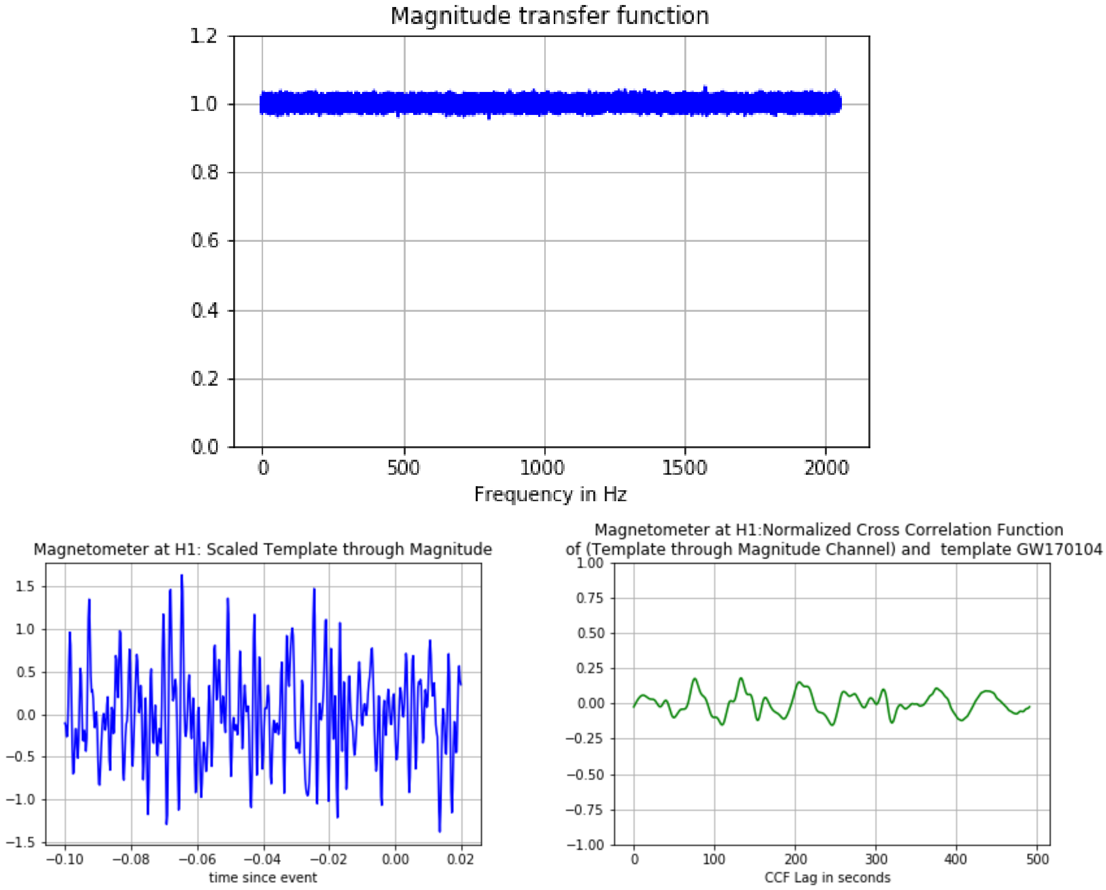

Figure 15. GW170104: Top: System 2: Example Magnitude Frequency response plots of Magnetometer Channel(GW channel assumed perfectly flat. The channels could be interchanged.). Bottom: Left: Template passed through System 2. Right: Normalzied CCF of (Template passed through System 1) vs Template.