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On the Matched Filter Signal to Noise Ratio in LIGO signals

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Abstract. This article analyzes the data for the three gravitational wave (GW) events detected in both its Hanford(H1) and Livingston(L1) detectors by the LIGO[1] collaboration. It is shown that GW151226 and GW170104 are very weak signals whose amplitude does not rise significantly during GW event duration and are indistinguishable from non-stationary detector noise. Using LIGO's Matched Filter to correlate H1 strain data with L1 strain data, it is shown that Matched Filter Signal to Noise Ratio(SNR) does not show expected peaky behaviour, as was observed while correlating H1 strain data with H1 template. It is also shown that LIGO's whitening code does not sufficiently remove strong impulsive interferences present in the strain signals and this may affect the performance of the Matched Filter. A classical implementation of Matched Filter is suggested and it is shown that the ratio of Matched Filter SNR for H1/L1 strains to the Matched Filter SNR for Ideal Template mixed with simulated noise is poor and hence GW151226, GW170104 and GW150914 should be rejected as candidates for GW signals.[2]

[1]The Laser Interferometer Gravitational-Wave Observatory

[2]All the results in this paper are demonstrated by modified version of LIGO's Python scripts.[14]

Contents

1 Introduction

The first GW signal observed was GW150914 [1] which was a relatively strong signal whose amplitude rose significantly, well over detector noise level, during the 0.2 second GW event duration, after whitening. In comparison, the second signal[2] GW151226[Fig. 2] and the third signal[3] GW170104[Fig. 3] were very weak signals, which look like noise after whitening and filtering[1] and whose signal amplitude does not rise above the detector noise level, during GW event of duration 1 second and 0.12 seconds respectively.(Fig. 1)

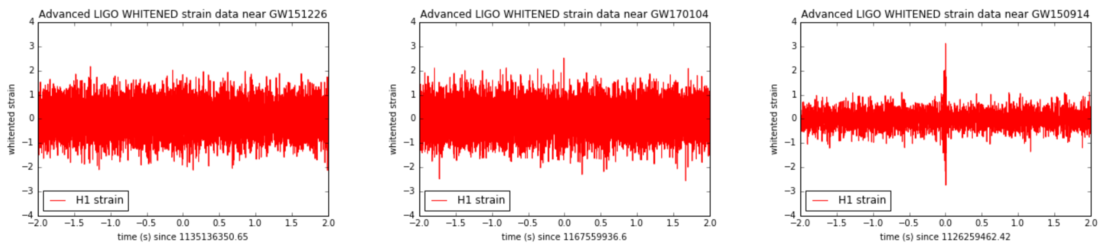

Figure 1. Plots of H1 whitened and filtered strain in GW151226, GW170104 and GW150914

This raises an interesting question if GW151226 and GW170104 could have been caused by non-stationary detector noise. Creswell et al.[5] have reported correlations in the detector noise which, at the time of the event, happen to be maximized for the same time lag as that found for

[1]Fourth order Butterworth Bandpass Filters used in frequency range 20-300Hz for GW150914, 43-800Hz for GW170104 and GW151226.

the GW event itself.

Because we are more often likely to observe weak signals which look like noise and whose amplitude does not rise during assumed GW event, it is of paramount importance that we should not classify Noise as GW events. We need **high** standards to classify an observed time series as a GW signal.

The organization of this paper is as follows. In Section 2 we consider the effect of non-stationarity of detector noise on the false alarm rate computation and argue that brief bursts in detector noise coincident at both detectors could cause high SNR at the Matched Filter output, despite whitened signal whose amplitude does not rise significantly over detector noise level in time domain, and could be mistaken for GW events, particularly for GW151226 and GW170104.

In Section 3, we examine the theory and implementation of LIGO Matched Filter which is the **core engine** that drives two independent analyses PYCBC and GSTLAL. It will be shown that Matched Filter SNR is far smaller than what we would expect if H1 and L1 strains were replaced by Template added with Additive White Gaussian Noise(AWGN) of comparable background noise power, for all 3 signals GW150914, GW151226 and GW170104, which is a **surprising** result. It will be also shown that Matched Filter SNR is very poor when correlating H1 and L1 whitened strain signals and hence all the 3 signals should be rejected.

In Section 4, Matched Filter is implemented using classical method mentioned in textbooks and it is shown that the results in Section 3 are corroborated. In Section 5, it is shown that H1 and L1 strains still show significant impulsive interferences even after LIGO's whitening procedure. In Section 6, the reasons for rejection of GW151226 and GW170104 is presented. In Section 7, a case for rejection of GW150914 is presented, despite its relatively strong signal amplitude during GW event window.

2 Non-stationary Detector Noise and its effect on False Alarm Rate

LIGO Detector Noise is non-stationary and non-Gaussian and has no analytical model and the background noise events which cross a specified SNR threshold, at the output of the matched filter, is empirically determined [4]. Creswell et al.[5] have pointed out that sources of non-stationary and non-Gaussian detector noise need to be identified and eliminated to ensure reliable GW detection and that analysis methods for stationary noise cannot be used for non-stationary noise in LIGO detectors.

It should be noted that, when LIGO papers claim high SNR for the 3 GW events [1–3], they mean SNR observed at the output of the Matched Filter. They **do not** necessarily mean high signal amplitude during GW event, which is significantly higher than background noise amplitude. Let us define Signal Power Ratio(**SPR**) which is defined as the Ratio of Signal power during GW event to the Ratio of Signal power outside the GW event(background detector noise). We can see in Fig. 1 that SPR is close to unity for weak signals GW151226(SPR=1.29) and GW170104(SPR=1.08) (Class A signals) while GW150914 has relatively higher SPR=4.122 (Class B signals).

LIGO's search software uses 2 independent analyses PYCBC and GSTLAL and False Alarm Computation in Eq.8 and Eq.12 in [4] experimentally measure the number of coincident events above a certain threshold. For example, in the PYCBC analysis, the False Alarm Rate is given by

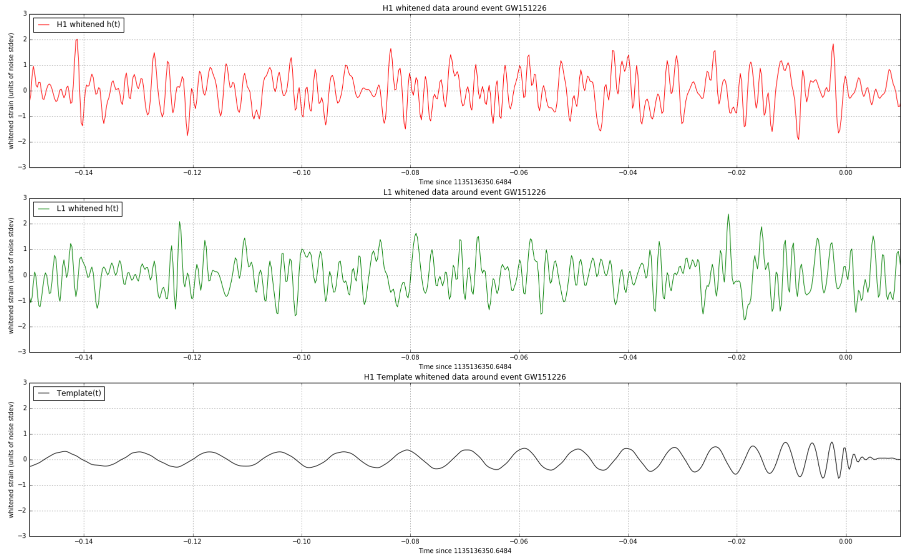

Figure 2. GW151226 whitened and filtered H1 and L1 strain, and Template

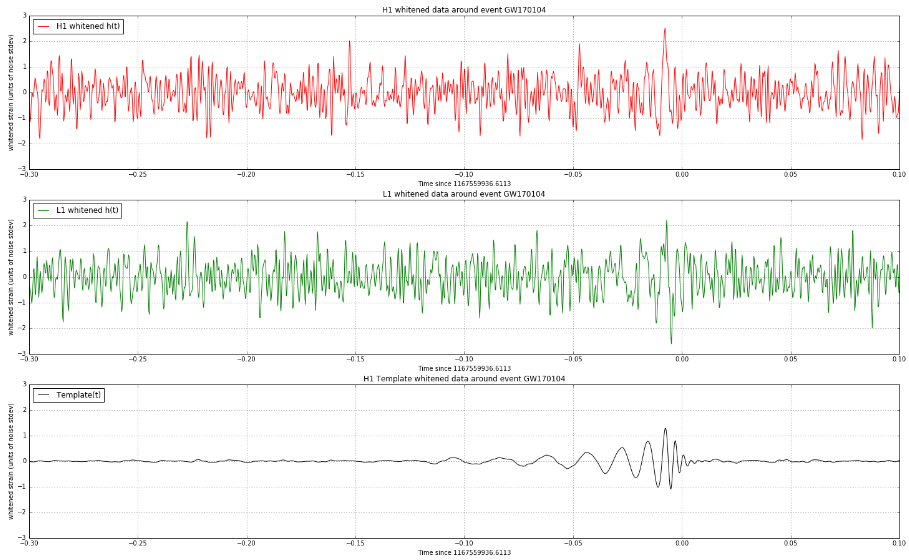

Figure 3. GW170104 whitened and filtered H1 and L1 strain, and Template

$F(\hat{\rho_c}) = 1 - e^{\frac{-T(1+n_b(\hat{\rho_c}))}{T_b}}$ where T is the observation time of the search and T_b is the background time and $n_b(\hat{\rho_c})$ is the number of background noise events that cross candidate event's re-weighted SNR threshold.

We may not observe high coincident SNR during 8 days of observation, but during next several months, by **Definition** of Non-Stationarity, its Mean and Variance can change unpredictably and simultaneously in both sites, causing high coincident SNR at the output of the matched filter, thus making the estimate of False Alarm Rate not applicable.

Hence, 5.1σ significance associated with False Alarm Rate computation may be true for Stationary White Gaussian Noise, but **not** for LIGO's non-stationary detector noise, which by definition, can change suddenly.

When high SNR is observed at the Matched Filter during GW event, while signal amplitude does not rise significantly, as is the case for GW151226 and GW170104, it may be due to non-stationary detector noise having high SNR at the output of the matched filter briefly at both sites as well.

For this reason alone[**Reason 1**], **independent** of other reasons, GW151226 and GW170104 should be rejected.

3 LIGO Matched Filter Implementation

The **core engine** of the LIGO software for identification of GW signals is Matched Filter as described in Eq.1-4 in [4], which is used in 2 independent search methods PYCBC and GSTLAL analysis[2]. For the case of PYCBC analysis, Matched Filter SNR(**MF-SNR**) $\rho^2(t)$ is given as follows.

$$\begin{aligned} \rho^2(t) &= \frac{1}{|\langle h|h\rangle|}|\langle s|h\rangle(t)|^2 \\ \langle s|h\rangle(t) &= 4\int_0^\infty \frac{\widehat{s}(f)\widehat{h}^*(f)}{S_n(f)}e^{i2\pi ft}df \end{aligned} \tag{3.1}$$

where $s(t), h(t)$ are the strain signal and template respectively and $\widehat{s}(f)$ and $\widehat{h}^*(f)$ are the Fourier Transforms of $s(t), h(-t)$ respectively and $S_n(f)$ is the power spectral density of the detector noise.

In time domain, this is equivalent to the convolution of whitened version of $s(t)$ with $h(-t)$, which is equivalent to **Cross-Correlation Function(CCF)** of the whitened strain signal $s(t)$ and the template $h(t)$, with a normalization scale factor $\langle h|h\rangle = 4\int_0^\infty \frac{\widehat{h}(f)\widehat{h}^*(f)}{S_n(f)}df$.

It is noted that the above CCF is **not** normalized to give MF-SNR =1 for an ideal template correlated with itself. What is needed is a Reference System with strain signal replaced by $s^{'}(t) = h(t) + w(t)$, and is correlated with an ideal template $h(t)$ in the above Matched Filter, where $w(t)$ is Additive White Gaussian Noise(AWGN) simulated in software whose power is comparable to background noise. Then we can compare how the MF-SNR of the practical system compares with this Reference system.

[2] GSTLAL analysis also uses Matched Filter search, and as per Page 7 in [4], "the data s(t) and templates h(t) are each whitened in the frequency domain by dividing them by an estimate of the power spectral density of the detector noise." and also "By the convolution theorem, $\rho(t)$ obtained in this manner is the same as the $\rho(t)$ obtained by frequency domain filtering in Eq. (1)."[in PYCBC analysis]. These equations are implemented in lines 662-740 in Matched Filter section of LIGO's tutorial python script. [12].

3.1 Test 3a: LIGO Matched Filter SNR by comparison with Reference System

Fig. 4 shows the LIGO Matched Filter SNR of the Reference system [$s^{'}(t)$ Vs $h(t)$] in the top left plot for GW150914. The top second plot correlates 2 noisy signals, $s(t)$ and $h(t)$ for GW150914 where $s(t)$ is a combination of H1 Strain outside GW event and Template, and $h(t)$ is the Template, and computes Matched Filter SNR as per LIGO's implementation.

We can see that Matched Filter SNR shows strong peaks in both these plots. This means if we correlate an ideal template with a noisy template(either template+AWGN or template+detector noise), we should expect to see very strong peak in Matched Filter SNR.

The 3 plots in Column 1 show the LIGO Matched Filter SNR of the Reference system for GW150914, GW151226 and GW170104. The plots in the 2 rightmost columns show the correlation of H1 with Template and L1 with template respectively. We can see that the Matched Filter SNR of the 3 GW signals, GW150914, GW151226 and GW170104, are lower than that of the Reference system[$s^{'}(t)$ Vs $h(t)$] by a factor of 5 at least.

Table 1 summarizes the Matched Filter SNR and SNR Ratios($\frac{SNR_{H1}}{SNR_{ref}}$, $\frac{SNR_{L1}}{SNR_{ref}}$) computed for the 3 GW signals. Given the high standards required for declaration of GW signal, signals with SNR Ratios < 0.5 should be rejected.

For this reason alone[**Reason 2**], **independent** of other reasons, GW150914, GW151226 and GW170104 should be rejected, because we need high standards to classify strain signals as GW signals.

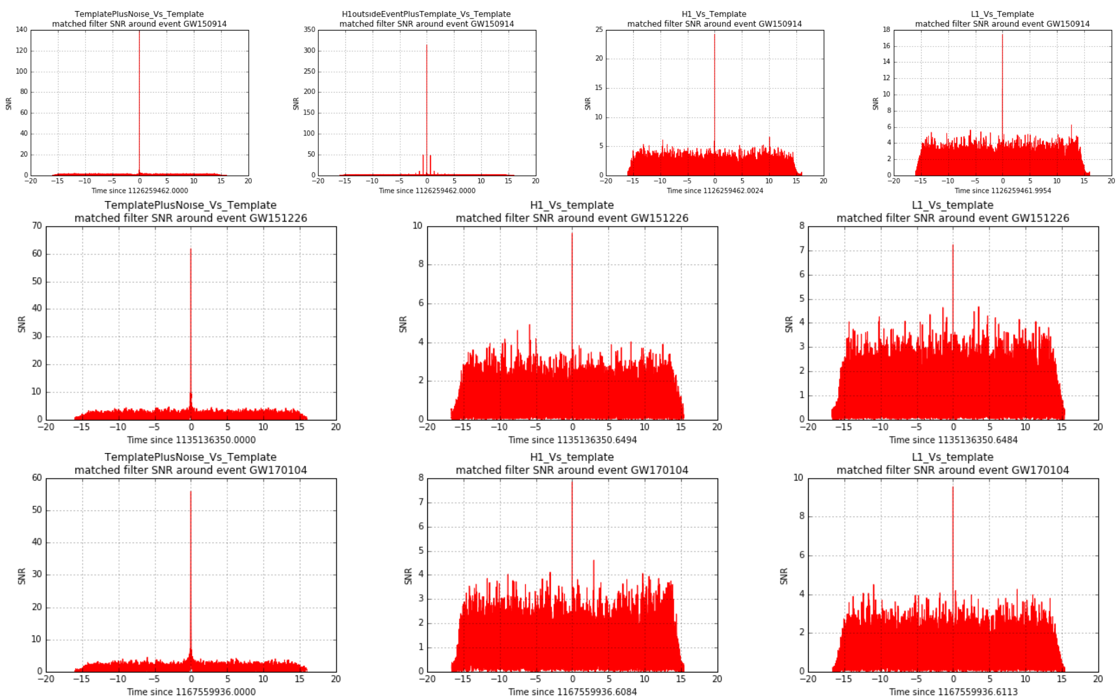

Figure 4. Plots of LIGO Matched Filter SNR for GW150914, GW151226 and GW170104

3.2 Test 3b: LIGO Matched Filter SNR by Correlating H1 with L1

The next step is to cross-correlate the strain signal H1 with L1 and test whether Matched Filter SNR shows the peaky behaviour observed in Fig. 4. Given that the same GW signal is expected to be received in both sites, these signals should give a high SNR at the matched filter output when correlated with each other. This is a **crucial** test which must be performed.

We **can** cross-correlate two noisy signals and expect a peaky CCF, if the signals are correlated. In fact, wireless communication with sensors routinely use cross-correlation of two noisy signals [8].

The Matched Filter Implementation in textbooks [9] is slightly different from LIGO's implementation mentioned above. It is described in Section 4 and it correlates a noisy signal $s(t) = h(t) + w_s(t)$ with a known signal(template) $h(t)$ where $w_s(t)$ is the detector noise.

We wish to correlate two noisy signals $s(t) = h(t) + w_s(t)$[H1] and $h^{'}(t) = h(t) + w_h(t)$[L1] where $w_h(t)$ is the detector noise. This is equivalent to correlating $s(t) = h^{'}(t) + w_s(t) - w_h(t) = h^{'}(t) + w(t)$ and $h^{'}(t)$. Matched Filter theory imposes no constraints on the characteristics of the template.

Fig. 5 shows a Reference System in the top left plot, which correlates 2 noisy signals, $s(t)$ and $h^{'}(t)$ where $s(t)$ is a combination of Template and AWGN, and $h^{'}(t)$ is a combination of Template and independent AWGN, and computes Matched Filter SNR as per LIGO's implementation. We can see that Matched Filter SNR shows strong Peaky behaviour.

The 3 rightmost plots in the top row show results for correlating H1 with L1 in LIGO matched Filter for the 3 GW signals, GW150914, GW151226 and GW170104. We can see clearly that the 3 GW signals **do not** show Peaky Correlation.

Let us consider a Normalized Cross Correlation Function(CCF) $CCF(t) = \int_{-T}^{T} s(\tau)h(t - \tau)d\tau$, which normalizes both $s(t)$ and $h(t)$ over the time window [-T,T] during which GW signal was observed, such that CCF of each signal with itself gives a result of unity for zero lag. The bottom 3 plots in Fig. 5 show that LIGO's Matched Filter SNR results in the top plots are **corroborated** for GW151226 and GW170104, while GW150914 shows relatively peaky CCF in the bottom plot. The lack of Peaky CCF for GW150914 in the top row plots is a surprising result and will be explored in Section 7

For this reason alone[**Reason 3**], **independent** of other reasons, GW151226 and GW170104 should be rejected.

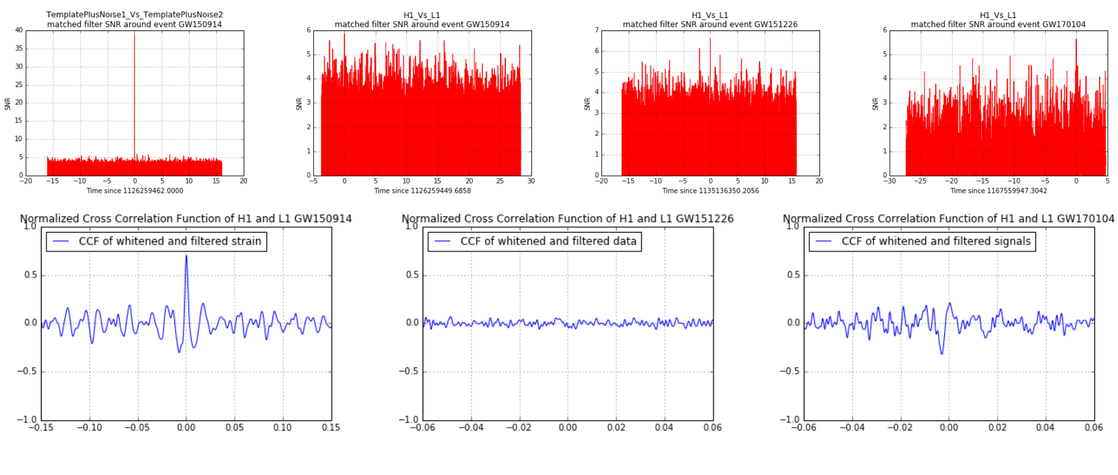

Figure 5. Plots of H1 vs L1 LIGO Matched Filter SNR for GW150914, GW151226 and GW170104

GW signal	SNR_{ref} (AWGN+Template) Vs Template	SNR_{H1} H1 Vs Template	SNR_{L1} L1 Vs Template	$RatioSNR_{H1}$ $\frac{SNR_{H1}}{SNR_{ref}}$	$RatioSNR_{L1}$ $\frac{SNR_{L1}}{SNR_{ref}}$
GW150914	138.47	24.23	17.43	0.175	0.126
GW151226	61	9.63	7.22	0.1578	0.1183
GW170104	56	7.83	9.54	0.1398	0.1703

Table 1. LIGO's Matched Filter SNR for GW150914, GW151226 and GW170104.

4 Classical Matched Filter Implementation

The Classical Matched Filter Implementation [9] is slightly different from LIGO's implementation of matched filter described in Section 3. Classical Matched Filter SNR is given by $SNR_{out} = \frac{|\phi_0(T)|^2}{E[n^2(t)]}$ where $n(t) = \int_{-\infty}^{\infty} w(\tau)h(t-\tau)d\tau$ and $\phi_0(t) = \int_{-\infty}^{\infty} \phi(\tau)h(t-\tau)d\tau$ and $\phi(t)$ is the known signal and $w(t)$ is the receiver noise and received signal $x(t) = \phi(t) + w(t)$ in the time window [0,T]. Matched filter impulse response $h(t)$ is matched to the known signal $\phi(t)$ and is given by $h(t) = \phi(T-t)$.

Let us consider discrete time implementation[10] where $x[n] = s[n] + w[n]$ is the strain signal H1/L1, $s[n]$ is the desired Template Reference signal and $w[n]$ is the detector noise. $h[n]$ is the matched filter impulse response matched to Template signal. For a symmetric template, we can shift the time window of $x[n]$ to [-T,T] and so we can choose $h[n] = s[n]$. Matched Filter SNR SNR_{out} is given by

$$\begin{aligned} x[n] &= s[n] + w[n] \\ &h[n] = s[n]; v[n] = w[n] = x[n] - s[n] \\ y_s &= \sum_{n=1}^{N} h[n] * s[n] \\ y_v &= \sum_{n=1}^{N} h[n] * v[n]; SNR_{out} = \frac{|y_s|^2}{|y_v|^2} \end{aligned} \tag{4.1}$$

In this section, we use whitened and filtered strain signals H1 and L1, within the time window in which GW signals were observed. GW150914 is observed in a 0.3 second window, GW151226 in a 1 second window and GW170104 in a 0.12 second window.

4.1 Test 4a: Classical Matched Filter SNR by comparison with Reference System

Table 2 summarizes the Classical Matched Filter SNR and SNR Ratios($\frac{SNR_{H1}}{SNR_{ref}}$, $\frac{SNR_{L1}}{SNR_{ref}}$) computed for the 3 GW signals, as per textbook definition. When Additive White Gaussian Noise(AWGN) added to a Template is correlated with Template, it is run over several iterations and Average SNR for the iterations is considered in second column in Table 2.

We can see that the SNR Ratio for L1 $\frac{SNR_{L1}}{SNR_{ref}}$ is < 0.5 for all 3 GW signals. Given the high standards required for declaration of GW signal, signals with SNR Ratios < 0.5 should be rejected.

For this reason alone[**Reason 4**], **independent** of other reasons, GW150914, GW151226 and GW170104 should be rejected.

GW signal	SNR_{ref} AWGN+Template Vs Template	SNR_{H1} H1 Vs Template	SNR_{L1} L1 Vs Template	$SNR_{H1,L1}$ H1 Vs L1	RatioSNR-H1 $\frac{SNR_{H1}}{SNR_{ref}}$	RatioSNR-L1 $\frac{SNR_{L1}}{SNR_{ref}}$
GW150914	5040	108	11.5	14.85	0.021	0.0022
GW151226	331	223190	15.47	0.98	674	0.0467
GW170104	168	300	6.18	1.78	1.78	0.0367

Table 2. Classical Matched Filter SNR for GW150914, GW151226 and GW170104. Average SNR computed for AWGN.

4.2 Test 4b: Classical Matched Filter SNR by Correlating H1 with L1

Table 2 shows that Classical Matched Filter SNR by correlating H1 with L1 ($SNR_{H1,L1}$) is comparatively poorer for GW151226 and GW170104. This may be due to the fact that both detectors have high detector noise and when cross-correlating H1 and L1 strain signals, the SNR is very poor. The only solution to improve this scenario is to clean up the impulsive interferences in the detectors.

5 Problems in LIGO Whitening Code

LIGO detectors have very strong impulsive interferences in 60Hz and harmonics and also in 300Hz-2000Hz range. Creswell et al.[5] have pointed out that whitening section of LIGO's Matched filter does not remove the narrow resonances in the strain signals well enough.

Fig. 6 shows the Amplitude Spectral Density(ASD) for 512 seconds of H1 strain data, H1 strain whitened outside matched filter and H1 strain whitened inside matched filter. We can see that LIGO's whitening code does not remove impulsive interferences well enough.

It is important to clean up the impulsive interferences in the detectors, so when we compute Matched Filter SNR by correlating H1 and L1 signals, it can give high values for valid GW signals.

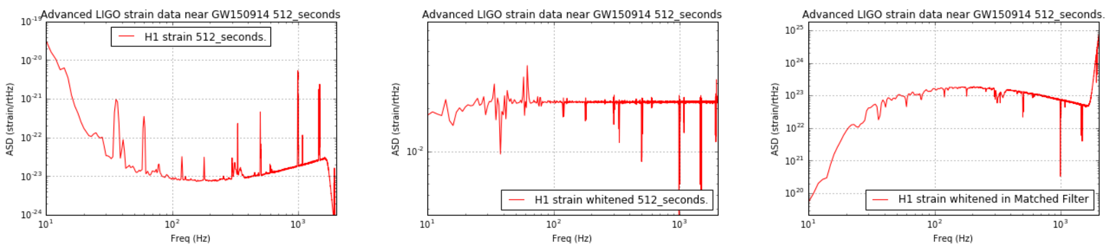

Figure 6. Amplitude Spectral Density(ASD) Plots of H1 strain in GW150914

6 Case for Rejection of GW151226 and GW170104

GW151226 and GW170104 belong to a class of signals with Signal Power Ratio(SPR) around unity. We need to have much higher standards for classifying such signals as GW signals, than signals like GW150914 which have $SPR > 4$.

For all the 3 signals, we need to make sure that Ratio $R_1 = \frac{SNR_c}{SNR_{ref}} > 0.5$ where SNR_c is the Matched Filter SNR of candidate signal and SNR_{ref} is the Matched Filter SNR of a reference signal which is a combination of template and AWGN of comparable detector noise power.

Given the high standards required for declaration of GW signal, GW151226 and GW170104 should be rejected due to each of the following Reason 1, 2, 3 and Reason 4 **independently**.

Reason 1: Being weak signals, non-stationary and non-Gaussian detector noise at the two sites could have caused the weak GW signal. When high SNR is observed at the Matched Filter during GW event, while signal amplitude does not rise significantly, as is the case for GW151226 and GW170104, it may be due to non-stationary detector noise having high SNR at the output of the matched filter briefly at both sites as well.

Reason 2: In Section 3, we observed in Table 1 that the LIGO Matched Filter SNR Ratios for H1 and L1 were < 0.5 for GW151226 and GW170104.

Reason 3: In Section 3, we observed in Fig. 5 that the LIGO Matched Filter SNR and Normalized CCF were very poor when correlating H1 and L1, for GW151226 and GW170104.

Reason 4: In Section 4, we observed in Table 2 that the Classical Matched Filter SNR Ratios for H1 and L1 were < 0.5 for GW151226 and GW170104.

7 Case for Rejection of GW150914

In Section 3, we observed a contradiction. If we consider the bottom left plot of Normalized Cross Correlation Function(CCF) in Fig. 5, GW150914 shows relatively peaky CCF. This agrees with visual correlations observed in the relatively strong signal GW150914. But the second plot in the top row in LIGO's Matched Filter plots in Fig. 5 shows that GW150914 does not show Peaky Correlation.

It is possible that LIGO's Matched Filter SNR correlation over a 32 second window is too long to bury the correlations in the presence of detector noise at both sites. Normalized CCF in the bottom plots of Fig. 5 is performed over 0.3 second window over which GW150914 was observed and hence the short window may have captured the correlation between H1 and L1 in GW150914.

Fig. 7 compares visual correlations of H1 and L1 strains with Classical Matched Filter SNR computed using textbook definition over the 3 windows and they agree roughly. We can see that the correlations look good only in a short 0.06 second window in the first plot. Correlations do not look good in the second and third plots in the rest of $0.3 - 0.06 = 0.24$ seconds.

GW150914 should be rejected due to Reason 2 and Reason 4.

Reason 2: In Section 3, we observed in Table 1 that the LIGO Matched Filter SNR Ratios for H1 and L1 were < 0.5 for GW150914.

Reason 4: In Section 4, we observed in Table 2 that the Classical Matched Filter SNR Ratios for H1 and L1 were < 0.5 for GW150914, Given the high standards required for declaration of GW signal, GW150914 should be rejected.

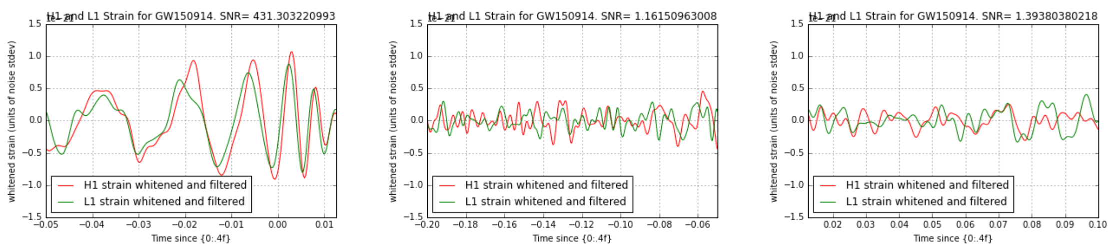

Figure 7. Plots of H1 vs L1 whitened and filtered strain in GW150914

8 Concluding remarks

Section 7 gives the reasons why GW150914 should be rejected. Section 6 gives the reasons why weak signals GW151226 and GW170104 should be rejected.

Reiterating the point made earlier, because we are more often likely to observe weak signals which look like noise and whose amplitude does not rise during assumed GW event, it is of paramount importance that we should not classify Noise as GW events. We need **high** standards to classify an observed time series as a GW signal.

LIGO detectors have very strong impulsive interferences in 60Hz and harmonics and also in 300Hz-2000Hz range. This may affect the performance of the matched filter. It is very important to clean up the impulsive interferences in the detectors.

Acknowledgments

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[12] LOSC LIGO GW151226 Tutorial Matched Filter equations are implemented in the section ”Matched filtering to find the signal”

[13] LIGO Tutorials

[14] Zip Files of Modified LIGO Tutorials which demonstrate the results in this paper