On the matched filter Signal to Noise Ratio in LIGO signals

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Abstract.   This article analyzes the data for the three gravitational wave (GW) events detected in both Hanford(H1) and Livingston(L1) detectors by the LIGO[1] collaboration. It is shown that GW151226 and GW170104 are very weak signals whose amplitude does not rise significantly during the GW event, and they are indistinguishable from non-stationary detector noise. Using LIGO's matched filter to correlate H1 strain data with L1 strain data, it is shown that the matched filter Signal to Noise Ratio(SNR) does not show expected peaky behaviour for GW151226, GW170104 and GW150914, as observed when correlating H1 strain data with the H1 template. It is also shown that LIGO's whitening code does not sufficiently remove strong impulsive interference present in the strain signals and that this may affect the performance of the matched filter. Normalized Cross Correlation Function(CCF) method is implemented, and it is shown that the normalized CCF is poor for GW151226 and GW170104, when correlating H1 and L1. Hence it is suggested that GW151226 and GW170104 be questioned as candidates for GW signals. It is shown that LIGO's matched filter misfires with high SNR, even when correlating a sine wave with the template. The implication of these results for GW150914 is also examined. All the results in this paper are demonstrated using modified versions of LIGO's Python scripts[13].

[1]The Laser Interferometer Gravitational-Wave Observatory

Contents

1   Introduction

The first GW signal observed was GW150914 [1] which was a relatively strong signal whose amplitude, after whitening and filtering[1], rose significantly, well over detector noise level, during the 0.2 second GW event duration. In comparison, the second signal[2] GW151226[Fig. 2] and the third signal[3] GW170104[Fig. 3] were very weak signals, which look like noise after whitening and filtering and whose signal amplitude does not rise above the detector noise level during the GW event of duration 1 and 0.12 seconds, respectively.(Fig. 1)
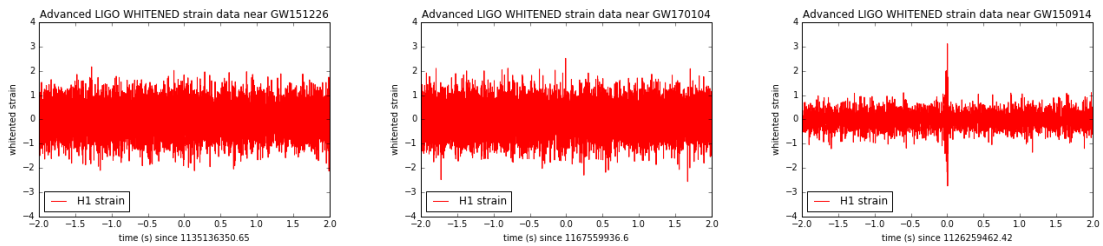
Figure 1. Plots of H1 whitened and filtered strain in GW151226, GW170104 and GW150914

     This raises the important question of whether GW151226 and GW170104 could have been caused by non-stationary detector noise. Creswell et al.[5] have reported correlations in the detector noise which, at the time of the event, happen to be maximized for the same time lag as that found for the GW event itself.

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[1]Fourth order Butterworth bandpass filters used in frequency range 20-300Hz for GW150914, 43-800Hz for GW170104 and GW151226.
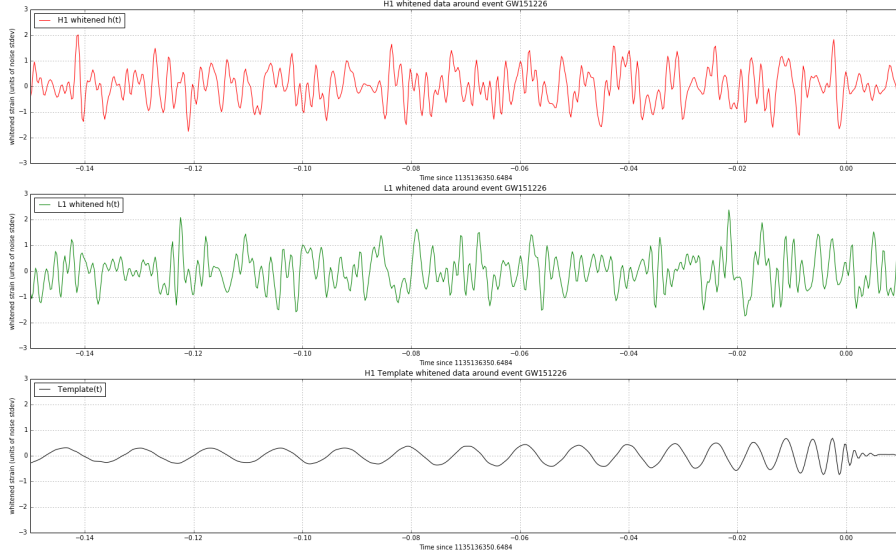
Figure 2. GW151226 whitened and filtered H1 and L1 strain, and the template

Because we are more often likely to observe weak signals which look like noise and whose amplitude does not rise during assumed GW event, it is of paramount importance that we should not classify noise as GW events. We must insist on high standards before classifying an observed time series as a GW signal.

The organization of this paper is as follows. In Section 2 we consider the effect of non-stationarity of detector noise on the false alarm rate computation and argue that brief bursts in detector noise coincident at both detectors could produce a high SNR in the matched filter, even though the amplitude of the whitened signal does not rise significantly over detector noise level in the time domain. Such bursts could be mistaken for GW events, particularly for GW151226 and GW170104.

In Section 3, we examine the theory and implementation of LIGO's matched filter which is the **core engine** driving the two independent analyses, PYCBC and GSTLAL. It will be shown that the matched filter SNR is very poor when correlating H1 and L1 whitened strain signals for GW151226 and GW170104 and also for GW150914. Using Normalized Cross Correlation Function(CCF) method, it will be shown that the normalized CCF is very poor, when correlating whitened and filtered H1 and L1 strain signals for GW151226 and GW170104.

In Section 4, LIGO's matched filter SNR performance is tested with a sine wave added to the detector noise and it will be shown that it misfires with high SNR, when correlated with the template. In Section 5, it is shown that H1 and L1 strains still show significant impulsive interference even after following LIGO's whitening procedure. In Section 6, the case is presented for questioning GW151226 and GW170104 as candidates for GW signals. In Section 7, a case for questioning GW150914 is presented, despite its relatively strong signal amplitude during the GW event window.
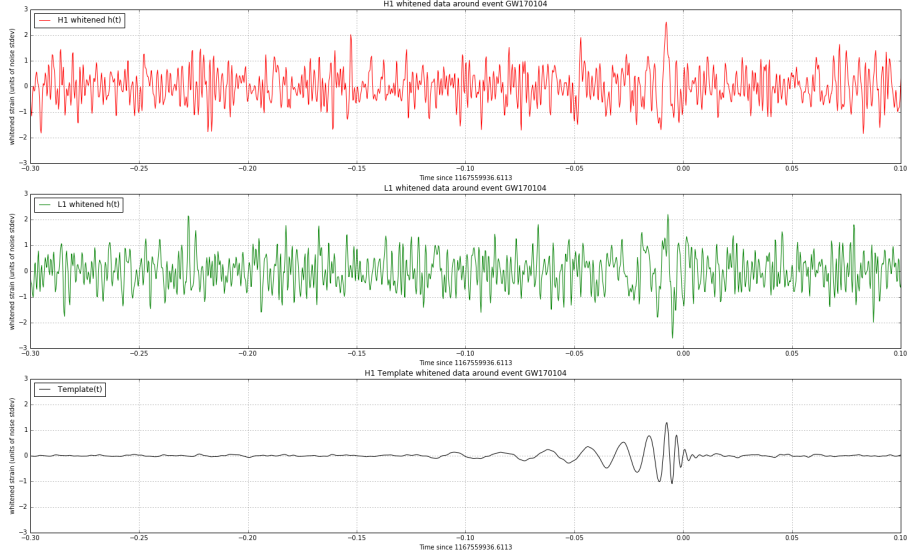
Figure 3. GW170104 whitened and filtered H1 and L1 strain, and the template

2 Non-stationary Detector Noise and its effect on False Alarm Rate

LIGO detector noise is non-stationary and non-Gaussian and has no analytical model. The number of background noise events which cross a specified SNR threshold, at the output of the matched filter, is determined empirically [4]. Creswell et al.[5] have pointed out that the sources of non-stationary and non-Gaussian detector noise need to be identified and eliminated to ensure reliable GW detection and that analysis methods for stationary noise cannot be used for non-stationary noise in LIGO detectors.

It should be noted that, when LIGO papers claim high SNR for the three GW events [1–3], they mean SNR observed at the output of the matched filter. This does not necessarily mean that the signal amplitude during the GW event is significantly higher than the background noise amplitude. Let us define Signal Power Ratio (**SPR**) which is defined as the ratio of signal power during the GW event to the signal power outside the GW event (background detector noise). We can see in Fig. 1 that SPR is close to unity for weak signals GW151226(SPR=1.29) and GW170104(SPR=1.08), while GW150914 has relatively higher SPR=4.122.

LIGO's search software uses two independent analyses, PYCBC and GSTLAL, and false alarm computations in Eq.8 and Eq.12 in [4] estimate empirically the number of coincident events above a certain threshold. For example, in the PYCBC analysis, the false alarm rate is given by the equation below, where T is the observation time of the search, T_b is the background time, $n_b(\hat{\rho}_c)$ is the number of background noise events that cross the candidate event's re-weighted SNR threshold.

$$F(\hat{\rho}_c) = 1 - e^{\frac{-T}{T_b}(1+n_b(\hat{\rho}_c))} \tag{2.1}$$

We may not observe high coincident SNR during 8 days of observation, but during next several months, by **definition** of non-stationarity, there is a possibility that the mean and variance of detector noise **may** change unpredictably and simultaneously in both sites, producing high coincident SNR in the output of the matched filter, even though the signal amplitude does not rise during the GW event, as is the case for GW151226 and GW170104 (Fig. 1). This will result in the estimate of the false alarm rate not applicable, particularly for weak signals GW151226 and 170104.

Hence, 5.1σ significance associated with the false alarm rate computation may be true for stationary white gaussian noise, but **not** necessarily for LIGO's non-stationary detector noise, which by definition, may change unexpectedly. We cannot be sure that the high SNR is produced by the GW template and **not** by non-stationary detector noise changing unexpectedly. For this reason alone[Reason 1], **independent** of other reasons, GW151226 and GW170104 should be questioned.

3 LIGO Matched Filter Implementation

The **core engine** of the LIGO software for identification of GW signals is the matched filter as described in Eq.1-4 in [4], which is used in two independent search methods, PYCBC and GSTLAL analysis[2]. For the case of PYCBC analysis, matched filter SNR(MF-SNR) $\rho^2(t)$ is given as follows.

$$\rho^2(t) = \frac{1}{|\langle h|h\rangle|}|\langle s|h\rangle(t)|^2$$

$$\langle s|h\rangle(t) = 4\int_0^\infty \frac{\widehat{s}(f)\widehat{h}^*(f)}{S_n(f)}e^{i2\pi ft}df$$

(3.1)

where $s(t)$ and $h(t)$ are the strain signal and the template respectively and $\widehat{s}(f)$ and $\widehat{h}^*(f)$ are the Fourier Transforms of $s(t)$ and $h^*(-t)$ respectively and $S_n(f)$ is the power spectral density of the detector noise. In the time domain, this is equivalent to the convolution of whitened version of $s(t)$ with real $h(-t)$, which is equivalent to the Cross-Correlation Function(CCF) of the whitened strain signal $s(t)$ and the template $h(t)$, with a normalization scale factor as follows.

$$\langle h|h\rangle = 4\int_0^\infty \frac{\widehat{h}(f)\widehat{h}^*(f)}{S_n(f)}df$$

(3.2)

It is noted that the above CCF is **not** normalized to give MF-SNR =1 for an ideal template correlated with itself. Hence we will use reference systems as follows.

Reference System A: An ideal template $h(t)$ is correlated with itself, after normalization, to give the maximum value of CCF = 1 at zero lag.

Reference System 1: Each strain signal H1 and L1 is replaced by $s'(t) = h(t) + w(t)$, where $h(t)$ is the ideal template and $w(t)$ is Additive White Gaussian Noise(AWGN) simulated in software, with power comparable to the background detector noise power.

Reference System 2: Each strain signal is replaced by $s'(t) = h(t) + w_d(t)$, where $h(t)$ is the ideal template and $w_d(t)$ is the actual non-stationary detector noise outside the GW events. Then we can compare how the MF-SNR of the practical system compares with the reference systems 1 and 2.

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[2] GSTLAL analysis also uses matched filter search, and as per Page 7 in [4], "the data s(t) and templates h(t) are each whitened in the frequency domain by dividing them by an estimate of the power spectral density of the detector noise." and also "By the convolution theorem, $\rho(t)$ obtained in this manner is the same as the $\rho(t)$ obtained by frequency domain filtering in Eq. (1)."[in PYCBC analysis]. These equations are implemented in lines 662-740 in the matched filter section of LIGO's tutorial python script. [11].

Given that GW signals are assumed to be the sum of an ideal template and detector noise, this comparison is reasonable.

3.1 Test 3a: LIGO matched filter SNR by Correlating H1 with L1
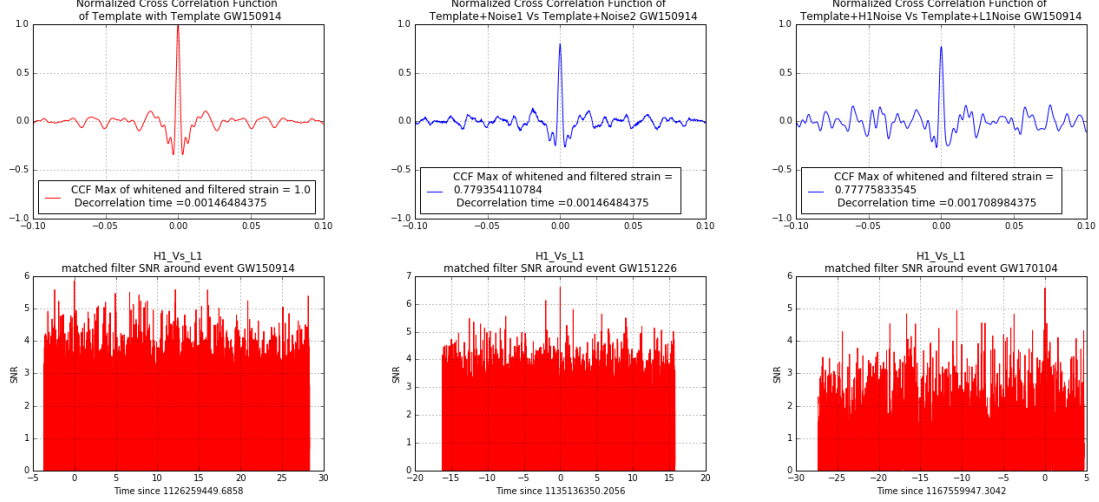
Figure 4. Bottom Row: Plots of H1 vs L1 LIGO matched filter SNR for GW150914, GW151226 and GW170104. Top Row: Reference Systems with Normalized CCF Plots. Top left: Reference System A. Top Middle: Reference System 1. Top Right: Reference System 2.

The first step is to cross-correlate the strain signal H1 with L1 and test whether LIGO's matched filter SNR shows the peaky behaviour observed in the reference plots in the top row of Fig. 4. Given that the same GW signal is expected to be received in both sites, these signals should give a high SNR at the matched filter output when correlated with each other. This is a **crucial** test which must be performed.

We **can** cross-correlate two noisy signals and expect a peaky CCF, if the signals are correlated. In fact, wireless communication with sensors routinely use cross-correlation of two noisy signals [8]. We wish to correlate two noisy signals $s(t) = h(t) + w_s(t)$[H1] and $h'(t) = h(t) + w_h(t)$[L1] where $w_s(t)$ and $w_h(t)$ represent the detector noise. This is equivalent to correlating $s(t) = h'(t) + w_s(t) - w_h(t) = h'(t) + w(t)$ and $h'(t)$. The theory of matched filter imposes no constraints on the characteristics of the template.

By peaky CCF, we mean that the ratio, R, of the the absolute value of CCF for any lag greater than the decorrelation time of the template, to the absolute maximum value of CCF at zero lag, should be less than a certain threshold. Decorrelation time of the template τ_0 is defined as the time taken for the autocorrelation of the template to fall to $\frac{1}{e}$ of the maximum value at zero lag [9] and $\tau_0 = 0.001464$ seconds for the template of GW150914. If we look at the reference plots in the top row of Fig. 4, we observe that $R < \frac{1}{e} = 0.36$.

Reference System 1: Fig. 4 shows this system in the top middle plot, which correlates two noisy signals, $s(t)$ and $h'(t)$, where $s(t)$ is the sum of the template and AWGN, and $h'(t)$ is the sum of the template and an independent AWGN. Then it computes the normalized $CCF(t) = \int_{-T}^{T} s(\tau) h'(t - \tau) d\tau$. We can see that the CCF shows strong peaky behaviour. Decorrelation time for this system is $\tau_1 = 0.001464$ seconds.

Reference System 2: Fig. 4 shows a reference system 2 in the top right plot, which correlates two noisy signals, $s(t)$ and $h'(t)$, where $s(t)$ is the sum of the template and H1 detector noise outside the GW event, and $h'(t)$ is the sum of the template and and L1 detector noise outside the GW event, and computes the normalized CCF. We can see that the CCF shows strong peaky behaviour. Decorrelation time for this system is $\tau_2 = 0.0017$ seconds.

The three plots in the bottom row in Fig. 4 show the results for correlating H1 with L1 in LIGO's matched filter for the three GW signals, GW150914, GW151226 and GW170104. We can see clearly that the three GW signals **do not** show peaky correlations ($R > \frac{1}{e}$), comparable to the reference plots in the top row of Fig. 4 ($R < \frac{1}{e}$).

This result is **surprising** for GW150914 because it shows very good visual correlations. This suggests LIGO's matched filter implementation may have problems. The lack of peaky CCF for GW150914 will be explored in Section 7.

For this reason alone[Reason 2], **independent** of other reasons, GW151226 and GW170104 should be questioned, because we need high standards to classify strain signals as GW signals.

3.2 Test 3b: Normalized CCF Method
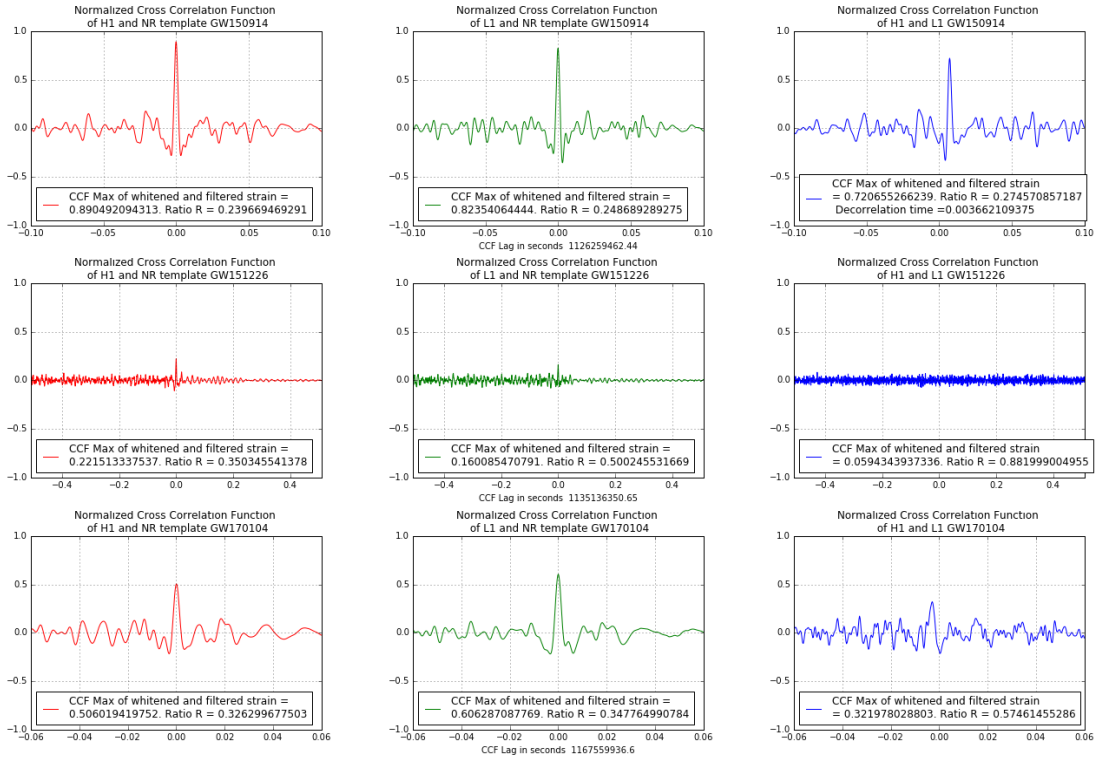
Figure 5. Normalized CCF Plots for GW150914, GW151226 and GW170104

Let us consider a normalized Cross Correlation Function(CCF) $CCF(t) = \int_{-T}^{T} s(\tau)h(t-\tau)d\tau$, where both $s(t)$ and $h(t)$ are normalized over the time window [-T,T] during which GW signal was

GW signal	R_3	R_3	R_3
	H1 vs template	L1 vs template	H1 vs L1
GW150914	0.2396	0.2486	0.2745
GW151226	0.35	0.50	0.88
GW170104	0.3262	0.3477	0.5746

Table 1. Normalized CCF Ratio for GW150914, GW151226 and GW170104. R_3 = Ratio of the absolute value of CCF at any lag greater than $\tau_0 * 3$ to the absolute maximum value of CCF at zero lag. τ_0 is the decorrelation time of the template. Lag greater than $\tau_0 * 3$ is taken to allow for some cushion.

GW signal	R_2	R_2	R_2
	H1 vs template	L1 vs template	H1 vs L1
GW150914	0.3137	0.4281	0.4530
GW151226	0.3878	0.50	0.8819
GW170104	0.3369	0.3490	0.6487

Table 2. Normalized CCF Ratio for GW150914, GW151226 and GW170104. R_2 = Ratio of the absolute value of CCF at any lag greater than $\tau_0 * 2$ to the absolute maximum value of CCF at zero lag.

observed, such that CCF of each signal with itself gives a result of unity for zero lag. GW150914 was observed in the window of duration 0.2 seconds, GW151226 in a duration of 1 second and GW170104 in a duration of 0.12 seconds[1–3].

We observed in the top row of Fig. 4 that the normalized CCF of the template with itself, gives a maximum value of 1 at zero lag and has a decorrelation time $\tau_0 = 0.001464$ seconds. The two reference systems 1 and 2, also have normalized CCF with the ratio $R < \frac{1}{e} = 0.36$. It is reasonable to expect a similar ratio $R < \frac{1}{e}$ from GW150914, GW151226 and GW170104, when correlating H1 with L1.

Fig. 5 plots the normalized CCF for GW150914(top row), GW151226(middle row) and GW170104 (bottom row) by correlating H1 with the template, L1 with the template and H1 with L1. We can see from the rightmost 3 plots that GW151226 and GW170104 show very poor CCF peaks ($R > \frac{1}{e}$) when correlating H1 with L1, while GW150914 shows relatively higher CCF peak. We can also see from the middle row in Fig. 5 that GW151226 shows very poor CCF peaks ($R > \frac{1}{e}$) when correlating L1 with the template.

Table 1 summarizes the ratio, R_3, of the absolute value of CCF at any lag greater than $\tau_0 * 3$ to the absolute maximum value of CCF at zero lag, for the three GW signals, where τ_0 is the decorrelation time of the template. GW151226 has $R_3 > \frac{1}{e}$ when correlating L1 with the template. GW151226 and GW170104 have $R_3 > \frac{1}{e}$ when correlating H1 with L1. For this reason alone[Reason 3], **independent** of other reasons, GW151226 and GW170104 should be questioned.

Table 2 summarizes the ratio, R_2, of the absolute value of CCF at any lag greater than $\tau_0 * 2$ to the absolute maximum value of CCF at zero lag, for the three GW signals, where τ_0 is the decorrelation time of the template. GW150914 also has $R_2 > \frac{1}{e}$ when correlating L1 with the template and H1 with L1. Decorrelation time is $\tau_3 = 0.0036$ seconds, when correlating H1 with L1, which is more than twice the decorrelation time for the reference system 2 with $\tau_2 = 0.0017$ seconds. Hence it is suggested that GW150914 be questioned.

4 LIGO's Matched Filter Misfires for Sine Wave
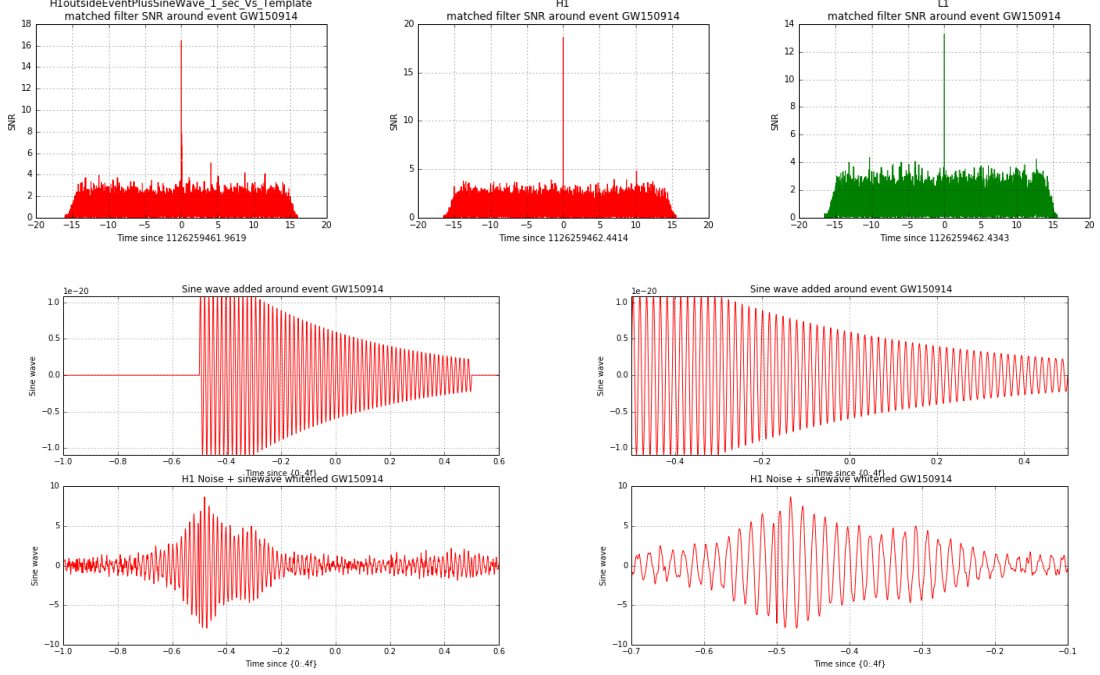
Figure 6. Top row: LIGO matched filter SNR for Sinewave + detector noise vs template. Bottom row: whitened and filtered signal in the time domain.

The lack of peaky CCF for GW150914 in the bottom left plot in Fig. 4 raises questions on whether LIGO's matched filter operates correctly. Let us test it by replacing the GW150914 signal with the signal $w_d(t) + a(t)$, where $w_d(t)$ is the H1 detector noise, obtained from a 32 second block of data which does not contain the GW signal and $a(t)$ is a sine wave of duration 1 second and frequency 60 Hz, scaled so that the standard deviation of $a(t)$ is 10 times lower than standard deviation of $w_d(t)$ and has an exponential decay. Then we correlate $w_d(t) + a(t)$ with the template $h(t)$ in the LIGO matched filter.

Fig. 6 shows the results, and we can see from the top left plot that even a sine wave added to the detector noise can produce strong peaky CCF in LIGO's matched filter, similar to what actual GW150914 produces in the top right plots. This result is not unique to any specific sine wave frequency but similar results are observed for other frequencies as well and also for GW151226 and GW170104.

This means that either the template or a sine wave of any frequency could have produced peaky CCF in LIGO's matched filter, and we **cannot be sure** that GW150914 was caused by an astrophysical event. It should be noted that we do not claim that a sine wave produced high SNR in the matched filter for GW150914.

It is clear that LIGO's matched filter may have problems that need to be fixed, and given the fact that LIGO's matched filter SNR is the core engine used in PYCBC and GSTLAL analyses and false alarm rates, this calls into question results for GW150914, GW151226 and GW170104.

For this reason alone[Reason 4], **independent** of other reasons, GW151226, GW170104 and GW150914 should be questioned.

5 Problems in the LIGO Whitening Code
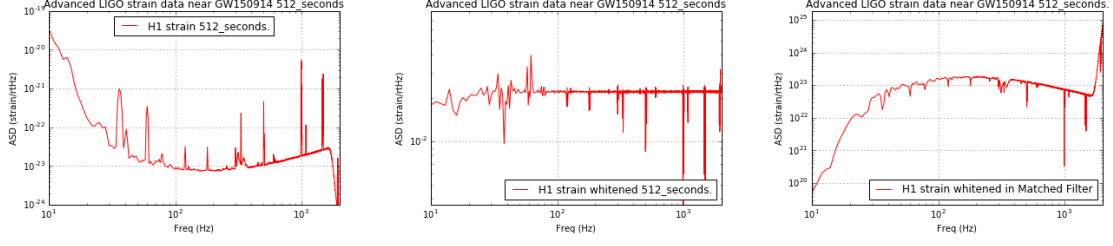
Figure 7. Amplitude Spectral Density(ASD) Plots of H1 strain in GW150914

LIGO detectors have very strong impulsive interference in 60Hz and harmonics and also in 300Hz-2000Hz range. Creswell et al.[5] have pointed out that the whitening section of LIGO's matched filter does not remove the narrow resonances in the strain signals well enough.

Fig. 7 shows the Amplitude Spectral Density(ASD) for 512 seconds of H1 strain data, H1 strain whitened outside the matched filter and H1 strain whitened inside the matched filter. We can see that LIGO's whitening code does not remove impulsive interference well enough.

It is important to clean up the impulsive interference in the detectors, so when we compute the matched filter SNR by correlating H1 and L1 signals, it can give high values for valid GW signals.

6 Case for Questioning of GW151226 and GW170104

GW151226 and GW170104 belong to a class of signals with Signal Power Ratio(SPR) around unity. We need to have much higher standards for classifying such signals as GW signals, than signals like GW150914 which have $SPR > 4$.

Given the high standards required for declaration of GW signal, it is suggested that GW151226 and GW170104 be questioned, due to each of the following Reason 1, 2, 3 and Reason 4 **independently**.

Reason 1: Being weak signals, non-stationary and non-Gaussian detector noise at the two sites could have caused the weak GW signal. When high SNR is observed at the matched filter during the GW event, while signal amplitude does not rise significantly, as is the case for GW151226 and GW170104, it may be due to non-stationary detector noise having high SNR at the output of the matched filter briefly at both sites as well.

Reason 2: In Section 3, we observed in Fig. 4 that the LIGO matched filter SNR is very poor ($R > \frac{1}{e}$) when correlating H1 and L1, for GW151226 and GW170104.

Reason 3: In Section 3, we observed in Table 1 and Fig. 5 that the normalized CCF is very poor ($R > \frac{1}{e}$) when correlating H1 and L1, for GW151226 and GW170104.

Reason 4: In Section 4, we observed in Fig. 6 that LIGO's matched filter misfiring for a sine wave, may have problems for GW151226 and GW170104.

7 Implications of Results for GW150914

In Section 3, we observed a contradiction. If we consider the top right plot of the Normalized Cross Correlation Function(CCF) in Fig. 5, GW150914 shows relatively peaky CCF. This agrees with visual correlations observed in the relatively strong signal GW150914. But the first plot in the bottom row in LIGO's matched filter SNR plots in Fig. 4 shows that GW150914 does not show peaky correlation.
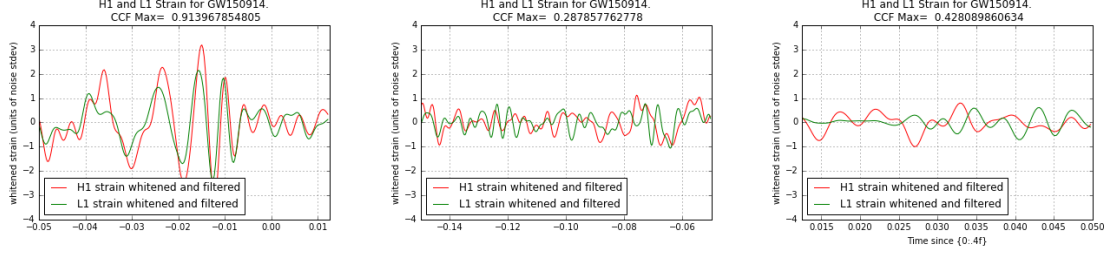
Figure 8. Plots of H1 vs L1 whitened and filtered strain in GW150914

It is possible that LIGO's matched filter SNR correlation over a 32 second window is too long to bury the correlations in the presence of detector noise at both sites. Normalized CCF in the plots of Fig. 5 is performed over 0.2 second window over which GW150914 was observed and hence the short window may have captured the correlation between H1 and L1 in GW150914.

Fig. 8 shows the visual correlations of whitened and filtered H1 and L1 strains over the 3 windows and we can see that the correlations are satisfyingly strong only in a short 0.06 second window in the first plot. Correlations in the second and third windows in the rest of $0.2 - 0.06 = 0.14$ seconds are not convincing.

This agrees with the maximum values of the normalized CCF, given by CCF_{max} computed for these 3 windows. We can see that $CCF_{max} = 0.91$ for the short 0.06 second window in the first plot, but $CCF_{max} < 0.45$ for the other 2 windows[Reason 5]. This is attributed to the high levels of detector noise which is comparable to the template power in these 2 windows. So, we **cannot be sure** if template is present in these 2 windows for GW150914[Reason 5]. This problem can be resolved only by reducing detector noise in future.

Further, the top plots for the reference systems 1 and 2 in Fig. 4 have a value of normalized CCF = 0.78. In comparison, normalized CCF = 0.72 for GW150914 in Table 1, which is relatively lower.

In Section 3.2, when correlating H1 and L1, we observed that the decorrelation time for this system is $\tau_3 = 0.0036$ seconds, which is more than twice the decorrelation time for the reference system 2 with $\tau_2 = 0.0017$ seconds. Hence it is suggested that GW150914 be questioned.

7.1 Effect of removal of 20-300Hz Bandpass filter
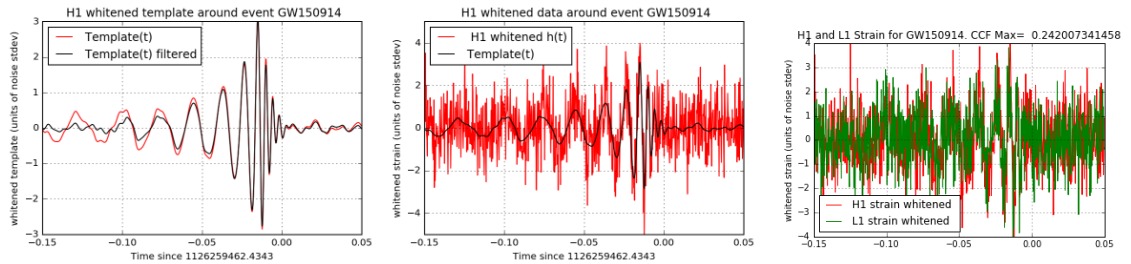
Figure 9. Left: Plots of templates with and without filtering. Middle: Plots of whitened H1 vs template without filtering. Right:Plots of H1 vs L1 whitened strain in GW150914 without filtering

Fourth order butterworth bandpass filters are used in the frequency range 20-300 Hz in plotting whitened and filtered H1 and L1 strains for GW150914 in Fig. 8. The template for GW150914 has 94 percent of its energy in this 20-300 Hz range and 6 percent of energy in the remainder of the total

frequency range of 0-2048 Hz, with sampling frequency of 4096 Hz. The first plot in Fig. 9 compares the templates inside 20-300 Hz band and the total range of 0-2048 Hz.

The second plot in Fig. 9 compares the whitened H1 strain without filtering and compares it with the unfiltered template. The third plot in Fig. 9 compares the whitened H1 and L1 strains without filtering. We can see that visual correlations and the normalized CCF values become much poorer if we do not filter the strains with 20-300 Hz filter, so that we can capture the 6 percent template energy outside 20-300 Hz[Reason 6]. This problem can be resolved only by reducing detector noise in future.

7.2   Summary of results

GW150914 should be questioned due to Reason 2, Reason 3, Reason 4, Reason 5 and Reason 6.

Reason 2: In Section 3, we observed in Fig. 4 that the LIGO matched filter SNR is very poor when correlating H1 and L1, for GW150914.

Reason 3: In Section 3.2, when correlating H1 and L1, we observed that the decorrelation time for this system is $\tau_3 = 0.0036$ seconds, which is more than twice the decorrelation time for the reference system 2 with $\tau_2 = 0.0017$ seconds.

Reason 4: In Section 4, we observed in Fig. 6 that LIGO's matched filter misfires for sine wave input with high SNR and hence we cannot be sure if GW150914 template was received.

Reason 5: We observed in Fig. 8 that the normalized CCF is high only for a short 0.06 second window when correlating H1 and L1, for GW150914. It is poor for the other 2 windows.

Reason 6: We observed in Fig. 9 that the normalized CCF is poor when correlating H1 and L1, for GW150914, without using 20-300 Hz filter, so that we can capture the 6 percent template energy outside 20-300 Hz.

8   Concluding remarks

Section 6 gives the reasons why weak signals GW151226 and GW170104 should be questioned. Section 7 gives the reasons why GW150914 should be questioned. Reiterating the point made earlier, because we are more often likely to observe weak signals which look like noise and whose amplitude does not rise during assumed GW event, it is of paramount importance that we should not classify noise as GW events. We need **high** standards to classify an observed time series as a GW signal. LIGO detectors have very strong impulsive interference in 60Hz and harmonics and also in 300Hz-2000Hz range. This may affect the performance of the matched filter. It is very important to clean up the impulsive interference in the detectors.

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[10] LOSC LIGO GW150914 Tutorial

[11] LOSC LIGO GW151226 Tutorial matched filter equations are implemented in the section "matched filtering to find the signal"

[12] LIGO Tutorials

[13] Zip Files of Modified LIGO Tutorials which demonstrate the results in this paper. Matched Filter SNR results are demonstrated with 32 second block of data, used in the original LIGO tutorial scripts. Results hold for 512 second block of data also.