Module - 1

=> Differential equation

An equation involving derivatives is called differential equation

=> Ordinary differential equation

A differential equation involving a single independent variable and hence only ordinary derivatives is called ordinary differential equation

=> Partial differential equation

A differential equation involving more than one independent variable and hence partial derivatives is called partial differential equation.

=> Order of differential equation

Order of D.E is the order of highest derivative occuring in it.

=> Degree of differential equation

Degree of D.E is the degree of highest derivative occuring in it.

=> General solution

The solution of a first order ODE contains a arbitrary constant is known as general solution.

=> Particular Solution

A Solution obtainable from the general Solution and giving particular value to the arbitrary Constant.

=> Exact differential equation

$$Mdx + Ndy = 0$$

-> working method for Solving the Exact equation

If D.E $Mdx + Ndy = 0$. Satisfies the Condition $\partial M/\partial y = \partial N/\partial x$. then exact Solution is given by

$$\boxed{\int M dx + \int \left[\text{terms in } N \text{ not involving } x\right] dy = c}$$

=> Integrating factors

method of finding integrating factor

1) if $\frac{M_y - N_x}{N} = f(x)$ is a function of x alone then $\mu = e^{\int f(x)dx}$ is the integrating factor of $Mdx + Ndy = 0$

2) if $\frac{N_x - M_y}{M} = f(y)$. is a function of y alone then $\mu = e^{\int f(y)dy}$

Bernoulli's theorm

A differential equation of the form $y' + f(x)y = r(x)y^n$ — ① where $f(x)$ & $r(x)$ are fns of x alone.

$$ZM = \int r\, M\, dx + c$$

Module - 2

=> Linearly dependent and independent

Two functions $y_1(x)$ and $y_2(x)$ are said to be linearly dependent on an interval I when both functions are defined, if they are proportional on I

ie $y_1(x) = k y_2(x)$

$y_2(x) = l y_1(x)$

=> Initial value Problem and boundary value Problem

in a general solution to a second order differential equation

$$y'' + ay' + by = 0 \quad \text{—} ①$$

Contains two arbitary Constants

To obtaining Particular solution in Some Cases these two Conditions are of the type $y(x_0) = k, y'(x_0) = L$ — ②

Condition — ② + equation ① = initial value Problem

Sometimes $y(A) = k_1, y(B) = k_2$ — ③

Condition ③ + equation ① = boundary value Problem

Second Order Homogeneous ODEs with Constant Coefficients

Case : 1 Two distinct real roots λ_1 & λ_2

$y_1 = e^{\lambda_1 x}$ & $y_2 = e^{\lambda_2 x}$

$G.S = y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

Case : 2 Double root

$y_1 = e^{\lambda x}$ & $y_2 = x e^{\lambda x}$ are solutions of ① then

$G.S = y = (c_1 + c_2 x) e^{x}$

Case : 3 Complex Conjugate roots

$\lambda_1 = p + iq$, & $\lambda_2 = p - iq$

$y_1 = e^{(p+iq)x}$, $y_2 = e^{(p-iq)x}$

$y_1 = e^{px} \cos qx$ & $y_2 = e^{px} \sin qx$

=> Euler-Cauchy equation

is of the form $x^2 y'' + axy' + by = 0$ —— ①

where a & b are arbitrary Constant

1) Distinct real root

m_1 & m_2

$y_1(x) = x^{m_1}$, $y_2 = x^{m_2}$

Non-homogeneous equation

$r(x)$	y_p
ke^{px}	ce^{px}
$P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$	$k_n x^n + k_{n-1}x^{n-1} + \dots + k_1 x + k_0$
$k \cos qx$	$A \cos qx + B \sin qx$
$k \sin qx$	
$P_n(x) e^{px}$	$(k_n x^n + k_{n-1}x^{n-1} + \dots + k_1 x + k_0) e^{px}$
$k e^{px} \cos qx$	$e^{px}(A \cos qx + B \sin qx)$
$k e^{px} \sin qx$	
$P_n(x) \cos qx$	$[k_n x^n + k_{n-1}x^{n-1} + \dots + k_0] \cos qx$
$P_n(x) \sin qx$	$+ [L_n x^n + L_{n-1}x^{n-1} + \dots + L_0] \sin qx$
$P_n(x) e^{px} \cos qx$	$[k_n x^n + k_{n-1}x^{n-1} + \dots + k_0] e^{px} \cos qx +$
$P_n(x) e^{px} \sin qx$	$[L_n x^n + L_{n-1}x^{n-1} + \dots + L_0] e^{px} \sin qx$

Module - 3

$f(t)$	$L[f(t)]$
0	0
1	$1/s$
t	$1/s^2$
t^2	$2!/s^3$
$t^n, n = 1, 2, 3 \dots$	$\frac{n!}{s^{n+1}}$

t^a, a is positive	$(a+1)/s^{a+1}$
e^{at}	$1/s-a$
e^{iat}	$1/s-ia$
$\cos at$	s/s^2+a^2
$\sin at$	a/s^2+a^2
$\cosh at$	s/s^2-a^2
$\sinh at$	a/s^2-a^2

=> inverse Laplace transform

If $L[f(t)] = F(s)$, then $f(t)$ is called inverse Laplace transform of $F(s)$

$$f(t) = L^{-1}[F(s)]$$

=> First shifting theorm

If $f(t)$ has the transform $F(s)$ where $s>r$ then $e^{at} f(t)$ has the transform $F(s-a)$ where $s-a>r$

$$L[f(t)] = F(s)$$

$$L[e^{at} f(t)] = F(s-a)$$

$f(t)$	$L[f(t)]$
$e^{at} t^n$ $n = 1, 2, 3 \ldots\ldots$	$\frac{n!}{(s-a)^{n+1}}$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
$e^{at} \sin bt$	$\frac{b}{(s-a)^2+b^2}$

=> Application of differential equations

$$L[y(t)] = Y(s)$$

$$L[y'(t)] = sY(s) - Y(0)$$

$$L[y''(t)] = s^2 y(s) - sy(0) - y(0)$$

=> 2nd shifting theorm

$$L[u(t-a)] = \frac{e^{-as}}{s} \quad , s > 0$$

=> Differentiation of transforms

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s)$$

$$= (-1)^n F^n(s) \qquad (n = 1, 2, 3 \ldots)$$

Integration of transforms

$$L\left[\frac{f(t)}{t}\right] = \int_0^{\infty} F(u)\,du$$

=> Convolution & integral equns

Convolution of $f(t)$ & $g(t)$ written $(f*g)(t)$

$$(f*g)(t) = \int_0^t f(u)\,g(t-u)\,du$$

=> Properties

i) Commutative :-

$$f*g = g*f$$

ii) Distributive :-

$$f*(g+h) = (f*g)+(f*h)$$

iii) Associative :-

$$f*(g*h) = (f*g)*h$$

=> Convolution theorm

If $f(t)$ and $g(t)$ are inverse transforms of $F(s)$ & $G(s)$ respectively the inverse transforms of the product $F(s)G(s)$ is the Convolution of $f(t)$ & $g(t)$ is $L^{-1}[F(s)\,G(s)] = [f*g](t)$

Module - 4

* Fourier Series of 2π Periodic fun $[-\pi, \pi]$

$$f(x) = a_0/2 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = 1/\pi \int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = 1/\pi \int_{-\pi}^{\pi} f(x) \cos nx\,dx$$

$$b_n = 1/\pi \int_{-\pi}^{\pi} f(x) \sin nx\,dx$$

* Fourier Series even and odd 2π Periodic function

1. $f(x)$ is an even

$$\int f(x)\,dx = 2\int f(x)\,dx$$

2. $f(x)$ is an odd

$$\int_{-a}^{a} f(x)\,dx = 0$$

$f(x)$ is an even

$$a_0 = 1/\pi \int_{-\pi}^{\pi} f(x)\,dx = 2/\pi \int_{0}^{\pi} f(x)$$

$$a_n = 1/\pi \int_{-\pi}^{\pi} f(x) \cos nx\,dx = 2/\pi \int_{0}^{\pi} f(x) \cos nx\,dx$$

* Fourier Cosine Series

$$f(x) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = 2/\pi \int_0^{\pi} f(x)\,dx$$

$$a_n = 2/\pi \int_0^{\pi} f(x)\cos nx\,dx$$

* Fourier Sine Series

$$f(x) = a_0/2 + \sum_{n=1}^{\infty} b_n \cos nx$$

$$a_0 = 2/\pi \int_0^{\pi} f(x)\sin nx\,dx$$

* Fourier series for even 2L periodic fun derived over the interval [-L, L]

$$f(x) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}\,dx$$

$$a_0 = 2/L \int_0^{L} f(x)\,dx$$

$$a_n = 2/L \int_0^{L} f(x) \cos \frac{n\pi x}{L}\,dx$$

odd:- $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$

$$b_n = 2/L\, f(x) \sin \frac{n\pi x}{L}\,dx$$

* Half range Fourier expansions of fun defined over $[0, L]$

i) Half range fourier sine series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n x}{L}$$

$$b_n = 2/L \int_0^L f(x) \sin \frac{n\pi x}{L} \, dx$$

ii) Half range fourier Cosine Series

$$f(x) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$$

$$a_0 = 2/L \int_0^l f(x) \, dx$$

$$a_n = 2/L \int_0^L f(x) \cos \left(\frac{n\pi x}{L}\right) dx$$