

Q1.

(a) We have,

$$(\lambda z.z)(\lambda y.y y)(\lambda x.x a)$$

$\downarrow \beta$

$$(\lambda y.y y)(\lambda x.x a)$$

$\downarrow \beta$

$$(\lambda x.x a)(\lambda x.x a)$$

$\downarrow \beta$

$$(\lambda x.x a) a$$

$\downarrow \beta$

$$(a a)$$

(b) We have,

$$(\lambda z.z)(\lambda z.z z)(\lambda z.z y)$$

$\downarrow \alpha$

$$(\lambda a.a)(\lambda z.z z)(\lambda z.z y)$$

$\downarrow \beta$

$$(\lambda z. z z) (\lambda z. zy)$$

$\downarrow \alpha$

$$(\lambda a. a a) (\lambda z. zy)$$

$\downarrow \beta$

$$(\lambda z. zy) (\lambda z. zy)$$

$\downarrow \beta$

$$(\lambda z. zy) y$$

$\downarrow \beta$

$$(y y)$$

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(c) We have,

$$(\lambda x. \lambda y. x y y) (\lambda a. a) b$$

$\downarrow \beta$

$$(\lambda y. ((\lambda a. a) y y)) b$$

$\downarrow \beta$

$$(\lambda y. y y) b$$

$\downarrow \beta$

$$(b b)$$

$$(d) (\lambda x. \lambda y. x y y) (\lambda y. y) y$$

$$\downarrow \alpha$$

$$(\lambda x. \lambda a x a a) (\lambda y. y) y$$

$$\downarrow \beta$$

$$(\lambda a ((\lambda y. y) a a)) y$$

$$\downarrow \beta$$

$$(\lambda a a a) y$$

$$\downarrow \beta$$

$$(y y)$$

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$$(e) (\lambda x. x x) (\lambda y. y x) z$$

$$\downarrow \beta$$

$$(\lambda y. y x) (\lambda y. y x) z$$

$$\downarrow \beta$$

$$((\lambda y. y x) x) z$$

$$\downarrow \beta$$

$$(x x) z$$

$$(f) (\lambda x. (\lambda y. (xy))y)z$$

$$\downarrow \beta$$

$$(\lambda y. (zy))y$$

$$\downarrow \beta$$

$$(zy)$$

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(g) We have,

$$((\lambda x. (\lambda y. (xy)) (\lambda y. y)) \omega)$$

$$\downarrow \alpha$$

$$(((\lambda x. (\lambda a. (xa)) (\lambda y. y)) \omega)$$

$$\downarrow \beta$$

$$((\lambda a. ((\lambda y. y) a)) \omega)$$

$$\downarrow \beta$$

$$((\lambda y. y) \omega)$$

$$\downarrow \beta$$

$$(\omega)$$

~~QED~~

PTO

Q2. (a) $Y = \lambda f. (\lambda x. f(x x)) (\lambda x. f(x x))$

Tri Product = TP

$= \lambda f. \lambda n \text{ (IF } n > 3 \text{ THEN } n * f(n-1) + f(n-2) + f(n-3)$

ELSE (IF $n = 3$ THEN 5 ELSE (

IF $n = 2$ THEN 2 ELSE (1))))

$= \lambda f. \lambda n. \text{FNC}$

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Tri Product (n)

$= (Y \text{ Tri Product}) n$

(b) TRI PRODUCT (4) = (YTP) 4

$((\lambda f. (\lambda x. f(x x)) (\lambda x. f(x x)) TP) 4$

$\downarrow \beta$

$((\lambda x. TP(x x)) (\lambda x. TP(x x)) 4$

$\downarrow \alpha$

$((\lambda t. TP(t t)) (\lambda x. TP(x x))) 4$

$\downarrow \beta$

PTO

01 E 2

$$TP ((\lambda x. TP (xx)) (\lambda x. TP (xx))) 4$$

$\downarrow \alpha$

$$TP ((\lambda t. TP (tt)) (\lambda x. TP (xx))) 4$$

$\downarrow \beta$

$$TP (TP ((\lambda x. TP (xx)) (\lambda x. TP (xx)))) 4$$

$\underbrace{\hspace{10em}}_z$

$$= TP (TP (z)) 4$$

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$$TP (TP (z)) 4$$

$$\Rightarrow \lambda f. \lambda n \left(n * f(n-1) + f(n-2) + f(n-3); \right. \\ \left. \text{if } n > 3 \right.$$

$$5 ; \text{ if } n = 3$$

$$2 ; \text{ if } n = 2$$

$$1 ; \text{ if } n = 1 \Big)$$

$$(TP(z)) 4$$

$$\Rightarrow \lambda n. \left(n * TP(z)(n-1) \right) + \left(TP(z)(n-2) \right) \\ \text{(}\beta\text{-redn)} \quad + \left(TP(z)(n-3) \right) 4$$

$$\Rightarrow \beta\text{-red}^n \quad 4^* (\text{TP}(z) \ 3) + (\text{TP}(z) \ 2) + (\text{TP}(z) \ 1)$$

$$= 4^* (\lambda f. \lambda n (\text{FNC}) \ z \ 3)$$

$$+ (\lambda f. \lambda n (\text{FNC}) \ z \ 2)$$

$$+ (\lambda f. \lambda n. (\text{FNC}) \ z \ 1)$$

$$\Rightarrow \beta\text{-red}^n \quad 4^* (\text{FNC} [f \rightarrow z, n \rightarrow 3])$$

$$+ (\text{FNC} [f \rightarrow z, n \rightarrow 2])$$

$$+ (\text{FNC} [f \rightarrow z, n \rightarrow 1])$$

$$= 4^* 5 + 2 + 1$$

$$= 20 + 2 + 1$$

$$= \underline{23}$$

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