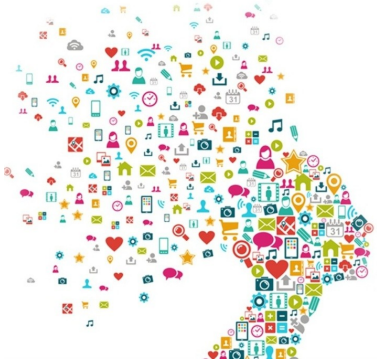

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Red Black Tree

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Contents

- What are Red Black trees
- RB Tree structure
- Terms used
- Rules/Properties
- Balancing in RB tree
- Tree Rotations
- Operations: Insert, Search, Delete

What are Red Black trees

- Self balancing binary search tree.
  - Invented in 1972 by Rudolf Bayer
- Each node has an extra attribute color(Red/Black).
- Color bit is used for balancing of the tree.
- Explained as extended binary search tree.
- In extended, NULL links are replaced by special nodes.

RB Tree Structure

```
typedef struct node
{
    struct node *parent;
    int data;
    char color;
    struct node *left;
    struct node *right;
}rbt;
```

Terms used

- Node - Each element of tree.
- Parent – Immediate predecessor of node.
- Grand parent – Parent’s parent node.
- Sibling – Other child of parent.
- Uncle – Sibling of parent node.
- Nephew – Children of sibling node.

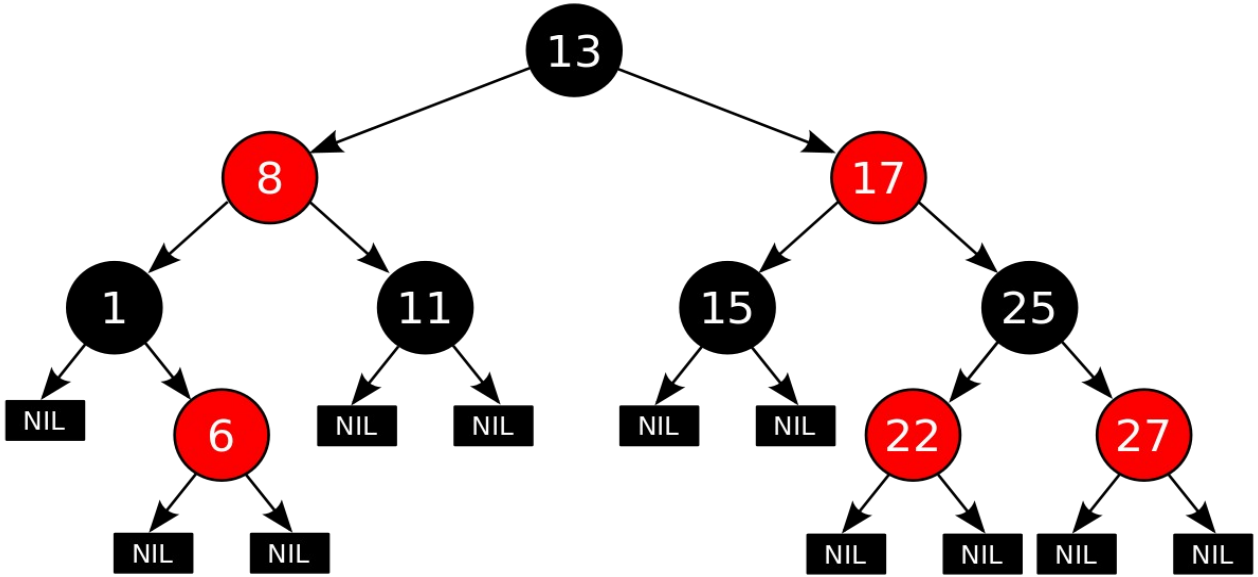
RB Tree Rules/Properties

- Root is always black.
- All extended nodes(NULL) are black.
- A red node cannot have red children, it can have only black children; A black node can have black or red children.
- For each node N, all paths from node N to external nodes contain the same number of black nodes (same black height).

Black height

- Number of black nodes on a path from a node to its leaf.
- While calculating black height of a node:
  - Leaf nodes are also counted.
  - Current node is not counted.
- Black height of tree = Black height of root.

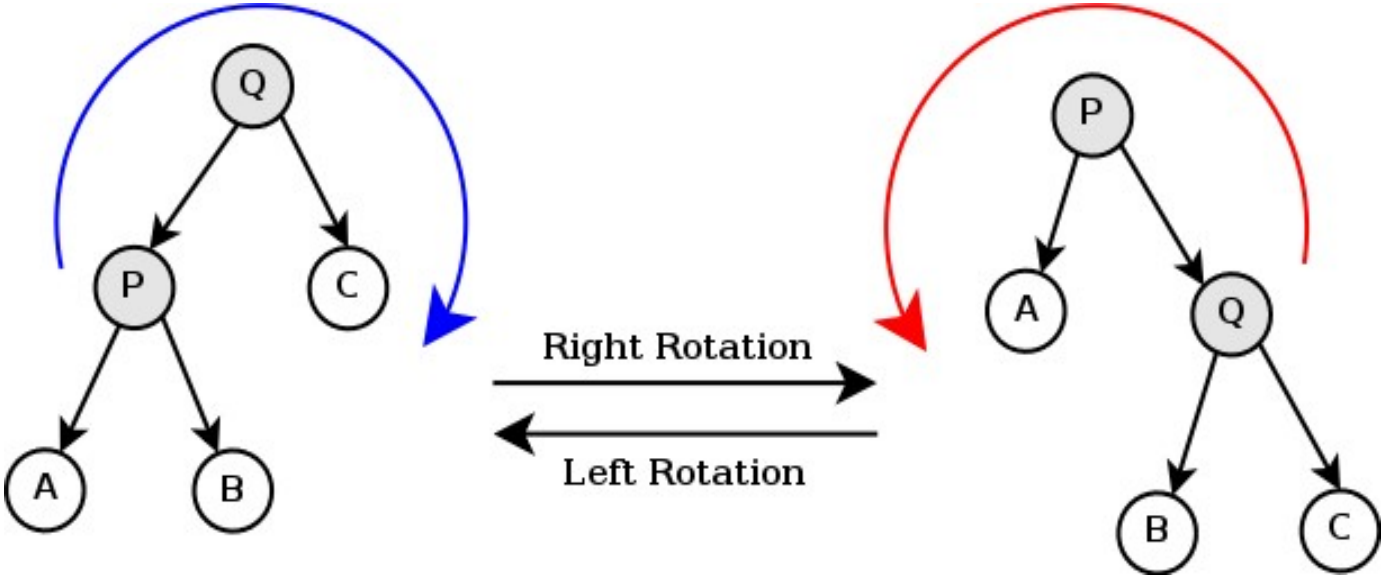
RB Tree Example



Balancing in RB Tree

- Suppose the black height of the root is k.
- Smallest path contains all black nodes i.e., k nodes
- Longest path contains alternate black and red nodes i.e., 2k nodes.
- So, no path in RBT, can be more than twice the another path.
- Hence, RBT is always balanced.

Tree Rotations



Operations

- Insertion
  - If new node is black then black height property will be violated.
  - So each new node inserted will be initially of red color.
  - Inserted red node may violate.
    i. Rule 1 if node is root.
    ii. Rule 3 if parent node is also red (double red problem).
  - Rule 1 violation is fixed by recoloring the node to black.
  - To fix Rule 3 violation there are some rules to be followed.

Insertion - Fixing of Rule 3 Violation

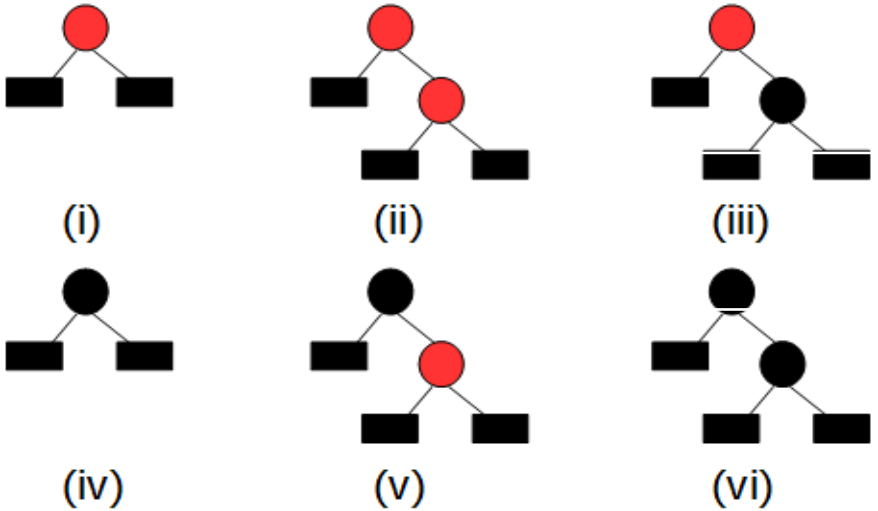
Parent Color	Parent Left/Right	Uncle Color	Node Left/Right	Case	How to balance?
Black	NA	NA	NA	NA	Already balanced
Red	Left (Case L)	Red (L_1)	NA	L_1	Recolor parent & uncle to black; Recolor grandparent to red; verify rule 3 for grandparent
		Black (L_2)	Right (L_2a)	L_2a	Rotate left about parent and Transform to case L_2b
			Left (L_2b)	L_2b	Recolor parent to black Recolor grandparent to red Right rotation is performed about the grandparent
	Right (Case R)	Red (R_1)	NA	R_1	Re-color parent & uncle to black, Re-color grandparent to red, verify rule 3 for grandparent
		Black (R_2)	Left (R_2a)	R_2a	Rotate right about parent and Transform to case R_2b
			Right (R_2b)	R_2b	Recolor parent to black, grandparent to red Left rotation is performed about the grandparent

Operations (Contd..)

- Searching
  - As in BST search starts from root by comparing node value.
  - Move left or right recursively till element is found.
- Deletion
  - Node can have no child, single child or two children.
  - In single child case node is replaced with child and colored black.
  - In two child case:
    i. Identify inorder successor and replace the node value with inorder successor.
    ii. Now delete inorder successor which can have no child or right child.
    iii. This gives 6 possible cases.

Deletion (Contd..)

The 6 possible cases for deletion:



Deletion Steps

- Case 1: Red node with no child
  - Delete the node
- Case 2: Black node with right red child.
  - Delete the node and replace it with child node colored black
- Case 3 : Black node with no children
  - By deleting the node black height property is violated.
  - To fix this we need to check some cases and then follow some rules.

Case 3-Black node with no children(Right case)

- Case 3.1: Sibling Red with black children
  - Right rotate from parent(P) and make sibling black.
  - Change color of near nephew to red.
  - If near nephew has red children apply left right or left left case accordingly.
- Case 3.2: Sibling black with red parent
  - 3 sub cases.
  - 1)No children for sibling.
  - 2)Left or right red child.
  - 3)Two red children.

Case 3.2 - Black sibling with red parent

- Case 3.21:Black sibling with no children
  - Color sibling red and parent black.
- Case 3.22: Black sibling with red left or right child
  - If left child only
    - Rotate right from parent.
  - If right child only
    - First left rotate about sibling and then right rotate about parent.
    - Change color of sibling to red and new parent to black.

Case 3.2 (Contd..)

- Case 3.23: Black sibling with two red children
  - Right rotate about sibling and make new parent red.
  - Left and right children of new parent is made black.
- Case 3.3: Black sibling with black parent
  - 1)No children
  - 2)Left or right red child only for sibling.
  - 3)Two red children for sibling

Case 3.3 (Contd..)

- Case 3.31: Black sibling with no children
  - Uncle black case
    - Left rotate from grand parent and change its color to red.
    - Change sibling color to red.
  - Uncle red case
    - Make uncle color black.
    - Left rotate from grand parent and change its color to red.

Case 3.3 (Contd..)

- Case 3.32: Black sibling with left or right red child
  - If left
    - Right rotate from parent and make far nephew black.
  - If right
    - Left rotate from sibling and right rotate from parent.
    - Make new parent black.
- Case 3.33: Black sibling with two red children
  - Right rotate from parent.
  - Far nephew is made black.

Applications of RB Tree

- Completely Fair Scheduler in Linux kernel
- Computational geometry data structures.
- C++ STL implementation of Map.

Summary

- Red Black Tree
- Rules/Properties of RB Tree
- Balancing in RB Tree
- Operations performed.
- Applications.

References

- https://www.cs.usfca.edu/~galles/visualization/RedBlack.html
- https://en.wikipedia.org/wiki/Red%E2%80%93black_tree

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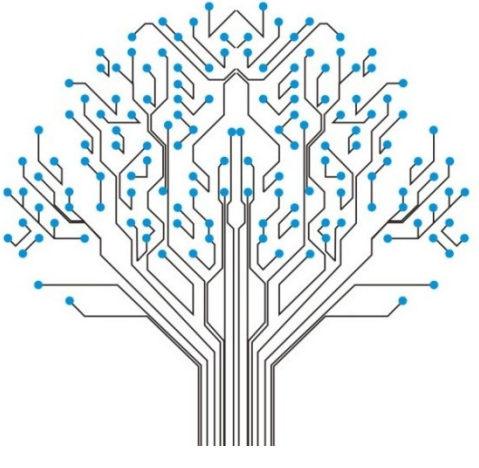

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Learning

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Respect