ME 498: Computational Modeling and Optimization

Projects 2+3

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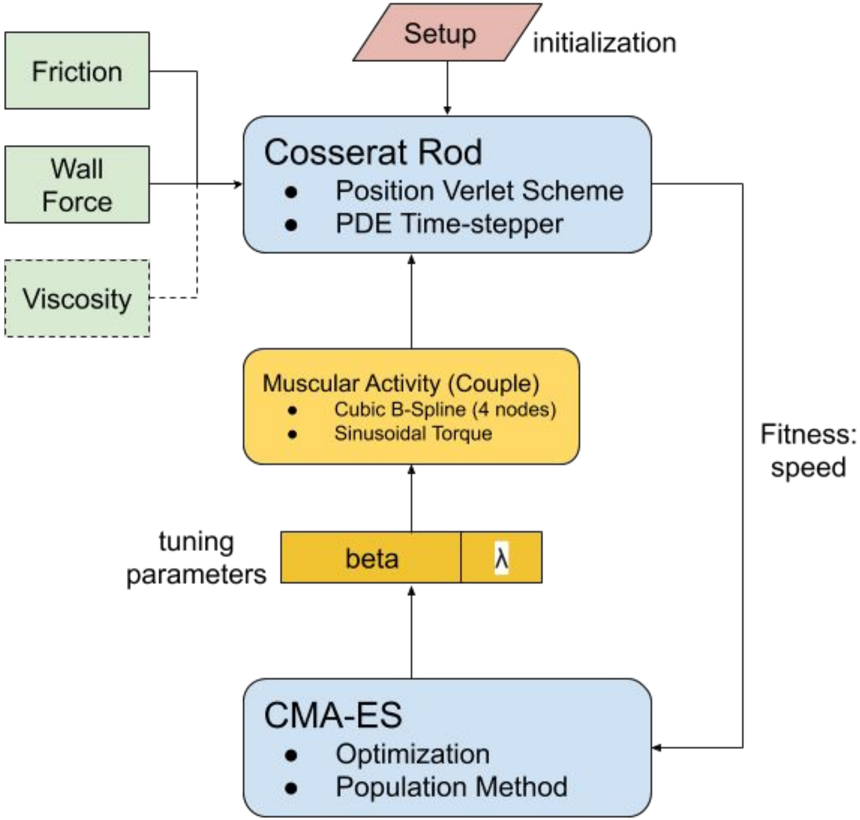
Problem Statement

- Optimize the velocity of the slithering snake.
- Goal
  - Design and implement numerical simulation of Cosserat rod
  - Apply internal muscular activity of the snake
  - Apply contact force and friction
  - Use CMA-ES algorithm to find parameters to maximize the speed
- Purpose
  - Investigate numerical snake movement
  - Compare with biological snake to learn dynamics

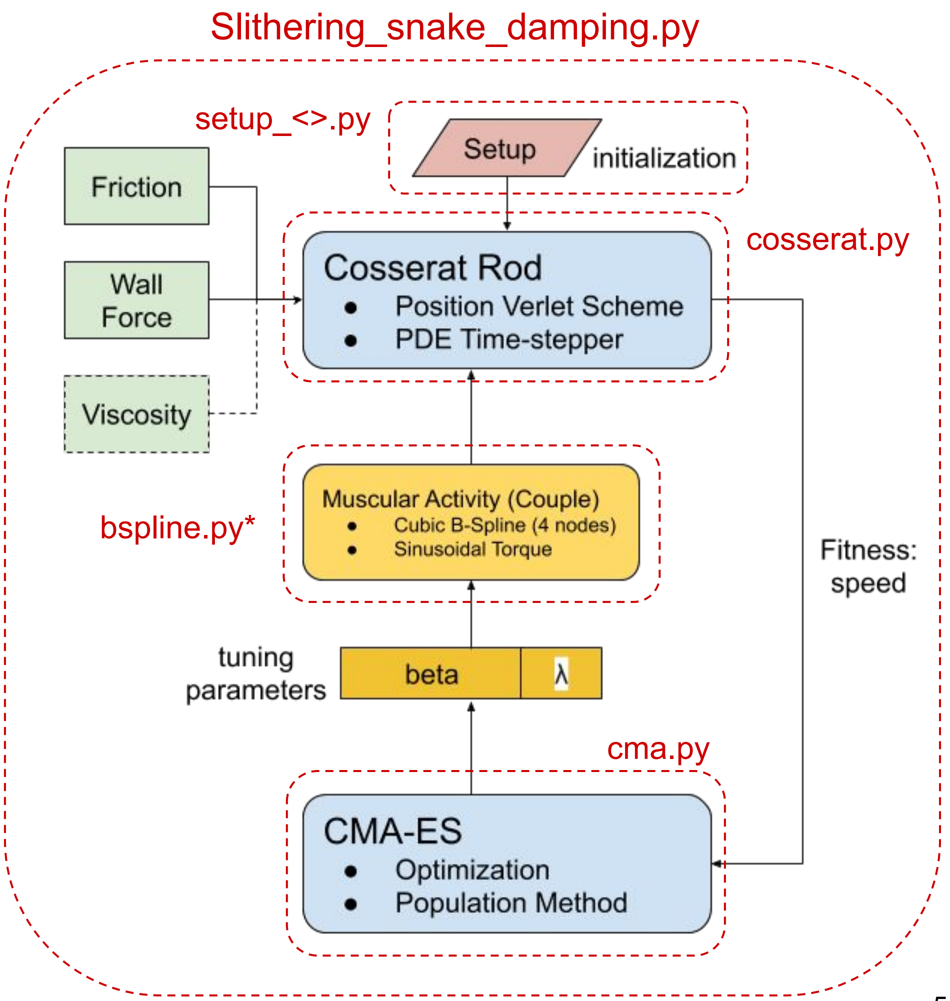
Method

- Snake: Cosserat rod model
  - 50 population
  - Iterate 100k steps
  - Compute average speed
- Optimization: CMA-ES
  - 50 generation
  - Optimize beta/lambda
    - beta: Strength at different location
    - lambda: Torque phase offset



Implementation

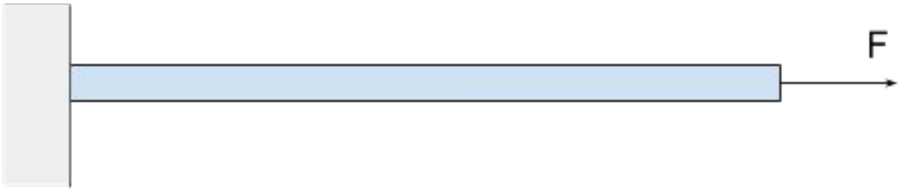
- Modules are designed to handle different initialization.
  - Problem initialization is separately defined in 'setup_<>.py' file
- Cosserat stepper function takes arguments:
  - dt: time step
  - force: instantaneous force
  - couple: instantaneous couple
- Friction and wall force is implemented separately.

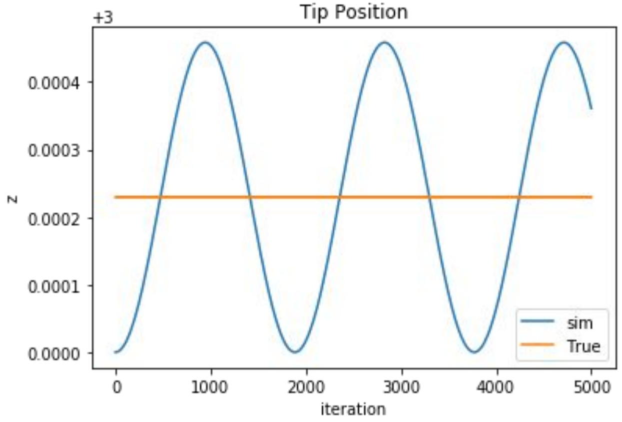


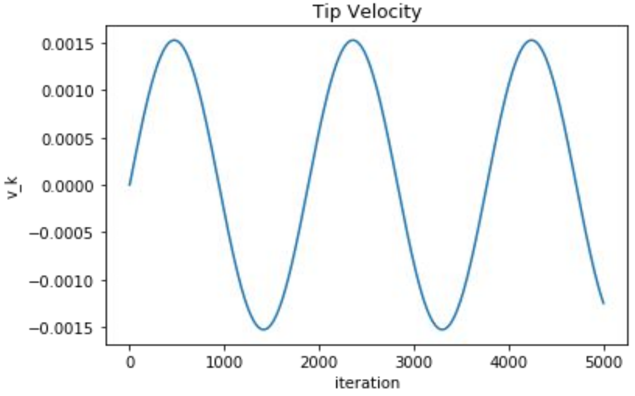
Validation: Solid Mechanics

- The Cosserat time-stepper module is validated with three basic cases.
  - 1. Linear Strain: Verify linear velocity/acceleration
  - 2. Prestressed Beam: Verify angular velocity/acceleration
  - 3. Timoshenko Beam: Verify linear and angular coupled calculation
- Main criteria for validation was:
  - Convergence of the numerical result against physical (true) solid-mechanic model
  - Stability of the numerical scheme
- Validation process helped to understand abnormal numerical behaviors.

Case 1: Linear Strain







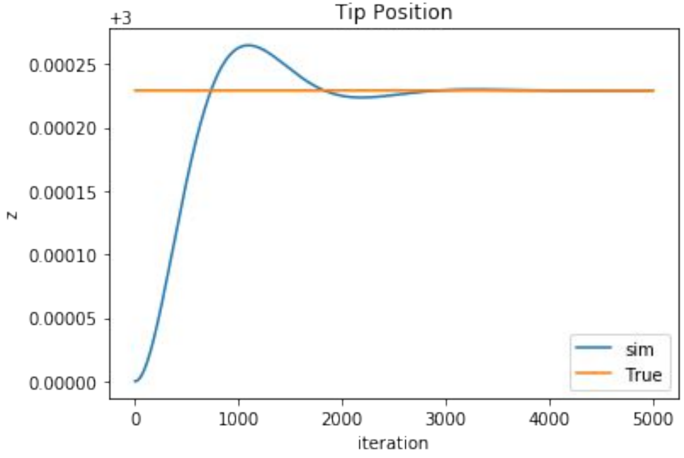
- ❑ Single Element
- ❑ Equivalent to spring-force system
- ❑ Expected to observe sinusoidal position and the velocity
- ❑ Oscillates about the actual extension value

Case 1: Linear Strain (Damped)

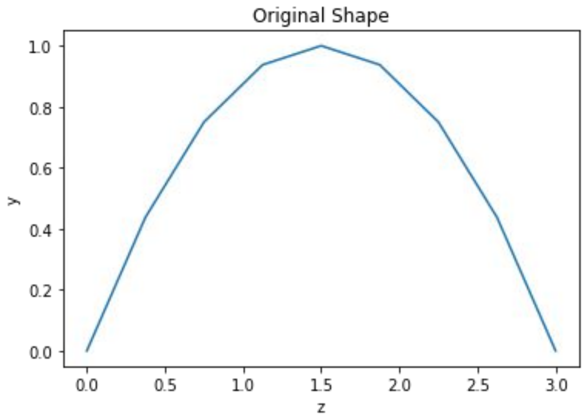
- Damping term is added

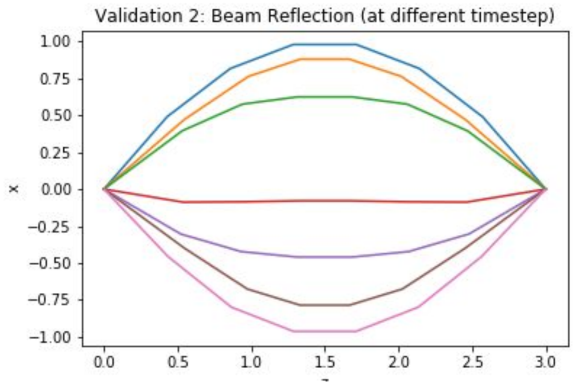
$$\mathrm{F}^t = -c_v v^t + \mathrm{F}_o$$

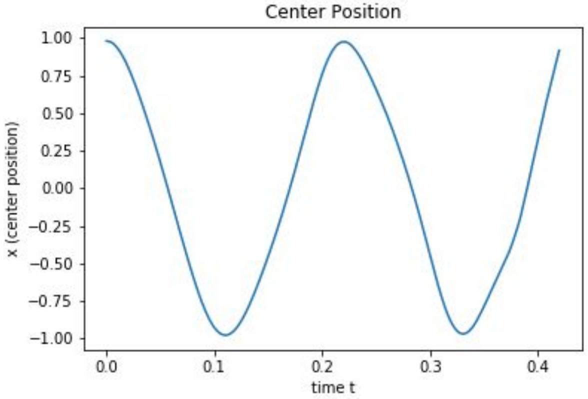
- Viscosity term is added to the total force with constant coefficient.
- Expected to observe steady state after oscillation decays
- *Damping has shown to improve the convergence and stability*



Case 2: Prestressed Beam







- Multiple Element (N=8)
- A prestressed rod (initial deformation)
  - Undeformed length of all elements are equal.

- Expected to observe sinusoidal motion
- Observed how curvature induces the internal torque

Case 3: Timoshenko Beam

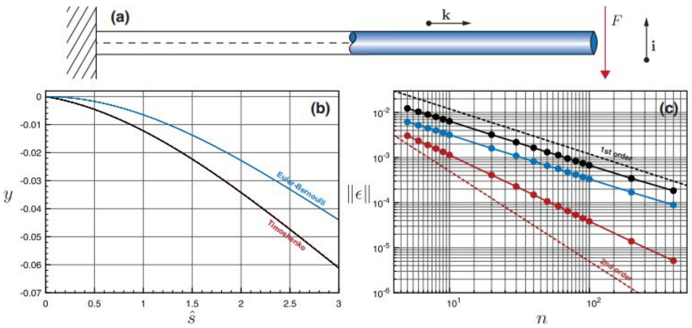
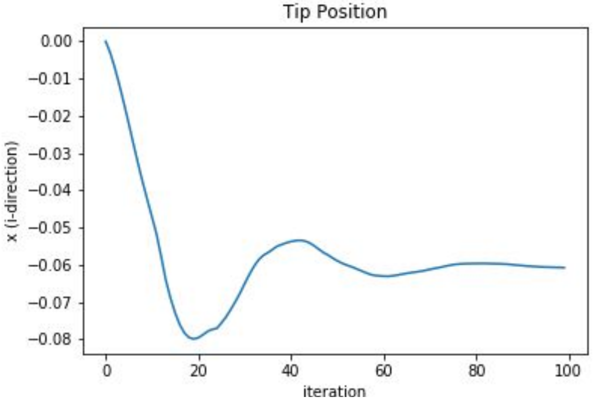
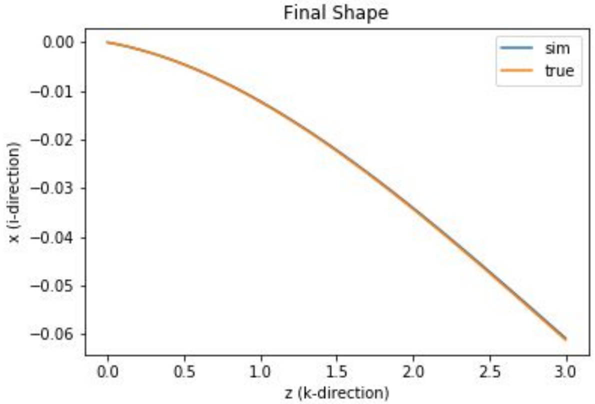


Figure 1: Validation—Deformation of a cantilever beam

*: Image borrowed from project description





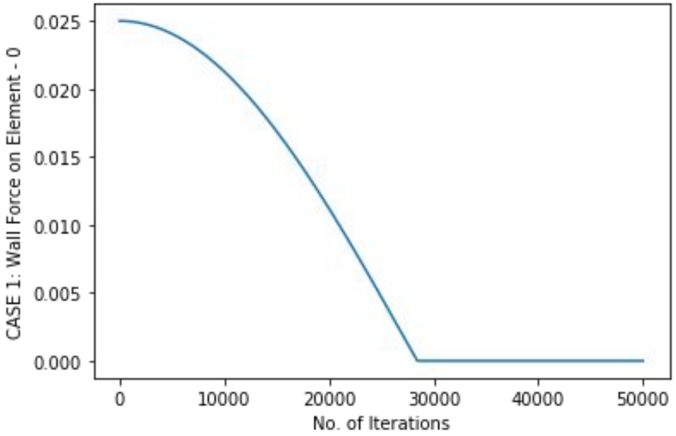
- Multiple Element (N=50)
- Damping term is added like Case 1.
  - Without damping, similar oscillation was observed.
  - Observed significant global error accumulation with long iteration without dampening.
- Confirmed Cosserat rod implementation

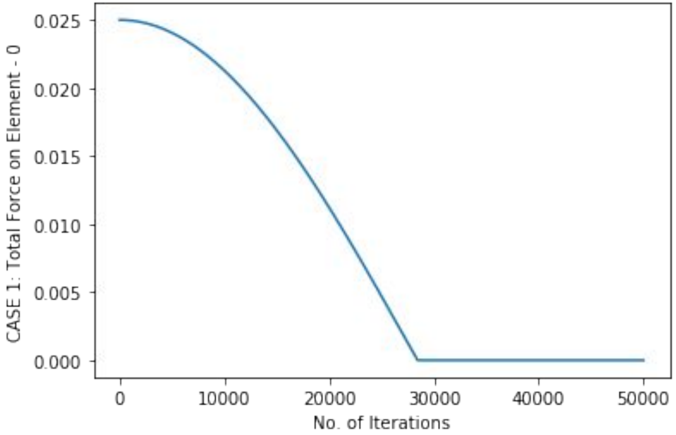
Validation: Wall Force

- Several validation test are designed to check wall force implementation.
- Cases showcase the behavior and variations of wall force and total force acting at the centre of a single element rod (length = 1m)
- Cases also validate the predicted behavior of velocity and position of one of the nodes (any one would be okay since both nodes behave identically)

Case 1: Penetration

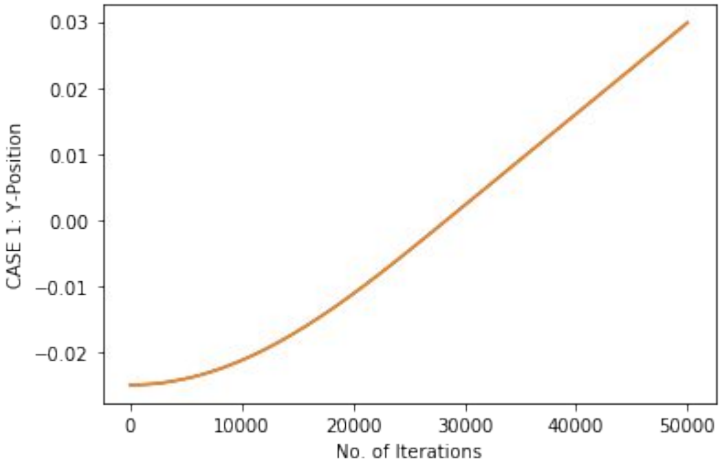
- Condition
    - A rod lying on ground with initial penetration equal to radius (25mm)
    - No gravity acts
- Behavior (Wall Force)
    - Instantaneous force pushing the rod upwards (top figure); wall force decreases as penetration decreases and rod moves upwards
    - When rod leaves contact, no force acts (fig-2)

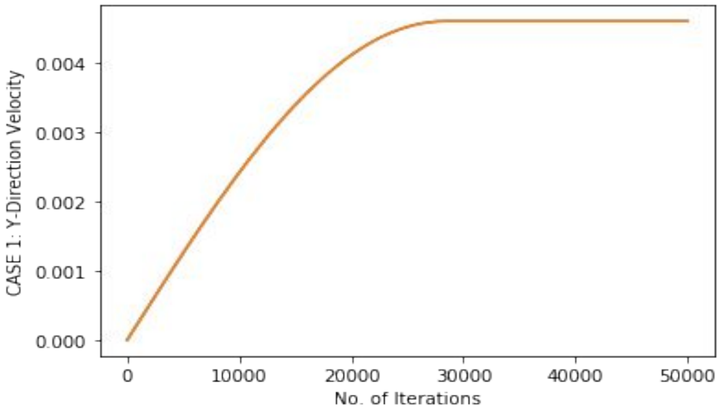




Case 1: Penetration

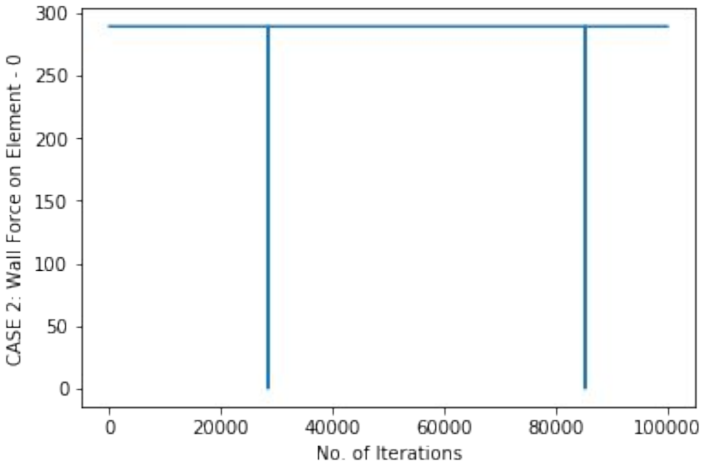
- Behavior (Velocity of Rod)
  - When the rod leaves contact, velocity remains constant because no net force acts, hence y-position keeps increasing linearly
  - This behavior should be fixed once the gravity is acting in y-direction.

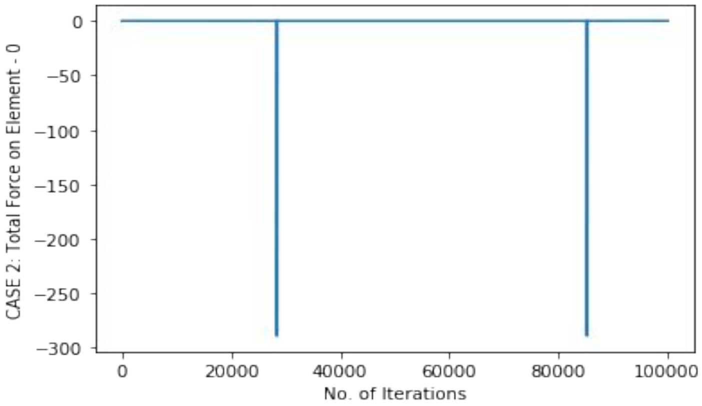




Case 2: Penetration w/ Gravity

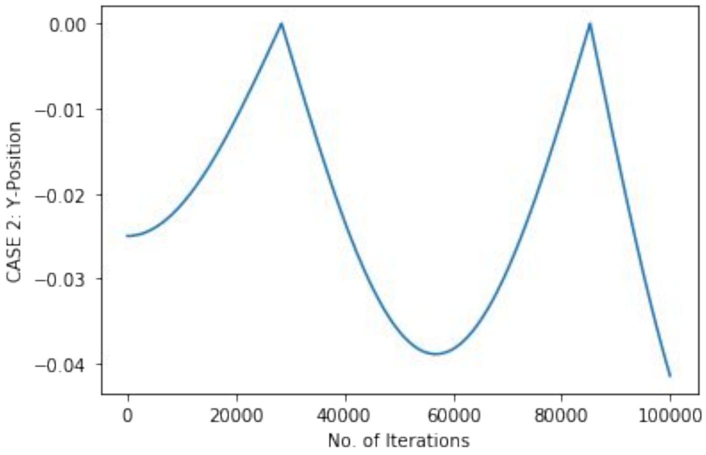
- Condition
  - A rod lying on ground with initial penetration equal to radius (25mm)
  - Gravity acts
- Behavior (Wall Force)
  - Instantaneous force pushes the rod upwards
  - When rod leaves the surface, only gravity acts and rod starts moving downwards
  - Wall force acts again

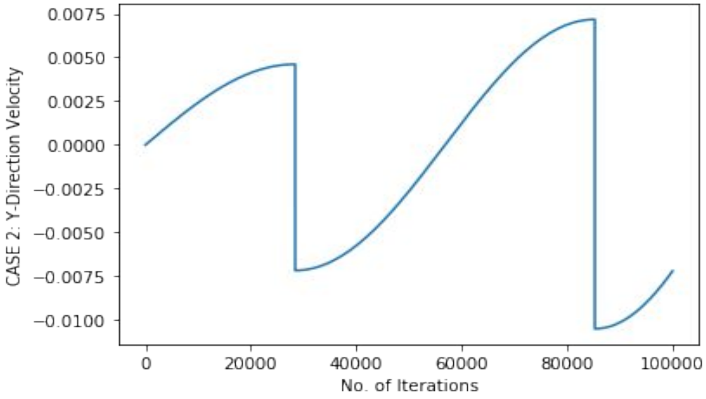




Case 2: Penetration w/ Gravity

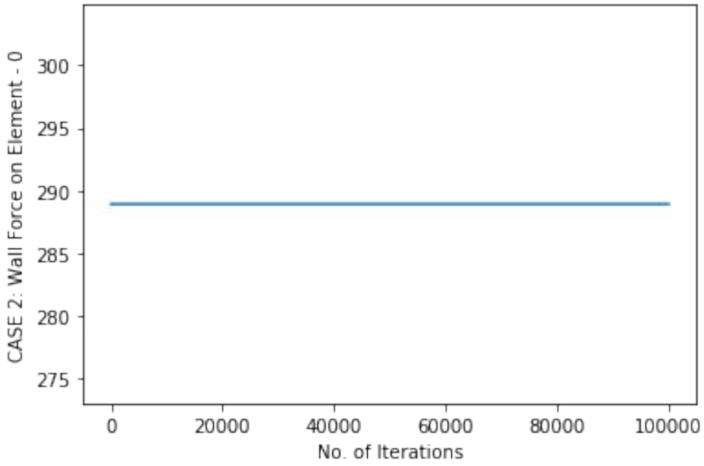
- Behavior (Velocity of Rod)
  - Initially, the rod travels upwards due to wall force
  - As it breaks contact, gravity pulls it back (y-position starts dec.)
  - It again penetrates the ground, and repeat
- Note
  - Increasing wall_stiffness would reduce penetration distance

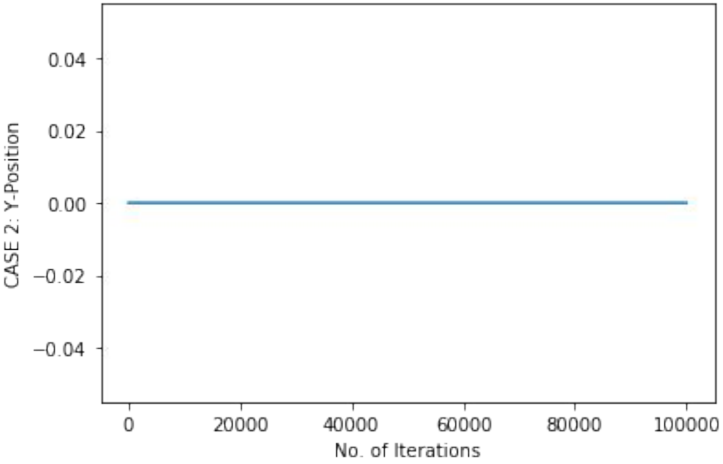




Case 3: Normal Force

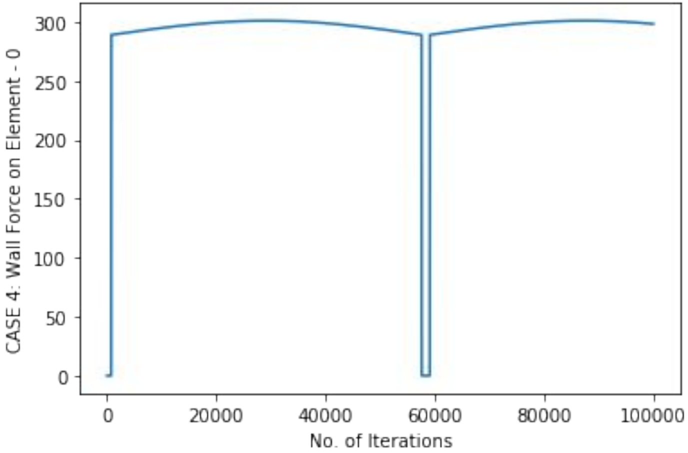
- Condition
  - No initial penetration
  - Gravity acts
- Behavior
  - Constant normal force induced by the ground
  - Zero net force on y-direction
  - No change in y-position and no velocity (rod remains stationary forever)

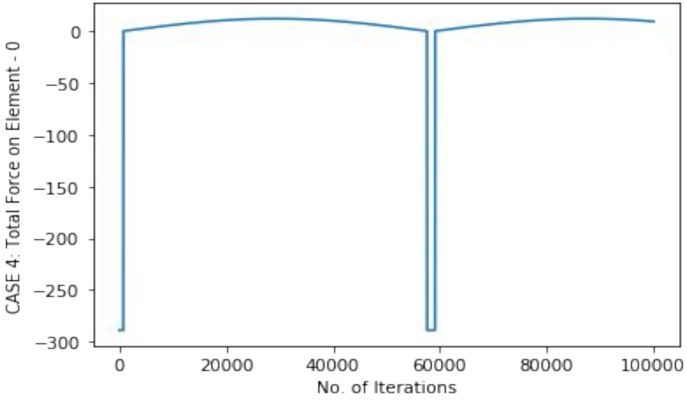




Case 4: Drop

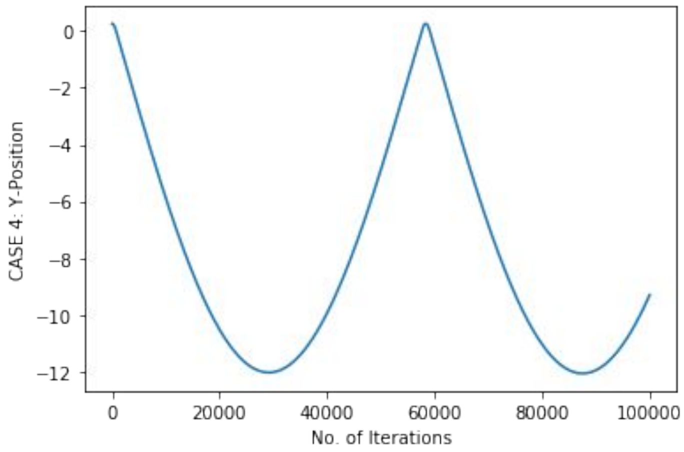
- Condition
  - Initial height of r (0.025 m)
  - Gravity acts
- Behavior (Wall Force)
  - Zero wall force at first until contact
  - Wall force spike and gradually increase
  - Wall force decrease as the body is pushed back

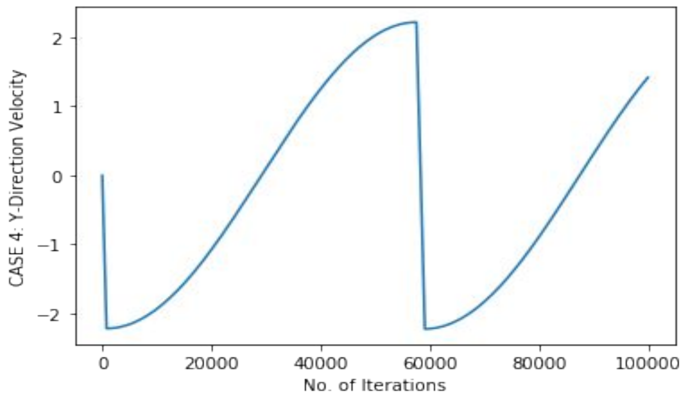




Case 4: Drop

- Behavior (Velocity of Rod)
  - Increase in downward velocity
  - Deceleration once rod hits the ground
  - Then wall force pushes rod upwards
  - Same cycle repeats





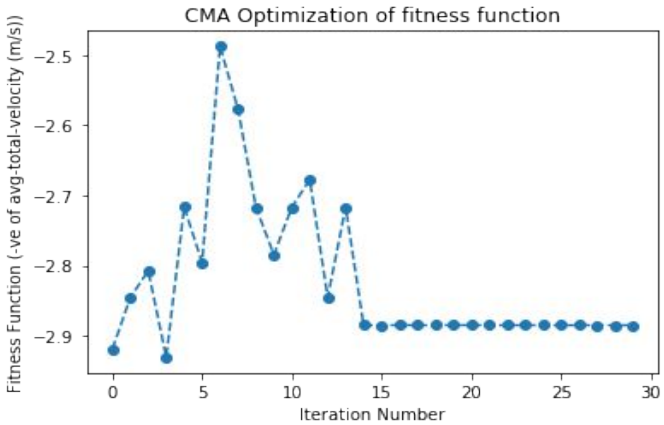
CMA-ES: Optimization

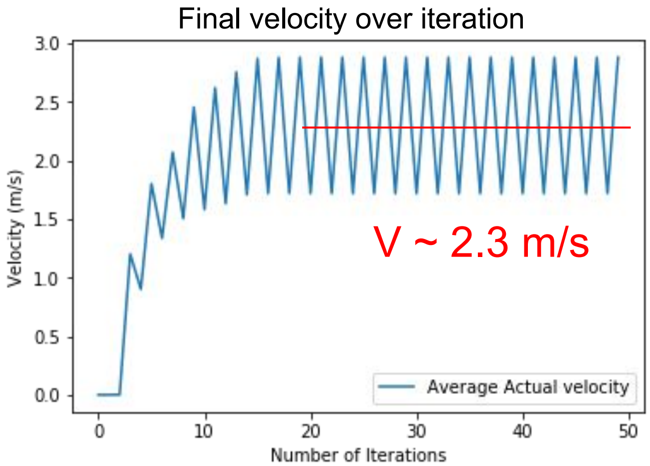
- Fitness function
  - Use CMA-ES algorithm to optimize body speed
  - Body speed: Average speed of all snake elements
  - Snake is optimized to travel in any direction
- Note
  - Wall friction coefficients were scaled up by 10^4 times
    - Faster convergence
    - Reduces simulation period

CMA-ES: Result

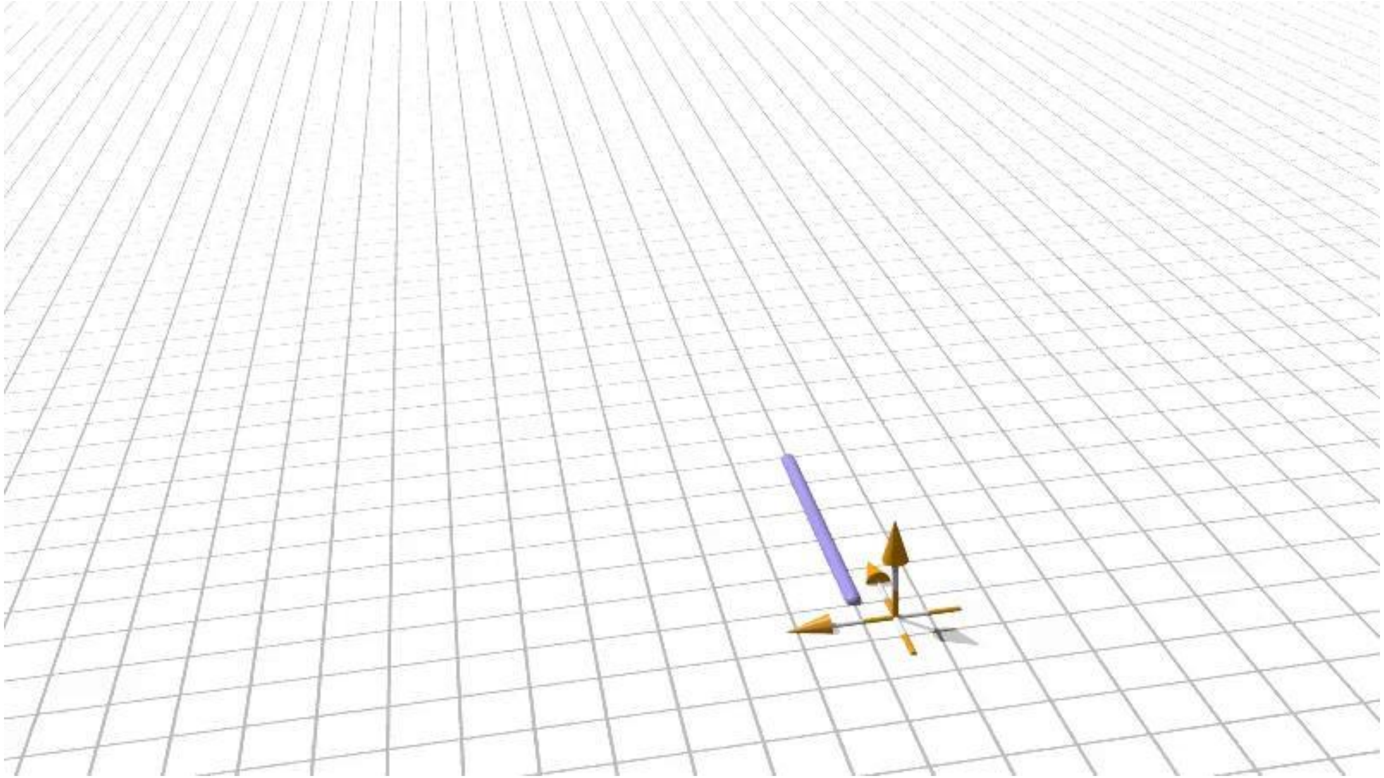
Maximizing the body speed

- A puzzling behavior observed is that in the first iteration, the fitness value is low, however, with subsequent iterations, the value increases (which is counter-intuitive to the working of CMA as seen from project 1)
- CMA does take the best fitness values in the first iteration but while iterating moves to a worse solution in the initial steps. One reason why this would be the case is: large value of sigma (~10). Due to lack of time, variation with different sigma values could not be studied





Visualization: Moving Snake (POV-Ray)



Project 3: Modeling and Simulation of a Brush's Bristle using cosserat rod theory

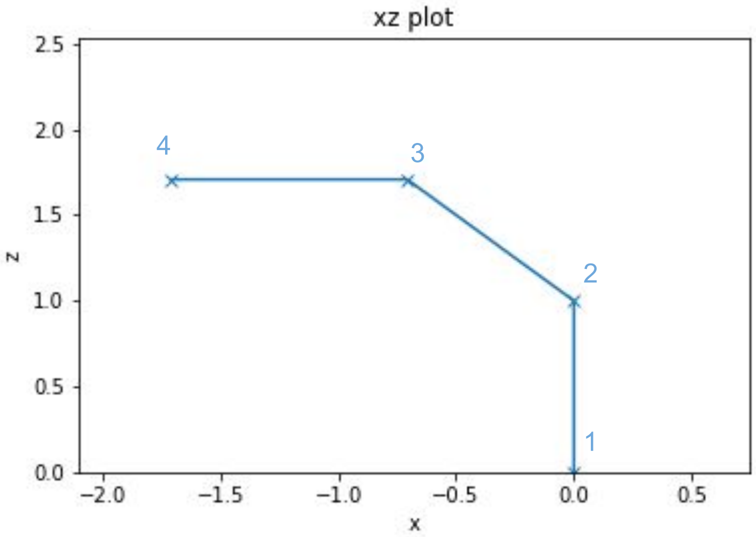
- Hair stylists and dressers can efficiently model and simulate hair for studying various patterns using cosserat rod theory
- This made us wonder if a similar concept can be applied to the bristles of a hair comb
- This led us to the bristles of a toothbrush as these bristles can be modeled using soft rods
- Trying to simulate and observe the behavior of something so common in our everyday life is what had attracted us to this problem

Problem Statement

- Bristle moving at a constant velocity
  - One end of the bristle is constrained to a constant velocity
- Fixed distance from a wall
  - Non-constrained end of the bristle begins in contact with the wall
- Goal is to view behavior of the rod and force on the wall at steady state
- Very little previous work has been completed on modelling toothbrush bristles
- General material properties found for current bristles in use:
  - Tensile modulus: 3.5GPa, Density: 1100 kg/m^3, Diameter: 0.18mm
  - These properties were modified in the simulation to produce a feasible result

Problem Setup

- 4 node / 3 element cosserat rod
  - Initialized with a 90 degree bend
- Elements begin unstrained axially
- Initialized with bending stress as the resting orientation is a straight line
- Nodes 1 and 2 move at a constant 3m/s
- Wall is at z-position of nodes 3 and 4



Result

- Steady state orientation shown
- Bend in the rod has opposite concavity as expected
- Wall stiffness was increased to 1e7 to create a reasonable behavior
    - This causes the large forces and velocities that are shown