Signals and Stochastic Processes

Module 12: Introduction to Sampling

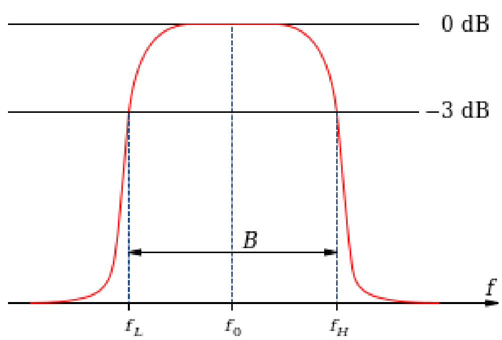
Objective: To Understand the concept of Sampling a signal its reconstruction and various methods of Sampling

Module Introduction:

In the applications of signal processing in real time systems, the mathematical description of signals will not be available. To find the characteristics of the signals, they must be measured and analyzed .If the signal is unknown, the process of analysis starts with the acquisition of the signal, which means measuring and recording the signals over a period of time.Sampling is the acquisition of a continuous signal at discrete time intervals. After Sampling, the analog signal is represented at discrete times only, with the values of the samples equal to those of the original analog signal at the discrete times. In the process of Analog to Digital conversion of a signal, the signal is first sampled, converting a continuous ,analog signal into a discrete time, continuous amplitude signal. Next comes the process of Quantization and digitization. The present Module focuses on the methods of sampling a given low pass signal and its reconstruction from its sampled version.

Module Description:

- A Band-pass signal is a signal containing a band of frequencies i.e. its magnitude spectrum ranges over two frequency limits i.e. $f_L(lower\ frequency\ limit \neq 0)\ and\ f_H(upper\ frequency\ limit \neq 0)$.
- A Band-pass Signal is a signal x (t) whose Fourier transform X(f) is nonzero only in some small band around some "central" frequency f o.



- A Low pass signal is also a band-pass signal with lower frequency limit $f_L = 0$. Then, the upper frequency limit $f_H(\neq 0)$ is referred to as Band-limiting frequency of the signal, and the Low pass signal is also referred to as band-limited signal.

- A signal x(t) band-limited to B Hz will have its Fourier transform $X(f) = 0 for\ |f| > B$

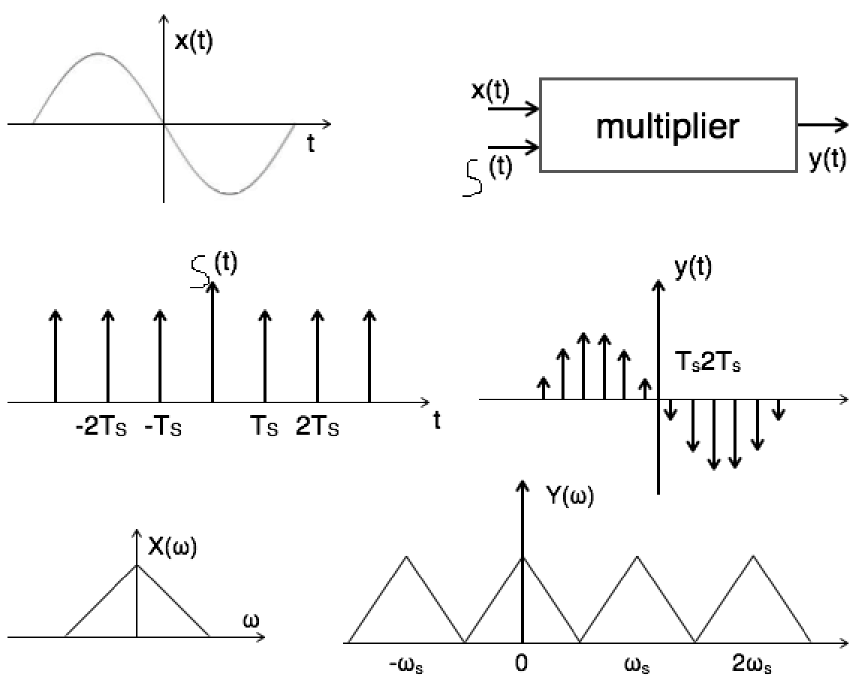
  Sampling Theorem for a Band limited signal is stated as:

  "A signal m(t) band limited to $f_m\ Hz$ can be specified in terms of its samples taken for every $T_s \leq \frac{1}{2f_m}\ sec$, where T_s is referred to as Sampling interval.
  The same can be expressed as $f_s \geq 2f_m\ Hz$ i.e. samples/sec where, f_s is the sampling frequency.

➢ **<u>Ideal Sampling(Instantaneous Sampling):</u>**

  ➢ Ideal Sampling describes a sampled signal as a weighted sum of impulses, with weights being equal to the values of the analog signal at the location of the impulses.

  ➢ An ideally sampled signal may be regarded as the product of an analog signal x(t) and a periodic impulse train.



Where $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ is the sampling signal which is a periodic impulse Train, with a period of T_s, which is referred to as Sampling period or Sampling interval.

➢ Sampled Signal y(t)= x(t).s(t) = x(t).$\sum_{n=-\infty}^{\infty} \delta\,(t - nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s) = \sum_{n=-\infty}^{\infty} x(n)\delta(t - nT_s)$

- Because of the sampling property of the unit impulse , multiplying x(t) by a unit impulse samples the value of the signal at the point at which the impulse is located i.e.$x(t).\delta(t-t_0) = x(t_0).\delta(t-t_0)$

- The discrete signal x(n) represents the sequence of sample values $x(nT_s)$
- The above y (t) in frequency domain is the convolution of the respective signals
- Periodic impulse train in time domain is also a periodic impulse train in frequency domain.
- Since, convolution with an impulse simply shifts a signal i.e{.$P(\omega) * \delta(\omega-\omega_0) = P(\omega-\omega_0)$}, it follows that $Y(\omega) = \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$.
- Thus, $Y(\omega)$ is a periodic function of ω, consisting of shifted replicas of $X(\omega)$

- The trigonometric Fourier series representation of S(t) is given by

$$s(t) = a_0 + \sum_{n=1}^{\infty} a_n\, .Cos(n\omega_s t) + a_n.Sin(n\omega_s t)$$

$$\text{Where } a_0 = \frac{1}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \mathrm{s(t).dt} = \frac{1}{T_s}$$

$$a_n = \frac{2}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \mathrm{s(t)}.Cos(n\omega_s t)\mathrm{dt} = \frac{2}{T_s}$$

$$b_n = \frac{2}{T_s}\int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \mathrm{s(t)}.Sin(n\omega_s t)\mathrm{dt} = 0$$

Hence,

$$s(t) = \frac{1}{T_s} + \sum_{n=1}^{\infty} \frac{2}{T_s}.Cos(n\omega_s t)$$

Sampled Signal y(t)=x(t).s(t)= x(t)[$\frac{1}{T_s} + \sum_{n=1}^{\infty}\frac{2}{T_s}.Cos(n\omega_s t)$]

$$y(t) = \frac{1}{T_s}.x(t) + \frac{2}{T_s}\, x(t).Cos(\omega_s t) + \frac{2}{T_s}x(t).Cos(2\omega_s t) + \frac{2}{T_s}x(t).Cos(3\omega_s t) + ---$$

Taking Fourier transform on both sides

$$Y(\omega) = \frac{1}{T_s}[X(\omega) + X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s) + X(\omega+2\omega_s) + ---]$$

$$= \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s), where\ n = 0, \pm1, \pm2, \pm3, \dots etc.$$

- Thus, sampling in time domain results in periodicity in frequency domain.
- This is called ideal sampling or impulse sampling or Instantaneous sampling.
- Here, the sampling signal is a true impulse train.

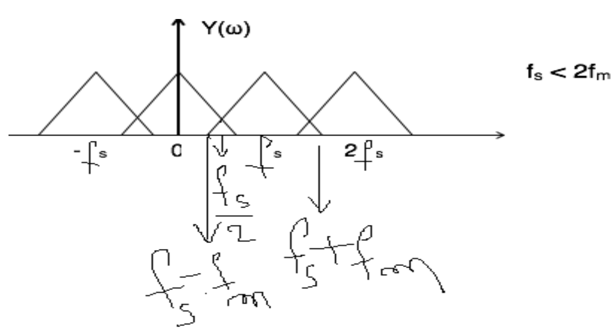
<u>Reconstruction of Signal from its Sampled Version</u>

- Consider the signal

$$y(t) = \frac{1}{T_s}.x(t) + \frac{2}{T_s}\,x(t).Cos(\omega_s t) + \frac{2}{T_s}x(t).Cos(2\omega_s t) + \frac{2}{T_s}x(t).Cos(3\omega_s t) + ---$$

- The first term in the above y(t) is the baseband signal$x(t)$ itself. The remaining terms give rise to DSB-SC signals with carrier frequencies $f_s, 2f_s, 3f_s - etc.$ whose spectra are symmetric about the respective carrier frequencies.
- The spectrum of y (t) is same as original analog spectrum, but repeats at multiples of sampling frequency f_s.
- These higher order components which are centered on the multiples of f_s are referred to as image frequencies.

- Let the Sampling Frequency $f_s < 2f_m$, where f_m is the band limiting frequency of the signal $x(t)$.
- Various frequency components in the sampled signal are $f_m, f_s - f_m$ $(which\ is\ less\ than\ f_m),$ $f_s + f_m, 2f_s \pm f_m, 3f_s \pm f_m$ $etc.$



- The image frequencies centered about f_s will fold over or alias into the baseband frequencies.

- This results an overlap between the spectrum of $x(t)$ itself and the spectrum of the DSB-SC signal centered around f_s.
- The overlapping or aliasing occurs about the point $f = \frac{f_s}{2}$, and this is referred to as Folding frequency
- Accordingly, no filtering operation allow an exact recovery of $x(t)$.
- Since $f_s < 2f_m \left(\frac{f_s}{2} < f_m\right)$ $(referred\ to\ undersampling)$results in aliasing, to avoid aliasing, $f_s \geq 2f_m\ i.e. \frac{f_s}{2} \geq f_m$
- This can be eliminated by selecting a higher sampling rate f'_s.
- The higher sampling rate separates the translated spectra.
- The baseband signal $x(t)$ is filtered prior to sampling so that $x(t)$ is band limited to a new frequency f'_m (<f_m) and is reduced to $\frac{f_s}{2}$ or less.
- Thus, aliasing terms are eliminated before sampling.
- Anti aliasing filters are used to avoid aliasing effect.
- The anti- aliasing filter ideally should remove all frequency components above the fold-over frequency i.e. it should provide sufficient attenuation at frequencies above the folding frequency.

- Let the sampling frequencies $f_s = 2f_m$

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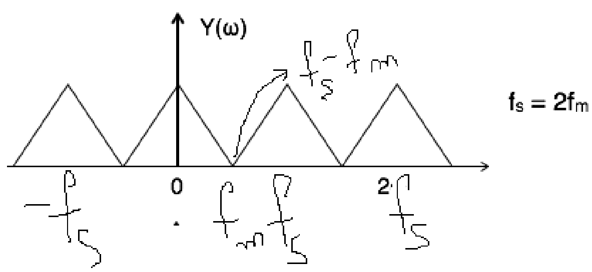
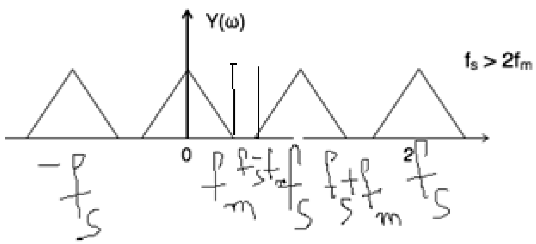


- When the sampled signal is passed through an ideal LPF with cutoff frequency f_m, the filter would pass the signal (t) .
- But, Ideal Filters are not physically realizable, as they are not causal.

- Let the sampling frequencies $f_s > 2f_m$

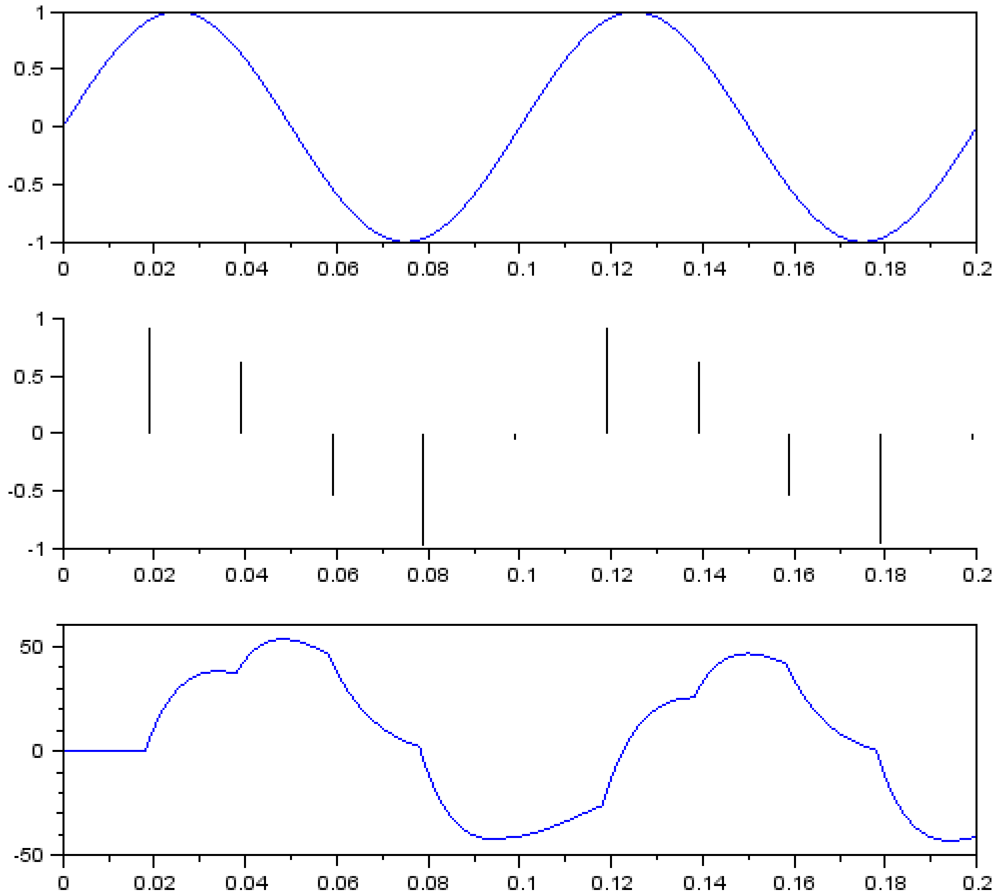


- There is a gap between the upper limit f_m of the spectrum of the base band signal and the lower limit of the DSB-SC spectrum centered around $f_s > 2f_m$.
- Thus, the LPF used to select m(t) need not have an infinitely sharp cutoff , rather, the filter attenuation may begin at f_m , and neet not attain a high value until the frequency $f_s - f_m$.
- This range from f_m to $f_s - f_m$ is referrd to as Guard band , which is always required in practice.
- The reconstruction filter is made more practical.

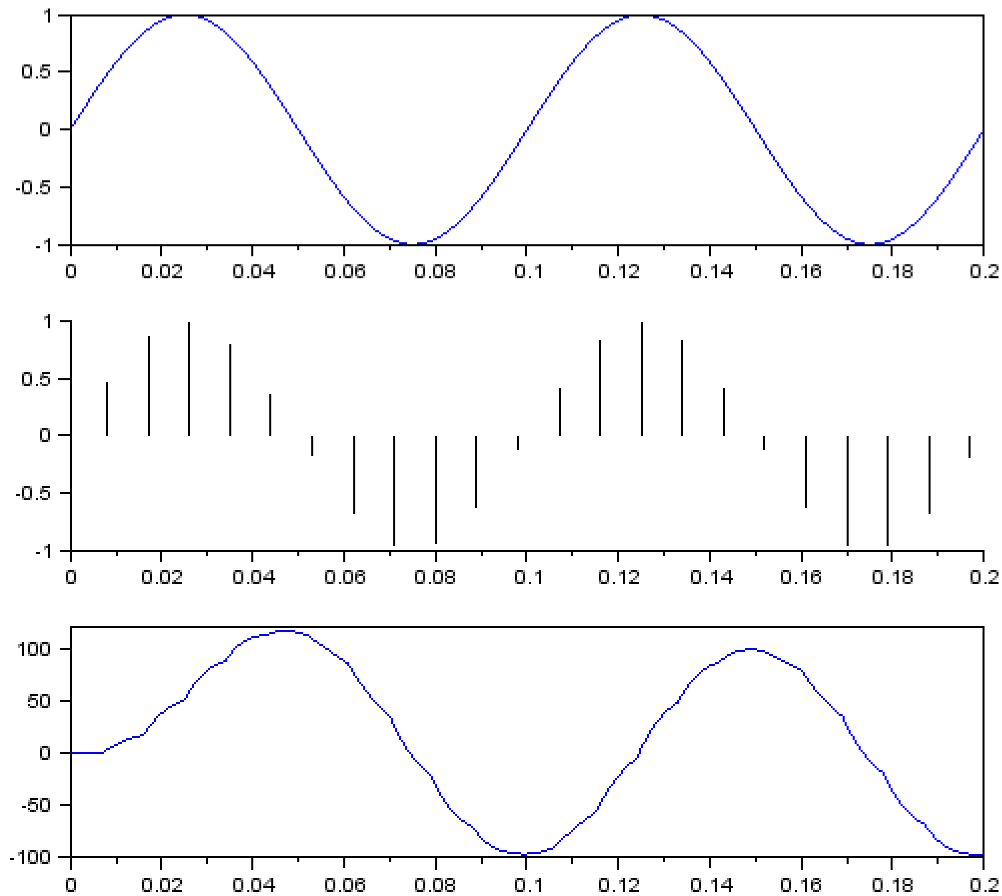
Scilab code for generating an instantaneously sampled Signal

```
n1=input('enter the lower limit of time axis')
n2=input('enter the upper limit of time axis')
s=input('enter the sapacing between the adjacent value of time axis')
t=n1:s:n2;
t1=zeros(1,length(t));
f=input('enter the baseband signal frequency')
x=sin(2*%pi*f*t);
n=input('enter the integer which decides the sampling frequency')
for i=1:length(t)
if n*i<=length(t)
t1(n*i)=1;
end
end
s1=x.*t1;
RC=1/(2*%pi*f)
h=(1/RC)*exp(-t/RC)
y=conv(h,conv(h,s1))
subplot(3,1,1)
plot(t,x)
subplot(3,1,2)
plot2d3(t,s1)
subplot(3,1,3)
plot(t,y(1:length(t))/length(y))
```

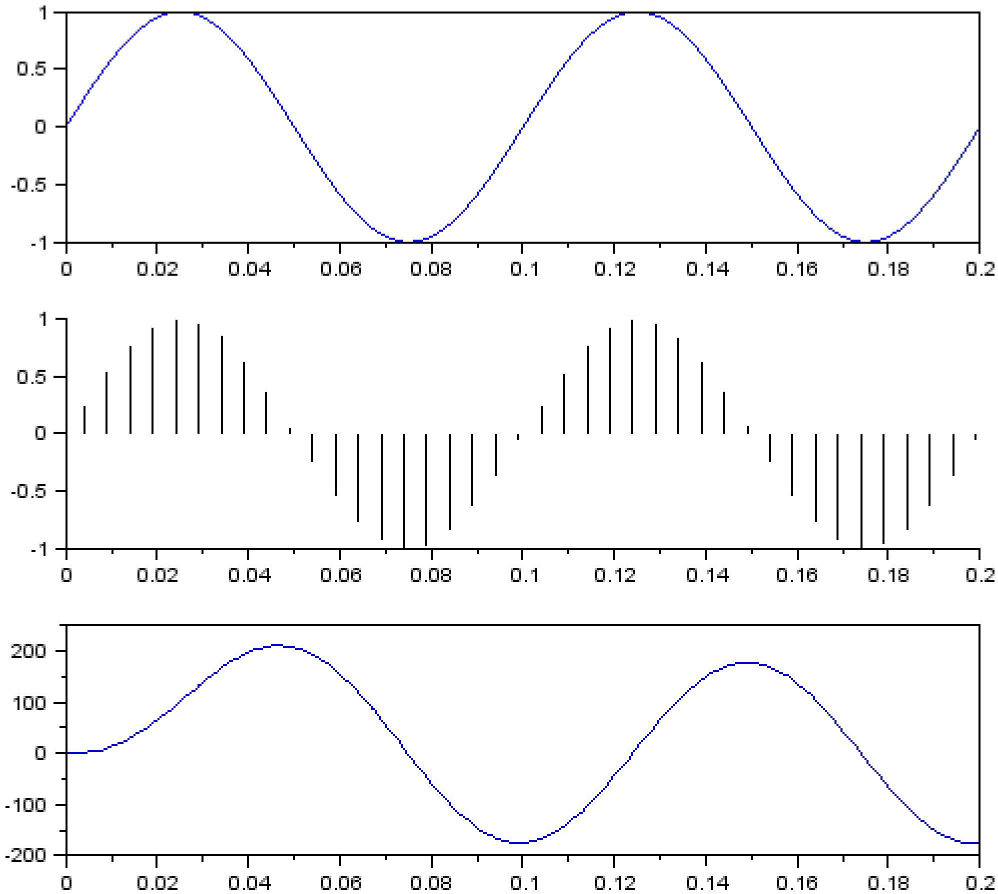
```
1 enter the lower limit of time axis    0
enter the upper limit of time axis    0.2
enter the spacing between the adjacent value of time axis   0.001
enter the baseband signal frequency   10
enter the integer which decides the sampling frequency   20
```



2. enter the lower limit of time axis   0
enter the upper limit of time axis   0.2
enter the spacing between the adjacent value of time axis   0.001
enter the baseband signal frequency   10
enter the integer which decides the sampling frequency   9



3. enter the lower limit of time axis 0
enter the upper limit of time axis 0.2
enter the spacing between the adjacent value of time axis 0.001
enter the baseband signal frequency 10
enter the integer which decides the sampling frequency 5



Sampling Theorem for Band-pass Signal

A band pass signal m(t) having a band width of B Hz and an upper frequency limit of f_H can be specified in terms of its samples taken a t a minimum rate of $\frac{2f_H}{N}$, where N = $\left\lfloor\frac{f_H}{B}\right\rfloor$, i.e. largest integer not exceeding $\frac{f_H}{B}$.

All the higher sampling rates are not necessarily usable, unless they exceed $2f_H$.

Solved Examples:

1. A baseband signal is recovered from its sampled version using an Ideal LPF of cutoff frequency of 1000π. Which of the following sampling periods enables the recovery of the signal from its sampled version using appropriate filtering?

(i) 0.5msec (ii) 2msec. (iii) 1.5msec

Soln.: The cutoff frequency of an ideal LPF used for recovery of the baseband signal from its sampled version, will be equal to the band limiting frequency of the signal, i.e .$\omega_c = \omega_m = 1000\pi$. Hence, $f_m = 500Hz$.As per sampling Theorem, practically used sampling intervals should be less than or equal to Nyquist's interval.

Since, Nyquist's interval is $T_s = \frac{1}{2f_m} = 1msec$, all sampling intervals less that 1msec will enable the recovery of the signal from its sampled version.

2. If m(t) is having a Nyquist's rate of $f_0 Hz$, find the Nyquist's rate for (i) $\int m(t).dt$ (ii) $\frac{d}{dt}m(t)$

(iii) $m^2(t)$

Soln.: Since m(t) is having a Nyquist's rate of $f_0 Hz$, it's band limiting frequency is $\frac{f_0}{2}\ Hz$.

The band limiting frequency of integral of a signal and its derivative will be same as that of the signal. Hence, the Nyquist'srate in the case of (i) and (ii) will be same as that of m(t) i.e. $f_0 Hz$.

If m(t) is band limited to F Hz, then, $m^n(t)$ will be band limited to n.F Hz. Hence, in the present case, $m^2(t)$ will be band limited to $2\left(\frac{f_0}{2}\right) = f_0\ Hz$. Hence, its Nyquist's rate is $2f_0\ Hz$.

3.Find the Nyquist's rate for $x(t) = \frac{sin\,200\pi t}{\pi t}$

Soln: The Fourier Transform of $\frac{sinat}{\pi t}$is $G_{2a}(\omega)$, a Gate function of ω, extending from ω =–a to ω =+a i.e. of width 2a.

By comparing the given x(t) with $\frac{sinat}{\pi t}$, $a = 200\pi$. Thus, x(t) is band limited to $\omega_m = 200\pi\ i.e. f_m = 100Hz$. Thus, Nyquist's rate for x(t) is 200Hz.

4. For the signal $x(t) = 10.cos200\pi t.cos800\pi t$, find the Nyquist's rate using (i) Sampling Theorem for Low pass signal (ii) Sampling Theorem for Band Pass signal.

Soln: $x(t) = 10.cos200\pi t.cos800\pi t = 5[cos1000\pi t + cos600\pi t]$

(i) The highest frequency of x(t) is 500Hz and hence, the Nyquist's rate is 1KHz.

(ii) The signal is having two frequency limits i.e. $f_L = 300Hz\ and\ f_H = 500Hz$. Nyquist's rate is $\frac{2f_H}{N}$, $where\ N\ is\ gthe\ largest\ integer\ not\ exceeding\ \frac{f_{H,}}{B}\ where\ B\ is\ the\ band\ width\ of\ the\ signal.$ Thus, $N = \frac{500}{200} = 2.5\ i.e. N = 2.$ Hence, the corresponding Nyquist's rate is 500Hz.

5.The signals $x(t) = 10.cos1000\pi t\ and\ y(t) = 10.cos50\pi t$, are both sampled with a sampling frequency of 75Hz. Verify that the two sequences of samples so obtained are identical.

Soln: The sampled x(t) is written as $x(n) = 10.cos1000\pi(nT_s) = 10.cos1000\pi\left(\frac{n}{f_s}\right) = 10.cos1000\pi\left(\frac{n}{75}\right) = 10.\cos\left(\frac{200\pi}{15}\right)n.$

Similarly, $y(n) = 10.cos50\pi(nT_s) = 10.cos50\pi\left(\frac{n}{f_s}\right) = 10.cos50\pi\left(\frac{n}{75}\right) = 10.\cos\left(\frac{10\pi}{15}\right)n =$

$$x(n) = 10.\cos\left(\frac{200\pi}{15}\right)n = 10.\cos\left[\left(14\pi - \frac{10\pi}{15}\right)n\right] = 10.\cos\left(\frac{10\pi}{15}\right)n = y(n)$$

Hence, verified.

6.A band pass signal $X(t) = Cos\ 10\omega_0 t + Cos11\omega_0 t + Cos12\omega_0 t$ is multiplied by an impulse train $S(t) = I.\sum_{k=-\infty}^{\infty}\delta(t - \frac{k}{8f_0})$. The sampled signal is passed $X_s(t) = S(t).X(t)$ is passed through a rectangular LPF with a bandwidth of $2f_0$. Write the expression for the output of the filter.

Soln.:

Since, S(t) is the periodic impulse train, its Fourier series representation is given by:

$a_0 = 8f_0 I; an = 2(8f_0 I);\ bn = 0$and hence $S(t) = 8f_0 I[1 + 2\sum_{k=1}^{\infty} Cos(8k\omega_0 t)]$

The sampled signal $X_s(t) = S(t).X(t) =$

$8f_0 I[1 + 2\sum_{k=1}^{\infty} Cos(8k\omega_0 t)].[\ Cos\ 10\omega_0 t + Cos11\omega_0 t + Cos12\omega_0 t]$

=$8f_0 I[\ Cos\ 10\omega_0 t + Cos11\omega_0 t + Cos12\omega_0 t]$ +$8f_0 I[Cos\ 18\omega_0 t + Cos2\omega_0 t + Cos19\omega_0 t + Cos\ 3\omega_0 t + Cos2\omega_0 t + Cos20\omega_0 t + Cos\ 4\omega_0 t] + \cdots$

The output of the LPF is $8f_0 I.Cos2\omega_0 t$

7.The signal $X(t) = cos5\pi t + 0.5Cos10\pi t$ is instantaneously sampled. The interval between samples is T_s. (i) Find the maximum allowable time for T_s. (ii) The sampling signal is $S(t) = 5.\sum_{k=-\infty}^{\infty}\delta(t - 0.1k)$. The sampled signal $X_s(t) = S(t).X(t)$ =$\sum_{k=-\infty}^{\infty} I_k\delta(t - 0.1k)$consists of a Train of impulses, each with a different strength I_k. Find $I_0, I_1, I_2\ and$ show that $I_k = I_{k+4}$

Son.:

(i) The given signal $X(t)$ is a Band pass signal with an upper frequency limit of 5Hz. The Nyquist's rate for the Band pass signal is $\frac{2f_H}{N}$, where N is the largest integer not exceeding $\frac{f_H}{B}$, where B is the band width of the signal.

$$N = \frac{f_H}{B} = \frac{5}{2.5} = 2 \; the\ respective\ Nyquist's\ rate\ (f_s)_{min}\ is\ 5Hz.$$

Hence, $(T_s)_{max} = \frac{1}{(f_s)_{min}} = 0.2sec.$

(ii)$X_s(t) = S(t).X(t)$ =$[cos5\pi t + 0.5Cos10\pi t][\sum_{k=-\infty}^{\infty} 5\delta(t - 0.1k)]$

$$=5.\sum_{k=-\infty}^{\infty}[cos5\pi(0.1k) + 0.5Cos10\pi(0.1k)]\delta(t - 0.1k)$$

By comparing this with the given expression for the sampled signal,

$$I_k = 5[cos5\pi(0.1k) + 0.5Cos10\pi(0.1k)]$$

Hence, $I_0 = 5;\ I_1 = -2.5;\ I_2 = -2.5$

$$I_{k+4} = 5[\cos\{0.5\pi(k+4)\} + 0.5Cos\{\pi(k+4)\}]$$

$$=5[\cos\{2\pi + 0.5k\pi\} + 0.5Cos\{4\pi + k\pi\}]$$

$$5[cos5\pi(0.1k) + 0.5Cos10\pi(0.1k)] = I_k$$

8. A modulated signal is given by $Y(t) = m(t).Cos(40000\pi t)$, where the baseband signal m(t) has frequency components less than 5kHz.Find the minimum required sampling frequency for $Y(t)$.

Soln.: Any Modulated signal is a Band pass signal and the given signal is a DSB-SC signal with Lower frequency limit $f_L = 15KHz$ and the Upper frequency limit $f_H = 25KHz$

The corresponding Nyquist's rate is $\frac{2f_H}{N}$ $and\ N = \frac{25}{10} = 2.5\ i.e.2\ and\ hence$ the minimum sampling rate required is 25KHz.

9. Find the Nyquist's rate of sampling for $X(t) = 16X10^4 sinc^2(400t) * 10^6 sinc^3(100t)$

Soln.: Considering $X(t) = X_1(t) * X_2(t),\ X(f) = X_1(f).X_2(f)$

The Fourier Transform of y(t)= $\frac{sinat}{\pi t}$ is $G_{2a}(\omega)$, a Gate function of ω, extending from ω =–a to ω =+a i.e. of width 2a and Fourier Transform of $y^2(t)$ will be extending from $\omega = -2a$ to $\omega = 2a$.

$$X_1(t) = 16X10^4 sinc^2(400t) = 16X10^4.\left(\frac{\sin 400\pi t}{400\,\pi t}\right)^2 \quad and\ its\ Fourier\ Transform$$

extends over $f = -400\ Hz\ to\ f = +400Hz.$

Similarly, the Fourier Transform of $X_2(t)$ extends over $f = -150\ Hz\ to\ f = +150Hz.$

The product $X(f) = X_1(f).X_2(f)$ extends over $f = -150\ Hz\ to\ f = +150Hz$ and the corresponding Nyquist's rate of sampling is 300Hz.

10. A 1KHz sinusoidal signal is ideally sampled at 1500 samples/sec and the sampled signal is passed through an ideal LPF with cutoff frequency of 800Hz.Find the frequency of the output of the filter.

Soln.: The frequencies present in the sampled signal are 0.5Khz, 1KHz, 1.5KHz, 2KHz, 4KHz etc. Hence, the output of the filter is having a frequency of 0.5KHz.

Exercise Problems:

1. Consider a sampled signal$(t) = 5X10^{-6}.X(t).\sum_{k=-\infty}^{\infty}\delta(t - 0.1k\,T_s)]$, where $X(t) = 10.Cos(8\pi x10^3 t)$ and $T_s = 100\mu sec$. When $Y(t)$ is passed through an ideal LPF with a cut off frequency of 5KHz. Find the output of the filter.

2.Find the Nyquist's rate of sampling for the signal $X(t) = sinc(700t) + sinc(500t)$

3.A signal $X(t) = 100.Cos(24\pi x10^3 t)$ is ideally sampled with a sampling period of $50\mu sec$ of 15KHz. and then passed through an ideal LPF with cutoff frequency. Find the frequencies at the output of the filter.

4.A signal m(t) with band width 500Hz is first multiplied by a signal $g(t) = \sum_{k=-\infty}^{\infty}(-1)^k\delta(t - 0.5x10^{-4}\,k)]$. The resulting signal is passed through an ideal LPF with band width 1KHz. Find the output of the filter.

5.Find the minimum sampling rate required to construct the following signal from its samples without distortion.

$$X(t) = 5\left(\frac{sin2000\pi t}{\pi t}\right)^3 + 7\left(\frac{sin2000\pi t}{\pi t}\right)^2$$

6.Find the minimum sampling rate required to construct the following signal from its samples without distortion.

$$X(t) = \frac{sin500\pi t}{\pi t}.\frac{sin700\pi t}{\pi t}$$

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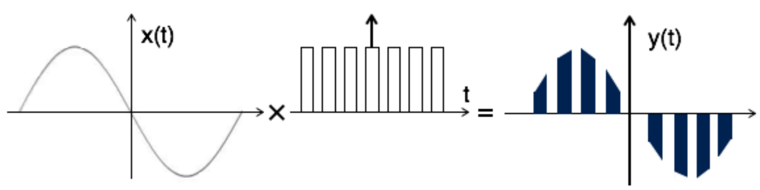
Module 13: Practical aspects of Sampling

Objective: To Understand various practical SamplingMethods

Module Description: In verifying the Sampling Theorem, ideal samples are assumed, which are obtained my multiplying a signal x(t) by an impulse train which is physically non existent. In practice, the x(t) is multiplied with by a train of pulses of finite width. Similar to Ideal Sampling, the signal x(t) can be recovered from its sampled version, by low-pass filtering the sampled signal. The present Module introduces various sampling methods that are used in practice.

- **<u>Natural Sampling</u>**
  - Also referred to as Pulse Amplitude Modulation
  - Since, an impulse is characterized with zero width, this can notbe used practically, as pulse width cannot be zero.
  - For all practical purposes, an impulse is approximated by a rectangular pulse of infinitesimal width, and even then, the energy(strength) of each sample is low, as width is low. $[strength\ of\ a\ sample = (width)X(Height)]$
  - The strength of the sample can be increased by increasing its width.
  - Natural sampling is similar to impulse sampling, except the impulse train is replaced by
  - pulse train of period T_s $i.e. the\ sampling\ signal\ is\ s(t) = \sum_{n=-\infty}^{\infty} P(t - nT_s)$



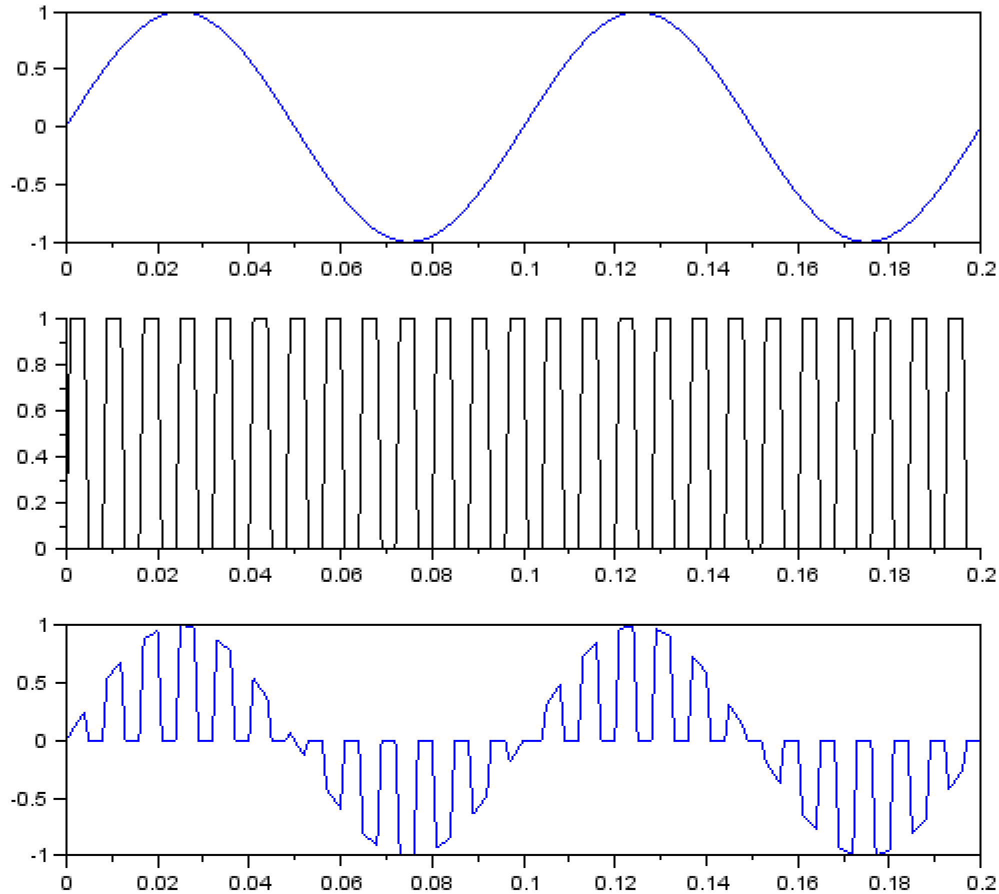
Sampled Signal $y(t) = x(t)\ X\ s(t) = x(t)X\ \sum_{n=-\infty}^{\infty} P(t - nT_s)$

The top of the sample follows the envelope of the base band signal.

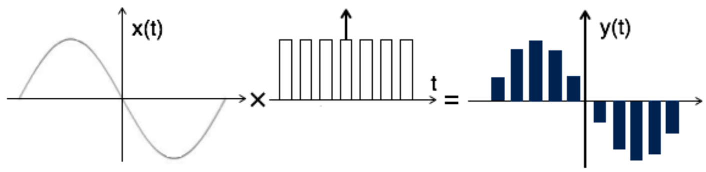
Scilab code for generating a Naturally sampled Signal

```
n1=input('enter the lower limit of time axis')
n2=input('enter the upper limit of time axis')
s=input('enter the spacing between the adjacent value of time axis')
t=n1:s:n2;
t1=ones(1,length(t));
f=input('enter the baseband signal frequency')
x=sin(2*%pi*f*t);
n=input('enter the integer which decides the width of the pulse')
s=[0 ones(1,n) zeros(1,n)]
while length(s)<=length(t)
   s=[s ones(1,n) zeros(1,n)]
end
s(length(t)+1:length(s))=[];
NAT=s.*x;
subplot(3,1,1)
plot(t,x)
subplot(3,1,2)
plot2d1(t,s)
subplot(3,1,3)
plot(t,NAT)
```

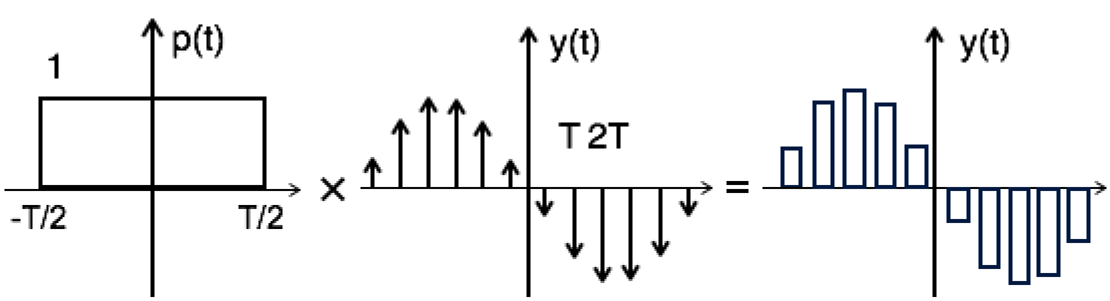
```
enter the lower limit of time axis     0
enter the upper limit of time axis    0.2
enter the spacing between the adjacent value of time axis     0.001
enter the baseband signal frequency     10
enter the integer which decides the width of the pulse   4
```



- **Flat Top Sampling**
- In a Naturally sampled signal, even though the strength of the sample is increased, it is not uniform throughout the sample, since the top of the sample is following the envelope of the base band signal.
- The strength can be made uniform, if the top of the sample is flat.
- During transmission, noise is introduced at top of the transmission pulse which can be easily removed if the pulse is in the form of flat top.
- Here, the top of the samples are flat i.e. they have constant amplitude. Hence, it is called as flat top sampling or practical sampling.
- Flat top sampling makes use of sample and hold circuit.



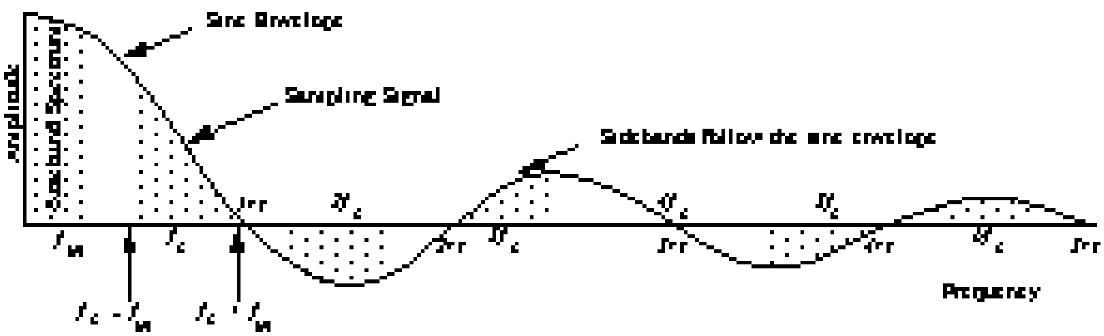
- Theoretically, the sampled signal can be obtained by convolution of rectangular pulse p(t) with ideally sampled signal say $y_\delta(t)$ i.e. $y(t) = P(t) * y_\delta(t)$
- To get the sampled spectrum, consider Fourier transform on both sides for $y(t) = P(t) * y_\delta(t)$ i.e. $Y(\omega) = F[P(t) * y_\delta(t)]$.
- From Convolution Property, $Y(\omega) = P(\omega) * Y_\delta(\omega)$, where $P(\omega) = T.sa(\frac{\omega T_s}{2})$



- <u>Extent of Distortion in Flat Top Sampling:</u>
- A flat top sampled pulse can be generated by passing the instantaneously sampled signal through a network which broadens a pulse of duration 'dt' (i.e. an impulse) into a pulse of duration τ.
- Fourier Transform of impulse of strength 'dt' at t=0 =dt
- Fourier Transform of pulse of unit amplitude and width $'\tau'$ $=\tau.\frac{\sin(\frac{\omega\tau}{2})}{(\frac{\omega\tau}{2})}$

- Transfer Function of pulse broadening network is $H(w) = \frac{\tau}{dt} . \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)}$
- Let the x(t) with Fourier Transform X(ω) be bandlimted to f_m and sampled .
- In the range 0 to f_m, the transform of the flat-topped sampled signal is $H(\omega).X(\omega)$ i.e.x

  F[Flat Top sampled x(t)]=$\frac{\tau}{dt} . \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)} . X(\omega)$.
- Each component follows sinc envelope

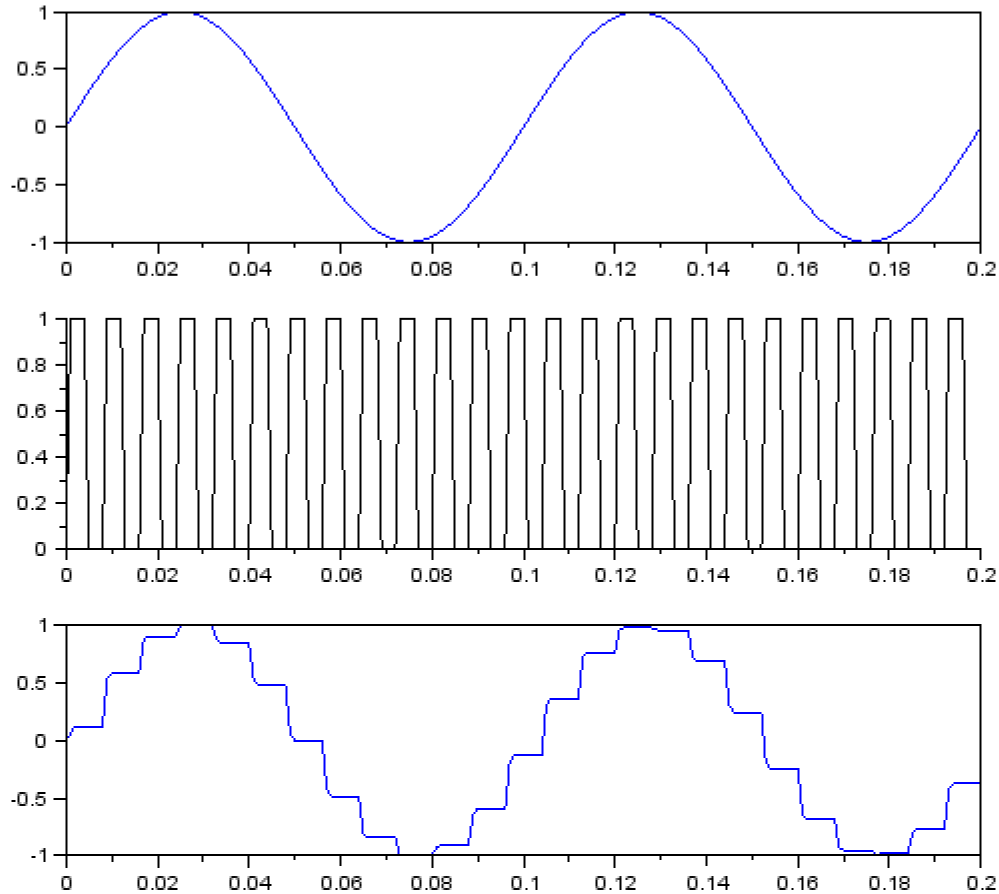


- The distortion is referred to as Aperture effect Distortion.
- This may corrected by using an equalizer in cascade with the reconstruction filter.
- To avoid the loss of recovery in the baseband signal, the width of each sample should be bounded as $\tau \ll \frac{1}{f_m}$.

Scilab code for generating a Flattop sampled Signal

```
n1=input('enter the lower limit of time axis')
n2=input('enter the upper limit of time axis')
s=input('enter the spacing between the adjacent value of time axis')
t=n1:s:n2
f=input('enter the baseband signal frequency')
x=sin(2*%pi*f*t);
n=input('enter the integer which decides the width of the pulse')
s=[0 ones(1,n) zeros(1,n)]
while length(s)<=length(t)
   s=[s ones(1,n) zeros(1,n)]
end
s(length(t)+1:length(s))=[];
FLA=s.*x;
for i=1:length(s)
if s(i)==1
FLA(i+1:i+n)=FLA(i+1)
end
end
subplot(3,1,1)
plot(t,x)
subplot(3,1,2)
plot2d1(t,s)
subplot(3,1,3)
plot(t,FLA)
```

```
enter the lower limit of time axis    0
enter the upper limit of time axis   0.2
enter the spacing between the adjacent value of time axis    0.001
enter the baseband signal frequency   10
enter the integer which decides the width of the pulse   4
```



1
0.5
0
-0.5
-1
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
1
0.8
0.6
0.4
0.2
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
1
0.5
0
-0.5
-1
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

&&&&&&&&&&