

## Unit - IV

# Probability

~~Introduction to Probability~~  
~~Random Variables; Expectation (Discrete & Continuous)~~

### Problems:

- ① A die is tossed thrice. A success is getting '1 or 6' on a toss. Find the mean and variance of the number of successes. [Mean = 1, var =  $2/3$ ] [Bsg Pg-953]
- ② A random variable  $X$  has the following probability func.
- |        |   |     |      |      |      |       |        |          |
|--------|---|-----|------|------|------|-------|--------|----------|
| $x$    | 0 | 1   | 2    | 3    | 4    | 5     | 6      | 7        |
| $P(x)$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2+k$ |
- i) Find  $k$       ii) Evaluate  $P(X < 6)$ ,  $P(X \geq 6)$   
 iii)  $P(0 < X < 5)$   
 [Ans: i)  $k = 1/10$     ii)  $8/100$ ,  $19/100$     iii)  $4/5$ ] [Bsg-954]
- ③ The probability density func of a variate  $X$  is
- |        |     |      |      |      |      |       |       |
|--------|-----|------|------|------|------|-------|-------|
| $x$    | 0   | 1    | 2    | 3    | 4    | 5     | 6     |
| $P(x)$ | $k$ | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |
- i) Find  $P(X < 4)$ ,  $P(X \geq 5)$ ,  $P(3 < X \leq 6)$  [Bsg-953]  
 ii) What will be the minimum value of  $k$  so that  $P(X \geq 2) > 3$
- ④ Four coins are tossed. What is the expectation of the number of heads? [Ans 2] [Bsg-960]
- ⑤ A R.V.  $X$  has the following probability func.
- |        |     |     |     |      |     |     |
|--------|-----|-----|-----|------|-----|-----|
| $x$    | -2  | -1  | 0   | 1    | 2   | 3   |
| $P(x)$ | 0.1 | $k$ | 0.2 | $2k$ | 0.3 | $k$ |
- Find  $k$  and find Mean & Var [Pg-960]  
 [Ans  $k = 0.1$ , Mean = 0.8, Var = 2.232]
- ⑥ Obtain the distribution function of the total no. of heads occurring in 3 tosses of an unbiased coin. [Pg-960 Bsg]

# Probability Distribution

~~Binomial Distribution~~ (Statement)

Binomial Distribution :

- 1) Find Mean & Variance (Derivation)
- 2) The probability that a pen manufactured by a company will be defective is  $\frac{1}{10}$ . If 12 such pens are manufactured find the probability that
  - i) Exactly two will be defective. (Ans: 0.2301)
  - ii) At least two will be defective. (Ans: 0.3412)
  - iii) None will be defective (Ans: 0.2833) [BSG Pg 963]
- 3) In 256 sets of 12 tosses of a coin in how many cases one can expect 8 heads & 4 tails. (Ans 30.9531) [BSG 963]
- 4) Compute the probability at least two 'six' in rolling a fair die four times [13.2%] [BSG Kreyzig-1080]
- 5) Determine the binomial dist<sup>n</sup> for which ~~mean = 2~~ (var) mean = 2 (variance). And mean + var = 3. Also find  $P(X \leq 3)$  [BSG Hint:  $n=4, p=q=\frac{1}{2}$ ] [Ans: 15/16 BSG-964].
- 6) Three fair coins are tossed simultaneously, find the probability of the random variable  $X = \text{No. of heads}$  and compute the probabilities of obtaining no heads, precisely one head, at least one head, not more than three heads. [Kreyzig - 1083].
- 7) Fit a B.D the following freq<sup>n</sup> distribution

$X$	0	1	2	3	4	5	6
$f(x)$	13	25	52	58	32	16	4

[BSG 965]  
[200 (0.554 + 0.446)<sup>6</sup>]

## Poisson Distribution

- 1) Find ...
- 2) If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get bad reaction  
[BSG-966]  
[Ans - 0.32]
- 3) If a random variable has a P.D such that  $P(1) = P(2)$  find i) mean ii)  $P(4)$   
[BSG-967]  
[Mean = 2,  $P(4) = 2/3 e^{-2}$ ]
- 4) If the probability of producing a defective screw is  $P = 0.01$  what is probability a lot of 100 screws will contain more than 2 defectives? [Kreyszig-1081]  
[Ans: 0.108]
- 5) Parking Problem:  
If on the average, two cars enter certain parking lot per minute what is the probability that during any given minute four or more cars enter the lot? [Kreyszig-1082, Ans: 1403-1]
- 6) Fit a Poisson distribution to the following.  
[BSG-967]
- | x    | 0  | 1  | 2  | 3 | 4 |
|------|----|----|----|---|---|
| f(x) | 46 | 38 | 22 | 9 | 1 |

## Exponential Distribution

- 1) Find ... (Exponential)
- 2) If  $x$  is an exponential variate with mean 3. find  
i)  $P(x < 1)$  ii)  $P(x < 3)$  (KSC Pg 275)  
Ans i) 0.7165 ii) 0.6321
- 3) The length of a telephone conversation in a booth has been an exponential distribution & found on an average to be 5 minutes. Find the probability that a random call made from this booth i) Ends less than 5 min ii) betn 5 to 10 min



4) Life of a battery is a random variable which has an exponential distribution with mean 200 hrs. Find the probability that the life of battery is

i) less than 100 hrs Ans: 0.3935

ii) between 400 to 600 hrs Ans 0.0855

5) The duration of shower in a certain town during the period of depression is exponentially distributed with mean 5 minutes. What is the probability that the duration of a shower is

i) 10 min or more ii) less than 10 min

\* Normal distributions

1) ~~Find the mean, variance~~

2) Find  $X$  is a normal variate with mean 30 & S.D 5. Find the probabilities that

i)  $26 \leq X \leq 40$  ii)  $X \geq 45$  & iii)  $|X - 30| > 5$   
 Ans 0.7653 0.0044 0.3174 (BSG 971)

3) In a test on <sup>2000</sup> electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hrs. & S.D of 60 hrs. Estimate the number of bulbs likely to burn for

a) more than 2150 hrs b) less than 1950 hrs  
 & c) more than 1920 hrs & but less than 2160 hrs  
 Ans a) 67 b) 184 c) 1909

4) The mean height of 500 students is 151 cm & the standard deviation is 15 cm. Assuming that the heights are normally distributed, find how many students' heights lie between 120 & 155 cm. (BSG 974)

Ans: ~~291~~ 291