UNIT - IV

PROBABILITY

* A random variable X has the following probability function for various values of x

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

i) Find K ii) Evaluate $P(x<6)$, $P(x \geq 6)$ and $P(3 \leq x \leq 6)$

Sol^n: We have i) $\boxed{\sum P(x) = 1}$

$\therefore 0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$\Rightarrow 10K^2 + 9K - 1 = 0 \quad \therefore K = \frac{1}{10}$

Hence

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$2/100$	$17/100$

ii) $P(x<6) = 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} = 0.81$

$P(x \geq 6) = P(6) + P(7) = \frac{2}{100} + \frac{17}{100} = 0.19$

$$P(3 \leq x \leq 6) = P(3) + P(4) + P(5) + P(6)$$
$$= \frac{2}{10} + \frac{3}{10} + \frac{1}{100} + \frac{2}{100}$$
$$= 0.53$$

* A Random variable x has the following probability function

X	: -2	-1	0	1	2	3
P(X)	: 0.1	K	0.2	2K	0.3	K

Find K and find mean and variance.

Soln: We have $\sum P(x) = 1$

$$0.1 + K + 0.2 + 2K + 0.3 + K = 1$$

$$\Rightarrow K = 0.1$$

$$\text{Mean} = E(X) = \sum x\, P(x)$$

$$= -2(0.1) + (-1)(0.1) + 0(0.2) + (1)(0.2) + 2(0.3) + 3(0.1)$$

$$E(X) = 0.8$$

$$\text{Variance} = E(X^2) - [E(X)]^2$$

Now $E(X^2) = \sum x^2 \cdot P(x)$

$$= (-2)^2(0.1) + (-1)^2(0.1) + 0(0.2) + (1)^2(0.2) + (2)^2(0.3) + (3)^2(0.1)$$

$$= 2.8$$

$$\therefore \text{Variance} = 2.8 - (0.8)^2$$

$$= 2.16$$

* From a sealed box containing a dozen apples it was found that 3 apples are perished. Obtain the probability distribution of the no. of perished apples when 2 apples are drawn at random. Also find mean and variance of this distribution.

Soln: Let X : No. of perished apples

2 apples out of 12 can be selected in $^{12}C_2$ ways

Good apples are 9

$\therefore X = 0, 1, 2$

$p(X=0)$ = Probability of getting 0 perished apple $= \dfrac{3C_0 \cdot 9C_2}{12C_2} = \dfrac{6}{11}$

$$P(X=1) = \frac{3C_1 \cdot 9C_1}{12C_2} = \frac{9}{22}$$

$$p(x=2) = \frac{3C_2 \cdot 9C_0}{12C_2} = \frac{1}{22}$$

The prob. distrn is

x :	0	1	2
$P(x)$:	$6/11$	$9/22$	$1/22$

Mean $= \sum x \cdot P(x) = 1/2$ $\quad$ $Var(x) = E(x^2) - [E(x)]^2 = 15/44$

Binomial Distribution

*

m = Mean = np   Var = npq

Probability fun of B.D is $P(x) = {}^nC_x \, p^x q^{n-x}$

p - success, q - failure, n - trials.

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* 256 set of 12 tosses of a coin in how many cases one can expect 8 heads & 4 tails ?

soln: $p = 0.5$, $q = 0.5$, $n = 12$, 8

$P(x) = {}^nC_x \, p^x q^{n-x}$

$P(x=8) = {}^{12}C_8 \, (0.5)^8 \, (0.5)^4 = 0.30$

Expected no of such cases in 256 sets

is $= 256 \times 0.30$

$= 76.80 \simeq 77$

⑤

* In sampling a large no of parts manufactured by a company, the mean no. of defectives in samples of 20 is 2. Out of 1000 such samples how many would be expected to contain atleast 3 defective parts.

Soln: Mean $= np = 2$ when $n = 20$

$\Rightarrow 20p = 2 \Rightarrow p = 1/10 = 0.1$

$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$= {}^{20}C_{x}\, (0.1)^{x} (0.9)^{20-x}$

Prob' of at least 3 defectives is $= P(3) + P(4) \ldots + P(20)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \{ {}^{20}C_{0} (0.1)^{0} + (0.9)^{20-0} +$

${}^{20}C_{1} (0.1)^{1} (0.9)^{19} + {}^{20}C_{2} (0.1)^{2} (0.9)^{18} \}$

$= 0.323$

No of defectives in 1000 samples =

$1000 \times 0.323 = 323$

* Five dice were thrown 96 times and the number of times an odd no actually turned out in the experiment is given. Fit B.D to this data & calculate expected frequencies.

No of dice showing 1 or 3 or 5	0	1	2	3	4	5
observed freq	1	10	24	35	18	8.

Soln: Prob' of getting 1 or 3 or 5 $= 3/6 = 1/2$

$$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x} \quad ; \quad n = 5$$

$$= {}^{5}C_{x} (1/2)^{x} (1/2)^{5-x}$$

Now $f(x) = 96 \times P(x)$

$$f(0) = 96 \times {}^{5}C_{0} (1/2)^{0} (1/2)^{5-0} = 3$$

$$f(1) = 96 \times {}^{5}C_{1} (1/2)^{1} (1/2)^{5-1} = \cancel{30}\ 15$$

$$f(2) = 96 \times {}^{5}C_{2} (1/2)^{2} (1/2)^{5-2} = 30$$

$$f(3) = 96 \times {}^{5}C_{3} (1/2)^{3} (1/2)^{5-3} = 30$$

$$f(4) = 96 \times {}^{5}C_{4} (1/2)^{4} (1/2)^{5-4} = 15$$

$$f(5) = 96 \times {}^{5}C_{5} (1/2)^{5} (1/2)^{5-5} = 3$$

Expected frequencis are

3, 15, 30, 30, 15, 3.

Poisson Distribution

$$P(x) = \frac{e^{-m} m^x}{x!}$$

[Mean = m] [Var = m]

* The no of persons joining a cinema que in a minute has poisson distribution with parameters 5.8. Find the probability
i) no one joins the queue in a particular minute ii) 2 or more persons join the queue

Soln: X : No of persons joining the queue

$$P(x) = \frac{e^{-m} m^x}{x!} \quad \text{where } m = 5.8 \ \& \ x = 0, 1, 2 \ldots$$

$$P(x) = \frac{e^{-5.8} (5.8)^x}{x!}$$

i) $P(x=0) = \frac{e^{-5.8} (5.8)^0}{0!} = 0.003$

ii) $P(x \geq 2) = 1 - P(x < 2)$

$= 1 - \{P(x=0) + P(x=1)]$

$= 1 - [0.003 + 6.8]$

$= 0.9796$

* For a Poisson variable (8)

$3 \times P[X=2] = P[X=4]$ find standard deviation.

Sol^n: $3 \times P[X=2] = P[X=4]$

$$3 \times \frac{e^{-m} m^2}{2!} = \frac{e^{-m} m^4}{4!}$$

$$m^2 = 36 \Rightarrow m = 6$$

$$S.D = \sqrt{var} = \sqrt{6} = 2.449 \quad [mean = var = 6]$$

(9)

- Fit a Poisson distribution

x :	0	1	2	3	4
f :	122	60	15	2	1

soln : Mean = $\dfrac{\Sigma fx}{\Sigma f} = \dfrac{0+60+30+6+4}{200} = 0.5$

$$P(x) = \frac{e^{-m} m^x}{x!} \quad \& \quad f(x) = 200 \times P(x)$$

$$f(0) = 200 \times \frac{e^{-0.5}(0.5)^0}{0!} \simeq 121$$

$$f(1) = 200 \times \frac{e^{-0.5}(0.5)^1}{1!} \simeq 61$$

$$f(2) = 200 \times \frac{e^{-0.5}(0.5)^2}{2!} \simeq 15$$

$$f(3) = 200 \times \frac{e^{-0.5}(0.5)^3}{3!} \simeq 3$$

$$f(4) = 200 \times \frac{e^{-0.5}(0.5)^4}{4} \simeq 0.$$

$\therefore$ Expected freq' are 121, 61, 15, 03, 0.

Cumulative distribution fun

* The diameter of electric cable is assumed to be continuous with p.d.f $f(x) = \begin{cases} 6x(1-x) & 0\le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

verify that $f(x)$ is a p.d.f & find mean & var

sol: $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx.$

$$= 0 + \int_0^1 6x(1-x)\,dx + 0$$

$$= \int_0^1 (6x - 6x^2)\,dx = [3x^2 - 2x^3]_0^1 = 1$$

$\therefore f(x)\,dx$ is pdf.

$$\text{Mean} = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^1 x \cdot 6x(1-x)\,dx$$

$$= \int_0^1 (6x^2 - 6x^3)\,dx = \left[2x^3 - \frac{3}{2}x^4\right]_0^1$$

$$= 1/2$$

$$\text{Var} = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = \int_0^1 (x - 1/2)^2\, 6x(1-x)\,dx.$$

$$= \int_0^1 \left(-6x^4 + \frac{9}{2}x^2 + \frac{3}{2}x\right)dx$$

$$= \frac{1}{20}$$

Normal Distribution

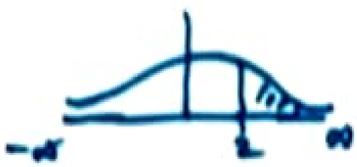
* If x is normally distributed with mean 12 and S.D 4 find the following
i) $P(x \geq 20)$ ii) $P(x \leq 20)$

Soln: let $z = \frac{x-\mu}{\sigma} = \frac{x-12}{4}$

$P(x \geq 20) = P\left[\frac{x-12}{4} \geq \frac{20-12}{4}\right]$

$= P[z \geq 2]$

= area from 2 to ∞



= [area from 0 to ∞] − [area from 0 to 2]

$= 0.5 - 0.52$

$= 0.0228$

$P(x \leq 20) = P\left[\frac{x-12}{4} \leq \frac{20-12}{4}\right]$

$= 0.9772$