# Mean, Variance & S.D of Binomial Distribution

* Mean ($\mu$):

$$\mu = \sum_{x=0}^{n} x\, p(x)$$

$$= \sum_{x=0}^{n} x\, {}^nC_x\, p^x q^{n-x}$$

$$= 0 + 1 \cdot {}^nC_1 p^1 q^{n-1} + 2 \cdot {}^nC_2 p^2 q^{n-2} + 3 \cdot {}^nC_3 p^3 q^{n-3} + \cdots + n \cdot {}^nC_n p^n q^{n-n}$$

$$= npq^{n-1} + 2 \cdot \frac{n!}{(n-2)!\,2!} p^2 q^{n-2} + 3 \cdot \frac{n!}{(n-3)!\,3!} p^3 q^{n-3} + \cdots + np^n$$

$$= npq^{n-1} + 2\left[\frac{n(n-1)(n-2)!}{(n-2)! \cdot 2} p^2 q^{n-2}\right] + 3\left[\frac{n(n-1)(n-2)(n-3)!}{(n-3)! \cdot 3\times 2\times 1}\right] p^3 q^{n-3} + \cdots + np^n$$

$$= npq^{n-1} + n(n-1)p^2 q^{n-2} + \frac{n(n-1)(n-2)}{2!} p^3 q^{n-3} + \cdots + np^n$$

$$= np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2!} p^2 q^{n-3} + \cdots + p^{n-1}\right]$$

$$= np\,[q+p]^{n-1}$$

$$\mu = np\,(1)^{n-1}$$

$$\therefore \boxed{\mu = np}$$

## Variance (V)

$$V = \sum_{x=0}^{n} x^2 p(x) - \mu^2$$

Consider, $\sum_{x=0}^{n} x^2 p(x) = \sum_{x=0}^{n} [x(x-1) + x]\, p(x)$

$$= \sum_{x=0}^{n} x(x-1)\, p(x) + \sum_{x=0}^{n} x\, p(x)$$

$$= \sum_{x=0}^{n} x(x-1)\, {}^nC_x\, p^x q^{n-x} + np$$

$$= 0 + 0 + 2\, {}^nC_2\, p^2 q^{n-2} + 3\cdot 2\, {}^nC_3\, p^3 q^{n-3} + 4\cdot 3\cdot {}^nC_4\, p^4 q^{n-4} + \dots +$$

$$n(n-1)\, {}^nC_n\, p^n q^0 + np$$

$$= 2\left[\frac{n!}{(n-2)!\,2!} p^2 q^{n-2}\right] + 3\cdot 2\left[\frac{n!}{(n-3)!\,3!} p^3 q^{n-3}\right] + 4\cdot 3\left[\frac{n!}{(n-4)!\,4!} p^4 q^{n-4}\right]$$

$$+ \dots + n(n-1)\, p^n + np$$

$$= \frac{n!}{(n-2)!} p^2 q^{n-2} + \frac{n!}{(n-3)!} p^3 q^{n-3} + \frac{n!}{(n-4)!\,2!} p^4 q^{n-4} + \dots + n(n-1) p^n$$

$$+ np$$

$$= \frac{n(n-1)(n-2)!}{(n-2)!} p^2 q^{n-2} + \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} p^3 q^{n-3} +$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!\,2!} p^4 q^{n-4} + \dots + n(n-1)p^n + np$$

$$= n(n-1)p^2\left[\cancel{p}\, q^{n-2} + (n-2)p q^{n-3} + \frac{(n-2)(n-3)}{2!} p^2 q^{n-4} + \dots + p^{n-2}\right]$$

$$+ np$$

$$= n(n-1)p^2\, [q+p]^{n-2} + np$$

$$= n(n-1)p^2\,(1) + np$$

$= n^2p^2 - np^2 + np$

Now, $V = \sum_{x=0}^{n} x^2 p(x) - \mu^2$

$= n^2p^2 - np^2 + np - (np)^2$

$= n^2p^2 - np^2 + np - n^2p^2$

$= np(1-p)$

$\boxed{V = npq}$

* $S.D = \sqrt{V} = \sqrt{npq}$

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## Mean, Variance and S.D of ~~Exponen~~ Poisson Distribution

* Mean ($\mu$)

$\mu = \sum_{x=0}^{\infty} x\, p(x)$

$= \sum_{x=0}^{\infty} x \frac{e^{-m} m^x}{x!}$

$= 0 + 1.\frac{me^{-m}}{1!} + 2.\frac{m^2e^{-m}}{2!} + 3.\frac{m^3e^{-m}}{3!} + 4.\frac{m^4e^{-m}}{4!} + \cdots$

$= me^{-m}\left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \cdots\right]$

$= me^{-m}.e^{m}$

$\boxed{\mu = m}$

* Variance (v)

$$V = \sum_{x=0}^{\infty} x^2 p(x) - \mu^2$$

Consider, $\sum_{x=0}^{\infty} x^2 p(x) = \sum_{x=0}^{\infty} \left(x[x-1]+x\right) p(x)$

$$= \sum_{x=0}^{\infty} x(x-1) p(x) + \sum_{x=0}^{\infty} x p(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-m} m^x}{x!} + m$$

$$= \left[0 + 0 + 2.1 \frac{e^{-m} m^2}{2!} + 3(2) \frac{e^{-m} m^3}{3!} + 4(3) \frac{e^{-m} m^4}{4!} + \dots\right] + m$$

$$= m^2 e^{-m} \left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \dots\right] + m$$

$$= m^2 e^{-m} . e^{m} + m$$

$$\boxed{\sum_{x=0}^{\infty} x^2 p(x) = m^2 + m}$$

$$V = \sum_{x=0}^{\infty} x^2 p(x) - \mu^2$$

$$= m^2 + m - m^2$$

$$\boxed{V = m}$$

$$\therefore \; S.D = \sqrt{V}$$

$$\boxed{S.D = \sqrt{m}}$$

The mean & variance are equal in Poisson Distribution