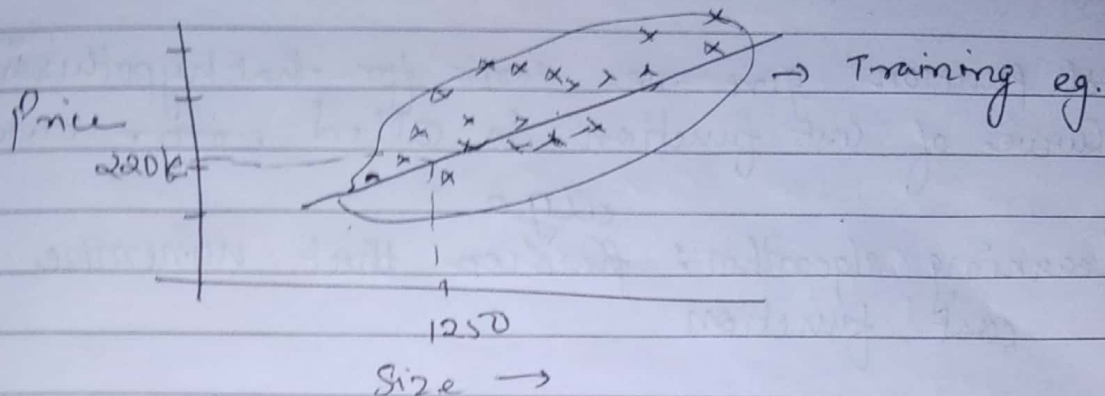


# Linear Regression

Page No.

Date

→ One variable



for a given size car / find price for price

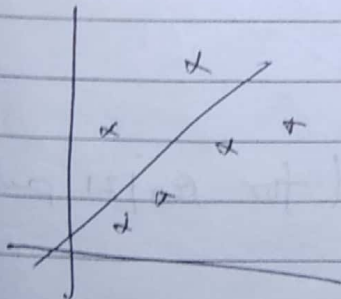
Training set :- Use to catch relation  $f()$  in  $y = f(x)$   
Supervised

$x \rightarrow h(\text{hypothesis}) \rightarrow \text{Estimated price}$

$$h = \theta_0 + \theta_1 x$$

Univariate linear regression

Now we want best <sup>linear</sup> curve to fit



We have to minimize  $\sum (h_0(x_i) - y_i)^2$   
 $\theta_0, \theta_1$

choose  $k$  such that  $h$  is close to  $y$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_0(x_i) - y_i)^2 = \text{Mean Square error}$$

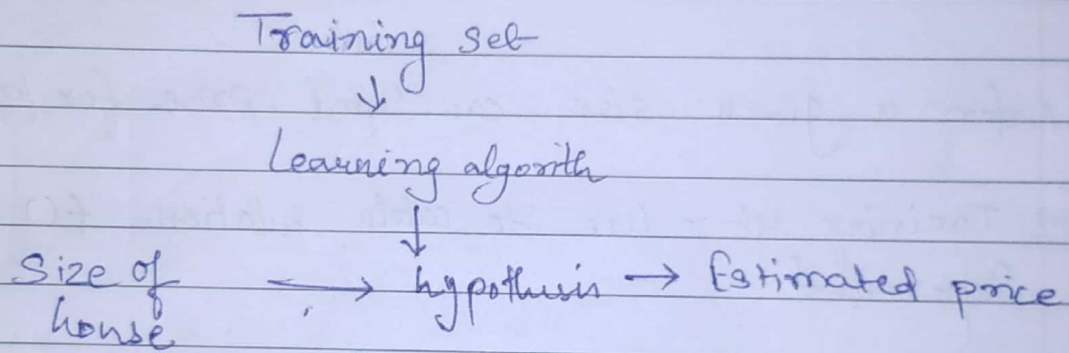
$J_{\theta_0, \theta_1} \equiv \text{Cost function}$

hypothesis gives me curve

Cost function give me error for that hypothesis.

Curve of cost function is called error curve.

Learning algorithm: <sup>algo</sup> function that minimize the cost function



## Gradient Descent

### Outline

- start with any  $\theta_0, \theta_1$  value.
- keep  $\theta_0, \theta_1$  to reduce  $J(\theta_0, \theta_1)$  until we hopefully end up at minimum

repeat until convergence {

$$\theta_j = \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad \text{for } j=1 \text{ and } j=0$$

$$\text{temp0 } \theta_0 = \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

$$\text{temp0 } \theta_0 = \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

$$\text{temp1 } \theta_1 = \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

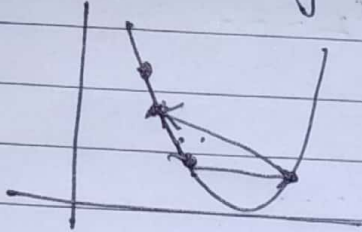


$\theta_0 = \text{temp } 0$

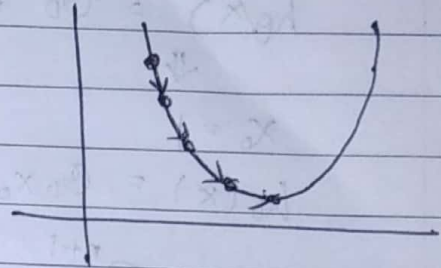
$\theta_1 = \text{temp } 1$

As

when  $\alpha$  is large



when  $\alpha$  is small



Repeat until convergence  
(Vector representation)

②

$$\theta = \theta - \alpha \cdot \frac{1}{m} \sum (h(x) - y)^2$$

Batch is size of training set.

When ' $\alpha$ ' is large it may bounce around minima and finally diverge.

when ' $\alpha$ ' is too low it may take high amount of time and

So right ' $\alpha$ ' is found by trial and error method

plus as we move toward the minima some researcher suggest to decrease the  $\alpha$  after some epochs.

Some suggest to keep ' $\alpha$ ' same, as derivative decrease.

# Linear Regression with Multiple features

## Multivariate Linear Regression.

$$h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots$$

↓

$$x_0 = 1$$

$$h_{\theta}(x) = \theta_0 x_0 + \theta_1 x_1 + \dots$$

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \end{bmatrix} \in \mathbb{R}^{n+1} \quad x = \begin{bmatrix} x_0 = 1 \\ x_1 \\ x_2 \\ \vdots \end{bmatrix}$$

$$h_{\theta} x = \theta^T x$$

## Gradient descent

$$J(\theta_0, \theta_1, \dots) = \frac{1}{2m} \sum_{i=1}^n (h_{\theta}(x^i) - y^i)^2$$

## Gradients

$$\theta_j = \theta_j - \alpha \frac{1}{m} \sum_{i=1}^n (h_{\theta}(x^i) - y^i) x_j^i$$

## Feature Scaling

Make Sure features are on same scale



Try to make  $-1 \leq x_j \leq 1$

2 ways to do that

- 1) Dividing max
- 2) Mean normalization

$$x_i^0 = \frac{x_i^0 - x_{\min}}{x_{\max}}$$

this makes new mean to zero.

#3)

$$x_i^1 = \frac{x_i^0 - x_{\min}}{x_{\max} - x_{\min}}$$

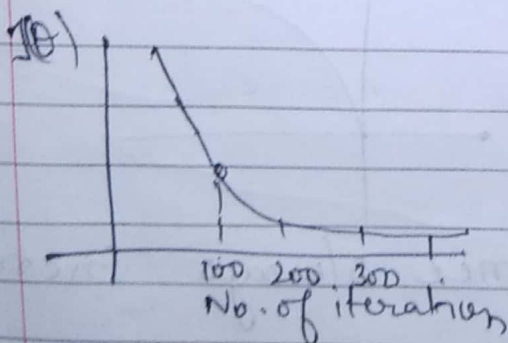
#4)

converting to their 2-scores  
as we know <sup>formula for</sup> 2-scores

$$x_i^1 = z_i^0 = \frac{x_i^0 - \mu}{\sigma}$$

$\sigma$   $\rightarrow$  standard deviation of sample

How to choose ' $\alpha$ '



Declare convergence if  $J(\theta)$  decreases by less than some  $\epsilon$

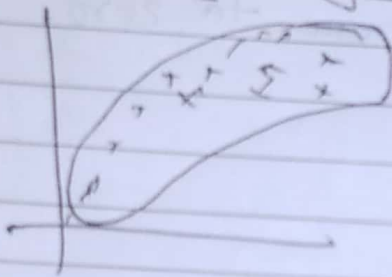
$\epsilon$  is generally taken  $10^{-3}$ .

To choose  $\alpha$ , try

different  $\alpha$ ,  $10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$

for small number of epochs.

Poly nomial Regression

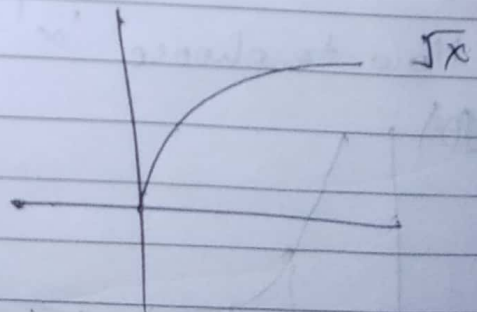
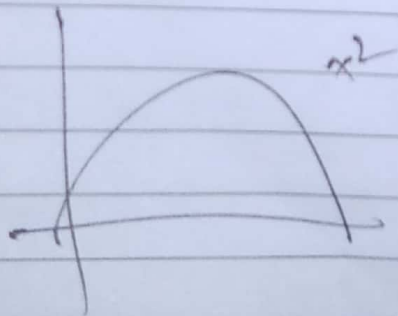


$$h_0(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$$

$$= \theta_0 + \theta_1 x_1 + \theta_2 x_1^2 + \theta_3 x_1^3$$

Choice of feature

$\sqrt{x}$  instead of  $x^2$



~~Common intuition: price always increases as size increases~~



Loss function / Cost function  
MSE/RMS

why ~~RM~~ ~~MSE/RMS~~ is preferred over other

other possible cost function

$$MAE = \frac{\sum |h(x_i) - y_i|}{m}$$

$$\text{mean log error} = \frac{\sum \hat{y}_i \log h(x_i)}{m}$$

MSE is preferred over other losses because it penalizes object far from result more than object near to ~~log~~ label.

MAE penalizes all distance equally.

mean log error is generally known as Cross entropy loss used in classification task. ~~it~~ ~~generally~~ ~~has~~

MBE

Mean Bias error

Generally used when need to determine ~~what~~ model have positive or negative bias

$$MBE = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)}{n}$$

$$y_i = h(x_i)$$