

Instruction set of the processor

- *1 step for execution:*

- Arithmetic and logical operation: $A=B+C...$
- Jumps and conditions: go to if $a>b$ then y
- Pointer instruction: $B=*C, *C=B$
- 1 D array operation : $A[i]=B$ or $B= C[i]$

Complex algorithmic instruction

$A= B+C*D-F$

$A[i]=B[i]+C[i]$ $\{x=B[i], y=C[i], z=x+y, A[i]=z\}$

Example

- For $i=1$ to n
- $C[i] = A[i] + B[i]$
- End for
- How many steps are required to execute?
- What will be running time?

Example

1 times

- $i=1$

$n+1$ times

- If $i > n$ go to last

- $X=A[i]$
- $Y=B[i]$
- $Z=X+y$
- $C[i]=Z$

n times

- $i=i+1$
- Go to ... n times

Running time = $b \cdot n + 2n + n + 1 + 1 = bn + 3n + 2, = 2 + n(b+3)$

Analysis of Algorithm

Matrix Multiplication

- **Input:** M and N with $n \times n$
- **Output:** O with $n \times n$
- $O = M \times N$
- $O_{ij} = \sum_k m_{ik} * n_{kj}$
- For $i=1$ to n

for $j=1$ to n

$c[i,j]=0$

Analysis of Algorithm

- For $i=1$ to n
 for $j= 1$ to n
 $O[i,j]=0$
 for $k= 1$ to n
 $O[i,j]= O[i,j]+M[i,k]*N[k,j]$
 End
 End
End

Analysis of Algorithm

Body: b steps

Iteration: n

Time taken : $2+n(b+3)$

Accessing of 2 dimensional array: 4 steps

Analysis of Algorithm

```
for(i=0; i < N; i++)  
{  
    statement;  
}
```

Complexity?

Analysis of Algorithm

```
for(i=0; i < N; i++)  
{  
    for(j=0; j < N; j++)  
    {  
        statement;  
    }  
}
```

Complexity?

Analysis of Algorithm

```
int a = 0;
for (i = 0; i < N; i++) {
    for (j = N; j > i; j--) {
        a = a + i + j;
    }
} Complexity?
```

Analysis of Algorithm

```
while(low <= high)
{
    mid = (low + high) / 2;
    if (target < list[mid])
        high = mid - 1;
    else if (target > list[mid])
        low = mid + 1;
    else break;
}
```

Complexity?

Analysis of Algorithm

```
void sort(int list[], int left, int right)
{
    int pivot = partition(list, left, right);
    sort(list, left, pivot - 1);
    sort(list, pivot + 1, right);
}
```

Complexity?

Analysis of Algorithm

```
int i, j, k = 0;
for (i = n / 2; i <= n; i++) {
    for (j = 2; j <= n; j = j * 2) {
        k = k + n / 2;
    }
```

} Complexity?

Solution

- If you notice, j keeps doubling till it is less than or equal to n . Number of times, we can double a number till it is less than n would be $\log(n)$.

Let's take the examples here.

for $n = 16$, $j = 2, 4, 8, 16$

for $n = 32$, $j = 2, 4, 8, 16, 32$

So, j would run for $O(\log n)$ steps.

i runs for $n/2$ steps.

So, total steps = $O(n/2 * \log(n)) = O(n * \log n)$

- $f(n) = 7n + 8$ and $g(n) = n$.
- Is $f(n) \in O(g(n))$?

- For $7n + 8 \in O(n)$, we have to find c and n_0 such that $7n + 8 \leq c \cdot n$, $\forall n \geq n_0$. By inspection, it's clear that c must be larger than 7.
- Let $c = 8$. Now we need a suitable n_0 . In this case, $f(8) = 8 \cdot g(8)$.
- Because the definition of $O()$ requires that $f(n) \leq c \cdot g(n)$, we can select $n_0 = 8$, or any integer above 8 – they will all work.
- We have identified values for the constants c and n_0 such that $7n + 8 \leq c \cdot n$ for every $n \geq n_0$, so we can say that $7n + 8$ is $O(n)$.