$$\times \sim N(0,1) \Rightarrow \times =$$

$$(X \cap N(0,1)) \Rightarrow X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$(1) \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = A$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$Y = A \times = \begin{bmatrix} 2 \times_1 \\ 2 \times_2 \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

$$\Sigma = AA^{T} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, |\Sigma| = 16$$

$$\Sigma' = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$$

$$f_{y}(y_{1},y_{2}) = e^{-(y-y)^{T}\Sigma^{-1}(y-y)}$$

$$(y)^T z^{-1}(y) = [y, y_2] \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{y_1}{4} & \frac{y_2}{4} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$f_{y}(y_{11}y_{2}) = e^{-\left(\frac{y_{1}^{2}+y_{2}^{2}}{8}\right)} = e^{-\left(\frac{y_{1}^{2}+y_{2}^{2}}{8}\right)} = e^{-\left(\frac{y_{1}^{2}+y_{2}^{2}}{8}\right)} = e^{-\left(\frac{y_{1}^{2}+y_{2}^{2}}{8}\right)}$$

$$\frac{\partial f_{y}}{\partial y_{i}} = \frac{e^{-(y_{i}^{2} + y_{2}^{2})} (-y_{i})}{8\pi}$$

 $\frac{\partial y_{2}}{\partial y_{2}} = e^{-\frac{(y_{1}^{2} + y_{2}^{2})}{8} \left(-\frac{y_{2}}{y_{2}}\right)}$ Along y=x, taking directional derivative,  $\left\lfloor \left(\frac{\partial f_{y}}{\partial y_{i}}\right)^{2} + \left(\frac{\partial f_{y}}{\partial y_{i}}\right)^{2} \right\rfloor \cdot \left\lfloor \frac{2}{2} \right\rfloor^{2} + \left\lfloor \frac{2}{2} \right\rfloor^{2}$  $= \left[ \frac{e^{-\frac{y_1^2}{4}y_1^2} - e^{-\frac{y_1^2}{4}y_2^2}}{32\pi\sqrt{2}} \right]$  $=-\int \frac{e^{-\frac{y_1}{4}y_1^2}}{16\pi52}$ along y=-x, taking idirectional iderivative;  $\left[ \left( \frac{\partial f_y}{\partial y_i} \right) \hat{i} + \left( \frac{\partial f_y}{\partial y_i} \right) \hat{j} \right] \cdot \left[ \frac{2}{\sqrt{2}} \hat{i} - \frac{4}{\sqrt{2}} \hat{j} \right]$  $= \left[ \frac{-e^{-\frac{y^2}{4}}y_1^2}{32\pi\sqrt{2}} - e^{-\frac{y_1^2}{4}}y_1^2 \right]$  $= - \left[ \frac{e^{-\frac{y_1^2}{4}y_1^2}}{1677\sqrt{2}} \right]$ us the directional iderivative is equal valong y=x equi-contours of joint identity is circle. (Is mariance ialong x-axis = Mariance along y-axis)

ii) 
$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\sum = AA^{T} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}, |\Sigma| = 9$$

$$\sum_{y=1}^{2} = \frac{1}{9} \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$f_{y}(y_{1}, y_{2}) = e^{-(y-4)^{\frac{7}{2}} \ge \frac{-1}{2} (y-4)}$$

$$\sqrt{(2\pi i)^{\frac{1}{2}} |x|}$$

$$\sqrt{(2\pi)^{k}|\Xi|}$$

$$|y|^{T} \Sigma^{-1}(y) = [y_1 \quad y_2] \left[\frac{5}{9} \quad -\frac{1}{9} \\ -\frac{1}{9} \quad \frac{5}{9} \right] \left[\frac{y_1}{y_2}\right] = \left[\frac{5y_1 - 4y_2}{9} \quad -\frac{1}{9} \frac{1}{9} \frac{1}{9} \right]$$

$$= \int \frac{5y_1^2 - 8y_1y_2 + 5y_2^2}{9}$$

$$f_y(y_1, y_2) = e^{-\left(\frac{5y_1^2 - 8y_1y_2 + 5y_2^2}{2\times 9}\right)}$$

× y

$$\frac{\partial f_{y}}{\partial y_{1}} = e^{-\left(\frac{5y_{1}^{2} - 8y_{1}y_{2} + 5y_{2}^{2}}{2\times 9}\right)} \left[\frac{10y_{1} + 8y_{2}}{2\times 9\times 6\pi}\right] [6\pi]$$

$$\frac{\partial f_{y}}{\partial y_{1}} = e^{-\left(\frac{5y_{1}^{2} - 8y_{1}y_{2} + 5y_{2}^{2}}{2\times 9}\right)} \left[\frac{8y_{1} - 10y_{2}}{2\times 9\times 6\pi}\right] [6\pi]$$

$$\frac{\partial f_{y}}{\partial y_{2}} = e^{-\left(\frac{5y_{1}^{2} - 8y_{1}y_{2} + 5y_{2}^{2}}{2\times 9}\right)} \left[\frac{8y_{1} - 10y_{2}}{2\times 9\times 6\pi}\right] [6\pi]$$

$$\begin{bmatrix} \frac{2}{6}y \\ 1 \end{bmatrix}^{\frac{1}{2}} + \frac{2}{6}y \\ \frac{1}{2}y \end{bmatrix}^{\frac{1}{2}} \cdot \begin{bmatrix} \frac{x}{15} + \frac{x}{15} \end{bmatrix}^{\frac{1}{2}} \\
= \begin{bmatrix} \frac{e^{-(5y_{1}^{2} - 8y_{1}y_{2} + 5y_{3}^{2})}}{(2\pi)(3)} & \begin{bmatrix} \frac{8y_{2} - 10y_{1}}{2x} \\ \frac{1}{2x} \end{bmatrix}^{\frac{1}{2}} \end{bmatrix} \\
+ \frac{e^{-(5y_{1}^{2} - 8y_{1}y_{2} + 5y_{3}^{2})}}{(2\pi)(3)} & \begin{bmatrix} \frac{10y_{2} + 8y_{1}}{9x_{2} \times 6\pi} \end{bmatrix} \begin{pmatrix} \frac{y_{1}}{12} \end{pmatrix}^{\frac{1}{2}} \end{pmatrix}^{\frac{1}{2}} \\
= -\frac{1}{1}y_{1}^{2} e^{-(5y_{1}^{2} - 8y_{1}y_{2} + 5y_{3}^{2})} & \begin{bmatrix} \frac{10y_{2} + 8y_{1}}{18x_{2}} \\ \frac{108\pi x_{1}}{12} \end{bmatrix}^{\frac{1}{2}} \\
= -\frac{1}{1}y_{1}^{2} e^{-(5y_{1}^{2} - 8y_{1}y_{2} + 5y_{3}^{2})} & \begin{bmatrix} \frac{x}{12} - \frac{x}{12} \end{bmatrix}^{\frac{1}{2}} \\
= -\frac{1}{1}y_{1}^{2} e^{-(5y_{1}^{2} - 8y_{1}y_{2} + 5y_{3}^{2})} & \begin{bmatrix} \frac{x}{12} - \frac{x}{12} \end{bmatrix}^{\frac{1}{2}} \\
= \begin{bmatrix} e^{-(5y_{1}^{2} - 8y_{1}y_{2} + 5y_{3}^{2})} & \begin{bmatrix} \frac{x}{12} - \frac{x}{12} \end{bmatrix}^{\frac{1}{2}} \\
= -\frac{1}{1}y_{1}^{2} e^{-(5y_{1}^{2} + 5y_{3}^{2} + 5y_{3}^{2})} & \begin{bmatrix} \frac{10y_{2} + 8y_{1}}{18x_{2}} \end{bmatrix}^{\frac{1}{2}} \\
= -\frac{1}{1}y_{1}^{2} e^{-(5y_{1}^{2} + 5y_{3}^{2} + 5y_{3}^{2})} & \begin{bmatrix} \frac{10y_{2} + 8y_{1}}{18x_{2}} \end{bmatrix}^{\frac{1}{2}} \\
= -\frac{1}{1}y_{1}^{2} e^{-(5y_{1}^{2} + 5y_{1}^{2} + 16y_{1}^{2})} & (2) = -\frac{36y_{1}^{2}}{108\pi x_{1}^{2}} e^{-\frac{18y_{1}^{2}}{108\pi x_{1}^{2}} \\
= \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} \\
= \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} \\
= \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} \\
= \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} \\
= \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} \\
= \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} \\
= \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi x_{1}^{2}} \\
= \frac{1}{108\pi x_{1}^{2}} & \frac{1}{108\pi$$

Along y=xs taking directional derivative,

iii) 
$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$|||| A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$S = AA^{T} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\Sigma = AA^{T} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}, |\Sigma| = 9$$

$$\begin{bmatrix} -4 & 5 \end{bmatrix}$$

$$\sum^{-1} = \frac{1}{9} \begin{bmatrix} 5 & 4 \end{bmatrix}$$

$$\sum_{i=1}^{n} \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$$f_{i}(u, u) = e^{-1}$$

$$(y_1, y_2) = 0$$

$$(y_1, y_2) = 1$$

$$(y_1, y_2) = 0$$

$$f_{y}(y_{1}, y_{2}) = e^{-(y-4)^{T} \sum_{j=1}^{-1} (y-4)}$$

$$y_{11}y_{2} = e^{-\frac{1}{2}}$$

 $f_{y}(y_{1},y_{2}) = e^{-(5y_{1}^{2} + 8y_{1}y_{2} + 5y_{2}^{2})}$ 

27(3)

=  $\left[ \frac{5y_1^2 + 8y_1y_2 + 5y_2^2}{9} \right]$ 

$$\begin{bmatrix} \frac{5}{9} \end{bmatrix}$$

$$y^{T} \Sigma y = [y_1, y_2] \begin{bmatrix} \frac{5}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{5}{9} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\frac{\partial f_{x}}{\partial y_{1}} = \frac{e^{-\left(5y_{1}^{2} + 8y_{1}y_{2} + 5y_{2}^{2}\right)}}{(2\pi)(3)} \left[ \frac{-\log_{1} - 8y_{2}}{18} \right]$$

$$\frac{\partial f_{y}}{\partial y_{2}} = \frac{e^{-\left(5y_{1}^{2} + 8y_{1}y_{2} + 5y_{2}^{2}\right)}}{(3\pi)(3)} \left[ \frac{-8y_{1} - \log_{2}}{18} \right]$$

$$\frac{\log y}{(3\pi)(3)} = \frac{\log y}{(3\pi)(3)} \left[ \frac{2}{18} \right] + \frac{2}{\sqrt{2}} \left[ \frac{1}{\sqrt{2}} \right] +$$

Along 
$$y = -x$$
, taking directional iderinative,
$$\begin{bmatrix} (\frac{\partial f_y}{\partial y_1}) \hat{i} + (\frac{\partial f_y}{\partial y_2}) \hat{j} \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{5} \hat{i} - \frac{2}{5} \hat{j} \end{bmatrix}$$

$$= \underbrace{ \left( e^{-(5y_1^2 + 8y_1y_2 + 5y_3^2)} \underbrace{ \left[ -10y_1 - 8y_2 \right] \left( \frac{y_1}{5} \right) \right)}_{18}$$

$$- \underbrace{ \left( e^{-(5y_1^2 + 8y_1y_2 + 5y_3^2)} \underbrace{ \left[ -8y_1 - 10y_2 \right] \left( \frac{y_1}{5} \right) \right)}_{18}$$

$$= -\sqrt{2} \underbrace{ y_1^2 e^{-\frac{y_1^2}{q}}}_{18}$$

: Rate of fall is greater valory 
$$y=x$$
 than  $y=-x$  initially, hence the vortour is valigned valory  $y=-x$