1. $f_X = \begin{cases} 1, & 0 \le X \le 1 \\ 0, & \text{rest } x \end{cases}$ $\quad F_Y = \begin{cases} 1, & 0 \le Y \le 1 \\ 0, & \text{rest } Y \end{cases}$

$$F_{r<c} = F_{\sqrt{X^2+Y^2}<c} = \int_{x=0}^{x=1} \int_{y=0}^{y=\sqrt{c^2-x^2}} f_X f_Y \, dx \, dy = \int_{x=0}^{x=1} \int_{y=0}^{y=\sqrt{c^2-x^2}} dx \, dy$$

$$= \int_{x=0}^{x=1} \sqrt{c^2-x^2} \, dx = \frac{\sqrt{c^2-1}}{2} + \frac{c^2}{2} \sin^{-1}\left(\frac{1}{c}\right)$$

$$f_r = r \sin^{-1}\left(\frac{1}{r}\right)$$

$$F_{\theta<c} = F_{\tan^{-1}\left(\frac{Y}{X}\right)<c} = \int_{x=0}^{x=1} \int_{y=0}^{y=\tan c(x)} dx \, dy$$

$$= \frac{\tan c}{2}$$

$$\therefore f_\theta = \frac{\sec^2 \theta}{2}$$

2.

a) Given $f_{x,y,z}$ be joint pdf.

Also, $\iiint_V f_{x,y,z}\,dx\,dy\,dz = 1$

$\Rightarrow c \iiint_V dx\,dy\,dz = 1$ [As the fly is equally likely to be anywhere in volume].

$\Rightarrow c = 1 = f_{x,y,z}$ [$\because \iiint_V dx\,dy\,dz = 1$]

$$f_{x,y} = \int_0^1 f_{x,y,z}\,dz = 1$$

$$f_{y,z} = \int_0^1 f_{x,y,z}\,dx = 1$$

$$f_{x,z} = \int_0^1 f_{x,y,z}\,dy = 1$$

$$f_x = \int_0^1\int_0^1 f_{x,y,z}\,dy\,dz = 1$$

$$f_y = \int_0^1\int_0^1 f_{x,y,z}\,dx\,dz = 1$$

$$f_z = \int_0^1\int_0^1 f_{x,y,z}\,dx\,dy = 1$$

Here, as $f_{x,y,z} = f(x)f(y)f(z)$ and $f(x,y) = f(x)f(y)$, $f(y,z) = f(y)f(z)$ and $f(x,z) = f(x)f(z)$; X, Y, Z are independent.

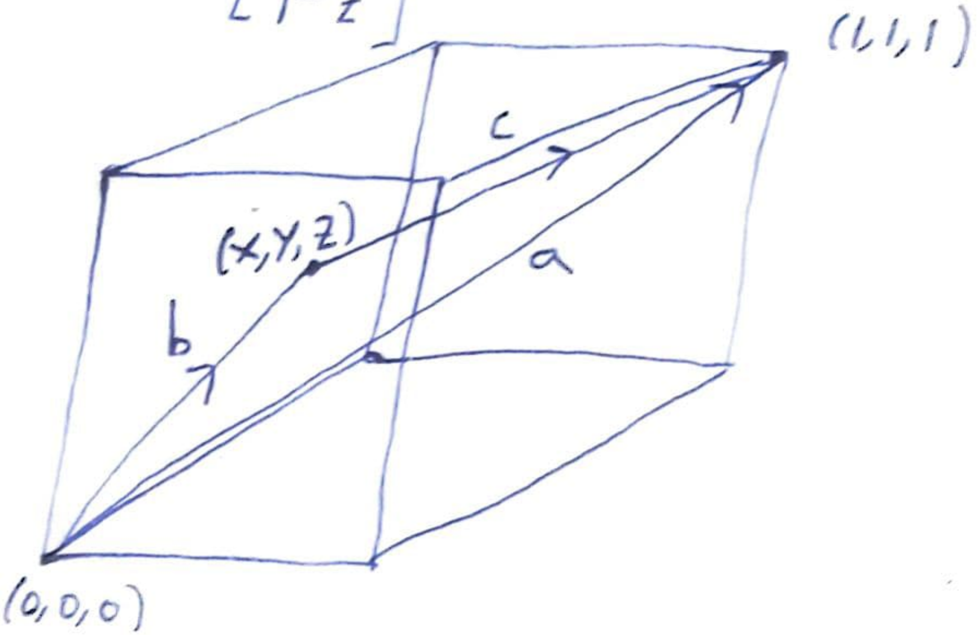
$\therefore \operatorname{cov}(X,Y) = \operatorname{cov}(X,Z) = \operatorname{cov}(Y,Z) = 0$

b)

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = I + A \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad I = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -x \\ -y \\ -z \end{bmatrix}$$

$$= \begin{bmatrix} 1-x \\ 1-y \\ 1-z \end{bmatrix}$$



$\vec{c} = (1-x, 1-y, 1-z) = \vec{a} - \vec{b}$

Hence, the animal is sitting at (1,1,1) and observing the fly.

c) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$I + A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} x \\ 2y \\ 3z \end{bmatrix} = \begin{bmatrix} 1+x \\ 1+2y \\ 1+3z \end{bmatrix}$$

The x-coordinate of the room remains the same, y-coordinate is doubled and z-coordinate tripled.

d) The probability density looks like a 3-D dimensional Gaussian with mean at $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$.