

$$P(T=m) = p(1-p)^{m-1}$$

$$E(T) = \sum_{m=1}^{\infty} T P(T=m)$$

$$= \sum_{m=1}^{\infty} T (1-p)^{m-1} p = \sum_{m=1}^{\infty} m (1-p)^{m-1} p$$

$$\Rightarrow S = p + 2(1-p)p + 3(1-p)^2 p + 4(1-p)^3 p + \dots$$

$$(1-p)S = p(1-p) + 2p(1-p)^2 + 3p(1-p)^3 + \dots$$

$$\Rightarrow pS = p + p(1-p) + p(1-p)^2 + \dots$$

$$\Rightarrow S = \frac{1}{1-(1-p)} = \frac{1}{p} = E(T)$$