Sponsored Search and Market Power*

Akhil Ilango†

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[*Preliminary and Incomplete*]

I develop a theory of digital marketplaces where an intermediary provides a platform for firms to advertise their product and where consumers need to engage in costly search if they want to learn more about the products. First, I show that when prices are observable prior to search, a more prominent firm charges a higher price and earns a higher revenue. Second, I augment this model by allowing the intermediary to determine endogenously the order in which products are displayed and the advertising commissions (per-click) to be paid, through an auction. I show that the pass-through from these commissions to product prices is higher for a non-prominent firm, thus restricting its ability to compete using price. This asymmetry in equilibrium lowers competition, consumer surplus and total transactions in the product market.

Keywords: Digital Economy, Market Power, Ad Auction, Consumer Search.

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†Department of Economics, Universitat Pompeu Fabra and BSE. Email: akhil.ilango@upf.edu

1 Introduction

While competition *among* platforms has received extensive attention from policymakers and academia,[1] understanding the competition *within a* platform can be instrumental in informing effective and comprehensive regulations. The rise of dominant digital platforms has been accompanied by concerns of increase in product market concentration, across different sectors.[2] This counters initial sentiment that the Internet would facilitate highly competitive markets due to lower search costs and enable more firms to reach consumers.[3] Ensuing studies that address the above changes in market structure have largely devoted their attention to the strategic interaction between firms and platforms.[4] However, consumers remain a crucial understudied determinant of market outcomes. In this paper, I study an important device through which a platform affects the online market - Position Auction or Ad Auction - and its interaction with product demand, competition and consumer surplus.

Selling advertisements (ads) are a key element in many platforms' business models. For instance, Google ad revenue accounts for more than 80% of Alphabet's total revenue and Amazon reports more than 40% annual growth in ad revenue.[5] Since ad revenues play a major role in monetising a platform's market power, apart from gaining insights into the effects on competition and welfare in the two-sided market, this paper also sheds light on a platform's incentives in accumulating market power.

This ad revenue is determined through an auction that platforms conduct to determine the order in which ads (of firms) are displayed and the ad commission to be paid at each position. However, I show that this ordered display can evoke significant reactions from consumers and firms which in turn endogenously affect product market concentration and platform revenue. Specifically, I highlight a novel channel - the asymmetry across ad positions in pass-through from ad commissions to product prices - which leads to lower product market competition and consumer surplus.

I build a model of the e-commerce environment with three key features. First, consumer behaviour follows recent empirical evidence which suggests that prominence plays an important role in determining consumer demand.[6] For example, consumers (*they*) may

[1] See, for instance, Broadbent (2020) for an overview of the Digital Markets Act and the Digital Services Act.
[2] See, for instance, Khan (2018), ACCC (2019), Crémer, de Montjoye, and Schweitzer (2019), Scott-Morton, Bouvier, Ezrachi, Julien, Katz, Kimmelman, Melamed, and Morgenstern (2019) and CMA (2020) for dominance of platforms. And Gutiérrez and Philippon (2017), Grullon, Larkin, and Michaely (2019) , Autor, Dorn, Katz, Patterson, and Van Reenen (2020), De Loecker, Eeckhout, and Unger (2020), Tambe, Hitt, Rock, and Brynjolfsson (2020) and Affeldt, Duso, Gugler, and Piechucka (2021) for rise in concentration.
[3] See, for instance, Brynjolfsson and Smith (2000) and Brynjolfsson, Hu, and Smith (2003) for some early empirical evidence of this sentiment and Pozzi (2012) for some later counter-evidence.
[4] See Evans and Schmalensee (2013) for a survey.
[5] See http://investor.google.com/ for Google data. See https://ir.aboutamazon.com/ for Amazon data. During the same period, Amazon's total revenue increased by 33.5%.
[6] See, for instance, Granka, Joachims, and Gay (2004) for eye-tracking evidence, Ghose and Yang (2009) for

start their search from a firm (*he*) because his ad occupies the top position, a larger banner space or earlier spot than others.[7] Another example could be that of a platform (*she*) which provides an affiliated brand a prominent position in the market through display characteristics, ease of access, etc.[8] In equilibrium, I show that this behaviour is also consistent with the rationale of consumers starting their search from the lower-priced firm.

Second, some product information is available to consumers even before they *search* to learn about other product characteristics. For example, when a consumer sees a list of ads, they see some information about the product on the ad (often the price of the product), while they have to click on the ad to learn the other attributes of the product. I will refer to the visible part as the price of the product, and the part that is discovered after clicking as the value of the product.[9]

Third, a platform's business model of ad sales is captured using a Generalised Second Price auction which is conducted by the platform to maximise her revenue.[10] The platform's revenue equals 'pay per-click' commission times the number of clicks or visitors to each firm.[11] This auction, in turn, determines the order of firms displayed to the consumers.

Using this framework, I endogenously determine the search and demand behaviour of consumers over products, as well as the pricing decisions of firms (for their product) and the platform (for her ad position). This allows me to reassess the effect of a firm's prominence on his profits and on consumer demand, in the presence of a rent-seeking platform. A product's price is determined by both consumers' preference and the commission paid by the firm to the platform (that is, the ad cost). In turn, a firm's commission is a function of his position on the list. As a result, the number of clicks (or visitors) a firm gets and the cost per-click varies with position. These differences in cost, and their effect on product pricing, allows me to micro-found the 'value of a click' based on position and firm characteristics, thus avoiding the common assumption of position homogeneity.[12]

evidence from panel data at Google, Agarwal, Hosanagar, and Smith (2011) for evidence from a field experiment at Google, Narayanan and Kalyanam (2015) for evidence from an RDD approach, Ursu (2018) for evidence from a field experiment at Expedia, Simonov and Hill (2021) for quasi-experimental evidence from Bing, and Moshary (2021) for evidence from a field experiment.

[7] See figure A.1 for some examples. Although I will use the context of search ads to describe my model, the insights can be interpreted in other contexts of advertising (e.g., display ads and video ads) and 'product' markets (e.g., finance: insurance ads, labour: job search ads) with some qualifications.

[8] See, for instance, https://support.google.com/google-ads/answer/6381002?hl=en for Google's definition of prominence.

[9] See figure A.1 for some examples.

[10] This is one of the most common auction formats employed by digital platforms to determine ad positions. See, for instance, Edelman, Ostrovsky, and Schwarz (2007) and Varian (2007).

[11] This formulation of platform revenue is motivated by Google ranking firms based on PPC times the estimated click-through rate. See https://support.google.com/google-ads/answer/142918?hl=en (Accessed May 14, 2020) - "The most important thing to remember is that even if your competition bids higher than you, you can still win a higher position – at a lower price – with highly relevant keywords and ads."

[12] See, for instance, Goldman and Rao (2016) for evidence from Bing on heterogeneous effect of position on

The contributions of this model are twofold. First, I analyse the market outcomes in the absence of an auction. For this exercise, I will assume that consumers always start their search from the prominent firm to highlight the effect of prominence and price visibility on product pricing and competition. I show that, when a pure-strategy equilibrium exists, firm 1 (in prominent position) charges a higher price than firm 2 (in non-prominent position), that is $p_1 > p_2$. Further, this price dispersion increases with search cost. These results are in contrast with the standard result from the literature on prominence where firms' prices increase with order and converge at high search costs (see, for instance, Armstrong, Vickers, and Zhou (2009)). When prices are free to observe (hereafter, *in-sight* prices), a lower p_2 not only retains more visitors, but is also likely to attract more of them. The latter effect makes his demand more elastic. Hence, search cost acts as a self-selection device and generates a heterogeneous composition of demand for two firms that are symmetric except for their position.

Second, I introduce endogenous ad cost (or commission) for firms. The platform chooses the auction reserve price such that her revenue is maximised, which determines the commissions and the order of firms. Her choice, in turn, is governed by the participation and incentive constraints of the two firms. Here, I allow for consumers to choose the firm at which they start their search. I show that there exist search costs for which prices are increasing with order ($p_2 > p_1$). This result reconciles with the rationale, of consumers starting their search from a lower-priced firm, from previous work on directed search and prominence. However, the order in my model is determined by non-price device and the endogenous asymmetry in commission structure. A duopoly model helps me clearly contrast the effects on a prominent versus on a non-prominent firm. Specifically, commission paid by the prominent firm is analogous to a fixed cost since the number of visitors it gets is constant. However, commission is analogous to marginal cost for the non-prominent firm since part of the population is filtered and selected out as described earlier. This again asymmetrically forces firm 2 to raise p_2 to compensate for the rise in its 'marginal' cost, thus diminishing firms 2's ability to compete using p_2.

Essentially, higher *conversion rate* (ratio of buyers to visitors) is what matters for firms. The intuition here is that a prominent firm has a lower conversion rate, and hence, the link between clicks and purchases is weaker. On the one hand, this is comfortingly in line with the interest of consumers, because this means the market would deliver an outcome that avoids wasteful searches. In particular, the *conversion rate* for firm 2 is more important for welfare as commission plays the role of a marginal cost. However, burdened by the commission cost, firm 2 is forced to raise p_2 which lowers both his conversion rate and the overall transactions in the market, thereby exacerbating the dispersion in firms' profits. Further, this rise in p_2 gives room for firm 1 to raise p_1 without losing consumers,

click-through rates.

thus increasing its market power. This asymmetric nature of the market provides a novel rationale for growing market concentration. A comparative statics exercise with search cost provides falsifiable predictions that can be tested empirically.

As an extension, I introduce an additional dimension of product heterogeneity due to vertical differentiation. This exercise can highlight situations where preferences of consumers, firms and the platform may not be well-aligned. For instance, since the total commission depends on the number of searches, a platform can place a firm of lower quality at the first position, thus inducing more searches. However, this could be suboptimal for the consumers and firms. Results from section 5 suggest that this is not the case. A firm that is of higher quality stands to earn higher in the first position, and therefore, bids higher and procures the prominent position. Note that this need not be the case when prices are *out-of-sight*.[13]

Above results suggest that a policy-candidate to improve consumer welfare is that of monitoring price visibility. This is promising, on the one hand, because even when consumers are forced to search (if their realisation at firm 1 is low), *in-sight* prices force firm 2 to lower his price and prevent surplus extraction from the held-up consumers. Further, this is straightforward to implement in practice by holding platforms liable to minor design changes. However, it warrants caution, since, on the other hand, *in-sight* prices reduce the number of searches or product exploration, and in the long run, it might induce entry barriers. Therefore, further analysis is required to understand its implications, across different settings.

1.1 Literature Review

. This paper contributes to three broad strands of literature, namely those of firm-intermediary interaction, ordered consumer search and advertising.[14,15]

Firm-intermediary interaction. The papers - Edelman, Ostrovsky, and Schwarz (2007) and Varian (2007) - on position auctions explore the question of how firms bid to place themselves at their preferred position. They approach the problem from the perspective of optimal auctions. My paper differs in terms of both motivation and modelling. They omit the analysis of pricing behaviour, while my main interest lies in the competition structure resulting from the pricing behaviour. De Corniere and Taylor (2016) studies the effects of collusion (bias) between an intermediary and a firm. My paper, though, studies any

[13] See, for instance, Jerath, Ma, Park, and Srinivasan (2011).

[14] Academic work on *hybrid* platforms also analyses competition *within a* platform, focusing on a different set of trade-offs from this paper. See, for instance, Hagiu, Teh, and Wright (2020), Anderson and Bedre-Defolie (2021) and Shelegia and Motta (2021).

[15] The literature on multi-homing also explores the strategic interactions between consumers and platforms, focusing on a different set of trade-offs from this paper. See, for instance, Doganoglu and Wright (2006) and Belleflamme and Peitz (2019). See Jullien and Sand-Zantman (2021) for a recent survey on the effect of network externalities on consumers and platform competition. See Bergemann and Bonatti (2019) for a recent survey on platforms as information intermediaries.

endogenous bias that might arise in a competitive setting. Inderst and Ottaviani (2012) studies the role of commissions in determining the intermediary's advice. The paper uses Hotelling's framework, while mine embeds ordered search and position auctions, which allow me to microfound the value of clicks at each position. Hagiu and Jullien (2011) studies the incentives for an intermediary to divert search. Unlike my paper, it assumes exogenous contracts between intermediaries and firms.

Anderson and Renault (2021), Kang (2021) and Motta and Penta (2021) are also interested in the effect of ad auction on firms and consumers. My paper analyses the pass-through from ad commission to product pricing, unlike Anderson and Renault (2021). My paper analyses a GSP auction where a competitor's product prices affect a firm's bidding strategy, unlike Kang (2021). Motta and Penta (2021) also shows that ad auction favours a prominent firm, where a prominent firm can be interpreted as a brand that a consumer is interested in, while a non-prominent firm would be a competitor brand. Unlike Motta and Penta (2021), my paper captures consumer behaviour using a sequential search model, and highlights the channel of asymmetric commission structure in affecting product market competition. Another difference is that firms in my model commit to bids before the prices. This can be interpreted as firms correctly anticipating their equilibrium prospects and setting their auction budget in advance.[16] Further, my paper also analyses the effect of prominence and price visibility.

Ad auctions have also received attention in the Computer Science literature, largely coming from two perspectives. Papers that employ game-theoretic modelling (see, for instance, Cary, Das, Edelman, Giotis, Heimerl, Karlin, Mathieu, and Schwarz (2007), Aggarwal, Muthukrishnan, Pál, and Pál (2009), Xu, Gao, Yang, and Liu (2013)) assume an exogenous value for display position and for sale of a product. My paper endogenises this value by deriving consumers' demand from modelling their search behaviour. Recent work has used a data-driven approach (see, for instance, Broder, Gabrilovich, Josifovski, Mavromatis, and Smola (2011), Cui, Zhang, Li, and Mao (2011), Long, Dong, Pan, Huangfu, Gou, and Wang (2015), Ren, Zhang, Chang, Rong, Yu, and Wang (2017), Liu, Liu, Basu, and Crespo (2019)) where the relevant factors for firms' optimal bidding is obtained using machine learning techniques. My paper analytically derives the optimal bids as an implicit function of demand and commission structure and illustrates the channel of asymmetric commission structure.

Ordered consumer search. Haan, Moraga-González, and Petrikaitė (2018) and Choi, Dai, and Kim (2018) study a directed search model where product prices determine consumers' path. My paper considers an intermediary that determines the price-path endogenously, which then affects the consumers. Chen and He (2011) carries out a similar exercise and also endogenises firms' pricing behaviour. Athey and Ellison (2011)

[16]See, for instance, Edelman, Ostrovsky, and Schwarz (2007) and Varian (2007) for an interpretation of the static auction as a long-term approximation of the highly dynamic ad auction environment.

incorporates consumers and elicits welfare implications of different auctions. However, there is no price competition in Chen and He (2011) and Athey and Ellison (2011), and hence, no role for prominence to affect market prices. Further, my paper shows that even though firms compete to direct consumers using an additional new device (ad commission), in equilibrium, consumer behaviour follows the path of prices.

Dai (2017) uses the framework of limited commitment and Arbatskaya (2007) uses that of heterogeneous search cost to study the effect of transparent prices. My paper, though, disentangles the channels which affect consumers' price elasticity when prices are *in-sight* from when they are *out-of-sight*. This shows that the order of prices can be reversed in a model of prominence, just by making prices visible prior to costly search.

Armstrong and Zhou (2011) considers commission payments, which inflate a supplier's marginal cost, and hence, can inefficiently drive up retail prices, similar to my framework. Ding and Zhang (2018) analyses a search model with exogenous product suitability. However, both papers assume that all consumers have identical valuations of all products. My paper relaxes this assumption and assumes that consumers have idiosyncratic valuations at each firm. Moreover, my paper analyses the case of price visibility and characterises the effect of intermediary's decision on other players.

A number of papers in the literature on Marketing have documented evidence for ordered search and prominence. For instance, Ghose and Yang (2009) documents that click-through rates are highest for the firms listed higher and decrease as we go down the list. My paper, though, uses a game-theoretic model to capture the interactions between consumers, firms and an intermediary. This can help understand the underlying mechanism behind some of the above empirical findings and also highlight some of its implications.

Advertising. Hristakeva and Mortimer (2021) considers the television advertising industry where ads are contracts written long before their broadcast, and are therefore a fixed cost to the firms. The paper also shows that a prominent firm is better-off, where a prominent firm can be interpreted as those that have a long-term relationship with the TV channels, while non-prominent firms are smaller and new to the market. Varian (2021) also studies the ad auction environment and treats the ad commissions as a fixed cost. My paper allows prices to affect the search decision of consumers. And since commissions are paid per-click, ad commissions need not be a fixed cost.

My paper highlights a persuasive channel of sponsored ads unlike the informative role of sponsored ads highlighted in Sahni, Stanford, and Zhang (2019) and Moshary (2021).[17] My paper shows that even if firms are ordered by their average value to consumers, being prominent can give sufficient market power to a firm to nullify and overturn this benefit of efficient matching by platforms.

[17]See, for instance, Clark (2007), Janssen and Non (2009), Rauch (2013) for some earlier work on informative vs persuasive advertising.

The remainder of this paper is organized as follows. Section 2 presents the key features of the model. Section 3 analyses consumer demand and firm pricing in the absence of an auction. Section 4 augments this framework with an intermediary (platform) that endogenises firm positions and commissions, and derives the equilibrium outcomes. Section 5 extends the model to the case of asymmetric firms. Section 6 concludes by discussing some avenues for future research. Proofs are provided in the appendix.

2 Model

Consider a market that consists of one intermediary (m), two firms (i, j) and a unit mass of consumers. Firms and intermediary maximise their own profits while consumers maximise their utility. Numeric subscripts denote the position of the firm - 1 denotes the prominent position and 2 denotes the non-prominent.

The intermediary (*she*) conducts a Generalised Second Price (GSP) auction to determine the order in which firms will be listed on her platform. She chooses the reserve price of the auction, denoted by $\widehat{b} \in \mathbb{R}_+$, such that her revenue is maximised. The revenue that she generates equals per-click commission for each firm times the number of clicks (visitors) the respective firm gets. I assume that she is unaware of consumers' individual product valuations apriori.[18]

Firms (*he*) compete with each other for the position in which they are displayed to consumers. Firms bid the per-click commission they are willing to pay and are, in turn, displayed (ordered) based on the outcome of the GSP auction. Each firm produces one product and sets its price after his position is known (see section 4.5 for discussion on timing). Let $b_i,\ b_j \in \mathbb{R}_+$ denote the bids and $p_i,\ p_j \in \mathbb{R}_+$ denote the prices that each firm sets. Let $r_i(b_i, b_j, \widehat{b}),\ r_j(b_i, b_j, \widehat{b}) \in \mathbb{R}_+$ be the ad cost or commission per consumer visit, paid by each firm to the intermediary. Note that the bids are an input to the auction while the per-click commissions are the outputs. To simplify the analysis, the marginal cost of production for both firms are assumed to be zero.

Consumers (*they*) face a search cost $s \in \mathbb{R}_+$. As widely observed on e-commerce platforms, I assume that they observe the order of listed firms and the prices of each product without paying the search cost.[19] However, they discover their idiosyncratic valuation of a product only after they visit the seller, thereby paying the search cost. As an example, imagine a website where consumers see the price on each ad on the homepage but one has to click on it and visit the product-specific webpage to learn about its features (see figure A.1 for some examples). Let $v_i,\ v_j$ be the consumer's idiosyncratic valuations of firms (or products) i and j respectively. The outside option for consumers is normalized

[18]This assumption implies that the intermediary does not customise the order for each consumer.

[19]I compare my results with a benchmark model where consumers do not observe prices prior to paying the search cost. I find that free price visibility plays a crucial role in understanding demand and pricing. See section 3.4

to 0 and recall is assumed to be costless.

Assumption 1 (Symmetric Firms). *Consumers draw their valuations for products sold by firms i, j from an identical distribution F.*

$$v_i, v_j \sim F[\underline{v}, \overline{v}]$$

F is twice differentiable and $f(\cdot) > 0$ in this domain.

Assumption 2. *Consumer utility for product i is given by $u(p_i) = v_i - p_i$.*

Firms and the intermediary are assumed to be risk neutral and have an outside option of 0. The order of events is as follows: First, the intermediary sets the auction reserve price. Second, firms place their bids. Third, the position of firms are revealed. Fourth, firms set product prices. Finally, consumers make purchase decisions. I use the solution concept of Sub-game Perfect Nash Equilibrium.

3 Consumer and Firm Behaviour

I begin the analysis by focusing on the firms' pricing strategies in the absence of commissions, taking the order of firms as exogenous. In section 4, I introduce the role of intermediary, which endogenises the position of a firm and his commission cost.

3.1 Consumer Search

To capture the notion of prominence in a simple yet stark manner, I assume that the first search is free (this assumption does not qualitatively affect my main result; see section 4.5 for a discussion) and a search cost of $s > 0$ applies for visiting firm 2. This induces consumers to start their search from the prominent firm.[20]

Consumers know their (indirect) utility from product 1 immediately as they begin to search.

$$u(p_1) \;=\; v_1 - p_1$$

Then, the only search decision that consumers have to make is whether to visit firm 2. The expected gains of visiting firm 2 are

$$l(v_1, s, \mathbf{p}) \;=\; \int_{v_1 - p_1 + p_2}^{V} (v_2 - p_2 - \max\{v_1 - p_1, 0\})\, f(v_2)\, dv_2 \;-\; s \tag{1}$$

I define the reservation value, $\widehat{v}(s, \mathbf{p})$, as the solution to $l(v_1 = \widehat{v}, s, \mathbf{p}) = 0$. It measures the value of opportunity cost that a consumer has to forego if they decide to purchase the offer

[20]Although it is not uncommon to treat search decisions such as order of firms to visit as part of consumers' choice vector, recent evidence supports the view that consumers often search by the order displayed. See, for example, Ursu, Simonov, and An (2021).

in hand. A rational consumer determines this value by taking into account all available information at the time of making the search decision. Hence, prices have a direct effect on search. This is in contrast with the case of *out-of-sight* prices where consumers base their search decision on their expectation of p_2.[21]

Consumers buy at firm 1 without visiting firm 2 whenever their offer in-hand exceeds their opportunity cost, $v_1 > \widehat{v}(v_1, s, \mathbf{p})$. Otherwise, they visit firm 2. For brevity, I write $\widehat{v}(v_1, s, \mathbf{p})$ as $\widehat{v}$. The price difference is denoted by $\Delta p = p_1 - p_2$.

Lemma 1 (Reservation Value ($\widehat{v}$))**.** *Reservation value is obtained by solving for the marginal consumer indifferent between searching and staying,*

$$\underbrace{\mathbb{E}[v_2 | v_1 - \Delta p \leq v_2 \leq \overline{v}]}_{\textit{Expected benefit from visiting firm 2}} \;-\; \underbrace{(v_1 - \Delta p) \cdot [F(\overline{v}) - F(v_1 - \Delta p)]}_{\textit{Oppurtunity cost of visiting firm 2}} \;\geq\; \underbrace{s}_{\textit{Search cost}}$$

- $\frac{\partial \widehat{v}}{\partial s} < 0$
- $\frac{\partial \widehat{v}}{\partial p_1} > 0$
- $\frac{\partial \widehat{v}}{\partial p_2} < 0$

I illustrate the results (throughout the paper) using $F = U[0, 1]$ as a working example. The explicit form for reservation value is then given by[22]

$$\widehat{v} \;=\; 1 + \Delta p \;-\; \sqrt{2s}$$

where prices play a direct role in the search decision.

Some consumers realise a value at firm 1 such that $v_1 - p_1 < 0$. These are consumers who prefer the outside option to product 1. If at all they visit firm 2, they would only compare his product with the outside option value of zero. Since their incentives to search are slightly different, it is determined by whether

$$\int_{p_2}^{\overline{v}} [(v_2 - p_2)] f(v_2) dv_2 \;\geq\; s$$

Let

$$A := \mathbb{1}\{\mathbb{E}[v_2 | p_2 \leq v_2 \leq \overline{v}] \;-\; p_2 \cdot [1 - F(p_2)] \;\geq\; s\} \tag{2}$$

denote the condition under which consumers who realised $v_1 < p_1$ continue to search.

[21]When prices are not free to observe prior to costly search, even though consumers' expectations are true in equilibrium, any deviation in firm 2's price will not affect the number of its visitors (consumers who decide to search).

[22]Note that the expression for the case of *out-of-sight* prices is $\widehat{v} = 1 - \sqrt{2s}$.

Intuitively, when this constraint binds, it imposes an upper bound on the price that the second firm can charge if he desires to attract those who discovered a low valuation at firm 1. For $F = U[0,1]$, this condition becomes $p_2 < 1 - \sqrt{2s}$. Let me denote the region where this constraint binds by $s \geq \underline{s}$.[23]

There are two forces that determine $\widehat{v}$. First, there is the standard force (from the literature) where $\widehat{v}$ is decreasing in s. When the search cost is high, the opportunity cost of staying put is not as worrying. Hence, the reservation value is lower and the consumer stops searching for lower draws of v_1. Secondly, there is a force due to *in-sight* prices, where the reservation value is increasing in the difference between prices (Δp). Relatively cheaper the first firm is, the sooner a consumer stops searching. This may cause a downward force on product prices by inducing an additional competitive element in attracting visitors, not just retaining them. This second force plays a key role in determining consumers' decision and subsequently, firms' pricing.

Search rule. Combining the above steps, we can characterise search behaviour. For consumers who find product 1 affordable ($v_1 > p_1$), one would continue searching only if $v_1 < \widehat{v}$. For consumers who find product 1 unaffodable ($v_1 < p_1$), their decision is determined by equation 2. When both conditions fail, consumers exit the market. This rule similar to the classic result in Wolinsky (1986).

Demand. I denote by D_{xy} the demand for firm x among consumers who visit y number of firms.[24] For example, D_{12} denotes those individuals who visit both firms but finally purchase from firm 1. Thus, demand for firm 1 is given by $D_1 = D_{11} + D_{12}$ and demand for firm 2 is given by $D_2 = D_{22}$.

D_{11} denotes the consumers who do not continue searching. They discover their valuation at firm 1 such that $v_1 > \widehat{v}$. Hence,

$$D_{11} = 1 - F(\widehat{v}) \tag{3}$$

D_{12} consumers satisfy two conditions. They want to search ($v_1 < \widehat{v}$) and prefer the first firm after discovering their valuation at firm 2 ($v_1 - p_1 > v_2 - p_2$). Hence,

$$D_{12} = \int_{p_1}^{\widehat{v}} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1 \tag{4}$$

D_{22} consumers also satisfy two conditions. They also want to search ($v_1 < \widehat{v}$) but prefer the second firm after discovering their valuation at firm 2 ($v_1 - p_1 < v_2 - p_2$). Additionally,

[23] For a general F, this threshold can be expressed as

$$\underline{s} = \{t | \ \mathbb{E}[v_2 | p_2 \leq v_2 \leq \overline{v}] - p_2 \cdot [1 - F(p_2)] > s \ \forall s < t\}$$

[24] For brevity, I write $D_{xy}(\mathbf{p}, \mathbf{b}, \widehat{v}(s, \mathbf{p}), \widehat{b})$ as D_{xy} throughout the paper.

D_{22} also includes consumers who held an outside option of zero ($v_1 < p_1$) and continued to search and also prefer the second firm after discovering their valuation at firm 2 ($v_2 > p_2$). Hence,

$$D_{22} \;=\; A \cdot [1 - F(p_2)]F(p_1) + \int_{p_1}^{\widehat{v}} [1 - F(v_1 - \Delta p)] \cdot f(v_1) \cdot dv_1 \tag{5}$$

where A denotes the condition from equation 2.

Following the terminology commonly used in the literature on prominence, D_{11} and D_{22} are also referred to as *fresh demand* since these buyers are visiting the respective firms for the first time. D_{12} is also referred to as *returning demand* since they are visiting the firm for the second time. Each of these demands are driven by different combinations of forces. This asymmetry in firms' demand invites careful consideration and, in fact, turns out to be the key element behind the equilibrium outcome derived in theorem 1. Lemma 2 shows comparative statics of these demands with respect to search cost.

Lemma 2. *For $s < \underline{s}$*

$$\frac{\partial D_{11}}{\partial s} > 0 \quad , \quad \frac{\partial D_{12}}{\partial s} < 0 \quad , \quad \frac{\partial D_{22}}{\partial s} < 0$$

An increase in search cost leads to a decrease in the reservation value. Therefore, fewer consumers search, thus, implying a rise in D_{11}. This, in turn, reduces the number of visitors to firm 2 and exerts a downward force on D_{12} and D_{22}. For consumers who still decide to search, we can deduce that they had a low valuation at firm 1. This would have a negative impact on firm 1's returning demand but a positive impact on firm 2's demand. The two-fold negative force lowers D_{12}. For D_{22}, the above two forces act in opposite directions. Overall, we see that the first effect dominates in the region where firm 2 can adjust his price freely (when equation (2) does not bind).

3.2 Firms' problem

Baseline. Firms 1 and 2 solve the following objective functions. Subscript NA denotes 'No Auction'.

$$\begin{aligned} \max_{p_{1,NA}} \; Rev_1 &= p_{1,NA}(D_{11} + D_{12}) \\ \max_{p_{2,NA}} \; Rev_2 &= p_{2,NA} D_{22} \end{aligned} \tag{6}$$

Below, I list some benchmark results to compare with.

Price for monopoly.

$$p^{Mon} \;=\; \frac{1 - F(p^{Mon})}{f(p^{Mon})}$$

It requires $pf'(p) < 2f(p)$ (or hazard rate is increasing in p) for p^{Mon} to be a global maximum.

Price for random search and two firms.

$$p^{Ran} = \frac{1 - F(p^{Ran})^2}{f(p^{Ran})}$$

It requires $pf'(p) < 2f(p)$ (or hazard rate is increasing in p) for p^{Ran} to be a global maximum.

3.3 Equilibrium

Using inverse demand functions from equations (3), (4) and (5), I solve the objective functions in (6) simultaneously, to obtain equilibrium prices. Lemma 3 shows that even when firms are symmetric in terms of their marginal cost and distribution of value to consumers, there is no symmetric pure-strategy equilibrium in prices. This result is driven by the asymmetry in composition of demand due to their respective positions.

Lemma 3. *There exists no symmetric equilibrium in pure strategies for* $0 < s \leq \overline{s}$*.*

To further understand the effect of this asymmetry, I solve for the asymmetric equilibrium in pure strategies. Theorem 1 characterises the equilibrium prices.

Theorem 1 (*In-sight* prices)**.** *For a distribution F under assumptions 1 and 2, asymmetric equilibrium in pure strategies is given by*

$$p^*_{1,NA} = \frac{1 - F(\widehat{v}^*) + \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1}{f(\widehat{v}^*) - \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1}$$

$$p^*_{2,NA} = \frac{A \cdot [1 - F(p^*_{2,NA})]F(p^*_{1,NA}) + F(\widehat{v}^*) - F(p^*_{1,NA}) - \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1}{A \cdot f(p^*_{2,NA})F(p^*_{1,NA}) + f(\widehat{v}^*) - F(p^*_{2,NA})f(p^*_{1,NA}) - \int_{p^*_{1,NA}}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1}$$

The intuition behind this result is as follows. Consider the unit mass of consumers plotted in figure 1. The co-ordinates of the square classify consumers by their valuation of the two products. For a small change in p_2, we see that demand for firm 2 changes along both line segments LM and MN. Since firm 1 shares those line segments, these consumers are taken over by him. On the other hand, imagine prices are *out-of-sight*. For a small change in p_2, we see that demand for firm 2 changes along line segment LM only. In other words, demand for firm 2 is now less elastic compared to the case of *in-sight* prices. This reduction in elasticity is sufficient to overturn the order of prices. See section 3.4 for further discussion.

To further illustrate the intuitions behind the above result, I make use of my working example in proposition 1.

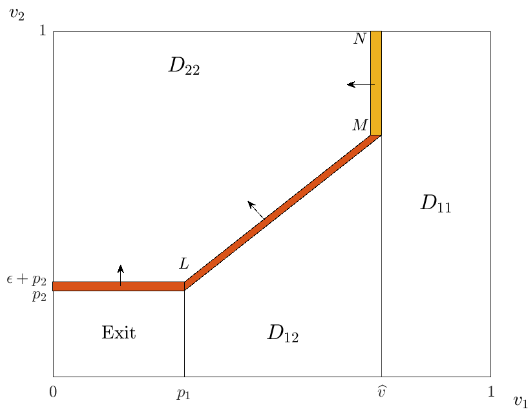


Figure 1: Elasticity of demand

Proposition 1. *For $F = U[0,1]$, $\widehat{v} \ = \ 1 + \Delta p - \sqrt{2s}$ and*

$$\begin{aligned} p^*_{1,NA} &= 1 - \widehat{v}^* + \frac{(\widehat{v}^* - \Delta p^*_{NA})^2 - p^*_{2,NA})^2}{2} \\ p^*_{2,NA} &= \frac{[1 - p^*_{2,NA}]p^*_{1,NA} + \widehat{v}^* - p^*_{1,NA} - \frac{(\widehat{v}^* - \Delta p^*_{NA})^2 - p^*_{2,NA})^2}{2}}{1 + \Delta p^*_{NA}} \end{aligned} \tag{7}$$

- *Conditions for global maxima are satisfied.*
- $\underline{s}_{NA} = \frac{1}{4}, \overline{s}_{NA} = \frac{1}{2}$
- $\frac{dp^*_{1,NA}}{ds} > 0, \frac{dp^*_{2,NA}}{ds} < 0$
- $p^*_{1,NA} > p^*_{2,NA}$

where market breaks down for $s > \overline{s}$.

Corollary 1. *For $F = U[0,1]$*

$$\begin{aligned} p^*_{1,NA} &> p^*_{2,NA} \\ p^*_{1,NA} &\in [p^{Ran}, p^{Mon}] \\ p^*_{2,NA} &\in [0, p^{Ran}] \end{aligned}$$

Figure 2 summarises the results from proposition 1. At zero search cost, the model collapses to the case of random search. Thus, we have the full information price as the equilibrium. As search cost increases, fewer consumers search. The reservation value, $\widehat{v}$, is equal to 1 at $s = 0$ and gradually decreases to $\widehat{v} = p_1$ at $s = \underline{s}$. Since $\widehat{v} > p_1$ in this region, there is no constraint on firm 2's price. This means that competition reduces gradually

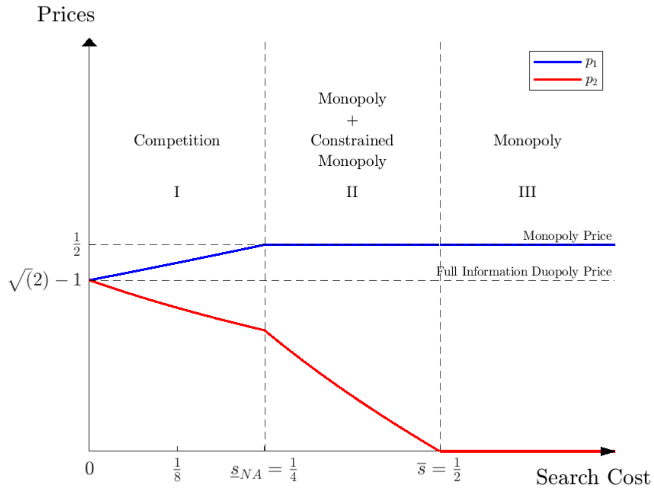


Figure 2: Prices (without auction)

with s, in region I. This can be seen from the increase in the price of firm 1. However, since prices are *in-sight*, firm 2 reacts by reducing its price to attract consumers. Crucially, *in-sight* prices eliminates any possibility for firm 2 to deviate to a higher price as it would immediately deter consumers from visiting him. Therefore, even though competition is reducing, we see that p_2 is decreasing with s in equilibrium.

At $\underline{s}$, firm 1 reaches the monopoly price and $\widehat{v} = p_1$. Now, all consumers who buy from firm 1 are precisely those that never searched (didn't visit product 2) and hence, $D_{12} = 0$. This implies that a certain fraction of consumers in the market buy product 1 without considering the other product while the rest of them find it unaffordable at any price. In other words, there is no longer any competition between the two firms. The intuition for this is as follows. When $\widehat{v} = p_1$, firm 2's demand consists only of consumers who hold in-hand an option of utility zero ($\max\{\widehat{v} - p_1, 0\} = 0$). At this point, firm 2's demand could potentially only comprise of consumers with $v_1 < p_1$, that is, those who will not return to firm 1 for any offer they get at firm 2. They will either buy product 2 or exit the market. This is as-if there is complete market segmentation as none of the consumers compare the two products while buying. Therefore, firm 2 would like to set the monopoly price as well. But since constraint A binds for $s > \underline{s}$, a high price would not attract any visitors to firm 2. Therefore, firm 2 is forced to choose a constrained-optimal price to attract atleast some visitor. Since the profits from this are still positive, firm 2 chooses to participate in the market. Note that this region also sees an increase in volume of transactions at firm 2 with s because price is decreasing while the consumers have the same outside option and expected benefit of search.

Diamond (1971) showed that, with homogeneous products and positive search costs, rivalry between firms had no impact on price. Subsequent research has shown that in search models with product differentiation, there are some consumers who are ill matched with their initial choice of supplier and this pushes them to search further. Similarly, I avoid the above well-known modelling difficulty (Diamond paradox) due to the existence of a pro-competitive benefit of search and hence, prices do not jump to the monopoly value for any small positive search cost. However, a situation similar to the one illustrated in Diamond (1971) does arise in my model. At intermediate search costs, that is when $\underline{s} \leq s \leq \overline{s}$, competition is eliminated. As in Diamond paradox, consumers do not search and compare. This market segmentation means that consumers only visit and buy (if at all) from one particular firm and that both firms prefer the monopoly price. Due to constraint A, firm 2 settles for the constrained-optimum.

When $s = \overline{s}$, firm 2 has exhausted its ability to attract consumers to search. The price hits the lower bound of zero. This leaves only one active firm in the market which continues to function as a monopoly for $s > \overline{s}$.[25]

3.4 Price Visibility

Equilibrium prices in a market where they are free to observe (*in-sight*) are lower than the corresponding market with *out-of-sight* prices, *ceteris paribus*. Intuitively, a seller's demand is more elastic with respect to changes in *in-sight* prices than when price is only discovered after the consumer pays the search cost (see figure 1). In models of random search (Wolinsky (1986), Anderson and Renault (1999)) and prominence (Armstrong, Vickers, and Zhou, 2009), firms might lower prices not to attract consumers, but to retain them once they visit. In my model, prices serve both roles: they help to both attract and retain consumers.[26]

Specifically, a lower price for firm 2 not only retains more visitors, but is also more likely to attract them; the latter effect makes his demand more elastic. This suggests that when consumers observe prices before searching, prices decrease with search costs. This can be observed from the price of firm 2. Since price of firm 1 is observed costlessly by the consumer at the beginning of search, this effect is absent in its case. Therefore, price dispersion increases with search cost in my model.

To further understand effect of *in-sight* prices, I disentangle the channels which influence firm 2's decision. When consumers visit firm 2, he learns that they, on average, have a lower valuation of product 1. This makes those consumers more likely to stay at firm 2, thus applying an upward force on p_2. I refer to this as the *information channel*.

When prices are *out-of-sight*, consumers visit firm 2 based on their rational belief about his price. However, he is free to surprise them as it doesn't change their incentives to visit

[25] Note that this is also the search cost at which market breaks down in Wolinsky (1986) and Armstrong, Vickers, and Zhou (2009).

[26] Some previous studies consider prices as free to observe under different conditions. For instance, Dai (2017) uses it to analyse limited commitment and Arbatskaya (2007) uses it to analyse heterogeneous search cost.

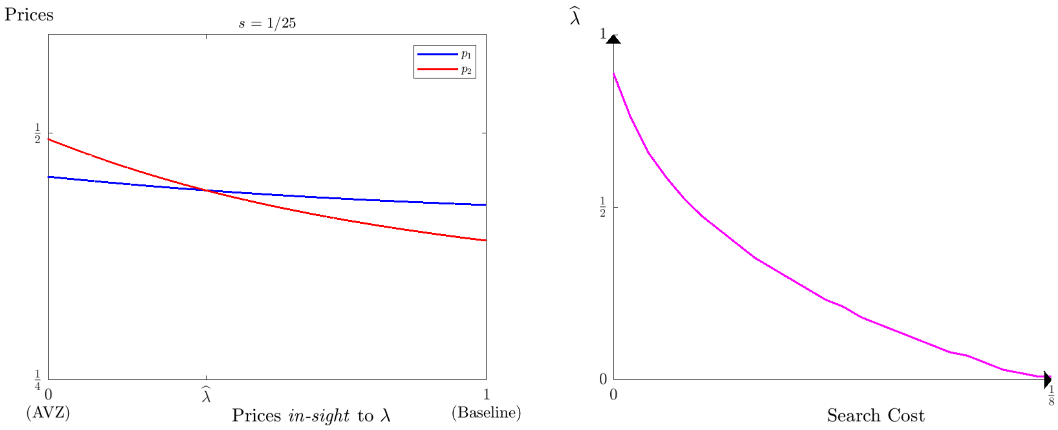


Figure 3: Weakening of the Hold-up channel

firm 2. This also applies an upward force on p_2. I refer to this as the *hold-up channel*. Note that, the case of *out-of-sight* prices influences through both channels. However, making prices free to observe eliminates firm 2's ability to surprise and shuts down the *hold-up channel*. Therefore, I find that only having the information channel is not sufficient to generate increasing order of prices.

To illustrate this effect, I consider a market where $\lambda \in [0, 1]$ fraction of consumers can see both prices before starting their search while the rest discover prices by search. This exercise nests equilibrium prices from my benchmark model at $\lambda = 1$ and those from the Armstrong, Vickers, and Zhou (2009) (AVZ) model (which predicts $p_1 < p_2$) at $\lambda = 0$.

Figure 3 shows that as prices become free to observe to more consumers (increase in λ), the role of p_2 in attracting consumers gains significance. The *hold-up* force, which had caused higher prices for firm 2 in AVZ, is weaker now due to an increase in the price elasticity of firm 2's demand. This reverses the price order.

As search cost increases, it makes the firms incentives to attract consumers more important. Therefore, even when a smaller fraction of consumers see prices before search, we see a reversal in price rankings. Moreover, this exercise of parametrizing λ resembles the 'clearing-house' models of search (see, for instance, Varian (1980), Perloff and Salop (1985)) where a fraction of the population have access to prices. In those models, we usually see a mixed-strategy equilibrium. However, in my model, the underlying consumer heterogeneity allows for the existence of pure-strategy equilibrium.

3.5 Profits

In this section, I derive the equilibrium revenues for both firms. Since the marginal cost of production is normalised to zero, this is equivalent to their profits.

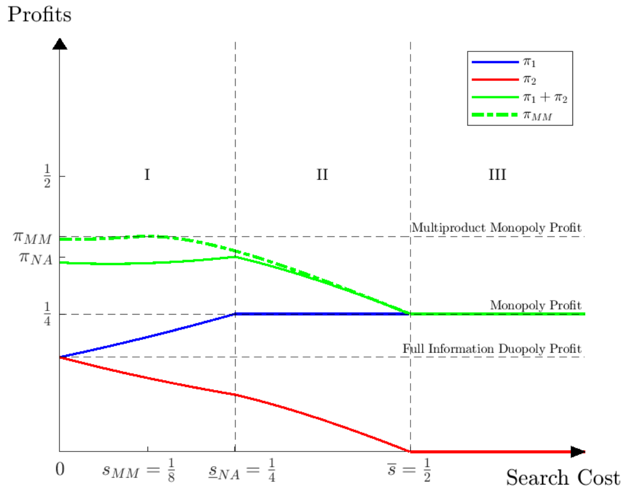


Figure 4: Profits (without auction)

Proposition 2. *Equilibrium revenue for $s < \underline{s}$ and $F = U[0, 1]$*

$$\frac{dRev_1^*}{ds} > 0 \ , \ \frac{dRev_2^*}{ds} < 0$$

Figure 4 plots the characterisation of firm revenues, following proposition 2. As search cost increases, there is a decrease in competition (region I). This increases the profits as seen for firm 1 and reaches the monopoly level at $\underline{s}$. For $s > \underline{s}$, firm 1 maintains its optimal price at the monopoly level.

On the other hand, due to *in-sight* prices, firm 2 has lost its ability to extract the hold-up rent and the price decreases along with its (region I). In region II, firm 2's demand increases with s. This is because he now has a constant population of visitors to attract and retain and as he decreases his price, more consumers find the offer feasible and purchase. However, the profits are well below the optimum due to constraint A. Therefore, profit continues to decrease.

The industry profits (sum of profits) is maximum at the lowest search cost where both firms have segmented demand. This is because $\underline{s}$ would impose the loosest constraint on firm 2 while there is no competition (both firms have their own set of visitors).

Figure 4 also compares the equilibrium industry profits from my baseline model with the case where a single firm is selling both products. I refer to this second case as the Multi-product Monopoly (MM).[27] Drawing parallels between the intermediary's function and MM can be helpful while considering policy tools applicable to the online market.

[27] See, for instance, Petrikaitė (2018) and Gamp (2016). Both papers characterise the optimal search cost such that a multi-product monopoly firm chooses to maximise its profits.

From the perspective of the intermediary, this benchmark may also be useful to assess alternate position-allocation mechanisms.

The MM uses search cost to (imperfectly) screen consumers based on their preferences. Therefore, MM finds it optimal to set some $s > 0$. Only the consumers with lower valuations for product 1 are catered to by the second product. This lets the MM charge a higher price for product 1, $(p_1)_{MM} > (p_2)_{MM}$. The MM does not lose as many consumers - if it raises its prices - as when firms are separate because some consumers buy from the other firm now (which is also a subsidiary of MM). Hence, prices of both products are higher in the case of MM than in the my baseline model.

Figure 4 also shows that the optimal s for the MM (s_{MM}) is lower than $\underline{s}$. This is because the MM is able to reach region II quicker. Since MM internalises the fact that some consumers that leave firm 1 for high prices buy product 2 instead, MM prices the goods more aggressively and there is a segmentation of the market for lower s. Thus, the maximum profit for MM is also larger than the industry profit when firms are separate.

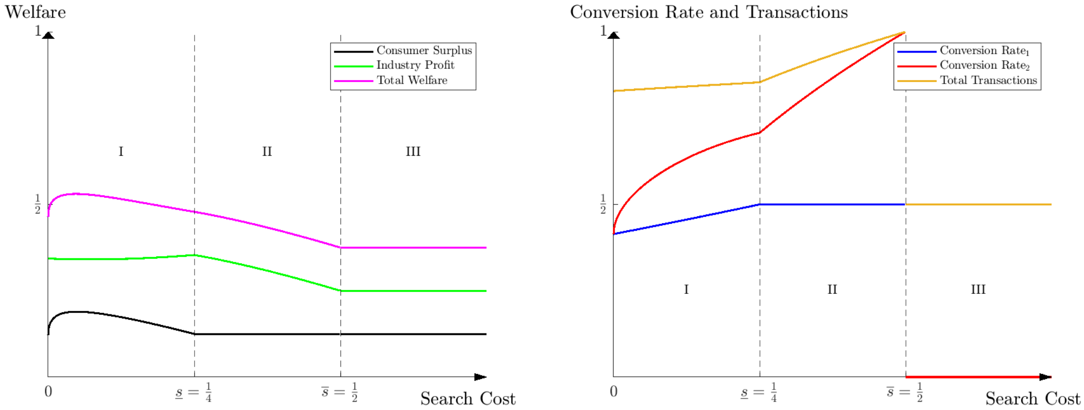
3.6 Welfare

In this subsection, I analyse the welfare implications in equilibrium. Figure 5 shows Consumer Surplus (CS_{NA}) and Total Welfare (TW_{NA}) for $F = U[0, 1]$. As search cost increases, searches are costlier and overall CS is affected negatively. On the other hand, buyers at firm 1 who don't search (D_{11}) avoid this cost. Their numbers increase with rising search cost and hence, contribute positively to CS_{NA}. Since firm 2 has lost its ability to surprise consumers (to extract surplus) due to *in-sight* prices, p_2 falls with search cost, countering the negative effect of search cost on his visitors. In region II, market is segmented and the situation at firm 1 remains constant. However, firm 2 now faces a constraint which forces it to charge a lower price as search cost increases. This favours the consumers substantially until he exits the market in region III. This benefit exactly counteracts the negative impact of search cost for customers of firm 2. Overall, we see a flat curve in region II. TW_{NA} is largely driven by CS_{NA} and reaches its peak at a much lower search cost than the Industry Profits. This point of maxima for TW_{NA} is also beneficial for firm 2 while firm 1 would be worse-off.

Figure 5 also plots a measure of positional performance of firms. The *conversion rate* (q) signifies the fraction of visitors that each firm manages to convert into successful sales. This gives us a measure of efficiency in the matching market.

$$\begin{aligned} (q_1)_{NA} &= D_{11} + D_{12} \\ (q_2)_{NA} &= \frac{D_{22}}{1 - D_{11}} \end{aligned}$$

As search cost increases consumers prefer the option in-hand to searching firm 2 and this



(a) Welfare (b) Conversion Rate

Figure 5: Welfare and Efficiency: 'No Auction'

drives the conversion rate of firm 1 up. On the other hand, the *information channel* grows stronger with search cost and those who do search probably have a bad offer at firm 1. This can be seen from the increase in firm 2's conversion rate. In region II, firm 2 gradually lowers price to attract consumers which makes his product affordable to more visitors. This culminates with a full conversion rate as $p_2 \to 0$ at $s = \overline{s}$.

A measure of overall market efficiency is the total volume of transactions which is given by the total demand in equilibrium $TR = D_{11} + D_{12} + D_{22}$. This is also equivalent to $1 - p_1 p_2$. The two efficiency measures provide clear model predictions to test empirically. Total transactions reach a maximum at $s = \overline{s}$, driven by the sales at firm 2.

4 Market Equilibrium With Auction

In this section, I introduce to the firms' problem an ad cost or commission. This induces firms to care more about 'quality' (conversion rate) than the quantity of clicks (visitors). Formally, the commissions charged per-click affect the optimal pricing problem. First, I consider the case where firms face a constant commission for any search cost. Second, I introduce a revenue-maximising intermediary that allocates ad positions and determines their commissions endogenously, in equilibrium.

4.1 Constant Commission

Firms' problem. Firms maximise their profits by choosing product prices ($\mathbf{p}$) and r_1 and r_2 are per-click commission to be paid by the respective firms. The analysis in section 3 was a special case of this framework where $r_1 = r_2 = 0$. In subsection 4.2, r_1 and r_2 will be endogenous objects of the auction. Firms maximise profit.

$$\max_{p_{1,C}} \pi_1 = p_{1,C}(D_{11} + D_{12}) - r_1(b_i, b_j, \widehat{b}) \tag{8}$$

$$\max_{p_{2,C}} \pi_2 = p_{2,C}D_{22} - r_2(b_i, b_j, \widehat{b}) \cdot (1 - D_{11}) \tag{9}$$

Subscript C denotes 'constant commission'. This framework with two firms parsimoniously allows me to highlight the differences in the cost structure of firms, due to positioning. From the above equation, we see the different roles per-click commissions play. Since all consumers visit firm 1, the ad commission plays the role of a fixed cost for firm 1, while it is analogous to a marginal cost per visitor for firm 2. This asymmetry, again a consequence of asymmetry in ad position, plays a crucial role in determining the equilibrium outcome, as shown in proposition 3 and later in the full equilibrium with auction in theorem 2.

Proposition 3 (Constant commission)**.** *For a distribution F under assumptions 1 and 2, asymmetric equilibrium prices in pure strategies are given by*

$$\begin{aligned} p^*_{1,C} &= \frac{1 - F(\widehat{v}^*) + \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f(v_1) \cdot dv_1}{f(\widehat{v}^*) - \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f'(v_1) \cdot dv_1} \\ p^*_{2,C} &= \frac{A \cdot [1 - F(p^*_{2,C})]F(p^*_{1,C}) + F(\widehat{v}^*) - F(p^*_1) - \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f(v_1) \cdot dv_1 + r_2 f(\widehat{v}^*)}{A \cdot f(p^*_{2,C})F(p^*_{1,C}) + f(\widehat{v}^*) - F(p^*_2)f(p^*_1) - \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f'(v_1) \cdot dv_1} \end{aligned}$$

For $F = U[0,1]$,

$$\begin{aligned} p^*_{1,C} &= 1 - \widehat{v}^* + \frac{(\widehat{v}^* - \Delta p^*_C)^2 - p^{*}_{2,C})^2}{2} \\ p^*_{2,C} &= \frac{[1 - p^*_{2,C}]p^*_{1,C} + \widehat{v}^* - p^*_{1,C} - \frac{(\widehat{v}^* - \Delta p^*_{1,C})^2 - p^*_{2,C})^2}{2} + r_2}{1 + \Delta p^*_C} \end{aligned}$$

and conditions for maxima are satisfied.

- $\underline{s}_C = \frac{1-2r_2}{4} < \underline{s}_{NA}$
- $\frac{dp^*_{1,C}}{ds} > 0$, $\frac{dp^*_{2,C}}{ds} < 0$
- *There exists* $\widehat{s}$ *such that* $p^*_{1,C} - p^*_{2,C} \begin{cases} < 0 & \text{if } s < \widehat{s} \\ > 0 & \text{if } s > \widehat{s} \end{cases}$

Proposition 3 shows that for sufficiently low commission rates, there exist search costs for which $p_2 > p_1$ (see, for example, figure 6). The marginal cost for visits for firm 2 increases his price which, in turn, gives room for firm 1 to also raise its price without the fear of

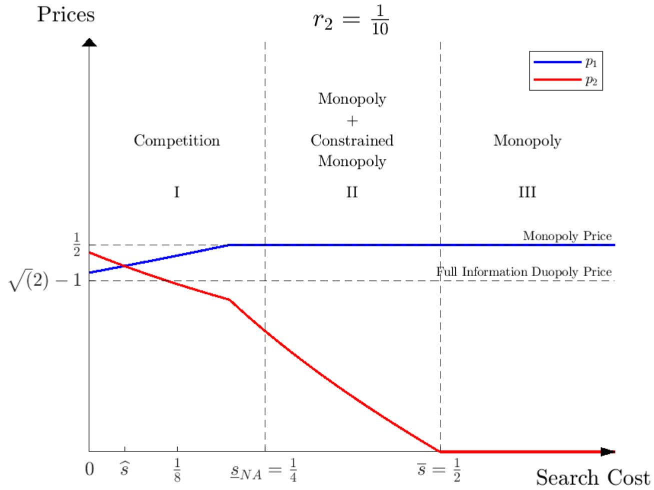


Figure 6: Prices (with constant commission)

losing consumers. However, the effect on firm 1 is of second-order. Note that firm 1 faces a fixed cost of commission. Hence, there is an asymmetric rise in p_2. This result reconciles with markets observed with increasing order of prices. However, the order is determined in my model due to the asymmetry in the cost structure. Thus, a non-prominent firm can charge a higher price, even when the *hold-up channel* has been eliminated.

The equilibrium prices diverge with search cost, following the properties highlighted in section 3. Note that $\underline{s}_C < \underline{s}_{NA}$ as prices are higher compared to the 'No Auction' case. The rise of p_2 due to marginal cost-like commission weakens competition as it damages firm 2's ability to attract and retain consumers. This gives firm 1 more power in the market for all s. Hence, p_1 shifts closer to the monopoly price and leads to a segmented market for a smaller s.

4.2 Position Auction

Firms' problem. Firms participate in a one-shot auction conducted by intermediary, to procure a position for their ad. Firms pay a per-click commission equal to the bid of the next firm in the ordering. The total commission to the intermediary is a function of the per-click commission (r_1, r_2). Firms maximise their profits by choosing their respective product prices (p_1, p_2) and auction bids (b_1, b_2). Firms place their bids, learn their positions and commissions and finally, set their product prices (see section 4.5 for a discussion on timing).

$$\max_{p_1,b_1} p_1(D_{11}+D_{12}) - r_1(b_i,b_j,\widehat{b}) \tag{10}$$

$$\max_{p_2,b_2} p_2 D_{22} - r_2(b_i,b_j,\widehat{b}) \cdot (1-D_{11}) \tag{11}$$

Intermediary's problem. Intermediary conducts a one-shot Generalised Second Price Auction (GSP) with the objective of maximising her own revenue. She ranks firms based on the expected payment from each firm, that is "Pay Per-Click" (PPC) commission times the number of visitors. Ties are broken by displaying each possible order with equal probability. The intermediary maximises its revenue by setting the optimal reserve price.

$$\max_{\widehat{b}} \ \pi_m \ = \ r_1(b_i,b_j,\widehat{b}) + r_2(b_i,b_j,\widehat{b}) \cdot (1-D_{11})$$

In a GSP auction, we have $r_1 = b_2, \ r_2 = \widehat{b}$ when firms bid b_1, b_2. See section 4.5 for a discussion on auction format.

4.3 Equilibrium

Given consumer demand and firm revenues for each position (Rev_1, Rev_2), I solve for the optimal bidding behaviour of the two firms. Consider $\widehat{b}$, the auction reserve price, a feature of the game, to be common knowledge. Once I have the optimal bids, I can solve for the optimal auction reserve price. As commonly found in the literature, this specification leads to multiple equilibria in pure strategies. Table A.1 shows the market outcome for different values of the auction reserve price ($\widehat{b}$). The condition on optimal reserve price is derived from firms' participation and incentive compatibility constraints.

When firms bid symmetrically,

$$b_i = b_j \ = \ Rev_1 - \left(Rev_2 - \widehat{b}(1-D_{11})\right)$$

This is feasible only when

$$\widehat{b} \ \leq \ \frac{Rev_2}{1-D_{11}}$$

When firms bid asymmetrically,

$$b_i \ \in \ \left(Rev_1 - \left(Rev_2 - \widehat{b}(1-D_{11})\right) \ , \ \infty\right)$$
$$b_j \ \in \ \left[\widehat{b} \ , \ Rev_1 - \left(Rev_2 - \widehat{b}(1-D_{11})\right)\right]$$

This is feasible only when

$$\widehat{b} \leq \min\left\{\frac{Rev_1 - Rev_2}{D_{11}}, \frac{Rev_2}{1 - D_{11}}\right\}$$

When a similar issue of multiplicity arises in the standard second-price (or Vickrey) auction, it is commonly dealt with by focusing on the weakly dominant strategy equilibrium. Analogously, I consider the set of equilibria in undominated strategies. The intuition here is that the bidder plays a strategy such that he will have no regret of losing the top position even if the opponent (other firm) deviates to a slightly lower bid.[28] Similar to the outcome in second-price auction, I find that this refinement rules out asymmetric bidding and we are left with a unique equilibrium outcome in pure strategies, for a given value of search cost (an exogenous parameter). Theorem 2 characterises this outcome.

Theorem 2 (Position Auction). *For a distribution F under assumptions 1 and 2, optimal bids and auction reserve price in the GSP auction are given by*

$$\begin{aligned}
\widehat{b}^* &= \min\left\{Rev_1^* - Rev_2^*, \frac{Rev_2^*}{F(\widehat{v}^*)}\right\} \\
b_i^* &= \left(Rev_1^*, \infty\right) \\
b_j^* &= Rev_1^*
\end{aligned} \tag{12}$$

Firms occupy each position with a probability of one-half. Equilibrium prices in pure strategies for each position are given by

$$\begin{aligned}
p_1^* &= \frac{1 - F(\widehat{v}^*) + \int_{p_1^*}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1}{f(\widehat{v}^*) - \int_{p_1^*}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1} \\
p_2^* &= \frac{A \cdot [1 - F(p^*)]F(p^*) + F(\widehat{v}^*) - F(p_1^*) - \int_{p_1^*}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1 + \widehat{b}^* f(\widehat{v}^*)}{A \cdot f(p^*)F(p^*) + f(\widehat{v}^*) - F(p_2^*)f(p_1^*) - \int_{p_1^*}^{\widehat{v}^*} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1}
\end{aligned} \tag{13}$$

Although there might still multiplicity due to the fact that firm 1's range of bids doesn't have an upper bound, this doesn't play a role in the equilibrium. Nor does it enter into any firms' commission cost. Theorem 2 finds the same outcome as if imposing the refinement of 'locally envy-free' strategies (Edelman, Ostrovsky, and Schwarz, 2007). See section 4.5 for further discussion.

Equation 13 shows how the asymmetry in commission structure enters the pricing equation. The non-prominent firm faces a marginal-cost like term added to the baseline expression from the 'No Auction' case (see equation 7). This forces firm 2 to raise its price

[28]This requires a technical assumption that a bidder (firm) exercises caution and doesn't *completely* rule out any action of the other firm. For further discussion, see Brandenburger, Friedenberg, and Keisler (2008) and Hillas and Samet (2020).

in order to cover this cost. As a result, firm 1 finds some space to increase his own price without the risk of becoming less attractive to consumers.

To further illustrate the intuitions behind the above result, I make use of my working example in proposition 1. Figure 7 plots the results.

Proposition 4. *For $F = U[0,1]$,*

$$\begin{aligned}
\widehat{b} &= \frac{Rev_2}{1 - D_{11}} \\
b_i &= (Rev_1, \infty) \\
b_j &= Rev_1 \\
\pi_1 = \pi_2 &= 0 \\
\pi_m &= Rev_1 + Rev_2 \\
p_1^* &= 1 - \widehat{v}^* + \frac{(\widehat{v}^* - \Delta p^*)^2 - p_2^*)^2}{2} \\
p_2^* &= \frac{[1 - p_2^*]p_1^* + \widehat{v}^* - p_1^* - \frac{(\widehat{v}^* - \Delta p_1^*)^2 - p_2^*)^2}{2} + \widehat{b}^*}{1 + \Delta p^*}
\end{aligned}$$

and conditions for maxima are satisfied.

- $\underline{s} = \frac{1 - 2\widehat{b}^*}{4} < (\underline{s})_{NA}$
- $p_1^* > p_2^*$

Since both prices are higher, they hit the monopoly mark at a lower search cost. Further, the reservation value drops with a steeper slope due to relatively larger increase in p_2. Therefore, region I shrinks and market segments for lower value of search cost (see figure 7).

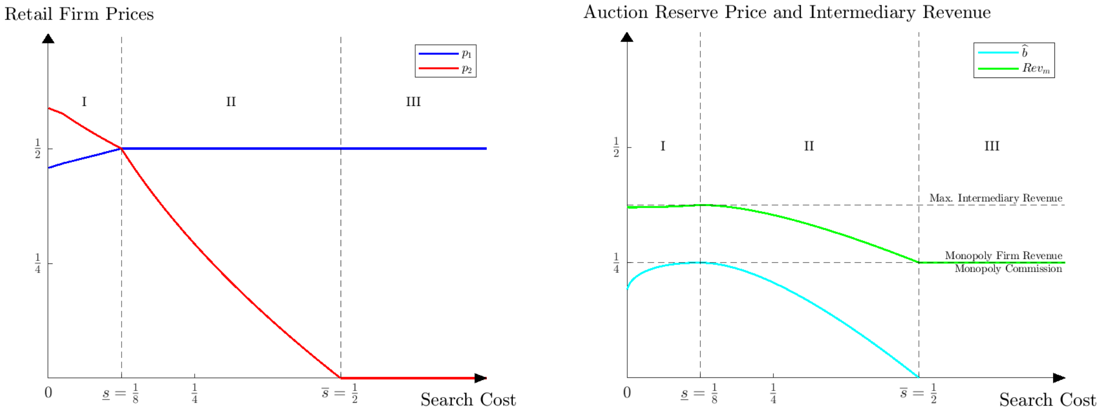
I find that the prices are higher in an equilibrium with an auction than without. Consequently, the main difference is the shrinkage of region I.

Proposition 5. *Comparison with 'No-Auction' case ($r_1 = r_2 = 0$):*
$p_1 \geq p_{1,NA}$ and $p_2 \geq p_{2,NA}$

4.4 Welfare

In this subsection, I redo the welfare analysis in equilibrium with auction. Analogous to section 3.6, I plot the results for $F = U[0,1]$ in figure 8. The results are similar but with a shift in curves to the left as the market segments for a lower search cost ($\underline{s} < \underline{s}_{NA}$). Note that the levels of consumer surplus is lower compared to figure 5 since the prices are higher in the model with auction.

The main takeaway in terms of empirical predications is that the conversion rate of firm 2 falls substantially with commission cost (which is driving p_2 up) compared to the case

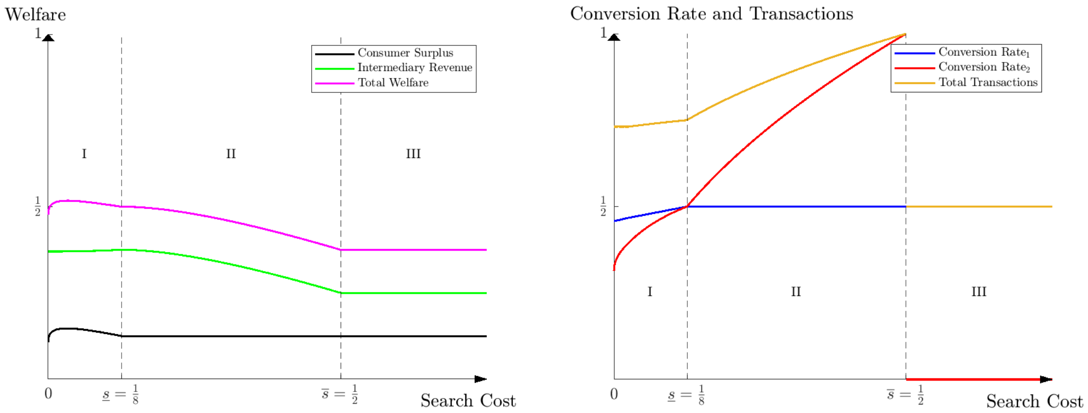


(a) Prices

(b) Revenue, commission and profit

Figure 7: Equilibrium in pure strategies

of 'No Auction'. Total transactions has also fallen significantly in region I but eventually catches up in region II.



(a) Welfare

(b) Conversion Rate

Figure 8: Welfare and Efficiency: With Auction

4.5 Discussion

4.5.1 Are the equilibria self-fulfilling?

Following the early work in Stahl (1989), treating the first search as free has been a common assumption in the consumer search literature.[29] Moreover, since consumers encounter online ads free of charge, their search costs are mainly the amount of time spent browsing the product on the Internet. In this context, the assumption of free visit to the prominent firm is motivated by ad listings where websites often host a larger advertisement of a particular firm with product details while the others are shown as smaller thumbnails. Therefore, options may be presented to consumers in an exogenously restricted order, and it costs lesser for consumers to learn the details of the first firm. Another motivating example is that of websites that make some ads less prominent by placing them at a less immediate location, requiring the consumer to scroll down or click on a hyperlink, thus entailing a search cost to visit the second firm. To extend the fit of my model to more general settings, I consider alternative assumptions in this subsubsection using my working example of $F = U[0,1]$.

If $s > 0$ for the prominent as well? Note that consumers still have to follow the displayed order. A positive search cost for firm 1 imposes an additional constraint which determines what fraction of the population prefers to visit firm 1 rather than choose the outside option of zero. Formally, it can be represented as

$$\int_{p_1}^{1} (v_1 - p_1) f(v_1) dv_1 - s > 0$$

This constraint does not bind when $p_1 < 1 - \sqrt{2s}$. For low search cost (region I), the equilibrium prices are the same as in figure 7 as the above constraint does not bind. For $s > \bar{s}$ (region III), the market breaks down as nobody is interested in searching. The action happens in region II.

Recall that the reservation value for a consumer considering between staying at firm 1 and visiting firm 2 is given by

$$\hat{v} = 1 + p_1 - p_2 - \sqrt{2s}$$

The market would be active only if the reservation value exceeds the outside option. For $\underline{s} < s < \bar{s}$, this constraint binds (region II) and can be written as $p_2 < 1 - \sqrt{2s}$. The only question that remains now is to determine if the firms would prefer to deviate from $p_1 = p_2 = 1 - \sqrt{2s}$ (the equilibrium prices at $s = \underline{s}$) and profits of $\pi_1 = (1 - \sqrt{2s})\sqrt{2s}$ and $\pi_2 = (1 - \sqrt{2s})^2\sqrt{2s}$.

[29]Recent work on consumer search has often assumed first search as free. See, for instance, Janssen, Moraga-González, and Wildenbeest (2007), Janssen and Parakhonyak (2013), Ding and Zhang (2018), Janssen and Shelegia (2020).

They do not deviate to a higher price because they would then not have any visitors. Let firm 1 charge $p_1 = 1 - \sqrt{2s}$ and firm 2 deviate to $p_2 = 1 - \sqrt{2s} - \epsilon$. Then, $\widehat{v} > p_1$ and we are in the competitive framework (á la region I). Firm 1 gets a measure 1 of visitors and a measure $\sqrt{2s}$ of buyers among them. As an approximation, let me ignore the ϵ visitors that consider both firms, since their contribution to firms' demand would only be second-order. So, the remaining visit firm 2 and he gets a measure $(1 - \sqrt{2s})(\sqrt{2s} + \epsilon)$ of buyers. Then, firm 2 gets a profit of $(1 - \sqrt{2s})(1 - \sqrt{2s} - \epsilon)(\sqrt{2s} + \epsilon)$.

$$\pi_2 + (1 - 2\sqrt{2s} + 2s)\epsilon - (\sqrt{2s} - 2s)\epsilon - \epsilon^2(\cdot) \;\approx\; \pi_2 + (1 - 4\sqrt{2s} + 4s)\epsilon > \pi_2$$

Hence, he will prefer to deviate. Now, imagine firm 1 deviates instead. His profit would be $(1 - \sqrt{2s} - \epsilon)(\sqrt{2s} + D_{12})$ where $\epsilon > D_{12} = \mathbb{P}[v_1 > v_2 - \epsilon] = (\frac{1}{2} - \epsilon)$.

$$\pi_1 - \sqrt{2s}\epsilon + (\frac{1}{2} - \epsilon)(1 - \sqrt{2s} - \epsilon) \;\approx\; \pi_1 + \frac{1 - \sqrt{2s} - \epsilon}{2} - \epsilon > \pi_1$$

Hence, he also prefers to deviate. Now, imagine $p_1 = p_2 = 0$ and firm 2 deviates to $p_2 = \epsilon$. This would generate a positive profit for firm 2. Therefore, although it requires a more formal analysis, it seems like there is no equilibrium in pure-strategies for $\underline{s} < s < \overline{s}$.

If given a choice, do consumers search in the displayed order? For low search cost (region I), p_2 is greater than p_1 and consumers prefer to follow the order in which firms are displayed.

At $s = \underline{s}$, firms charge an identical price, $p_1 = p_2$. Hence, consumers are indifferent between starting from either firm and if consumers can choose which firm to visit, we will have random search. Thus, firms' prices will follow Wolinsky (1986) and have a value of $1 - \sqrt{2s}$ for $\underline{s} < s < \overline{s}$. At $s = \underline{s} = \frac{1}{8}$, this value coincides with the monopoly price, $\frac{1}{2}$.

4.5.2 Timing

In the baseline model, firms place their bids and learn their position before setting product prices. This is motivated by the intuition that the one-shot auction in my model is an approximation of a long dynamic game which has revealed all the unknown information about the auction outcome. Hence, firms set prices as if they know their position and commission. To extend the fit of my model to a more general setting, I explore alternate timings below.

Auction reserve price → firms bid and set prices simultaneously: This case is relevant to situations where auctions are nearly as frequent as product price adjustments. In symmetric equilibrium, the firms now have to set one price for both positions they will take up with equal probability. In such situations, firms' objective can be represented by

$$
\begin{aligned}
\max_{p_i}\ Rev_i &= p_i \cdot \frac{D_1 + D_2}{2} = p_i \cdot \frac{1 - p_i p_j}{2} \\
\text{Symmetry} \implies p &= \frac{1}{\sqrt{3}} \, (= p^{MM}) \\
\implies Rev &= \frac{1}{3\sqrt{3}} \approx 0.192 \\
\implies \widehat{b} &= \frac{1}{\sqrt{3}} \\
\implies Rev_m &\approx 0.385 > \max\{Rev_m^{baseline}(s)\} \\
\pi &= 0
\end{aligned}
$$

Note that this outcome is not a function of search cost. The asymmetric equilibria are identical to the baseline model, since we look at subgame-perfect Nash equilibria and hence, beliefs are true in equilibrium. However, we do not have the symmetric equilibrium we derived for the baseline model.

The crucial difference here is that firms do not know their position when they are setting a price. In the symmetric equilibrium of the baseline model, even though firms get each position with equal probability (due to tie-breaking), they know their position when they set prices. For example, consider that there are 100 instances of the same two firms competing for the top spot. In the baseline model, the position-allocation is revealed to the firms before the price setting. However, this is not true when firms set prices and bid simultaneously. In each of the 100 instances, the firms have to set prices without knowing their position, even though they end up getting each position with equal probability.

Auction reserve price → firms set prices → firms bid: This case is relevant to situations where auctions are rare (for e.g., quarterly) compared to product price adjustments. Solving backwards, we know that demand at position 1 is higher, even though they are more price elastic. Both firms prefer the first position and they both bid $b = Rev^{Mon}$. Therefore, they set prices equal to $\frac{1}{\sqrt{3}}$ since they do not know their position when they set prices. The logic from the previous case applies here as well.

Firms set prices → auction reserve price → firms bid: This case is relevant to situations where ad display designs changes at a higher frequency than product prices. In other words, pricing strategies are much more stickier than advertisement strategies.

The outcome here is same as in the previous case. Exchanging the timing of setting auction reserve price does not change outcomes for subgame-perfect Nash equilibria.

4.5.3 Auction

Equilibrium refinement: The tradition of restricting the set to undominated strategies is much older and attributed to analyses of co-operative game theory where the 'core' of a game is given by the undominated outcomes (Roth and Sotomayor, 1990). If as a planner or auctioneer, one would like to advice the firms to avoid some 'bad strategies', one may prescribe playing the undominated strategies. Note that this may not necessarily lead to a unique equilibrium.

Equilibrium in undominated strategies coincides with a unique locally-envy free refinement in my model. Edelman, Ostrovsky, and Schwarz (2007) develops this refinement informed by the markets' dynamic structure. Although there is still multiplicity due to the fact that firm 1's range of bids does not have an upper bound, this does not play a role in the equilibrium outcome.

Further, Varian (2007) also independently addresses the multiplicity and illustrates two possible refinements. The paper describes the intuition behind an aggressive approach - "what is the highest bid I can set so that if I happen to exceed the bid of the agent above me and I move up by one slot, I am sure to make at make at least as much profit as I make now?" - and a defensive approach - "If I set my bid too high, I will squeeze the profit of the player ahead of me so much that he might prefer to move down to my position". The paper also derives a lower-bound of outcomes from a set of sub-game perfect Nash equilibria which coincides with the 'locally envy-free' refinement mentioned above.

Microfoundation: Standard models of position auction oversee a certain form of interdependency in object (position in this case) values across bidders (firms in this case). The value of a click for a firm is assumed to be constant across positions (see, for instance, Varian (2009)). Although it is common to assume that the click-through rate as a product of firm-specific factor times a position-specific factor, consumer behaviour remains simplified. For instance, Athey and Ellison (2011) assumes the number of clicks to be exogenous. These simplifications may not be innocuous but definitely helped gain insights on various features of the position auctions.

In my model, I derive consumer behaviour as a function of the price, which in turn, is a function of both value of a buyer and number of clicks and the commission (or the cost per click). Previous literature assumed the shape of these functions. Using consumer search as a model of consumer behaviour, we now know how these functions would look like. More importantly, now we know not just the functions but that there is a feedback, and what we want and can find is the fixed point.

Reserve Price: Edelman and Schwarz (2010) assesses the welfare effects of the auction reserve price. They separate the effects into direct (causing the lowest value bidder to face a higher payment - in my model it is firm 2) and indirect (inducing other bidders to increase their bids, thereby increasing others' payments - in my model it is firm 1). They find that most of the incremental revenue from setting a reserve price optimally comes not from a direct effect, but rather from the indirect effects on high bidders. I find a similar result that the larger share of intermediary revenue comes from firm 1 for any increment in $\widehat{b}$ (see, for instance, figure A.2). Although top bidders' large valuations place them 'furthest' from the reserve price, they contribute more as commission. This is due to two forces. First, firm 2 is now forced to charge a higher price to cover his commission cost which allows firm 1 to also increase (relatively smaller than firm 2's increase) his price without loss. Second, this makes firm 2 less attractive. Thus, overall revenues of firm 1 increases substantially, to the benefit of the intermediary. Further, characterisation of the auction reserve price taking into account endogenous consumer behaviour has shown the existence of a strategy-proof and optimal allocation mechanism.

Similarly, on the other side, the auction reserve price also has an effect on the overall conversion rate in the market and how many consumers choose outside option.

One-shot auction: Note that the assumption of one-shot simultaneous bidding is a deviation from the possibility of continuous asynchronous bidding allowed in auctions run by some popular search engines. As in previous theoretical work on position auctions, I make this assumption for simplicity. Edelman, Ostrovsky, and Schwarz (2007) shows that the "lowest revenue envy-free" equilibrium generates the upper bound of revenues from dynamic auction models. However, having relaxed the assumption of homogenous position effects in my model, this may no longer clear be the case and requires further study. This concern is also shared in Goldman and Rao (2016). Unfortunately, an analysis of this dynamic auctions is beyond the scope of this paper.

4.5.4 Superstar and Fringe

In my model, the intermediary would prefer to design the market at a higher search cost than the welfare-maximising level. Also, note that firms' revenues get more dispersed with search cost. Another factor that affects equity is the role played by the per-click commission cost. This asymmetry in cost structure significantly changes firms' price setting problem, as captured starkly in the duopoly model. Potentially, a superstar firm may have sufficient funds to *enter* the top positions by paying the *fixed cost*. This pushes the fringe firms to lower positions. Since the profits are distributed in order of position, this result highlights a mechanism through which a profit-maximising intermediary may exacerbate the gap between superstar and fringe firms and provide a rationale for growing market concentration.

5 Asymmetric Firms

In this section, I relax the symmetric nature of the firms and consider two vertically differentiated firms. I capture this by varying the maximum value that any consumer can realise at a firm. I refer to this dimension as the *quality* of a firm (or product).

Theorem 6 characterises the equilibrium for asymmetric firms. Table ?? shows the market outcome for different values of the auction reserve price ($\widehat{b}$). Without loss of generality, let firm i have the ability to generate higher revenue than firm j under the same conditions (position and auction reserve price). This can be due to either firm i having more relevance or selling a higher quality product. The intuition here is that the non-existence of symmetric equilibrium is a result of the additional asymmetry in firms' abilities on top of the positional asymmetry. The positional heterogeneity and quality are in resonance and firm i procures the first position by bidding higher and earning higher in equilibrium.

Proposition 6. *There exists no symmetric equilibrium. Locally envy-free asymmetric equilibrium for firms i, j such that firm i is more relevant or has higher quality.*

$$\begin{aligned}
\widehat{b} &= \frac{Rev_{2,j}}{1 - D_{11,i}} \\
b_i &= (Rev_{1,j} - \pi_{2,j},\ \infty) \\
b_j &= Rev_{1,i} - \pi_{2,i} \\
\pi_1 &= 0 \\
\pi_2 &= 0 \\
\pi_m &= Rev_{1,i} + Rev_{2,j}
\end{aligned}$$

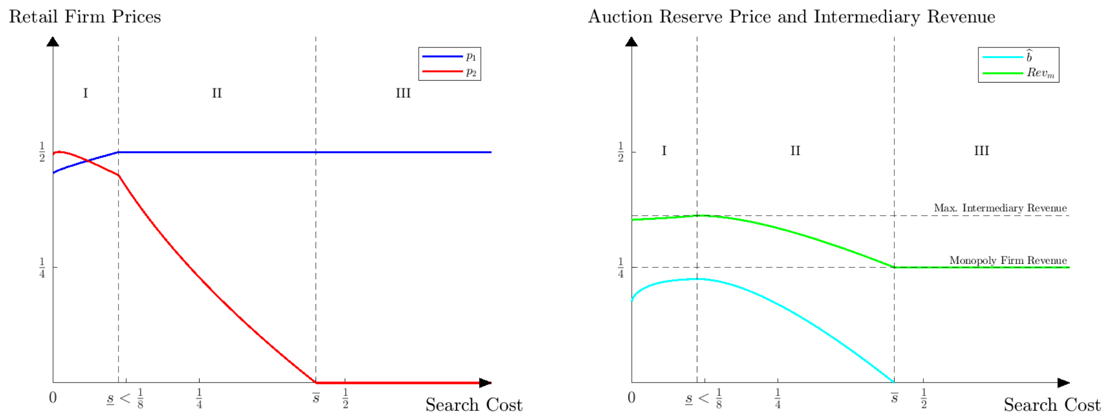
Below, I illustrate the results with my working example.

Ads often display some technical specifications about the product which signal its quality depending on the consumers' purpose. In this subsection, I characterise the equilibrium for when firms offer products of heterogeneous quality. I assume that one of the products is on average of higher quality. Let the valuations be drawn from a distribution $v_1 \sim U[0,1]$ and $v_2 \sim U[0,\alpha]$ where $\alpha \in \mathbb{R}_+$.

Deriving the reservation value for the search rule (for $F = U[0,1]$), we get

$$\widetilde{v}_\alpha = \alpha + \Delta p - \sqrt{2\alpha s}$$

Figure 9 plots the equilibrium results for $F = U[0,1]$. See figure A.3 for equilibrium results for the 'No Auction' case for various α.



(a) Prices

(b) Commission

Figure 9: Quality: With auction

Using the above exercise, I show that there is now much less of threat to the prominent firm from firm 2 because firm 2 is less appealing to consumers due to lower *quality*. Further, the intermediary prefers a monopoly to a duopoly for a smaller search cost ($\overline{s} < \frac{1}{2}$) as the ability of firm 2 to generate revenue exhausts for lower s.

6 Conclusion

The two-sided nature of online marketplaces makes the intermediary a key player in influencing economic outcomes. This paper presents a model of consumer-firm and firm-intermediary interaction to illustrate the tradeoffs for each player in this setting. I consider an ordered search model where consumers discover independent valuations of each product by paying the search cost while the intermediary lists the products by conducting an auction. Product prices being observable prior to search plays an important role in determining consumer demand as well profits of firms and the intermediary. Moreover, the order of product prices in the list are determined by the search cost and auction reserve price. Since the ad commission takes up the form of fixed cost for the first firm and marginal cost for the second, this changes the pricing equation for each and in turn, the demand and profits. My model gives predictions for two measures, the conversion rate of firms and the total volume of transaction. This can provide a benchmark for the empirical estimation of the impact of prominence and ad commissions.

Some of the insights gained from this model may also apply to other situations, especially where we see firm-intermediary interaction of the similar kind where sellers pay for their products to be displayed in a prominent position. For example, in offline supermarkets, firms advertise at the entrance or near the cash counter. Further, the store may order these ads in different positions based on the flow of traffic within a store and the commission promised by each manufacturer. Some other examples include publishers paying book-stores to be promoted, more prominent adverts being more expensive in brochures/menus, and eBay offering sellers the option to list their products prominently for an extra fee. With suitable adaptation, my framework might have application to a related set of circumstances where market participants or public authorities seek to influence consumer choice by framing the order in which choices are presented. For instance, the presentation of choices about savings plans, healthy eating, advertising and its regulation, or the operation of commission schemes for *e-commerce* platforms.[30]

The topic of digital marketplaces deserves further research. Making a firm prominent will have an impact on firm entry. Since the top firm has to pay a fixed cost and, at high search costs, can enjoy a large market share, one may be concerned with the potential for entry deterrence which may affect product variety negatively. On the other hand, this might increase efficiency because free entry may result in excess entry, for instance, in the random-search case (Anderson and Renault, 1999). Further, we need welfare analyses which take into account the distribution of outcomes over heterogeneous agents. For instance, in my model, I find that the gap between firms' revenues widens with search cost. This result appeals to recent discussions on increasing market concentration and implores further research. Studying a dynamic model to highlight the effect of endogenous consumer loyalty can also help understand the long-run implications and can provide additional insights on firm strategy in the initial periods.

[30] See, for instance, Thaler and Sunstein (2009) for an overview of some of these issues.

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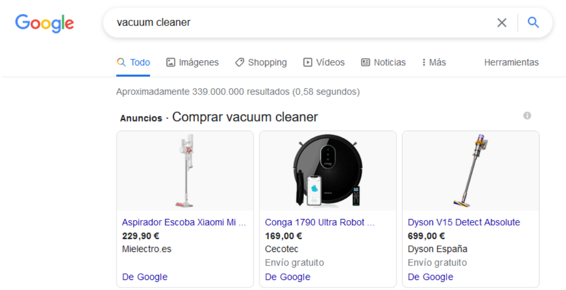
A Figures and Tables



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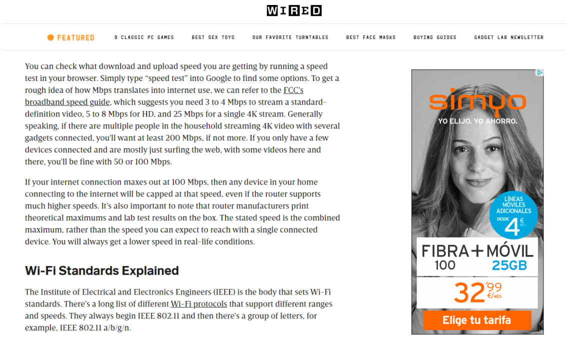

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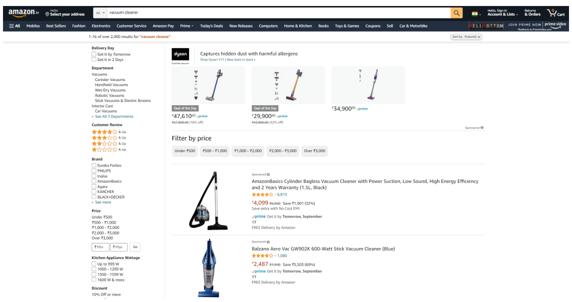


Source: https://www.amazon.in/ (Accessed: 16 Sep, 2021)

Figure A.1: A snapshot of sponsored results: Prices are often observable costlessly on digital platforms

Table A.1: Outcomes for different auction reserve prices

Auction Reserve Price	**Firms find ...**	**Firms' Bids**	**Firms' Profits**
$\widehat{b} > Rev^{Mon}$	neither feasible*	$b_1 = \times$ $b_2 = \times$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = 0$
$\max\left\{\frac{Rev_1 - Rev_2}{D_{11}},\ \frac{Rev_2}{1-D_{11}}\right\} < \widehat{b} < Rev^{Mon}$	position 1 feasible	$b_1 = Rev^{Mon}$ $b_2 = Rev^{Mon}$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = Rev^{Mon}$
$\frac{Rev_2}{1-D_{11}} < \widehat{b} < \frac{Rev_1 - Rev_2}{D_{11}}$	position 1 feasible	$b_1 = Rev^{Mon}$ $b_2 = Rev^{Mon}$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = Rev^{Mon}$
$\max\left\{\frac{Rev_2}{2},\ \frac{Rev_1 - Rev_2}{D_{11}}\right\} < \widehat{b} < \frac{Rev_2}{1-D_{11}}$	neither feasible	$b_1 = \times$ $b_2 = \times$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = 0$
$\frac{Rev_1 - Rev_2}{D_{11}} < \widehat{b} < \frac{Rev_2}{2} < \frac{Rev_2}{1-D_{11}}$ (this interval has zero measure for $U[0,1]$)	position 2 feasible	$b_1 = \widehat{b}$ $b_2 = \widehat{b}$	$\pi_1 = -\widehat{b}$ $\pi_2 = \min\left\{\frac{1}{2},\ 1 - \sqrt{2s}\right\} - \widehat{b}$ $Rev_m = \widehat{b} \leq \frac{Rev^{Mon}}{2}$
$\widehat{b} \in \left[0,\ \frac{Rev_2}{1-D_{11}}\right]$	both feasible symmetric bids	$b_1 = Rev_1 - \pi_2$ $b_2 = Rev_1 - \pi_2$	$\pi_1 = Rev_1 - b_2$ $\pi_2 = Rev_2 - \widehat{b}(1 - D_{11})$ $Rev_m = \frac{b_2 + \widehat{b}(1-D_{11})}{2}$
$\widehat{b} \in \left[0,\ \min\left\{\frac{Rev_1 - Rev_2}{D_{11}},\ \frac{Rev_2}{1-D_{11}}\right\}\right]$	both feasible asymmetric bids	$b_1 \in (Rev_1 - \pi_2,\ \infty)$ $b_2 \in \left[\widehat{b},\ Rev_1 - \pi_2\right]$	$\pi_1 = Rev_1 - b_2$ $\pi_2 = Rev_2 - \widehat{b}(1 - D_{11})$ $Rev_m = b_2 + \widehat{b}(1 - D_{11})$

Rev^{Mon}: Revenue of monopoly, Rev_k: Revenue of firm in position k, *feasible: Firm profit ≥ 0

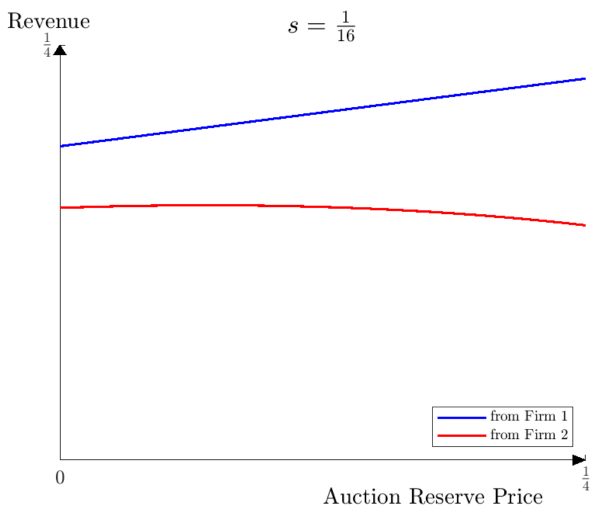
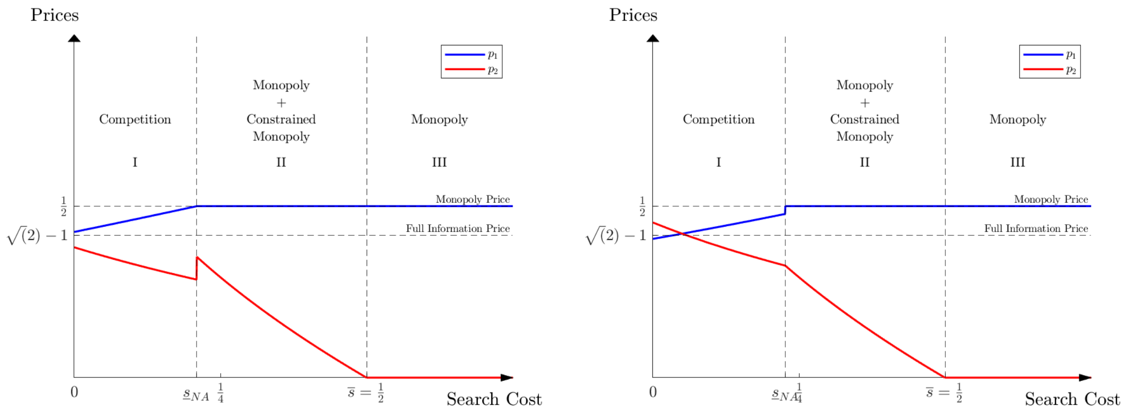


Figure A.2: Indirect effect of reserve price is stronger than direct effect

Table A.2: Outcomes for different auction reserve prices: Asymmetric firms

Auction Reserve Price	**Firms find ...**	**Firms' Bids**	**Firms' Profits**
$\widehat{b} > Rev_{1,j}$	neither feasible*	$b_1 = \times$ $b_2 = \times$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = 0$
$\max\left\{\frac{Rev_{1,i}-Rev_{2,i}}{D_{11,j}}, \frac{Rev_{2,j}}{1-D_{11,i}}\right\} < \widehat{b} < Rev_{1,j}$	position 1 feasible	$b_1 = \frac{Rev^{Mon}}{2}$ $b_2 = \frac{Rev^{Mon}}{2}$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = \frac{Rev^{Mon}}{2}$
$\frac{Rev_{2,j}}{1-D_{11,i}} < \widehat{b} < \frac{Rev_{1,i}-Rev_{2,i}}{D_{11,j}}$	position 1 feasible	$b_1 = \frac{Rev^{Mon}}{2}$ $b_2 = \frac{Rev^{Mon}}{2}$	$\pi_1 = 0$ $\pi_2 = 0$ $Rev_m = \frac{Rev^{Mon}}{2}$
$\widehat{b} \in \left[0, \min\left\{\frac{Rev_{1,i}-Rev_{2,i}}{D_{11,j}}, \frac{Rev_{2,j}}{1-D_{11,i}}\right\}\right]$	both feasible asymmetric bids	$b_1 \in (Rev_{1,j} - \pi_{2,j}, \infty)$ $b_2 \in \left[\widehat{b}, Rev_{1,i} - \pi_{2,i}\right]$	$\pi_1 = Rev_{1,i} - b_2$ $\pi_2 = Rev_{2,j} - \widehat{b}(1 - D_{11,i})$ $Rev_m = b_2 + \widehat{b}(1 - D_{11,i})$

Rev^{Mon}: Revenue of monopoly, Rev_k: Revenue of firm in position k, *feasible: Firm profit ≥ 0



(a) $\alpha = 0.9$

(b) $\alpha = 1.1$

Figure A.3: Quality: Without auction

B Proofs

B.1 Reservation value

Lemma 1.

$$\begin{aligned}
\int_{\hat{v}-\Delta p}^{\overline{v}} [(v_2 - p_2) - (\hat{v} - p_1)] f(v_2) dv_2 &= s \\
\int_{\hat{v}-\Delta p}^{\overline{v}} v_2 f(v_2) dv_2 - (\hat{v} - \Delta p)[F(\overline{v}) - F(\hat{v} - \Delta p)] &= s \\
\mathbb{E}[v_2 | \hat{v} - \Delta p \leq v_2 \leq \overline{v}] - (\hat{v} - \Delta p)[F(\overline{v}) - F(\hat{v} - \Delta p)] &= s
\end{aligned}$$

- Using Leibniz's rule[31],

$$\begin{aligned}
0 - (\hat{v} - \Delta p) f(\hat{v} - \Delta p) \frac{\partial \hat{v}}{\partial s} + 0 \; - \; \frac{\partial \hat{v}}{\partial s}[1 - F(\hat{v} - \Delta p)] & \\
-(\hat{v} - \Delta p) \left[-f(\hat{v} - \Delta p) \frac{\partial \hat{v}}{\partial s} \right] &= 1 \\
\frac{\partial \hat{v}}{\partial s} \left[-(\hat{v} - \Delta p) f(\hat{v} - \Delta p) - [1 - F(\hat{v} - \Delta p)] + (\hat{v} - \Delta p) f(\hat{v} - \Delta p) \right] &= 1
\end{aligned}$$

$$\frac{\partial \hat{v}}{\partial s} = -\frac{1}{1 - F(\hat{v} - \Delta p)} < 0$$

- Using Leibniz's rule,

$$\begin{aligned}
0 - (\hat{v} - \Delta p) f(\hat{v} - \Delta p)(\frac{\partial \hat{v}}{\partial p_1} - 1) + 0 \; - \; (\frac{\partial \hat{v}}{\partial p_1} - 1)[1 - F(\hat{v} - \Delta p)] & \\
-(\hat{v} - \Delta p) \left[-f(\hat{v} - \Delta p)(\frac{\partial \hat{v}}{\partial p_1} - 1) \right] &= 0 \\
\left(\frac{\partial \hat{v}}{\partial p_1} - 1 \right) \left[-(\hat{v} - \Delta p) f(\hat{v} - \Delta p) - [1 - F(\hat{v} - \Delta p)] + (\hat{v} - \Delta p) f(\hat{v} - \Delta p) \right] &= 0
\end{aligned}$$

$$\frac{\partial \hat{v}}{\partial p_1} = 1 > 0$$

[31]When both the function $f(x,t)$ and its partial derivative $f_x(x,t)$ are continuous in both x and t,

$$\frac{d}{dx} \left(\int_{a(x)}^{b(x)} f(x,t) dt \right) = f(x, b(x)) \cdot \frac{d}{dx} b(x) \; - \; f(x, a(x)) \cdot \frac{d}{dx} a(x) \; + \; \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t) dt$$

- Using Leibniz's rule,

$$
\begin{aligned}
0 - (\hat{v} - \Delta p) f(\hat{v} - \Delta p)(\frac{\partial \hat{v}}{\partial p_2} + 1) + 0 \; - \; (\frac{\partial \hat{v}}{\partial p_2} + 1)[1 - F(\hat{v} - \Delta p)] & \\
-(\hat{v} - \Delta p)\left[-f(\hat{v} - \Delta p)(\frac{\partial \hat{v}}{\partial p_2} + 1)\right] &= 0 \\
\left(\frac{\partial \hat{v}}{\partial p_1} + 1\right)[-(\hat{v} - \Delta p) f(\hat{v} - \Delta p) - [1 - F(\hat{v} - \Delta p)] + (\hat{v} - \Delta p) f(\hat{v} - \Delta p)] &= 0
\end{aligned}
$$

$$
\frac{\partial \hat{v}}{\partial p_2} = -1 < 0
$$

■

Given $v_k \sim U[0, V]$.

$$
\begin{aligned}
l(\hat{v}, s, \mathbf{p}) &= 0 = \int_{\hat{v}-\Delta p}^{V} (v_2 - \hat{v} + \Delta p) f(v_2) dv_2 \; - \; s \qquad [f(v_2) > 0 \implies 0 < \hat{v} - \Delta p < V] \\
&\implies \frac{1}{V}\left[\frac{v_2^2}{2} - \hat{v} v_2 + \Delta p v_2\right]_{\hat{v}-\Delta p}^{V} \; - \; s = 0 \\
&\implies \frac{1}{V}\left(\frac{V^2}{2} - \hat{v}V + \Delta p V + \frac{\hat{v}^2}{2} - \Delta p \hat{v} + \frac{\Delta p^2}{2}\right) - s = 0 \\
&\implies \frac{\hat{v}^2}{2} - \hat{v}(V + \Delta p) + \frac{V^2}{2} + \Delta p V + \frac{\Delta p^2}{2} - Vs = 0
\end{aligned}
$$

Quadratic roots:

$$
\begin{aligned}
\hat{v} &= V + \Delta p \; \pm \; \sqrt{(V + \Delta p)^2 \; - \; 2\left(\underbrace{\frac{V^2}{2} + \Delta p V + \frac{\Delta p^2}{2}}_{\left(\frac{V+\Delta p}{\sqrt{2}}\right)^2} - Vs\right)} \\
\implies \hat{v} &= V + \Delta p \; \pm \; \sqrt{2Vs}
\end{aligned}
$$

Deriving constraints for $0 < \hat{v} < V$, we get

$$
\hat{v} = \begin{cases} V + \Delta p \; - \; \sqrt{2Vs} & \text{when } \Delta p \in [\sqrt{2Vs} - V, \sqrt{2Vs}] \\ V + \Delta p \; + \; \sqrt{2Vs} & \text{when } \Delta p \in [-\sqrt{2Vs} - V, -\sqrt{2Vs}] \end{cases}
$$

Since the two constraints on Δp do not overlap for non-zero values of s, V, we have a unique value for $\hat{v}$ when either of the constraints are satisfied.

But applying the constraint $0 < \hat{v} - \Delta p < V$, we only have

$$\hat{v} \;=\; V + \Delta p \;-\; \sqrt{2Vs} \text{ when } \Delta p \in [\sqrt{2Vs} - V, \sqrt{2Vs}]$$

B.2 Consumer Demand

Lemma 2. For D_{11},

$$\frac{\partial D_{11}}{\partial s} \;=\; -f(\hat{v})\frac{\partial \hat{v}}{\partial s} \;=\; \frac{f(\hat{v})}{1 - F(\hat{v} - \Delta p)} \;>\; 0$$

For D_{12}, using Leibniz's rule,

$$\begin{aligned}
\frac{\partial D_{12}}{\partial s} &= F(\hat{v} - \Delta p)f(\hat{v})\frac{\partial \hat{v}}{\partial s} - 0 + 0 \\
&= -\frac{F(\hat{v} - \Delta p)f(\hat{v})}{1 - F(\hat{v} - \Delta p)} \;<\; 0
\end{aligned}$$

For D_{22}, using Leibniz's rule,

$$\begin{aligned}
\frac{\partial D_{22}}{\partial s} &= f(\hat{v})\frac{\partial \hat{v}}{\partial s} - \left[F(\hat{v} - \Delta p)f(\hat{v})\frac{\partial \hat{v}}{\partial s} - 0 + 0\right] \\
&= \frac{-f(\hat{v}) + F(\hat{v} - \Delta p)f(\hat{v})}{1 - F(\hat{v} - \Delta p)} \;<\; 0
\end{aligned}$$

$\blacksquare$

B.3 Equilibrium - No Auction

Random Search: Solving for symmetric eq.

$$\max_p pD(p) \;=\; p\frac{D_{11} + D_{12} + D_{22}}{2}$$

For some deviation p and equilibrium price p^{Ran}

$$\begin{aligned}
\frac{\partial \pi^{Ran}(p^{Ran})}{\partial p} &= \frac{\partial}{\partial p}\left(p(1 - pp^{Ran})\right) \\
&= 1 - 2(p^{Ran})^2 \;=\; 0 \\
\implies \quad p^{Ran} &= \frac{1}{\sqrt{2}}
\end{aligned}$$

Lemma 3. Solving for firm 1 when prices are symmetric,

$$p^* \;=\; \sqrt{2 + 2s} - 1$$

Solving for firm 2 when prices are symmetric,

$$p^* \ = \ \sqrt{2-2s}-1$$

They are equal only if $s=0$. ■

Theorem 1. Solving First order conditions, we get

$$p^*_{1,NA} \ = \ \frac{1-F(\widehat{v}^*)+\int_{p^*_{1,NA}}^{\widehat{v}^*}F(v_1-\Delta p)\cdot f(v_1)\cdot dv_1}{f(\widehat{v}^*)-\int_{p^*_{1,NA}}^{\widehat{v}^*}F(v_1-\Delta p)\cdot f'(v_1)\cdot dv_1}$$

$$p^*_{2,NA} \ = \ \frac{A\cdot[1-F(p^*_{2,NA})]F(p^*_{1,NA})+F(\widehat{v}^*)-F(p^*_{1,NA})-\int_{p^*_{1,NA}}^{\widehat{v}^*}F(v_1-\Delta p)\cdot f(v_1)\cdot dv_1}{A\cdot f(p^*_{2,NA})F(p^*_{1,NA})+f(\widehat{v}^*)-F(p^*_{2,NA})f(p^*_{1,NA})-\int_{p^*_{1,NA}}^{\widehat{v}^*}F(v_1-\Delta p)\cdot f'(v_1)\cdot dv_1}$$

For $p^*_{1,NA}, p^*_{2,NA}$ to be global maxima, we require

$$p_1\cdot\left(f'(\widehat{v})-\int_{p_1}^{\widehat{v}}f(v_1-\Delta p)\cdot f''(v_1)\cdot dv_1\right)\le 2\left(f(\widehat{v})-\int_{p_1}^{\widehat{v}}F(v_1-\Delta p)\cdot f'(v_1)\cdot dv_1\right)$$

$$p_2\cdot\left(A\cdot f'(p^*_{2,NA})F(p^*_{1,NA})-f'(\widehat{v})-f(p_2)f(p_1)+F(\widehat{v}-\Delta p)\cdot f'(\widehat{v})-\int_{p_1}^{\widehat{v}}f(v_1-\Delta p)\cdot f'(v_1)\cdot dv_1\right)$$
$$\le 2\left(A\cdot f(p^*_{2,NA})F(p^*_{1,NA})+f(\widehat{v})-F(p_2)f(p_1)-\int_{p_1}^{\widehat{v}}F(v_1-\Delta p)\cdot f'(v_1)\cdot dv_1\right)$$

■

Proposition 1. To find demand:

$$D_{11} \ = \ 1-\frac{\widehat{v}}{V}$$

D_{12} requires $p_1 < v_1 < \widehat{v}, \ \ v_2-p_2 < v_1-p_1$.

$$\begin{aligned} D_{12} \ &= \ \int_{p_1}^{\widehat{v}} F_2(v_1-\Delta p)\ f(v_1)\ dv_1 \\ &= \ \frac{(\widehat{v}_1-\Delta p)^2}{2V^2}-\frac{p_2^2}{2V^2} \end{aligned}$$

D_{22} requires $v_1 < \widehat{v}, \;\; v_2 - p_2 > \max\{0, v_1 - p_1\}$.

$$D_{22} \;=\; \underbrace{\left(1 - \frac{p_2}{V}\right)\frac{p_1}{V} \cdot \mathbb{1}_{p_2 < \frac{V}{2} - s}}_{v_1 < p_1} \;+\; \underbrace{\int_{p_1}^{\widehat{v}} [1 - F_2(v_1 - \Delta p)]\, f(v_1)\, dv}_{v_1 > p_1}$$

$$= \left(1 - \frac{p_2}{V}\right)\frac{p_1}{V} \;+\; \frac{(\widehat{v} - p_1)}{V} - \frac{(\widehat{v} - \Delta p)^2}{2V^2} - \frac{p_2^2}{V^2}$$

Profit Maximisation:

$$\begin{aligned} p_1 \;&=\; \frac{V}{4} + \frac{s}{2} + \frac{p_2}{2} - \frac{p_2^2}{4V} \\ p_2 \;&=\; \frac{1}{3}\left(2p_1 + 2V - \sqrt{10p1^2 + 2p1V + 6sV + V^2}\right) \end{aligned}$$

- At $\underline{s}_{NA}$, $\hat{v} = p^*_{1,NA}$. Solving for s using the equations for prices, we get $\underline{s}_{NA} = \frac{1}{4}$.
-

$$\frac{dp^*_{1,NA}}{ds} \;=\; \frac{1 + (1 - p^*_{2,NA})\frac{dp^*_{2,NA}}{ds}}{2}$$

$$\frac{dp^*_{2,NA}}{ds} \;=\; \frac{(1 - p^*_{2,NA})\left(-1 + \frac{dp^*_{1,NA}}{ds}\right)}{\underbrace{2(1 + \Delta p^*_{NA})^2 - D^*_{22}}_{>0}}$$

Note that $(1 - p_2) \geq 1 - p^{Ran}$ and $D_{22} \leq \frac{1 - (p^{Ran})^2}{2}$ whose bounds are realised when there is random search. This implies that the denominator of the RHS of the above expression is bounded below by zero.
Substituting this result in the previous expression, we get

$$\frac{dp^*_{1,NA}}{ds} \;=\; \frac{1 + (1 - p^*_{2,NA})\frac{(1 - p^*_{2,NA})\left(-1 + \frac{dp^*_{1,NA}}{ds}\right)}{2(1 + \Delta p^*_{NA})^2 - D^*_{22}}}{2}$$

Note that

$$\frac{(1-p^*_{2,NA})^2}{2(1-p^*_{2,NA})^2 - D^*_{22}} = \frac{1}{2 - \underbrace{\frac{D^*_{22}}{(1-p^*_{2,NA})^2}}_{\in[0,1)}} < 1$$

$$\Longrightarrow \frac{dp^*_{1,NA}}{ds} \in (0,1) \ , \ \frac{dp^*_{2,NA}}{ds} < 0$$

- Since prices are continuous and p_1 is monotonically increasing while p_2 is monotonically decreasing, they cross each other at one point atmost (which happens at $s = 0$).
- At the lower bound, $s = 0$, oredered consumer search is equivalent to that of random search as shown below. At $s = 0$,

$$p^*_{1,NA} = p^*_{2,NA} = \frac{1-(p^*_{NA})^2}{2} = p^{Ran} = \sqrt{2} - 1$$

At $p^*_{2,NA} = 0$, $F(\hat{v}^*) - F(p^*_{1,NA}) - \int_{p_1}^{\hat{v}^*} F(v_1 - \Delta p) \cdot f(v_1) \cdot dv_1 = 0$. This implies that

$$p^*_{1,NA} = 1 - p^*_{1,NA} = p^{Mon} = \frac{1}{2}$$

This upper bound for $p^*_{1,NA}$ is hit at $(\underline{s})_{NA} = \frac{1}{4}$. For $s > \underline{s}$, condition A binds

$$p_2 < 1 - \sqrt{2s}$$

Therefore, $\overline{s} = \frac{1}{2}$.

$\blacksquare$

Proposition 2. Using Envelope Theorem and substituting the expressions for the derivatives,

$$
\begin{aligned}
\frac{dRev_1^*}{ds} &= p^*_{1,NA}\left(\frac{dD^*_{11}}{ds}+\frac{dD^*_{12}}{ds}\right)\\
&= p^*_{1,NA}\left(-\frac{dp^*_{1,NA}}{ds}+(1-p^*_{2,NA})\frac{dp^*_{2,NA}}{ds}+1\right)\\
&= p^*_{1,NA}\left(-\frac{1+(1-p^*_{2,NA})\frac{dp^*_{2,NA}}{ds}}{2}+(1-p^*_{2,NA})\frac{dp^*_{2,NA}}{ds}+1\right)\\
&= p^*_{1,NA}\left(\frac{1+(1-p^*_{2,NA})\frac{dp^*_{2,NA}}{ds}}{2}\right)\\
&= p^*_{1,NA}\left(\frac{dp^*_{1,NA}}{ds}\right) \geq 0\\
\frac{dRev_2^*}{ds} &= p^*_{2,NA}\left(\frac{dD^*_{22}}{ds}\right)\\
&= p^*_{2,NA}\left(-1\underbrace{-(1-p^*_{2,NA})\frac{dp^*_{2,NA}}{ds}}_{\leq 1 \text{ from proposition } 1}\right) \leq 0
\end{aligned}
$$

■

B.4 Constant Commission

Proposition 3. Prices are obtained by solving the first-order conditions for equation 8. For $(p_1^*)_C$, $(p_2^*)_C$ to be global maxima:

$$
p_1\cdot\left(f'(\widehat{v})-\int_{p_1}^{\widehat{v}} f(v_1-\Delta p)\cdot f''(v_1)\cdot dv_1\right)\leq 2\left(f(\widehat{v})-\int_{p_1}^{\widehat{v}} F(v_1-\Delta p)\cdot f'(v_1)\cdot dv_1\right)
$$

$$
\begin{aligned}
p_2\cdot&\left(A\cdot f'(p^*_{2,NA})F(p^*_{1,NA})-f'(\widehat{v})-f(p_2)f(p_1)+F(\widehat{v}-\Delta p)\cdot f'(\widehat{v})-\int_{p_1}^{\widehat{v}} f(v_1-\Delta p)\cdot f'(v_1)\cdot dv_1\right)\\
&\leq 2\left(A\cdot f(p^*_{2,NA})F(p^*_{1,NA})f(\widehat{v})-F(p_2)f(p_1)-\int_{p_1}^{\widehat{v}} F(v_1-\Delta p)\cdot f'(v_1)\cdot dv_1\right) - r_2 f'(\widehat{v})
\end{aligned}
$$

At $s=0$,

$$
\begin{aligned}
(p_1^*)_C &= \sqrt{2}-1 = p^{Ran}\\
(p_2^*)_C &= \sqrt{2(1+r_2)}-1 > (p_1^*)_C
\end{aligned}
$$

■

B.5 GSP Auction

<u>**Symmetric bids**</u>

Symmetric Bidding. Let η_k denote the total commission paid by firm k. Assume there exists a symmetric equilibrium where $b_i = b_j = b_{sym}$. In this case, each firm gets the first position with probability. Profit for each firm is

$$\pi_i = \pi_j =: \pi = \frac{Rev_1 + Rev_2}{2} - \frac{\eta_1 + \eta_2}{2}$$

where

$$\begin{aligned} \eta_1 &= b_{sym} \\ \eta_2 &= \widehat{b}(1 - D_{11}) \end{aligned}$$

Individual Rationality: Firms need to make non-negative profits.

$$\begin{aligned} \pi &\geq 0 \\ \iff \quad b_{sym} &\leq Rev_1 + Rev_2 - \widehat{b}(1 - D_{11}) \end{aligned} \tag{14}$$

Therefore, $b_{sym} \in [\widehat{b},\ Rev_1 + Rev_2 - \widehat{b}(1 - D_{11})]$. For any value in this set, consider a deviation.

Case One: Let $b_i > b_j = b_{sym}$ be the bids of the two firms. Assume that the positions assigned are $i \longrightarrow 1,\ j \longrightarrow 2$.

Individual Rationality: Firm needs to make non-negative profit.

$$\begin{aligned} \pi_1 &\geq 0 \\ \iff \quad Rev_1 &\geq b_j \end{aligned}$$

Incentive Compatibility: Firm deviates only if there is a gain from doing so.

$$\begin{aligned} \pi_1 &\geq \pi = \frac{\pi_1 + \pi_2}{2} \\ \iff \quad \frac{\pi_1}{2} &\geq \frac{\pi_2}{2} \\ \iff \quad \pi_1 &\geq \pi_2 \\ \iff \quad Rev_1 - b_j &\geq Rev_2 - \widehat{b}(1 - D_{11}) \\ \iff \quad b_j &\leq Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big) \end{aligned} \tag{15}$$

Combining (14) and (15), there exists a deviation if

$$\begin{aligned} Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big) &> Rev_1 + \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big) \\ \iff \quad Rev_2 - \widehat{b}(1 - D_{11}) &< 0 \\ \iff \quad \widehat{b} &> \frac{Rev_2}{1 - D_{11}} \end{aligned}$$

This gives the sufficient condition to find a deviation. Then, we see that there exists no deviation of the kind $b_i > b_j$ iff

$$b_j \in \Big[Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big) \ , \ Rev_1 + \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big)\Big] \quad \text{and} \quad \widehat{b} \leq \frac{Rev_2}{1 - D_{11}} \tag{16}$$

Case Two: Consider the deviation $b_i < b_j$ and position assignments $i \longrightarrow 2, \ j \longrightarrow 1$. Following similar steps, we find that there exists no such deviation iff

$$b_j \in \Big[0, Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big)\Big) \qquad \text{and} \qquad \widehat{b} \leq \frac{Rev_2}{1 - D_{11}} \tag{17}$$

Combining (16), and (17), we get the optimal symmetric bid

$$b_i = b_j = Rev_1 - \Big(Rev_2 - \widehat{b}(1 - D_{11})\Big) \tag{18}$$

This is feasible only when

$$\widehat{b} \leq \frac{Rev_2}{1 - D_{11}} \tag{19}$$

Asymmetric bids

Consider the bids $b_i > b_j$ and position assignments $i \longrightarrow 1, \ j \longrightarrow 2$.

Individual Rationality: Firms need to make non-negative profits.

$$\begin{aligned} & \pi_1 \geq 0 \\ \iff \quad & b_j \leq Rev_1 \end{aligned} \tag{20}$$

$$\begin{aligned} & \pi_2 \geq 0 \\ \iff \quad & \widehat{b} \leq \frac{Rev_2}{1 - D_{11}} \end{aligned} \tag{21}$$

Incentive Compatibility: Firm i deviates to position 2 iff there is a gain from doing so.

$$\begin{aligned} & Rev_1 - \eta_1 \geq Rev_2 - \eta_2 \\ \iff \quad & Rev_1 - b_j \geq Rev_2 - \widehat{b}(1 - D_{11}) \\ \iff \quad & b_j \leq Rev_1 - (Rev_2 - \widehat{b}(1 - D_{11})) \end{aligned} \tag{22}$$

Firm j doesn't want position 1, given the above bids.

$$\begin{aligned} & Rev_1 - \widetilde{\eta}_1 \leq Rev_2 - \eta_2 \\ \iff \quad & Rev_1 - b_i \leq Rev_2 - \widehat{b}(1 - D_{11}) \\ \iff \quad & b_i \geq Rev_1 - (Rev_2 - \widehat{b}(1 - D_{11})) \end{aligned} \tag{23}$$

Combining (22) and (23), we get the optimal asymmetric bids

$$b_i \in \left(Rev_1 - \left(Rev_2 - \widehat{b}(1 - D_{11})\right) \ , \ \infty\right) \tag{24}$$

$$b_j \in \left[\widehat{b} \ , \ Rev_1 - \left(Rev_2 - \widehat{b}(1 - D_{11})\right)\right] \tag{25}$$

This is feasible only when

$$\widehat{b} \le \min\left\{\frac{Rev_1 - Rev_2}{D_{11}}, \frac{Rev_2}{1 - D_{11}}\right\} \tag{26}$$

Auction reserve price

The intermediary's objective is to maximise its revenue.

$$\max_{\widehat{b}} \ Rev_m = b_j + \widehat{b}(1 - D_{11})$$

Note that

$$\frac{\partial\left[Rev_1 - \left(Rev_2 - \widehat{b}(1 - D_{11})\right)\right]}{\partial \widehat{b}} \ge 0$$

This implies that b_j would take larger values when $\widehat{b}$ increases for during both symmetric and asymmetric bidding.

$$\frac{\partial Rev_m}{\partial \widehat{b}} \ge 0$$

Theorem 2. Prices are obtained by solving the first-order conditions for equation 10. For p_1^*, p_2^* to be global maxima:

$$p_1 \cdot \left(f'(\widehat{v}) - \int_{p_1}^{\widehat{v}} f(v_1 - \Delta p) \cdot f''(v_1) \cdot dv_1\right) \le 2\left(f(\widehat{v}) - \int_{p_1}^{\widehat{v}} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1\right)$$

$$p_2 \cdot \left(A \cdot f'(p_{2,NA}^*)F(p_{1,NA}^*) - f'(\widehat{v}) - f(p_2)f(p_1) + F(\widehat{v} - \Delta p) \cdot f'(\widehat{v}) - \int_{p_1}^{\widehat{v}} f(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1\right)$$
$$\le 2\left(A \cdot f(p_{2,NA}^*)F(p_{1,NA}^*)f(\widehat{v}) - F(p_2)f(p_1) - \int_{p_1}^{\widehat{v}} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1\right) - r_2 f'(\widehat{v})$$

■

Proposition 5.

$$\frac{dp_1^*}{ds} > \frac{dp_{1,NA}^*}{ds} \ , \ \frac{dp_2^*}{ds} > \frac{dp_{2,NA}^*}{ds}$$

■

B.6 Asymmetric Firms

Theorem 6. There exists no symmetric equilibrium as the conditions on b_i, b_j cover disjoint intervals in $\mathbb{R}$. For asymmetric equilibrium, consider the bids $b_i > b_j$ and position assignments $i \rightarrow 1,\ j \rightarrow 2$.

Individual Rationality: Firms need to make non-negative profits.

$$\begin{aligned} \pi_{1,i} &\geq 0 \\ \iff \quad b_j &\leq Rev_{1,i} \end{aligned} \tag{27}$$

$$\begin{aligned} \pi_{2,j} &\geq 0 \\ \iff \quad \widehat{b} &\leq \frac{Rev_{2,j}}{1 - D_{11,i}} \end{aligned} \tag{28}$$

Incentive Compatibility: Firm i deviates to position 2 only if there is a gain from doing so.

$$\begin{aligned} Rev_{1,i} - \eta_{1,i} &\geq Rev_{2,i} - \eta_{2,i} \\ \iff \quad Rev_{1,i} - b_j &\geq Rev_{2,i} - \widehat{b}(1 - D_{11,j}) \\ \iff \quad b_j &\leq Rev_{1,i} - (Rev_{2,i} - \widehat{b}(1 - D_{11,j})) \end{aligned} \tag{29}$$

Firm j doesn't want position 1, given the above bids when

$$\begin{aligned} Rev_{1,j} - \widetilde{\eta}_{1,j} &\leq Rev_{2,j} - \eta_{2,j} \\ \iff \quad Rev_{1,j} - b_i &\leq Rev_{2,j} - \widehat{b}(1 - D_{11,i}) \\ \iff \quad b_i &\geq Rev_{1,j} - (Rev_{2,j} - \widehat{b}(1 - D_{11,i})) \end{aligned} \tag{30}$$

Combining (29) and (30), we get the optimal asymmetric bids

$$b_i \in \left(Rev_{1,j} - \left(Rev_{2,j} - \widehat{b}(1 - D_{11,i})\right)\ ,\ \infty\right) = \left(Rev_{1,j} - \pi_{2,j},\ \infty\right) \tag{31}$$

$$b_j \in \left[\widehat{b}\ ,\ Rev_{1,i} - \left(Rev_{2,i} - \widehat{b}(1 - D_{11,j})\right)\right] = \left[\widehat{b},\ Rev_{1,i} - \pi_{2,i}\right] \tag{32}$$

This is feasible only when

$$\widehat{b} \leq \frac{Rev_{1,i} - Rev_{2,i}}{D_{11,j}} \tag{33}$$

■