Online Appendix

Sponsored Search : Theory and Evidence on How Platforms Exacerbate Product Market Concentration*

Akhil Ilango†

First draft: September 2021
This draft: July 29, 2022

Contents

*I thank Rosa Ferrer and Sandro Shelegia for their unwavering support and valuable suggestions throughout various stages of this paper. This paper also substantially benefited from discussions with Simon Anderson, Rodrigo Carril, Jan Eeckhout, Alexander Frug, Tobias Gamp, Juan José Ganuza, Massimo Motta, Antonio Penta, David Ronayne, Jaume Ventura, Pinar Yildrim, as well as seminar participants at DIW Berlin, Humboldt, UPF, 2022 CMID, AusMES, ICT ZEW Mannheim, AsMES, ISMS Chicago Booth, Warwick PhD, MWET, TADC, AfMES, SMYE, RES, 2021 Delhi WS, Stockholm IO workshop, and BSE Jamboree. I also thank my colleagues at UPF for their helpful comments.

†Department of Economics, Universitat Pompeu Fabra and BSE. Email: akhil.ilango@upf.edu

A Price Visibility

Equilibrium prices in a market where consumers are free to observe prices prior to visiting a firm's webpage (*in-sight*) are lower than the equilibrium prices in the same market with *out-of-sight* prices, *ceteris paribus*. Intuitively, a seller's demand is more elastic with respect to changes in *in-sight* prices than when price is only discovered after the consumer pays the search cost. Consider the unit mass of consumers plotted in Figure A.1. The co-ordinates of the square classify consumers by their valuation of the two products. For a small change in p_2, we see that demand for firm 2 changes along both line segments LM and MN. Since firm 1 shares those line segments, these consumers are taken over by it. On the other hand, imagine a situation where prices are *out-of-sight*. For a small change in p_2, we see that demand for firm 2 changes along line segment LM only. In other words, demand for firm 2 is now less elastic compared to the case of *in-sight* prices. This reduction in elasticity is sufficient to overturn the order of prices.

In models of random search (Wolinsky, 1986, Anderson and Renault, 1999) and prominence (Armstrong, Vickers, and Zhou, 2009), firms might lower prices not to attract consumers, but to retain them once they visit. In my model, prices serve both roles: they help to both attract and retain consumers.[1] Specifically, a lower price for firm 2 not only retains more visitors, but is also more likely to attract them; the latter effect makes its demand more elastic. This suggests that when consumers observe prices before searching, prices decrease with search costs. This can be observed from the price of firm 2. Since price of firm 1 is observed costlessly by the consumer at the beginning of search, this effect is absent in its case. Therefore, price dispersion increases with search cost in my model.

To further understand effect of *in-sight* prices, I disentangle the channels which influence firm 2's decision. When consumers visit firm 2, it learns that consumers, on average, have a lower valuation of product 1. This makes those consumers more likely to stay at firm 2, thus applying an upward force on p_2. I refer to this as the *information channel*.

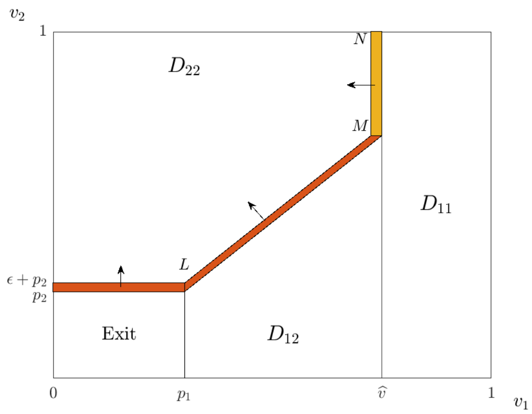


Figure A.1: Elasticity of demand: Intuition for prices

[1]Some previous studies consider prices as free to observe under different conditions. For instance, Dai (2017) uses it to analyse limited commitment and Arbatskaya (2007) uses it to analyse heterogeneous search cost.

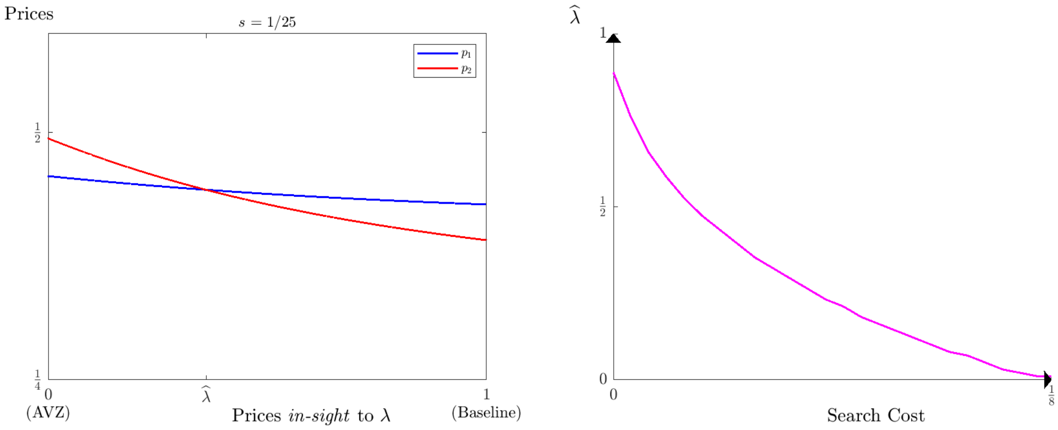


Figure A.2: Weakening of the Hold-up channel

When prices are *out-of-sight*, consumers visit firm 2 based on their rational belief about its price. However, it is free to surprise them as it doesn't change their incentives to visit firm 2. This also applies an upward force on p_2. I refer to this as the *hold-up channel*. Note that, the case of *out-of-sight* prices influences through both channels. However, making prices free to observe eliminates firm 2's ability to surprise and shuts down the *hold-up channel*. Therefore, I find that only having the information channel is not sufficient to generate increasing order of prices.

To illustrate this effect, I consider a market where $\lambda \in [0, 1]$ fraction of consumers can see both prices before starting their search while the rest discover prices by search. This exercise nests equilibrium prices from my benchmark model at $\lambda = 1$ and those from the Armstrong, Vickers, and Zhou (2009) (AVZ) model (which predicts $p_1 < p_2$) at $\lambda = 0$.

Figure A.2 shows that as prices become free to observe to more consumers (increase in λ), the role of p_2 in attracting consumers gains significance. The *hold-up* force, which had caused higher prices for firm 2 in AVZ, is weaker now due to an increase in the price elasticity of firm 2's demand. This reverses the price order.

As search cost increases, it makes the firms incentives to attract consumers more important. Therefore, even when a smaller fraction of consumers see prices before search, we see a reversal in price rankings. Moreover, this exercise of parametrizing λ resembles the 'clearing-house' models of search (see, for instance, Varian, 1980, Perloff and Salop, 1985) where a fraction of the population have access to prices. In those models, we usually see a mixed-strategy equilibrium. However, in my model, the underlying consumer heterogeneity allows for the existence of pure-strategy equilibrium.

B Revenues: 'No Auction'

In this section, I derive the equilibrium revenues for both firms, in the absence of the auction.

Proposition 1. *When match values are drawn from a Uniform distribution ($v \sim F = U[0, 1]$), the* *equilibrium revenue for firms diverge with search cost.*

$$\frac{dRev_1^*}{ds} > 0 \ , \ \frac{dRev_2^*}{ds} < 0$$

Proof. Using Envelope Theorem and substituting the expressions for the derivatives,

$$\begin{aligned}
\frac{dRev_1^*}{ds} &= p_{1,NA}^*\left(\frac{dD_{11}^*}{ds} + \frac{dD_{12}^*}{ds}\right) \\
&= p_{1,NA}^*\left(-\frac{dp_{1,NA}^*}{ds} + (1 - p_{2,NA}^*)\frac{dp_{2,NA}^*}{ds} + 1\right) \\
&= p_{1,NA}^*\left(-\frac{1 + (1 - p_{2,NA}^*)\frac{dp_{2,NA}^*}{ds}}{2} + (1 - p_{2,NA}^*)\frac{dp_{2,NA}^*}{ds} + 1\right) \\
&= p_{1,NA}^*\left(\frac{1 + (1 - p_{2,NA}^*)\frac{dp_{2,NA}^*}{ds}}{2}\right) \\
&= p_{1,NA}^*\left(\frac{dp_{1,NA}^*}{ds}\right) \geq 0 \\
\frac{dRev_2^*}{ds} &= p_{2,NA}^*\left(\frac{dD_{22}^*}{ds}\right) \\
&= p_{2,NA}^*\left(-1 \underbrace{-(1 - p_{2,NA}^*)\frac{dp_{2,NA}^*}{ds}}_{\leq 1 \text{ from Proposition 1}}\right) \leq 0
\end{aligned}$$

■

Figure B.1 visualises the comparative statics of firm revenues, following proposition 1. As search cost increases, there is a decrease in competition (region I). This increases the revenues as seen for firm 1 and reaches the monopoly level at $\underline{s}$. For $s > \underline{s}$, firm 1 maintains its optimal price at the monopoly level.

On the other hand, due to *in-sight* prices, firm 2 has lost its ability to extract the hold-up rent and the price decreases along with its (region I). In region II, firm 2's demand increases with s. This is because it now has a constant population of visitors to attract and retain and as it decreases its price, more consumers find the offer feasible and purchase. However, the revenues are well below the optimum due to constraint A. Therefore, revenue continues to decrease.

The industry revenue (sum of revenues) is maximum at the lowest search cost where both firms have segmented demand. This is because $\underline{s}$ would impose the loosest constraint on firm 2 while there is no competition (both firms have their own set of visitors).

Figure B.1 also compares the equilibrium industry revenues from my baseline model with the case where a single firm is selling both products. I refer to this second case as the Multi-

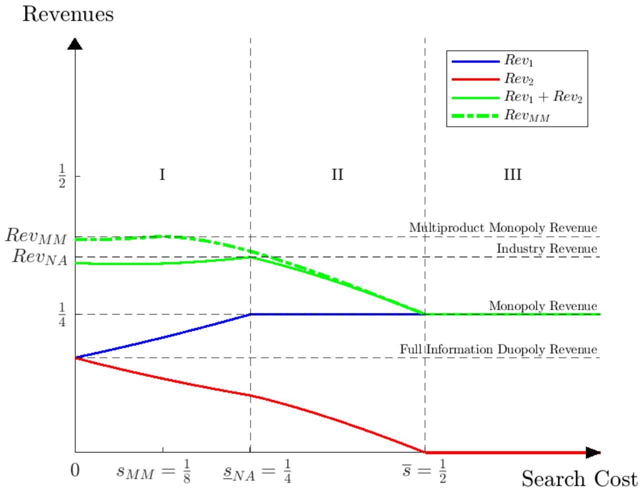


Figure B.1: Revenues (without auction)

product Monopoly (MM).[2] Drawing parallels between the intermediary's function and MM can be helpful while considering policy tools applicable to the online market. From the perspective of the intermediary, this benchmark may also be useful to assess alternate position-allocation mechanisms.

The MM uses search cost to (imperfectly) screen consumers based on their preferences. Therefore, MM finds it optimal to set some $s > 0$. Only the consumers with lower valuations for product 1 are catered to by the second product. This lets the MM charge a higher price for product 1, $(p_1)_{MM} > (p_2)_{MM}$. The MM does not lose as many consumers - if it raises its prices - as when firms are separate because some consumers buy from the other firm now (which is also a subsidiary of MM). Hence, prices of both products are higher in the case of MM than in the my baseline model.

Figure B.1 also shows that the optimal s for the MM (s_{MM}) is lower than $\underline{s}$. This is because the MM is able to reach region II quicker. Since MM internalises the fact that some consumers that leave firm 1 for high prices buy product 2 instead, MM prices the goods more aggressively and there is a segmentation of the market for lower s. Thus, the maximum revenue for MM is also larger than the industry revenue when firms are separate.

C Constant Commission

In this section, I introduce to the firms' problem an ad cost or commission. I consider the case where firms face a constant commission for any search cost.

Firms' problem. Firms maximise their profits by choosing product prices ($\mathbf{p}$) and r_1 and r_2 are per-click commission to be paid by the respective firms. The analysis in Section 3 of the main text is a special case of this framework where $r_1 = r_2 = 0$. In Section 4 of the main text, r_1 and r_2 are

[2]See, for instance, Petrikaitė (2018) and Gamp (2016). Both papers characterise the optimal search cost such that a multi-product monopoly firm chooses to maximise its revenues.

endogenous objects derived from the auction. Firms maximise profit.

$$\max_{p_{1,C}} \pi_1 = p_{1,C}(D_{11} + D_{12}) - r_1(b_i, b_j, \widehat{b}) \tag{1}$$

$$\max_{p_{2,C}} \pi_2 = p_{2,C} D_{22} - r_2(b_i, b_j, \widehat{b}) \cdot (1 - D_{11}) \tag{2}$$

Subscript C denotes 'constant commission'. This framework with two firms parsimoniously allows me to highlight the differences in the cost structure of firms, due to positioning. From the above equation, we see the different roles per-click commissions play. Since all consumers visit firm 1, the ad commission plays the role of a fixed cost for firm 1, while it is analogous to a marginal cost per visitor for firm 2. This asymmetry, again a consequence of asymmetry in ad position, plays a crucial role in determining the equilibrium outcome, as shown in Proposition 1 and later in the full equilibrium with auction in Theorem 2 of the main text.

Proposition 1 (Constant commission). *Under an exogenous per-click commission fees of r_1 and r_2 is imposed, there is a unique asymmetric equilibrium of prices in pure strategies and are given by*

$$p^*_{1,C} = \frac{1 - F(\widehat{v}^*) + \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f(v_1) \cdot dv_1}{f(\widehat{v}^*) - \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f'(v_1) \cdot dv_1}$$

$$p^*_{2,C} = \frac{A \cdot [1 - F(p^*_{2,C})]F(p^*_{1,C}) + F(\widehat{v}^*) - F(p^*_1) - \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f(v_1) \cdot dv_1 + r_2 f(\widehat{v}^*)}{A \cdot f(p^*_{2,C})F(p^*_{1,C}) + f(\widehat{v}^*) - F(p^*_2) f(p^*_1) - \int_{p^*_1}^{\widehat{v}^*} F(v_1 - \Delta p^*_C) \cdot f'(v_1) \cdot dv_1}$$

When match values are drawn from a Uniform distribution ($v \sim F = U[0,1]$),

- *the search cost at which the "Attraction" condition starts binding is lower than the 'No auction' case and is given by $\underline{s}_C = \frac{1-2r_2}{4} < \underline{s}_{NA}$*
- *the price of prominent firm increases with search cost $\frac{dp^*_{1,C}}{ds} > 0$, and the price of non-prominent firm decreases with search cost $\frac{dp^*_{2,C}}{ds} < 0$*
- *and there exists $\widehat{s}$ such that the price difference is positive for lower search costs, that is $p^*_{1,C} - p^*_{2,C} \begin{cases} < 0 & \text{if } s < \widehat{s} \\ > 0 & \text{if } s > \widehat{s} \end{cases}$*

Proof. Prices are obtained by solving the first-order conditions for equation 1. For $(p^*_1)_C, (p^*_2)_C$ to be global maxima:

$$p_1 \cdot \left(f'(\widehat{v}) - \int_{p_1}^{\widehat{v}} f(v_1 - \Delta p) \cdot f''(v_1) \cdot dv_1 \right) \leq 2 \left(f(\widehat{v}) - \int_{p_1}^{\widehat{v}} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right)$$

$$p_2 \cdot \left(A \cdot f'(p^*_{2,NA})F(p^*_{1,NA}) - f'(\widehat{v}) - f(p_2)f(p_1) + F(\widehat{v} - \Delta p) \cdot f'(\widehat{v}) - \int_{p_1}^{\widehat{v}} f(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right)$$
$$\leq 2 \left(A \cdot f(p^*_{2,NA})F(p^*_{1,NA}) f(\widehat{v}) - F(p_2) f(p_1) - \int_{p_1}^{\widehat{v}} F(v_1 - \Delta p) \cdot f'(v_1) \cdot dv_1 \right) - r_2 f'(\widehat{v})$$

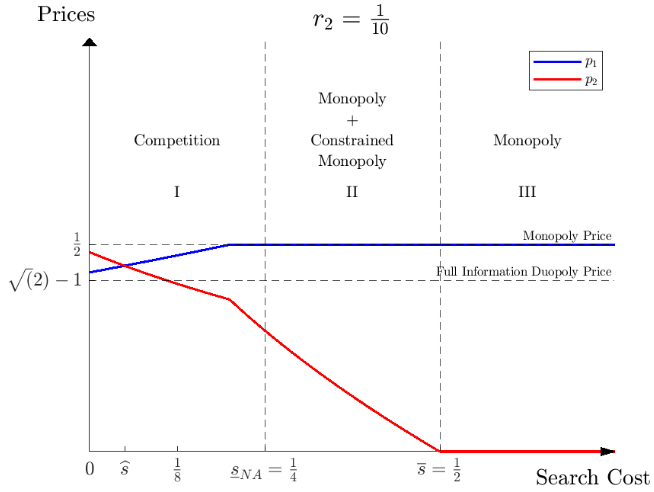


Figure C.1: Prices (with constant commission)

At $s = 0$,

$$
\begin{aligned}
(p_1^*)_C &= \sqrt{2} - 1 = p^{Ran} \\
(p_2^*)_C &= \sqrt{2(1 + r_2)} - 1 > (p_1^*)_C
\end{aligned}
$$

$\blacksquare$

Proposition 1 shows that for sufficiently low commission rates, there exist search costs for which $p_2 > p_1$ (see, for example, figure C.1). The marginal cost for visits for firm 2 increases its price which, in turn, gives room for firm 1 to also raise its price without the fear of losing consumers. However, the effect on firm 1 is of second-order. Note that firm 1 faces a fixed cost of commission. Hence, there is an asymmetric rise in p_2. This result reconciles with markets observed with increasing order of prices. However, the order is determined in my model due to the asymmetry in the cost structure. Thus, a non-prominent firm can charge a higher price, even when the *hold-up channel* has been eliminated.

The equilibrium prices diverge with search cost, following the properties highlighted in Section 3 of the main text. Note that $\underline{s}_C < \underline{s}_{NA}$ as prices are higher compared to the 'No Auction' case. The rise of p_2 due to marginal cost-like commission weakens competition as it damages firm 2's ability to attract and retain consumers. This gives firm 1 more power in the market for all s. Hence, p_1 shifts closer to the monopoly price and leads to a segmented market for a smaller s.

D Costly first search

For $s > \overline{s}$ (region III), the market breaks down as nobody is interested in searching. The action happens in region II.

Recall that the reservation value for a consumer considering between staying at firm 1 and visiting firm 2 is given by

$$\hat{v} = 1 + p_1 - p_2 - \sqrt{2s}$$

The market would be active only if the reservation value exceeds the outside option. For $\underline{s} < s < \overline{s}$, this constraint binds (region II) and can be written as $p_2 < 1 - \sqrt{2s}$. The only question that remains now is to determine if the firms would prefer to deviate from $p_1 = p_2 = 1 - \sqrt{2s}$ (the equilibrium prices at $s = \underline{s}$) and profits of $\pi_1 = (1 - \sqrt{2s})\sqrt{2s}$ and $\pi_2 = (1 - \sqrt{2s})^2\sqrt{2s}$.

They do not deviate to a higher price because they would then not have any visitors. Let firm 1 charge $p_1 = 1 - \sqrt{2s}$ and firm 2 deviate to $p_2 = 1 - \sqrt{2s} - \epsilon$. Then, $\widehat{v} > p_1$ and we are in the competitive framework (á la region I). Firm 1 gets a measure 1 of visitors and a measure $\sqrt{2s}$ of buyers among them. As an approximation, let me ignore the ϵ visitors that consider both firms, since their contribution to firms' demand would only be second-order. So, the remaining visit firm 2 and it gets a measure $(1-\sqrt{2s})(\sqrt{2s}+\epsilon)$ of buyers. Then, firm 2 gets a profit of $(1-\sqrt{2s})(1-\sqrt{2s}-\epsilon)(\sqrt{2s}+\epsilon)$.

$$\pi_2 + (1 - 2\sqrt{2s} + 2s)\epsilon - (\sqrt{2s} - 2s)\epsilon - \epsilon^2(\cdot) \;\approx\; \pi_2 + (1 - 4\sqrt{2s} + 4s)\epsilon > \pi_2$$

Hence, it will prefer to deviate. Now, imagine firm 1 deviates instead. its profit would be $(1 - \sqrt{2s} - \epsilon)(\sqrt{2s} + D_{12})$ where $\epsilon > D_{12} = \mathbb{P}[v_1 > v_2 - \epsilon] = (\frac{1}{2} - \epsilon)$.

$$\pi_1 - \sqrt{2s}\epsilon + (\frac{1}{2} - \epsilon)(1 - \sqrt{2s} - \epsilon) \;\approx\; \pi_1 + \frac{1 - \sqrt{2s} - \epsilon}{2} - \epsilon > \pi_1$$

Hence, it also prefers to deviate. Now, imagine $p_1 = p_2 = 0$ and firm 2 deviates to $p_2 = \epsilon$. This would generate a positive profit for firm 2. Therefore, although it requires a more formal analysis, it seems like there is no equilibrium in pure-strategies for $\underline{s} < s < \overline{s}$.

E Timing of the game

In the baseline model, firms place their bids and learn their position before setting product prices. This is motivated by the intuition that the one-shot auction in my model is an approximation of a long dynamic game which has revealed all the unknown information about the auction outcome. Hence, firms set prices as if they know their position and commission. See also, for instance, Gorodnichenko and Talavera (2017) for recent evidence on the high frequency of price changes in online markets. To extend the fit of my model to a more general setting, I explore alternate timings below.

Auction reserve price → firms bid and set prices simultaneously: This case is relevant to situations where auctions are nearly as frequent as product price adjustments. In symmetric equilibrium, the firms now have to set one price for both positions they will take up with equal probability. In such situations, firms' objective can be represented by

$$\begin{aligned}
\max_{p_i} \; Rev_i &= p_i \cdot \frac{D_1 + D_2}{2} = p_i \cdot \frac{1 - p_i p_j}{2} \\
\text{Symmetry} \implies p &= \frac{1}{\sqrt{3}} \; (= p^{MM}) \\
\implies Rev &= \frac{1}{3\sqrt{3}} \approx 0.192 \\
\implies \widehat{b} &= \frac{1}{\sqrt{3}} \\
\implies Rev_m &\approx 0.385 > \max\{Rev_m^{baseline}(s)\} \\
\pi &= 0
\end{aligned}$$

Note that this outcome is not a function of search cost. The asymmetric equilibria are identical to the baseline model, since we look at subgame-perfect Nash equilibria and hence, beliefs are true in equilibrium. However, we do not have the symmetric equilibrium we derived for the baseline model.

The crucial difference here is that firms do not know their position when they are setting a price. In the symmetric equilibrium of the baseline model, even though firms get each position with equal probability (due to tie-breaking), they know their position when they set prices. For example, consider that there are 100 instances of the same two firms competing for the top spot. In the baseline model, the position-allocation is revealed to the firms before the price setting. However, this is not true when firms set prices and bid simultaneously. In each of the 100 instances, the firms have to set prices without knowing their position, even though they end up getting each position with equal probability.

Auction reserve price → firms set prices → firms bid: This case is relevant to situations where auctions are rare (for e.g., quarterly) compared to product price adjustments. Solving backwards, we know that demand at position 1 is higher, even though they are more price elastic. Both firms prefer the first position and they both bid $b = Rev^{Mon}$. Therefore, they set prices equal to $\frac{1}{\sqrt{3}}$ since they do not know their position when they set prices. The logic from the previous case applies here as well.

Firms set prices → auction reserve price → firms bid: This case is relevant to situations where ad display designs changes at a higher frequency than product prices. In other words, pricing strategies are much more stickier than advertisement strategies.

The outcome here is same as in the previous case. Exchanging the timing of setting auction reserve price does not change outcomes for subgame-perfect Nash equilibria.

F Position auction

Equilibrium refinement: The tradition of restricting the set to undominated strategies is much older and attributed to analyses of co-operative game theory where the 'core' of a game is given by the undominated outcomes (Roth and Sotomayor, 1990). If as a planner or auctioneer, one would like to advice the firms to avoid some 'bad strategies', one may prescribe playing the undominated strategies. Note that this may not necessarily lead to a unique equilibrium.

Equilibrium in undominated strategies coincides with a unique locally-envy free refinement in my model. Edelman, Ostrovsky, and Schwarz (2007) develops this refinement informed by the markets' dynamic structure. Although there is still multiplicity due to the fact that firm 1's range of bids does not have an upper bound, this does not play a role in the equilibrium outcome.

Further, Varian (2007) also independently addresses the multiplicity and illustrates two possible refinements. The paper describes the intuition behind an aggressive approach - "what is the highest bid I can set so that if I happen to exceed the bid of the agent above me and I move up by one slot, I am sure to make at make at least as much profit as I make now?" - and a defensive approach - "If I set my bid too high, I will squeeze the profit of the player ahead of me so much that it might prefer to move down to my position". The paper also derives a lower-bound of outcomes from a set of sub-game perfect Nash equilibria which coincides with the 'locally envy-free' refinement mentioned above.

Microfoundation: Standard models of position auction oversee a certain form of interdependency in object (position in this case) values across bidders (firms in this case). The value of a click for a firm is assumed to be constant across positions (see, for instance, Varian, 2009). Although it is common to assume that the click-through rate as a product of firm-specific factor times a position-specific factor, consumer behaviour remains simplified. For instance, Athey and Ellison (2011) assumes the number of clicks to be exogenous. These simplifications may not be innocuous but definitely helped gain insights on various features of the position auctions.

In my model, I derive consumer behaviour as a function of the price, which in turn, is a function of both value of a buyer and number of clicks and the commission (or the cost per click). Previous literature assumed the shape of these functions. Using consumer search as a model of consumer behaviour, we now know how these functions would look like. More importantly, now we know not just the functions but that there is a feedback, and what we want and can find is the fixed point.

Reserve Price: Edelman and Schwarz (2010) assesses the welfare effects of the auction reserve price. They separate the effects into direct (causing the lowest value bidder to face a higher payment - in my model it is firm 2) and indirect (inducing other bidders to increase their bids, thereby increasing others' payments - in my model it is firm 1). They find that most of the incremental revenue from setting a reserve price optimally comes not from a direct effect, but rather from the indirect effects on high bidders. I find a similar result that the larger share of intermediary revenue comes from firm 1 for any increment in $\widehat{b}$ (see, for instance, figure F.1). Although top bidders' large valuations place them 'furthest' from the reserve price, they contribute more as commission. This is due to two forces. First, firm 2 is now forced to charge a higher price to cover its commission cost which allows firm 1 to also increase (relatively smaller than firm 2's increase) its price without loss. Second, this makes firm 2 less attractive. Thus, overall revenues of firm 1 increases substantially, to the benefit of the intermediary. Further, characterisation of the auction reserve price taking into account endogenous consumer behaviour has shown the existence of a strategy-proof and optimal allocation mechanism.

Similarly, on the other side, the auction reserve price also has an effect on the overall conversion rate in the market and how many consumers choose outside option.

One-shot auction: Note that the assumption of one-shot simultaneous bidding is a deviation from the possibility of continuous asynchronous bidding allowed in auctions run by some popular search engines. As in previous theoretical work on position auctions, I make this assumption for

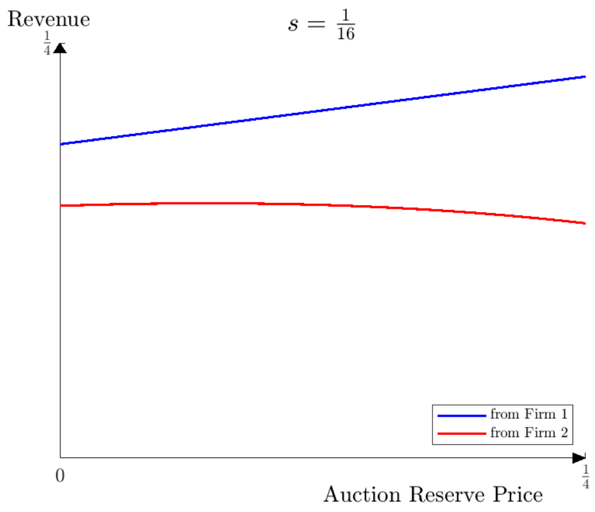


Figure F.1: Indirect effect of reserve price is stronger than direct effect

simplicity. Edelman, Ostrovsky, and Schwarz (2007) shows that the "lowest revenue envy-free" equilibrium generates the upper bound of revenues from dynamic auction models. However, having relaxed the assumption of homogenous position effects in my model, this may no longer clear be the case and requires further study. This concern is also shared in Goldman and Rao (2016). Unfortunately, an analysis of this dynamic auctions is beyond the scope of this paper.

G Asymmetric Firms: Branding/Relevance

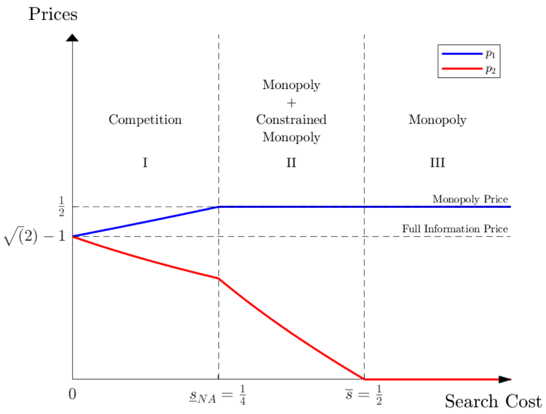
Imagine a consumer searching for a particular brand as the keyword. Competitors of this 'focal brand' may also bid for ad positions. Consumers are more likely to visit these firms if their products are more relevant to their initial search.[3] However, it is *ex-ante* unclear how many of these consumers the firms will be able to convert and how their equilibrium prices will be. One can also interpret this as the intermediary targeting its display ads to a demography that is interested in a particular brand. I explore this extension of asymmetric firms in this section.[4]

Let firms have relevance $\beta_1 > \beta_2$ (abuse of notation anticipates the equilibrium outcome). Let $\widetilde{D}$ denote the demand for firms when there is heterogeneous relevance. The value of product $k = \{i, j\}$ (from firm k) for each consumer is drawn from a mixture

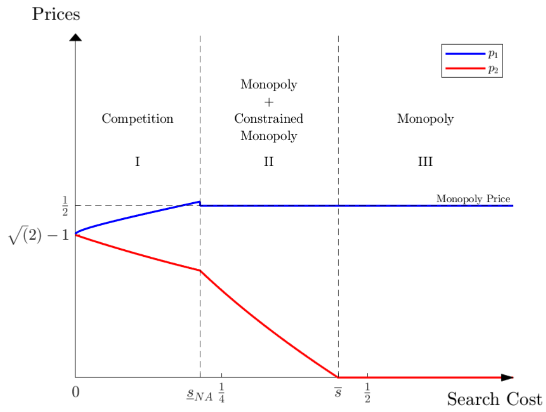
$$\begin{cases} = \ 0 & \text{with prob. } 1 - \beta_k \\ \sim \ U[0,1] & \text{with prob. } \beta_k \end{cases}$$

[3]See, for instance, Simonov and Hill (2021).

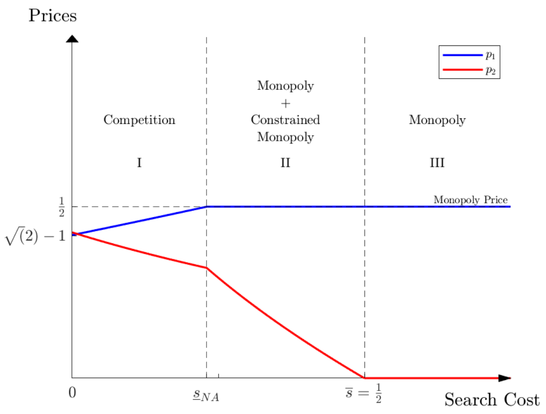
[4]Another way to think about this extension is that some fraction of consumers who search for a particular brand find a competitor brand's ad to be less relevant. This kind of heterogeneity in ordering by relevance might also happen when firms misunderstand the keyword yet, mistakenly, participate. On the other hand, a platform may even do this deliberately for two reasons. One, when they have seen many consumers who enter a certain keyword but end up buying a slightly different product, or second, when they want to increase the visibility of a certain firm. I do not take a stand on this and model the reduced form.



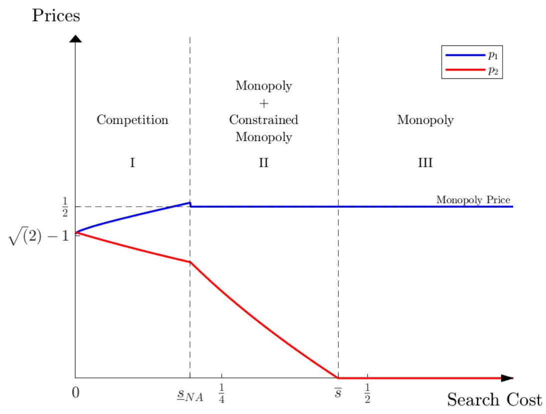
(a) $\beta_1 = 1,\ \beta_2 = 1$



(b) $\beta_1 = 0.9,\ \beta_2 = 1$



(c) $\beta_1 = 1,\ \beta_2 = 0.9$



(d) $\beta_1 = 0.9,\ \beta_2 = 0.9$

Figure G.1: Relevance: Without auction

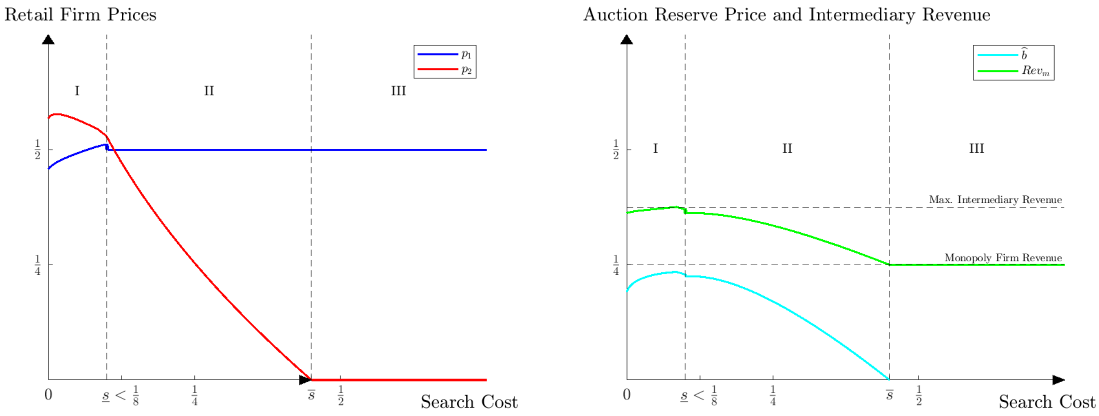


Figure G.2: Relevance ($\beta_1 = 1,\ \beta_2 = 0.9$): With auction

Deriving the reservation value for the search rule, we get

$$\widetilde{v}_\beta \;=\; \beta_2 + \widetilde{\Delta p} - \sqrt{2\beta_2 s}$$

where $\widetilde{\Delta p} = p_1 - \beta_2 p_2$. Note that the effect of p_2 on $\widetilde{v}_\beta$ is dampened but β_2 is now an additional characteristic about firm 2 that consumers consider before deciding whether to search. Figure G.2 plots the equilibrium results for $F = U[0,1]$.

H Alternative Auction Format

Digital platforms are constantly innovating in their technology to sell ads, especially in the type of auction conducted.[5] In the following exercise, I compare the baseline format of Generalised Second Price auction with the Generalised First Price auction (implemented widely by Yahoo!).[6] I analyse the implications on welfare when the intermediary conducts an auction and both firms *pay their own bid* as the per-click ad commission.

Proposition 1. *Symmetric pricing equilibrium does not exist (due to the tie-breaking rule). Asymmetric pricing equilibrium in pure strategies resembles that of the baseline model (Proposition 2 of the main text).*

Corollary 1. *Following Proposition 1, Proposition 4 of the main text on the equilibrium outcomes in a pay per-sale model also holds in a first-price auction setting.*

Firstly, there is non-existence of a symmetric equilibrium due to cyclic deviations: each firm would like to reduce its bid in order to pay less commission, but for a low enough competing bid, it would like to raise its bid to obtain the prominent position. Secondly, there is no difference between using first or second price auction from the firms' point of view, since under the symmetric bidding equilibrium, the highest bid and the second highest bid coincide. Since I resolve this symmetry using the tie-breaking rule to assign positions, the sub-game that follows is also identical.

References

[Citing pages are listed after each reference.]

Anderson, Simon P, and Regis Renault. Pricing, product diversity, and search costs: A bertrand-chamberlin-diamond model. *The RAND Journal of Economics*, pages 719–735, 1999. [see page: 2]

Arbatskaya, Maria. Ordered search. *The RAND Journal of Economics*, 38(1):119–126, 2007. [see page: 2]

Armstrong, Mark, John Vickers, and Jidong Zhou. Prominence and consumer search. *The RAND Journal of Economics*, 40(2):209–233, 2009. [see pages: 2, 3]

Athey, Susan, and Glenn Ellison. Position auctions with consumer search. *The Quarterly Journal of Economics*, 126(3):1213–1270, 2011. [see page: 10]

CMA. Online platforms and digital advertising market study. *Market study final report*, 2020. [see page: 13]

[5]See, for instance, CMA (2020) and Ferrer, Ilango, and Richter (2022) for an overview of recent technological developments and Decarolis, Goldmanis, and Penta (2018) for a discussion of auction formats.

[6]Note that for the case of two firms, a Vickrey-Clarkes-Grove auction (implemented widely by Facebook) coincides, by design, with the GSP auction (implemented widely by Google).

Dai, Anovia Yifan. Consumer search and optimal pricing under limited commitment, 2017. [see page: 2]

Decarolis, Francesco, Maris Goldmanis, and Antonio Penta. Desarrollos recientes en las subastas de publicidad online. *Papeles de Economía Española*, (157):85–108, 2018. [see page: 13]

Edelman, Benjamin, and Michael Schwarz. Optimal auction design and equilibrium selection in sponsored search auctions. *American Economic Review*, 100(2):597–602, 2010. [see page: 10]

Edelman, Benjamin, Michael Ostrovsky, and Michael Schwarz. Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords. *American economic review*, 97(1):242–259, 2007. [see pages: 10, 11]

Ferrer, Rosa, Akhil Ilango, and Paul Richter. Publicidad online. In *Reformas para impulsar la competencia en España*. CNMC, COMISIÓN NACIONAL DE LOS MERCADOS Y LA COMPETENCIA, 2022. [see page: 13]

Gamp, Tobias. Guided search, 2016. [see page: 5]

Goldman, Matthew, and Justin Rao. Experiments as instruments: Heterogeneous position effects in sponsored search auctions. *EAI Endorsed Trans. Serious Games*, 3(11):e2, 2016. [see page: 11]

Gorodnichenko, Yuriy, and Oleksandr Talavera. Price setting in online markets: Basic facts, international comparisons, and cross-border integration. *American Economic Review*, 107(1):249–82, 2017. [see page: 8]

Perloff, Jeffrey M, and Steven C Salop. Equilibrium with product differentiation. *The Review of Economic Studies*, 52(1):107–120, 1985. [see page: 3]

Petrikaitė, Vaiva. Consumer obfuscation by a multiproduct firm. *The RAND Journal of Economics*, 49(1):206–223, 2018. [see page: 5]

Roth, Alvin E., and Marilda A. Oliveira Sotomayor. *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*. Econometric Society Monographs. Cambridge University Press, 1990. doi:10.1017/CCOL052139015X. [see page: 9]

Simonov, Andrey, and Shawndra Hill. Competitive advertising on brand search: Traffic stealing and click quality. *Marketing Science*, 2021. [see page: 11]

Varian, Hal R. A model of sales. *The American Economic Review*, 70(4):651–659, 1980. [see page: 3]

———. Position auctions. *international Journal of industrial Organization*, 25(6):1163–1178, 2007. [see page: 10]

———. Online ad auctions. *American Economic Review*, 99(2):430–34, 2009. [see page: 10]

Wolinsky, Asher. True monopolistic competition as a result of imperfect information. *The Quarterly Journal of Economics*, 101(3):493–511, 1986. [see page: 2]