Application Compatibility in the Presence of Preference for Variety

Abstract

With the rising relevance of digital products, there has been an increased regulatory focus on understanding the competitive dynamics of digital ecosystems. Using a theoretical model, we examine competition between two multi-product firms, each selling hardware and an application, where each firm decides on the compatibility of its application with the rival hardware device. Distinct from previous work, we take into account consumers' preferences for variety of applications. We find that, even with ex-ante symmetric firms, an asymmetric compatibility regime (with one firm choosing compatibility and the other firm choosing incompatibility) can arise in equilibrium. Moreover, the likelihood of an asymmetric compatibility regime is higher in markets with a higher fraction of users consuming both hardware and applications, weaker hardware device differentiation, higher per-user advertising revenue, and lower per-unit nuisance cost of advertisements. Also, from a welfare point of view, we find that full compatibility is not always socially optimal.

Key Words: Platforms, Compatibility, Preference for Variety, Regulation.

1 Introduction

The importance of a highly competitive and dynamic market for digital products such as virtual reality headsets, tablets, etc., and the associated software is significant to improve benefits to consumers and businesses. A crucial strategic choice shaping the competitive dynamics of digital ecosystems is the compatibility decisions of big tech firms (e.g., Meta, Microsoft, Apple, Amazon) of their application software with rival hardware devices. Take, for example, Microsoft's recent decision to make its MS Office and Xbox gaming store compatible with Meta's VR (virtual reality) headset (TechCrunch, 2023). Users of Meta's VR headsets can access and consume Microsoft's exclusive content (e.g., Xbox games) on Meta's VR headset.[1] In contrast, Meta games are still incompatible with Microsoft's VR headsets, and the users of Microsoft's VR headsets cannot access and consume Meta games on them. Another notable example is the contrasting approaches taken by Amazon and Apple. Amazon's Kindle application is compatible with Apple's iPad, allowing users to access and read exclusive Kindle e-books on the iPad device. However, Apple's iBooks application is not compatible with Amazon's Kindle devices, limiting users' ability to read iBooks content on Kindle e-readers (Forbes, 2015). However, Amazon applications (e.g., Amazon Prime, Amazon Music) are compatible with Apple devices (e.g., iPhone, iPad) and Apple applications (e.g., Apple TV, Apple Music) are compatible with Amazon devices (e.g., Fire Tablets). Thus, based on these examples, we observe both full compatibility and asymmetric compatibility in digital markets.

Previous literature has examined various aspects of compatibility in digital ecosystems, such as the role of multi-homing users (Doganoglu and Wright, 2006), asymmetry in hardware qualities (Adner et al., 2020), developer costs (Maruyama and Zennyo, 2015), and product life cycles (Maruyama and Zennyo, 2013). However, there is limited work examining an important market characteristic: product variety, i.e., each platform has *exclusive* application content, and consumers prefer a variety of application content. In addition, CERRE Report 2022 argues that compatibility costs such as the technical challenges and resource investments required to make applications compatible with rival hardware are important in affecting firms' compatibility choices.

[1] See, "Xbox Cloud Gaming app comes to Meta Quest VR headsets, lets you play hundreds of games," *Indian Express*, available at https://indianexpress.com/article/technology/gaming/xbox-cloud-gaming-app-comes-to-meta-quest-vr-headsets-lets-you-play-hundreds-of-games-9067747/ (accessed on 18-04-2024).

Our paper aims to contribute to the theory of compatibility choice by examining the following research questions: (1) What are the market conditions for different compatibility regimes to exist? (2) How do market parameters affect firms' compatibility strategies? (3) What is the impact of firms' compatibility decisions on social welfare? To answer these questions, we develop a game-theoretic model with two firms offering differentiated and paid hardware devices and paid software applications (containing exclusive content) to a unit mass of users. The user base is segmented into two groups: a fraction of users are single-product users, consuming only hardware, while the remaining fraction of users are multi-product users, consuming both hardware and applications. All users decide which firm's hardware device to consume. However, crucially, multi-product users have a preference for a variety of application content and consume all applications available on a hardware device. Additionally, a unit mass of advertisers seek to reach users and decide whether or not to place advertisements in a firm's hardware device. The timing is sequential: first, firms make application compatibility decisions, followed by setting prices for hardware, applications, and advertisements. Finally, advertisers and consumers make adoption decisions.

Using this tractable model, we derive a set of results. To observe the intuition behind the equilibrium outcomes, note that the impact of application compatibility on the firm's profit is a double-edged sword. A preference for variety of applications, in our framework, leads to two distinct effects on the firm that chooses compatibility. One, its application demand expands. Two, its hardware demand decreases, which in turn, decreases hardware and advertising revenue. Moreover, a compatible firm incurs a cost to make its application compatible with the rival hardware. Our analysis evaluates these trade-offs yielding four major results.

First, in contrast to previous studies (e.g., Adner et al., 2020; Maruyama and Zennyo, 2015), we show that an asymmetric compatibility regime leads to lower hardware prices relative to a full compatibility or incompatibility regime. Intuitively, since users have a preference for variety, an asymmetric compatibility regime results in vertical product differentiation which, in turn, intensifies hardware price competition. Thus, hardware prices are lower under the asymmetric compatibility regime relative to other compatibility regimes.

Second, a compatible firm can charge a higher application price in an asymmetric compatibility regime, where it faces an incompatible rival, relative to the full compatibility regime, where it faces a compatible rival. As discussed, in the former case, the application variety leads to vertical product differentiation, which, in turn, leads to lower hardware prices. Thus, the compatible firm, facing an inelastic demand for its application, can charge a sufficiently large application price to extract more user surplus under an asymmetric compatibility regime relative to the full compatibility regime.

Third, we show that, even with ex-ante symmetric firms, an asymmetric compatibility regime can arise in equilibrium. Previous work examining compatibility decisions with symmetric firms (e.g., Innocenti and Menicucci, 2021) has shown that for each firm, choosing *compatibility* weakly dominates choosing *incompatibility*, and asymmetric compatibility never arises in equilibrium. Extending this to platform markets, Adner et al. (2020) shows that an asymmetric compatibility regime arises in equilibrium only when platforms are ex-ante asymmetric; otherwise, not. In contrast, we show that even with ex-ante symmetric platforms, asymmetric compatibility can arise in the presence of a preference for variety. Firms can exploit the preference for variety effect asymmetrically, with the firm that chooses compatibility focusing on application demand expansion, and the firm that chooses incompatibility focusing on vertical product differentiation to maximize profits. This equilibrium outcome, we find, is more likely in markets with a large fraction of users consuming both hardware and applications, weak hardware device differentiation, large per-user advertising revenue, and small per-unit nuisance cost of advertisements. Another crucial determinant of the compatibility regimes is the compatibility costs. Our analysis shows that changes in firms' compatibility decisions with respect to changes in compatibility costs depend on their initial level of compatibility costs. For instance, starting from sufficiently large compatibility costs, firms respond asymmetrically to a decrease in compatibility cost because firms' profit focus becomes asymmetric and they are better off in an asymmetric compatibility regime.

Finally, motivated by recent policy initiatives aimed to address compatibility issues in digital markets (e.g., Digital Markets Act, 2021; ACCESS Act, 2021),[2] we conduct a welfare analysis and derive results distinct from previous work on platform markets (e.g., Adner et al., 2020; Maruyama and Zennyo, 2015). First, in contrast to Adner et al. (2020), our major finding is that full compatibility is not always desirable. Moreover, unlike Maruyama and Zennyo (2015), only for sufficiently small compatibility costs and a large fraction of multi-product users, full compatibility is socially desirable; otherwise, either asymmetric compatibility or incompatibility can yield higher social welfare relative to full compatibility. Also, asymmetric compatibility is socially more desirable in markets with weak product differentiation and where users obtain a large intrinsic value from consuming applications. From a policy point of view, we compare market outcomes with socially optimal outcomes and show that a blanket application of full compatibility is not always desirable and that the nature of the regulatory intervention, i.e., taxing or subsidizing compatibility, should depend on the market conditions.

The remainder of the paper is structured as follows. Section 2 discusses the relevant literature. Section 3 sets up the game-theoretic model. Section 4 characterizes the equilibrium outcomes, analyzes the impact of market parameters on firms' compatibility strategies, and discusses managerial implications. Section 5 examines the welfare implications of firms' compatibility choices and discusses policy implications. Section 6 concludes the paper. All proofs are deferred to the appendices.

[2] The European Union's Digital Markets Act (DMA) proposal aims to promote fair competition and consumer choice by requiring large digital platforms to ensure interoperability with third-party services. In the United States, the ACCESS Act proposed by Congress seeks to mandate data portability and interoperability among online platforms to reduce switching costs and foster competition. These initiatives reflect a growing recognition among policymakers of the importance of compatibility in shaping the competitive dynamics of digital ecosystems.

2 Contributions to the Literature

Broadly, our paper builds on and contributes to two strands of literature: the first strand examines platform competition and its impact on pricing and design decisions, and the second strand examines the nature of co-opetition between platforms in digital markets.

2.1 Platform Competition

First, our paper is related to the stream of work on platform competition which studies platform pricing strategy (e.g., Rochet and Tirole, 2003; Armstrong, 2006; Armstrong and Wright, 2007), advertising decisions (e.g., Anderson and Coate, 2005; Ambrus et al., 2016), business model choice (e.g., Peitz and Valletti, 2008), platform quality (e.g., Llanes and Casadesus-Masanell, 2015), etc. Our paper contributes to the literature on platform competition by examining the implications of each platform's application compatibility decision on hardware, application, and advertising prices.

2.2 Co-opetition Relationship in Digital Markets

Second, our paper contributes to the literature that examines the co-opetition relationship between firms where two or more firms engage in both competition and cooperation in a variety of settings, such as the partnership between two complementary product manufacturers on research and development investment while competing on prices (e.g., Casadesus-Masanell and Yoffie, 2007; Niu et al., 2019), competition between a cluster of competing firms who could benefit from a common complementary product (Yuan et al., 2021), etc.

An important co-opetition relationship in digital markets is platform compatibility decisions. Hagiu et al. (2020) investigates the incentives of a multi-product firm to profit from hosting rival sellers. They find conditions under which hosting can be unilaterally and jointly profitable for both the multi-product firm and the rival. Boudreau (2010) documents evidence on how the degree of openness of a new platform to complementary applications increases the user base of the platform. In contrast, we study a setting in which the platform decides whether to design its own complementary application to be compatible with a competitor platform. However, we also find that more variety in applications attracts a larger user base, consistent with the empirically documented mechanism.

In a system market with multi-product firms, a set of papers has studied firms' compatibility decisions and its social optimality. One strand of work has examined compatibility decisions in markets without network effect. A series of successive papers (e.g., Matutes and Regibeau, 1988; Matutes and Regibeau, 1992) examine the compatibility incentives of symmetric multi-product firms. They show that, in equilibrium, firms adopt compatibility because it decreases firms' benefits from price cuts as they are partially appropriated by rivals, which in turn, leads to higher prices relative to incompatibility regime. Kim and Choi (2015) extends this framework to more than two firms to show that alignment of private and social preference towards compatibility fails with many firms, due to intensification of price competition with a dense market. Hurkens et al. (2019) studies bundling decisions of two multi-product firms with asymmetric quality. It shows that for a sufficiently large asymmetry in quality, both firms adopt bundling.

Another strand of work has examined compatibility decisions in markets with network effect. In an early paper, Katz and Shapiro (1985) show that due to more variety stemming from network externalities, consumer demand shifts upward when firms choose to design compatibility, thus providing a rationale for industry-wide 'standardization' of technology. Liang et al. (2023) studies competing platforms of asymmetric scale, where the large platform has the ability to host the small platforms for users' access. It shows that compatibility is optimal when demand through the compatible channel is in the intermediate range. Closer to our work, Adner et al. (2020) studies the decision of two competing platforms, that are asymmetric by quality, on whether to allow their complementary product to be compatible with their competitor's platform. They find that an asymmetric compatibility regime arises in equilibrium only when platforms are ex-ante asymmetric; otherwise, not. In a different setting, both Maruyama and Zennyo (2015) and Innocenti and Menicucci (2021) derive a similar result and show that, with ex-ante symmetric firms, asymmetric compatibility can never arise as an equilibrium outcome. In contrast to Adner et al. (2020), Innocenti and Menicucci (2021), and Maruyama and Zennyo (2015), we find that, even with ex-ante symmetric firms, an asymmetric compatibility regime can arise in equilibrium. Our result hinges on two important market characteristics, not examined in these papers. One, each platform has *exclusive* application content, and consumers prefer a variety of application content. Two, the effect of different costs of compatible-product development in optimal market strategy. Moreover, using this setting, unlike Adner et al. (2020), we also show that an asymmetric compatibility regime leads to lower hardware prices relative to a full compatibility or incompatibility regime. Finally, from a social point of view, unlike Adner et al. (2020) and Maruyama and Zennyo (2015), our major finding is that full compatibility is not always desirable. For instance, for sufficiently small compatibility costs and small to intermediate fraction of multi-product users, either asymmetric compatibility or incompatibility can yield higher social welfare relative to full compatibility.

3 Model Preliminaries

3.1 Market Structure

Our model comprises three different types of agents: (i) two firms 1 and 2 (e.g., Meta and Microsoft), each producing a hardware device H_i, $i \in \{1, 2\}$ (e.g., Oculus VR headset and Hololens VR headset) and an application software A_i, $i \in \{1, 2\}$ (e.g., Meta game store and Xbox game store), (ii) a unit mass of advertisers, and (iii) a unit mass of users with a fraction $\alpha > 0$ users deriving utility from consuming both hardware device and application(s) available on it (referred to as multi-product users), and the remaining $1 - \alpha$ users deriving utility from consuming only hardware device (referred to as single-product users). Both hardware devices and applications are paid, and, in addition, firms obtain revenue from placing advertisements in the hardware.

3.2 Firms' Profit

Each firm $i \in \{1, 2\}$ offers a hardware device H_i and an application A_i. A firm $i \in \{1, 2\}$ obtains revenue from three sources: (i) hardware sales at price p_i per unit, (ii) application sales at price r_i per unit, and (iii) advertisement revenue from placing a_i advertisements in its hardware with price s_i per user charged to an advertiser. Let N_i be the total number of users (both single-product and multi-product) who consume hardware H_i, and let D_i be the total number of multi-product users who consume both – hardware H_i and application(s) available on it.

In addition, each firm $i \in \{1, 2\}$ chooses whether to make its application A_i compatible with the rival hardware H_j, $j \neq i$ at a fixed compatibility cost $F \geq 0$.[3] If firm i doesn't make its application compatible with rival hardware H_j, $j \neq i$, then a multi-product user of rival hardware H_j can consume only application A_j, $j \neq i$, available on H_j. Whereas, if firm i makes its application compatible with rival hardware H_j, $j \neq i$, then a multi-product user of rival hardware H_j can consume both applications A_i and A_j available on H_j. Therefore, the profit of firm $i \in \{1, 2\}$ is

$$\pi_i(p_i, r_i, s_i) = \begin{cases} p_i N_i + r_i D_i + s_i a_i N_i, & \text{if } A_i \text{ is not compatible with } H_j, \text{ and} \\ p_i N_i + r_i(D_i + D_j) + s_i a_i N_i - F, & \text{if } A_i \text{ is compatible with } H_j. \end{cases} \tag{1}$$

3.3 User Utility

There is a unit mass of users. Among them, the users are divided into two types. A fraction $\alpha > 0$ users obtain utility from consuming both the hardware device and the total number of applications available on it. For them, the hardware and application(s) available on it

[3]The assumption of fixed compatibility cost is in line with previous work (e.g., Maruyama and Zennyo, 2015; Boom, 2001).

are complementary products. The remaining fraction $1-\alpha>0$ users only decide which hardware to consume and they derive zero utility from the applications. This assumption is in line with empirical observations. For instance, a 2022 survey shows that 30% of virtual reality (VR) device consumers used them for VR activities (e.g., watching movies, browsing the internet), and the remaining 70% of VR device consumers used them for gaming. This indicates that the usage of VR devices varies across consumers.[4] To proceed, we will refer to $\alpha>0$ users as multi-product users and $1-\alpha>0$ users as single-product users.

A user (single-product or multi-product) obtains a gross intrinsic value equal to $V>0$ from consuming a hardware device. Moreover, she is exposed to a_i advertisements when consuming the hardware product H_i, incurring a total dis-utility of δa_i from viewing a_i advertisements, where $\delta \geq 0$ is the per-unit nuisance cost of advertisement. We assume firms compete à la Hotelling for both hardware devices. Hardware devices can be differentiated in the eyes of the users because of their intended use, user interface, etc. For instance, Meta and Microsoft virtual reality headsets are differentiated because they provide different virtual experiences. Meta's Oculus allows users to step into fully virtual environments and leave the real world behind, whereas Microsoft's HoloLens allows users to access virtual information without stepping away from their physical environment.[5] Similarly, Apple iPad is mainly used for work related activities (e.g., lectures, taking notes), whereas Amazon Fire tablet is more suitable for entertainment related purposes (e.g., games, watching movies).[6] User preferences for the hardware devices are represented using a Hotelling interval [0, 1] with firm 1's hardware device located at point 0 and firm 2's hardware device located at point 1 of the Hotelling line. A user is characterized by her location x on the Hotelling interval [0, 1], where x represents her preference for the ideal product. Each user located at $x \in [0,1]$

[4] See "Beyond Reality 2022," *Group, N.R.*, available at https://assets.ctfassets.net/4ivt4uy3jinr/12b92XBfBiZSYVRBttBLdk/3b47b91d2ba4fa333186f2c3bd69e278/Beyond_Reality_April_2022.pdf (accessed on 19-04-2024).

[5] See, "HoloLens 2 vs Oculus Quest 2: Which is Best?," *XR Today*, available at https://www.xrtoday.com/mixed-reality/hololens-2-vs-oculus-quest-2-which-is-best/ (accessed on 19-04-2024).

[6] See, "Amazon Fire Tablet vs. iPad: Which Is Right for You?," *Lifewire*, available at https://www.lifewire.com/amazon-fire-tablet-vs-ipad-5270471 (accessed on 19-04-2024).

incurs a constant per-unit transportation cost (dis-utility) t from consuming a hardware. Thus, she faces a transportation cost of tx (respectively, $t(1-x)$), if she consumes hardware H_1 (respectively, H_2).

In addition, a multi-product user also obtains an additional utility from consuming application(s) available on hardware H_i. We assume that a multi-product user consumes all applications that are available on a hardware device because of variety-seeking behavior, i.e., they will purchase and consume all applications available on hardware. This assumption is in line with empirical observations For instance, in a 2022 survey, a common consumer complaint while using VR headsets was that there isn't a wide enough selection of games to play on it.[7] An important factor affecting the purchase of multiple applications (e.g., games) is that there exists an incremental value in doing so. This is particularly important in the gaming industry as both Meta and Microsoft own exclusive VR games (e.g., Meta Lone Echo and Microsoft Assasin's Creed). For simplicity, we assume that there is no overlap in the firms' application content. Therefore, if rival application A_j, $j \neq i$, is not compatible with hardware H_i, then she obtains a gross intrinsic value $W > 0$ from consuming the available application A_i on hardware H_i. However, if rival application A_j, $j \neq i$, is compatible with hardware H_i, then she obtains a gross intrinsic value $2W$ from consuming both applications A_i and A_j available on hardware H_i.[8]

A single-product user's net utility depends on her hardware consumption. She pays hardware price p_i for the hardware and is exposed to a_i advertisements that generate a

[7]See,"Beyond Reality 2022," *Group, N.R.*, page 14, available at https://assets.ctfassets.net/4ivt4uy3jinr/12b92XBfBiZSYVRBttBLdk/3b47b91d2ba4fa333186f2c3bd69e278/Beyond_Reality_April_2022.pdf (accessed on 19-04-2024).

[8]Our main results remain unchanged if we allow for partial content overlap among multiple applications. Suppose, for simplicity, the total measure of content/features available on each application is equal to 1 with $\theta \in (0, 1]$ units of exclusive content and $1-\theta$ units are non-exclusive/overlapping content. Also, let W be the intrinsic value (utility) obtained from an additional unit of content/feature available on application A_i, $i \in \{1, 2\}$. Then, if a user consumes either application A_1 or A_2, the intrinsic value obtained is $(1-\theta)W + \theta W = W$. Whereas, if a user consumes both applications A_1 and A_2, then the intrinsic value obtained is $W + \theta W = (1+\theta)W$, where $\theta \in (0, 1]$ can be interpreted as the incremental benefits from consuming both applications. In this modified framework, our main results remain unchanged. The details are available from authors upon request.

nuisance cost δa_i. Therefore, her net utility is

$$U_i(x) \;=\; \begin{cases} V - p_1 - \delta a_1 - tx, & \text{if she consumes } H_1 \ (i = 1), \text{ and} \\ V - p_2 - \delta a_2 - t(1-x), & \text{if she consumes } H_2 \ (i = 2). \end{cases} \tag{2}$$

A multi-product user's net utility depends on the hardware and the number of applications available on it, which in turn, depends on the compatibility regime. She pays price p_i for the hardware H_i, faces a_i advertisements that generate nuisance cost δa_i, and pays application price r_i (if A_j is not compatible with H_i) or $r_i + r_j$ (if A_j is compatible with H_i). Depending on the compatibility regime, we can have four different scenarios. If application A_j is not compatible with rival hardware H_i, then we are in an incompatible regime NN. A multi-product user's utility is

$$U_i(x) \;=\; \begin{cases} V - p_1 - \delta a_1 + W - r_1 - tx, & \text{if she consumes } H_1 \text{ and } A_1 \ (i = 1), \text{ and} \\ V - p_2 - \delta a_2 + W - r_2 - t(1-x), & \text{if she consumes } H_2 \text{ and } A_2 \ (i = 2). \end{cases} \tag{3}$$

If application A_j is compatible with rival hardware H_i, then we are in full compatibility regime CC. A multi-product user's utility is

$$U_i(x) \;=\; \begin{cases} V - p_1 - \delta a_1 + 2W - r_1 - r_2 - tx, & \text{if she consumes } H_1,\ A_1 \text{ and } A_2 \ (i = 1), \text{ and} \\ V - p_2 - \delta a_2 + 2W - r_2 - r_1 - t(1-x), & \text{if she consumes } H_2,\ A_1 \text{ and } A_2 \ (i = 2). \end{cases} \tag{4}$$

If application A_1 is incompatible with hardware H_2, whereas application A_2 is compatible with hardware H_1, then we are in an asymmetric compatibility regime NC. A multi-product user's net utility is

$$U_i(x) = \begin{cases} V - p_1 - \delta a_1 + 2W - r_1 - r_2 - tx, & \text{if she consumes } H_1\text{, } A_1 \text{ and } A_2 \ (i=1)\text{, and} \\ V - p_2 - \delta a_2 + W - r_2 - t(1-x), & \text{if she consumes } H_2 \text{ and } A_2 \ (i=2). \end{cases} \tag{5}$$

If application A_1 is compatible with hardware H_2, whereas application A_2 is incompatible with hardware H_2, then we are in an asymmetric compatibility regime CN. A multi-product user's net utility is

$$U_i(x) = \begin{cases} V - p_1 - \delta a_1 + W - r_1 - tx, & \text{if she consumes } H_1 \text{ and } A_1 \ (i=1)\text{, and} \\ V - p_2 - \delta a_2 + 2W - r_1 - r_2 - t(1-x), & \text{if she consumes } H_2\text{, } A_1 \text{ and } A_2 \ (i=2). \end{cases} \tag{6}$$

3.4 Advertisers' Profit

A unit mass of advertisers decides on whether or not to place an advertisement in firm i's hardware. An advertiser obtains a revenue (benefit) q per user from placing an advertisement in hardware H_i. For simplicity, we assume that q is homogeneous. A similar assumption is made in the previous literature (e.g., Amaldoss et al., 2021) to focus on consumer-side competition while simplifying the advertising market. Note that we work with a reduced form expression for per-user advertising revenue q. As shown later in Section 4, this implies that each firm also earns a per-user advertising revenue q in equilibrium. In an alternate setting, based on a second price auction model on the advertising side, in Appendix C.1, we provide a microfoundation for the equilibrium outcome that each firm earns per-user advertising revenue q. Let s_i denote the price paid per user for placing an advertisement in firm i's hardware H_i. Thus, the net payoff of an advertiser q from advertising in firm i's hardware H_i, $i \in \{1, 2\}$ is

$$(q - s_i)N_i, \tag{7}$$

where N_i is the total number of users consuming hardware H_i.

3.5 Timeline of the Game

We consider the following four-stage game:

Stage 1: Each firm chooses between compatibility (C) and incompatibility (N). As a result, four market regimes are possible. In the first regime NN, both firms choose incompatibility. In the second regime CC, both firms choose compatibility. In the third regime NC, only firm 2 chooses compatibility, while firm 1 does not, and in the fourth regime CN, only firm 1 chooses compatibility, while firm 2 does not.

Stage 2: With full information on the above choice, firms 1 and 2 simultaneously and non-cooperatively choose hardware prices p_1 and p_2, application prices r_1 and r_2, and advertising prices, s_1 and s_2, respectively.

Stage 3: With full information on the above choices, advertisers decide whether or not to advertise in firm i's hardware H_i, $i \in \{1, 2\}$.

Stage 4: With full information on the above choices, a single-product user decides whether to consume hardware H_1 or H_2. Whereas, a multi-product user decides whether to consume hardware H_1 or H_2 along with the application(s) available on it.

The solution concept used is the subgame perfect Nash equilibrium (henceforth equilibrium).

4 Equilibrium Analysis

We begin by introducing three assumptions that will hold throughout the analysis. Assumption 1 below assumes that the intrinsic value of hardware V is sufficiently large so as to ensure full market coverage for hardware.

Notation	Description
Parameters	
V	Standalone/intrinsic value of firm i's hardware H_i.
W	Standalone/intrinsic value of firm i's application A_i.
δ	Per-unit nuisance cost of an advertisement.
t	Per-unit transportation cost.
q	Per-user advertising revenue.
F	Cost of making application A_i compatible with rival firm j's hardware H_j.
Variables	
p_i	Price paid by a user to consume firm i's hardware H_i.
r_i	Price paid by a user to consume firm i's application A_i.
s_i	Price paid per-user by advertiser for placing an advertisement in firm i's hardware H_i.
a_i	Level of advertisements in firm i's hardware H_i.
N_i	Demand for firm i's hardware H_i.
D_i	Demand for firm i's application A_i.

Table 1: Model parameters and variables

Assumption 1 (Covered Market)**.**

$$V \geq \frac{3t}{2} - q$$

Assumption 2 imposes a parametric restriction on the per-unit transportation cost t to ensure non-negative equilibrium prices for both hardware and applications.

Assumption 2 (Non-negative prices)**.**

$$t \geq q + \frac{\alpha W}{3}$$

Assumption 3 imposes a parametric restriction on the per-user advertising revenue to advertisers, to ensure non-negative equilibrium revenue to the platform from selling advertisement slots.

Assumption 3 (Non-negative per-user advertising surplus)**.**

$$1 \geq q \geq \delta$$

We begin by analyzing *Stages 2*, *3* and *4* of the game, and derive the equilibrium prices, advertisements, and profits for three broad scenarios of competition. (i) Regime NN: incompatibility regime in which neither firm's application is compatible with its rival's hardware, (ii) Regime CC: full compatibility regime in which each firm's application is compatible with its rival's hardware, and (iii) Regime NC or CN: asymmetric compatibility regime in which firm i's application is not compatible with rival j's hardware, $j \neq i$, whereas rival j's application is compatible with firm i's hardware.

4.1 Incompatibility

When neither firm's application is compatible with the rival's hardware, a multi-product user can consume only firm i's own application on its hardware H_i, $i \in \{1, 2\}$. The utility of a single-product user (respectively, a multi-product user) from consuming either hardware is defined by Equation (2) (respectively, Equation (3)). At *Stage 4*, using Equations (2) and (3), (details relegated to Appendix A), we obtain the demand functions for the hardware devices and applications as

$$N_1 = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{\alpha(r_2 - r_1)}{2t} + \frac{\delta(a_2 - a_1)}{2t}, \quad N_2 = \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha(r_1 - r_2)}{2t} + \frac{\delta(a_1 - a_2)}{2t},$$

$$D_1 = \alpha\left[\frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{(r_2 - r_1)}{2t} + \frac{\delta(a_2 - a_1)}{2t}\right], \text{ and } D_2 = \alpha\left[\frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha(r_1 - r_2)}{2t} + \frac{\delta(a_1 - a_2)}{2t}\right].$$

At *Stage 3*, advertisers make the participation decisions. Given advertising price $s_i, i \in \{1, 2\}$, an advertiser q places an advertisement in hardware $H_i, i \in \{1, 2\}$ if $q \geq s_i$. This gives the level of advertisements a_i in hardware $H_i, i \in \{1, 2\}$ as

$$a_i = \begin{cases} 1, & \text{if } \ q \geq s_i \text{ and} \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

Using Equation (1), firms' profits are $\pi_1 = p_1 H_1 + r_1 D_1 + s_1 a_1 N_1$ and $\pi_2 = p_2 H_2 + r_2 D_2 + s_2 a_2 N_2$. At *Stage 2*, firm $i \in \{1, 2\}$ chooses the hardware price p_i, application price r_i and advertising price s_i to maximize its profits. Let the equilibrium value of a variable y, where y

could imply price, advertisement, market share, or profit, be denoted by y^{NN}. The following lemma characterizes the equilibrium.

Lemma 1. *When neither firm's application is compatible with rival's hardware, i.e., both choose incompatibility at Stage 1, then the equilibrium prices, demands, advertisements, and profits, are as follows:*

$$\begin{aligned}
p_1^{NN} &= p_2^{NN} = t - q; \;\; r_1^{NN} = r_2^{NN} = 0; \;\; s_1^{NN} = s_2^{NN} = q, \\
N_1^{NN} &= N_2^{NN} = \frac{1}{2}; \;\; D_1^{NN} = D_2^{NN} = \frac{\alpha}{2}; \; a_1^{NN} = a_2^{NN} = 1, \;\; \textit{and} \\
\pi_1^{NN} &= \pi_2^{NN} = \frac{t}{2}.
\end{aligned}$$

We highlight three important properties of the preceding equilibrium. First, hardware prices are subsidized by the amount of per-user advertising revenue q. This is a key property of the two-sided market structure with users and advertising sides, in which, in equilibrium, hardware products are subsidized to compete intensively for users and generate more revenue on the advertising side (see, for e.g., Armstrong and Wright, 2007; Peitz and Valletti, 2008). Thus, the higher the per-user advertising revenue q, the lower the price charged for the hardware. Second, firms offer applications for free in the incompatibility case. Intuitively, charging a higher application price increases the surplus extracted from each multi-product user. However, it also decreases the number of multi-product users consuming the firm's hardware and application. On the net, in the incompatible case, the latter effect dominates the former, driving application prices to zero for both firms. Finally, firms' profits are independent of the advertising profit, because there is a full pass-through of advertising profits to the users in the form of lower hardware prices.

4.2 Full Compatibility

When both firms choose to make their application compatible with the rival's hardware, a multi-product user can consume both firms' applications on hardware H_i, $i \in \{1, 2\}$. The utility of a single-product user (respectively, a multi-product user) from consuming either hardware is defined by Equation (2) (respectively, Equation (4)). At *Stage 4*, using Equations (2) and (4), (details relegated to Appendix A), we obtain the demand functions for the hardware devices and applications as

$$N_1 = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{\delta(a_2 - a_1)}{2t}, \quad N_2 = \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\delta(a_1 - a_2)}{2t}, \text{ and}$$

$$D_1 = D_2 = \alpha.$$

At *Stage 3*, advertisers make participation decisions, and advertising demand is as defined by Equation (8). Using Equation (1), firms' profits are $\pi_1 = p_1 H_1 + r_1(D_1 + D_2) + s_1 a_1 N_1$, and $\pi_2 = p_2 H_2 + r_2(D_1 + D_2) + s_2 a_2 N_2$. At *Stage 2*, firm $i \in \{1, 2\}$ chooses the hardware price p_i, application price r_i and advertising price s_i to maximize its profits. Let the equilibrium value of a variable y, where y could imply price, advertisement, market share, or profit, be denoted by y^{CC}. The following lemma characterizes the equilibrium.

Lemma 2. *When both firms choose to make their applications compatible with the rival's hardware, i.e., both choose compatibility at Stage 1, then the equilibrium prices, demands, advertisements, and profits, are as follows:*

$$\begin{aligned}
p_1^{CC} &= p_2^{CC} = t - q; \;\; r_1^{CC} = r_2^{CC} = \frac{1}{2}\left[V - \frac{3t}{2} + 2W + q - \delta\right]; \;\; s_1^{CC} = s_2^{CC} = q, \\
N_1^{CC} &= N_2^{CC} = \frac{1}{2}; \;\; D_1^{CC} = D_2^{CC} = \alpha; \;\; a_1^{CC} = a_2^{CC} = 1, \;\; \textit{and} \\
\pi_1^{CC} &= \pi_2^{CC} = \frac{t}{2} + \frac{\alpha}{2}\left[V - \frac{3t}{2} + 2W + q - \delta\right] - F.
\end{aligned}$$

The intuition for equilibrium hardware prices remains the same as in the incompatible regime. However, when both applications are compatible, firms choose positive application prices. Since all multi-product users consume both applications available on the hardware they choose to consume, the demand for each application is inelastic. As a result, when a firm increases application price, then it increases the surplus extracted from each multi-product user, however, the number of multi-product users consuming the applications remains unchanged because demand is inelastic. Thus, each firm has full market power over the users. Therefore, in equilibrium, given the assumption of full market coverage (Assumption 1), the application prices are charged to extract the maximum possible consumer surplus keeping the market covered. In a Hotelling framework, the indifferent multi-product user obtains the lowest surplus. Thus, the symmetric equilibrium application price is set such that the indifferent multi-product user is left with zero surplus.

4.3 Asymmetric Compatibility

Without loss of generality, suppose firm 2 chooses to make its application A_2 compatible with hardware H_1, whereas firm 1 chooses to make its application A_1 incompatible with hardware H_2. The utility of a single-product user (respectively a multi-product user) from consuming either hardware is defined by Equation (2) (respectively, Equation (5)). At *Stage 4*, using Equations (2) and (5), (details relegated to Appendix A), we obtain the demand functions for the hardware devices and applications as

$$N_1 = \frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W}{2t} + \frac{\delta(a_2 - a_1)}{2t}, \quad N_2 = \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha r_1}{2t} - \frac{\alpha W}{2t} + \frac{\delta(a_1 - a_2)}{2t},$$

$$D_1 = \alpha\left[\frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{r_1}{2t} + \frac{W}{2t} + \frac{\delta(a_2 - a_1)}{2t}\right], \text{ and } D_2 = \alpha.$$

At *Stage 3*, advertisers make participation decisions, and the advertising demand is as defined by Equation (8). Using Equation (1), firms' profits are $\pi_1 = p_1 H_1 + r_1 D_1 + s_1 a_1 N_1$, and $\pi_2 = p_2 H_2 + r_2(D_1 + D_2) + s_2 a_2 N_2$. At *Stage 2*, firm $i \in \{1, 2\}$ chooses the hardware price p_i, application price r_i and advertising price s_i to maximize its profits. Let the equilibrium value of a variable y, where y could imply price, advertisement, market share, or profit, be denoted by y^{NC}. The following lemma characterizes the equilibrium.

Lemma 3. *When, at Stage 1, firm 1 chooses to make its application* A_1 *incompatible with hardware* H_2*, whereas firm 2 chooses to make its application* A_2 *compatible with hardware*

H_1, *then the equilibrium prices, demands, advertisements, and profits, are as follows:*

$$p_1^{NC} = t - q - \frac{\alpha W}{6},\ p_2^{NC} = t - q - \frac{\alpha W}{3};\ r_1^{NC} = \frac{W}{2},\ \ r_2^{NC} = V - \frac{3t}{2} + \frac{\alpha W}{4} + \frac{3W}{4} + q - \delta;\ \ s_1^{NC} = s_2^{NC} =$$

$$N_1^{NC} = \frac{1}{2} + \frac{\alpha W}{6t}, N_2^{NC} = \frac{1}{2} - \frac{\alpha W}{6t};\ \ D_1^{NC} = \alpha\left[\frac{1}{2} + \frac{(3-\alpha)W}{12t}\right], D_2^{NC} = \alpha;\ a_1^{NC} = a_2^{NC} = 1,$$

$$\pi_1^{NC} = \frac{t}{2} + \frac{\alpha W}{3} + \frac{\alpha W^2}{8} - \frac{5\alpha^2 W^2}{72t},\ \textit{and}\ \pi_2^{NC} = \frac{t(1-3\alpha)}{2} + \frac{5\alpha W}{12} + \frac{\alpha^2 W}{4} + \frac{\alpha^2 W^2}{18t} + \alpha V + \alpha q - \alpha\delta - F.$$

A few comments regarding the preceding equilibrium are in place. The equilibrium hardware prices are subsidized to attract users for the advertisers and multi-product users for the consumption of the application on the hardware. The former effect is the same as discussed in the preceding two regimes and decreases hardware prices by per-user advertising revenue q. However, the latter effect is novel and specific to asymmetric compatibility regimes. Under an asymmetric compatibility regime, in equilibrium, a new force emerges to attract multi-product users for the application consumption on hardware for firm 1 that chose *incompatibility* (incompatible firm 1, hereafter). Thus, firm 1 subsidizes hardware prices to attract multi-product users to its device. This cross-subsidization force, in turn, leads to intense hardware competition between the two firms, and firm 2 also subsidizes its hardware price to attract multi-product users on its device. Therefore, both firms end up subsidizing hardware prices to attract multi-product users. As a result, each firm's hardware price decreases in the fraction of the multi-product of users α and the intrinsic value W obtained from consuming an application. Next, consider application prices. Firm 1 sets the price of its application, A_1, balancing the higher surplus generation effect and demand elasticity of multi-product users. However, since application A_2 is compatible with hardware H_1, demand for application A_2 is inelastic, and firm 2 charges application price to extract the maximum possible consumer surplus. In a Hotelling framework, the indifferent multi-product user obtains the lowest surplus. Thus, the equilibrium application A_2 price is set such that the indifferent multi-product user is left with zero surplus.

4.4 Comparison of Prices and Profits

Having examined *Stages 2, 3* and *4* of the game, we now examine *Stage 1* (firms' compatibility decisions). We begin by comparing equilibrium prices. In the next proposition, we examine the differences in the equilibrium prices of hardware devices, applications, and advertisements under different market regimes. Comparing Lemmas 1, 2 and 3 shows the following.

Proposition 1 (Comparison of Optimal Prices). *(i) Firms charge* $(\mathfrak{a})$ *equal hardware prices under incompatibility and full compatibility regimes, and* $(\mathfrak{b})$ *lower hardware prices under asymmetric compatibility regimes relative to incompatibility and full compatibility regimes.*

(ii) Firms charge lower application prices under the incompatibility regime relative to the full compatibility regime. Moreover, under an asymmetric compatibility regime, the compatible firm (say firm 2) charges a higher application price relative to both incompatibility and full compatibility regimes, whereas, the incompatible firm (say firm 1) charges a lower application price relative to full compatibility regime, but higher relative to incompatibility regime.

(iii) Advertising prices are equal across all regimes.

Consider hardware prices. The hardware prices are determined by three distinct forces. One, the strength of competition between the firms measured by per-unit transportation cost, $\mathfrak{t}$, as is standard under Hotelling price competition. Two, firms compete to attract users for advertisers on their hardware devices because an additional user generates per-user advertising revenue equal to $\mathfrak{q}$. This implies that firms subsidize hardware prices to attract users and make their platforms attractive to advertisers. Third, if the rival firm is incompatible, then the compatible firm becomes an inferior hardware firm and further subsidizes users to make its hardware device more attractive for the multi-product users, monetizing them through application sales on its hardware.

First, consider the comparison of hardware prices under two symmetric regimes: incompatibility and compatibility regimes. Under these market regimes, the subsidization of hardware to attract multi-product users for application sales vanishes. The final change in hardware prices is determined by the interplay between the strength of consumer preferences for hardware devices and the use of advertising revenue to subsidize prices. Since firms are symmetric, both effects lead to symmetric hardware prices under the two regimes, i.e., $p_i^{NN} = p_i^{CC}$, $i \in \{1, 2\}$. Next, consider the comparison of hardware prices under an asymmetric compatibility regime with either incompatibility or full compatibility regime. Without loss of generality, under an asymmetric compatibility regime, suppose firm 1 chooses incompatibility and firm 2 chooses compatibility. In this market regime, in addition to subsidization of the hardware to attract users for the advertisers, firm 2 further subsidizes hardware to attract multi-product users because its hardware has fewer applications, and thus reduces its hardware price below the hardware price under an incompatibility or compatibility regime, i.e., $p_2^{NC} < p_2^{NN} = p_2^{CC}$. This also results in firm 1 reducing its hardware price below the one charged under incompatibility or compatibility regime so as to compete effectively with firm 2, i.e., $p_1^{NC} < p_1^{NN} = p_1^{CC}$. Also, since firm 1's hardware has more applications available on it than firm 2's hardware, the former is perceived as a better quality product by the multi-product users. This, in turn, implies that firm 1 reduces its hardware price by a smaller amount than firm 2 to attract multi-product users, i.e., $p_1^{NC} > p_2^{NC}$. Our result contrasts with previous work (e.g., Adner et al., 2020; Maruyama and Zennyo, 2015). Under an asymmetric compatibility regime with independent preferences for hardware and software, Adner et al. (2020) show that both incompatible and compatible platforms charge higher hardware prices relative to the incompatibility regime. Maruyama and Zennyo (2015) also show that hardware prices are lowest under the incompatibility regime. In contrast to these studies, we show that hardware prices are lowest under an asymmetric compatibility regime. This results stems from a novel aspect of our model, i.e., multi-product users have a preference for a variety of application content.

Now, consider application prices. In determining the optimal application pricing strategy, there are two forces at play. First, charging a higher price increases the surplus extracted from each multi-product user. However, it also reduces the number of multi-product users consuming the firm's application. The optimal content price is determined by the strength of these two forces, which, in turn, depends on the market regime. Under an incompatibility regime, a firm optimizes its profit by attracting the maximum possible multi-product users by charging zero application prices and monetizing them through hardware sales. This is because demand for applications from multi-product users is sufficiently elastic, leading to a loss in revenue from increasing and charging a positive application price. Thus, we have application prices, $r_1^{NN} = r_2^{NN} = 0$, to be lower than application prices under both full compatibility and asymmetric compatibility regimes. Next, consider the comparison of application prices under asymmetric compatibility and full compatibility regimes. Without loss of generality, under an asymmetric compatibility regime, suppose firm 1 chooses incompatibility and firm 2 chooses compatibility. The demand for applications is inelastic under the full compatibility regime because all multi-product users consume both applications, whereas it is relatively elastic for firm 1 and still inelastic for firm 2 under the asymmetric compatibility regime. Therefore, under the full compatibility regime, firm 1 charges the maximum possible application price to extract the maximum possible surplus per multi-product user, and thus, its application price under the full compatibility regime is higher than under the asymmetric compatibility regime, i.e., $r_1^{CC} > r_1^{NC}$. Firm 2 can charge the maximum possible application price to extract the maximum possible surplus per multi-product user under both regimes. However, since hardware prices are lower under the asymmetric compatibility regime relative to the full compatibility regime, a multi-product user is left with a higher surplus under the former regime relative to the latter regime. This, in turn, implies that firm 2 can charge a higher application price under the asymmetric compatibility regime relative to the full compatibility regime, i.e., $r_2^{NC} > r_2^{CC}$. Finally, consider advertising prices. Given that advertisers are homogeneous, in equilibrium, advertising prices remain unchanged across all market regimes.

Now, we analyze the potential profits for each of firms' choices at *Stage 1*, when the firms decide whether or not to choose compatibility. The matrix below summarizes the firms' profits associated with each decision.

		Firm 2	
		Incompatibility	*Compatibility*
Firm 1	*Incompatibility*	$(\pi_1^{NN},\ \pi_2^{NN})$	(π_1^{NC}, π_2^{NC})
	Compatibility	(π_1^{CN}, π_2^{CN})	(π_1^{CC}, π_2^{CC})

Table 2: Payoff matrix

Note that the expression for firms' profits π_i^{NN}, π_i^{CN}, π_i^{NC} and π_i^{CC} are defined in Lemmas 1, 2 and 3. The comparison of profits are summarized in the following proposition and illustrated in Figure 1.

Proposition 2 (Comparison of Profits)**.** *There exist thresholds* $F_1(\alpha)$*, and* $F_2(\alpha)$*, with* $F_1(\alpha) \geq F_2(\alpha)$ *and* $\partial F_1(\alpha)/\partial\alpha > \partial F_2(\alpha)/\partial\alpha > 0$*, such that in a market characterized by compatibility cost* (F) *and a fraction of multi-product users* (α)*, the following holds.*

(i) For sufficiently small compatibility cost, i.e., $F < F_2(\alpha)$*, both firms choose compatibility, i.e., regime* CC*, is the equilibrium outcome. Whereas, for sufficiently large compatibility cost, i.e.,* $F \geq F_1(\alpha)$*, both firms choose incompatibility, i.e., regime* NN*, is the equilibrium outcome.*

(ii) Otherwise, there exist multiple equilibria with either firm 1 choosing incompatibility and firm 2 choosing compatibility, i.e., regime NC*, or firm 1 choosing compatibility and firm 2 choosing incompatibility, i.e., regime* CN*, as the equilibrium outcome.*

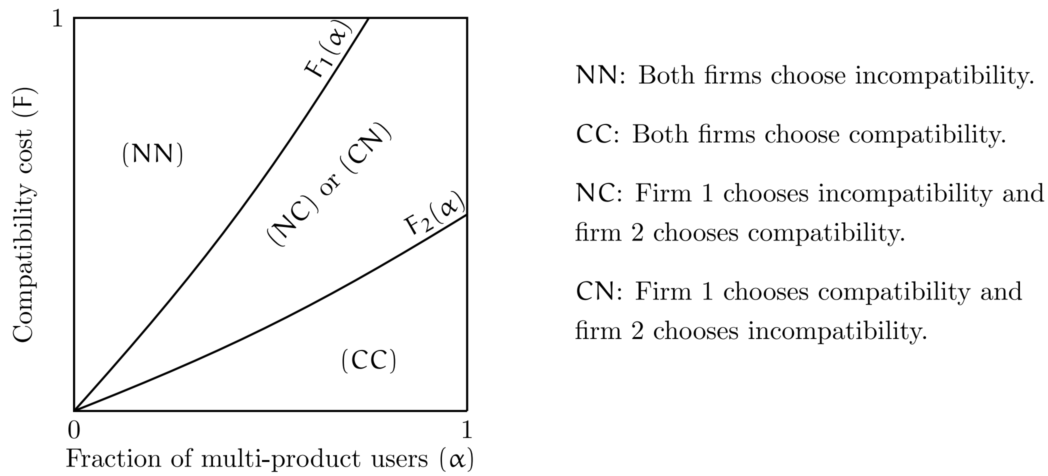


Figure 1: Optimal compatibility regimes under market equilibrium

The figure is based on parameter values $V = 1.2$, $W = 1$ $\delta = 0.3$, $q = 0.6$ and $t = 1$. The threshold $F_1(\alpha)$ ($F_2(\alpha)$) represents the loci of points along which a firm is indifferent between choosing compatibility and incompatibility when it believes that rival has chosen incompatibility (compatibility).

Firm i's decision, $i \in \{1, 2\}$, to make its application compatible with rival hardware depends on the effect of its decision on application revenue, hardware revenue, and advertising revenue. The forces that arise as a result of firm i choosing *compatibility* stem from consumers' preference for variety in our framework. First, demand for compatible firm i's application increases (positive effect). This raises the ability to charge a higher application because of inelastic application demand and the strength of price increase depends on rival firm j's decision. Overall, application revenue always increases with compatibility. Second, perceived hardware quality of compatible firm i changes, which in turn, depends on rival firm j's compatibility choice. If rival firm j chooses incompatibility, then the compatible firm i's hardware is perceived of lower quality because of fewer applications on it, decreasing demand and price for firm i's hardware, and thus, hardware revenue decreases. If rival firm j chooses compatibility, then the compatible firm i's perceived hardware quality difference vanishes because both devices have all applications, decreasing hardware demand, but increasing price of firm i's hardware. However, the net hardware revenue decreases. Third, perceived change in compatible firm i's hardware quality also changes advertising revenue earned because of its effect on hardware demand. Since hardware demand always decreases, irrespective of rival firm j's decision, advertising revenue decreases with compatibility (negative effect). Finally, if firm i chooses compatibility, then it incurs a compatibility cost F to make application A_i compatible with rival hardware H_j. Therefore, the optimal compatibility decision of firm $i \in \{1, 2\}$ depends on the strength of these trade-offs, which in turn, depends on the levels of compatibility costs (F) and fraction of multi-product user (α).

To evaluate firms' compatibility decisions, we compare the increase in application revenue against the decrease in hardware and advertising revenues, and the compatibility cost associated with choosing compatibility. For sufficiently small compatibility cost F, i.e., $F < F_2(\alpha)$, if firm i chooses compatibility, then the increase in application revenue is sufficient to compensate for the decrease in hardware and advertising revenues, and compatibility cost. Also, this holds true, irrespective of its belief about the compatibility decision of rival firm j, $j \neq i$. Thus, firm i's profit increases with adopting compatibility, irrespective of its belief about the rival's compatibility decision, i.e., $\pi_i^{CN} > \pi_i^{NN}$ or $\pi_i^{CC} > \pi_i^{NC}$. As a result, each firm's decision to choose compatibility is a dominant strategy, irrespective of the rival's decision, leading to regime CC with both firms choosing compatibility as the equilibrium outcome. For sufficiently large compatibility cost F, i.e., $F \geq F_1(\alpha)$, if firm i chooses compatibility, then the increase in application revenue is insufficient to compensate for the decrease in hardware and advertising revenues, and compatibility cost. Also, this holds true, irrespective of its belief about the compatibility decision of rival firm j, $j \neq i$. Thus, firm i's profit decreases with adopting compatibility, irrespective of its belief about the rival's compatibility decision, i.e., $\pi_i^{CN} < \pi_i^{NN}$ or $\pi_i^{CC} < \pi_i^{NC}$. As a result, each firm's decision to choose incompatibility is a dominant strategy, irrespective of the rival's decision, leading to regime NN with both firms choosing incompatibility as the equilibrium outcome.

Otherwise, for $F_2(\alpha) \leq F < F_1(\alpha)$, the decision of firm i to choose compatibility depends upon its belief about the decision of rival firm j, $j \neq i$. This is because the strength of changes in revenue from choosing compatibility depends upon its belief about the rival's decision. In this case, firms' profits are affected asymmetrically by increased application demand and the perceived quality difference associated with compatibility. If firm i believes that rival firm j chooses incompatibility, then the increase in application revenue is sufficient to compensate for the decrease in hardware and advertising revenues, and compatibility cost. Therefore, if firm i chooses compatibility, its profit increases relative to the case when it chooses incompatibility, i.e., $\pi_i^{CN} > \pi_i^{NN}$, and, thus, it chooses compatibility. If firm i believes that rival firm j chooses compatibility, then the increased application revenue is insufficient to compensate for the decrease in hardware and advertising revenues, and compatibility cost. Therefore, if firm i chooses compatibility, its profit decreases relative to the case when it chooses incompatibility, i.e., $\pi_i^{CC} < \pi_i^{NC}$, and, thus, it chooses incompatibility. Therefore, we have multiple equilibria, with either firm 1 choosing incompatibility and firm 2 choosing compatibility, i.e., regime NC, or firm 1 choosing compatibility and firm 2 choosing incompatibility, i.e., regime CN.

Interestingly, our result on asymmetric compatibility in platform markets with ex-ante symmetric firms is not seen in the extant literature examining platform compatibility. Adner et al. (2020) examines compatibility decisions of vertically differentiated platforms and shows that asymmetric compatibility can arise only for sufficiently large differences in hardware qualities; otherwise, either full compatibility or incompatibility is the equilibrium outcome. Moreover, recently, in markets without network effects, Innocenti and Menicucci (2021) show that asymmetric compatibility can never arise as an equilibrium outcome when firms are ex-ante symmetric. In contrast, we show that even with ex-ante symmetric firms, asymmetric compatibility may arise as an equilibrium. In contrast to these studies, we introduce a *new mechanism*, i.e., a preference for variety. This, in turn, generates two distinct effects for a compatible firm: application demand expansion and vertical differentiation. For intermediate compatibility cost, the two firms exploit preference for variety asymmetrically. Facing an incompatible firm, the compatible firm improves profit by increasing demand for its application, and thus, higher application revenue offsets the decrease in hardware and advertising revenue, and the compatibility cost associated with choosing compatibility. Facing a compatible firm, the incompatible firm improves its profit by creating a vertical hardware differentiation, generating higher hardware demand, and thus, higher hardware and advertising revenues.

Next, we examine the impact of market parameters, i.e., t, q, and δ, on the compatibility strategies of the firms.

Proposition 3 (Comparative Statics). *(i) In markets with higher (lower) per-unit transportation cost* t*,* (a) *the region where both firms prefer incompatibility, i.e., regime* NN*, is larger (smaller), and* (b) *the region where one firm chooses compatibility and the other firm chooses incompatibility, i.e., regime* NC *or* CN*, is smaller (larger).*

(ii) In markets with higher (lower) per-user advertising revenue q*,* (a) *the region where both firms prefer incompatibility, i.e., regime* NN*, is smaller (larger), and* (b) *the region where one firm chooses compatibility and the other firm chooses incompatibility, i.e., regime* NC *or* CN*, is larger (smaller).*

(iii) In markets with higher (lower) per-unit nuisance cost δ*,* (a) *the region where both firms prefer incompatibility, i.e., regime* NN*, is larger (smaller), and* (b) *the region where one firm chooses compatibility and the other firm chooses incompatibility, i.e., regime* NC *or* CN*, is smaller (larger).*

The intuition behind the preceding proposition is as follows. Consider the impact of per-unit transportation cost t. An increase in t increases the strength of user preferences for the hardware devices. As a result, competition between firms to attract users for hardware becomes less intense. This, in turn, implies that hardware revenue becomes more significant in firms' profit foci. As a result, each firm is more likely to choose incompatibility, irrespective of the belief about the rival firm's decision. This is because choosing compatibility leads to a change in perceived hardware quality difference, which decreases hardware revenue. Therefore, both firms choose incompatibility for a larger range of parameter values. Moreover, the region over which firms' profits are affected asymmetrically by increased application demand

and the perceived quality difference associated with compatibility becomes smaller because hardware revenue becomes more significant in firms' profit foci. As a result, the region with an asymmetric compatibility regime becomes smaller.

Consider the impact of per-user advertising revenue q. An increase in q has three distinct effects on firms' profits. First, advertising revenue increases. Second, hardware prices reduce, because competition among firms to attract multi-product users for advertisers becomes more intense. Third, users are left with more surplus (because of lower hardware prices), which a compatible firm can extract by charging a higher application price. This, in turn, implies that hardware revenue becomes less significant in firms' profit foci, whereas advertising revenue and application revenue become more significant in firms' profit foci. As a result, each firm is less likely to choose incompatibility, irrespective of the belief about the rival firm's decision. Therefore, both firms choose incompatibility for a smaller range of parameter values. Moreover, the region over which firms' profits are affected asymmetrically by increased application demand and the perceived quality difference associated with compatibility becomes larger because hardware revenue becomes less significant in firms' profit foci, with incompatible firm focusing on advertising revenue and the compatible firm focusing on application revenue. As a result, the region with an asymmetric compatibility regime becomes larger.

Finally, consider the impact of per-unit nuisance cost δ. An increase in δ reduces the surplus of multi-product users from consuming hardware. This, in turn, implies that a compatible firm charges a lower application price. Thus, hardware revenue becomes more significant in firms' profit foci, whereas application revenue becomes less significant in firms' profit foci. As a result, each firm is more likely to choose incompatibility (to keep its hardware device differentiated), irrespective of the belief about the rival firm's decision. Therefore, both firms choose incompatibility for a larger range of parameter values. Moreover, the region over which firms' profits are affected asymmetrically by increased application demand and the perceived quality difference associated with compatibility becomes smaller because hardware revenue becomes more significant in firms' profit foci. As a result, the region with an asymmetric compatibility regime becomes smaller.

4.5 Implications for Managers

4.5.1 How should firms price their hardware and applications?

First, we provide guidance on how a compatible firm should price its applications. As our results show, if a firm chooses compatibility, the application pricing crucially depends on the rival's strategy. When facing an incompatible firm, it should implement a higher application price relative to the case when it faces a compatible firm. This insight is important given that there is an emergence of both full compatibility and asymmetric compatibility regimes in digital markets. Another implication of our result is that, in an asymmetric compatibility regime, firms can exploit different revenue components by reducing hardware prices relative to other regimes, and then capturing value through the pricing of advertisements and applications. Thus, we should observe a decline in hardware revenue in these regimes. For instance, Meta revenues from virtual reality declined to 1.9 billion U.S. dollars in 2023 down from 2.1 billion USD in 2022.[9]

4.5.2 How should firms choose their compatibility strategies?

Based on our results, whether firms should compete or cooperate depends on the opportunity to exploit different revenue streams. This, in turn, depends on how firms can exploit preference for variety behavior. In markets with sufficiently small or large compatibility cost, the opportunity to capture value through different revenue components is lacking, and the preference for variety is exploited by firms employing similar strategies to generate either greater hardware revenues or non-hardware revenues. However, with intermediate compatibility costs, firms can exploit consumers' preference for variety behaviour asymmetrically,

[9]See "Annual revenue generated by Meta Reality Labs segment from 2019 to 2023," *Statista*, available at https://www.statista.com/statistics/1290133/meta-reality-labs-annual-revenue (accessed on 20-04-2024).

and the opportunity to capture value through different revenue components arises. As argued above, in such markets with asymmetric compatibility, managers can shift from hardware revenue to application or advertising revenue to improve firm profit. Based on our results, in an asymmetric compatibility regime, the compatible firm focuses on application revenue, whereas the incompatible firm focuses on advertising revenue. Moreover, both firms' hardware revenue is lower relative to other regimes. This insight is relevant given that, in many digital markets, technology firms that are becoming asymmetrically compatible. Recently, Microsoft made its Xbox Cloud Gaming application compatible with Meta's Oculus headset, whereas Meta's VR games are still incompatible with Microsoft's Hololens headset. This strategy is in line with the revenue focus of the two technology firms, with Meta focusing on improving advertising revenue[10] and Microsoft focusing on improving its application revenue (e.g., from VR games, productivity software) in the VR space.[11]

Another important managerial insight is that as the cost of compatibility decreases, firms' responses can vary depending on the level of compatibility costs. This insight is important given the increased regulatory focus on reducing compatibility costs, for instance, through the adoption of open, standardized, and well-documented APIs.[12] For sufficiently large compatibility costs, firms' profit foci can change with a decrease in compatibility cost, and both firms are better off in an asymmetric compatibility regime. For intermediate compatibility costs, a decrease in compatibility cost will have symmetric effects on firms' profit foci, and both firms can benefit by choosing compatibility and improving hardware and advertising revenue.

[10]See "Here's how Zuckerberg thinks Facebook will profit by building a metaverse," *CNBC*, available at https://www.cnbc.com/2021/07/29/facebook-metaverse-plans-to-make-money.html (accessed on 19-04-2024).

[11]See "Analysis of the metaverse strategies of Meta and Microsoft," *Design4real*, available at https://design4real.de/en/the-differences-between-the-metaverse-concept-of-microsoft-and-meta/ (accessed on 20-04-2024).

[12]See, "Building a European Data Economy," *European Commission*, available at https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=COM:2017:9:FIN (accessed on 22-04-2024).

5 Welfare Analysis and Policy Implications

5.1 Welfare Analysis

In this section, we examine and compare social welfare among different market regimes. Social welfare is defined as the sum of users' surplus, advertisers' profit and firms' profit. Since prices are just transfers in the model, it equals the sum of gross users' surplus from consumption of hardware devices, the sum of gross multi-product users' surplus from consumption of applications, total transportation costs, and net advertisers' surplus (net of nuisance costs). To proceed, we define a socially optimal outcome as the market regime which maximizes social welfare. The following proposition summarizes the main results on welfare analysis and is illustrated in Figure 2.

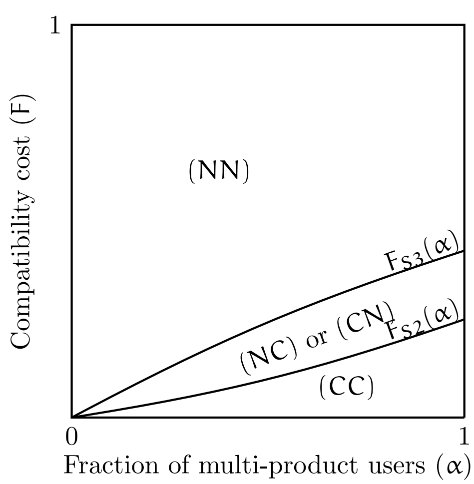
Proposition 4 (Comparison of Social Welfare)**.** *There exist thresholds* $\mathsf{F}_{\mathsf{S2}}(\alpha)$ *and* $\mathsf{F}_{\mathsf{S3}}(\alpha)$*, with* $0 \leq \mathsf{F}_{\mathsf{S2}}(\alpha) \leq \mathsf{F}_{\mathsf{S3}}(\alpha)$*,* $\partial \mathsf{F}_{\mathsf{S2}}(\alpha)/\partial\alpha > 0$*, and* $\partial \mathsf{F}_{\mathsf{S3}}(\alpha)/\partial\alpha > 0$*, such that in a market characterized by compatibility cost* (F) *and a fraction of multi-product users* (α)*, the following holds.*

(i) For a sufficiently small compatibility cost, i.e., $0 \leq \mathsf{F} < \mathsf{F}_{\mathsf{S2}}(\alpha)$*, full compatibility regime* CC *is socially optimal.*

(ii) For an intermediate compatibility cost, i.e., $\mathsf{F}_{\mathsf{S2}}(\alpha) \leq \mathsf{F} < \mathsf{F}_{\mathsf{S3}}(\alpha)$*, asymmetric compatibility regimes* NC *and* CN *are socially optimal.*

(iii) For a sufficiently large compatibility cost, i.e., $\mathsf{F} \geq \mathsf{F}_{\mathsf{S3}}(\alpha)$*, incompatibility regime* NN *is socially optimal.*

In order to explain the intuition behind the preceding proposition, four important observations must be highlighted. First, the sum of gross multi-product users' surplus from applications is the largest under full compatibility regime CC. Second, total compatibility costs are also the largest under full compatibility regime CC. Third, total transportation costs are lower under symmetric full compatibility regime CC and incompatibility regime NN relative to asymmetric compatibility regimes NC and CN. Fourth, net advertising surplus remains the same across all regimes. Therefore, for a sufficiently small compatibility cost, i.e., $0 \leq F < F_{s2}(\alpha)$, full compatibility regime maximizes gross multi-product users' surplus from applications, and together with weakly lower transportation costs relative to other regimes, maximizes social welfare. However, for a sufficiently large compatibility cost, i.e., $F \geq F_{s3}(\alpha)$, social welfare under regimes CC, NC, and CN are lower relative to the incompatibility regime NN, and thus, incompatibility regime NN maximizes social welfare. For intermediate compatibility cost, i.e., $F_{S2}(\alpha) \leq F < F_{S3}(\alpha)$, social welfare under full compatibility regime CC is still lower relative to incompatibility regime NN because of large total compatibility costs in the former case relative to the latter. However, higher gross multi-product users' surplus from applications under asymmetric compatibility regimes NC and CN offsets higher total transportation costs and compatibility costs, thus, increasing social welfare relative to the incompatibility regime NN. Therefore, in this case, asymmetric compatibility regime NC or CN maximizes social welfare.



NN: Both firms choosing incompatibility is socially optimal.

CC: Both firms choose compatibility is socially optimal.

NC: Firm 1 chooses incompatibility and firm 2 chooses compatibility is socially optimal.

CN: Firm 1 chooses compatibility and firm 2 chooses incompatibility is socially optimal.

Figure 2: Socially optimal compatibility regimes

The figure is based on parameter values $V = 1.2$, $W = 1$ $\delta = 0.3$, $q = 0.6$ and $t = 1$. The threshold $F_{S2}(\alpha)$ ($F_{S3}(\alpha)$) represents the loci of points along which regime CC (regime NN) and regime NC or CN yield the same welfare.

Next, we examine the impact of market parameters, i.e., t, α, and W, on the socially optimal compatibility regimes.

Proposition 5. *Asymmetric compatibility regime as the socially optimal outcome is more (less) likely in markets with* (a) *small (large) per-unit transportation cost* t*,* (b) *large (small) intrinsic value of application* W*, and* (c) *large (small) fraction of multi-product users* α*.*

An increase in per-unit transportation cost t increases total transportation costs. This, in turn, has two effects. First, for larger values of F, asymmetric compatibility regimes NC and CN lead to higher total transportation costs and compatibility costs, relative to the incompatibility regime NN, thus shifting threshold $F_{S3}(\alpha)$ downward. For smaller values of F, similar effects lead to lower social welfare under asymmetric compatibility regimes NC and CN relative to the incompatibility regime CC, thus shifting threshold $F_{S2}(\alpha)$ upward. Therefore, the region over which asymmetric compatibility regime is the socially optimal outcome becomes smaller.

Next, consider an increase in the intrinsic value of application W or an increase in the fraction of multi-product users α. This implies that gross users' surplus from consuming applications increases. First, for larger values of F, asymmetric compatibility regimes NC and CN lead to a higher gross users' surplus from consuming applications relative to the incompatibility regime NN, thus shifting threshold $F_{S3}(\alpha)$ upward. However, for smaller values of F, gross users' surplus under asymmetric compatibility regimes NC and CN, can either offset or be offset by total transportation costs and compatibility costs, relative to the incompatibility regime CC. This implies that threshold $F_{S2}(\alpha)$ can either shift downward or upward. In net, as shown in the appendix, the upward shift in threshold $F_{S3}(\alpha)$ is larger than the shift in threshold $F_{S2}(\alpha)$. Therefore, the region over which asymmetric compatibility regime is the socially optimal outcome becomes larger.

Our results on socially optimal compatible choices contrast with the existing work on platform compatibility (e.g., Adner et al., 2020; Maruyama and Zennyo, 2015). First, unlike Adner et al. (2020), we show that full compatibility is not always desirable. An asymmetric compatibility regime can be socially optimal for small to intermediate compatibility costs.

Moreover, unlike Maruyama and Zennyo (2015), we find that even for sufficiently small compatibility costs, an asymmetric compatible regime can be socially optimal for a small to intermediate fraction of multi-product users. Finally, distinct from Maruyama and Zennyo (2015), in our paper, the likelihood of asymmetric compatibility increases with a decrease in per-unit transportation cost t, and an increase in the intrinsic value of the application W.

5.2 Comparison of Market and Socially Optimal Outcome

We now compare the market and socially optimal outcomes and examine whether firms' strategic compatibility decisions are socially optimal. Since a closed-form solution cannot be derived, we conduct a graphical analysis, illustrated in Figures 3(a) and 3(b) and draw important conclusions. First, in markets with sufficiently large or small compatibility costs, market outcome coincides with social optimum. Second, for small to intermediate compatibility costs, there can be a (mis)alignment of incentives of firms and policymakers (maximizing social welfare) regarding the optimal compatibility regimes. Based on graphical analysis, we find that there can be socially excessive compatibility. However, the nature of comparison depends on the level of compatibility costs. First, consider $F_{S2}(\alpha) \leq F < F_2(\alpha)$. Intuitively, firms' profits are affected symmetrically by compatibility decisions. Thus, both firms choose compatibility to increase application revenue, i.e., regime CC is the equilibrium outcome. However, for the same range of parameter values, from a social point of view, either asymmetric compatibility, i.e., regime CN or NC (refer Figure 3 (a)), or incompatibility regime NN (refer Figure 3 (b)), maximizes social welfare. Next, consider either $F_{S3}(\alpha) \leq F < F_1(\alpha)$ (refer Figure 3 (a)) or $F_2(\alpha) \leq F < F_1(\alpha)$ (refer Figure 3 (b)). Firms' profits are affected asymmetrically by compatibility decisions and depend on the rival's compatibility choice. Thus, firms make asymmetric compatible decisions with one firm choosing compatibility and the other firm choosing incompatibility, i.e., regime NC or CN, is the equilibrium outcome. However, for the same range of parameter values, from a social point of view, compatibility costs are large, and regime NN maximizes social welfare.

Another important insight is that the region over which there is socially excessive compatibility depends on the market conditions (e.g., fraction of multi-product users, per-unit transportation cost, per-user advertising benefit, per-unit nuisance cost). We consider the role of firms net advertising surplus (net of nuisance costs), i.e., $q - \delta$ on the likelihood of socially excessive compatibility. First, consider markets with sufficiently small per-user advertising revenue or sufficiently large per-unit nuisance cost. In such markets, firms' net advertising surplus (net of nuisance costs), i.e., $q - \delta$, is small (refer Figure 3(a)). Therefore, firms' profits are affected asymmetrically by compatibility decisions for a smaller range of compatibility costs, and asymmetric compatibility is less likely. This, in turn, implies that the region over which there is socially excessive compatibility is smaller, as shown in Figure 3(a). Next, consider markets with sufficiently large per-user advertising revenue or sufficiently small per-unit nuisance cost. In such markets, firms' net advertising surplus (net of nuisance costs), i.e., $q - \delta$, is large (refer Figure 3(b)). Therefore, firms' profits are affected asymmetrically by compatibility decisions for a larger range of compatibility costs, and asymmetric compatibility is more likely. This, in turn, implies that the region over which there is socially excessive compatibility is larger, as shown in Figure 3(b).

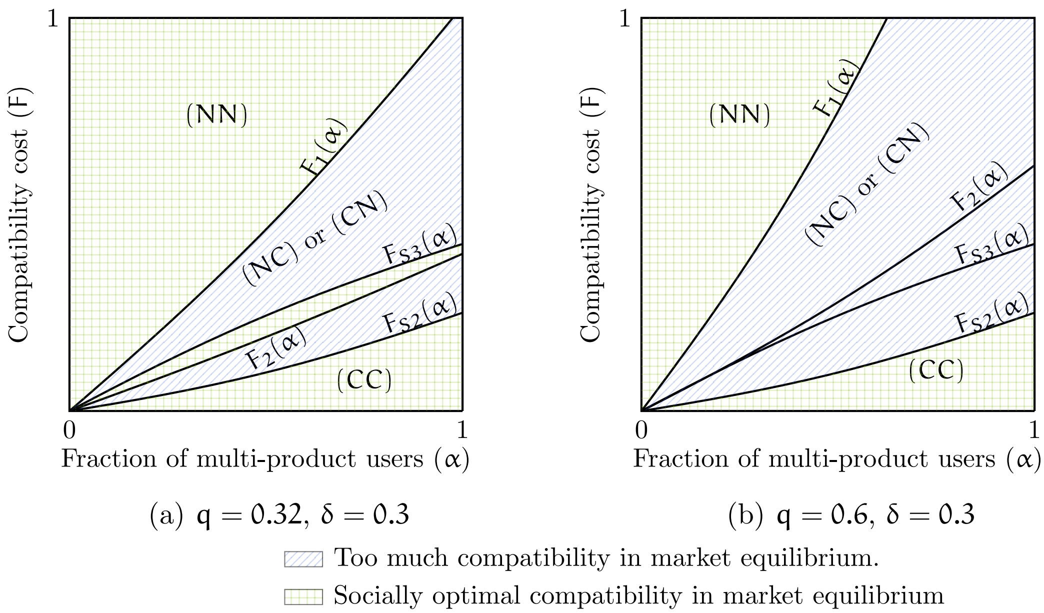


Figure 3: Comparison of optimal compatible regimes under market equilibrium and social optimum

The figure is based on parameter values $V = 1.2$, $W = 1$, and $t = 1$. The thresholds $F_1(\alpha)$ and $F_2(\alpha)$ ($F_{S2}(\alpha)$ and $F_{S3}(\alpha)$) are as defined in Proposition 2 (Proposition 4).

5.3 Policy Implications

5.3.1 Is full compatibility always desirable?

There has been an increased focus on regulating platforms and, in particular, implementing mandated compatibility rules (e.g., Digital Markets Act) to increase platform competition. However, our analysis shows that in markets where compatibility costs are intermediate, consumer preferences for hardware are weak, and the intrinsic value of application usage is large, full compatibility is not always desirable. In such markets (with intermediate compatibility costs) a regulator can better achieve regulatory objectives by implementing asymmetric compatibility.

5.3.2 In which markets, regulatory intervention is required?

Based on our analysis, regulatory intervention is required in markets with intermediate compatibility costs. However, there is no universally applicable regulatory intervention. This is because market conditions crucially determine the difference between firms' strategic choices and socially optimal choices. Our analysis shows that a stringent full compatibility regulation that requires all firms' applications to be compatible with their rival devices can be socially sub-optimal in most cases. For instance, a policy intervention mandating full compatibility can decrease social welfare in markets with intermediate to large compatibility costs. Therefore, the regulator needs to carefully examine market conditions and accordingly adopt compatibility policies, which is what we have attempted in this article.

Another important insight that our analysis can provide is on the unintended consequences of existing regulations in digital markets. This is important in the presence of the recent wave of other platform regulations to deal with different aspects of digital markets. For instance, there is a recent focus on strengthening user privacy and enactment of regulations (e.g., General Data Protection Regulations), which can significantly affect the surplus that a firm can generate from online advertising. Recent studies have shown that privacy regulations can reduce firms' advertising revenue (e.g., Wang et al., 2024). Our analysis shows that, in such markets, it is less likely that regulatory intervention can be required to deal with the issue of excessive compatibility, and, if required, it should, counter-intuitively, focus on taxing compatibility.

6 Conclusion

Co-opetition, a phenomenon involving simultaneous competition and cooperation, is becoming more prevalent among firms in digital markets. A key facet of co-opetitive behavior is when a digital platform makes its application compatible with the rival firm's hardware device. We examine the issue of application compatibility using a theoretical model with two firms, each having a hardware device and application, a set of users with a fraction of single-product users, and the remaining multi-product users, and a set of advertisers. We examine equilibrium market outcomes, their properties, and socially optimal compatibility choices. We find several interesting insights. First, hardware and application prices vary across compatibility regimes. Second, in markets with intermediate compatibility costs, firms can have different incentives to adopt compatibility, leading to asymmetric compatibility; otherwise, we have either symmetric full compatibility or incompatibility regimes. Third, in markets with intermediate compatibility costs, the compatibility choices in the market equilibrium can be misaligned with socially optimal compatible regimes, giving a rationale for regulatory intervention. We discuss insights based on our analysis and derive important managerial and policy implications.

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Appendix for "Application Compatibility in the Presence of Preference for Variety"

This appendix contains proofs of lemmas and propositions in the main text.

A Derivation of Demand Functions

In this section, we derive the demand functions for both the hardware and the application of each firm, in each of the possible market regimes: incompatibility regime NN, full compatibility regime CC, and asymmetric compatibility regime NC or CN.

Demand from single-product users $(1-\alpha > 0)$

In all regimes, a fraction $1-\alpha > 0$ of single-product users derive utility from consuming only hardware H_i, $i \in \{1,2\}$ and no utility from consuming application A_i, $i \in \{1,2\}$. That is, the single-product users compare the benefits and costs of H_1 and H_2 to make their decision. Using Equation (2), the single-product user indifferent between hardware H_1 and H_2 is located at $\hat{x}$, such that (i) $U_1(\hat{x}) = U_2(\hat{x})$, (ii) $U_1(x) > U_2(x)$ for all $x < \hat{x}$, and (iii) $U_1(x) < U_2(x)$ for all $x > \hat{x}$, where

$$\hat{x} \;=\; \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{\delta(a_2 - a_1)}{2t}. \tag{9}$$

Demand from multi-product users $(\alpha > 0)$

Incompatibility (NN)

When each firm $i \in \{1,2\}$ chooses to make its applications A_i incompatible with the rival firm's hardware $H_j, j \neq i$, a fraction $\alpha > 0$ of multi-product users can consume application A_i only through its parent firm's hardware H_i. Using Equation (3), the multi-product user indifferent between hardware H_1 and H_2 is located at $\bar{x}$, such that (i) $U_1(\bar{x}) = U_2(\bar{x})$, (ii) $U_1(x) > U_2(x)$ for all $x < \bar{x}$, and (iii) $U_1(x) < U_2(x)$ for all $x > \bar{x}$, where

$$\bar{x} \;=\; \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{r_2 - r_1}{2t} + \frac{\delta(a_2 - a_1)}{2t}. \tag{10}$$

Combining Equations (9) and (10) gives demand functions for hardware H_1 and H_2 as

$$
\begin{aligned}
N_1 &= (1-\alpha)\hat{x} + \alpha\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \alpha\frac{(r_2 - r_1)}{2t} + \frac{\delta(a_2 - a_1)}{2t}, \text{ and} \\
N_2 = 1 - N_1 &= \frac{1}{2} + \frac{p_1 - p_2}{2t} + \alpha\frac{(r_1 - r_2)}{2t} + \frac{\delta(a_1 - a_2)}{2t}.
\end{aligned}
\tag{11}
$$

Using Equation (10) gives demand for applications A_1 and A_2 as

$$
\begin{aligned}
D_1 &= \alpha\bar{x} = \alpha\left[\frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{r_2 - r_1}{2t} + + \frac{\delta(a_1 - a_2)}{2t}\right], \text{ and} \\
D_2 &= \alpha(1-\bar{x}) = \alpha\left[\frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{r_1 - r_2}{2t} + \frac{\delta(a_2 - a_1)}{2t}\right].
\end{aligned}
\tag{12}
$$

Full compatibility (CC)

When each firm $i \in \{1, 2\}$ chooses to make its applications A_i compatible with the rival firm's hardware H_j, $j \neq i$, a fraction $\alpha > 0$ of multi-product users can consume application A_i through either hardware H_i or H_j. Using Equation (4), the multi-product user indifferent between hardware H_1 and H_2 is located at $\bar{x}$, such that (i) $U_1(\bar{x}) = U_2(\bar{x})$, (ii) $U_1(x) > U_2(x)$ for all $x < \bar{x}$, and (iii) $U_1(x) < U_2(x)$ for all $x > \bar{x}$, where

$$
\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{\delta(a_2 - a_1)}{2t}. \tag{13}
$$

Combining Equations (9) and (13) gives demand functions for hardware H_1 and H_2 as

$$
\begin{aligned}
N_1 &= (1-\alpha)\hat{x} + \alpha\bar{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{\delta(a_2 - a_1)}{2t}, \text{ and} \\
N_2 &= 1 - N_1 = \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\delta(a_1 - a_2)}{2t},
\end{aligned}
\tag{14}
$$

and demand for application A_1 and A_2 are

$$
D_1 = D_2 = \alpha. \tag{15}
$$

Asymmetric compatibility (NC *or* CN)

Without loss of generality, suppose that firm 1 chooses to make its application A_1 incompatible with rival firm's hardware H_2, whereas firm 2 chooses to make its application A_2 compatible with rival firm's hardware H_1. This implies that a multi-product user can consume application A_1 only through H_1, but can consume application A_2 either through hardware H_1 or H_2. Using Equation (5), the multi-product user indifferent between hardware

H_1 and H_2 is located at $\bar{x}$, such that (i) $U_1(\bar{x}) = U_2(\bar{x})$, (ii) $U_1(x) > U_2(x)$ for all $x < \bar{x}$, and (iii) $U_1(x) < U_2(x)$ for all $x > \bar{x}$, where

$$\bar{x} \;=\; \frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{r_1}{2t} + \frac{W}{2t} + \frac{\delta(a_2 - a_1)}{2t}. \tag{16}$$

Using Equations (16), demand functions for hardware H_1 and H_2 are

$$\begin{aligned} N_1 \;&=\; (1-\alpha)\hat{x} + \alpha\bar{x} \;=\; \frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W}{2t} + \frac{\delta(a_1 - a_2)}{2t}, \text{ and} \\ N_2 \;&=\; 1 - N_1 \;=\; \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha r_1}{2t} - \frac{\alpha W}{2t} + \frac{\delta(a_2 - a_1)}{2t}. \end{aligned} \tag{17}$$

Combining Equations (9) and (16) gives demand functions for application A_1 and A_2 as

$$D_1 \;=\; \alpha\bar{x} \;=\; \alpha\left[\frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{r_1}{2t} + \frac{W}{2t} + \frac{\delta(a_2 - a_1)}{2t}\right], \text{ and } D_2 \;=\; \alpha. \tag{18}$$

This completes the derivation of demand functions of hardware and applications.

B Proofs of the Technical Results

Proof of Lemma 1

First, note that using the advertising demand function (defined by Equation (8)), we have that $s_i = q$ and $a_i = 1$. Now, using $s_i = q$ and $a_i = 1$ in Equation (1), the profit of firm $i \in \{1, 2\}$ is

$$\pi_i \;=\; p_i N_i + r_i D_i + q N_i.$$

Substituting demands from Equations (11) and (12) in the preceding profit functions, the first-order conditions to firm i's profit maximization are

$$\frac{\partial \pi_i}{\partial p_i} \;=\; \frac{1}{2} + \frac{p_j - p_i}{2t} + \alpha\frac{(r_j - r_i)}{2t} + p_i\left(\frac{-1}{2t}\right) + r_i\left(\frac{-\alpha}{2t}\right) + q\left(\frac{-1}{2t}\right) \;\leq\; 0, \text{ and} \tag{19}$$

$$\frac{\partial \pi_i}{\partial r_i} \;=\; p_i\left(\frac{-\alpha}{2t}\right) + \frac{\alpha}{2} + \alpha\frac{(p_j - p_i)}{2t} + \alpha\frac{r_j - r_i}{2t} + r_i\left(\frac{-\alpha}{2t}\right) + q\left(\frac{-1}{2t}\right) \;\leq\; 0, \tag{20}$$

where subscript $j \neq i$ denotes the rival firm. Since profit functions are strictly concave in hardware prices, the solution constitutes a unique maximum. Note that at $p_i = 0, r_i = 0$, we have $\frac{\partial \pi_i}{\partial p_i} > 0$ and $\frac{\partial \pi_i}{\partial r_i} > 0$, under the assumption of non-negative prices (Assumption 2). This implies that first-order conditions in Equations (19) and (20) bind and we have an interior solution. Solving first-order conditions in Equations (19) and (20), and imposing symmetry (since firms have symmetric objective functions), we obtain the optimal hardware prices as

$$p_1^{NN} = p_2^{NN} = t - q, \tag{21}$$

which are non-negative, given the assumption of non-negative prices (Assumption 2), and application prices are

$$r_1^{NN} = r_2^{NN} = 0. \tag{22}$$

The equilibrium demand for hardware H_i is $N_i = \frac{1}{2}, i \in \{1, 2\}$ and the equilibrium demand for application A_i is $D_i = \frac{\alpha}{2}, i \in \{1, 2\}$. Using Equation (1), the profits are $\pi_1^{NN} = \pi_2^{NN} = \frac{t}{2}$. This completes the proof. ∎

Proof of Lemma 2

Note that using the advertising demand function (defined by Equation (8)), we have that $s_i = q$ and $a_i = 1$. Using $s_i = q$ and $a_i = 1$ in Equation (1), the profit of firm $i \in \{1, 2\}$ is

$$\pi_i = p_i N_i + r_i D_i + q N_i - F.$$

Substituting demands from Equations (11) and (12) in the preceding profit functions, the first-order conditions to firm i's profit maximization are

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{2} + \frac{p_j - p_i}{2t} + p_i \left(\frac{-1}{2t}\right) + 0 + q\left(\frac{-1}{2t}\right) \leq 0, \text{ and} \tag{23}$$

$$\frac{\partial \pi_i}{\partial r_i} = \alpha > 0. \tag{24}$$

Since profit functions are strictly concave in hardware prices, the solution constitutes a unique maximum. Note that at $p_i = 0$, we have $\frac{\partial \pi_i}{\partial p_i} > 0$, under the assumption of non-negative prices (Assumption 2). This implies that first-order conditions in Equation (23) bind and we have an interior solution. Solving Equation (23), and imposing symmetry (since firms have symmetric objective functions), we obtain the optimal hardware prices as

$$p_1^{CC} = p_2^{CC} = t - q, \tag{25}$$

which are non-negative, given the assumption of non-negative prices (Assumption 2). Equation (24) shows that the profits are strictly increasing in application prices. Thus, firm $i \in \{1, 2\}$ sets r_i to extract as much consumer surplus as possible, while also ensuring that all multi-product users join the market (Assumption 1). In the Hotelling model, the indifferent multi-product user located at $\bar{x} \in (0, 1)$ obtains the lowest utility. This implies that firm i chooses r_i so that the indifferent multi-product user obtains a non-negative payoff from using the hardware-application bundle (so that consuming the bundle is better than the outside option of not purchasing anything gives zero payoff). Using individual rationality condition, the utility of indifferent multi-product user located at $\bar{x} \in (0, 1)$, $U(\bar{x}, H_1) = U(\bar{x}, H_2) = V + 2W - t + q - r_1^{CC} - r_2^{CC} - \frac{t}{2} - \delta = 0$, which gives

$$r_1^{CC} = r_2^{CC} = r^{CC} = \frac{V}{2} - \frac{3t}{4} + W + \frac{q - \delta}{2}, \tag{26}$$

which are non-negative given the assumption of fully covered market (Assumption 1). The equilibrium demand for hardware H_i is $N_i = \frac{1}{2}, i \in \{1, 2\}$ and the equilibrium demand for application A_i is $D_i = \alpha, i \in \{1, 2\}$. Using Equation (1), the profits are $\pi_1^{CC} = \pi_2^{CC} = \frac{t}{2} + \alpha \left[\frac{V}{2} - \frac{3t}{4} + W + \frac{q-\delta}{2}\right] - F$. This completes the proof. ■

Proof of Lemma 3

Without loss of generality, suppose that firm 1 chooses to make its application A_1 incompatible with rival firm's hardware H_2, whereas firm 2 chooses to make its application A_2 compatible with rival firm's hardware H_1. Note that using the advertising demand function (defined by Equation (8)), we have that $s_i = q$ and $a_i = 1$. Using $s_i = q$ and $a_i = 1$ in Equation (1), the profit of firm $i \in \{1, 2\}$ is

$$\pi_1 = p_1 N_1 + r_1 D_1 + q N_1 \text{ and } \pi_2 = p_2 N_2 + r_2 D_2 + q N_2 - F.$$

Substituting demand from Equations (17) and (18) in the preceding profit functions, the first-order conditions to firm 1 and firm 2's profit maximization are

$$\frac{\partial \pi_1}{\partial p_1} = \frac{1}{2} + \frac{p_2 - p_1}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W}{2t} + p_1\left(\frac{-1}{2t}\right) + r_1\left(\frac{-\alpha}{2t}\right) + q\left(\frac{-1}{2t}\right) \leq 0, \tag{27}$$

$$\frac{\partial \pi_1}{\partial r_1} = p_1\left(\frac{-\alpha}{2t}\right) + \frac{\alpha}{2} + \alpha\frac{(p_2 - p_1)}{2t} - \frac{\alpha r_1}{2t} + \frac{\alpha W}{2t} + \alpha\left(\frac{-r_1}{2t}\right) + q\left(\frac{-\alpha}{2t}\right) \leq 0, \tag{28}$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2} + \frac{p_1 - p_2}{2t} + \frac{\alpha r_1}{2t} - \frac{\alpha W}{2t} + p_2\left(\frac{-1}{2t}\right) + 0 + q\left(\frac{-1}{2t}\right) \leq 0, \text{ and} \tag{29}$$

$$\frac{\partial \pi_2}{\partial r_2} = \alpha > 0. \tag{30}$$

Since the profit functions of firms 1 and 2 are concave in hardware and application prices, the solution constitutes a unique maximum. Note that at $p_1 = 0, r_1 = 0$, we have $\frac{\partial \pi_1}{\partial p_1} > 0$ and $\frac{\partial \pi_1}{\partial r_1} > 0$, and at $p_2 = 0$, we have $\frac{\partial \pi_2}{\partial p_2} > 0$, under the assumption of non-negative prices (Assumption 2), and $\frac{\partial \pi_2}{\partial r_2} > 0$ always,. This implies that first-order conditions in Equations (27), (28) and (29) bind and we have an interior solution. Solving Equations (27), (28) and (29), we obtain the optimal hardware H_1 price as

$$p_1^{NC} = t - q - \frac{\alpha W}{6}, \tag{31}$$

and the optimal hardware H_1 price as

$$p_2^{NC} = t - q - \frac{\alpha W}{3}, \tag{32}$$

which are non-negative, given the assumption of non-negative prices (Assumption 2). The optimal application A_1 price is

$$r_1^{NC} = \frac{W}{2}. \tag{33}$$

Equation (30) shows that the profit of firm 2 is strictly increasing in application price r_2. Thus, firm 2 sets r_2 to extract as much consumer surplus as possible, while also ensuring that all multi-product users join the market (Assumption 1). In the Hotelling model, the indifferent multi-product user located at $\bar{x} \in (0, 1)$ obtains the lowest utility. This implies that firm 2 chooses r_2 so that the indifferent multi-product user obtains a non-negative payoff from using the hardware-application bundle (so that consuming the bundle is better than the outside option of not purchasing anything which gives zero payoff). Using individual rationality condition, the utility of indifferent multi-product user located at $\bar{x} \in (0,1)$, $U(\bar{x}, H_1) = U(\bar{x}, H_2) = V - t + q + \frac{\alpha W}{3} + W - r_2^{NC} - t\left(\frac{1}{2} + \frac{\alpha W}{12t} + \frac{W}{4t} - \frac{W}{2t}\right) - \delta = 0$, which gives

$$r_2^{NC} = V - \frac{3t}{2} + \frac{\alpha W}{4} + \frac{5W}{4} + q - \delta. \tag{34}$$

The equilibrium demand is obtained by substituting Equations (31), (32), and (33) in Equations (17) and (18) to get

$$N_1^{NC} = \frac{1}{2} + \frac{\alpha W}{6t}, N_2^{NC} = \frac{1}{2} - \frac{\alpha W}{6t};\ \ D_1^{NC} = \alpha\left[\frac{1}{2} + \frac{(3-\alpha)W}{12t}\right], \text{ and } D_2^{NC} = \alpha.$$

Using Equation (1), the profits are $\pi_1^{NC} = \frac{t}{2} + \frac{\alpha W}{3} + \frac{\alpha W^2}{8t} - \frac{5\alpha^2 W^2}{72t}$, and $\pi_2^{NC} = \frac{t(1-3\alpha)}{2} + \frac{11\alpha W}{12} + \frac{\alpha^2 W}{4} + \frac{\alpha^2 W^2}{18t} + \alpha(V + q - \delta) - F$. This completes the proof. ■

Proof of Proposition 1

We proceed through a series of steps. In *Step 1*, we compare the hardware prices under different market regimes to show that hardware prices are equal under incompatibility, and full compatibility regimes, and lower under asymmetric compatibility regimes relative to other regimes. Further, within the asymmetric compatibility regimes, the firm choosing to remain incompatible charges a higher hardware price. In *Step 2*, we compare the application prices under different market regimes to show that application prices are lowest under incompatibility regime. Moreover, under the asymmetric compatibility regimes, the firm choosing to make its application compatible (incompatible) charges a higher (lower) application price relative to the application price under the full compatibility regime.

Step 1: Comparison of hardware prices.

Using Equations (21) and (25), we find that the hardware prices are equal for both firms in the incompatibility and full compatibility regimes, i.e.,

$$p_1^{NN} = p_2^{NN} = p_1^{CC} = p_2^{CC} = t - q. \tag{35}$$

Using Equation (31) and (32), we compare the hardware prices of the firms in the asymmetric compatibility regimes, which gives

$$p_1^{NC} = p_2^{CN} = t - q - \frac{\alpha W}{6} \geq t - q - \frac{\alpha W}{3} = p_2^{NC} = p_1^{CN}. \tag{36}$$

Combining Equations (35) and (36), we have

$$p_1^{NN} = p_2^{NN} = p_1^{CC} = p_2^{CC} \geq p_1^{NC} = p_2^{CN} \geq p_2^{NC} = p_1^{CN}, \tag{37}$$

where the inequality holds for all values of $\alpha \in [0, 1]$, given the assumption of non-negative prices (Assumption 2).

Step 2: Comparison of application prices.

Using Equations (22), (33) and (34), we find that the application price that a firm set in the incompatibility regime is lower than the application price set by the firm in the asymmetric compatibility regimes, i.e.,

$$\begin{aligned} r_1^{NN} &= r_2^{NN} = 0 \leq \frac{W}{2} = r_1^{NC} = r_2^{CN}, \text{ and} \\ r_1^{NN} &= r_2^{NN} = 0 \leq V - \frac{3t}{2} + \frac{\alpha W}{4} + \frac{3W}{4} + q - \delta = r_1^{CN} = r_2^{NC}. \end{aligned}$$

Using Equations (26) and (33), we find that the application price set by the firm that chooses incompatibility in the asymmetric compatibility regimes is lower than the application price it sets in the full compatibility regime, i.e.,

$$r_1^{NC} = r_2^{CN} = \frac{W}{2} \leq \frac{V}{2} - \frac{3t}{4} + W + \frac{q-\delta}{2} = r_1^{CC} = r_2^{CC},$$

which requires

$$\frac{V}{2} - \frac{3t}{4} + \frac{W}{2} + \frac{q-\delta}{2} \geq 0,$$

where the preceding inequality holds, given the assumption of full market coverage (Assumption 1). Using Equations (26) and (34), we find that the application price that the firm sets in the full compatibility regime is lower than the application price it sets if it chooses compatibility in the asymmetric compatibility regime, i.e.,

$$r_1^{CC} = r_2^{CC} = \frac{V}{2} - \frac{3t}{4} + W + \frac{q-\delta}{2} \leq 2\left(\frac{V}{2} - \frac{3t}{4} + \frac{(5+\alpha)W}{8} + \frac{q-\delta}{2}\right) = r_1^{CN} = r_2^{NC},$$

which requires

$$\frac{V}{2}-\frac{3t}{4}+\frac{(1+\alpha)W}{4}+\frac{q-\delta}{2}\geq 0,$$

where the preceding inequality holds given the assumption of full market coverage (Assumption 1). Comparing the application prices across all market regimes, we have

$$r_1^{NN} = r_2^{NN} < r_1^{NC} = r_2^{CN} < r_1^{CC} = r_2^{CC} < r_2^{NC} = r_1^{CN}, \tag{38}$$

where the inequalities hold for all values of $\alpha \in [0,1]$, given the assumption of full market coverage (Assumption 1). This completes the proof. ■

Proof of Proposition 2

We proceed through a series of steps. In *Step 1*, we characterize the conditions for the adoption of compatibility by the firms. In *Step 2*, we compare the thresholds obtained from *Step 1*. In *Step 3*, we characterize firms' optimal strategies and show that there exists multiple equilibria.

Step 1: Conditions for choosing compatibility by the firms.

Suppose firm $i \in \{1,2\}$ believes that firm $j \neq i$ chooses incompatibility. Then, using Lemmas 1 and 3, we find that firm i is indifferent between choosing incompatibility and compatibility when compatibility cost is $F_1(\alpha)$, such that (i) $\pi_i^{NN}(F)=\pi_i^{CN}(F)$, at $F=F_1(\alpha)$, (ii) $\pi_i^{NN}(F)<\pi_i^{CN}(F)$, for all $F<F_1(\alpha)$, and (iii) $\pi_i^{NN}(F)>\pi_i^{CN}(F)$, for all $F>F_1(\alpha)$, where $F_1(\alpha)$ is obtained by solving

$$\frac{t}{2} = \frac{t(1-3\alpha)}{2}+\frac{11\alpha W}{12}+\frac{\alpha^2 W}{4}+\frac{\alpha^2W^2}{18t}+\alpha(V+q-\delta)-F_1(\alpha),$$

$$F_1(\alpha) = -\frac{3\alpha t}{2}+\frac{11\alpha W}{12}+\frac{\alpha^2 W}{4}+\frac{\alpha^2W^2}{18t}+\alpha(V+q-\delta). \tag{39}$$

In summary, $F_1(\alpha)$ is the threshold for compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to make its application incompatible.

Suppose firm $i \in \{1,2\}$ believes that firm $j \neq i$ chooses compatibility. Then, using Lemmas

2 and 3, we find that firm i is indifferent between choosing incompatibility and compatibility when its compatibility cost is $F_2(\alpha)$, such that (i) $\pi_i^{NC}(F) = \pi_i^{CC}(F)$, at $F = F_2(\alpha)$, (ii) $\pi_i^{NC}(F) < \pi_i^{CC}(F)$, for all $F < F_2(\alpha)$, and (iii) $\pi_i^{NC}(F) > \pi_i^{CC}(F)$, for all $F > F_2(\alpha)$, where $F_2(\alpha)$ is obtained by solving

$$\frac{t}{2} + \frac{\alpha W}{3} + \frac{\alpha^2 W^2}{8t} - \frac{5\alpha^2 W^2}{72t} = \frac{t}{2} + \alpha \left[\frac{V}{2} - \frac{3t}{4} + W + \frac{q-\delta}{2}\right] - F_2(\alpha),$$

$$F_2(\alpha) = \alpha \left[\frac{V}{2} - \frac{3t}{4} + \frac{q-\delta}{2}\right] + \frac{2\alpha W}{3} - \frac{\alpha W^2}{8t} + \frac{5\alpha^2 W^2}{72t}. \tag{40}$$

In summary, $F_2(\alpha)$ is the threshold for compatibility cost above which a firm prefers to switch from compatibility to incompatibility, given its belief that the rival firm chooses to makes its application compatible.

Step 2: Comparison of thresholds $F_1(\alpha)$ *and* $F_2(\alpha)$*.*

First, consider the threshold $F_1(\alpha)$. Using Equation (39), and differentiating $F_1(\alpha)$ with respect to α gives

$$\frac{\partial F_1(\alpha)}{\partial \alpha} = -\frac{3t}{2} + \frac{11W}{12} + \frac{\alpha W}{4} + \frac{\alpha W^2}{9t} + V + q - \delta > 0,$$

where the inequality holds for all values of $\alpha \geq 0$. Now, consider the threshold $F_2(\alpha)$. Using Equation (40), and differentiating $F_2(\alpha)$ with respect to α gives

$$\frac{\partial F_2(\alpha)}{\partial \alpha} = \frac{5W^2}{72t} > 0,$$

where the inequality holds for all values of $\alpha \geq 0$.

Claim 1. *For all* $\alpha \in [0, 1]$, $F_1(\alpha) \geq F_2(\alpha)$.

Proof. First note that, at $\alpha = 0$, using Equations (39) and (40), $F_1(\alpha = 0) = F_2(\alpha = 0) = 0$. Next, for $\alpha \in (0, 1]$, we have

$$\begin{aligned} F_1(\alpha) - F_2(\alpha) &= -\frac{3\alpha t}{2} + \frac{11\alpha W}{12} + \frac{\alpha^2 W}{4} + \frac{\alpha^2 W^2}{18t} + \alpha(V + q - \delta) \\ &\quad - \alpha \left[\frac{V}{2} - \frac{3t}{4} + \frac{q-\delta}{2}\right] - \frac{2\alpha W}{3} + \frac{\alpha W^2}{8t} - \frac{5\alpha^2 W^2}{72t}, \\ &= -\frac{3\alpha t}{4} + \frac{\alpha W}{4} + \frac{\alpha^2 W}{4} + \frac{\alpha W^2}{8t} - \frac{\alpha^2 W^2}{72t} + \frac{\alpha}{2}(V + q - \delta) \geq 0, \end{aligned}$$

where the final inequality holds, given the assumption of full market coverage (Assumption 1). □

Step 3: Optimal strategies and equilibrium outcomes.

Now, we use the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ on compatibility costs, to characterize firms' optimal strategies. For sufficiently small compatibility costs, i.e., $F < F_2(\alpha)$, it is optimal for a firm to choose compatibility, irrespective of its belief about the rival firm's choice. Thus, the dominant strategy for each firm is to choose compatibility. This implies that, at *Stage 1*, the Nash equilibrium - the firms' best responses to each other's individual optimal strategies - is given by both firms choosing compatibility, i.e., regime CC. For sufficiently large compatibility costs, i.e., $F \geq F_1(\alpha)$, it is optimal for a firm to choose incompatibility, irrespective of its belief about the rival firm's choice. Thus, the dominant strategy for each firm is to choose incompatibility. This implies that, at *Stage 1*, the Nash equilibrium is given by both firms choosing incompatibility, i.e., regime NN. For the intermediate range of compatibility costs, i.e., $F_2(\alpha) \leq F < F_1(\alpha)$, it is optimal for a firm to choose compatibility if it believes that its rival chooses incompatibility, and it is optimal for a firm to choose incompatibility if it believes that its rival chooses compatibility. This implies that, at *Stage 1*, we have two Nash equilibria: (i) firm 1 choosing incompatibility and firm 2 choosing compatibility, i.e., regime NC, or (ii) firm 1 choosing compatibility and firm 2 choosing incompatibility, i.e., regime CN. This completes the proof. ■

Proof of Proposition 3

Proposition 2 characterized the region in which each regime is preferred by firms, using thresholds $F_1(\alpha)$ and $F_2(\alpha)$ on compatibility costs that firms face. In this proposition, we examine the change in the parameter space where each regime is the equilibrium outcome, with respect to model parameters. We proceed through a series of steps. In *Step 1*, we characterize the sensitivity of the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ with respect to per-unit transportation cost t that users face. In *Step 2*, we characterize the sensitivity of the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ with respect to the per-user advertising revenue q to the advertisers. In *Step 3*, we characterize the sensitivity of the thresholds $F_1(\alpha)$ and $F_2(\alpha)$ with respect to the per-unit nuisance cost of an advertisement δ to users.

Step 1: Impact of per-unit transportation cost t *to users on the thresholds* $F_1(\alpha)$ *and* $F_2(\alpha)$.

First, consider threshold $F_1(\alpha)$. Using Equation (39), we have

$$\frac{\partial F_1(\alpha)}{\partial t} = -\frac{3\alpha}{2} - \frac{\alpha^2 W^2}{18t^2} \leq 0, \tag{41}$$

where the inequality holds for all values of $\alpha \geq 0$.

Next, consider threshold $F_2(\alpha)$. Using Equation (40), we have

$$\frac{\partial F_2(\alpha)}{\partial t} = -\frac{3\alpha}{4} + \frac{\alpha W^2}{8t^2} - \frac{5\alpha W^2}{72t^2} \leq 0, \tag{42}$$

where the inequality holds for all values of $\alpha \in [0, 1]$.

This shows that both thresholds on compatibility costs decrease with an increase in per-unit transportation cost t. Also, comparing Equations (41) and (42), we have

$$\left|\frac{\partial F_1(\alpha)}{\partial t} - \frac{\partial F_2(\alpha)}{\partial t}\right| = \frac{3\alpha}{4} + \frac{\alpha W^2}{8t^2} - \frac{\alpha^2 W^2}{72t^2}, \tag{43}$$

Claim 2. $\frac{\partial\left|\frac{\partial F_1(\alpha)}{\partial t} - \frac{\partial F_2(\alpha)}{\partial t}\right|}{\partial \alpha} \geq 0$

Proof. From Equation (43), we have

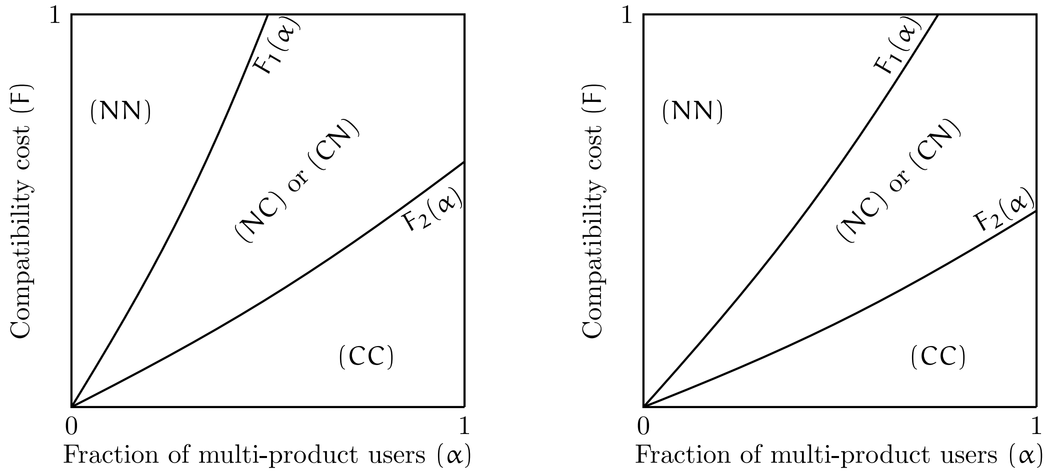
$$\frac{\partial\left|\frac{\partial F_1(\alpha)}{\partial t} - \frac{\partial F_2(\alpha)}{\partial t}\right|}{\partial \alpha} = \frac{3}{4} + \frac{W^2}{8t^2} - \frac{\alpha W^2}{36t^2} \geq 0$$

where the last inequality holds for all values of $\alpha \in [0, 1]$. □

Note that at $\alpha = 0$, Equation (43) takes the value of zero. Combining this observation with the result from Claim 2, we have that

$$\left|\frac{\partial F_1(\alpha)}{\partial t}\right| \geq \left|\frac{\partial F_2(\alpha)}{\partial t}\right|,$$

where the inequality holds for all values of $\alpha \geq 0$. Therefore, as both thresholds fall, the parameter space over which firms choose incompatibility in equilibrium increases. Further, as threshold $F_1(\alpha)$ falls at a higher rate than threshold $F_2(\alpha)$, the region with $F_2(\alpha) \leq F \leq F_1(\alpha)$, where asymmetric compatibility regimes arise in equilibrium becomes smaller. As an illustration (refer Figure 4), we use the numerical values with $V = 1.2$, $W = 1$, $\delta = 0.3$, $q = 0.6$, $t = 0.5$ and $t' = 1$, and show the change in thresholds $F_1(\alpha)$ and $F_2(\alpha)$ as t increases from $t = 0.5$ to $t' = 1$.



(a) $t = 0.5$ (b) $t' = 1$

Figure 4: Comparative statics of optimal compatibility regimes with respect to per-unit transportation cost t.

The figure is based on parameter values $V = 1.2$, $W = 1$ $\delta = 0.3$, and $q = 0.6$. The threshold $F_1(\alpha)$ ($F_2(\alpha)$) represents the loci of points along which a firm is indifferent between choosing compatibility and incompatibility when it believes that rival has chosen incompatibility (compatibility).

Step 2: Impact of per-user advertising revenue q on the thresholds $F_1(\alpha)$ and $F_2(\alpha)$.

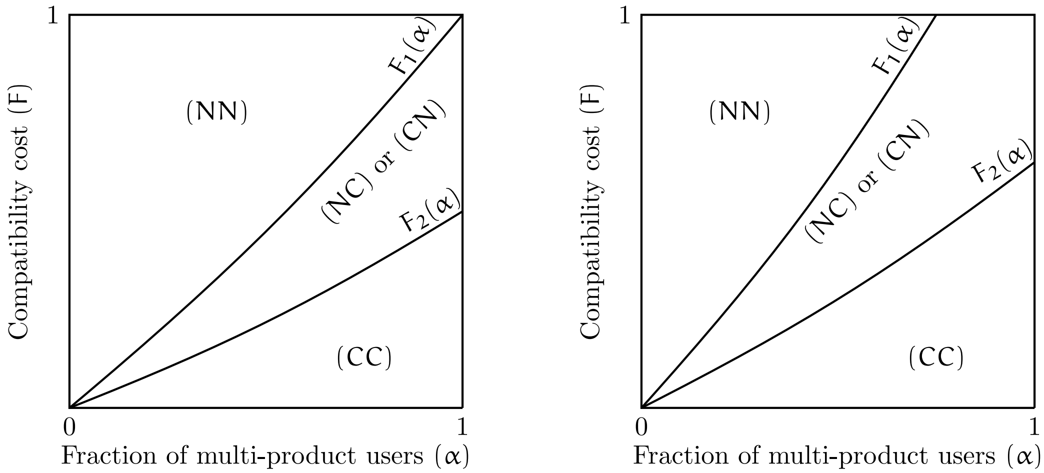
First, consider the threshold $F_1(\alpha)$. Using Equation (39), we have

$$\frac{\partial F_1(\alpha)}{\partial q} = \alpha \geq 0, \tag{44}$$

where the inequality holds for all values of $\alpha \geq 0$. Now, consider the threshold $F_2(\alpha)$. Using Equation (40), we have

$$\frac{\partial F_2(\alpha)}{\partial q} = \frac{\alpha}{2} \geq 0, \tag{45}$$

where the inequality holds for all values of $\alpha \geq 0$. This shows that both thresholds on compatibility costs increase with an increase in per-user advertising revenue to advertisers. Moreover, comparing Equations (44) and (45), we can see that $|\alpha| > \left|\frac{\alpha}{2}\right|$, where the inequality always holds. Therefore, as both thresholds rise, the parameter space over which firms choose incompatibility in equilibrium decreases. Further, as threshold $F_1(\alpha)$ rises at a higher rate than thresholds $F_2(\alpha)$, the region with $F_2(\alpha) \leq F < F_1(\alpha)$, where asymmetric compatibility regimes arise in equilibrium becomes larger. As an illustration (refer Figure 5), we use the numerical values with $V = 1.2$, $W = 1$, $\delta = 0.3$, $t = 1$, $q = 0.35$ and $q' = 0.6$, and show the change in thresholds $F_1(\alpha)$ and $F_2(\alpha)$ as q increases from $q = 0.35$ to $q' = 0.6$.



(a) $q = 0.35$ (b) $q' = 0.6$

Figure 5: Comparative statics of optimal compatibility regimes with respect to per-user advertising revenue q.

The figure is based on parameter values $V = 1.2$, $W = 1$ $\delta = 0.3$, and $t = 1$. The threshold $F_1(\alpha)$ ($F_2(\alpha)$) represents the loci of points along which a firm is indifferent between choosing compatibility and incompatibility when it believes that rival has chosen incompatibility (compatibility).

Step 3: Impact of per-unit nuisance cost of advertisement δ on the thresholds $F_1(\alpha)$ and $F_2(\alpha)$.

First, consider threshold $F_1(\alpha)$. Using Equation (39), we have

$$\frac{\partial F_1(\alpha)}{\partial \delta} = -\alpha \leq 0, \tag{46}$$

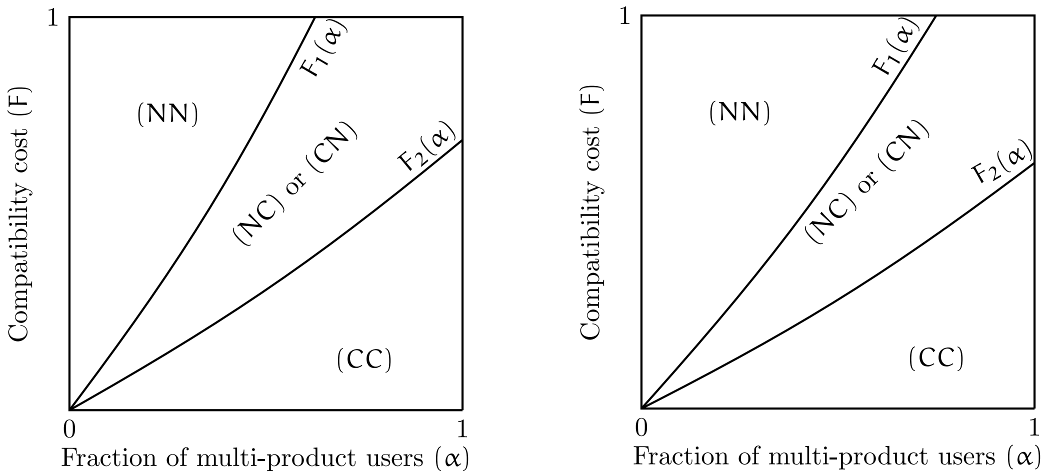
where the inequality holds for all values of $\alpha \geq 0$.

Now, consider threshold $F_2(\alpha)$. Using Equation (40), we have

$$\frac{\partial F_2(\alpha)}{\partial \delta} = -\frac{\alpha}{2} \leq 0, \tag{47}$$

where the inequality holds for all values of $\alpha \geq 0$.

This shows that both thresholds on compatibility costs decrease with an increase in nuisance cost to users. Moreover, comparing Equations (46) and (47), we can see that $|-\alpha| > \left|-\frac{\alpha}{2}\right|$, where the inequality always holds. Therefore, as both thresholds fall, the parameter space over which firms choose incompatibility in equilibrium expands. Further, as threshold $F_1(\alpha)$ falls at a higher rate than thresholds $F_2(\alpha)$, the region with $F_2(\alpha) \leq F < F_1(\alpha)$, where asymmetric compatibility regimes arise in equilibrium becomes smaller. As an illustration (refer Figure 6), we use the numerical values with $V = 1.2$, $W = 1$, $q = 0.3$, $t = 1$, $\delta = 0.1$ and $\delta' = 0.3$, and show the change in thresholds $F_1(\alpha)$ and $F_2(\alpha)$ as q increases from $\delta = 0.1$ to $\delta' = 0.3$. This completes the proof. ■



(a) $\delta = 0.1$ (b) $\delta' = 0.3$

Figure 6: Comparative statics of optimal compatibility regimes with respect to per-unit nuisance cost of advertisement δ.

The figure is based on parameter values $V = 1.2$, $W = 1$ $q = 0.6$, and $t = 1$. The threshold $F_1(\alpha)$ ($F_2(\alpha)$) represents the loci of points along which a firm is indifferent between choosing compatibility and incompatibility when it believes that rival has chosen incompatibility (compatibility).

Proof of Proposition 4

We proceed through a series of steps. In *Step 1*, we define social welfare under different market regimes. In *Step 2*, we compare social welfare under different market regimes and derive conditions under which incompatibility or full compatibility, or asymmetric compatibility regime is socially optimal. In *Step 3*, we compare thresholds obtained from *Step 2*. In *Step 4*, we characterize the socially optimal compatibility regimes.

Step 1: Social welfare under different market regimes.

If both firms choose incompatibility, then social welfare is given by

$$\begin{aligned} SW^{NN} &= (1-\alpha)\left[\int_0^{\hat{x}}(V-tx-\delta)dx + \int_{\hat{x}}^1 (V-t(1-x)-\delta)dx\right] \\ &+ \alpha\left[\int_0^{\bar{x}}(V+W-tx-\delta)dx + \int_{\bar{x}}^1 (V+W-t(1-x)-\delta)dx\right] + q. \end{aligned}$$

Using Equations (9) and (10) on the user demand under incompatibility, i.e., regime NN, we obtain

$$SW^{NN} = V + q - \delta - \frac{t}{4} + \alpha W. \tag{48}$$

If both firms choose compatibility, then social welfare is given by

$$\begin{aligned} SW^{CC} &= (1-\alpha)\left[\int_0^{\hat{x}}(V-tx-\delta)dx + \int_{\hat{x}}^1 (V-t(1-x)-\delta)dx\right] \\ &+ \alpha\left[\int_0^{\bar{x}}(V+2W-tx-\delta)dx + \int_{\bar{x}}^1 (V+2W-t(1-x)-\delta)dx\right] + q - 2F. \end{aligned}$$

Using Equations (9) and (13) on the user demand under full compatibility, i.e., regime CC, we obtain

$$SW^{CC} = V + q - \delta - \frac{t}{4} + 2\alpha W - 2F. \tag{49}$$

Without loss of generality, suppose firm 1 chooses incompatibility and firm 2 chooses compatibility, i.e., regime NC. Under an asymmetric compatibility regime, the social welfare is given by

$$\begin{aligned} SW^{NC} &= (1-\alpha)\left[\int_0^{\hat{x}}(V-tx-\delta)dx + \int_{\hat{x}}^1 (V-t(1-x)-\delta)dx\right] \\ &+ \alpha\left[\int_0^{\bar{x}}(V+2W-tx-\delta)dx + \int_{\bar{x}}^1 (V+W-t(1-x)-\delta)dx\right] + q - F. \end{aligned}$$

After algebraic calculations, we obtain

$$S^{NC} = V + q - \delta - \frac{t}{4} + (1+\bar{x})\alpha W - (1-\alpha)t\left(\frac{1}{2}-\hat{x}\right)^2 - \alpha t\left(\frac{1}{2}-\bar{x}\right)^2 - F. \tag{50}$$

Step 2: Comparison of social welfare under different market regimes.

Using Equations (48) and (49), we find that the social planner is indifferent between choosing incompatibility and full compatibility regimes when the firms' compatibility cost is $F_{S1}(\alpha)$, such that (i) $SW^{CC}(F) = SW^{NN}(F)$ at $F = F_{S1}(\alpha)$, (ii) $SW^{CC}(F) > SW^{NN}(F)$ for all $F < F_{S1}(\alpha)$, and (iii) $SW^{CC}(F) < SW^{NN}(F)$ for all $F > F_{S1}(\alpha)$, where

$$F_{S1}(\alpha) = \frac{\alpha W}{2}. \tag{51}$$

In summary, $F_{S1}(\alpha)$ is the threshold for firms' compatibility cost below which a social planner prefers full compatibility over incompatibility.

Using Equations (49) and (50), we find that the social planner is indifferent between choosing full compatibility and asymmetric compatibility regimes when the firms' compatibility cost is $F_{S2}(\alpha)$, such that (i) $SW^{CC}(F) = SW^{NC}(F)$, at $F = F_{S2}(\alpha)$, (ii) $SW^{CC}(F) > SW^{NC}(F)$, for all $F < F_{S2}(\alpha)$, and (iii) $SW^{CC}(F) < SW^{NC}(F)$, for all $F > F_{S2}(\alpha)$, where

$$F_{S2}(\alpha) = \frac{\alpha W}{2} - \frac{\alpha W^2}{144t}(27 - 7\alpha). \tag{52}$$

In summary, $F_{S2}(\alpha)$ is the threshold for compatibility below above which a social planner prefers full compatibility regime CC over asymmetric compatibility NC or CN.

Using Equations (48) and (50), we find that the social planner is indifferent between choosing incompatibility and asymmetric compatibility when the firms' compatibility cost is $F_{S3}(\alpha)$, such that (i) $SW^{NN}(F) = SW^{NC}(F)$, at $F = F_{S3}(\alpha)$, (ii) $SW^{NN}(F) < SW^{NC}(F)$, for all $F < F_{S3}(\alpha)$, and (iii) $SW^{NN}(F) > SW^{NC}(F)$, for all $F > F_{S3}(\alpha)$, where

$$F_{S3}(\alpha) = \frac{\alpha W}{2} + \frac{\alpha W^2}{144t}(27 - 7\alpha). \tag{53}$$

In summary, $F_{S3}(\alpha)$ is the threshold for compatibility cost above which a social planner prefers incompatibility regime NN over asymmetric compatibility regime NC or CN.

Step 3: Comparison of thresholds $F_{S1}(\alpha)$*,* $F_{S2}(\alpha)$ *and* $F_{S3}(\alpha)$*.*

Using Equations (51), (52), and (53), the thresholds can be ordered as follows:

$$F_{S2}(\alpha) \;\leq\; F_{S1}(\alpha) \;\leq\; F_{S3}(\alpha), \tag{54}$$

where the inequality holds for all values of $\alpha \geq 0$.

Step 4: Socially optimal compatibility regimes.

Now, we use Equation (54) and thresholds $F_{S1}(\alpha)$, $F_{S2}(\alpha)$, and $F_{S3}(\alpha)$ on compatibility cost, to characterize the socially optimal compatibility regimes. For a sufficiently small compatibility cost, i.e., $F < F_{S2}(\alpha)$, both firms choose compatibility, i.e., regime CC, is socially optimal. For sufficiently large compatibility costs, i.e., $F \geq F_{S3}(\alpha)$, both firms choose incompatibility, i.e., regime NN, is socially optimal. For intermediate range of compatibility costs, i.e., $F_{S2}(\alpha) \leq F < F_{S3}(\alpha)$, firm 1 chooses incompatibility while firm 2 chooses compatibility, i.e., regime NC, or firm 1 chooses compatibility while firm 2 chooses incompatibility, i.e., regime CN, is socially optimal. This completes the proof. ■

Proof of Proposition 5

Proposition 4 characterized the region in which each regime is socially optimal, using thresholds $F_{S2}(\alpha)$ and $F_{S3}(\alpha)$ on compatibility costs that firms face. In this proposition, we examine the change in the parameter space where each regime is socially optimal, with respect to model parameters. We proceed through a series of steps. In *Step 1*, we characterize the sensitivity of the thresholds $F_{S2}(\alpha)$ and $F_{S3}(\alpha)$ with respect to per-unit transportation cost t that users face. In *Step 2*, we characterize the sensitivity of the thresholds $F_{S2}(\alpha)$ and $F_{S3}(\alpha)$ with respect to the intrinsic value of application W to users. In *Step 3*, we characterize the sensitivity of the thresholds $F_{S2}(\alpha)$ and $F_{S3}(\alpha)$ with respect to the fraction of multi-product users α in the market.

Step 1: Impact of per-unit transportation cost t on the thresholds $F_{S2}(\alpha)$ and $F_{S3}(\alpha)$.

First, consider threshold $F_{S2}(\alpha)$. Using Equation (52), we have

$$\frac{\partial F_{S2}(\alpha)}{\partial t} = \frac{\alpha W^2}{144t^2}(27 - 7\alpha) \geq 0, \tag{55}$$

where the inequality holds for all values of $\alpha \in [0, 1]$.

Next, consider threshold $F_{S3}(\alpha)$. Using Equation (53), we have

$$\frac{\partial F_{S3}(\alpha)}{\partial t} = -\frac{\alpha W^2}{144t^2}(27 - 7\alpha) \geq 0, \tag{56}$$

where the inequality holds for all values of $\alpha \in [0, 1]$. Since $\frac{\partial F_{S2}(\alpha)}{\partial t} \geq$ and $\frac{\partial F_{S3}(\alpha)}{\partial t} \leq 0$, the region with $F_{S2}(\alpha) \leq F < F_{S3}(\alpha)$ becomes smaller, and the likelihood of asymmetric compatibility regime as the socially optimal regime decreases.

Step 2: Impact of intrinsic value of application W on the thresholds $F_{S2}(\alpha)$ and $F_{S3}(\alpha)$.

First, consider threshold $F_{S2}(\alpha)$. Using Equation (52), we have

$$\frac{\partial F_{S2}(\alpha)}{\partial W} = \frac{\alpha}{2} - \frac{2\alpha W(27-7\alpha)}{144t}, \tag{57}$$

where the sign of the preceding expression depends on the value t, α, and W.

Next, consider threshold $F_{S3}(\alpha)$. Using Equation (53), we have

$$\frac{\partial F_{S3}(\alpha)}{\partial W} = \frac{\alpha}{2} + \frac{2\alpha W(27-7\alpha)}{144t} \geq 0, \tag{58}$$

where the inequality holds for all values of $\alpha \in [0,1]$. Therefore, the region with $F \geq F_{S3}(\alpha)$, where incompatibility regime is socially optimal becomes smaller.

Moreover, comparing Equations (57) and (58), we have

$$\frac{\partial F_{S3}(\alpha)}{\partial W} - \frac{\partial F_{S2}(\alpha)}{\partial W} = \frac{4\alpha(27-7\alpha)}{144t} \geq 0, \tag{59}$$

where the preceding inequality holds for all values of $\alpha \in [0,1]$. Therefore, the region with $F_{S2}(\alpha) \leq F < F_{S3}(\alpha)$ becomes larger, and the likelihood of asymmetric compatibility regime as the socially optimal regime increases.

Step 3: Impact of fraction of multi-product users α, on the thresholds $F_{S2}(\alpha)$ and $F_{S3}(\alpha)$.

First, consider threshold $F_{S2}(\alpha)$. Using Equation (52), we have

$$\frac{\partial F_{S2}(\alpha)}{\partial \alpha} = \frac{W}{2} - \frac{(27-14\alpha)W^2}{144t}, \tag{60}$$

where the sign of the preceding expression depends on the value t, α and W.

Next, consider threshold $F_{S3}(\alpha)$. Using Equation (53), we have

$$\frac{\partial F_{S3}(\alpha)}{\partial \alpha} = \frac{W}{2} + \frac{(27-14\alpha)W^2}{144t} > 0, \tag{61}$$

where the inequality holds for all values of $\alpha \in [0,1]$. Therefore, as the threshold F_{S3} increases, the region with $F_{S3}(\alpha) \leq F$, where incompatibility is socially optimal becomes smaller.

Moreover, comparing Equations (60) and (61), we have

$$\frac{\partial F_{S3}(\alpha)}{\partial \alpha} - \frac{\partial F_{S2}(\alpha)}{\partial \alpha} = \frac{(27-14\alpha)W^2}{72t} > 0, \tag{62}$$

where the preceding inequality holds for all values of $\alpha \in [0,1]$. Therefore, the region with $F_{S2}(\alpha) \leq F < F_{S3}(\alpha)$ becomes larger, and the likelihood of asymmetric compatibility regime as the socially optimal regime increases. This completes the proof. ■

C Extensions

C.1 A Microfoundation of the Advertising Side

In this extension, based on a second price auction model, we provide a microfoundation of the advertising side and derive the result that a firm earns per-user advertising revenue q in equilibrium. Consider $K \geq 2$ advertisers competing for the advertisement space available on a firm's hardware for each user. Let each advertiser's value of displaying his advertisement to a single user be denoted by q_i. We assume that advertisers draw their valuations privately and independently from a common distribution $q_i \in Q()$.

Following industry practice, we assume that the firm conducts a personalized second price auction for each user that uses its hardware, such as the one employed by Apple.[13] Each advertiser draws a valuation and submits their bid. Since it is weakly dominant strategy for each advertiser to bid their own value, at each auction, the firm (auctioneer) would obtain a profile of bids $\{q_k\}_{k\in\{1,\ldots K\}}$ such that they can be ordered as $q_1 \geq q_2 \geq \ldots q_K$ without loss of generality.[14]

The truthful equilibrium described above is the unique symmetric Bayesian Nash equilibrium of the second price auction. Since each advertiser will bid their value, the firm's revenue per-user (the amount paid in equilibrium per-auction) will be equal to the second highest value. Then, firm i's total expected advertising revenue is $\mathbb{E}[S^{(2)}]N_i$, where $\mathbb{E}[S^{(2)}]$

[13] See, for example, "How Do You Win The Auction in Apple Search Ads," *SearchAds*, available at https://www.searchads.com/blog/how-to-win-auction-apple-search-ads (accessed on 10-05-2024). Although advertisers pay Apple for each tap they receive on the ad displayed, with significant personalisation, we assume that the probability of tap from each user is close to 1.

[14] See Athey and Ellison (2011) for an earlier application of order statistics to analyse position auctions in digital markets.

denotes the second order-statistic of the distribution $Q()$, and N_i denotes the demand for the hardware of firm i. For brevity, we denote $\mathbb{E}[S^{(2)}]$ by q in our main analysis.

Further, since firms are symmetric and we have a measure 1 of users in the market (and therefore, a measure 1 of auctions conducted), winning outcome in the auctions are also equally distributed across the advertisers. To investigate the effect of variation in our auction model primitive, that is variation in the valuation distribution $Q()$, we analyse the sensitivity of market outcomes to changes in summary statistic $q := \mathbb{E}[S^{(2)}]$ (see Propositions 3 for more details).