Coset Groups

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Existence Proof

We define a **normal subgroup** of a group G to be a subgroup $H \leq G$ such that for all $a \in G$, $aH = Ha$. Equivalently, $aHa^{-1} = H$ for all $a \in G$.

We claim that the set of all left cosets of a normal subgroup H of a group G is itself a group (denote this H_l, where $(aH) \cdot (bH) = (a * b)H$. The right coset group (denoted H_r) is symmetric. We'll show this is a group.

1. The operation is well defined. Let $a, b \in G$. Then $aH \cdot bH = (Ha) \cdot (bH) = H(a * b)H = (a * b)HH = (a * b)H$.

2. The identity is $eH = H$. For any aH, Hb we have $(aH) \cdot H = (a)(HH) = aH$, $H \cdot aH = (HH)a = Ha = aH$ and symmetric arguments apply to Hb.

3. For any aH, Hb their inverses are precisely $a^{-1}H$ and Hb^{-1}.

4. The group operation on G is associative, from which it follows that it is on H_l, H_r respectively.