Akhil Kumar Donka
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Project - 1

(1) $\boxed{\text{1st timestep}}$:—

initial Belief : $Bel(x) = \left[\frac{1}{101}, \frac{1}{101}, \cdots \frac{1}{101}\right]_{1\times 101}$

@ action $\Rightarrow (u = u_1)$

$$Bel'(x) = \sum P(x|u, x') Bel(x')$$

$$Bel'(x_0) = P(x_0|u, x_0) Bel(x_0) + \cancelto{0}{P(x_0|u, x_1)} Bel(x_1) + \cdots$$

$$= 0.05 \times \frac{1}{101} = 4.95 \times 10^{-4}$$

$$Bel'(x_1) = P(x_1|u, x_0) Bel(x_0) + P(x_1|u, x_1) Bel(x_1) + \cdots$$

$$= 0.9 \times \frac{1}{101} + 0.05 \times \frac{1}{101} + 0 \times \frac{1}{101} + \cdots$$

$$= 0.00940$$

$$Bel'(x_2) = P(x_2|u, x_0) Bel(x_0) + P(x_2|u, x_1) Bel(x_1) + P(x_2|u, x_2) Bel(x_2) + \cdots$$

$$= 0.05 \times \frac{1}{101} + 0.9 \times \frac{1}{101} + 0.05 \times \frac{1}{101} + 0 \ldots$$

$$= 0.0099$$

$$Bel'(x_3) = Bel'(x_4) = \cdots = Bel'(x_{100}) = 0.0099$$

@ Measurement $\Rightarrow (Z = Z_1)$; η : for ~~measurement~~ normalization

$$Bel'(x) = P(Z|x) Bel(x)$$

$$Bel'(x_0) = P(Z_1|x_0) Bel(x_0) = \frac{0.1}{100} \times 4.95 \times 10^{-4}$$

$$Bel'(x_1) = P(Z_1|x_1) Bel(x_1) = 0.9 \times 0.0094$$

$$Bel'(x_2) = P(Z_1|x_2) Bel(x_2) = \frac{0.1}{100} \times 0.0099$$

$$\eta = \sum_i Bel'(x_i) = 0.0094$$

$$\frac{Bel'(x)}{\eta} = [5.24\times10^{-5} \quad 8.96\times10^{-1} \quad 1.4\times10^{-3} \cdots 1.4\times10^{-3}]$$

$\boxed{\text{2nd Timestep}}$:-

@ action $\Rightarrow (u = u_2)$

$$Bel'(x) = \sum P(x|u, x') \, Bel(x')$$

$$Bel'(x_0) = P(x_0|u, x_0)\,Bel(x_0) + P(x_0|u, x_1)\,Bel(x_1) + \cdots$$

$$= 0.05 \times 5.24\times10^{-5} + 0 + 0 \cdots$$

$$Bel'(x_1) = P(x_1|u, x_0)\,Bel(x_0) + P(x_1|u, x_1)\,Bel(x_1) + \cdots$$

$$= 0.9 \times 5.24\times10^{-5} + 0.05 \times 8.96\times10^{-1}$$

$$Bel'(x_2) = P(x_2|u, x_0)\,Bel(x_0) + P(x_2|u, x_1)\,Bel(x_1) + P(x_2|u, x_2)\,Bel(x_2) + \cdots$$

$$= 0.05 \times 5.24\times10^{-5} + 0.9 \times 8.96\times10^{-1} + 0.05 \times 1.4\times10^{-3} + 0 \cdots$$

$\vdots$

@ measurement $\Rightarrow (z = z_2)$

$$Bel'(x_0) = P(z_2|x_0)\,Bel(x_0) = \frac{0.1}{100} \times 2.62\times10^{-6}$$

$$Bel'(x_1) = \frac{0.1}{100} \times 4.48\times10^{-2}$$

$$Bel'(x_2) = 0.9 \times 8.066\times10^{-1}$$

$\vdots$

$$Bel'(x_{100}) = 1.047\times10^{-6}$$

after normalizing :-

$$Bel'(x) = [3.6\times10^{-9},\ 6.19\times10^{-5},\ 9.9\times10^{-1},\ 6.307\times10^{-5},\ 1.44\times10^{-6}, \cdots 1.44\times10^{-6}]$$

$\boxed{\text{3rd timestep}}$:-

@ action $\Rightarrow$ $u = u_3$

$Bel'(x_0) = 0.05 \times 3.608 \times 10^{-9} = 1.8 \times 10^{-10}$

$Bel'(x_1) = 0.9 \times 3.608 \times 10^{-9} + 0.05 \times 6.1 \times 10^{-5}$

$= 3.091 \times 10^{-6}$

$Bel'(x_2) = 5.004 \times 10^{-2}$

$\vdots$

@ measurement $\Rightarrow$ $z = z_3$

$Bel'(x_0) = P(z_3 | x_0)\, Bel(x_0)$

$= \frac{0.1}{100} \times 1.804 \times 10^{-10} = 1.8 \times 10^{-13}$

$Bel'(x_1) = P(z_3 | x_1)\, Bel(x_1) = \frac{0.1}{100} \times 3.09 \times 10^{-6}$

$\vdots$

$Bel'(x_{100}) = 1.44 \times 10^{-9}$

after normalization :-

$Bel'(x) = [2.2 \times 10^{-13}, 3.8 \times 10^{-9}, 6.1 \times 10^{-5},$
$9.98 \times 10^{-1}, 6.17 \times 10^{-5}, \ldots]$

$\boxed{\text{4th Timestep}}$:-

@ action $\Rightarrow$ $u = u_4$

$Bel'(x_0) = \frac{5}{100} \times 2.2 \times 10^{-13} = 1.11 \times 10^{-14}$

$Bel'(x_1) = \frac{90}{100} \times 2.2 \times 10^{-13} + \frac{5}{100} \times 3.8 \times 10^{-9}$

$Bel'(x_2) = \frac{5}{100} \times 2.27 \times 10^{-13} + \frac{90}{100} \times 3.8 \times 10^{-9}$

$+ \frac{5}{100} \times 6.17 \times 10^{-5} + 0$

$$Bel'(x) = [1.11 \times 10^{-14}, 1.91 \times 10^{-10}, 3.09 \times 10^{-6}, 5 \times 10^{-2},$$
$$8.99 \times 10^{-1}, \ldots\ldots]$$

@ measurement $\Rightarrow$ $z = z_4$

$$Bel(x_0) = P(z_4 | x_0)\, Bel(x_0) = \frac{0.1}{100} \times 1.1 \times 10^{-14}$$
$$= 1.11 \times 10^{-17}$$

$$Bel(x_1) = \frac{0.1}{100} \times 1.9 \times 10^{-10} = 1.91 \times 10^{-13}$$

$\vdots$

after normalization :-

$$Bel'(x) = [1.375 \times 10^{-17}, 2.35 \times 10^{-13}, 3.8 \times 10^{-9}$$
$$6.17 \times 10^{-5}, \ldots\ldots, 2.2 \times 10^{-12}]$$

(2) Let, plane's equation be :-

$$z = Ax + By + c$$

Say we have "n" data points :

then,

$$[\bar{z}] = A\bar{x}_1 + B\bar{y} + c \quad \left(\text{taking average}\right)$$

Now,

$$z_i - \bar{z} = A(x_i - \bar{x}) + B(y_i - \bar{y})$$

$$\hat{z}_i = A\hat{x}_i + B\hat{y}_i$$

from Least squares method :-

$$\sum \hat{z}_i \hat{x}_i = A\sum \hat{x}_i^2 + B\sum \hat{x}_i \hat{y}_i$$

$$\sum \hat{z}_i \hat{y}_i = A\sum \hat{x}_i \hat{y}_i + B \cdot \sum \hat{y}_i^2$$

$$\therefore \begin{bmatrix} \sum \hat{z}_i \hat{x}_i \\ \sum \hat{z}_i \hat{y}_i \end{bmatrix} = \begin{bmatrix} \sum x_i^2 & \sum \hat{x}_i \hat{y}_i \\ \sum \hat{x}_i \hat{y}_i & \sum y_i^2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

By cramer's rule :-

$$A = \frac{\begin{vmatrix} \sum \hat{z}_i \hat{x}_i & \sum \hat{x}_i \hat{y}_i \\ \sum \hat{z}_i \hat{y}_i & \sum \hat{y}_i^2 \end{vmatrix}}{\begin{vmatrix} \sum \hat{x}_i^2 & \sum \hat{x}_i \hat{y}_i \\ \sum \hat{x}_i \hat{y}_i & \sum \hat{y}_i^2 \end{vmatrix}}, \quad D \neq 0$$

$$A = \frac{\sum \hat{z}_i \hat{x}_i \sum \hat{y}_i^2 - \sum \hat{z}_i \hat{y}_i \sum \hat{x}_i \hat{y}_i}{\sum \hat{x}_i^2 \sum y_i^2 - \left(\sum \hat{x}_i \hat{y}_i\right)^2}$$

$$B = \frac{\begin{vmatrix} \sum \hat{x}_i^2 & \sum \hat{z}_i \hat{x}_i \\ \sum \hat{x}_i \hat{y}_i & \sum \hat{z}_i \hat{y}_i \end{vmatrix}}{\begin{vmatrix} \sum \hat{x}_i^2 & \sum \hat{x}_i \hat{y}_i \\ \sum \hat{x}_i \hat{y}_i & \sum \hat{y}_i^2 \end{vmatrix}} \quad D \neq 0$$

$$B = \frac{\sum \hat{x}_i^2 \sum \hat{z}_i \hat{y}_i - \sum \hat{x}_i \hat{y}_i \sum \hat{z}_i \hat{x}_i}{\sum \hat{x}_i^2 \sum \hat{y}_i^2 - \left(\sum \hat{x}_i \hat{y}_i\right)^2}$$

$$c = \bar{z} - A\bar{x} - B\bar{y}$$

Least squares fails when 3-D data points lie in a line.