Tutorial 6 : EE22MTECH02003

(1) Let x_1, x_2, x_3 be the proportion of time devoted each day to iPod, iPhone & ipad respectively.

And also,

Number of produced item of each type $\propto$ Time devoted in producing item

Formulation of Problem :—

$\therefore$ Item produced each day : $\begin{pmatrix} 6000\,x_1 \\ 5000\,x_2 \\ 3000\,x_3 \end{pmatrix}$

Item produced in a week : $\begin{pmatrix} 30000\,x_1 \\ 25000\,x_2 \\ 15000\,x_3 \end{pmatrix}$.

$\therefore$ Profit $= 4 \times 30000 x_1 + 6 \times 25000 x_2 + 10 \times 15000 x_3$

$= 120000 x_1 + 150000 x_2 + 150000 x_3$

Constraints :—

Proportions : $x_1 + x_2 + x_3 \leq 1$

Storage : $\frac{40}{1000} \times 30000 x_1 + \frac{45}{1000} \times 25000 x_2 + \frac{210}{1000} \times 15000 x_3 \leq 6000$

$\Rightarrow 1200 x_1 + 1125 x_2 + 3150 x_3 \leq 6000$

Minimum Sale : $30000 x_1 \geq 5000$

$15000 x_3 \geq 4000$

Demand : $30000 x_1 \leq 10000$

$25000 x_2 \leq 15000$

$15000 x_3 \leq 8000$

Positive constraint : $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(b) Let y_1, y_2, y_3 be number of covers produced over the week of iPod, iPhone and iPad respectively.

$\therefore$ Profit/week $= 4y_1 + 6y_2 + 10y_3$

Hence objective : $\{\max\ 4y_1 + 6y_2 + 10y_3\}$

Such that :—

$\frac{1}{30000y_1} + \frac{1}{25000y_2} + \frac{1}{15000y_3} \leq 1$

↳ Production constraint

storage : $0.04y_1 + 0.045y_2 + 0.21y_3 \leq 6000$

minimum sale : $y_1 \geq 5000$, $y_2 \geq 4000$

Demand : $y_1 \leq 10000$, $y_2 \leq 15000$, $y_3 \leq 8000$

Positive : $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(c) Let z_1, z_2 and z_3 be the number of hours devoted to the production of iPod, iPhone and iPad covers respectively over 1 week.

Given that in one day 8 hours of work is done.

objective function : $\max \frac{6000}{8} z_1 \times 4 + \frac{5000}{8} z_2 \times 6 + \frac{3000}{8} z_3 \times 10$

$= \max\ 3000z_1 + 3750z_2 + 3750z_3$

Subject to : $\frac{z_1}{40} \leq 1$, $\frac{z_2}{40} \leq 1$, $\frac{z_3}{40} \leq 1$

↳ Production constraints.

storage : $\frac{6000}{8} z_1 \times \frac{40}{1000} + \frac{5000}{8} z_2 \times \frac{45}{1000} + \frac{3000 z_3}{8} \times \frac{210}{1000} \leq 6000$

$= 30z_1 + 28.125z_2 + 78.75z_3 \leq 6000$

minimum sale :

$$750 Z_1 \geq 5000$$
$$375 Z_3 \geq 4000$$

Demand :

$$750 Z_1 \leq 10000$$
$$625 Z_2 \leq 15000$$
$$375 Z_3 \leq 8000$$

Positive constraint :

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(d) Since X represents number of hours devoted to each item per day and Z represents number of hours devoted to each item per week with working hours 8hr/day.

$\therefore$ working hours per week $= 5 \times 8 = 40$ hrs

Relation b/w Z and X are :—

$Z_1 = 40 x_1$, $Z_2 = 40 x_2$, $Z_3 = 40 x_3$