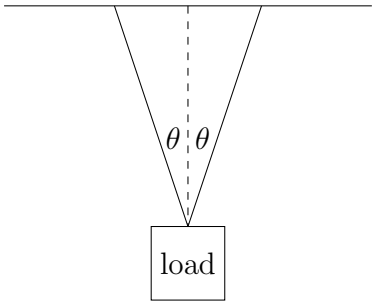
3. *Minimum time maneuver for a crane.* A crane manipulates a load with mass $m > 0$ in two dimensions using two cables attached to the load. The cables maintain angles $\pm\theta$ with respect to vertical, as shown below.



The (scalar) tensions T^{left} and T^{right} in the two cables are independently controllable, from 0 up to a given maximum tension $T^{\max}$. The total force on the load is

$$F = T^{\text{left}} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} + T^{\text{right}} \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} + mg,$$

where $g = (0, -9.8)$ is the acceleration due to gravity. The acceleration of the load is then F/m.

We approximate the motion of the load using

$$p_{i+1} = p_i + hv_i, \quad v_{i+1} = v_i + (h/m)F_i, \quad i = 1, 2, \ldots,$$

where $p_i \in \mathbf{R}^2$ is the position of the load, $v_i \in \mathbf{R}^2$ is the velocity of the load, and $F_i \in \mathbf{R}^2$ is the force on the load, at time $t = ih$. Here $h > 0$ is a small (given) time step.

The goal is to move the load, which is initially at rest at position p^{init} to the position p^{des}, also at rest, in minimum time. In other words, we seek the smallest k for which

$$p_1 = p^{\text{init}}, \quad p_k = p^{\text{des}}, \quad v_1 = v_k = (0, 0)$$

is possible, subject to the constraints described above.

(a) Explain how to solve this problem using convex (or quasiconvex) optimization.

(b) Carry out the method of part (a) for the problem instance with

$$m = 0.1, \quad \theta = 15^\circ, \quad T^{\max} = 2, \quad p^{\text{init}} = (0, 0), \quad p^{\text{des}} = (10, 2),$$

with time step $h = 0.1$. Report the minimum time $k^\star$. Plot the tensions versus time, and the load trajectory, *i.e.*, the points $p_1, \ldots, p_k$ in $\mathbf{R}^2$. Does the load move along the line segment between p^{init} and p^{des} (*i.e.*, the shortest path from p^{init} and p^{des})? Comment briefly.