Image Reflection Suppression using Convex Optimization

Course Project

Akhil Kumar Donka[1] U. Venkata Sai Anand Mohan[2]
Nitish Kumar[3] Sourish Chatterjee [4]

[1]EE22MTECH02003 [2]EE21MTECH14022 [3]EE22MTECH02005

[4]EE22MTECH02002

April 23, 2022

Table of Contents

Table of Contents

Introduction

- Reflections are a common artifact in images taken through glass windows, an undesired experience whose removal has practical significance
- From image processing perspective, reflection suppression must be fast responsive and user friendly
- Algorithm must be computationally efficient for portable devices
- Automatically removing the reflection artifacts after the picture is taken is an ill-posed problem, instead focus must be on suppression

About Proposed Methodology

- Proposed model is convex, solution is guaranteed to be the global optimal of the model
- Optimal solution is in closed form and doesn't rely on iterative algorithms, obtained through solving a partial differential equation, which can be done efficiently using Discrete Cosine Transform.
- Method doesn't require any external dataset or training time as in the neural network approaches

Table of Contents

Literature Survey

- Process can be categorized into two types :
  1. Multiple Image Reflection Removal
  2. Single Image Reflection Removal
- For multiple images case,
  1. Separate transmission and reflection layers by taking images of objects at different angles through polarizers
  2. Use images taken with and without flash to reduce reflection
- But these approaches faces practical limitations of deployment

Single image reflection removal

- Existing approaches rely on different prior assumptions on transmission and reflection layers
  1. Employ the gradient sparsity prior with user assisted labels to distinguish between layers
  2. Exploit the relative smoothness of different layers to separate them using a probabilistic framework
  3. Utilize multi-scale depth of field to classify edges into different layers
- Instead of separating layers, we can suppress the reflection component, using Laplacian-based data fidelity term and gradient sparsity prior.

Background on Suppression

- Consider two natural assumptions :
  1. Compared to transmission edges, reflection edges are of smaller magnitude and they are less in focus,

$$Y = (W)o(T) + (1 - W)o(k * R) \tag{1}$$

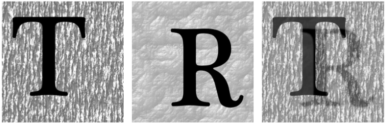
o : elementwise multiplication
k : blurring kernel
W : weights contribution of Txn layer (varying as per lighting and camera positions)

  2. Human visual system discounts the intensity modulations due to the reflection. This helps in constraint formulation.

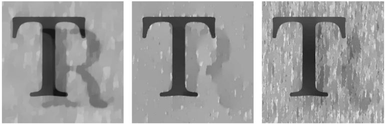
Image smoothing priors

- Priors helps in transforming the histograms into its desired ones.
- Gradient based priors helps in suppressing artifacts (objects in reflection)
- Different image smoothing priors :
  1. Conventional total variation (l_1-based) priors smooth edges by reducing gradients, But large magnitude edges are smoothed out by increasing the regularization parameter
  2. l_0-based sparsity prior has the effect of flattening a signal in order to affect signal smoothness, but even with small λ, we lose texture of Txn layer

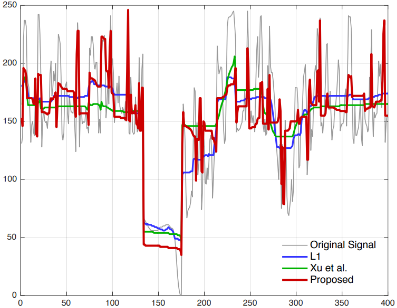
Illustration on Priors



(a) Transmission layer (b) Reflection layer (c) Synthetic blend, $w = 0.7, \sigma = 2$



(d) $l1$, $\lambda = 0.5$ (e) Xu et al. [23] $\lambda = 0.05$ (f) Proposed, $\lambda = 0.05$



(g) 1-d image scanline from the middle of the synthetic blend.

Reflection Suppression Formulation

- To account above two natural assumptions, we formulate problem as:

$$T^* = \arg\min_{T} ||T - Y||_2^2 + \lambda C(T) \tag{2}$$

$$C(T) = \#(i,j) : |\nabla_x T_{i,j}| + |\nabla_y T_{i,j}| \neq 0 \tag{3}$$

Eq. 2 refers to Data fidelity term and Eq. 3 refers to prior term (encourages smoothing)

- Data fidelity term with the l_0 prior on the image gradients ensures that the algorithm removes edges in increasing order of magnitudes
- λ controls number of gradients getting removed

- However, their combination eliminates high frequency details which is not desirable during reflection suppression
- To avoid loosing high frequency details, we introduce Laplacian based data fidelity,

$$T^* = \arg\min_{T} ||\mathcal{L}(T) - \mathcal{L}(Y)||_2^2 + \lambda C(T) \tag{4}$$

where,

$$\mathcal{L}(Y) = \nabla_{xx}(Y) + \nabla_{yy}(Y)$$

Table of Contents

Algorithm

- In above model, λ acts as regularization whose increase leads to removal of gradients due to l_0 prior. Thus acting as threshold above which gradients are removed
- Rather than solving the minimization problem and threshold the gradient from the solution, put the gradient-thresholding step directly into the objective function

$$\min_{T} \frac{1}{2}||\mathcal{L}(T) - div(\delta_h(\nabla Y))||_2^2 \tag{5}$$

where,

$$\delta_h(X_{i,j}) = \begin{cases} X_{i,j}, & \text{if } |X_{i,j}| \geq h \\ 0, & \text{otherwise} \end{cases}$$

- So data fidelity term in eq 5 imposes the gradient-thresholding step on the input image Y before taking the divergence of ∇Y
- Since data fidelity term is second order in T, we add another term which guarantee uniqueness of solution

$$\min_{T} \frac{1}{2}||\mathcal{L}(T) - div(\delta_h(\nabla Y))||_2^2 + \frac{\epsilon}{2}||T - Y||_2^2 \tag{6}$$

where, small ϵ ensures no affect on performance of data fidelity term

Solving Model

- Gradient of objective function :

$$\nabla_T = \mathcal{L}(\mathcal{L}(T) - div(\delta_h(\nabla Y))) + \epsilon(T - Y) \tag{7}$$

- Equating it to zero :

$$(\mathcal{L}^2 + \epsilon)T = \mathcal{L}(div(\delta_h(\nabla Y))) + \epsilon Y \tag{8}$$

- Above equation is variation in 2D Poisson's Equation associated with Neumann boundary condition or zero gradient at the boundary. This is solved using Discrete Cosine Transform (DCT) and its inverse.

Theorem

- Discretization of 2D poisson's equation :

$$\mathcal{L}(T) = f \tag{9}$$

with Neumann boundary condition on an M $\times$ N grid is solved by

$$T_{m,n} = DCT^{-1}\left(\frac{[DCT(f)]_{m,n}}{K_{m,n}}\right) \tag{10}$$

where,

$$T, f, K \in \mathbb{R}^{MxN}$$
$$K_{m,n} = 2\left(cos\left(\frac{m\pi}{M}\right) + cos\left(\frac{n\pi}{N}\right) - 2\right)$$
$$0 \leq m \leq M-1, 0 \leq n \leq N-1$$

Solving Equation 8

- Above theorem states that, DCT on left side of eq 9 gives:

$$DCT(\mathcal{L}(T)) = (K)o[DCT(T)]$$

  But above equation has singularity at (m,n) = (0,0), say at $T_{0,0}$

- Applying DCT to equation 8 :

$$((K)o(K) + \epsilon E)o(DCT(T)) = DCT(P) \tag{11}$$

where,
$P \in \mathbb{R}^{MxN}$ represents RHS of equation 8
$E \in \mathbb{R}^{MxN}$ is matrix of all 1's

$$T_{m,n} = DCT^{-1}\left(\frac{[DCT(P)]_{m,n}}{K_{m,n}^2 + \epsilon}\right) \tag{12}$$

Algorithm

- Uniqueness of the solution is automatically guaranteed because of the presence of ϵ in the denominator, which is the consequence of adding the $\frac{\epsilon}{2}$ term
- Algorithm:
  1. Input : Y, h, ϵ
  2. return :

$$T_{m,n} = DCT^{-1}\left(\frac{[DCT(P)]_{m,n}}{K_{m,n}^2 + \epsilon}\right)$$

  3. Output : T

Table of Contents

Reflection Suppression Results

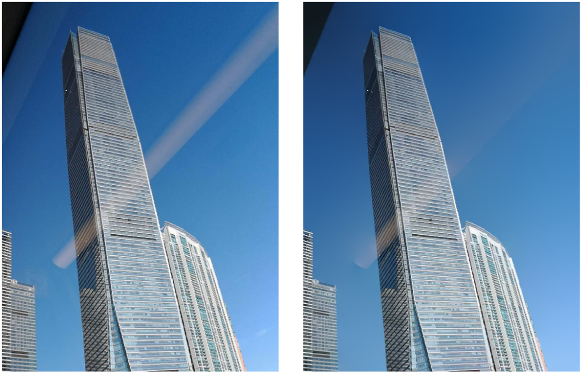


Figure: Proposed method with h = 0.033

Figure: Proposed method with $h = 0.033$

Conclusion & Future Work

- Proposed an efficient approach for single image reflection suppression
- Formulated as a convex problem, which is solved via gradient thresholding and solving a variation of 2D Poisson's equation using DCT
- Able to output desirable dereflected smartphone images in seconds
- Still cases where current approaches fail to completely remove the reflection
- Future work includes designing effective and efficient algorithms to handle sharp and strong reflections for large images

Table of Contents

References

- N. Arvanitopoulos, R. Achanta and S. Süsstrunk, "Single Image Reflection Suppression," 2017 IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2017, pp. 1752-1760, doi: 10.1109/CVPR.2017.190.
- Y. Yang, W. Ma, Y. Zheng, J. -F. Cai and W. Xu, "Fast Single Image Reflection Suppression via Convex Optimization," 2019 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), 2019, pp. 8133-8141, doi: 10.1109/CVPR.2019.00833.
- W. Ma, J. Morel, S. Osher and A. Chien, "An L1-based variational model for Retinex theory and its application to medical images," CVPR 2011, 2011, pp. 153-160, doi: 10.1109/CVPR.2011.5995422.
- L. Xu, C. Lu, Y. Xu and J. Jia, " Image smoothing via l 0 gradient minimization ", ACM Transactions on Graphics (TOG), vol. 30, pp. 174, 2011.