Road Gradient problem

A company ' XYZ ' is constructing a road through a hilly area. The height of the roadbed should be chosen in such a way that the total cost of construction is minimized. The construction cost depends on the difference between height of the roadbed and the current elevation of the road. h_i gives the height of the roadbed at a distance di meters down the road, where $d > 0$ is a given discretization. The existing elevation at a point di meters down the road is given by e_i.

The construction cost is mainly affected by the cuts(roadbed below existing elevation) and fills(roadbed above existing elevation) present in the road. The cut cost ϕ^{cut} and fill cost ϕ^{fill} are the ~~linear~~ functions of the difference between the existing elevation of the road and height of the roadbed. The overall cost (C) is a linear combination of the cut cost and fill cost. .

$C = \phi^{fill} + \phi^{cut}$

$$\phi^{fill}(u) = 2(u)_+^2 + 30(u)_+ \tag{1}$$

$$\phi^{cut}(u) = 12(u)_+^2 + (u)_+ \tag{2}$$

Where $(a)_+ = max\{a, 0\}$

The goal is to minimize C subject to the following constraints.

* The maximum allowable road slope(first derivative) is $D^{(1)}$.
* The maximum allowable curvature(second derivative) is $D^{(2)}$.
* The maximum allowable third derivative is $D^{(3)}$.

Formulate the optimization problem and verify the convexity of cut and fill functions by plotting for u in range (1:0.1:10). Find the optimal grading plan for the problem with data given in the file ~~data.m or data.py~~ . Plot h_i,e_i and $h_i - e_i$ for the optimal grading plan and report the associated cost.

Data

n = 100

e = 5*sin((1:n)/n*3*pi) + sin((1:n)/n*10*pi)

d = 1 # descretizing unit

D1 = 0.08
D2 = 0.025
D3 = 0.005