Currency Exchange

Convex Optimization

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1 Problem Statement

2. *Currency exchange.* An entity (such as a multinational corporation) holds $n = 10$ currencies, with $c_i^{\text{init}} \geq 0$ denoting the number of units of currency i. The currencies are, in order, USD, EUR, GBP, CAD, JPY, CNY, RUB, MXN, INR, and BRL. Our goal is to exchange currencies on a market so that, after the exchanges, we hold at least c_i^{req} units of each currency i.

   The exchange rates are given by $F \in \mathbf{R}^{n\times n}$, where F_{ij} is the units of currency j it costs to buy one unit of currency i. We call $1/F_{ij}$ the bid price for currency j in terms of currency i, and F_{ji} the ask price for currency j in terms of currency i.

   For example, suppose that $F_{12} = 0.88$ and $F_{21} = 1.18$. This means that it takes 0.88 EUR to buy one USD, and it takes 1.18 USD to buy one EUR; the bid and ask prices for EUR in USD are 1.1364 USD and 1.1800 USD, respectively.

   We will value a set of currency holdings in USD, by valuing each unit of currency j at the geometric mean of the bid and ask price in USD, $\sqrt{F_{j1}/F_{1j}}$. In our example above, we would value one EUR as $\sqrt{1.1364 \cdot 1.1800} = 1.1580$ USD.

   We let $X \in \mathbf{R}_+^{n\times n}$ denote the currency exchanges that we carry out, with $X_{ij} \geq 0$ the amount of currency j we exchange on the market for currency i, for which we obtain X_{ij}/F_{ij} of currency i. (You can assume that $X_{ii} = 0$.) The total of each currency j that we exchange into other currencies cannot exceed our initial holdings, c_j^{init}. After the currency exchange, we must end up with at least c_i^{req} of currency i. (The post-exchange amount we hold of currency i is our original holding c_i^{init}, minus the total we exchange into other currencies, plus the total amount we obtain from exchanging other currencies into currency i.)

   The cost of the exchanges is the decrease in value between the currency holdings before and after the exchanges, in USD. The cost can be interpreted as the transaction costs incurred by crossing the bid-ask spread (*i.e.*, if the bid and the ask were the same, there would be no cost.)

   Find the currency exchanges $X^\star$ that minimize the currency exchange cost for the data in `currency_exchange_data.*`. (These data are based on real exchange rates, but with artificially large spreads, to make sure that you don't encounter any numerical issues.) Explain your method, and give the optimal value, *i.e.*, the cost obtained.

2 Solution Report

Let X be currency amount exchange matrix, F be exchange rates matrix, c_init be initial holdings of currencies, c_req be minimum required currency units matrix and c_final be matrix of final units of each currency holdings.

Now c would be such that from c_init, for a particular currency, units exchanged with other currency gets subtracted ($X^T.[Unitvectormatrix]$) and incoming units due to other currencies gets added ($X/F.[Unitvectormatrix]$).

$$c\_final = c\_init - X^T.1 + X/F.1 \text{ ..... [Constraint 1]}$$

Cost function to minimize is :

$$\sum_{n}^{i=1}(c\_init - c\_final).\sqrt{F_{i1}/F_{1i}}$$

Other constraints: Each and every exchanges in X Matrix must be greater than 0, currency exchanges within same currency must be zero, final units of each currency i.e c_final must be atleast equal to required unit matrix i.e c_init, every currency unit after exchanges must be less than corresponding initial units i.e c_init .

$$X_{ij} \geq 0 \text{ ..... [Constraint 2]}$$
$$\text{diag(X)} = 0 \text{ ..... [Constraint 3]}$$
$$c\_final \geq c\_req \text{ ..... [Constraint 4]}$$
$$X^T.1 \leq c\_init \text{ ..... [Constraint 5]}$$

3 Answer

After formulating it as convex optimization problem using *cvxpy* python library, Minimum cost required for currency exchange is : 7.72 USD.
And final currency units are:

c_final = [1257.49577907 1636.36363639 1771.6701903 727.27272732, 909.09091043, 1090.90909094
1272.72727321 1454.54545464 1636.3636367, 1818.1818182]
in the order: [USD, EUR, GBP, CAD, JPY, CNY, RUB, MXN, INR, BRL]