

Assignment 2 | Interpolation & Decimation | DSP Lab

Problem: Implementation of decimation and interpolation.

Design Specifications:

1. Decimation and interpolation by factor 2 ($M=L=2$):
 - *Anti aliasing Gain* = 1, *Anti imaging Gain* = L
 - *Cutoff frequency* (f_c) = 600 Hz
 - *Sampling frequency* (f_s) = 2400 Hz
 - *Digital cutoff frequency* (ω_c) = $\pi/2$
 - *Number of samples* (N) = 101
2. Decimation and interpolation by factor 4 ($M=L=4$):
 - *Anti aliasing Gain* = 1, *Anti imaging Gain* = L
 - *Cutoff frequency* (f_c) = 300 Hz
 - *Sampling frequency* (f_s) = 2400 Hz
 - *Digital cutoff frequency* (ω_c) = $\pi/4$
 - *Number of samples* (N) = 101
3. Decimation and interpolation by factor 8 ($M=L=8$):
 - *Anti aliasing Gain* = 1, *Anti imaging Gain* = L
 - *Cutoff frequency* (f_c) = 150 Hz
 - *Sampling frequency* (f_s) = 2400 Hz
 - *Digital cutoff frequency* (ω_c) = $\pi/8$
 - *Number of samples* (N) = 101

Understandings & Report:

Down sampling refers to limiting the no. of samples in a signal by a factor (Say M). Whereas Up sampling refers to increasing the no. of samples in a signal at regular intervals by a factor (Say L). Both down and up sampling system are Linear Time Variant systems and hence cannot be defined using any impulse response. So dealing in frequency domain is what we prefer.

Now directly applying down sampling to signal can lead to aliasing in frequency domain. To deal with this problem we pass input signal through an antialiasing Low pass filter (cut off frequency: π / M) so that necessary condition of signal bandwidth $< (2 \pi / M)$ is maintained. Then we pass the filtered output signal through down sampler to get desired output. This output is said to be decimated by factor of M .

Similarly, we cannot do interpolation directly because up sampling operation can lead to image frequencies in frequency domain. To deal with this, we pass up sampled signal through anti-imaging Low pass filter with cut off frequency $= \pi / L$, where L denotes interpolation factor.

Interpolation & Decimation are reversible operations whereas down sampling & up sampling are irreversible processes. This is because in down sampling case, directly samples are lost and cannot be retrieved back and in case of decimation we maintain the shape of signal by taking

care of aliasing effects. Similar operations are carried out with interpolation and up sampling as well.

We also have to take care of number samples beings generated after every operations and limit them accordingly.

Plots:

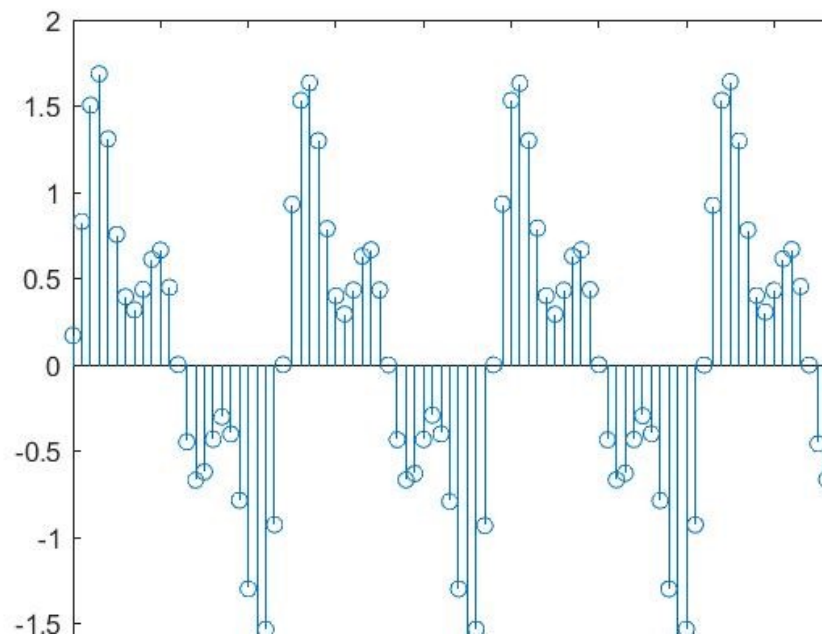


Fig1: $M=L=2$

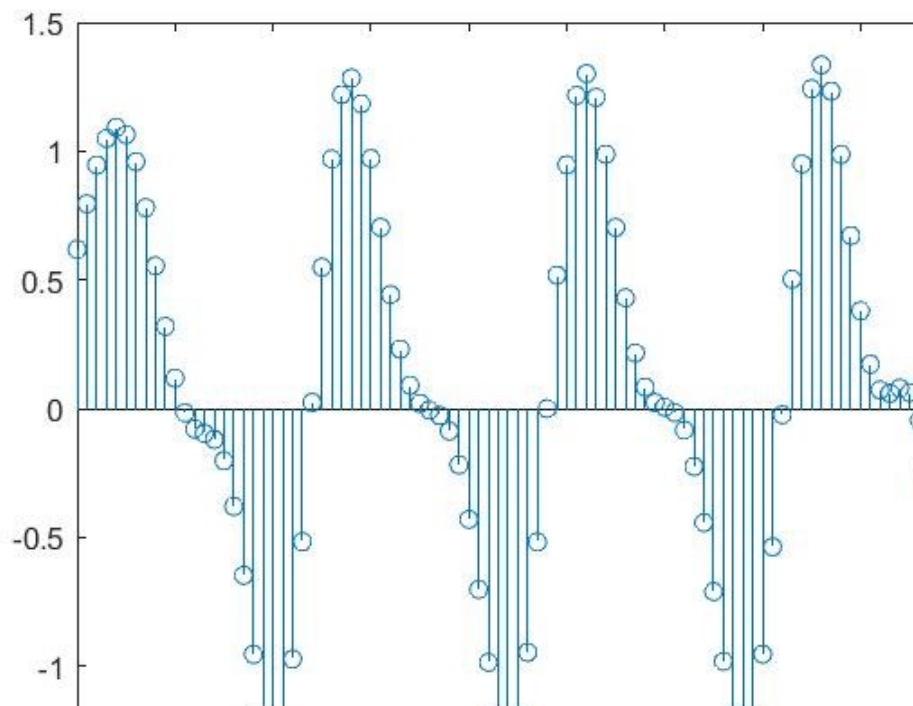


Fig2: $M=L=4$

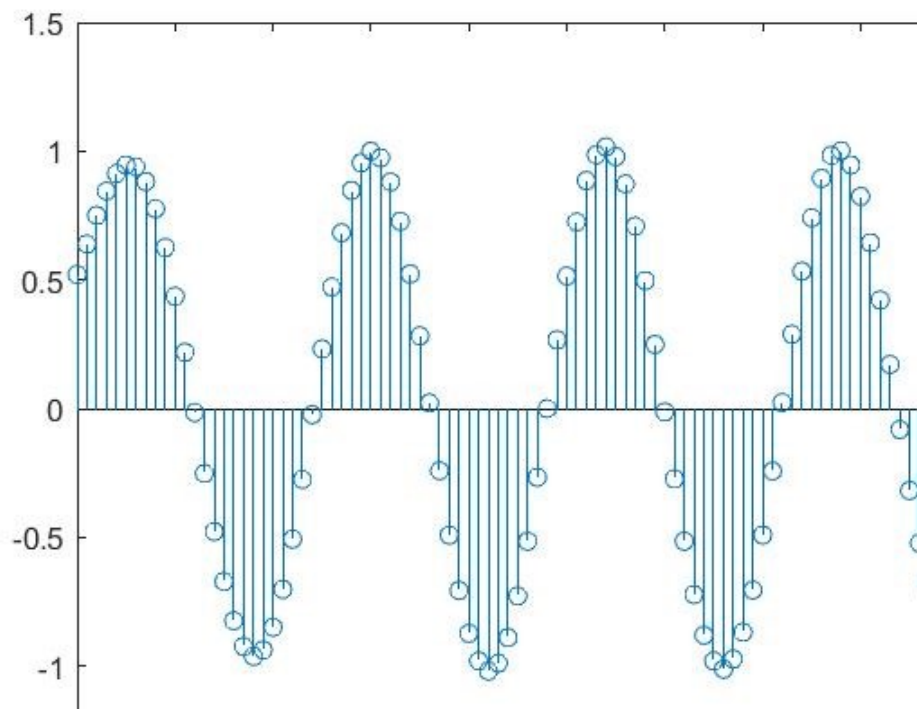


Fig3: $M=L=8$

Errors:

1. Decimation-Interpolation Error for $M=L=2$: 0.0027
2. Decimation-Interpolation Error for $M=L=4$: 0.0161
3. Decimation-Interpolation Error for $M=L=8$: 0.0244