EE2801: DSP Lab
(Jan – Apr 2022)

Lecture 1

Contents

- Analog vs Digital frequencies
- Sampling Theorem
- Convolution
- Correlation
- Downsampling
- Upsampling

Analog vs Digital frequencies

- Consider the continuous time signal $Sin(\Omega t)$,

  where $\Omega = 2\pi f, -\infty < f < \infty$, Ω is known as big omega
- To convert it into discrete time signal put $t = nT_s$, where $T_s =$ sampling interval and $f_s = \frac{1}{T_s} = sampling\ frequency$

$$Sin(2\pi f\ nT_s) = Sin\left(\frac{2\pi f}{f_s} n\right) = Sin(\omega n)$$

where ω is known as small omega

- Ω represents analog frequencies and ω represents digital frequencies.
- $\omega = \Omega T_s$, $\Omega \in (-\infty, \infty)$, $\omega \in [-\pi, \pi]$

Analog vs Digital frequencies(contd…)

- Why digital frequency ω has finite range?

Reason : As per sampling theorem $f_s \geq 2f$

We know, $\omega = \frac{2\pi f}{f_s}$

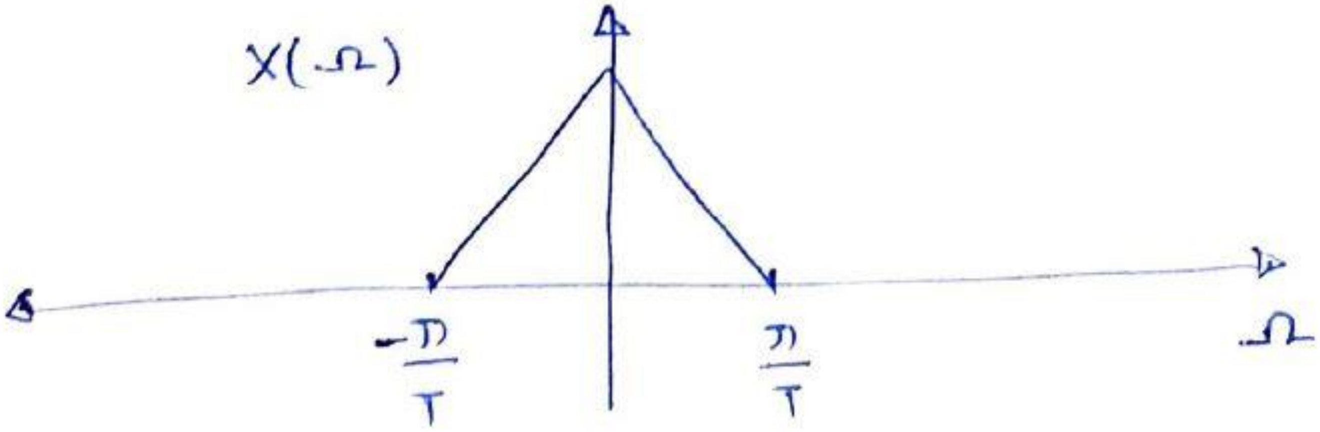
min value of f_s can be $2f$, so max value of $\omega = \frac{2\pi f}{2f} = \pi$

max value of f_s can be ∞ , so min value of $\omega \rightarrow 0$

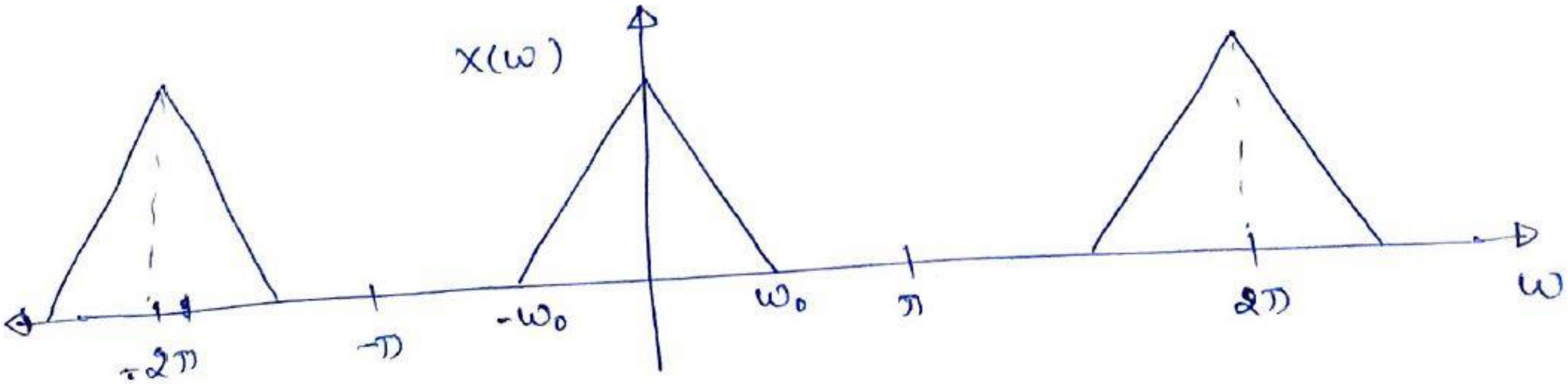
Similarly for –ve frequencies, min value is $-\pi$ and max is 0

So $-\pi \leq \omega \leq \pi$

- Spectrum of a analog signal



- Spectrum of a digital signal

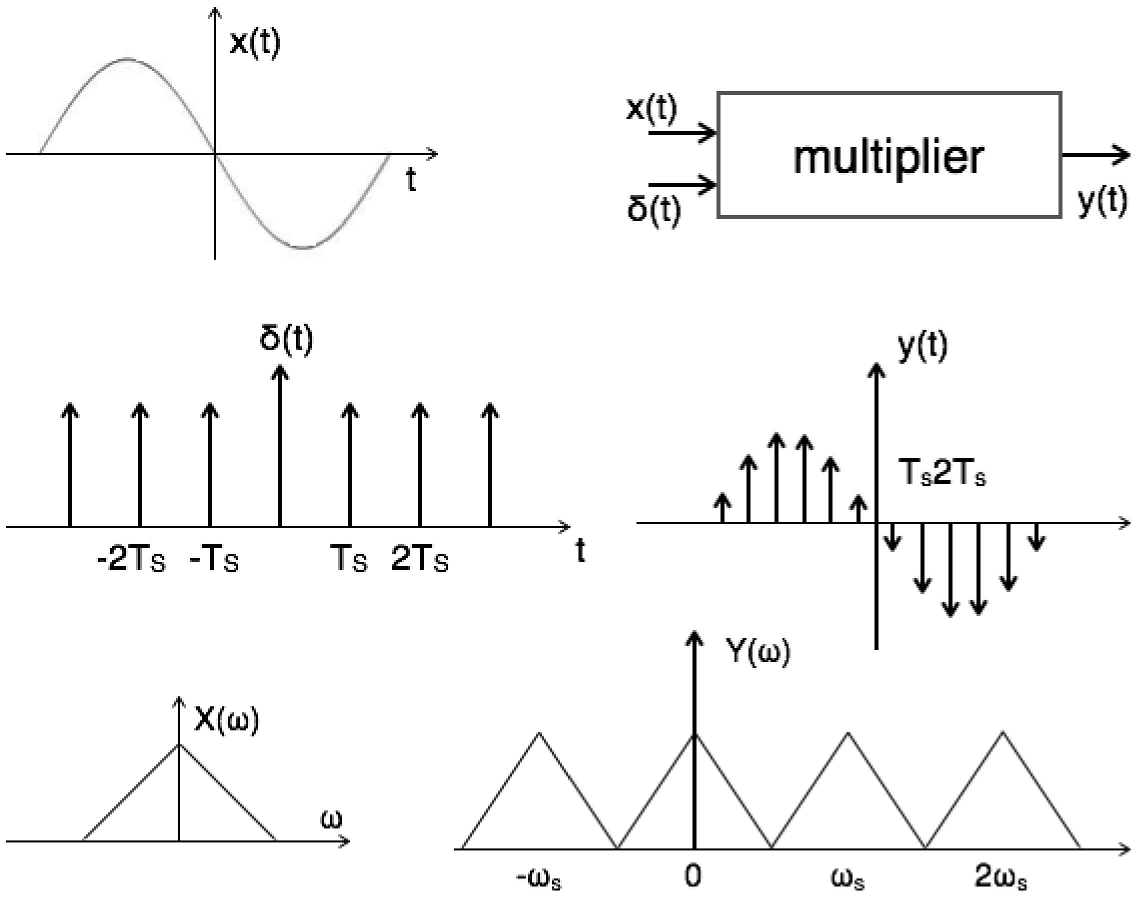


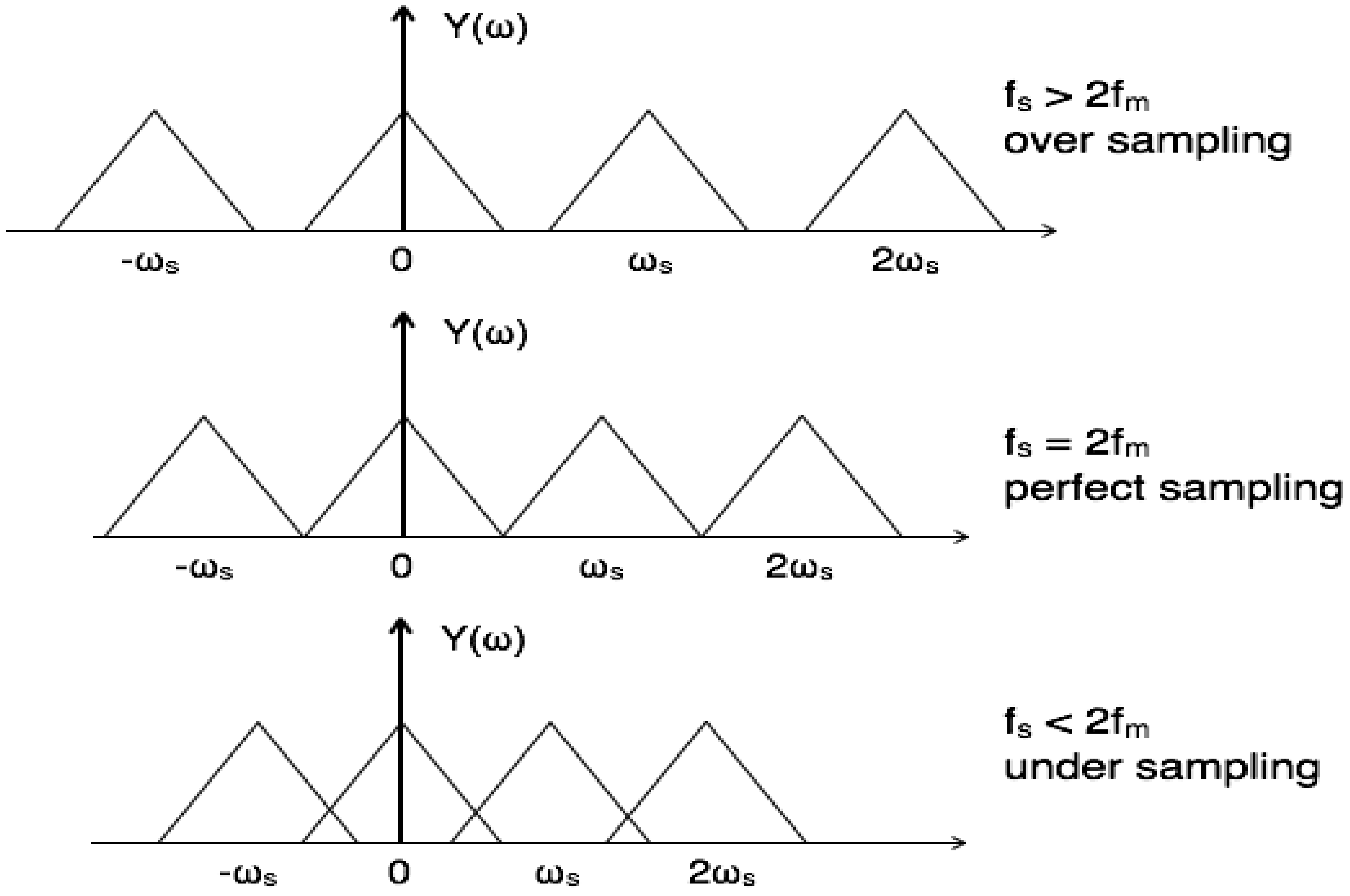
Sampling Theorem

- x(t) is analog signal.
- $\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$
- $y(t) = x(t)\,.\,\delta_T(t)$
- $Y(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_0)$
- Nyquist rate $f_s \geq 2f$

  where f is max freq of input

  signal.




Y(ω)
-ωs
0
ωs
2ωs
fs > 2fm
over sampling
Y(ω)
-ωs
0
ωs
2ωs
fs = 2fm
perfect sampling
Y(ω)
-ωs
0
ωs
2ωs
fs < 2fm
under sampling

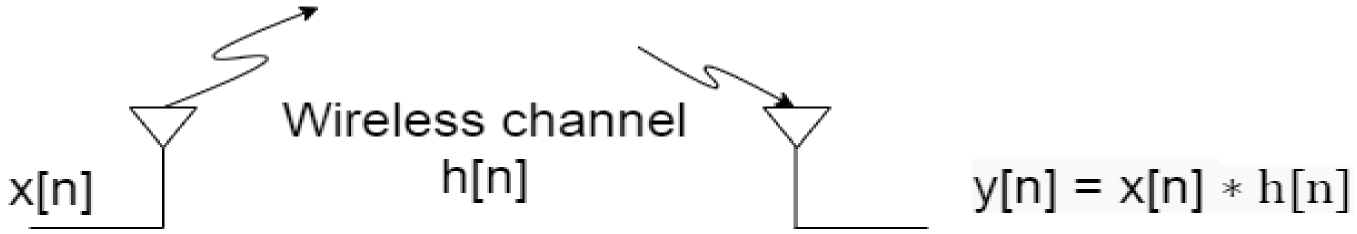
Convolution

Convolution of two sequence x[n] and h[n] is given as

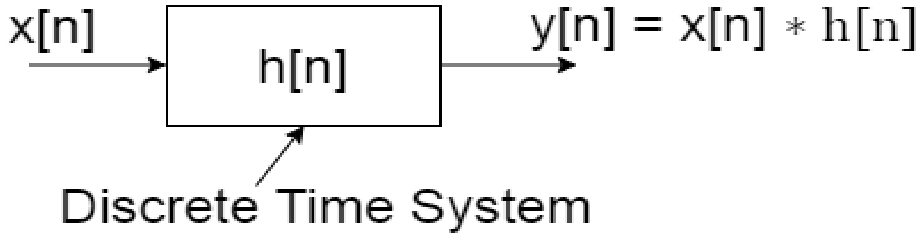
$$y[n] = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Application :

1. Wireless communication



2. Discrete time systems



Correlation

Correlation of two sequence x[n] and y[n] is given as

$$R_{xy}[k] = \sum_{n=0}^{\infty} x(n)\, y(n-k)$$

Application :

1. Finding Similarity between two signals or sequences.
2. Synchronization between base station and mobile station.

Downsampling

Given a discrete sequence x[n] , the downsampled signal y[n] is

$$\boldsymbol{y[n] = x[Mn]}$$

Application :

- To reduce the sampling rate of a signal.

Upsampling

Given a discrete sequence x[n] , the upsampled signal y[n] is

$$\boldsymbol{y[n] = \begin{cases} x[n/L], & if\ n\ is\ a\ multiple\ of\ L \\ 0, & otherwise \end{cases}}$$

Application :

- To Increase the sampling rate of a signal.