Linear Dynamical Systems & Kalman Filter

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April 27, 2022

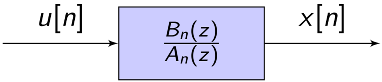
Linear Dynamical Systems

$$u[n] \longrightarrow \boxed{\frac{B_n(z)}{A_n(z)}} \longrightarrow x[n]$$

- LTI System

$$\sum_{k=0}^{M} a[k]x[n-k] = \sum_{k=0}^{N} b[k]u[n-k]$$

Linear Dynamical Systems



- LTI System

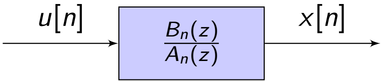
$$\sum_{k=0}^{M} a[k]x[n-k] = \sum_{k=0}^{N} b[k]u[n-k]$$

- Linear Dynamical System

$$\sum_{k=0}^{M} a_n[k]x[n-k] = \sum_{k=0}^{N} b_n[k]u[n-k] + w[n]$$

$$x[n] = -\sum_{k=1}^{M} a_n[k]x[n-k] + \sum_{k=0}^{N} b_n[k]u[n-k] + v_p[n]$$

Linear Dynamical Systems



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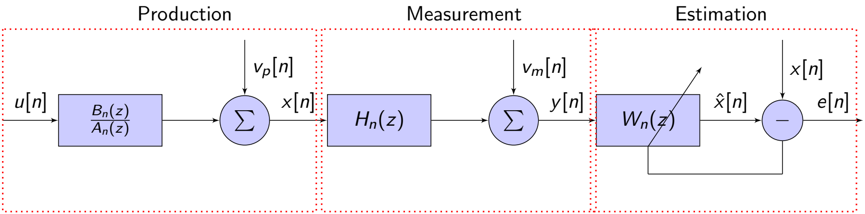
$$x[n] = \mathbf{a}^T\mathbf{x}[n-1] + \mathbf{b}^T\mathbf{u}[n] + v_p[n]$$

Basis for Kalman Filtering



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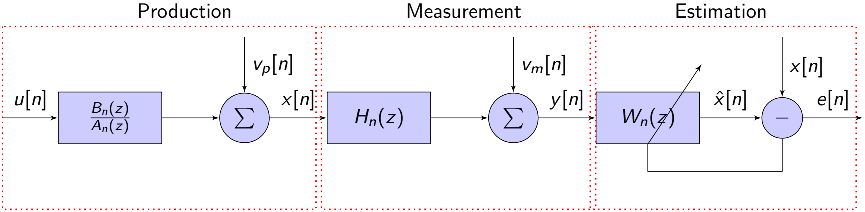

Production Model

$$\mathbf{x}[n] = \mathbf{A}_n\mathbf{x}[n-1] + \mathbf{B}_n\mathbf{u}_n + \mathbf{v}_p[n]$$

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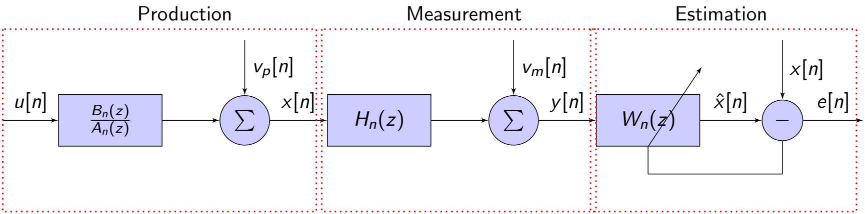

Production Model

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- $\mathbf{x}_n$ - State vector
- $\mathbf{A}_n$ - Transition matrix
- $\mathbf{u}_n$ - Control inputs
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- $\mathbf{v}_p[n] \sim \mathcal{N}(\mathbf{0}, \Sigma_p)$

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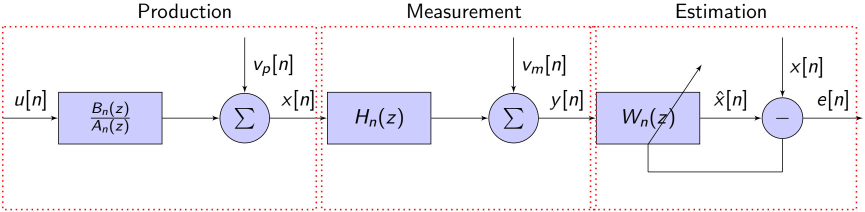

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Measurement Model

$$\mathbf{y}_n = \mathbf{H}_n \mathbf{x}_n + \mathbf{v}_m[n]$$

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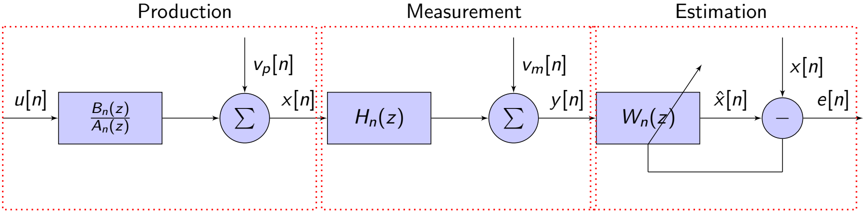

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Measurement Model

$$\mathbf{y}_n = \mathbf{H}_n\mathbf{x}_n + \mathbf{v}_m[n]$$

- $\mathbf{y}_n$ - Measurement vector
- $\mathbf{H}_n$ - Transformation matrix
- $\mathbf{v}_m[n] \sim \mathcal{N}(\mathbf{0}, \Sigma_m)$
- $\mathbf{v}_p[n] \perp \mathbf{v}_m[n]$

Vehicle Tracking

- Inst. position: $x[n]$
- Inst. velocity: $\dot{x}[n]$
- State vector: $\mathbf{x}[n] = [x[n] \ \dot{x}[n]]^{\mathsf{T}}$

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- Inst. position: $x[n]$
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- Mass of the vehicle: m
- Control input: $\mathbf{u}[n] = \frac{f_n}{m}$

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- State update:

$$x[n] = x[n-1] + \dot{x}[n-1]\Delta t + \frac{f_n}{2m}\Delta t^2$$

$$\dot{x}[n] = \dot{x}[n-1] + \frac{f_n}{m}\Delta t$$

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- Inst. position: $x[n]$
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$$\begin{bmatrix} x[n] \\ \dot{x}[n] \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x[n-1] \\ \dot{x}[n-1] \end{bmatrix} + \begin{bmatrix} \frac{(\Delta t)^2}{2} \\ \Delta t \end{bmatrix} \frac{f_n}{m}$$

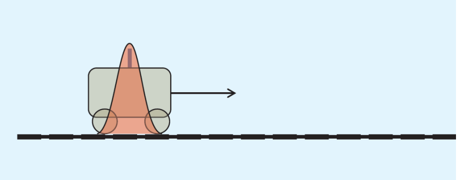
$$A_n = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, \; B_n = \begin{bmatrix} \frac{(\Delta t)^2}{2} \\ \Delta t \end{bmatrix}$$

Measurement

- Lidar reflections
- Shared GPS location
- H_n is the relation between $x[n$ and observation

Basic Idea behind Kalman Filtering

Position is known at $t = 0$



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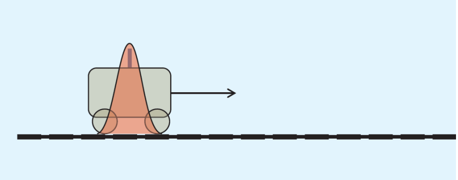
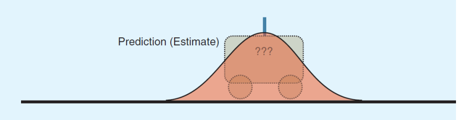


Predict the position at $t = 1$

$$\hat{x}[t = 1/t = 0] \sim \mathcal{N}(\mu_1, \sigma_1^2)$$



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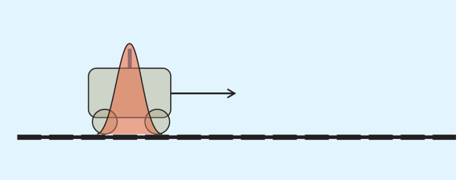

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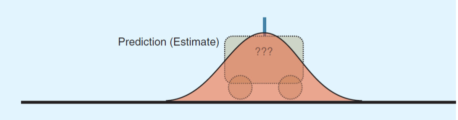
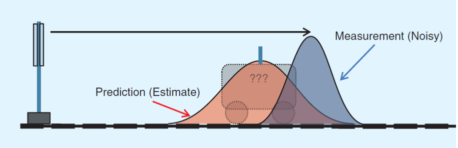


Noisy measurement: (Likelihood)

$y[1] = x[1] + v_m[1] \sim \mathcal{N}(\mu_2, \sigma_2^2)$



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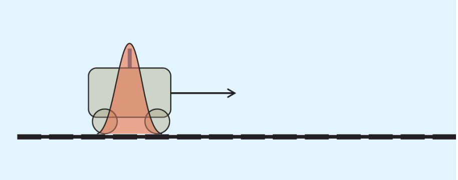

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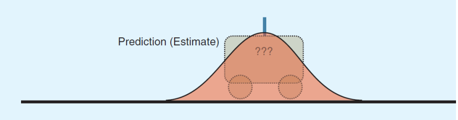

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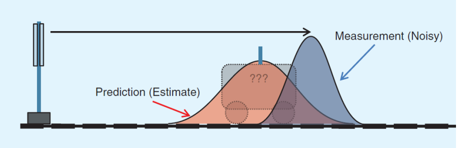
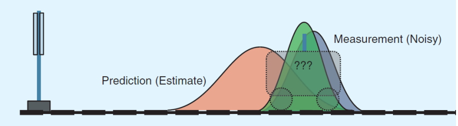


Posterior: $\hat{x}[1/1] \sim \mathcal{N}(\mu_f, \sigma_f)$



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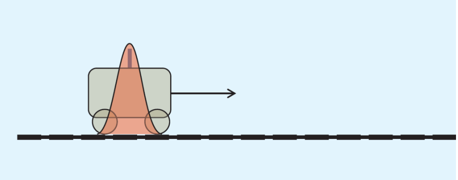

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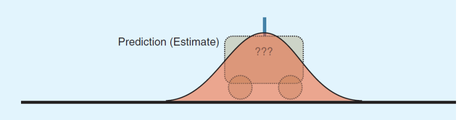

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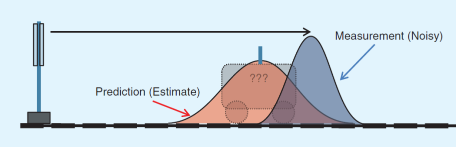

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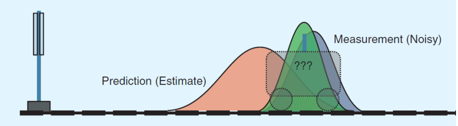

$$\mu_f = \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \qquad \sigma_f^2 = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

Kalman Filtering

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- Prediction Model: $\mathbf{x}[n] = \mathbf{A}[n]\mathbf{x}[n-1] + \mathbf{v}_p[n]$

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- A posteriori estimate ($\mathbf{y}[n]$ is observed)

$$\hat{\mathbf{x}}[n/n] = \mathbf{K}'[n]\hat{\mathbf{x}}[n/n-1] + \mathbf{K}[n]\mathbf{y}[n]$$

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- A posteriori error: $\mathbf{e}[n/n] = \mathbf{x}[n] - \hat{\mathbf{x}}[n/n]$
- Estimate $\mathbf{K}'[n]$ and $\mathbf{K}[n]$ to minimize MSE:

$$J[n] = \mathbb{E}[\|\mathbf{e}[n/n]\|^2]$$

Relation between $\mathbf{K}[n]$ and $\mathbf{K}'[n]$

$$\mathbf{e}[n/n] = \mathbf{x}[n] - \hat{\mathbf{x}}[n/n]$$

Relation between $\mathbf{K}[n]$ and $\mathbf{K}'[n]$

$$
\begin{aligned}
\mathbf{e}[n/n] &= \mathbf{x}[n] - \hat{\mathbf{x}}[n/n] \\
&= \mathbf{x}[n] - \mathbf{K}'[n]\hat{\mathbf{x}}[n/n-1] - \mathbf{K}[n]\mathbf{y}[n]
\end{aligned}
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&= \mathbf{x}[n] - \mathbf{K}'[n]\Big(\mathbf{x}[n] - \mathbf{e}[n/n-1]\Big) - \mathbf{K}[n]\Big(\mathbf{H}[n]\mathbf{x}[n] + \mathbf{v}_m[n]\Big)
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&= \Big(\mathbf{I} - \mathbf{K}'[n] - \mathbf{K}[n]\mathbf{H}[n]\Big)\mathbf{x}[n] + \mathbf{K}'[n]\mathbf{e}[n/n-1] - \mathbf{K}[n]\mathbf{v}_m[n]
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\end{aligned}$$

- For unbiased estimation of state vector, $\mathbb{E}[\mathbf{e}[n/n]] = 0$

$$\mathbf{K}'[n] = \mathbf{I} - \mathbf{K}[n]\mathbf{H}[n]$$

Relation between $\mathbf{K}[n]$ and $\mathbf{K}'[n]$

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- A posteriori estimate of the state is given by

$$\mathbf{x}[n/n] = \hat{\mathbf{x}}[n/n-1] + \mathbf{K}[n]\Big(\mathbf{y}[n] - \mathbf{H}[n]\hat{\mathbf{x}}[n/n-1]\Big)$$

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- $\alpha[n] = \mathbf{y}[n] - \hat{\mathbf{y}}[n/n-1]$ is referred to as innovation process

Estimating Kalman Gain

$$\mathbf{e}[n/n] = \Big(\mathbf{I} - \mathbf{K}[n]\mathbf{H}[n]\Big)\mathbf{e}[n/n-1] - \mathbf{K}[n]\mathbf{v}_m[n]$$

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\end{aligned}$$

$$J[n] = \mathbb{E}[\|\mathbf{e}[n/n]\|^2] = \mathbb{E}[\mathbf{e}^{\mathsf{T}}[n]\mathbf{e}[n]] = \mathrm{Tr}\{\mathbf{P}[n/n]\}$$

Estimating Kalman Gain

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\mathbf{e}[n/n] &= \Big(\mathbf{I} - \mathbf{K}[n]\mathbf{H}[n]\Big)\mathbf{e}[n/n-1] - \mathbf{K}[n]\mathbf{v}_m[n] \\
\mathbf{P}[n/n] &= \mathbb{E}[\mathbf{e}[n]\mathbf{e}^\mathsf{T}[n]] \\
&= \Big(\mathbf{I} - \mathbf{K}[n]\mathbf{H}[n]\Big)\mathbf{P}[n/n-1]\Big(\mathbf{I} - \mathbf{K}[n]\mathbf{H}[n]\Big)^\mathsf{T} + \mathbf{K}[n]\boldsymbol{\Sigma}_m\mathbf{K}^\mathsf{T}[n] \\
J[n] &= \mathbb{E}[\|\mathbf{e}[n/n]\|^2] = \mathbb{E}[\mathbf{e}^\mathsf{T}[n]\mathbf{e}[n]] = \mathrm{Tr}\{\mathbf{P}[n/n]\} \\
\nabla_\mathbf{K} J[n] &= -2\Big(\mathbf{I} - \mathbf{K}[n]\mathbf{H}[n]\Big)\mathbf{P}[n/n-1]\mathbf{H}^\mathsf{T}[n] + 2\mathbf{K}[n]\boldsymbol{\Sigma}_m = \mathbf{0}
\end{aligned}
$$

Estimating Kalman Gain

$$\mathbf{e}[n/n] = \Big(\mathbf{I} - \mathbf{K}[n]\mathbf{H}[n]\Big)\mathbf{e}[n/n-1] - \mathbf{K}[n]\mathbf{v}_m[n]$$

$$\mathbf{P}[n/n] = \mathbb{E}[\mathbf{e}[n]\mathbf{e}^T[n]]$$

$$= \Big(\mathbf{I} - \mathbf{K}[n]\mathbf{H}[n]\Big)\mathbf{P}[n/n-1]\Big(\mathbf{I} - \mathbf{K}[n]\mathbf{H}[n]\Big)^T + \mathbf{K}[n]\mathbf{\Sigma}_m\mathbf{K}^T[n]$$

$$J[n] = \mathbb{E}[\|\mathbf{e}[n/n]\|^2] = \mathbb{E}[\mathbf{e}^T[n]\mathbf{e}[n]] = \mathrm{Tr}\{\mathbf{P}[n/n]\}$$

$$\nabla_{\mathbf{K}} J[n] = -2\Big(\mathbf{I} - \mathbf{K}[n]\mathbf{H}[n]\Big)\mathbf{P}[n/n-1]\mathbf{H}^T[n] + 2\mathbf{K}[n]\mathbf{\Sigma}_m = \mathbf{0}$$

- Kalman gain is given by

$$\mathbf{K}[n] = \mathbf{P}[n/n-1]\mathbf{H}^T[n]\left[\mathbf{H}[n]\mathbf{P}[n/n-1]\mathbf{H}^T[n] + \mathbf{\Sigma}_m\right]^{-1}$$

- Posterior covariance $\mathbf{P}[n/n] = \Big(\mathbf{I} - \mathbf{K}[n]\mathbf{H}[n]\Big)\mathbf{P}[n/n-1]$

Implementing Kalman Filter

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- Observation vectors: $(\mathbf{y}[1], \mathbf{y}[2], \ldots, \mathbf{y}[n], \ldots)$

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$$\mathbf{K}[n] = \mathbf{P}[n/n-1]\mathbf{H}^\mathsf{T}[n]\Big(\mathbf{H}[n]\mathbf{P}[n/n-1]\mathbf{H}^\mathsf{T}[n] + \boldsymbol{\Sigma}_m\Big)^{-1}$$

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Extended Kalman Filter

- Nonlinear state equations
  - Prediction model: $\mathbf{x}[n] = \mathbf{A}_n(\mathbf{x}[n-1]) + \mathbf{v}_p[n]$
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  - $\mathbf{A}_n(.) : \mathbf{R}^{d_x} \rightarrow \mathbf{R}^{d_x}$ denotes nonlinear state-transition map
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  - $\bar{\mathbf{y}}[n] = \mathbf{H}_n(\mathbf{x}_n) - \mathbf{H}[n]\mathbf{x}[n]$ evaluated at $\hat{\mathbf{x}}[n/n-1]$
  - $\bar{\mathbf{x}}[n-1]$ and $\bar{\mathbf{y}}[n]$ are known at every n

Nonlinear State Equations in 2D

- Nonlinear prediction model

$$\begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix} = \begin{bmatrix} x_1[n-1] + x_2^2[n-1] \\ nx_1[n-1] - x_1[n-1]x_2[n-1] \end{bmatrix} + \begin{bmatrix} v_{p1}[n] \\ v_{p2}[n] \end{bmatrix}$$

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$$\mathbf{A}[n] = \frac{\partial \mathbf{A}_n(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} 1 & 2x_2 \\ n - x_2 & -x_1 \end{bmatrix} \Bigg|_{\mathbf{x}=\hat{\mathbf{x}}[n-1/n-1]}$$

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- Nonlinear measurement model

$$y[n] = x_1[n]x_2^2[n] + v_2[n]$$

$$\mathbf{H}[n] = \frac{\partial \mathbf{H}_n(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} x_2^2 & 2x_1x_2 \end{bmatrix} \Big|_{\mathbf{x}=\hat{\mathbf{x}}[n/n-1]}$$

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  Estimate linearized matrices $\mathbf{A}[n]$ & $\mathbf{H}[n]$

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Thank You!