भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Department of Electrical Engineering
IIT Hyderabad
EE5610/AI5000 - PRML

Classtest - 2 Max. Marks: 40

1. Consider two nonnegative numbers a and b, and show that, if $a \leq b$, then $a \leq \sqrt{ab}$. Use this result to show that, if the decision regions of a binary classifier are chosen to minimize the probability of misclassification, this probability will satisfy

$$p(\text{mistake}) \leq \int \{p(\mathbf{x}, C_1)\, p(\mathbf{x}, C_2)\}^{1/2}\, d\mathbf{x}$$

2. Consider a binary classification task in single dimension. The class conditional densities for the two classes are given by $\mathcal{N}(0, \sigma^2)$ and $\mathcal{N}(1, \sigma^2)$, respectively. Let the loss matrix associated with the classification task be

$$L = \begin{bmatrix} 0 & L_{12} \\ L_{21} & 0 \end{bmatrix}$$

Show that the threshold x_0 that minimizes the average risk is given by

$$x_0 = \frac{1}{2} - \sigma^2 \log \frac{L_{21} p[c_2]}{L_{12} p[c_1]}$$

where $p[c_k]$ is the prior probability of class k.

3. Consider a scenario, in which the points $\{(0,0), (0,1)\}$ belong to class C_1 and points $\{(1,1), (2,1)\}$ belong to C_2.
(a) Design a linear discriminant function using least squares approach. Sketch the decision boundary and identify the decision regions.
(b) Starting with an initial weight vector $\mathbf{0}$, show the perceptron weight updates till the algorithm converges. Sketch the resulting decision boundary, and identify the decision regions

4. A random variable X follows the Erlang density function given by

$$p(x/\theta) = \theta^2 x \exp(-\theta x)\, u(x)$$

where $u(x)$ denotes the unit-step function. Find the ML estimate of θ given N independent observations from the distribution.