KALMAN FILTER FOR LOCATION ESTIMATION

PRML
Final Project Presentation

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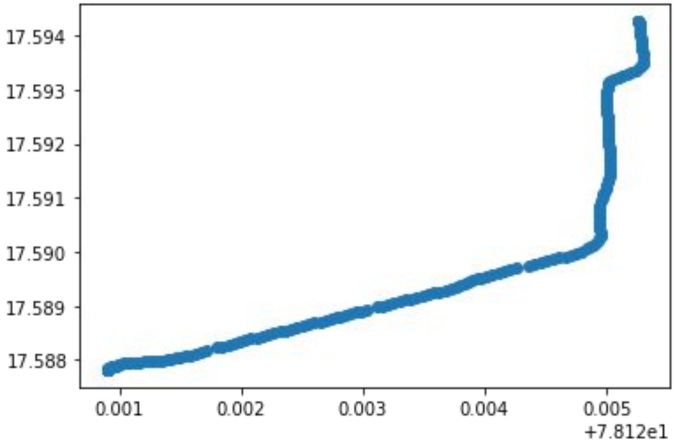
Introduction

- The Kalman Filter is one of the most important and common estimation algorithms. The Kalman Filter produces estimates of hidden variables based on inaccurate and uncertain measurements. Also, the Kalman Filter provides a prediction of the future system state based on past estimations.
- It is a **Recursive** data processing and **Optimal** estimation algorithm that predicts the parameters of interest such as location, speed and direction in the presence of noisy measurements.
- Doesn’t need to store all previous measurement and reprocess all data each time.

Project Overview

- In this project we are estimating the location of vehicle in XY plane
- Measurements are coming from noisy onboard location sensor (GPS) with standard deviation of 0.0003 in latitude and longitude



Kalman Filter Equations

Time Update ("Predict")

1. Extrapolate the state

$$\hat{\boldsymbol{x}}_{n+1,n} = \boldsymbol{F}\hat{\boldsymbol{x}}_{n,n} + \boldsymbol{G}\boldsymbol{u}_n$$

2. Extrapolate uncertainty

$$\boldsymbol{P}_{n+1,n} = \boldsymbol{F}\boldsymbol{P}_{n,n}\boldsymbol{F}^T + \boldsymbol{Q}$$

Measurement Update ("Correct")

1. Compute the Kalman Gain

$$\boldsymbol{K}_n = \boldsymbol{P}_{n,n-1}\boldsymbol{H}^T\left(\boldsymbol{H}\boldsymbol{P}_{n,n-1}\boldsymbol{H}^T + \boldsymbol{R}_n\right)^{-1}$$

2. Update estimate with measurement

$$\hat{\boldsymbol{x}}_{n,n} = \hat{\boldsymbol{x}}_{n,n-1} + \boldsymbol{K}_n\left(\boldsymbol{z}_n - \boldsymbol{H}\hat{\boldsymbol{x}}_{n,n-1}\right)$$

3. Update the estimate uncertainty

$$\boldsymbol{P}_{n,n} = (\boldsymbol{I} - \boldsymbol{K}_n\boldsymbol{H})\boldsymbol{P}_{n,n-1}(\boldsymbol{I} - \boldsymbol{K}_n\boldsymbol{H})^T + \boldsymbol{K}_n\boldsymbol{R}_n\boldsymbol{K}_n^T$$

Initial Estimate: $\hat{\boldsymbol{x}}_{0,0}$, $\boldsymbol{P}_{0,0}$

System Dynamics and Matrices

State Matrix

$$\boldsymbol{x_n} = \begin{bmatrix} \hat{x}_n \\ \hat{\dot{x}}_n \\ \hat{\ddot{x}}_n \\ \hat{y}_n \\ \hat{\dot{y}}_n \\ \hat{\ddot{y}}_n \end{bmatrix}$$

Production Model

$$\begin{bmatrix} \hat{x}_{n+1,n} \\ \hat{\dot{x}}_{n+1,n} \\ \hat{\ddot{x}}_{n+1,n} \\ \hat{y}_{n+1,n} \\ \hat{\dot{y}}_{n+1,n} \\ \hat{\ddot{y}}_{n+1,n} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & 0.5\Delta t^2 & 0 & 0 & 0 \\ 0 & 1 & \Delta t & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta t & 0.5\Delta t^2 \\ 0 & 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{n,n} \\ \hat{\dot{x}}_{n,n} \\ \hat{\ddot{x}}_{n,n} \\ \hat{y}_{n,n} \\ \hat{\dot{y}}_{n,n} \\ \hat{\ddot{y}}_{n,n} \end{bmatrix}$$

Measurement Model

$$\begin{bmatrix} x_{n,measured} \\ y_{n,measured} \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} x_n \\ \dot{x}_n \\ \ddot{x}_n \\ y_n \\ \dot{y}_n \\ \ddot{y}_n \end{bmatrix}$$

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Process Noise Matrix

$$\boldsymbol{Q} = \begin{bmatrix} \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} & \frac{\Delta t^2}{2} & 0 & 0 & 0 \\ \frac{\Delta t^3}{2} & \Delta t^2 & \Delta t & 0 & 0 & 0 \\ \frac{\Delta t^2}{2} & \Delta t & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} & \frac{\Delta t^2}{2} \\ 0 & 0 & 0 & \frac{\Delta t^3}{2} & \Delta t^2 & \Delta t \\ 0 & 0 & 0 & \frac{\Delta t^2}{2} & \Delta t & 1 \end{bmatrix} \sigma_a^2$$

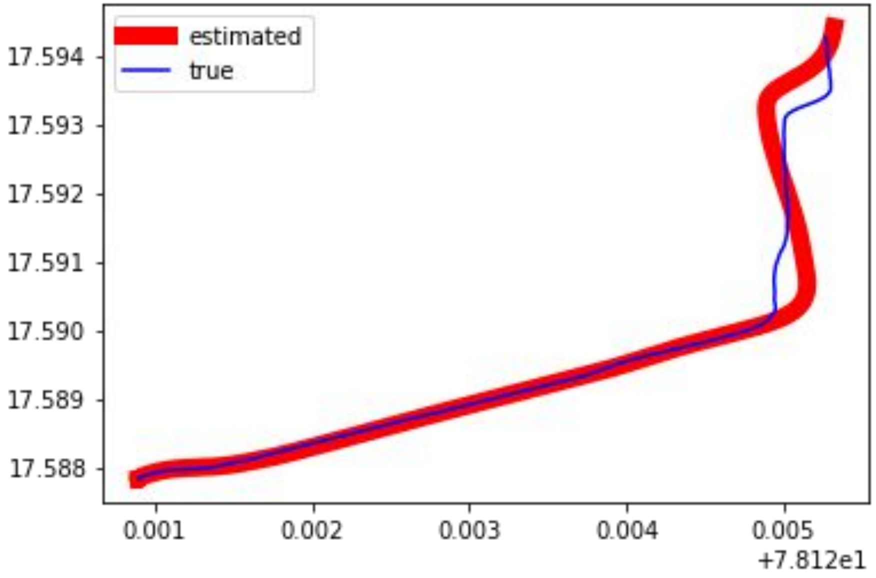
Measurement Covariance

$$\boldsymbol{R_n} = \begin{bmatrix} \sigma_{x_m}^2 & 0 \\ 0 & \sigma_{y_m}^2 \end{bmatrix}$$

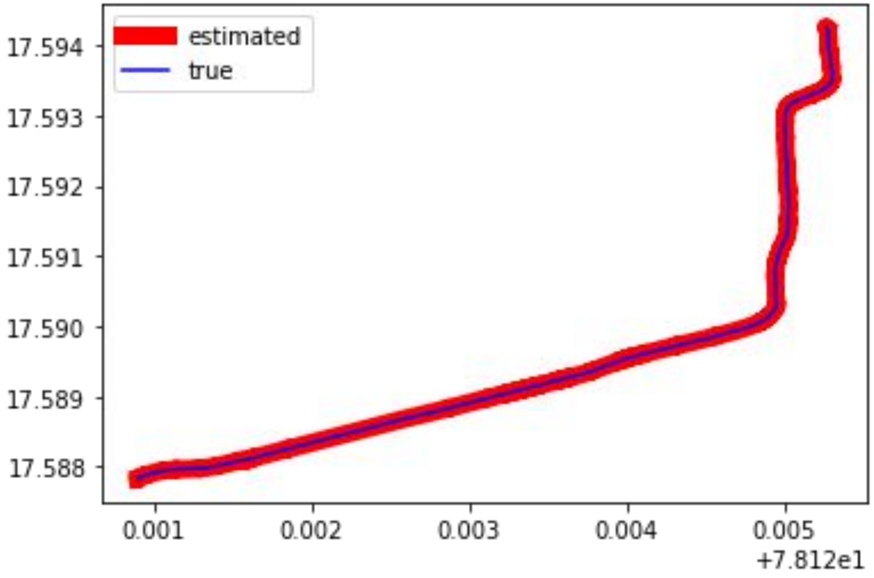
$\sigma_a = 0.0001$

$\sigma_x = 0.00003$ $\sigma_y = 0.00003$

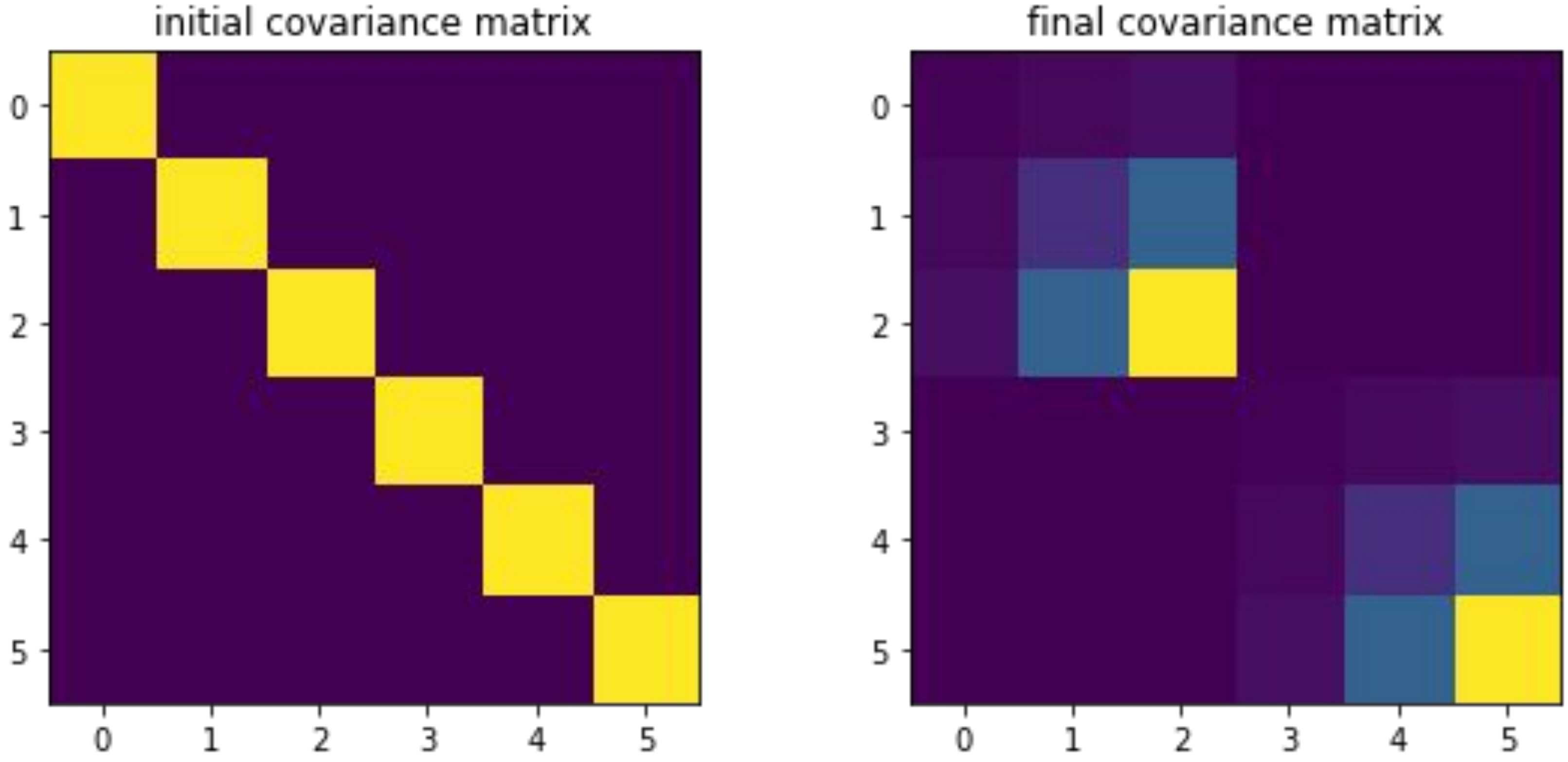
Observations



With Low Process Noise



With Increased Process Noise


initial covariance matrix
final covariance matrix
0
1
2
3
4
5