Single-Channel Speech Enhancement

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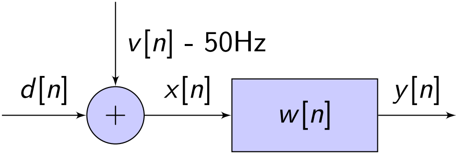
November 24, 2022

Classical Filters - Known Noise



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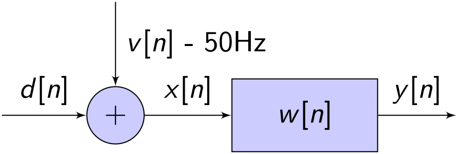

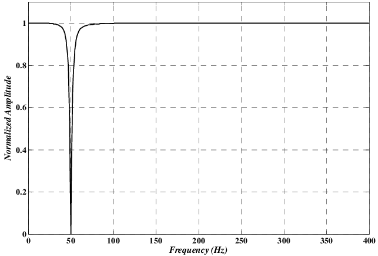

- **Known, stationary, and nonoverlapping** distortion at - 50Hz.
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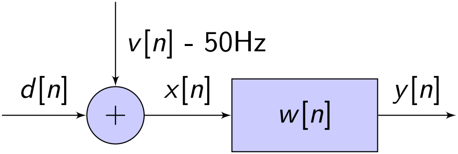

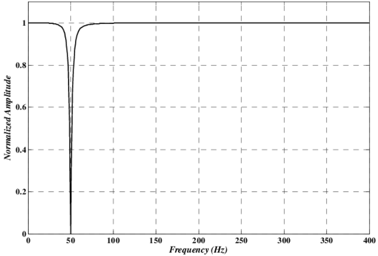

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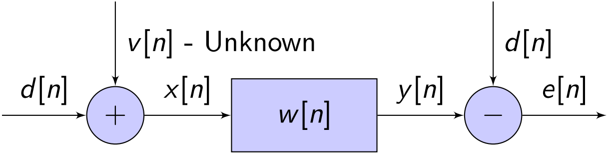
- The coefficients of w[n] are predetermined.
- What if noise characteristics are not known?

The Wiener Filter - Unknown Noise



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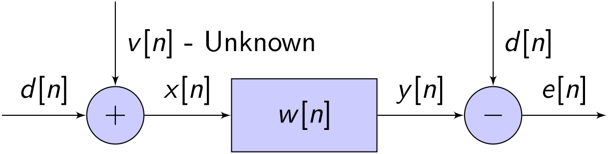

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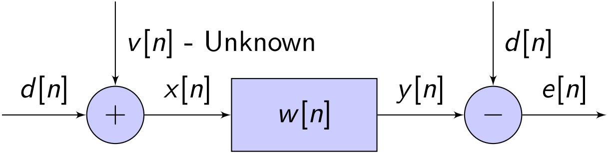

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- Error in the estimation (innovation process) is

$$e[n] = d[n] - \sum_{k=-\infty}^{\infty} w[k]\, x[n-k]$$

Wiener Estimates

- Estimate the filter coefficients that minimize mean squared error
$$J(\mathbf{w}) = \mathbb{E}[|e[n]|^2]$$
- Equating partial derivatives of $J(\mathbf{w})$ w.r.t $w[m]$ to zero

$$\frac{\partial J(\mathbf{w})}{\partial w[m]} = \frac{\partial J(\mathbf{w})}{\partial e[n]} \frac{\partial e[n]}{\partial w[m]} = 0 \implies \mathbb{E}[e[n]x[n-m]] = 0$$

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- $d[n]$ is estimated from noisy observation $x[n] = d[n] + v[n]$ as
$$\hat{d}[n] = w[n] * x[n] \qquad \hat{D}(j\omega) = W(j\omega)X(j\omega) = \frac{P_{XD}(j\omega)}{P_{XX}(\omega)}X(j\omega)$$

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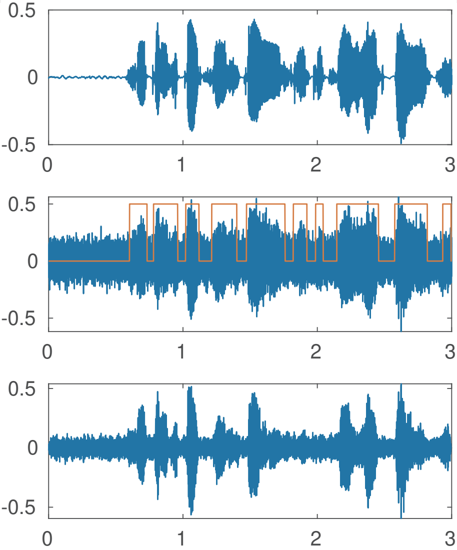
- Estimate ACS of the desired signal as $r_{DD}[k] = r_{XX}[k] - r_{VV}[k]$
- r_{XD} is estimated from temporal average. (Assumption: Ergodicity)

$$r_{XD}[k] = \mathbb{E}[X[n]D[n+k]] \approx \sum^{\infty} x[n]d[n+k]$$

Speech Enhancement



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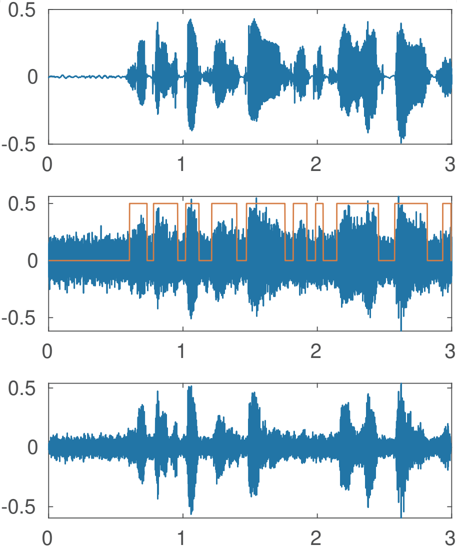

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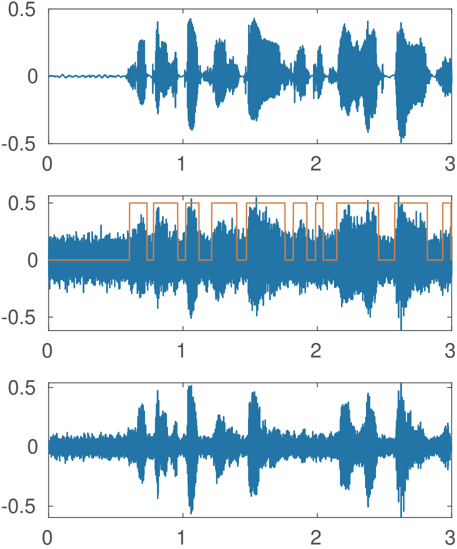

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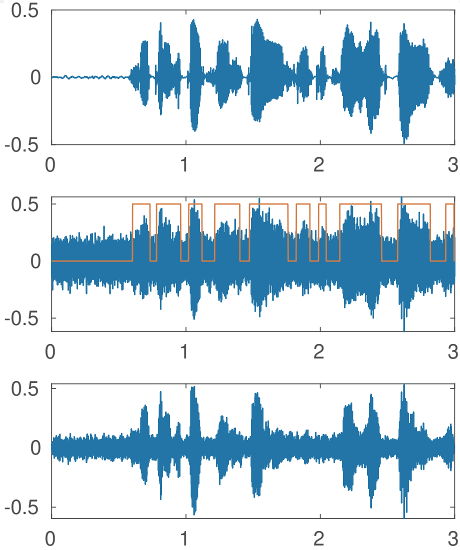

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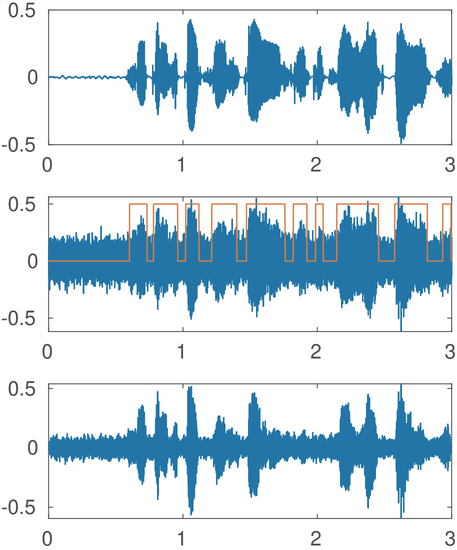

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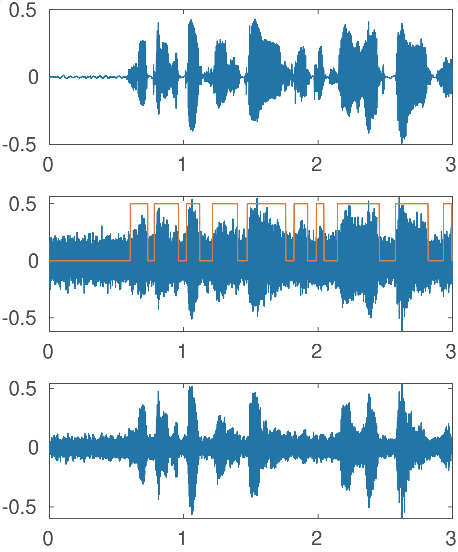

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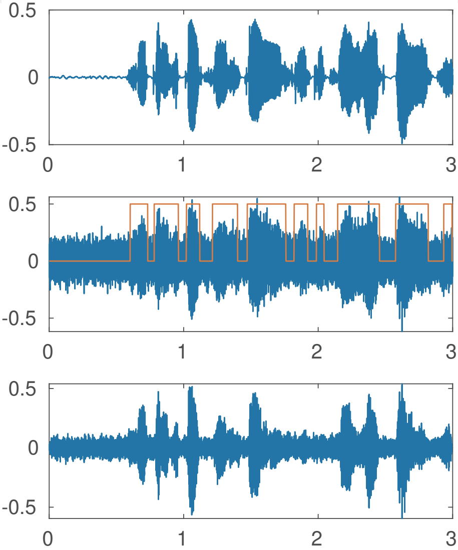

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Spectral subtraction!

Summary of Wiener Filters

- Desired signal is linearly related to the observed signal
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- Wiener filter can be interpreted as $\dfrac{\text{Clean Signal Power}}{\text{Noisy Signal Power}}$
- Wiener filter estimates this ratio from the given noisy observation
- DNNs estimates this ratio through supervised learning
- DNNs learn a nonlinear map between noisy speech signal and the ratio

DNN Approaches to Speech Enhancement

- Extract speech signal $x[n]$ from noisy mixture $x[n] = d[n] + v[n]$
- Time-Domain Approaches directly regress $\hat{d}[n] = f(x[n], \mathbf{W})$
- Frequency-Domain approaches operate in the STFT domain $X[n, k]$
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$$\hat{D}[n, k] = f(X(n, k], \mathbf{W})$$

  - Estimate a mask $M[n, k]$ to retrieve $\hat{D}[n, k]$ from $X[n, k]$

$$\hat{X}[n, k] = \hat{M}[n, k]Y[n, k]$$

- Frequency domain approaches operate either on magnitude spectrum or complex spectrum

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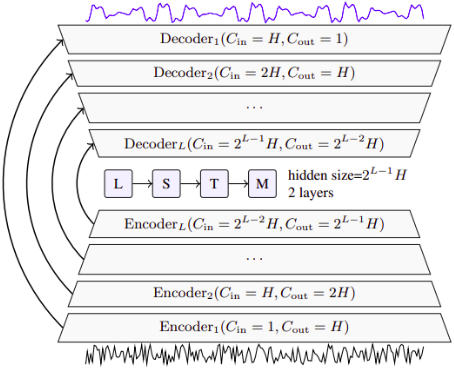
- Compact models with lesser parameters
  - Complexity can be high!
- Slower inference- sequential estimation even with in each hidden layer
- Vulnerable to linear changes: $x[n]$ and $\alpha x[n]$ lead to different results

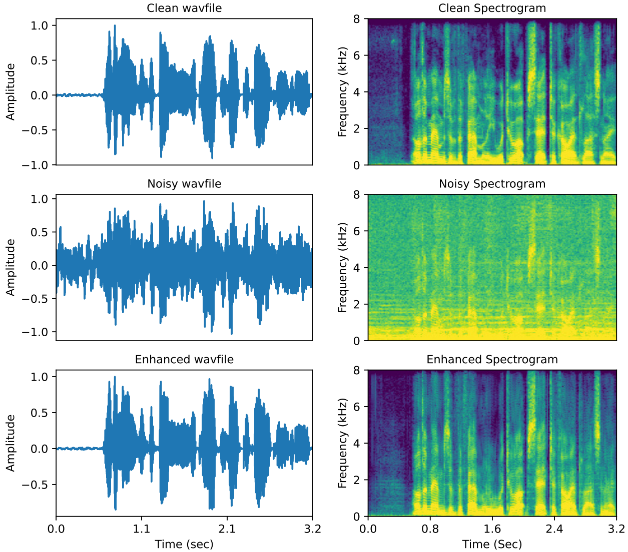
$$f(\alpha y, \mathbf{w}) \neq \alpha f(y, \mathbf{w}) \qquad 10dB \downarrow \text{for} \alpha = 2$$

Time-domain SE

- Encoder-decoder architectures with skip connections are more popularly used to learn the mapping function in time-domain.



Enhanced signals using DEMUCS



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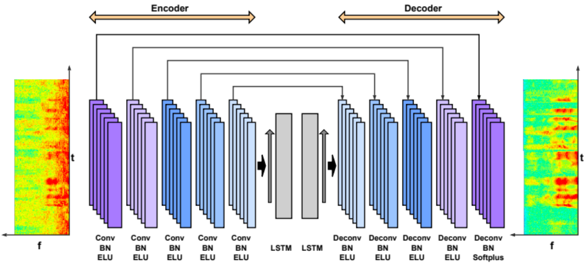
  - Loss function on spectral masks may not be perceptually relevant
- Magnitude-only vs Complex domain networks
  - In magnitude-only processing, noisy phase is reused for reconstruction
  - Complex networks offers phase enhancement - but complex masks are not bounded, definition of complex nonlinearity is not clear
  - Complex networks may offer improvement only for correlated noise - but bulkier networks

Regressing Clean Spectrum

- Encoder-decoder architecture with skip connections
  - Encoder projects the noisy spectrum to a lower dimensional space
  - Signal is compressible, not noise!
  - Decoder progressively upsamples the compressed representation
  - The skip connections are essential for energy mapping.
  - Issue: One-to-Many mapping.



Frequency Domain Masking

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- Spectral Magnitude Mask (SMM)

$$M[n,k] = \frac{|D[n,k]|}{|X[n,k]|}$$

Incorporating Phase Information

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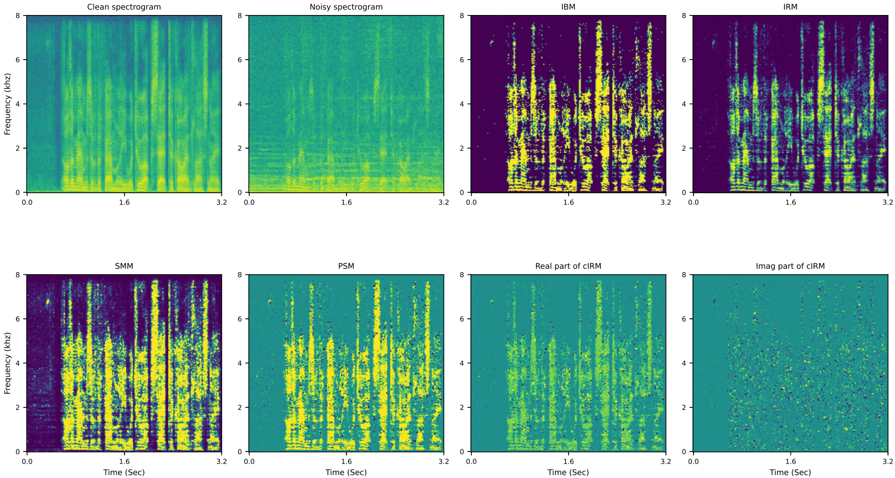
- Complex Ideal Ratio Mask (cIRM)

$$M[n,k] = \frac{D[n,k]}{X[n,k]}$$

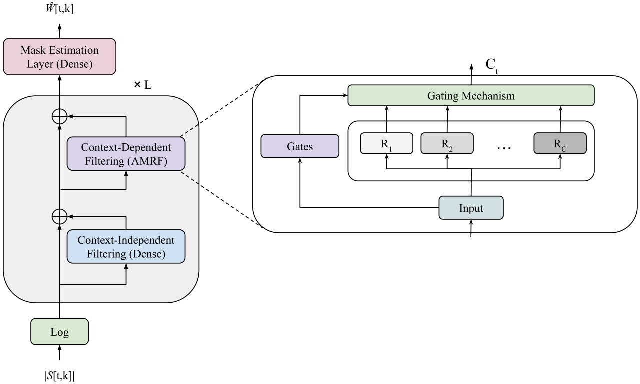
Need to estimate both real and imaginary parts of the cIRM

Illustration of Spectral Masks

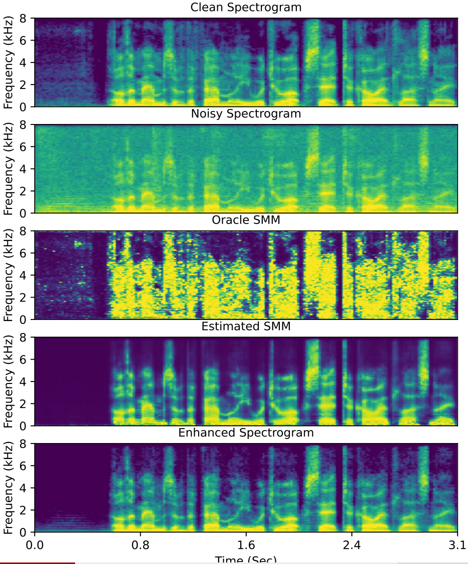


Spectral masking

- Mask estimation requires time-varying filter
- Estimate TV filter coefficients using CNN in log-spectral domain
- Gating mechanism to adaptively select the filter order



Enhancement using TVCN



Comparative Analysis

Method	Domain	PESQ	Remarks
Wiener Filter	T	2.32	Lightening Fast
TasNet	T	2.57	Linear scaling
DEMUCS	T	3.04	Huge Computation
C-UNet	F	2.87	Bulky
T-GSA	F	3.06	Parallel
TVCNN	F	3.08	Compact

Summary

- **Classical filters**
  - Desired filter characteristics are prespecified
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- **Neural Networks**
  - Data-dependent nonlinear transformation
  - RNNs offer nonlinear state-space models to capture sequence info.
  - FFNNs can be tweaked to incorporate sequence information
  - Missing theoretical guarantees & difficult to interpret its operation

Thank You!