Indian Institute of Space Science and Technology – Thiruvananthapuram

MA622 Pattern Recognition and Machine Learning

Assignment-1

Date: 23-01-2015

1. Show that the set X of all integers with metric defined by $d(m,n) = |m-n|$ is a complete metric space.

2. Show that $d(x,y) = \sqrt{|x-y}$ defines a metric on the set of all real numbers.

3. Show that the closure $\bar{Y}$ of a subspace Y of a normed space X is again a vector space.

4. Show that in an inner product space , $x \perp y$ iff $||x+\alpha y|| \geq ||x|| \forall \alpha \in \mathbb{R}$.

5. Find $\langle u,v \rangle$, where $v = (1+2i, 3-i)^T, u = (-2+i, 4)^T$.

6. Which of the following subsets of R^3 constitute a subspace of R^3?$[x = (\eta_1, \eta_2, \eta_3)^T]$
   (a) All x with $\eta_1 = \eta_2$ and $\eta_3 = 0$.
   (b) All x with $\eta_1 = \eta_2 + 1$

7. Show that the norm $||x||$ is the distance from x to 0.

8. If in an inner product space $\langle x,u \rangle = \langle x,v \rangle$ for all x, show that $u = v$.

9. Prove that $||T_1T_2|| \leq ||T_1|| \, ||T_2||; ||T^n|| \leq ||T||^n$.

10. For a real inner product space prove that $\langle x,y \rangle = 1/4(||x+y||^2 - ||x-y||^2)$.

11. Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x,y) = (x,0)$. Is T a linear operator?

12. Show that a discrete metric space is complete.

13. Descibe Weistrass appoximation theorem.

Notes

- **Assignment has to be written in latex**.
- All the files related with the assignment should be saved in a single folder and send to sumitra@iist.ac.in.
- Last date of submission: 02-02-2015.
- **As far as assignments are concerned, students are expected to observe academic honesty and integrity. Though the students can collaborate and discuss, copying directly other students' assignment or allowing your own assignment to be copied constitute academic dishonesty and is highly discouraged.**