8 Appendix C: Institutional details of orbit use

8.1 International laws regarding space traffic control

Orbits are inherently global resources, and space law is fragmented across nations and documents. Space law spans domestic policies, international treaties, bilateral agreements, and guidelines. Not all agreements are signed by all spacefaring nations, and many are non-binding. Most of the agreements are vague and suffer from enforcement problems. Four of the most relevant international agreements relating to orbit management are the 1967 Outer Space Treaty, the 1972 Liability Convention, the 1975 Registration Convention, and the 2007 COPUOS Guidelines.[25]

1967 Outer Space Treaty The Outer Space Treaty[26] established the legal framework for peaceful uses of outer space. Article 2 of the Treaty designates outer and orbital space as common pool resources, to be used "for the benefit of all" humankind. The only explicit restrictions are on military uses and claims of national sovereignty; the state of resource use is left ambiguous. The Treaty does not mention debris, only stating that nations should avoid causing (undefined) "harmful contamination" of outer space.

1972 Liability Convention The Liability Convention[27] established the framework for tort law of space activities. However, the Convention focused more on damage to terrestrial objects from re-entry than on damages to orbital objects which occur in space. "Damage" in this Convention is defined only in relation to realized outcomes for people and property, rather than potential outcomes caused by the environment. Additionally, the Convention places liability for such damages on the launching state rather than the launching entity. This has motivated nations like the US to require satellite owners insure their satellites, with the federal government indemnifying losses beyond a certain amount. The EU has different insurance requirements, with a similar motivation. There is no liability attached to producing debris in orbit, only to attributable damages. Liability extends to damage to people or property caused by re-entry. Such attribution is difficult in space, where damages may be caused by difficult-to-detect fragments of unknown origin.

1975 Registration Convention The Registration Convention[28] requires nations to register space objects launched from or by that nation with the UN Secretary-General. The responsibility for ensuring compliance lies with the launching state, with the UN being responsible for integrating

25 A more detailed analysis of these laws can be found in Akers (2012).

26 The "Treaty on Principles Governing the Activities of States in the Exploration and Use of Outer Space, Including the Moon and Other Celestial Bodies."

27 The "Convention on International Liability for Damage Caused by Space Objects."

28 The "Convention on Registration of Objects Launched into Outer Space."

all the registrations and publishing a publicly available international registry of objects in orbit. The Convention only requires basic orbital information to be provided: orbital parameters to ascertain the object's initial path, and the general function. It does not require more detailed information, such as orbit changes or satellite positions, or even continuous updates. The Convention does not offer a deadline by which a launched object must be registered, or specify a penalty or enforcement mechanism for noncompliance.

2007 COPUOS Guidelines The COPUOS Guidelines[29] are seven nonbinding guidelines for mitigating artificial space debris. This was the first international treaty to recognize the problem of orbital debris, though it is unenforceable and only focused on technical mitigation practices rather than economic control measures. Senechal (2007) discusses some features that an enforceable international space debris convention should possess; the COPUOS guidelines are a step in this direction, but they do not contain the kinds of clear definitions and enforceable provisions required.

In general, the legal doctrine of *nemo dat quod non habet* ("no one gives what he doesn't have") means that the Outer Space Treaty prevents states from issuing rights over orbital paths. Since they lack sovereignty in space, states do not have rights to give[30]. States retain their authority over launches within their borders, and satellites operated by firms within their borders.

8.2 US institutions regarding space traffic control

International law places responsibility for objects launched to space on the nations from which the objects are launched and the nations in which the launching entities are registered. This means that understanding the legal status of space objects also requires some background on national laws. A majority of currently-operational satellites were launched from the United States, Russia, and China. In this section, I focus on institutions in the United States.

In the US, the Federal Aviation Administration's Office of Commercial Space Launch handles issues related to launches and reentry, including issuing launch permits. The Department of Commerce currently regulates remote sensing satellite systems, with a focus on controlling the

[29] The "Committee on the Peaceful Uses of Outer Space (COPUOS) 2007 Nonbinding Guidelines for Space Debris Mitigation."

[30] Salter and Leeson (2014) argue that this poses no difficulty to efficient decentralized orbit use management, as privately enforced property rights have arisen in other common resource settings on Earth. Weeden and Chow (2012) offer suggestions guided by Ostrom's principles of commons management for developing decentralized protocols for orbit use. An interesting question, not pursued in detail here, is the degree to which physical dynamics allow cooperative mechanisms to operate. Section 6.1 in the Appendix examines the stability of cooperation with debris removal plans.

resolution and coverage of images that are sold. The DOC is set to take control of regulating on-orbit activities by US entities over the next two years. The FCC regulates satellite activities by US telecom entities through radio spectrum controls. Traffic management operations so far have therefore been limited to controlling launches or operations by entities primarily based within a country's national borders - for example, the FCC can use its control of spectrum rights to deny service to poorly-behaving telecom operators who want to provide service to North America, but have no leverage over providers who are interested in serving China. The patchwork of laws around the world has already led some firms to evade regulation in their home area by launching from another (for example, Dvorsky (2018)). Space Policy Directive-3, a presidential memorandum issued in June of 2018, directs federal agencies to cooperate in developing a framework for a national space traffic management policy centered on a new object tracking infrastructure to better predict the aggregate collision rate and coordinate collision avoidance maneuvers.

8.3 The militarization of space

Military use of space accounts for over 10% of known active satellites in orbit Union of Concerned Scientists (2018). Although I focus on commercial orbit use with no reference to military use, some understanding of military incentives is necessary to place military use in the appropriate context. Sweeney (1993) describes some of generic logic for the effects of national security concerns on nationally optimal depletable resource extraction, whereby the socially optimal extraction rate may exceed the privately optimal extraction rate. Similarly, national security concerns may drive a national fleet planner to launch more than their national space sector would.[31]

Military uses of space have been among the largest sources of new debris fragments, particularly anti-satellite missile testing Liou and Johnson (2009a); Bradley and Wein (2009). Anti-satellite missile uses can trigger Kessler Syndrome. The space traffic control policies described here are not designed to control anti-satellite missile use. International agreements around space traffic control in the future are likely to be shaped in large part by the military stances of major space-faring nations, most notably the United States, Russia, and China. The current lack of binding international space traffic agreements is partly due to a lack of science and consensus around how such controls should be designed and why, but also partly due to military-related incentives facing the governments of space-faring nations which would be party to such agreements.

[31] The depletable resource case may be more apt than the renewable resource one for some orbital regimes, particularly higher-altitude ones such as GEO or high-Earth orbits. Natural rates of orbital decay in these regimes can be on the order of millennia - long enough that Kessler Syndrome without removal technologies can render them effectively unusable (the orbital volume "completely extracted") for economic purposes.

Article 4 of the Outer Space Treaty declares that state parties "undertake not to place in orbit around the Earth any objects carrying nuclear weapons or any other kinds of weapons of mass destruction, install such weapons on celestial bodies, or station such weapons in outer space in any manner." It also forbids establishing military installations, conducting weapons tests, or any other non-peaceful activities on the Moon and other celestial bodies. Despite these provisions, the Outer Space Treaty does not explicitly prohibit using near-Earth space for reconnaissance, terrestrial warfare coordination, or even outright conflict so long as "weapons of mass destruction" are not used in orbit. The ongoing militarization of space has therefore involved these uses, with the US government being the largest such user of orbital space. The US government has not yet supported international treaty efforts to limit the militarization of space. Shimabukuro (2014) offers an explanation for the lack of more international regulation on space militarization in light of rising tensions between the US and China. The core of Shimabukuro's explanation is that the US benefits from high ambiguity over acceptable uses of outer space given the US's high level of military dependence on space systems. This allows the US to induce its adversaries to spend ever-increasing amounts on developing comparable space capabilities, potentially collapsing their warfighting capabilities without a battle.

9 Appendix D: Model extensions and supplemental properties

9.1 Spectrum use management and price effects

So far I have assumed that there is no spectrum congestion from satellites and that the entry of new firms does not affect the price of satellite services. In practice, radio frequency interference is one of the major concerns of space traffic control. However, policy to manage spectrum use is not a focus of this paper because it is generally handled well by existing institutions Johnson (2004). The effect of optimally managed spectrum congestion on the expected collision risk in a deterministic setting is shown in the appendix of Rao and Rondina (2018). In this section, I adapt the result to this paper's setting and show that permits or fees for spectrum use can approximate stock controls.

Spectrum congestion degrades the quality of the signals to and from satellites. This makes the per-period output from a satellite decreasing in the number of orbital spectrum users. For simplicity, suppose that all satellites in orbit use enough spectrum to have some congestion impact. The per-period return function is then $\pi = \pi(S), \pi'(S) < 0$, and the one-period rate of return on a satellite is $\pi(S)/F = r_s(S)$. Assuming spectrum is optimally managed, firms will account for their marginal impact on spectrum congestion when they launch their satellite. The open access equilibrium condition, equation 11, becomes

$$E_t[\ell_{t+1}] = r_s(S_{t+1}) - r + r_s'(S_{t+1}). \tag{102}$$

Satellite owners would internalize the final term, $r'(S_{t+1})$, through a permit or fee system. Though spectrum permits may be purchased before the satellite is launched, their continued use is contingent on the firm abiding by non-interference protocols and any other stipulations by the appropriate regulatory body. Similarly, an optimal fee for spectrum use would adjust to reflect the marginal spectrum congestion from another broadcasting satellite. In general, regulated spectrum use will adjust the equilibrium collision rate to be

$$E_t[\ell_{t+1}] = r_s(S_{t+1}) - r - q_{t+1}, \tag{103}$$

where q_{t+1} is the spectrum use fee or permit price. Note that equation 103 is similar to equation 23,

$$\begin{aligned} \pi &= rF + E_t[\ell_{t+1}]F + p^s_{t+1} \\ \implies E_t[\ell_{t+1}] &= r_s - r - \frac{p^s_{t+1}}{F}. \end{aligned} \tag{104}$$

This suggests another avenue for controlling the equilibrium collision rate. By setting the price of spectrum use, q_{t+1}, equal to the sum of marginal spectrum and collision risk congestion costs, $r_s'(S_{t+1}) + E_t[\xi(S_{t+1}, D_{t+1})]$, a spectrum regulator can implement an optimal stock control. More generally, this would be an optimal space traffic control in the sense of Johnson (2004), as it would account for both radio frequency and physical interference.

The same argument applies to price reductions due to downward-sloping demand. Some satellite-using industries, such as telecommunications in many regions, compete with terrestrial alternatives. Bertrand competition between satellite-provided and terrestrially-provided services, with the terrestrial alternatives acting as the low-price alternative, would place an upper bound on π. In regions where satellite-provided services are more expensive, π would be constant with respect to orbital congestion and debris growth. In industries where satellite-provided services compete with no or high-cost terrestrial alternatives, such as satellite imaging, the price effects of new entry would reduce collision risk and increase the sensitivity of the launch rate to the number of firms in orbit. Price changes on their own cannot prevent Kessler Syndrome and will not ensure optimal orbit use (Rao and Rondina, 2018), but they still approximate a stock control as described in equation 103.

9.2 Mandatory satellite insurance

Can insurance markets correct the orbital congestion externality in the absence of active debris removal? Suppose that satellites were required to be fully insured against loss once they reached orbit and the satellite insurance sector was perfectly competitive. The insurance payment will act as a stock control, so the only question remaining is how the insurance industry will price the product. Denote the price of insurance in period t by p_t, and the profits of the insurance sector by I_t.

$$Q(S_t, D_t, \ell_t, p_t) = \pi - \underbrace{p_t}_{\text{Insurance premium}} + (1-\ell_t)F + \underbrace{\ell_t F}_{\text{Insurance payout}} = \pi - p_t + F \tag{105}$$

$$I(S_t, \ell_t) = \underbrace{p_t S_t}_{\text{Inflow of premium payments}} - \underbrace{\ell_t S_t F}_{\text{Outflow of reimbursements}} . \tag{106}$$

Competitive insurance pricing With competitive insurance pricing, satellite insurance will be actuarially fair. Plugging this price into the open access equilibrium condition, we can solve for the loss rate under mandatory insurance:

$$p_t : I(S_t, \ell_t) = 0 \implies p_t = \ell_t F \tag{107}$$

$$\pi = rF + E_t[\ell_{t+1}]F + p_{t+1} \tag{108}$$

$$\implies E_t[\ell_{t+1}] = r_s - r. \tag{109}$$

Proposition 14. *(Competitive insurance won't change collision risk) The equilibrium collision risk given mandatory satellite insurance with actuarially fair pricing is the same as the equilibrium collision risk given uninsured open access.*

Proof. From equation 109, the equilibrium expected loss rate with actuarially fair insurance is

$$E_t[\ell_{t+1}] = r_s - r.$$

From equation 51, the equilibrium expected loss rate with no insurance is

$$E_t[\ell_{t+1}] = r_s - r.$$

□

Regulated insurance pricing As in the case of spectrum management policies, mandatory satellite insurance premiums approximate a stock control. This suggests another avenue by which a regulator could induce optimal orbit use without assigning property rights over orbits or levying an explicit satellite tax.

Suppose the regulator was able to give insurers a per-satellite penalty or subsidy of τ_t to ensure insurance would be priced at the marginal external cost ($p_{t+1} = E_t[\xi(S_{t+1}, D_{t+1})]$) while still allowing free entry into the insurance sector. When τ_t is positive the regulator would be issuing an underwriting subsidy, and when τ_t is negative the regulator would be issuing an underwriting penalty. The insurance sector's profit is then

$$I(S_t, \ell_t) = (E_{t-1}[\xi(S_t, D_t)] - \ell_t F + \tau_t)S_t \tag{110}$$

$$\tau_t : I(S_t, \ell_t) = 0 \implies \tau_t = \ell_t F - E_{t-1}[\xi(S_t, D_t)]. \tag{111}$$

Equation 111 shows that the socially optimal mandatory insurance pricing can be achieved by an incentive which imposes the difference between the actuarial cost of satellite insurance and the marginal external cost on the insurer. The insurer then passes the marginal external cost on to the satellite owner. Depending on the magnitude of the risk and the marginal external cost, this may be a net subsidy or tax on the insurer.

9.3 Competitive debris removal pricing

The profits of the cleanup industry, which supplies active debris removal, are

$$I_t(R_t) = \underbrace{c_t R_t}_{\substack{\text{Cleaning}\\\text{revenues}}} - \underbrace{\gamma R_t^2}_{\substack{\text{Cleaning}\\\text{costs}}}. \tag{112}$$

If the cleanup industry is competitive and a positive amount of debris is removed, debris will be removed from orbit until industry profits are zero,

$$R_t^s : I_t(R_t^s) = 0 \tag{113}$$

$$\implies R_t^s(c_t - \gamma R_t^s) = 0 \tag{114}$$

$$\implies R_t^s = \frac{c_t}{\gamma}, \tag{115}$$

subject to the constraint that $R_t^s \leq D_t$. Tkatchova (2018) examines the potential for debris removal markets.

Combining equation 56 with equation 115 and the market clearing condition $R_t^s = R_t$, the aggregate amount of debris removed from orbit will be

$$R_t = \frac{\partial E_t[\ell_t | S_t, D_t - R_t]}{\partial D_t} \frac{F}{\gamma} S_t. \tag{116}$$

9.4 Cost and congestion shifts in cooperative removal demands from new satellites

For brevity, I write $E_t[\ell_{t+1}]$ as $L(S_t, D_t - S_t R_{it})$ in this subsection and use S and D subscripts to indicate the respective partial derivatives. Since these results are intratemporal in nature, I also drop time subscripts.

To see the cost and congestion shifts in the cooperative private demand for removal, suppose there were two types of satellites, Kinds (k) and Unkinds (u). Kinds make sure their satellites can never collide with others and purchase debris removal, while Unkinds allow their satellites a non-zero chance of colliding with another satellite and never purchase debris removal. The private marginal benefit of debris removal for a Kind named i is

$$MB_i = L_D(u, D - kR_i)kF. \tag{117}$$

The congestion shift is the effect of another Unkind entering, while the cost shift is the effect of another Kind entering. Formally,

$$\text{Cost shift: } \frac{\partial MB_i}{\partial k} = -L_{DD}(u, D - kR_i)kR_iF + L_D(u, D - kR_i)F \tag{118}$$

$$\text{Congestion shift: } \frac{\partial MB_i}{\partial u} = L_{DS}(u, D - kR_i)kF \tag{119}$$

Adding them and normalizing by a function of the collisions rate's convexity in debris,

$$\frac{\frac{\partial MB_i}{\partial k} + \frac{\partial MB_i}{\partial u}}{L_{DD}k^2F} = \frac{(-L_{DD}kR_iF + L_DF) + (L_{DS}kF + L_DF)}{L_{DD}k^2F} \tag{120}$$

$$= \frac{L_D}{L_{DD}k^2} + \frac{L_{DS} - L_{DD}R_i}{L_{DD}k} = \frac{\partial R_i}{\partial S}. \tag{121}$$

The congestion shift may be positive or negative. It is the effect of increasing the number of satellites on the marginal collision risk from a unit of debris. If the collision rate were decoupled from the satellite stock, the congestion shift would disappear. If a marginal Unkind would increase the effect of a marginal unit of debris, the congestion shift will be positive. Reducing the amount of debris would greatly reduce the threat posed by the marginal satellite. If a marginal Unkind would decrease the effect of a marginal unit of debris, the congestion shift will be negative. This could be the case if the Unkind was well-shielded from debris but a threat to other satellites. Reducing the amount of debris would not change the risk of the marginal Unkind by much then.

The cost shift is the effect of increasing the number of customers in the market for debris removal on the marginal collision threat from a unit of debris. There are two pieces to this. First, the debris removed by each Kind reduces the collision risk for all owners. As long as the collision rate is increasing in debris, reducing debris is always a good thing for everyone. This will tend to make the cost shift positive. Second, the debris removed by each Kind changes the marginal benefit of the next Kind's removal. Since the collision rate must be locally convex in debris at an interior solution, this effect will tend to make the cost shift negative. If the collision rate is sufficiently locally convex, this effect can make the cost shift negative in total. Generic satellites are both Kinds and Unkinds.

9.5 Open access launch response to collisions

Lemma 6. *(Open access launch response to collision risk draws) The open access launch rate in t can be non-monotonic in the realized collision risk in t.*

Proof. From equation 11,

$$\mathscr{F} = r_s - r - E_t[\ell_{t+1}] = 0. \tag{122}$$

Applying the Implicit Function Theorem to $\mathscr{F}$,

$$\frac{\partial X_t}{\partial \ell_t} = -\frac{\partial \mathscr{F}/\partial \ell_t}{\partial \mathscr{F}/\partial X_t} \tag{123}$$

$$= \frac{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} S_t + \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}} \frac{\partial G(S_t, D_t, \ell_t)}{\partial \ell_t}}{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} + m \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}} \lesseqgtr 0 \tag{124}$$

□

Figure 15 illustrates three cases of Lemma 6: one where the open access launch rate is first decreasing and then increasing in the satellite-destroying collision risk draw, another where it is uniformly decreasing in the collision risk draw, and a third where it is uniformly increasing in the collision risk draw. There are two competing effects of collisions driving this behavior: collisions generate debris, but collisions also remove other satellites from orbit.

In the first case, the effect of additional debris dominates the launch decision when the collision risk draw is low, and the effect of fewer satellites dominates when the collision risk draw is high. In the second case shown in Figure 15, the new-debris effect dominates for all draws. In the third case shown in Figure 15, the fewer-satellites effect dominates for all draws. The third case is the least-realistic, as it implies that the number of fragments from a satellite destruction is tiny compared to the number of fragments created by a debris-debris collision. Debris modeling studies such as Liou (2006); Letizia et al. (2017) find the opposite: satellite destructions contribute much more debris to the orbital environment than collisions between debris fragments. The first and second cases are plausible under realistic relative orders of magnitude between the number of fragments created by satellite-satellite, satellite-debris, and debris-debris collisions.

[Figure 15 about here.]

9.6 Two details for flow control implementations

Lemma 7. *(A feature of flow controls with no end date) If the discount rate is positive, the price of a time-consistent infinite-horizon flow control is an exploding process.*

Proof. From equation 26, we can write the price in period $t+1$ as a function of the price in period t as

$$p_{t+1} = \frac{1+r}{1-E_t[\ell_{t+1}]} p_t - \frac{\pi - rF - E_t[\ell_{t+1}]F}{1-E_t[\ell_{t+1}]}. \tag{125}$$

Differentiating with respect to p_t,

$$\frac{\partial p_{t+1}}{\partial p_t} = \frac{1+r}{1-E_t[\ell_{t+1}]} > 1 \ \ \forall r > 0. \tag{126}$$

This holds along a generic equilibrium path as shown above, or along an optimal path. Along an optimal path, the numerator of the second term on the right-hand side would be replaced with the marginal external cost of a satellite in period $t+1$. Since the first term on the right-hand side is unchanged, the flow control process still has a unit or greater-than-unit root. □

Lemma 7 is likely to be practically relevant to designing a flow control policy, even though in theory such a control could be made optimal.[32]

Lemma 8. *(A limitation of flow controls) If the expected future loss rate is one, there is no value of the future flow control price which can affect the current launch rate.*

Proof. Rewriting equation 26 with $E_t[\ell_{t+1}] = 1$, the launch rate will satisfy

$$X_t : \pi = rF + E_t[\ell_{t+1}]F + (1+r)p_t. \tag{127}$$

[32] Though it seems unlikely to me, perhaps there exists or could exist a regulatory body with the credibility to commit to an exploding price path for the foreseeable future; they would find this result irrelevant.

Since equation 127 no longer contains p_{t+1}, the regulator cannot use it to affect X_t. $\square$

I present this result for completeness, though it is likely a moot point. The destruction of all active satellites in an orbit would likely trigger Kessler Syndrome there. The regulator and the space industry would then have bigger problems than whether or not a flow control in line with the pre-committed path exists. This may be relevant for low orbits where debris decays sufficiently quickly. If such destruction occurred in those orbits intentionally, for example missile tests or conflict, the regulator and space industry would again have bigger problems than deviation from prior commitments.

10 Appendix E: Computational framework

10.1 Stochastic and deterministic models

The dynamics of the satellite and debris stocks make the process for ℓ_t dependent and heterogeneously distributed over time. For exposition and to illustrate specific points (most often transition dynamics), I simulate deterministic versions of the models described in this paper. The basic deterministic orbit use model is analyzed in detail in Rao and Rondina (2018). Using deterministic models simplifies making "apples-to-apples" comparisons between different policies without collision rate draws complicating matters. Making the collision risk process independent and identically distributed would remove the economically interesting physics of the problem - open access becomes socially optimal because launch and removal choices have no impact on the collision risk. In the deterministic model, I use the expected collision risk without drawing from a distribution. In the stochastic model, I draw the collision risk from a binomial distribution with floor(S_t) many trials and mean equal to $\min\{\alpha_{SS}S^2 + \alpha_{SD}SD, 1\}$. Formally,

$$\text{Deterministic model collision risk: } \ell_t = \min\{\alpha_{SS}S^2 + \alpha_{SD}SD, 1\}, \tag{128}$$

$$\text{Stochastic model collision risk: } \ell_t \sim Bin(\text{floor}(S_t), \min\{aS_t^2 + bS_tD_t, 1\}). \tag{129}$$

Figure 16 compares sequences of launch rates, satellite and debris stocks, and expected collision risks under open access in the deterministic and stochastic models. The deterministic model behaves as expected - it is the mean of the stochastic model. The expected collision risk in each model is identical, which is unsurprising given that open access and the planner both target the expected collision risk.

[Figure 16 about here.]

Although the time paths are similar, there are some interesting properties of the stochastic values and policies (such as Lemma 6) which are not captured by the deterministic ones.

10.2 Value and policy functions

In general, the algorithms I use to compute decentralized solutions under open access are simpler than those used to compute the planner's solutions. This is because open access simplifies the continuation value to the cost of launching a satellite. I use R for all simulations, parallelizing where possible.

I compute optimal value functions by alternating between value and policy function iteration with a version of the one-way multigrid approach described in Chow and Tsitsiklis (1991). The multigrid approach involves computing the value function first on a coarse grid, then progressively refining the grid for further computation while using the previous results as initial guesses. I use linear interpolation to fill in new elements of the initial guess when moving to a finer grid. For simplicity, I use the same number of grid points, with the same limits, over S and D. Algorithm 1 describes how I compute the policy and value functions for a given grid $(grid_S, grid_D, grid_\ell)$ and given initial guess $guess(S, D, \ell)$.

Algorithm 1: Value function iteration with policy evaluation

Set

$$W_0(S,D,\ell) = guess(S,D,\ell)$$

for all $(S,D,\ell) \in (grid_S, grid_D, grid_\ell)$

Set $i = 1$ and $\delta = 100$ (some large value to begin).

while $\delta > \varepsilon$ **do**

At each grid point in $(grid_S, grid_D, grid_\ell)$, use a numerical global optimizer to obtain

$$X^* = \text{argmax}_X\{\pi S - FX + \beta \hat{W}_{i-1}(S',D',E[\ell'|S',D'])\},$$
$$W_i(S,D,\ell) = \pi S - FX^* + \beta \hat{W}_{i-1}(S',D',E[\ell'|S',D']),$$

where $\hat{W}_{i-1}(S',D',E[\ell'|S',D'])$ is computed using linear interpolation from $W_{i-1}(S,D,\ell)$, $S' = S(1-\ell)+X$, $D' = D(1-\delta)+G(S,D,\ell)+mX$, and $E[\ell'|S',D'] = \min\{\alpha_{SS}S'^2 + \alpha_{SD}S'D', 1\}$.

$\delta \leftarrow ||W_i(S,D,\ell) - W_{i-1}(S,D,\ell)||_\infty$.

if $\delta < \bar{\delta}$: **then**

Evaluate the policy. Compute $W_i^T(S,D,\ell) = \sum_{t=1}^{T-1}\beta^{t-1}(\pi S - FX^*) + W_i(S,D,\ell)$ by backwards induction, using the laws of motion for S and D and the form of $E[\ell'|S',D']$ and sufficiently large T. Set $W_i(S,D,\ell) = W_i^T(S,D,\ell)$ and return to step (a).

end

i ← i+1

end

My use of $\hat{W}_{i-1}(S',D',E[\ell'|S',D'])$ implies that the value function is linear in ℓ. If it is not, then the continuation value I interpolate is a lower bound due to concavity of the value function and Jensen's inequality. An alternative approach, which would be more computationally intensive, would be to numerically compute the expectation using the distribution function of the binomial evaluated at each grid point in grid$_\ell$. However, as the grid gets successively finer the approximation error due to Jensen's inequality shrinks to zero. Since each value function iteration until the final grid size is only used to provide a guess for the next-finer grid, the final approximation error can be made arbitrarily small by choosing a sufficiently fine final grid. There is no such approximation error for the deterministic model since ℓ is $E[\ell'|S',D']$ there.

Once the algorithm converges for a given grid, I add more points to the grid. I linearly interpolate the final value of $W_i(S,D,\ell)$ from the coarser grid onto the finer grid to obtain the new value of $guess(S,D,\ell)$. I then repeat the algorithm until convergence. I use the same size grid for all state variables. I begin with an initial grid of 2 points for each state variable (4 points in the deterministic model, 8 points in the stochastic model) and continue until a grid of 128 points

for each state variable (16384 points in the deterministic model). I use $\varepsilon = 1e^{-8}$ for the final grid, and $\varepsilon = h^{0.35} \cdot 1e^{-8}$ for the intermediate grids (where h is the number of points in a single dimension of the grid). This saves computation time for grids where the converged value function will only be used as an initial guess.[33] I set $\bar{\delta} = 1$ for all grids, and use a value for T between 10 and 100 depending on the grid size. These values were chosen by experimentation for different parameterizations to balance speed and accuracy.
The numerical global optimization can take some time. I use this approach because the first-order condition to the planner's Bellman equation can have multiple solutions (maxima and minima), and this approach is about as fast as storing each solution, evaluating the second-order condition at each solution, and comparing the value function level at each solution. This approach also makes it easier to code additional choice variables, such as the amount of debris removal, by adjusting the per-period return function and laws of motion. With debris removal, I use the numerical global optimizer to find the pair (X^*, R^*) which maximizes the value function. This approach allows the code to scale easily to multiple orbital shells or debris types for future work. However, for the problem with debris removal, I only compute solutions to the deterministic model.

Computing the open access equilibrium is much simpler than computing the planner's solution. Algorithm 2 describes how I compute the open access equilibrium with debris removal. As in the case of finding the planner's solution, I use a numerical global optimizer on the objective function rather than a rootfinder on the first-order condition to avoid issues of multiple optima and facilitate future extensions. To compute the open access equilibrium launch rate without debris removal, I skip the first step of the algorithm 2 and set $R, R' = 0$ in the laws of motion and expected collision risk.

[33] I use $||\cdot||_\infty$ for the sup norm.

Algorithm 2: Open access launch and removal plans

1 At each point on the final grid used in the planner's solution, use a numerical global optimizer to obtain

$$R_i^o = \text{argmax}_{R_i}\{\pi - cR_i + E[\ell|S, D - SR_i]F\},$$

and set $R^o = SR_i^o$.

2 Use a numerical rootfinder to find the X^o which solves

$$E[\ell'|S', D' - R'] = r_s - r,$$

where $E[\ell'|S', D' - R']$ is computed using linear interpolation from $S' = S(1-\ell) + X$, $D' = (D-R)(1-\delta) + G(S, D-R, \ell) + mX$, $E[\ell|S, D-R] = \min\{\alpha_{SS}S^2 + \alpha_{SD}S(D-R), 1\}$, and the mapping R^o from the prior step.

3 Approximate $W_i^\infty(S,D,\ell) = \sum_{t=1}^{\infty}\beta^{t-1}(\pi S - FX^*) + W_i(S,D,\ell)$ as $W_i^T(S,D,\ell) = \sum_{t=1}^{T-1}\beta^{t-1}(\pi S - FX^*) + W_i(S,D,\ell)$. Compute $W_i^T(S,D,\ell)$ by backwards induction, using the laws of motion for S and D and the form of $E[\ell'|S',D']$ and sufficiently large T.

To compute deterministic value and policy functions, the above algorithms can be modified to use only two state variables (S,D) instead of three (S,D,ℓ). The deterministic value and policy functions are much faster to compute than the ones with stochastic collision rates. I show deterministic value and policy functions in the paper for ease of exposition.

10.3 Time paths for the stochastic model

For all of the time path simulations, I simulate the path for a large enough T that finite-horizon effects can be ignored along most of the path. I then truncate the sequences at a t far enough into the sequence to observe convergence to a steady state.[34]

As with the value functions, the nature of open access simplifies computation of open access time paths. However, the addition of debris removal slightly complicates the open access time path computation. Algorithm 3 describes how I compute the open access launch path without debris removal. It is fast even for large T (fractions of a second for $T = 1000$).

[34] This convergence is not guaranteed; see Rao and Rondina (2018) for more details on how the collisions between satellites and debris make periodic or aperiodic oscillations around an open access steady state possible and economically plausible. I set parameter values so that this is not an issue in any simulations in this paper.

Algorithm 3: Stochastic open access launch time path

for t *in* $1,\ldots,T-1$ **do**
  Draw ℓ_t from $Bin(\text{floor}(S_t), \min\{aS_t^2 + bS_tD_t, 1\})$
  Use a numerical rootfinder to find the X_t^o which solves

$$E_t[\ell_{t+1}|S_{t+1}, D_{t+1}] = r_s - r,$$

  using the laws of motion for S_t, D_t, and the form of $E[\ell|S,D]$.
  Update S_{t+1} and D_{t+1} using their laws of motion.
end
Set $X_T^o = 0$.

Simulating time paths for the planner, or even open access time paths with removal, is slightly more complicated due to the nature of the stochastic process for collisions. As mentioned above, the dynamics of the satellite and debris stocks make the process for ℓ_t dependent and heterogeneously distributed over time. Open access time paths with debris removal, and the planner's time paths generally, depend on future values of choice variables (R_{t+1} for open access, and (X_{t+1}, R_{t+1}) for the planner), which in turn depend on future draws. Simulating these stochastic processes directly is computationally challenging even in the open access case. Instead, I simulate the analogous deterministic processes. Algorithm 4 describes how I compute the deterministic time path of open access launch rates and cooperative removal.

Algorithm 4: Deterministic open access launch time path with cooperative endogenous debris removal

Set $T = 100$ (some large value). Initialize $\{X_t^1, R_t^1\}_0^T$.

Set $i = 1$ and $\delta = 100$ (some large value to begin).

while $\delta < \varepsilon$ **do**

Compute the launch rate sequence:

for t *in* $1, \ldots, T-1$ **do**

Use a numerical rootfinder to find the X_t^{i+1} which solves

$$E_t[\ell_{t+1} | S_{t+1}, D_{t+1} - R_{t+1}^i] = r_s - r,$$

using the laws of motion for S_t, D_t, and the form of $E[\ell | S, D]$.

Update S_{t+1} and D_{t+1} using their laws of motion.

end

Set $X_T^{i+1} = 0$.

Compute the removal rate sequence:

Use a numerical global optimizer to find the $\{R_t^{i+1}\}_0^T$ which maximizes

$$W_i^T(S, D, \ell) = \sum_{t=1}^{T} \beta^{t-1} (\pi S_t - F X_t^{i+1} - c_t R_t^{i+1})$$

using the laws of motion for S_t, D_t, and the form of $E[\ell | S, D]$.

$\delta \leftarrow \frac{1}{2} ||X_t^i - X_t^{i+1}||_\infty + \frac{1}{2} ||R_t^i - R_t^{i+1}||_\infty$

i $\leftarrow$ i+1

end

11 Figures

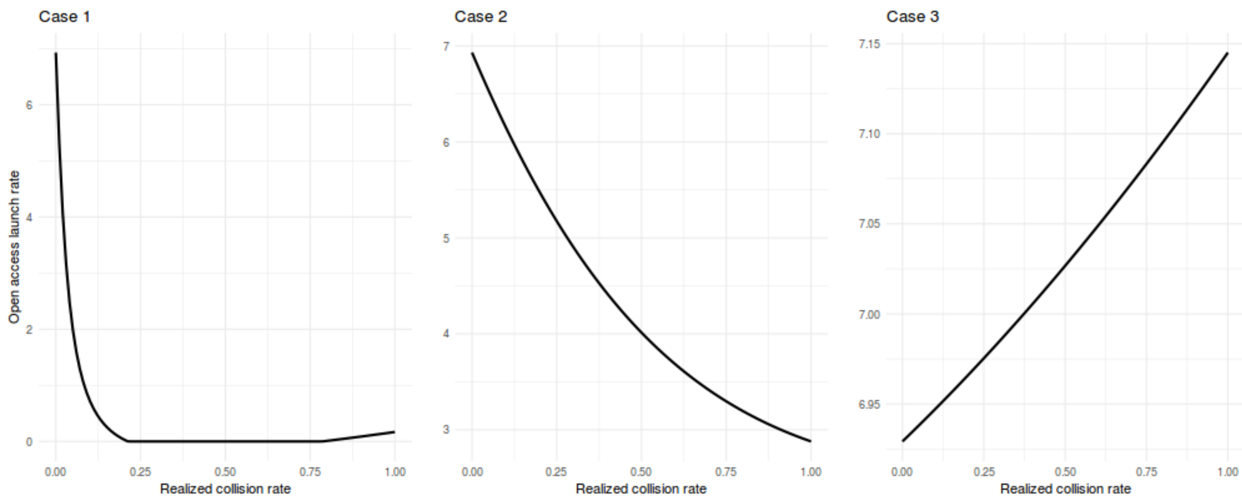

Figure 15: *Three ways the open access launch rate may respond to collision risk draws.*
Left panel: The open access launch rate is decreasing then increasing in the collision risk draw. The effect of additional debris dominates the launch decision when the collision risk draw is low, and the effect of fewer satellites dominates when the collision risk draw is high.
Middle panel: The open access launch rate is uniformly decreasing in the collision risk draw. The new-debris effect dominates for all draws.
Right panel: The open access launch rate is uniformly increasing in the collision risk draw. The fewer-satellites effect dominates for all draws.
The left panel and middle panel cases are more plausible than the right panel case under realistic relative orders of magnitude between the number of fragments created by satellite-satellite, satellite-debris, and debris-debris collisions.

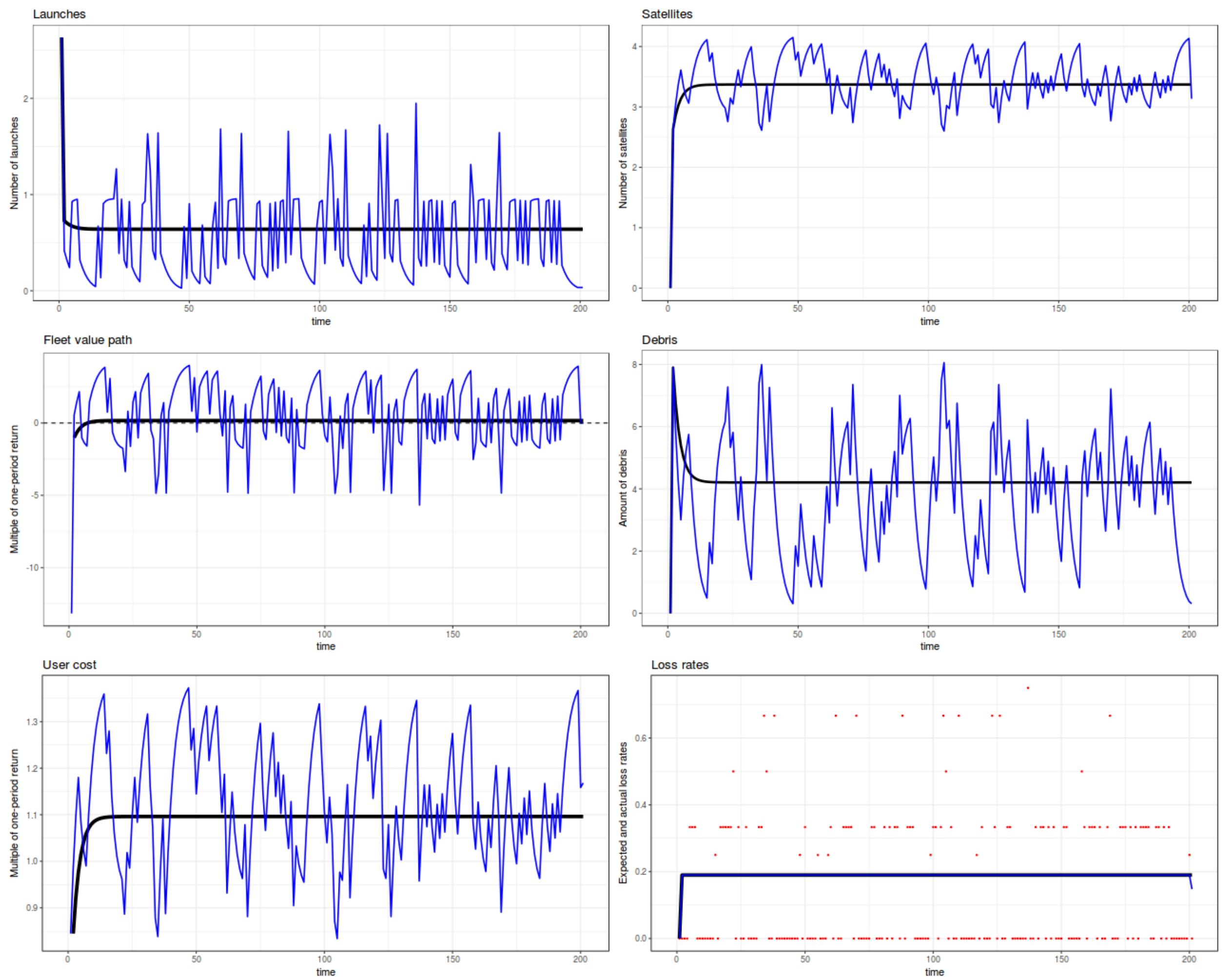

Figure 16: *Time paths under the stochastic (blue line) and deterministic (black line) models.* The red dots in the "collision rates" panel are the draws of ℓ_t.