ECON 1078-003
Problem set # 4

March 5, 2018

Due Friday 3/16 on D2L at the start of class. Please write clearly, draw a box around your final answer, and submit your answers in the order listed below. If there are no real solutions, write that a real solution does not exist.

This problem set will be graded out of 10 points. Extra credit problems will not count toward this total, so a score of 12/10 is possible.

Book Problems:

Section 4.5: Numbers 2 and 6

Section 4.6: Numbers 2, 3, 5, 7(b), 8

EXTRA CREDIT (2 points): Section 4.6, problem 9(b)

Section 4.7: Numbers 3, 6, 8

Section 4.8: Numbers 3, 4, 6

Word Problems:

1. A linear supply and demand system is described by the equations

$$\text{(Supply) } Q = a + bP$$
$$\text{(Demand) } Q = c - dP$$

where Q and P are the equilibrium quantity and price, and a, b, c, d are positive parameters. Derive a set of necessary conditions for the intersection of these equations to be in quadrant 1. (Suggested path: First solve the system for (Q, P) and label your solution Q^* and P^*. Then, try to answer "$Q^* > 0 \implies (blank)$" and "$P^* > 0 \implies (blank)$". Graphing the system may help you understand what's going on.)

2. In this problem, we'll study fisheries management.

The stock of fish in a fishery grows according to the equation $y(x) = rx(1 - \frac{x}{K}) - Ex$, where x is the level of the fish stock (i.e., how many fish are in the fishery), E is the level of harvest effort (bigger E means more fish are harvested), and y is the growth rate of the fish population as a function of x and E.

(a) Assume that r, K, and E are positive real numbers satisfying $r > E$, and plot the function $y(x)$.

(b) The "steady state" level of the fish stock is $\tilde{x} : y(\tilde{x}) = 0$. Expressed as a function of the harvest effort, $\tilde{x}$ is $\tilde{x}(E) = \frac{K}{r}(r - E)$. Plot $\tilde{x}(E)$.

i. Label the horizontal and vertical axis intercepts.

(c) This industry's profit as a function of harvest effort is $\pi(E) = pE\tilde{x}(E) - cE$, where p is the price per unit effort c is the cost per unit effort. Assume that $p > c > 0$. Plot $\pi(E)$.

i. When individual fishers behave noncooperatively, they will fish until industry profits are driven to zero. This is called *open access*; formally, open access effort is $E_0 : \pi(E_0) = 0$. Find E_0, the open access effort level.

ii. When individual fishers behave cooperatively, they will fish until profits have been maximized. This is called *good neighbors management*, or optimal management; formally, $E_{max} : \pi(E_{max}) > \pi(E)\ \forall E \neq E_{max}$ (equivalently, this can be written as $E_{max} = \text{argmax}\ \pi(E)$). Find E_{max}.

(d) How large is the fish stock under E_0 and E_{max}? (Calculate $\tilde{x}(E_0)$ and $\tilde{x}(E_{max})$.)

i. Which is larger, $\tilde{x}(E_0)$ or $\tilde{x}(E_{max})$?

ii. Do you need any restrictions (including what is stated in the problem) on the parameters r, K, p, c for your conclusion about $\tilde{x}(E_0)$ and $\tilde{x}(E_{max})$ to hold?

iii. Do you need any restrictions (including what is stated in the problem) on the parameters r, K, p, c for $\tilde{x}(E) \geq 0$?

(e) What is the growth rate of the fish stock under E_0 and E_{max}? (Calculate $y(\tilde{x}(E_0))$ and $y(\tilde{x}(E_{max}))$.)

i. Which level of harvest effort leads to a higher rate of fish population growth, E_0 or E_{max}?

ii. Do you need any restrictions (including what is stated in the problem) on the parameters r, K, p, c for your conclusion about the growth rates to be true?

iii. Do you need any restrictions (including what is stated in the problem) on the parameters r, K, p, c for $y \geq 0$?