Summation and induction worksheet

1. Consider the mn numbers $\{a_{ij}\}$ in a rectangular spreadsheet that has m rows and n columns. Denote the arithmetic mean of them by $\bar{a}$, and the mean of all the numbers in the jth column by $\bar{a}_j$, so that

$$\bar{a} = \frac{1}{mn}\sum_{r=1}^{m}\sum_{s=1}^{n} a_{rs}, \text{ and } \bar{a}_j = \frac{1}{m}\sum_{r=1}^{m} a_{rj}$$

Prove that $\bar{a}$ is the mean of the column sums $\bar{a}_j$, $(j = 1, \ldots, n)$. (Hint: The mean of a collection of n numbers, $\{x_1, x_2, \ldots, x_n\}$, is $\frac{1}{n}\sum_{i=1}^{n} x_i$.)

2. Now suppose $n = m$. Prove that $\sum_{r=1}^{m}\sum_{s=1}^{m}(a_{rj} - \bar{a})(a_{sj} - \bar{a}) = m^2(\bar{a}_j - \bar{a})^2$

3. Use induction to prove that $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$.

4. Use induction to prove that $\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$

5. Use induction to prove that $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$