Cost in Space:
Debris and Collision Risk in the Orbital Commons

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Abstract

As Earth's orbits fill with satellites and debris, the probability of collisions between orbiting bodies increases. Runaway debris growth, known as Kessler syndrome, may cause Earth's orbits to become unusable for millennia. We present the first long-run economic model of Earth orbit use which accounts for the risk of satellite-destroying collisions and Kessler syndrome. Orbital decay and profit maximization can prevent Kessler syndrome even in the absence of cleanup technologies, but open access will result in inefficiently high levels of launches, debris, and collision risk. Steady state debris levels and the equilibrium collision rate are increasing in the excess return on a satellite, and sustained increases in the excess return will lead open access to cause Kessler syndrome. Short-run rebound effects can also make open access debris levels increase as the rate of orbital decay increases and as launches generate less debris. These results suggest that careful attention to economic incentives is necessary to ensure orbital sustainability.

1 Introduction

Earth's orbits are the world's largest common-pool resource, and increasingly necessary for services that power the modern world. As society launches more satellites, the risk of collisions between orbiting objects increases. Such collisions can destroy satellites and produce orbital debris, further increasing the risk of future collisions. Collision risk and debris accumulation threaten active satellites and the future of human activity in space. How will open access affect orbital debris accumulation, satellite collision risk, and occurrence of Kessler syndrome? These questions have been explored very little in economics, and have not yet been addressed in the physics and engineering or law and policy literatures[1]. In this paper, we examine the consequences of open access to orbit in the first long-run dynamic economic model of satellite launch, show that the equilibrium collision rate is determined by the excess return on a satellite, and consider the economics of debris accumulation.

Satellites produce debris over their lifecycle. Launching satellites produces orbital debris (spent rocket stages, separation bolts), satellites can produce some debris while in orbit (paint chips, lost tools, etc.), and satellites which are not deorbited or shifted to disposal orbits at the end of their life become debris[2]. Objects in orbit move at velocities higher than 5 km/s, so debris as small as 10 cm in diameter can be hazardous to active satellites.

Though debris does deorbit naturally, this process can be very slow, especially at higher altitudes. Debris as low as 900 km above the Earth's surface can take centuries to deorbit, while debris at 36000 km can take many millennia (Weeden (2010)). Compounding the problem is the fact that collisions between debris objects can generate even more debris, also moving at high velocities and capable of damaging operational satellites[3]. It is possible for debris accumulation to cause a cascading series of collisions between orbital objects, resulting in an expanding field of debris which can render an orbital region unusable and impassable for thousands of years. Engineers and physicists call this phenomenon "collisional cascading" or "Kessler syndrome" (Kessler and Cour-Palais (1978)). Kessler syndrome can cause large economic losses, directly from damage to active satellites and indirectly from limiting access to space (Bradley and Wein (2009), Schaub, Jasper, Anderson, and McKnight (2015)). Proposed commercial uses of space, like low-Earth orbit (LEO) broadband internet constellations with global coverage, asteroid mining, and space-based solar power, could become infeasible if Kessler syndrome occurs. Existing estimates of debris growth indicate that the risk of Kessler syndrome is highest in LEO, placing imaging and future telecommunications satellites at risk and potentially reducing access to higher orbits (Kessler, Johnson, Liou, and Matney (2010)). Currently, there are more than 1,000 operating satellites in orbit, up to 600,000 pieces of debris large enough to cause satellite loss, and millions of smaller particles that can degrade satellite performance (Ailor, Womack, Peterson, and Lao (2010)). Approximately 49 percent of active satellites are in LEO, 41 percent are in geostationary orbit (GEO), and the remainder are in elliptical or other orbits.

[1] Wienzierl (2018) highlights a number of issues in the development of a space economy, from space debris to coordination problems and market design, which economists have the tools to address.

[2] Reusable rockets can significantly lower the cost of launching and the amount of launch debris generated. SpaceX and Blue Origin are currently the only launch providers who offer such vehicles.

[3] The risk of debris striking another satellite is generally orders of magnitude larger than the risk of debris deorbiting and harming consumers directly.

Existing legal frameworks for orbit use, such as the Outer Space Treaty, complicate the process of establishing of explicit orbital property rights and hinder cleanup efforts. For example, Article 2 of the OST forbids national appropriation or claims of sovereignty over outer space, which could be interpreted as prohibiting national authorities from unilaterally establishing orbital property rights[4] (Gorove (1969)). As launch and satellite costs decrease and more firms enter markets for satellite services, the debris problem is likely to get worse (Selk (2017)).

The congestibility of orbital resources has been recognized in economics as early as Sandler and Schulze's 1981 paper, *"The Economics of Outer Space"* (Sandler and Schulze (1981)). In it the authors present a series of models to analyze issues which may face a future space economy, including a static optimization program to manage orbital spectrum and positions as club goods. Their program accounts for congestion due to radio frequency interference and collision risk, but ignores debris accumulation. Despite this early recognition of economic externalities in orbit, economists have paid relatively little attention to orbital management, likely in part due to the lack of development in commercial space markets. More recent economic analyses have considered the static economic costs of inefficient electromagnetic spectrum and position use or the inefficiency of open access and voluntary debris mitigation under monopolistically competitive behavior in static or two-period settings (Macauley (1998), Macauley (2015), Adilov, Alexander, and Cunningham (2015)). The economic dynamics of orbit use under perfectly competitive open access and socially optimal management over longer time horizons have not been analyzed yet.

Orbital congestion has been studied more actively in physics and engineering, beginning with Kessler and Cour-Palais' analysis of the creation of a debris belt (Kessler and Cour-Palais (1978)). This community has focused on two areas: how orbital congestion might evolve in particular orbits as the numbers of satellites and debris fragments increase, and how satellite systems and trajectories should be designed to be more robust to spectral congestion and physical collision risk. Papers in the former literature have relied on launch rate models estimated from historical trends, and have abstracted away from optimizing or forward-looking behavior by the agents launching the satellites. Gordon's observation on early models of fisheries (Gordon (1954)) comes

[4]Weeden (2010) and Weeden and Chow (2012) are skeptical of what decentralized bargaining can achieve in this setting, given the difficulties in solving other global coordination problems, like climate change, and the number of current and potential orbit users and their incentives. Salter and Leeson (2014) and Salter (2015) take more optimistic views of what commercial users can achieve under celestial anarchy, the former based on the idea of self-enforcing property rights and the latter based on the Coase Theorem, technological advancement, and insurance markets.

to mind:

> On the whole, biologists tend to treat the fisherman as an exogenous element in their analytical model, and the behavior of fishermen is not made into an integrated element of a general and systematic 'bionomic' theory... the 'overfishing problem' has its roots in the economic organization of the industry.

Similarly, with the exception of Adilov, Alexander, and Cunningham (2015), current models of Earth orbit use do not account for the economic incentives involved in launching satellites.

First, satellites are assets, and collision risk reduces their expected present value. This gives firms an incentive to avoid launching if the collision risk becomes too high, potentially stabilizing orbit use by deterring entry. However, because orbits are an open access commons, firms will launch until the collision risk makes expected profits zero, rather than stopping when expected marginal profits are zero[5]. Entry until zero profits would be socially optimal if not for the presence of endogenous collision risk. Second, debris is an accumulating stock pollutant which only matters because it increases collision risk. This gives firms an incentive to prevent debris accumulation, but only to the extent to which they are directly affected by it now and in the future. Since open access makes the expected net present value of owning a satellite zero, firms' incentives to protect the future value of their satellites is reduced. The problem of debris accumulation is made uglier by the potential for Kessler syndrome, but it is not primarily a pollution problem.[6]

The feedback from collision risk to profits causes the equilibrium launch rate to decrease as collision risk increases. This can make the equilibrium launch rate also decrease as the debris stock increases, but the lack of feedback from future satellite operators to present satellite operators means that the equilibrium launch rate may not be responsive enough to debris accumulation to prevent Kessler syndrome. The

[5] The geosynchronous belt is an exception; though it is still a commons, it is not under open access. The International Telecommunication Union auctions orbital slots which national authorities then allocate. See Macauley and Portney (1984) and Jones, Skora, Monson, Jones, Jack, and Eby (2010) for more discussion of these mechanisms.

[6] Haveman (1973) describes the similarities and differences between open access, congestion, and pollution externalities in more detail. The key distinction between congestion and pollution is that congestion is a reciprocal externality stemming from crowding costs not being reflected in marginal use decisions, while pollution is a one-sided externality borne by agents who are different from the ones creating the pollution. Pollution is part of the problems of orbit use insofar as debris is an intertemporal externality imposed by current users on future users, but debris also imposes costs on current users, so the distinction is not clean-cut. Since debris is irrelevant in the absence of collision risk, it seems reasonable to argue that congestion, not pollution, is the heart of the problems of social cost in orbit.

appropriate property rights or system of charges could make the collision risk level efficient and prevent runaway debris growth.

We build on prior economic analysis of orbital debris by extending the setting from two periods to arbitrary finite and infinite horizons and by allowing more general analytical forms of satellite and debris accumulation. We contribute to the literature on common-pool resource management in the presence of environmental risk by incorporating a capital stock that is affected by congestion of the commons and a pollution stock which increases congestion. We do not consider spectrum use explicitly, though we comment on the effects of spectrum congestion in section 5.1.

Open access is characterized by too many launches and collisions and too much debris relative to the socially optimal plan, in the short-run and the long-run. While open access steady states cannot cause Kessler syndrome and firms always face incentives to limit satellite-destroying collisions, open access may cause Kessler syndrome while approaching a steady state. Like bioeconomic collapse in fisheries, Kessler syndrome becomes more likely under open access as the cost of launching satellites falls, or more generally as the excess return on a satellite rises. Regions with higher decay rates or lower incidence of launch debris may have higher long-run debris levels due to a combination of short-run rebound effects and local instability of long-run equilibria. Additionally, increases in the rate of excess return earned by a satellite will tend to locally destabilize open access steady states with higher levels of debris. These results suggest that purely technical solutions aiming to limit debris growth by targeting launch debris or debris decay rates may ultimately increase the amount of orbital debris or destabilize existing equilibria. Market-based instruments or a new legal paradigm for orbit use may be essential for orbital sustainability.

2 Social cost in orbit

2.1 Orbits as an environmental resource

Orbits share characteristics of renewable and nonrenewable resources. On any timescale relevant for human decision making, the Earth will continue to exert its gravitational pull. Bodies in its gravity well will eventually either exit the well or fall to the surface. In low-Earth orbits, the decay is fast enough (on the order of hours to years) that the gravity well and paths within it are renewable on human timescales. In higher orbits, such as the geosynchronous belt, the decay is slow enough (on the order of millions of years) that the well and its paths are nonrenewable on human timescales. Between the two lie a continuum of possible paths through the well, each with their own rates of renewal. Elliptical paths through the well, such as Molniya orbits[7], span multiple regions of renewability.

Fishing and mining offer useful intuition for the economics of orbit use. In fisheries, open access results in entry until expected profits are zero, and may drive the stock below the minimum level required for biological renewal. While fishing may become uneconomical at or past that level, once the threshold is crossed the natural dynamics of the stock will drive the fish population to zero anyway. The stock level at which fishing becomes uneconomical may also be above the minimum biological level, in which case open access will result in sustainable, if suboptimal, harvest rates. In standard open access fishery models, the lack of rights over the fish stock induces myopic fishing - though the fish stock evolves over time, fishers make a sequence of static decisions about harvest levels. Since future profits will be zero anyway due to open access, there is no incentive to conserve today for higher profits tomorrow. In contrast, a mine owner faces an inherently dynamic problem. Each decision to extract affects the amount they are able to extract in the future. As a result, a mine owner's profit-maximizing extraction path equalizes discounted returns over time. To do otherwise would be to leave money on the table in some periods.

As in an open access fishery, firms under open access launch satellites until expected profits are zero, and may fill the orbits past the maximum level at which natural decay can prevent net debris growth. While satellite launching may become uneconomical at or past that level, the natural dynamics of orbiting bodies will cause further collisions and debris growth if the number of objects in orbit is not reduced. If owning a satellite is not uneconomical before this occurs, it is likely to be uneconomical after. Orbit users equalize the expected net present value of owning a satellite across periods, similar to how a competitive mine owner sequences ore extraction.

The neoclassical growth model with pollution is another helpful reference point. Satellites are capital assets which produce a per-period payoff of π at zero marginal cost per period, but a fixed cost of F to build and launch[8]. The collision rate, $L(S, D)$,

[7] A Molniya orbit has a low perigee over the Southern Hemisphere and a high apogee over the Northern Hemisphere. Molniya orbits require less power to cover regions in the Northern Hemisphere (e.g. former Soviet Union countries) than geosynchronous orbits, due to the low incidence angles of rays from the Northern Hemisphere to geosynchronous positions.

[8] This is a simplification to focus on the margin of launch decisions. Operational costs like managing receiver stations on the ground or monitoring the satellite to perform stationkeeping are incurred each period. Since the decision to launch a satellite involves forecasting the operational costs, they can be viewed in this model as having been capitalized into the fixed cost.

is similar to the capital depreciation rate in how it enters the law of motion for satellites. Debris is a stock of residuals from production (launch debris) and "depreciation" (collision fragments). If debris did not cause collisions ($L(S, D) = L(S)$), it would be irrelevant and only the satellite stock would need management. If the collision rate were uncoupled from even the satellite stock ($L(S) = L$), then this model would reduce to the neoclassical growth model with an irrelevant pollution stock. Open access would then be efficient.

The coupling between the satellite stock and the collision rate ($L(S)$) implies congestion in orbit under open access, since firms will not internalize the cost of the risks their satellites pose to others[9]. Adding a coupling between the debris stock and the collision rate ($L(S, D)$) adds to the congestion.

When a firm launches a satellite, its entry to the orbit adds launch debris to the orbit. While its satellite is in orbit, the firm contributes to congestion in the orbit. If the satellite is lost in a collision, its removal reduces the risk to survivors, but the new fragments from its destruction increase the risk to survivors later. Firms ignore the congestion they cause through the debris created by their launch, the addition of their satellite to the orbit, and the new fragments created by their satellite's eventual destruction. The debris effect of launch is "dynamic" congestion imposed by a firm's entry on itself and others. The risk generated by a satellite's presence is "steady state" congestion imposed by a firm on others. The debris effects of destruction are also dynamic congestion imposed by a firm's exit on others. As long as launches create debris, marginal satellites in orbit are bundled with debris creation, so satellites and launch debris are also complements in producing social value. The essential tradeoff of the planner's problem is in balancing the lifetime value created by satellites and launch debris against the present and future congestion created by each.

2.2 A simple model of orbital mechanics

An orbital region is a set of closed paths around a central body. When the paths are chosen to form a closed spherical shell around the central body, the orbital region is also called an orbital shell. Paths which span shells are possible and useful for some applications, but highly elliptical orbits (such as Molniya orbits) are the exception rather than the rule. In what follows, we assume all bodies are in an orbital shell. This ap-

[9]This is a mirror of the difference between a competitive firm which doesn't internalize the effect of its entry on the price and a monopolist who internalizes the price effect of marginal units of production. As in other natural resource settings, an orbital monopolist acts as a "satellite conservationist" by restricting and resequencing launches to preserve the fleet.

proach is frequently used in debris modeling, e.g. Rossi, Anselmo, Cordelli, Farinella, and Pardini (1998) and Bradley and Wein (2009), though higher fidelity models use large numbers of small regions to track individual objects, e.g Liou, Hall, Krisko, and Opiela (2004), Liou and Johnson (2008), and Liou and Johnson (2009). We abstract from the composition of orbital stocks, and assume that all satellites and debris are identical.

The number of active satellites in orbit is the number of launches in the previous period plus the number of satellites which survived the previous period. The amount of debris in orbit is the amount from the previous period which did not decay, plus the number of new fragments created in collisions, plus the amount of debris in the shell created by new launches. The laws of motion for the satellite and debris stocks show this:

$$S_{t+1} = S_t(1 - L(S_t, D_t)) + X_t \tag{1}$$

$$D_{t+1} = D_t(1 - \delta) + G(S_t, D_t) + mX_t. \tag{2}$$

$L(S_t, D_t)$ is the proportion of orbiting active satellites which are destroyed in collisions, and $G(S_t, D_t)$ is the number of new fragments created by collisions between orbiting bodies. We assume that the collision rate is nonnegative, increasing in each argument, and bounded below by 0 and above by 1[10]. No satellites can be destroyed when there are none in orbit ($L(0, D_t) = 0 \; \forall D_t$). In a model with aggregate uncertainty in the number of losses, $L(S_t, D_t)$ would be the expected number of losses in period t.

We assume that the number of new fragments is nonnegative, increasing in each argument, and zero when there are no objects in orbit ($G(0, 0) = 0$). To derive results about the occurrence of Kessler syndrome in section 3.2, we also assume that the growth in new fragments due to debris alone will eventually be greater than δ. δ is the rate of orbital decay for debris, and m is the amount of launch debris created by new satellites.

To fix concepts, a form for $L(S_t, D_t)$ is helpful. We can model the average rate at which objects of type j are struck by objects of type k as

$$p_{jk}(k_t) = 1 - e^{-\alpha_{jk} k_t}, \tag{3}$$

where $\alpha_{jk} > 0$ is a physical parameter reflecting the relative mean sizes, speeds, and

[10]Firms try to avoid collisions by maneuvering their satellites when possible; the collision rate in this model should be thought of as the rate of collisions which could not be avoided, with easily avoided collisions optimized away. Collisions which could have been avoided but were not due to human error are included in this. Implicitly we are assuming that firms are operate their satellites as imperfect cost-minimizers.

inclinations of the object types. The rate at which satellites are destroyed is the sum of the rates at which they are struck by debris and by other satellites, adjusted for the number of satellites which are struck by both. For satellite-satellite and satellite-debris collisions, equation 3 gives us

$$\begin{aligned} L(S,D) &= p_{SS}(S) + p_{SD}(D) - p_{SS}(S)p_{SD}(D) \quad (4)\\ &= (1-e^{-\alpha_{SS}S}) + (1-e^{-\alpha_{SD}D}) - (1-e^{-\alpha_{SS}S})(1-e^{-\alpha_{SD}D}) \\ \implies L(S,D) &= 1 - e^{-\alpha_{SS}S-\alpha_{SD}D}. \quad (5) \end{aligned}$$

The form in equation 3 is convenient as it allows us to solve explicitly for the open access launch rate and is easy to manipulate. It is not derived from any physical model we are aware of, and does not satisfy our assumption that $L(0,D) \equiv 0$. We use this form where it facilitates exposition or computation, but our analytical results do not assume a specific functional form for the collision rate.

2.3 Satellites as a productive asset

Active satellites provide services to a variety of entities: individuals, firms, governments, research agencies, and others. The services provided tend to be information services like mobile broadband, images of the Earth, and GPS. We ignore such product differentiation, though it is an important feature of orbit use, to focus on the dynamics of the collision rate and debris growth. Adilov, Alexander, and Cunningham (2015) account for differentiation in a two-period setting.

We assume that satellites are identical, infinitely lived unless destroyed in a collision, and produce a single unit of output per period valued at $\pi > 0$. The market for satellite output is perfectly competitive, so that the price per unit service is the same as the social marginal benefit from that unit of service. The rate of return on a satellite is given by $\pi/F \equiv r_s > 0$. We discuss the effects of relaxing the assumption that satellites are infinitely lived in section 5.2. We assume that costs and returns are constant over time, and comment on the effect of the rate of return on a satellite varying over time in section 5.3.

There are two types of agents: a profit-maximizing firm which can own up to one satellite at a time, and a fleet planner who owns all satellites. A firm which owns a satellite collects a revenue of π every period that the satellite survives. A fraction $L(S_t, D_t)$ of the orbiting satellites are destroyed in collisions every period. Since they are identical, the probability that an individual satellite survives the period is $(1 - L(S_t, D_t))$. The discount factor used by all agents is $\beta = (1+r)^{-1}$, where $r > 0$

is the social discount rate. The value of a satellite, denoted $Q_t(S_t, D_t, X_t)$, is the sum of present returns and the expected discounted value of its remaining lifetime returns.

$$Q_t(S_t, D_t, X_t) = \pi + \beta(1 - L(S_t, D_t))Q_{t+1}(S_{t+1}, D_{t+1}, X_{t+1}) \tag{6}$$

where $X_t = \int_0^\infty x_{it} di$ is the aggregate launch rate based on each potential launcher's entry decision x_{it}. We assume for simplicity that firms cannot choose to deorbit satellites.

A firm which does not own a satellite in period t faces the decision to pay a fixed cost F and launch a satellite which will reach orbit and start generating revenues in period $t + 1$, or to wait and decide again whether or not to launch in period $t + 1$[11]. Assuming potential launchers are risk-neutral profit maximizers, the value of potential launcher i at period t is

$$V_{it}(S_t, D_t, X_t) = \max_{x_{it} \in \{0,1\}} \{(1 - x_{it})\beta V_{it+1}(S_{t+1}, D_{t+1}, X_{t+1}) + x_{it}[\beta Q_{t+1}(S_{t+1}, D_{t+1}, X_{t+1}) - F]\} \tag{7}$$

$$\begin{aligned} \text{s.t. } S_{t+1} &= S_t(1 - L(S_t, D_t)) + X_t \\ D_{t+1} &= D_t(1 - \delta) + G(S_t, D_t) + mX_t \end{aligned}$$

The launch Bellman has an i subscript to indicate that firms may or may not choose to launch. Since satellites are identical, there is no i subscript on the value of a satellite or the expectation operator.

2.4 Open access and the equilibrium collision rate

Under open access, firms launch satellites until the value of launching is 0,

$$X_t \geq 0 : \ \beta Q_{t+1} = F. \tag{8}$$

The resulting value of a satellite is the period return plus the expected value of its survival under open access,

$$Q_t = \pi + (1 - L(S_t, D_t))F. \tag{9}$$

The open access condition and the satellite value can be rewritten to show that in equilibrium the flow of benefits generated by a satellite is equated with the flow of

[11]Whether the lag reflects time-to-build or time-to-launch, there is a difference between the timescale of physical interactions in orbit and the timescale of launch decisions. The former occurs continuously, while the latter does not.

opportunity costs and expected collision costs (the marginal private costs):

$$\pi = rF + L(S_{t+1}, D_{t+1})F. \tag{10}$$

The Implicit Function Theorem and our assumptions on the derivatives of the collision rate and new fragment function allow us to obtain comparative statics on the launch rate from equation 10 without imposing a specific functional form for $L(S, D)$. We provide general analytical results in section 2.10, and a specific example in this section to illustrate the intuition. Let $L(S, D) = 1 - e^{-\alpha_{SS}S - \alpha_{SD}D}$. Define the following quantities:

$$\text{(Log rate of excess return:)}\ \ R = -\log(1 + r - r_s) \tag{11}$$

$$\text{(Launch contribution to collision rate:)}\ \ a^{-1} = \alpha_{SS} + \alpha_{SD}m \tag{12}$$

$$\text{(Carryover satellite stock:)}\ \ \mathscr{s}_t = S_t(1 - L(S_t, D_t)) \tag{13}$$

$$\text{(Carryover debris stock:)}\ \ \mathscr{d}_t = D_t(1 - \delta)) + G(S_t, D_t), \tag{14}$$

The forms of the first two terms come from $L(S, D)$. The first term is the negative log return differential, and is only defined when $r_s - r < 1$. Since the equilibrium collision rate under open access is $r_s - r$, this restriction imposes that some satellites survive each period. The second term is the direct effect of a new launch on the collision rate: each new launch adds 1 satellite to orbit with collision parameter α_{SS}, and m units of debris with collision parameter α_{SD}. Modern launches typically carry more than one satellite; a launch delivering k satellites would contribute ka^{-1} to the collision rate, though we do not model ridesharing on launches. The third and fourth terms are the present stock's contributions to the future stock.

Using the form for $L(S, D)$ in equation 10, we can solve for the equilibrium launch rate as

$$\bar{X}(S_t, D_t) = a[R - \alpha_{SS}\mathscr{s}_t - \alpha_{SD}\mathscr{d}_t].$$

Increases in the excess return cause more firms to want to own a satellite, and hence to launch. On the other hand, strengthening the collision rate couplings (measured in α_{SS} and α_{SD}, described more generally in section 2.6) increases the persistence of prior stocks and reduces the scope for firms to launch new satellites. Increasing the amount of launch debris, m, increases the risk that the newly launched satellites pose to themselves, and reduces the equilibrium launch rate. Since the launch rate cannot be negative, the open access launch policy is

$$X_t = \begin{cases} \bar{X}(S_t, D_t) & \text{if } R > \alpha_{SS} s_t + \alpha_{SD} d_t \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

As $\alpha_{SS} + \alpha_{SD} m \to 0$, launches become decoupled from the collision rate and debris evolution. If this happens, the launch rate goes to infinity when $r_s > r$. This is an extreme and unrealistic case, but it highlights the role of collisions in the model: without the risk of a collision, a satellite is a perfectly safe asset that provides a higher rate of return than the risk-free rate. In reality, the risk of a collision is just one of many factors to be weighed in the expected cost of launching a satellite.

2.5 Marginal and average collision rates

The marginal survival rate, $1 - L - \frac{\partial L}{\partial S} S$, measures the proportion of satellites which will survive collisions given the change in the collision rate caused by new satellites. Its sign is important in understanding orbit use and the marginal external cost, in particular the case when $1 - L - \frac{\partial L}{\partial S} S \geq 0$. Rearranging, we see that this statement relates two quantities: the average survival rate ($\frac{1-L}{S}$), and the marginal collision rate due to a satellite ($\frac{\partial L}{\partial S}$). In particular, when the marginal survival rate is nonnegative, the average survival rate exceeds the marginal collision rate due to another satellite ($1 - L - \frac{\partial L}{\partial S} S \geq 0 \implies \frac{1-L}{S} \geq \frac{\partial L}{\partial S}$). Since the average survival rate is a decreasing function of the average collision rate, the condition implies that the marginal collision rate due to another satellite is greater than the average collision rate, so another satellite will increase the collision rate.

The opposite statement, $1 - L - \frac{\partial L}{\partial S} S \leq 0$, implies that the marginal collision rate is lower than the average collision rate (greater than the average survival rate), so another satellite would decrease the collision rate. This could be plausible in cases where the "most likely destruction" after adding another satellite is less severe than it was before - for example, if the new satellite poses a significant threat to a dangerous already-orbiting satellite, the increased likelihood of that one collision may decrease the expected number of collisions. This seems unrealistic for most applications. For the rest of this paper, we rule out this case and assume $1 - L - \frac{\partial L}{\partial S} S \geq 0$.

Another way to motivate this assumption is through "careful placement": since firms launching satellites want them to survive for as long as possible, they will avoid launching in ways which are more likely to result in the satellite's destriction. As a result, the increase in the expected number of collisions from a marginal satellite should be smaller than the average probability a satellite survives. This assumption is formalized below:

Assumption 1. *(Launches increase the rate of collisions) The marginal survival rate is nonnegative,*

$$1 - L - \frac{\partial L}{\partial S} S \geq 0,$$

implying that a new launch will weakly increase the collision rate.

2.6 Three types of physical couplings

We use the term "physical coupling" here to refer to ways that objects in the laws of motion for satellite and debris stocks are connected to each other. These couplings give some structure to the laws of motion, and drive the physical dynamics of orbit use. There are three economically relevant physical couplings between objects in orbit. If all of these couplings were turned off, there would be no problem of excess congestion or interesting dynamics in orbit.

The couplings highlight the role of debris in the economics of orbit use. Without debris, the only congestion externality can be a "steady state" one created by the present stock of satellites being too high. With debris and some of the couplings (described below), the congestion externality can be "dynamic" in the sense that the debris stock makes present congestion a function of past launch rates.

The collision rate couplings The collision rate coupling refers to the arguments of the mean collision rate function, $L(\cdot)$. When the couplings are turned off, the collision rate is exogenous: $L(\cdot) = L$. It may vary over time, but since collision rate is exogenous with the coupling turned off, there is no congestion externality. Even if open access causes Kessler syndrome due to other couplings, the open access launch rate is efficient in the sense that the planner would produce the same outcome. With the collision rate coupling turned off, there are no consequences to excess satellite levels or debris growth.

If the collision rate is coupled with the satellite stock ($L(\cdot) = L(S)$), then there can be a "steady state" congestion externality. The strength of this coupling can be measured by $\frac{\partial L}{\partial S}$. Despite any other couplings and their effects on debris growth, the only source of inefficiency is that open access results in too many satellites being launched relative to the optimal plan. Though the other couplings may produce interesting dynamics in debris accumulation, those dynamics are irrelevant to socially optimal orbit use. Both the open access and socially optimal launch paths will then involve single-period jumps to the steady state. The optimality of the most rapid approach path (MRAP) is driven by the linearity of the objective function and the lack of persistent consequences due to satellite launches. Though the state equations make the setting dynamic, firms and the planner face a sequence of static decisions. This situation is explored further in section 2.7.

If the collision rate is coupled with the debris stock ($L(\cdot) = L(D)$) but not the satellite stock, there can be a "dynamic" congestion externality. The strength of this coupling can be measured by $\frac{\partial L}{\partial D}$. The presence of other couplings will influence the nature of this externality, but with just a coupling between debris and collision rate there can be persistent consequences for specific launch histories due to debris accumulation. Depending on the other couplings, the planner may no longer find the MRAP to the steady state optimal, although firms will continue to take the MRAP. As a result of this coupling and others, firms may end up overshooting the steady state, particularly if there are initially low levels of satellites and debris. The "fully coupled" case of collision rate, which is the main focus of this paper, links collision rate to both the satellite and debris stocks ($L(\cdot) = L(S, D)$), allowing for dynamic congestion where the MRAP is not optimal.

The launch debris coupling The launch debris coupling refers to the amount of launch debris created, i.e. the parameter m in the debris law of motion. The strength of this coupling can be measured by m, with higher values of m implying a stronger coupling. When $m = 0$, launches are uncoupled from the debris stock. When collision rate is coupled to the debris stock, this coupling strengthens the externality of launch, since when $m > 0$ launches create debris independent of any collision effects. However, when collision rate is coupled to the debris stock, the launch debris coupling can "stabilize" open access orbit use by forcing firms to internalize some of the persistent effects of their launch on orbital congestion.

The new fragment formation couplings The new fragment formation coupling refers to the arguments of the new fragment function, $G(\cdot)$. We assume that with any coupling, $G(\cdot)$ is a strictly increasing function of its arguments. When the coupling is turned off, the number of new fragments created is exogenous and there are no positive feedbacks between debris. With the collision rate coupling and the launch debris coupling activated, current congestion can still depend on past launch rates, but the persistence may be muted by of the lack of the new fragment formation coupling. If the new fragment function is coupled to the satellite stock, $G(\cdot) = G(S)$, then there can be persistent consequences of excess satellite levels. When the new fragment function is coupled to the debris stock, $G(\cdot) = G(D)$, there can be multiple equilibria: one with low debris and one with high debris. Of the two, only the former will be stable, while the latter will either return to the former or else explode to infinite debris. This multiplicity will exist independent of the collision rate and launch debris couplings. When the collision rate is only coupled to the satellite stock or uncoupled ($L(\cdot) = L(S)$ or $L(\cdot) = L$), the multiplicity is only in debris levels. When the collision rate is fully coupled ($L(\cdot) = L(S, D)$) and the new fragment function is coupled with only the debris stock, multiple equilibria in both satellites and debris can exist: one with low debris and high satellites, and one with high debris and low satellites. As in the previous case, only the low debris equilibrium is stable. As with the collision rate couplings, the strengths of these couplings can be measured by $\frac{\partial G}{\partial S}$ and $\frac{\partial G}{\partial D}$.

2.7 Steady state congestion: collisions without debris

Suppose the collision rate does not depend on the debris stock, i.e. $L_t = L(S_t)$. This could be because there is no debris, or because satellites are perfectly shielded against debris. In this case there are no interesting dynamics in the model: open access jumps to the steady state in a single period, as does the planner.

The lack of persistent consequences for specific launch or collision histories means that the congestion problem is effectively static. Present decisions do not affect future decisions. In this case, marginal satellites create a negative congestion externality if they increase the collision rate, i.e. cause a first-order stochastically dominant shift in the distribution of collisions. Satellites are always a good in this case (the marginal value of a satellite to the fleet is always positive). The linearity of the expected social welfare function in collision probabilities suggests that the planner will be indifferent to or like any shift in the collision distribution which does not increase the average collision rate.

2.8 Dynamic congestion: collisions with debris

When the collision rate depends on the debris stock, i.e. $L_t = L(S_t, D_t)$, debris creates persistent consequences for specific launch and collision histories. In this case open access can have interesting dynamics, and the planner's optimal satellite accumulation path will in general not be a jump to the steady state. Firms under open access still face a linear objective function and attempt to take a single-period jump to the steady state, but the delayed effect of period t launches on period $t + 2$ debris means the system dynamics may not allow such a jump.

Both firms and the planner will launch more satellites when the skies are clear.

While the planner will progressively decrease their launch rate to reach the optimal steady state, firms may overshoot the stable region of the state space and end up on a trajectory which results in Kessler syndrome. This happens because firms set their launch rates in response to debris accumulation one period ahead, but no farther. Open access removes any incentive for firms to care about longer-horizon effects. Even if firms do not launch into Kessler syndrome, they may still consistently miss the steady state. This can happen because firms will launch at rates which make the future satellite and debris stocks create collision risk greater than the excess rate of return on a satellite. When that happens, firms will stop launching, allowing the stocks to fall again.

2.9 Marginal external cost and the optimal launch plan

In this section, we use letter subscripts on functions to indicate partial derivatives with respect to a given argument, e.g. $L_S(S, D) \equiv \frac{\partial L(S,D)}{\partial S}$.

The fleet planner owns all of the satellites in orbit, and controls all launches to maximize the expected net present value of the satellite fleet. This is the "sole owner" benchmark used in other environmental and natural resource settings[12]. In doing so, the planner equates the marginal benefit of another satellite with its social marginal cost. The social marginal cost in this setting is the sum of the opportunity cost of the investment and the collision rate $(r + L(S_{t+1}, D_{t+1}))$, and the effect of the marginal satellite on future collisions and debris growth $(\xi(S_{t+1}, D_{t+1})/F)$. The first two terms are private marginal costs internalized by firms, and the final term is a marginal external cost not internalized by firms.

There are three important functions which are necessary to understand the marginal external cost: the marginal survival rate $(\mathscr{L}(S_t, D_t))$, the growth-launch fragment balance $(\Gamma_1(S_t, D_t))$, and the new fragments from the current stock $(\Gamma_2(S_t, D_t))$.

The marginal survival rate, introduced in section 2.5, is

$$\mathscr{L}(S_t, D_t) = 1 - L(S_t, D_t) - SL_S(S_t, D_t). \tag{16}$$

$\mathscr{L}(S_t, D_t)$ is the probability that an already-orbiting satellite will escape a collision if one more satellite is added to the fleet. In the standard growth model with an exogenous depreciation rate on capital, this would be the proportion of capital which

[12]For example, Gordon (1954) appeals to the sole owner's management of a fishery in showing that the competitive equilibrium is inefficient, and Scott (1955) appeals to the sole owner's management to show cases where short-run competitive equilibria can manage fisheries efficiently.

is undepreciated. With the couplings in $L(S_t, D_t)$, $\mathscr{L}(S_t, D_t)$ is the proportion of undepreciated capital after accounting for the marginal unit's effect on the depreciation rate and capital stock.

The growth-launch fragment balance is

$$\Gamma_1(S_t, D_t) = G_S(S_t, D_t) - m\mathscr{L}(S_t, D_t). \tag{17}$$

$\Gamma_1(S_t, D_t)$ is the balance between the growth in debris through collisions caused by the marginal satellite and the chance a satellite which survives the marginal satellite's launch is exposed to the debris created by that launch. When $\Gamma_1(S_t, D_t)$ is positive the marginal satellite causes more collision fragments between satellites and debris through its presence than through its launch debris. When $\Gamma_1(S_t, D_t)$ is negative it is the opposite case - the marginal satellite's launch debris is a bigger threat to satellites which survive collisions than the satellite's effect on the rest of the fleet and the debris.

The number of new fragments from the current stock is

$$\Gamma_2(S_t, D_t) = 1 - \delta + G_D(S_t, D_t) + mSL_D(S_t, D_t). \tag{18}$$

$\Gamma_2(S_t, D_t)$ is the number of debris fragments which will enter the next period due to this period's debris. It contains two pieces: the first $(1 - \delta + G_D(S_t, D_t))$ is the number of debris fragments which will not decay plus the number of new fragments which will be created in collisions between satellites and debris due to marginal units of debris, and the second $(mSL_D(S_t, D_t))$ is the increase in satellites lost due to launch debris.

With these terms defined, we can express the marginal external cost as

$$\xi(S_{t+1}, D_{t+1}) = \underbrace{S_{t+1}L_S(S_{t+1}, D_{t+1})F}_{\substack{\text{cost of collisions} \\ \text{caused by marginal} \\ \text{satellite } (\geq 0)}} - \underbrace{\frac{\Gamma_1(S_{t+1}, D_{t+1})}{\Gamma_2(S_{t+1}, D_{t+1})} S_{t+1}L_D(S_{t+1}, D_{t+1})F}_{\substack{\text{relative debris effect} \\ \text{weighted cost of collisions caused} \\ \text{by marginal debris } (\lesseqgtr 0)}}. \tag{19}$$

The first term of $\xi(S_{t+1}, D_{t+1})$ is always nonnegative, but the second term may be positive or negative. When the second term is negative the growth in new fragments due to a new satellite exceeds the risk caused by launch debris, and the planner would like to see the satellite stock reduced.

The fleet planner's problem is

$$W(S_t, D_t) = \max_{X_t \geq 0} \{\pi S - FX_t + \beta W(S_{t+1}, D_{t+1})\} \tag{20}$$

$$\text{s.t.} \quad S_{t+1} = S(1 - L(S_t, D_t)) + X_t \tag{21}$$

$$D_{t+1} = D(1-\delta) + G(S_t, D_t) + mX_t. \tag{22}$$

The planner's optimality condition, expressed in terms of the flow of marginal benefits and costs, is

$$\pi = rF + L(S_{t+1}, D_{t+1})F + \xi(S_{t+1}, D_{t+1}). \tag{23}$$

The above flow condition allows us to determine the socially optimal collision rate.

Proposition 1. *(Optimal collision rate) The planner launches so that the collision rate is equated to the rate of excess return net of the rate of marginal external cost, i.e.*

$$L(S_{t+1}, D_{t+1}) = r_s - r - \frac{\xi(S_{t+1}, D_{t+1})}{F}. \tag{24}$$

Proof. Rearranging equation 23 and dividing by F yields equation 24. □

Without further assumptions on the model primitives, the marginal external cost is not always positive. The possibility of a negative marginal external cost here comes from the physical function $\Gamma_1(S_t, D_t)$. $\Gamma_1(S_t, D_t)$ is the balance between the growth in debris through collisions caused by the marginal satellite ($G_S(S_t, D_t)$) and the chance a satellite which survives the marginal satellite's launch is exposed to the debris created by that launch ($m\mathscr{L}(S_t, D_t)$). When $\Gamma_1(S_t, D_t)$ is positive the marginal satellite causes more collision fragments between satellites and debris through its presence than through its launch debris. When $\Gamma_1(S_t, D_t)$ is negative it is the opposite case - the marginal satellite's launch debris is a bigger threat to satellites which survive collisions than the satellite's effect on the rest of the fleet and the debris. $\Gamma_2(S_t, D_t)$ is the number of debris fragments which will enter the next period due to this period's debris. It contains two pieces: the first $(1 - \delta + G_D(S_t, D_t))$ is the number of debris fragments which will not decay plus the number of new fragments which will be created in collisions between satellites and debris due to marginal units of debris, and the second ($mSL_D(S_t, D_t)$) is the increase in satellites lost due to launch debris. The fragment balance is weighted by the inverse number of new fragments generated by the current stock.

We assume that $m\mathscr{L}(S_t, D_t)$ is small enough that the marginal external cost is always positive in a steady state. One intuition for this assumption is that m is small. Launch integrators and vehicle providers have incentives to invest in this, as a strong launch coupling implies that a satellite faces some probability of being destroyed by the debris from its own launch. Launch services are generally competitive enough, and buyers of launch services sufficiently risk-averse, to make this reasonable[13].

Figure 1 shows how the satellite-debris dynamics shift private and social marginal cost curves over time. Increases in satellites and debris shift both costs up, with both marginal cost curves shifting up more under open access than under the optimal plan. This offers another intuition for the congestion wedge in orbit use: firms are unable to control how much marginal costs shift as well as the planner would.

[13] Figures 11, 12, and 13 in the appendix show cases where the MEC is negative, violating the assumption. These are cases where the collision rate couplings are very weak, the launch debris coupling is very strong, or the launch cost is very low so that the excess return on a satellite is very high.

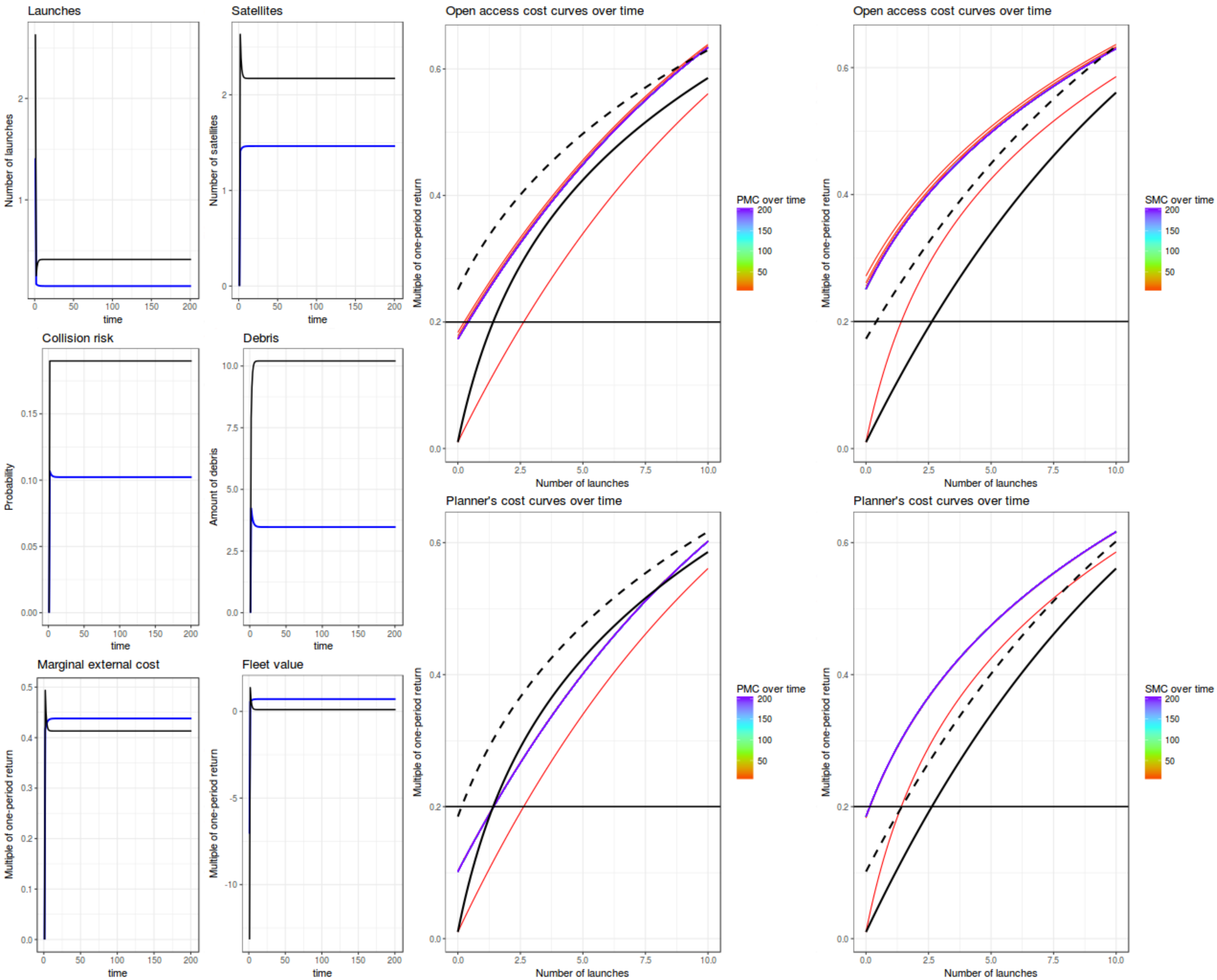


Figure 1: Left: time paths under open access (black) and the optimal plan (blue). Right: shifts in private and social marginal cost curves over time under each plan (open access above and the optimal plan below). The middle column of cost curves show how the private marginal cost shifts each period in color, with the initial social marginal cost shown in solid black and the final social marginal cost shown in dashed black. The right column of cost curves show how the social marginal cost shifts each period in color, with the initial private marginal cost shown in solid black and the final private marginal cost shown in dashed black. The marginal benefit of another satellite is the horizontal black line.

2.10 Properties of open access equilibria

In this section we explore the properties of open access equilibria. The main results are that the equilibrium collision rate is determined by the excess return on a satellite, that when the collision rate is increasing in new launches (Assumption 1) the collision rate and debris stocks are too large under open access relative to the optimal plan, and that exogenous increases in the debris decay rate or decreases in the amount of launch debris may increase the equilibrium debris stock due to rebound effects. The first two properties establish the nature of open access launching as well as the existence and magnitude of the congestion externality. The third property is relevant to policy proposals which seek to limit debris growth through technical changes, without addressing the economic incentives involved in launching satellites.

Proposition 2. *(Equilibrium collision rate) The equilibrium collision rate along the open access path is equal to the rate of excess return on a satellite over the market asset.*

Proof. Rearranging the open access equilibrium flow condition (equation 10) and dividing by F, the equilibrium collision rate can be written as

$$L(S_{t+1}, D_{t+1}) = r_s - r. \tag{25}$$

□

In the case where the economic parameters π, F, β are time-varying, Proposition 2 implies that the equilibrium collision rate in period t under open access will be determined by the values of the economic parameters in period $t-1$, i.e. the period when the newest satellites in orbit were launched. In this case, the equilibrium collision rate will not be a constant, but will still be determined by the economic fundamentals of launching satellites. Section 5.3 explores this in more detail.

Proposition 2 implies that the equilibrium collision rate is increasing in the profitability of a satellite, decreasing in the cost of launching a satellite, and increasing in the discount factor. Proposition 2 does not imply that the open access equilibrium must be a steady state. Any type of dynamics in the satellite and debris stocks which keep the next-period collision rate equal to the excess return on a satellite can be an equilibrium path.

Corollary 1. *There are multiple open access equilibria, defined by the isoquant where collision rate is equal to the excess return on a satellite.*

Corollary 1 follows from the form of the equilibrium condition. Since the collision rare is equal to excess return everywhere along that isoquant, every point on the isoquant is an open access equilibrium. The initial conditions and physical dynamics determine which equilibrium is reached from a non-equilibrium state.

Corollary 2. *When it is profit-maximizing to launch, firms under open access pursue the most rapid approach path to an equilibrium.*

Corollary 2 follows from the fact that equilibrium under open access requires choosing a launch rate to equate the rate of excess return on a satellite with the next-period collision rate. In a sense, this is an assumption of the model and not a result: if firms equilibriate the collision rate in every period, then they are necessarily following paths directly from the initial conditions to the equilibrium set. Since the laws of motion are linear in the launch rate, the open access paths from the initial condition to the equilibrium set are also linear in the state space. The implications of this type of approach path are more interesting. Unless the equilibrium reached is also a steady state, MRAPs from an initial condition to the equilibrium set are not MRAPs to steady states, and result in overshooting in at least one state variable. This is explored in more detail in Proposition 8.

Corollary 3. *Along the open access equilibrium path, the collision rate is*

1. *increasing in the per-period satellite return,*
2. *decreasing in the launch cost, and*
3. *increasing in the discount factor.*

Corollary 3 shows that policies which increase the cost or reduce the profitability of operating a satellite will reduce the equilibrium collision rate. If firms become more patient, they will be willing to tolerate a higher equilibrium collision rate. Proposition 3 shows the existence of the externality: equilibrium collision rate is too high.

Proposition 3. *(Externality) The open access equilibrium results in higher collision rate than the optimal plan.*

Proof. We denote objects along the open access path with hats, and objects along the optimal path with stars, e.g. $\hat{S}_t$ is the satellite stock in period t under open access and S^*_t is the satellite stock in period t under the optimal plan. The equilibrium collision rate is

$$L(\hat{S}_{t+1}, \hat{D}_{t+1}) = r_s - r, \tag{26}$$

while the socially optimal collision rate is

$$L(S^*_{t+1}, D^*_{t+1}) = r_s - r - \frac{\xi(S^*_{t+1}, D^*_{t+1})}{F}. \tag{27}$$

Since we assumed $G_S(S, D) > m\mathscr{L}(S, D)\ \forall S, D$, the marginal external cost $\xi(S^*_{t+1}, D^*_{t+1})$ is positive, and $L(S^*_{t+1}, D^*_{t+1}) < L(\hat{S}_{t+1}, \hat{D}_{t+1})$. $\square$

It is useful to know how the launch rate and debris stock respond to changes in the model parameters. Particularly policy-relevant parameters include the launch cost, the launch debris, and the decay rate. Pigouvian launch taxes have been suggested to reduce orbital debris, and would manifest as changes in the launch cost; command-and-control policies to reduce the amount of launch debris have been proposed, and reusable launch vehicles reduce the amount of launch debris; decay rates are typically decreasing as altitude increases and policies which reduce the lifespan of orbital debris can be modeled as increasing the decay rate[14]. Proposition 4 considers the effects of changes to the collision rate, satellite stock, and debris stock on the equilibrium launch rate. Proposition 5 considers the effects of changes in the launch costs, launch debris, and decay rates on the equilibrium launch rate and debris stock.

Proposition 4. *(Economic incentives to reduce congestion) Along the open access equilibrium path, the launch rate is*

1. *decreasing in the current satellite stock, and*
2. *decreasing in the current debris stock if and only if the marginal survival rate is nonnegative.*

The proof, shown in Appendix C, follows from applying the Implicit Function Theorem to equation 10, shown in the appendix. The key inequality from the proof determines how the launch rate varies with the debris stock (period t values are shown with no subscript, and period $t+1$ values are marked with a $'$, e.g. $S_t \equiv S, S_{t+1} \equiv S'$):

$\frac{\partial X}{\partial D} < 0$ if

$$(S,D): \quad \underbrace{\frac{\partial L(S',D')}{\partial S'} S}_{\substack{t+1 \text{ marginal collision probability from} \\ t+1 \text{ satellites} \cdot \text{ present satellite stock}}} < \underbrace{\frac{\frac{\partial L(S',D')}{\partial D'}}{\frac{\partial L(S,D)}{\partial D}}}_{\substack{\text{growth rate of} \\ \text{marginal collision rate} \\ \text{due to debris}}} \cdot \underbrace{\left(1 - \delta + \frac{\partial G(S,D)}{\partial D}\right)}_{\substack{\text{net growth in debris} \\ \text{due to debris}}} \tag{28}$$

Inequality 28 defines a set of values for (S,D) within which the launch rate is decreasing in the debris stock and outside of which the launch rate is increasing in the debris stock. The condition states that the marginal collision rate in period $t+1$ from satellites in that period across the stock of satellites in t is less than the product of the growth rate of the marginal collision rate from debris and the net growth in debris due to debris. If $1 - L - L_S S \geq 0$, as we have assumed, then $\frac{\partial X}{\partial D}$ is nonpositive. Intuitively, inequality 28 states that the launch rate should be decreasing in the debris

[14]Current FCC policy mandates that inactive satellites must be deorbited within 25 years of launch, which could be captured in the decay rate parameter of this model. Similar policies are encouraged by the IADC and other space agencies.

stock whenever the increase in the collision rate due to debris and debris growth exceeds the increase in collision rate from marginal satellites. This is the essence of "careful placement" of new satellites.

Proposition 5. *(Short-run rebound effects) Along the open access equilibrium path, the current launch rate is*

1. *increasing in the return on a satellite,*
2. *decreasing in the amount of launch debris, and*
3. *increasing in the debris decay rate.*

The next-period debris stock is

1. *increasing in the return on a satellite,*
2. *increasing in the amount of launch debris, and*
3. *decreasing in the debris decay rate.*

The proof is shown in Appendix C. The potential for policies intended to reduce future debris stocks to have perverse effects arises from a combination of Propositions 4 and 5. Policies which reduce amount of time before inactive satellites must deorbit (i.e. increase the debris decay rate) can lead to better steady states, shown in Proposition 9, but the convergence may be non-monotonic. The initial reduction in debris spurs an increase in the launch rate, which may be too large to reach the steady state right away. Figure 2 shows an example of how a non-monotonic steady state transition caused by an increase in the decay rate can lead to higher short-run levels of debris.

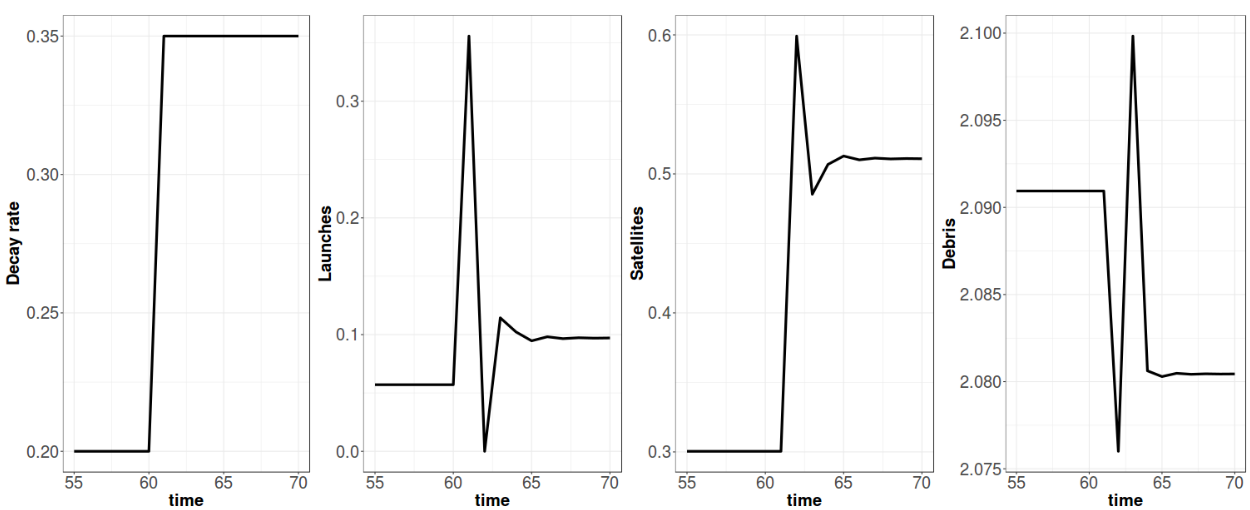


Figure 2: An example of an increase in the rate of debris decay, δ, causing non-monotonic steady state convergence. Though the debris stock initially falls, the launch response is large enough that the debris stock rises above its previous level before reaching the new steady state.

These effects stem from the substitutability of the rate of return on a satellite and the collision risk in the equilibrium condition (equation 10). Since firms will enter the commons until the expected profits are zero, reducing the risk along one margin (e.g. less launch debris) can result in firms increasing risk along another margin (e.g. more launches). Since the launch debris and decay rate effects are (a) the opposite of what a purely physical model which ignores open access would predict, and (b) they occur between steady states, we refer to them as short-run rebound effects. Proposition 9 in section 3.1 explores the long-run effects of the same parameter changes on debris levels. Figure 5 illustrates numerically how the statics results in Proposition 5 interact with the stability results in Proposition 7 across thousands of time series simulations.

Economic controls which increase the cost of launching will have the expected effects: fewer launches and less debris. The results in Proposition 5 do not account for the fact that command-and-control policies which decrease the amount of launch debris or increase the decay rate may also increase the cost of launching a satellite. Such feedbacks could reduce or remove the perverse effects. The takeaway is that in order for policies targeting launch debris generation and debris decay rates to be effective at reducing the equilibrium debris stock, they must affect the incentive to launch a satellite in the first place.

2.11 Action and inaction

Under open access, firms will not launch satellites if the excess return is insufficient to cover the congestion cost. Holding the launch rate constant, collisions and debris growth may drive an orbit from the action region, where some find it economical to launch, to the inaction region, where it is uneconomical for any to launch. Launch shutdown in some periods may be socially optimal. As long as Kessler syndrome doesn't occur, the physical dynamics of the system will eventually bring satellite and debris stocks back to the action region. Figure 3 shows an example of action and inaction regions in the open access and optimal launch policies, and figure 4 shows the associated fleet values[15].

Definition 1. *(Action region) The action region is the set of satellite and debris levels for which open access results in launches, i.e.*

$$(S_t, D_t) : L(S_{t+1}, D_{t+1}) \leq r_s - r.$$

[15]Figure 14 in the appendix shows time paths where open access bounces between the regions and the planner does not.

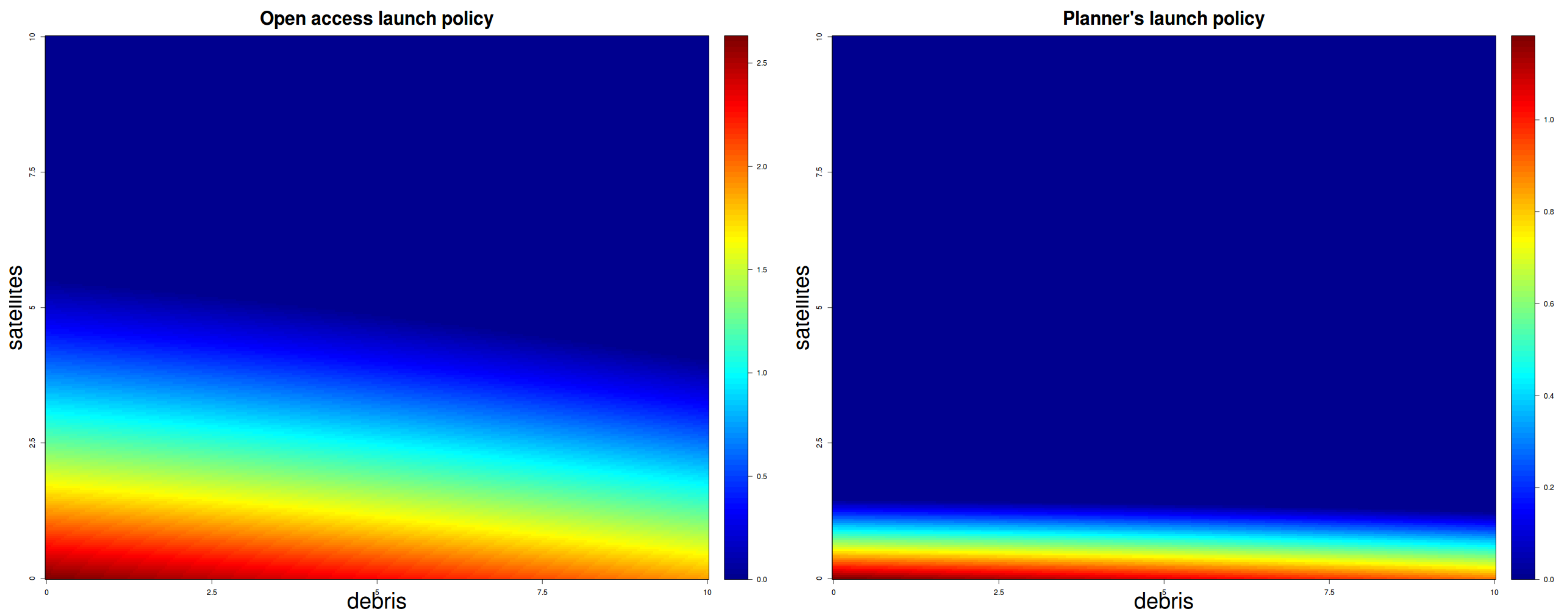


Figure 3: An example of the action and inaction regions under open access (left) and the optimal plan (right). Open access does not restrict launches in response to satellite or debris stocks as quickly as the planner or as severely.

The complement of the action region is the inaction region. Under open access, the inaction region is where the collision rate is greater than the excess return on a satellite. In the inaction region, aggregate dynamics are driven entirely by the collision rate, decay rate, and new fragment production (the "physical dynamics"). The laws of motion become

$$S_{t+1} = S_t(1 - L(S_t, D_t)) \tag{29}$$

$$D_{t+1} = D_t(1 - \delta) + G(S_t, D_t). \tag{30}$$

One possible steady state under these laws is to have no satellites or debris. Before that happens the stocks would again reach the action region, and the zero objects steady state will not be reached. The other possibility is that, as the satellite stock is driven to zero, the debris stock reaches the threshold at which Kessler syndrome occurs. Though open access reduces launches in response to exogenous increases in the steady state debris level, the point on the equilibrium isoquant reached by a MRAP from the initial conditions may be in a place where the physical dynamics lead to the Kessler region.

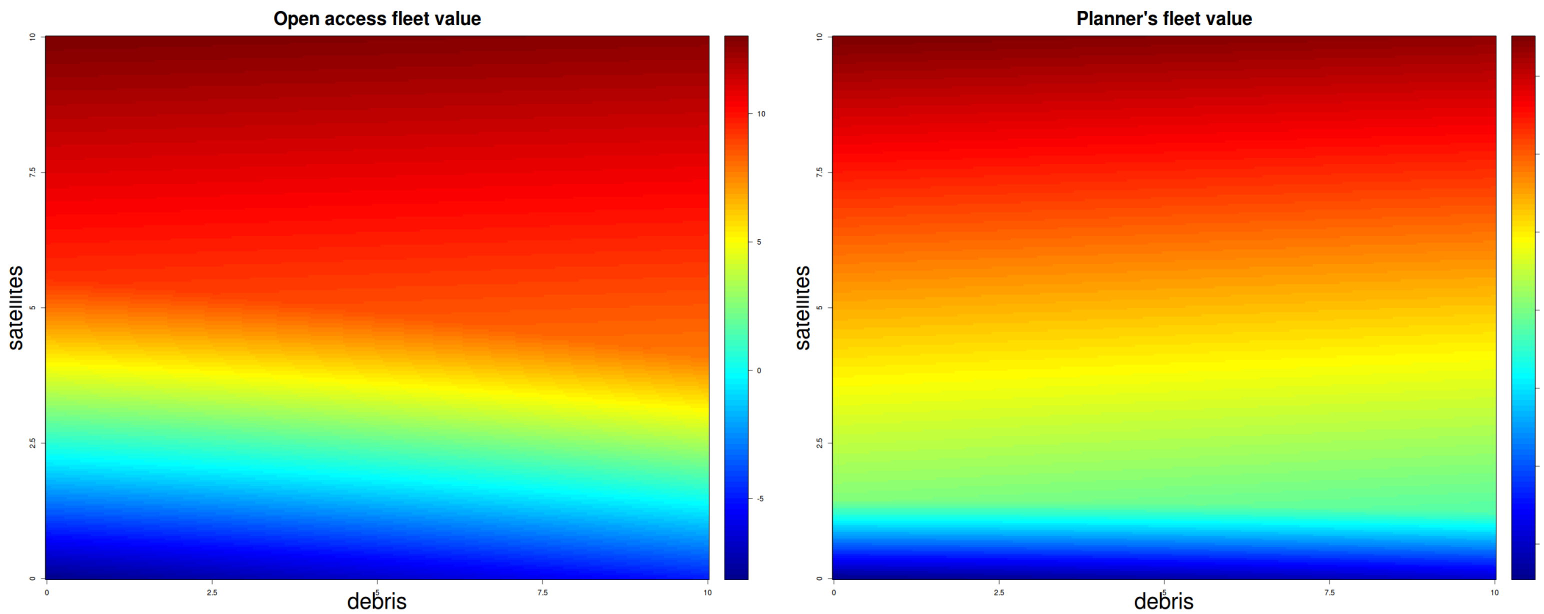


Figure 4: An example of the fleet value functions under open access (left) and the optimal plan (right). The planner's relatively muted launch policy creates a smaller region of negative profits and a larger region of positive profits. Losses under the open access plan are steeper, larger fleets less valuable, and higher debris stocks more costly than under the planner's policy.

3 The economic dynamics of orbital mechanics

Having described the nature of open access equilibria, we now turn to their dynamic properties. Two issues of first-order policy concern are the long-run properties of open access equilibria, and the conditions under which open access will prevent or cause Kessler syndrome. In this section, we focus on understanding these issues by analyzing open access steady states and by formally characterizing Kessler syndrome and its relation to open access equilibria. We also consider the effects of other dynamic issues affecting orbit use, namely time-varying economic parameters and space weather. Our main results are that there can be multiple steady states of varying stability, and that increases in the excess return on a satellite shift the equilibrium set outward and bring it close to the region of the state space where Kessler syndrome occurs. Allowing rates of return to vary over time does not change these properties, though it highlights the role of launch costs as a source of satellite value and suggest that Pigouvian launch fees will need to account for this in order to be efficient. Space weather does not change the properties of the equilibrium set, though it shifts its location and in doing so may cause Kessler syndrome.

3.1 Properties of open access steady states

The distinction between a "steady state" and an "equilibrium" is relevant here. The laws of motion for satellites and debris may be stationary under constant non-equilibrium launch rates, a case frequently analyzed in the physics and engineering literature. Another possibility is for firms to launch until zero profits in every period, while the satellite and debris stocks vary over time. As long as the collision rate in each such period is equal to the excess return on a satellite, the firms are in an open access equilibrium. What we are interested in here are states where both conditions are true: the physical aggregates are stationary over time and firms are earning zero economic profits. We refer to these as "open access steady states".

In an open access steady state, the collision rate must be equal to the excess return on a satellite ($L(S, D) = L(S', D') = r_s - r$) along with the usual conditions that the aggregate variables be stationary:

$$(S, D) : L(S, D) = r_s - r, \tag{31}$$

$$\begin{aligned} S' = S &\implies S = (1 - L(S, D))S + X \\ &\implies X = L(S, D)S, \end{aligned} \tag{32}$$

$$\begin{aligned} D' = D &\implies D = (1 - \delta)D + G(S, D) + mX \\ &\implies \delta D = G(S, D) + mX. \end{aligned} \tag{33}$$

Equations 31, 32, and 33 define the open access steady states. If there is no launch debris ($m = 0$), then steady states only require balancing natural debris decay (δD) against debris growth due to objects already in orbit ($G(S, D)$). As m increases, the effect of replacement satellites (reflected in mX) on the steady state level of debris also increases, requiring a lower rate of debris growth due to objects already in orbit.

Proposition 6. *(Multiplicity) There can exist multiple open access steady states.*

The proof is shown in Appendix C. The exact open access steady state reached is determined by the physical and economic dynamics. Not all open access steady states are locally stable. When there are multiple stable open access steady states, the one reached will depend on the initial conditions. Shocks to the debris stock, like weapons tests by national authorities, could move the system around in the same basin or to a different one.

Proposition 7. *(Local stability) Open access steady states may be locally unstable.*

The proof is shown in Appendix C. The key equation from the proof is

$$\frac{\partial \mathscr{Y}}{\partial D}(D) = -\delta - \frac{L_D(S,D)}{L_S(S,D)}(G_S(S,D) + m(r_s - r)) + G_D(S,D). \tag{34}$$

The stability of an open access steady state depends on three factors, shown on the right hand side of equation 34. The first is the decay rate (δ), which can ensure stability if it is high enough. Physically, higher decay rates indicate a region with greater renewability. The second is the equilibrium effect of new satellites on debris growth. The effect is increasing in the strength of the collision rate debris coupling (L_D), the new fragment satellite coupling (G_S), and the launch debris coupling (m). These are all launch byproducts deter future launches through debris creation and reduce the number of satellites which can be sustained. The effect is decreasing in the strength of the collision rate debris coupling (L_S), which determines the effect of new satellites on the collision rate through their presence in orbit. The third is the strength of the new fragment debris coupling, which creates the potential for local instability if it is large enough. Physically, it measures the strength of the positive feedback between debris. The same coupling creates the possibility of Kessler syndrome.

Open access steady states with less debris are more likely to be locally stable if the rate of excess return ($r_s - r$) is not too high, or if the coupling between new fragments and active satellites (G_S) is large enough. It depends linearly on the convexity of the new fragment satellite coupling (G_{SD} and G_{SS}), and nonlinearly on the strength and convexity of the collision rate couplings (L_S, L_D, L_{SD}, and LDD). Short-run dynamics of the type described in propositions 4 and 5 can interact with local instability to prevent the system from reaching a stable steady state, as shown in figure 5.

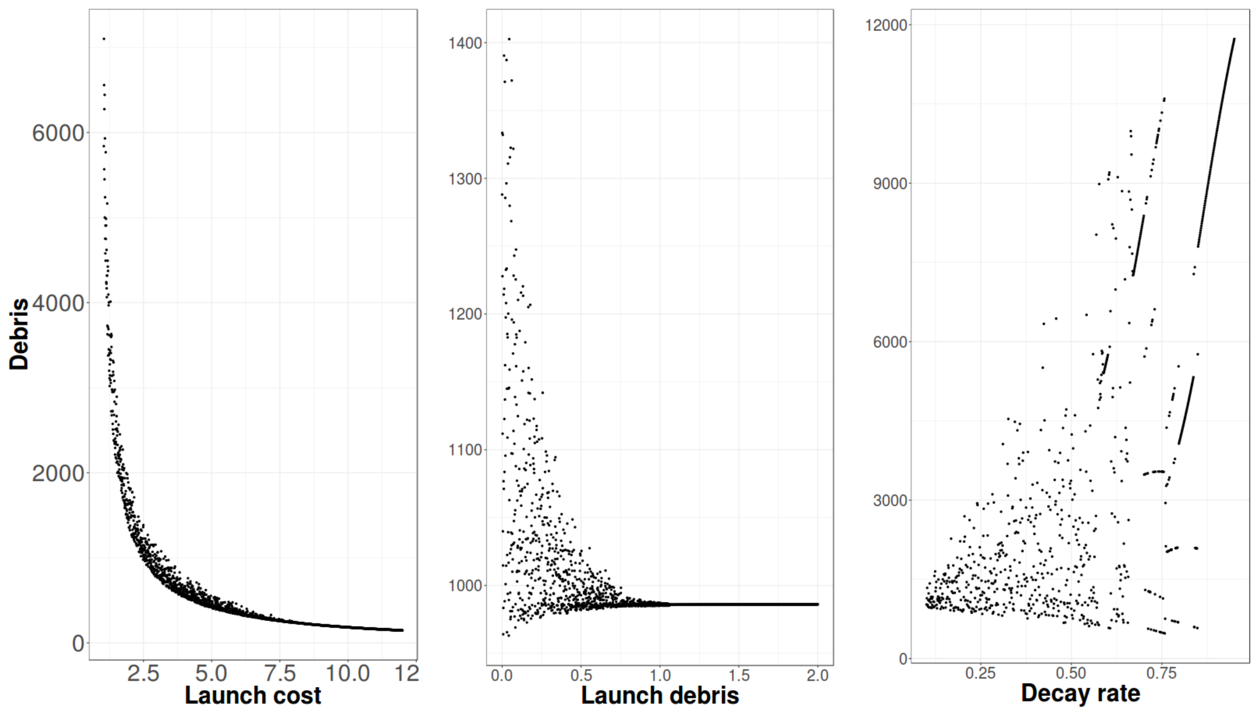


Figure 5: Each point is a long-run ($t = 1000$) debris level for different values of F, m, and δ under open access. The spreads indicate the presence and amplitudes of cycles in debris stocks.

Proposition 8. *(Overshooting) For each open access steady state, there are two disjoint sets in the action region. Paths from one will initially overshoot the open access steady state debris level, and paths from the other will initially overshoot the open access steady state satellite level.*

Proof. (Sketch) The full proof is shown in Appendix C. The intuition is that there is a limited set of points which can reach the open access steady state in one period, and that all other points in the action region are on paths which overshoot at least one state variable in approaching an steady state via the equilibrium isoquant. □

Proposition 8 highlights two properties of open access. First, open access attempts to immediately equilibriate the collision rate by moving to the isoquant where it is equal to the excess return, even if the point on the isoquant that can be reached in one period is not a steady state. This is analogous to a driver speeding to reach their destination and forgetting to decelerate as they approach. Second, this "uninternalized acceleration" problem is more severe at low levels of debris than at higher levels, provided the debris level is below the set leading directly to the steady state. In the driving analogy, this is the effect of starting location on the uninternalized acceleration: when the driver is farther away, they must accelerate more and go faster to reach their destination in the same amount of time, meaning that forgetting to decelerate has higher consequences. Economically, open access induces firms to ignore longer-term environmental consequences of their scramble to take advantage of cleaner orbits. Overshooting can cause Kessler syndrome if the portion of the equilibrium isoquant reached is in the basin of the Kessler region. Figure 6 shows a case where open access and the planner reach steady states.

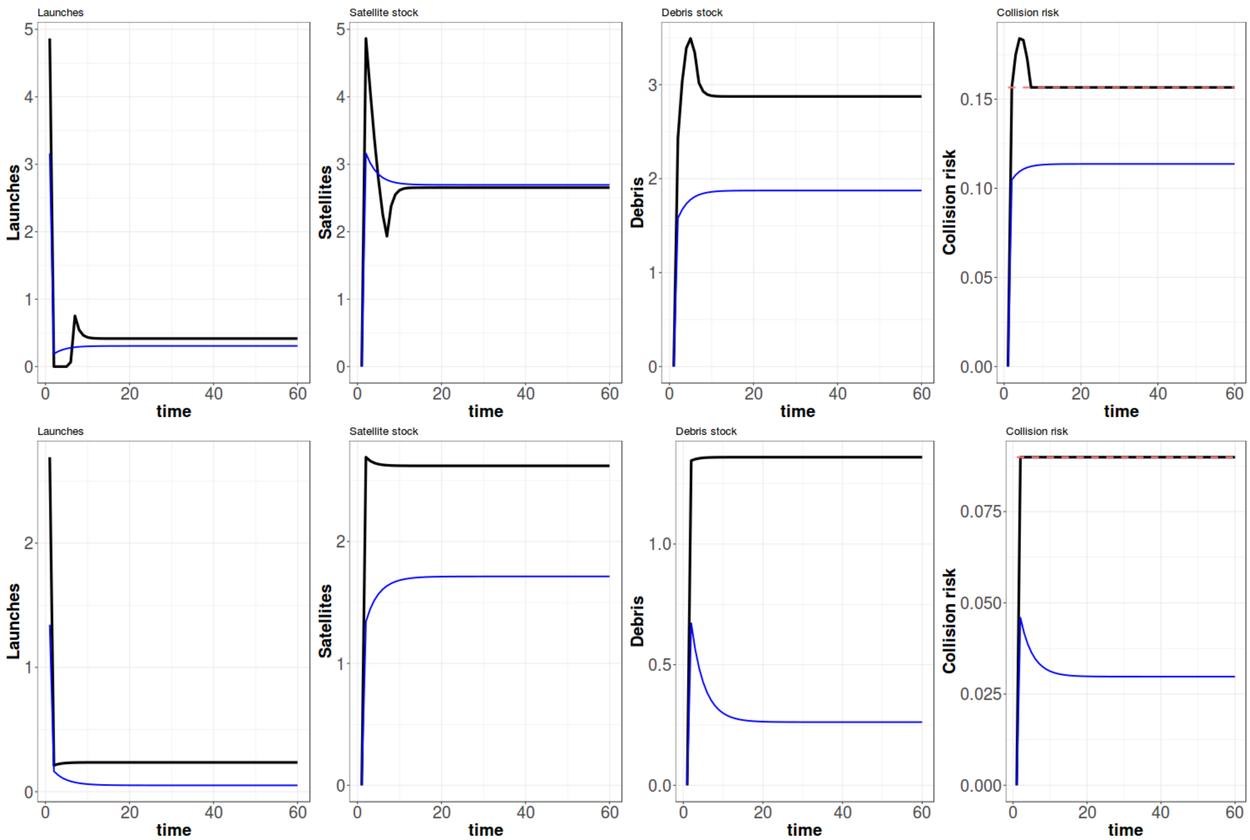


Figure 6: Examples paths which reach the steady state under open access (black) and the optimal plan (blue). Open access is not as responsive as the planner to delayed debris accumulation, which can result in firms under open access launching into the inaction region (left).

The overshooting becomes more severe as the initial condition and physical dynamics bring firms to points in the action region which are farther from the line segment leading to the open access steady state. To see this, note that all open access launch plans are parallel lines in the satellite-debris space. Points farther from the line segment leading to the open access steady state in question will therefore end up farther from that steady state. Given the potential multiplicity of open access steady states, however, these points may lead to a different open access steady state. Since they are all on the same isoquant, reaching one of many steady states implies that other steady states have been overshot. This result suggests some scope for orbit use stabilization policies which simply impose a fixed cap on the number of launches per period, since slowing the rate at which firms approach the equilibrium isoquant can reduce the amount of overshooting. Figures 7 and 8 show examples of overshooting in the open access and planner's phase diagrams.

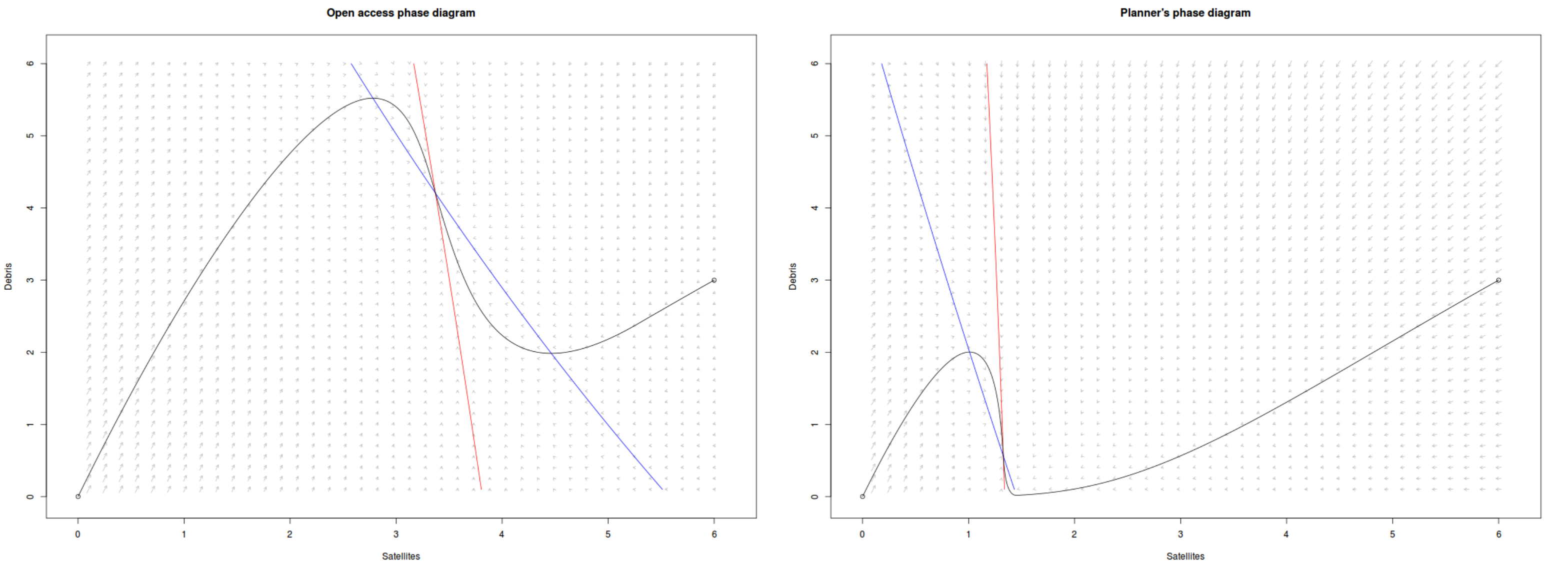
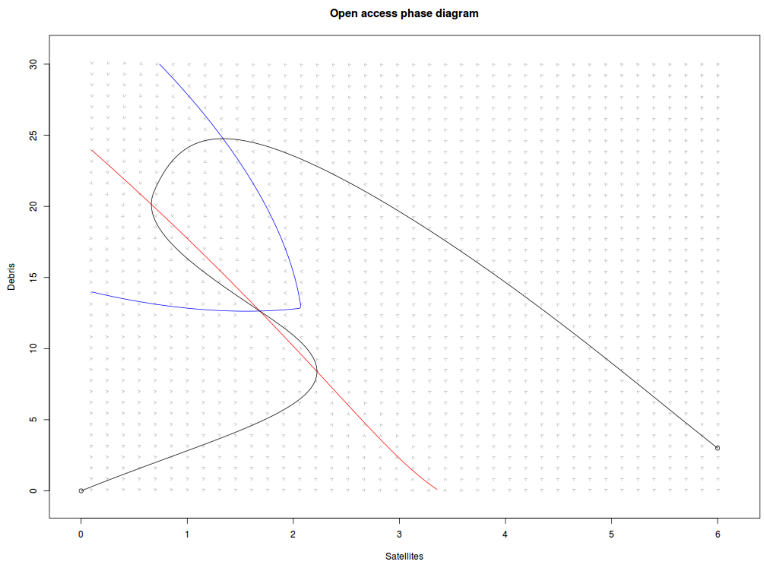


Figure 7: Left: open access phase diagram of the satellite-debris system with trajectories. Right: optimal plan phase diagram of the satellite-debris system. The blue line is the debris nullcline, the red line is the satellite nullcline, and the black lines are trajectories from $(0, 0)$ and $(6, 3)$.

In figure 7, both open access and the planner overshoot when starting from no satellites and no debris.



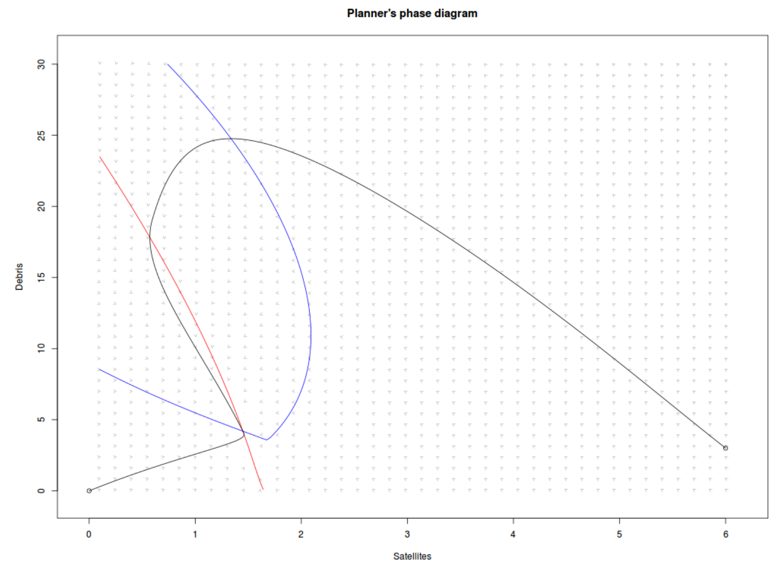


Figure 8: Left: open access phase diagram of the satellite-debris system with trajectories. Right: optimal plan phase diagram of the satellite-debris system. The blue line is the debris nullcline, the red line is the satellite nullcline, and the black lines are trajectories from $(0, 0)$ and $(6, 3)$. The parameterization is the same as figure 7 except for the numbers of new fragments created in collisions (stronger new fragment couplings with satellites and debris in this figure).

In figure 8, open access overshoots when starting from no satellites and no debris, but the planner doesn't.

Proposition 9. *(Long-run debris levels) Open access steady state debris levels are*

1. *increasing in the return on a satellite and the amount of launch debris if and only if the open access steady state is locally stable, and*
2. *decreasing in the decay rate if and only if the open access steady state is locally stable.*

The proof is shown in Appendix C. When the open access steady state is locally stable, long-run debris levels have the expected signs under small changes in return rates, decay rates, and launch debris levels. These signs are reversed when the open access steady state is locally unstable. Unstable steady states will not be where the system ends up in the long-run, so a thought experiment starting from one of these states is on questionable footing. Instead, consider a steady state near the boundary of stability. For such steady states, the effects of technology shocks affecting the return on a satellite or the amount of launch debris produced, or sunspots affecting the decay rate (see section 10), can be ambiguous. If a combination of shocks pushes an open access steady state over the boundary into instability - for example, an increase in debris due to military activities coinciding with sunspot activity - the system will move to a different steady state with more debris. These types of long-run effects can matter if the short-run dynamics preclude moving smoothly between nearby stable steady states. In such cases the short-run dynamics and the number and stability of open access steady states will determine the long-run behavior of the system.

3.2 Kessler syndrome

The occurrence of Kessler syndrome is a key concern in managing orbit use. If open access can prevent Kessler syndrome, regulating orbit use is not as important from an environmental perspective. Even though orbit use will be inefficient it won't cause irreversible environmental damage. On the other hand, if open access can cause Kessler syndrome, orbit use management becomes more urgent.

In this section we formally define Kessler syndrome and establish some properties of the debris threshold beyond which it occurs. Our main result, Proposition 10, is that open access debris levels are increasing in the excess return on a satellite while the Kessler threshold is constant, implying that sustained increases in the return on a satellite can cause Kessler syndrome under open access. Though the Kessler threshold is defined purely in terms of the system's physics, the occurrence of Kessler syndrome depends critically on the economics of orbit use.

Assumption 2. *(Debris growth) The growth in new fragments due to debris is larger than the decay rate for all levels of the satellite stock greater than some level $\bar{D} > 0$,*

$$\bar{D} : G_D(0, D) > \delta \ \forall D > \bar{D}.$$

Assumption 2 and $G(S, D)$ being increasing in both arguments, there is a unique threshold $D^\kappa \geq \bar{D}$ above which Kessler syndrome occurs. Figure 9 shows an example of an open access path which causes Kessler syndrome. Kessler syndrome occurs when the number of new fragments created by collisions between debris exceeds the amount which decays in a single period. For regimes where this condition doesn't hold at any level of debris, Kessler syndrome is impossible.

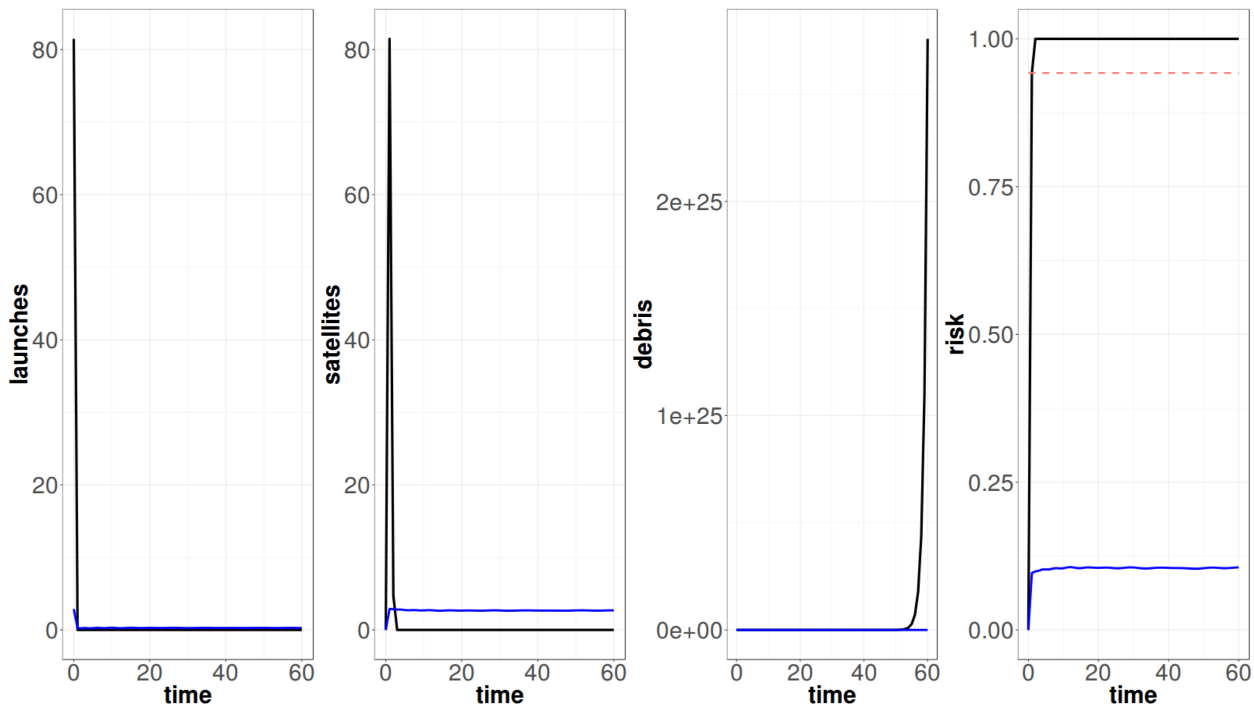


Figure 9: An example of open access causing Kessler syndrome (black) when the planner would avoid it (blue).

Definition 2. *(Kessler syndrome) The Kessler region is the set of debris levels for which cessation of launch activity and immediate deorbit of all active satellites cannot prevent continued debris growth, i.e.*

$$D^\kappa : G(0, D^\kappa) > \delta D^\kappa.$$

Kessler syndrome has occurred when the debris stock enters the Kessler region.

Our assumption that the new fragments function is monotonically increasing in both arguments, and the lack of debris removal technologies, imply that Kessler syndrome is an absorbing state.

The Kessler threshold is entirely determined by the physical properties of the orbit. As the open access steady state debris level rises, say due to a change in the decay rate, the open access launch rate decreases (Proposition 9). The open access steady state debris level is by definition less than or equal to the Kessler threshold, so the open access launch rate is decreasing as steady state debris levels approach the threshold. However, this does not imply that open access will never cause Kessler syndrome; open access launch policies may overshoot the steady state due to debris growth in period $t+1$ (Proposition 8). While steady states are inconsistent with Kessler syndrome, open access equilibria need not be.

Corollary 4. *The Kessler threshold is increasing in the decay rate.*

This is an intuitive property of the physical system: as its absorptive capacity grows, the level at which its capacity is overwhelmed also grows.

Proposition 10. *(Kessler syndrome and open access) If there are sustained increases in the return on a satellite, open access will eventually cause Kessler syndrome.*

Proof. The result follows from Proposition 5 and the fact that the Kessler threshold is independent of the economics of satellite use. Since D^κ is constant with changes in r_s, and the open access debris level $\hat{D}_{t+1}$ is increasing in r_s, sustained increases in r_s will eventually push $\hat{D}_{t+1}$ over D^κ. $\square$

Proposition 10 shows that profit motives are not necessarily sufficient to prevent Kessler syndrome, and may in fact cause it. While the open access steady state will avoid the Kessler region by definition, open access equilibria need not. Because of this, when the excess return on a satellite rises, Kessler syndrome may occur in the transition from the original steady state to the new one[16].

3.3 The risk to LEO

LEO is one of the fastest growing segments in the satellite industry today, particularly for smaller satellites with fewer guidance and control systems (Brodkin (2017), Selk (2017), Dvorsky (2018)). LEO users typically have shorter planning horizons than GEO users and face lower costs. Propositions 4, 5, 9, and 10 reinforce the conclusions from physical models of debris growth: LEO is at the highest risk of Kessler syndrome (Liou and Johnson (2008), Kessler, Johnson, Liou, and Matney (2010)). Our results show that this is not just a feature of the physics given current use patterns, but of the economics underlying those use patterns. While higher decay rates in LEO may protect it against Kessler syndrome, Proposition 10 shows that the economic properties of LEO - particularly the low cost of access - work in the opposite direction. Attempts by national authorities to unilaterally prevent satellites from being launched face a leakage problem similar to the one faced by national authorities attempting to unilaterally control pollution emitted by mobile capital - firms can leave the regulated area and launch from an unregulated one[17].

[16]Though we express this result elsewhere in the paper in terms of "chance" or "likelihood", these are only informal uses for expository purposes. There is no probability measure defined anywhere in the model, so no sense in which one outcome can be "more likely" than another.

[17]Dvorsky (2018) documents what may be the first instance of launch leakage: a California startup denied launch permission by the FAA went outside the FAA's jurisdiction and purchased a launch from an Indian launch provider instead. The FAA denied permission on the grounds that the startup's satellites were too small to be effectively tracked and would increase the risk of unavoidable collisions and debris growth.

Larger collision or fragmentation parameters are known to increase the chance of Kessler syndrome (e.g., Rossi, Anselmo, Cordelli, Farinella, and Pardini (1998), Kessler and Anz-Meador (2001), Liou (2006)). Adilov, Alexander, and Cunningham (2015) have shown that lower launch costs will increase the equilibrium amount of debris. We reaffirm these findings, and show that lower launch costs will also increase the equilibrium collision rate. An increase in the collision and fragmentation parameters could be caused by cost-minimizing satellite launchers opting to launch cheaper satellites with fewer guidance and control systems and less shielding. A reduction in launch costs could occur independent of changes in satellite characteristics, for example as more firms enter the launch market and drive launch prices down.

These issues are not as pronounced in GEO in part because property rights have been defined for GEO slots. Our model suggests that other characteristics of GEO, in particular the high cost of access and the low natural decay rate, create additional economic incentives to reduce debris accumulation and collision risk there.

4 Conclusion

There are potentially serious problems associated with open access to Earth's orbits. In this paper we present the first long-run economic model of orbit use, explore the nature of the congestion externality associated with launching satellites, and study policy implications of this externality. We highlight three key messages for readers concerned for the future of Earth's orbits.

First, too many firms will launch satellites because they won't internalize the risk they impose on other orbit users. Though profit maximizing satellite owners have incentives to reduce launches as risk of a collision grows, they do not respond to debris growth or the risk of collisions optimally. Second, rebound effects can result in higher debris levels when the rate of natural renewal is high and when the debris created by a launch is low. Combined with the low cost of access, these effects suggest that lower regions of LEO may end up more congested than higher regions. Third, Kessler syndrome is more likely under open access as the excess return on a satellite rises. As launch costs fall and new commercial satellite applications become viable, open access will be more likely to cause Kessler syndrome despite launch reductions in response to orbital congestion.

Economists tend to focus on property rights, corrective taxes, or other market-based mechanisms to solve externality problems. While these mechanisms may prevent

Kessler syndrome and ensure efficient orbit use, more study is needed to understand how orbital management policies should be designed in light of the unique physical features of this resource. Orbits are a global commons, and will likely require global policy solutions. These solutions may be enforced by states, arise as self-enforcing agreements between private actors, or some combination of the two. The problems of social cost in orbit, of interacting congestion and pollution with endogenous regime change, are similar to the problems of climate change. The fact that economic policy solutions to climate change have proven difficult to build consensus around and implement suggests enacting policy solutions to orbital externalities may not be easy either.

5 Appendix A: Extensions to the baseline model

5.1 The effects of spectrum congestion

Satellite applications generally require transmissions to and from the Earth. These transmissions may be the satellite's main output or incidental to its operation. In both cases, satellite operators must secure spectrum use rights from the appropriate national authorities for their broadcast and receiving locations. How will spectrum management affect collisions and debris growth?

Spectrum congestion degrades signal quality, making the per-period output of a satellite decreasing in the number of satellites in orbit, i.e. $\pi = \pi(S), \pi'(S) < 0$. If satellites are launched only when they have appropriate spectrum rights and spectrum use is optimally managed, then firms will be forced to account for their marginal effects on spectrum congestion in their decision to launch or not. The equilibrium condition becomes

$$L(S_{t+1}, D_{t+1}) = r_s(S_{t+1}) + r_s'(S_{t+1}) - r, \tag{35}$$

where $r_s'(S_{t+1}) = \pi'(S_{t+1})/F < 0$ by assumption. The equilibrium set would no longer be a collision rate isoquant, although it would still be a surface in the state space. Even if it isn't managed optimally[18], spectrum congestion will reduce the equilibrium collision rate by reducing the rate of excess return on a satellite.

Open access orbit use will still be inefficient. Although spectrum congestion can reduce the chance of Kessler syndrome, efficient spectrum management will not incorporate the marginal external cost of collisions and debris growth, $\xi(S_{t+1}, D_{t+1})$. Collision and debris growth management policies could be implemented through spectrum pricing. The rest of the analysis in this paper still goes through when spectrum congestion is considered, though some proofs become more complicated since the equilibrium set is no longer a collision rate isoquant.

5.2 The effects of limited satellite lifespans

Satellites do not, in general, produce returns forever until destroyed in a collision. Over 1967-2015, planned satellite lifetimes ranged from 3 months to 20 years, with longer lifetimes being more representative of larger and more expensive GEO satellites[19]. How

[18] If spectrum use were also under open access then the marginal congestion effect ($r_s'(S_{t+1})$) would not be in the equilibrium condition, but the equilibrium set would still not be a collision rate isoquant.

[19] These numbers are taken from the Union of Concerned Scientists' publicly available data on satellites. The data are available at https://www.ucsusa.org/nuclear-weapons/space-weapons/satellite-database.

would finite lifetimes affect the collision rate and debris growth problems?

Suppose satellite lifetimes are finite and exogenously distributed with mean μ^{-1}. Satellites live at least one time period on average, so that $\mu^{-1} > 1$. The equilibrium condition becomes

$$L(S_{t+1}, D_{t+1}) = \frac{r_s - r - \mu}{1 - \mu}, \tag{36}$$

which is lower than the equilibrium collision rate when satellites are infinitely lived. Intuitively, the fact that the satellite will stop generating returns at some point reduces its expected present value, and with it the incentive to launch. All else equal, shorter lifetimes reduce the equilibrium collision rate. The rest of the analysis in the paper goes through with minor modifications.

Satellites built for GEO tend to be longer lived than satellites launched for LEO. If shorter lifetimes have reduced satellite costs, then the downward shift in the collision rate isoquant from the shorter lifetimes will be balanced against the upward shift caused by higher rates of return. The net effect may be higher or lower equilibrium rates of collisions.

The distinction between exogenous and endogenous lifetimes is relevant here. The above analysis hinges on satellite lifetimes being exogenously set. This is not the case in reality. Satellite lifetimes are determined by cost minimization concerns, technological constraints, expectations of component failures, and expectations of future technological change. Incorporating all these features to realistically model the choice of satellite life along with launch decisions and their effects on orbital stock dynamics is beyond the scope of this paper, though it is an interesting area for future research. The assumption that the end-of-life is random simplifies the analysis, but does not change any conclusions over imposing a pre-specified end date in this model[20].

5.3 The effects of changes in satellite returns over time

Though it simplifies the analysis, the rate of return on a satellite is not constant over time. How would changes over time in these economic parameters affect our conclusions? For simplicity, suppose that costs and returns vary exogenously and are known in advance[21]. The open access equilibrium condition is then

[20] This simplification could matter in a model where firms own multiple satellites and have to plan replacements.

[21] Uncertainty over costs and returns doesn't change the qualitative results, though it introduces expectations over the changes. Endogeneity in the changes, for example due to investment in R&D or marketing, may have more significant consequences which are beyond the scope of this paper.

$$\pi_{t+1} = (1+r)F_t - (1 - L(S_{t+1}, D_{t+1}))F_{t+1} \tag{37}$$

Equation 37 can be rewritten as

$$L(S_{t+1}, D_{t+1}) = 1 + \frac{\pi_{t+1} - (1+r)F_t}{F_{t+1}}. \tag{38}$$

If $\pi_{t+1} > (1+r)F_t$, then the one period return on a satellite is greater than the gross return on the launch cost from the safe asset, and the collision rate will be 1. Ignoring that corner case, the equilibrium collision rate is decreasing in the current cost of launching a satellite, but increasing in the future cost of launching a satellite. All else equal, the collision rate in $t+1$ will be lower when F_{t+1} increases. This highlights the role of the launch cost under open access: if firms enter until zero profits in each period, future increases in the cost deter firms from entering in the future, increasing the value of satellites already in orbit by the amount of the cost increase.

When the costs and returns are time-varying, the equilibrium set is still a collision rate isoquant, though the isoquant selected may vary over time. These changes do not affect the physical dynamics or the Kessler threshold, though they may affect how close the selected equilibrium is to the threshold. If the parameters vary so that the ratio $\frac{\pi_{t+1}-(1+r)F_t}{F_{t+1}}$ is stationary, then the equilibrium set will stay on the same isoquant.

5.4 The effects of space weather

Sunspots have two effects: first, changes in radiation pressure force satellites in higher orbits to spend more fuel on stationkeeping; second, the Earth's atmosphere expands in response to the solar activity, increasing drag and debris decay in all but the highest orbits. The latter effect can be particularly significant for active satellites in LEO. Formally, suppose sunspots cause decay rates to vary periodically with mean $\bar{\delta}$, making the laws of motion

$$S_{t+1} = S_t(1 - L(S_t, D_t)) + X_t$$
$$D_{t+1} = D_t(1 - \delta_t) + G(S_t, D_t) + mX_t$$

If the variations are exogenous and publicly known, there are no changes to the firm or planner's Bellman equations. Figure 10 shows example time paths with and without sunspot activity.

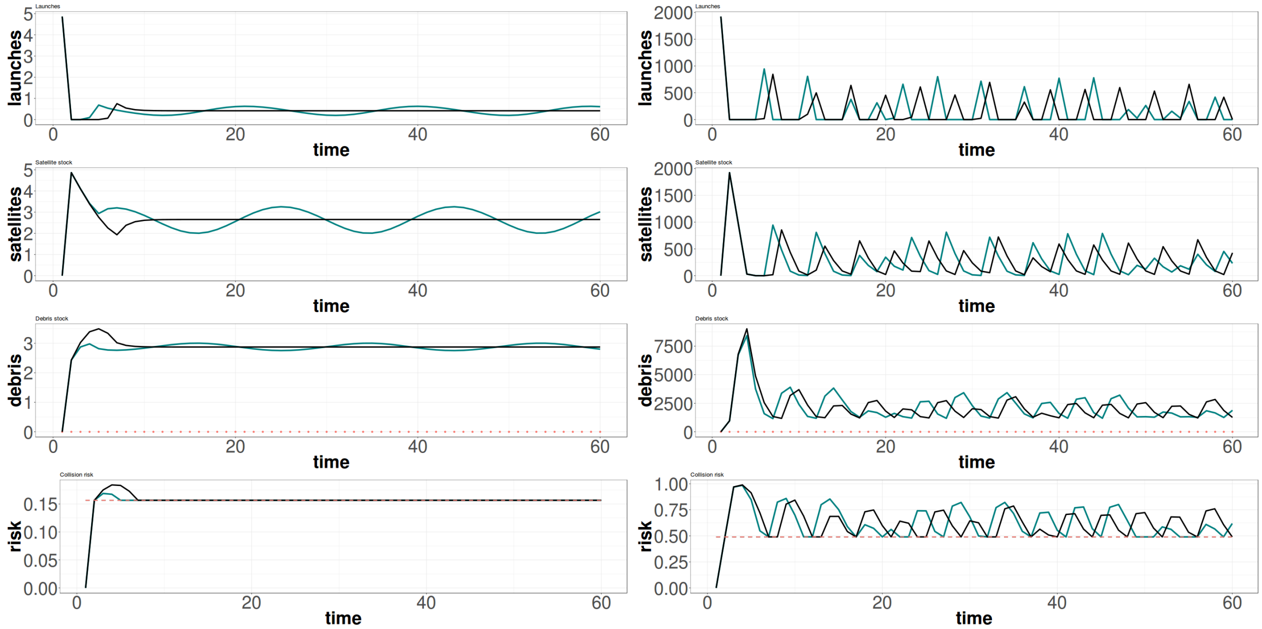


Figure 10: Open access equilibrium paths with no sunspots (black line) and with subspots driven by $\delta_t = \bar{\delta} + \omega_1 \sin(\omega_2 t)$ (teal line).

The results in Proposition 5 describe how open access equilibria will respond to increases in the decay rate caused by space weather, holding costs constant. As the activity increases, firms will have incentives to take advantage of the increased renewal by launching more satellites. This may increase or decrease the net amount of debris. As the activity decreases, firms will reduce the launch rate, decreasing or increasing the net amount of debris. These effects will be offset by the extent to which sunspot activity increases the cost of operating a satellite.

Increases in the decay rate have the benefit of shifting the Kessler threshold up (Proposition 4). If the net increase in debris due to the launch response is not too large, then increased sunspot activity may stabilize orbit use. On the other hand, if the rebound effect for decay rates is strong enough, sunspot activity may push the equilibrium debris level into the basin of the Kessler region. Even when the rebound effect is not very strong, a decrease in the decay rate at the end of a period of extra sunspot activity may make the original open access steady state locally unstable given the increase in debris due to the rebound effect.

6 Appendix B: Additional figures

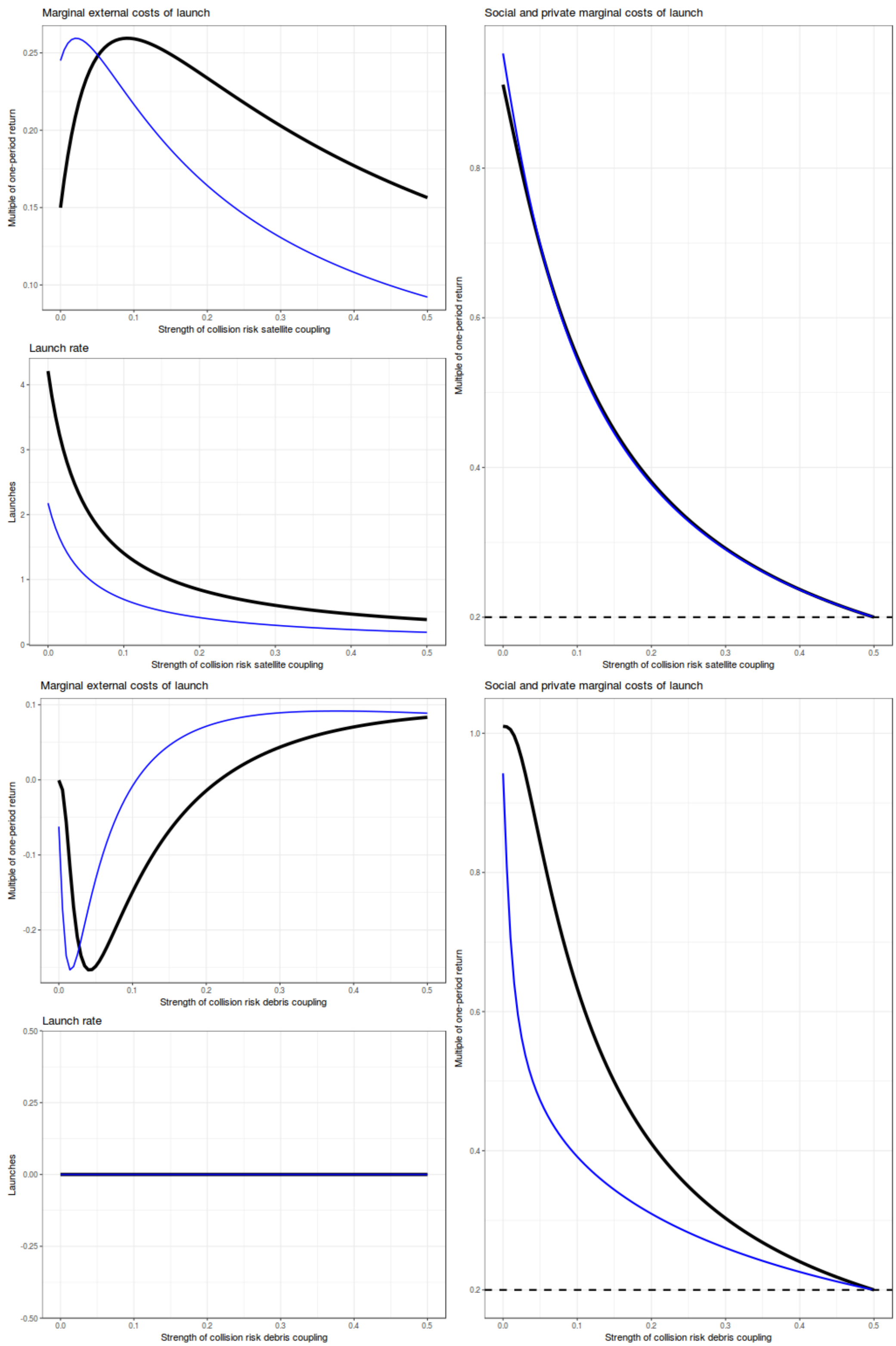


Figure 11: Left: open access (black) and planner (blue) MEC and launch rate; private marginal cost (blue) and social marginal cost (blue) over $L(S)$ coupling. Right: open access (black) and planner (blue) MEC and launch rate; private marginal cost (blue) and social marginal cost (blue) over $L(D)$ coupling

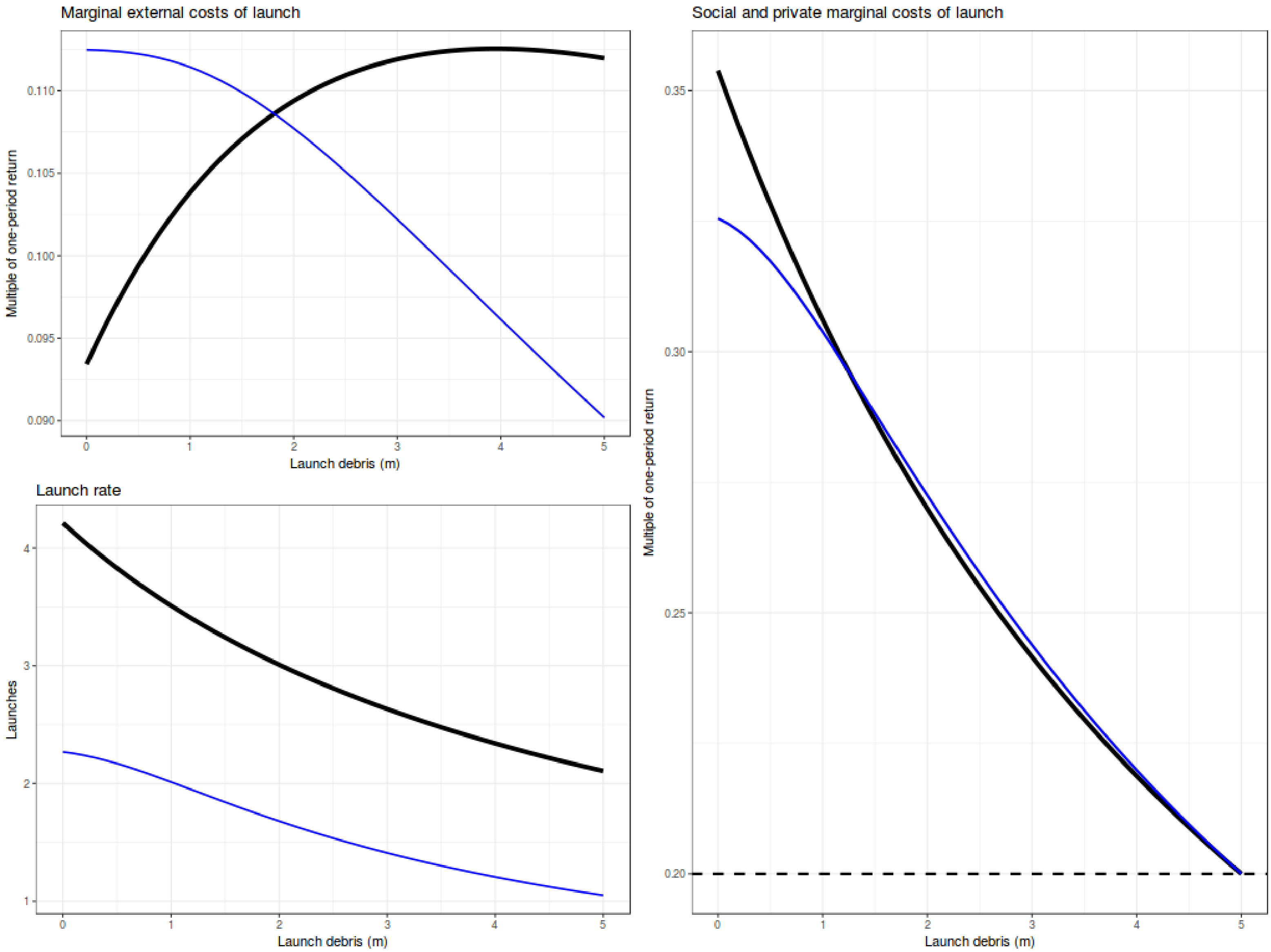


Figure 12: open access (black) and planner (blue) MEC and launch rate; private marginal cost (blue) and social marginal cost (blue) over launch debris coupling.

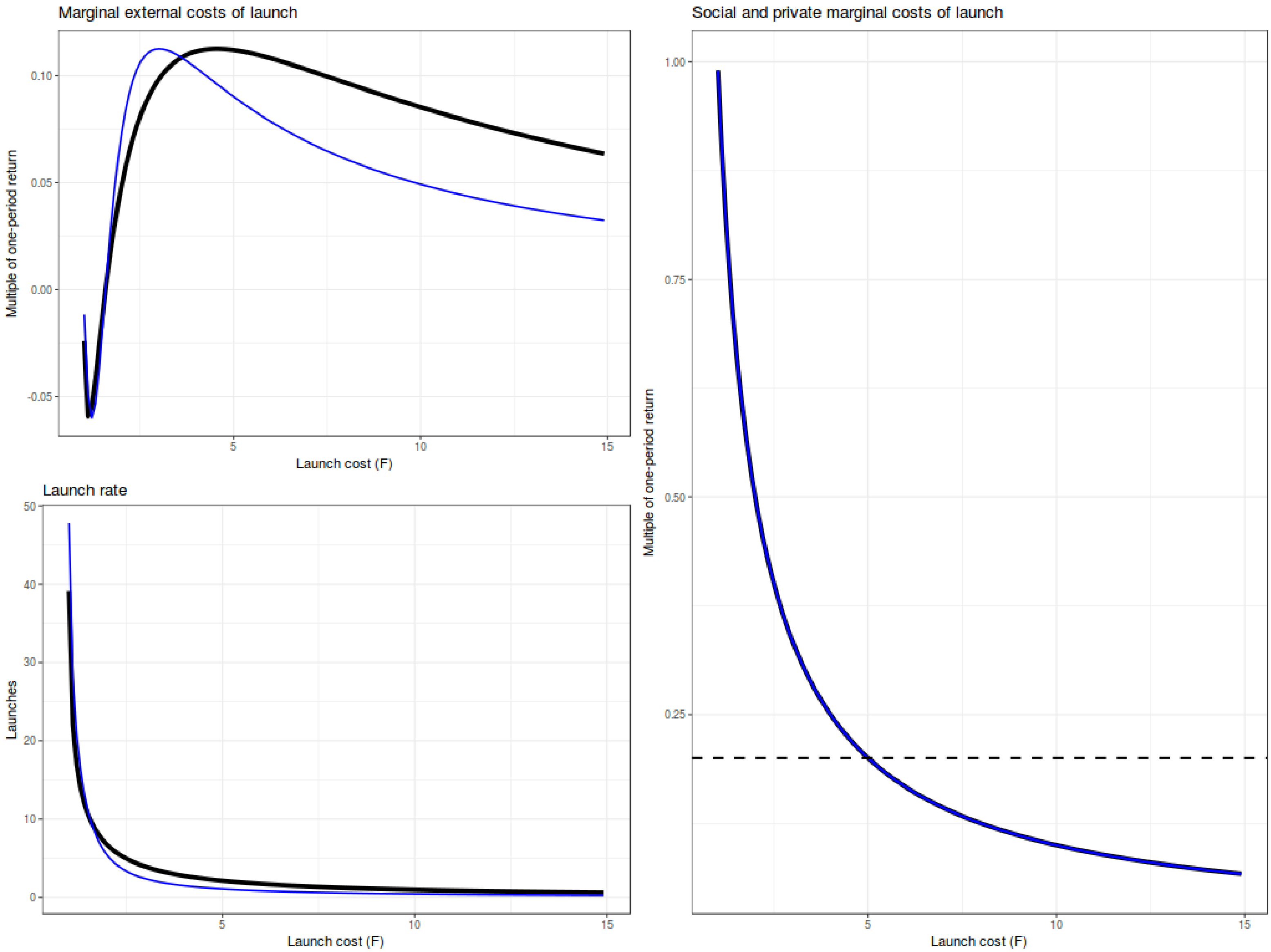


Figure 13: Open access (black) and planner (blue) MEC and launch rate; private marginal cost (blue) and social marginal cost (blue) over launch cost.

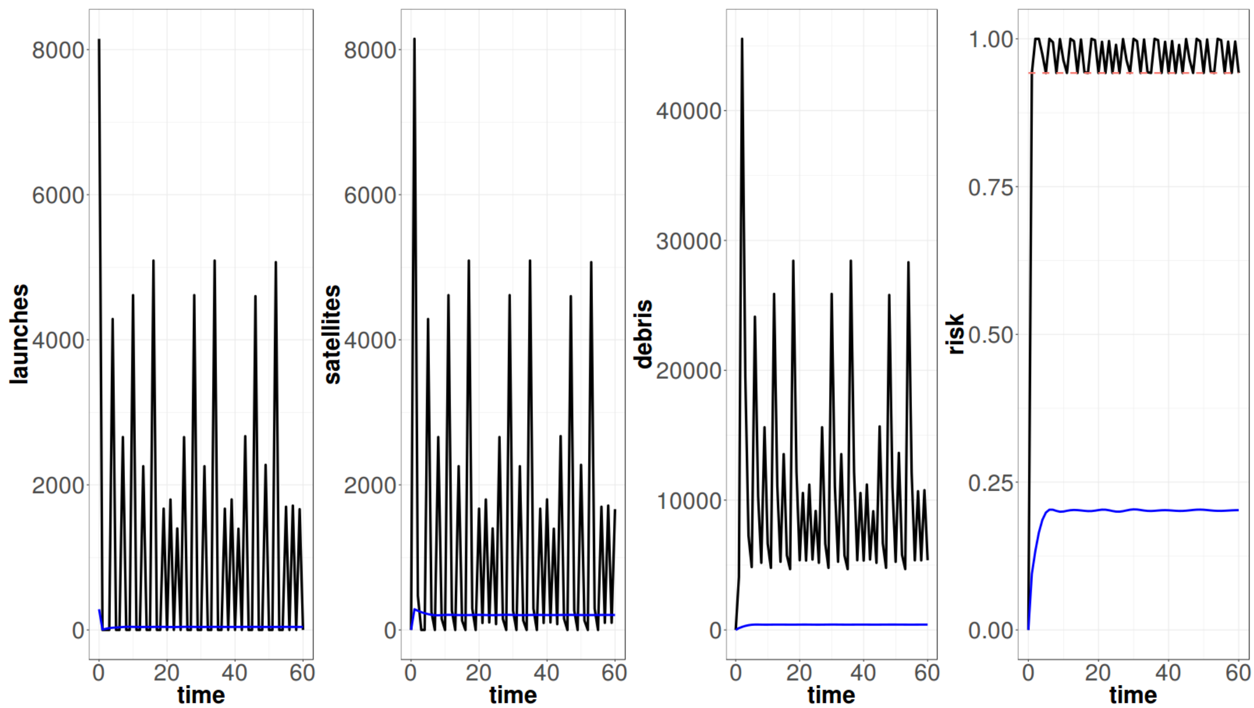


Figure 14: An example of open access time paths which bounce between action and inaction regions (black). The planner (blue) does not find this optimal.

Figures were generated using

$$L(S, D) = 1 - e^{-\alpha_{SS}S - \alpha_{SD}D},$$
$$G(S, D) = \beta_{SS}L(S, D)\left(\frac{S}{S+D}\right)S + \beta_{SD}L(S, D)\left(\frac{D}{S+D}\right)D + \beta_{DD}(1 - e^{-\alpha_{DD}D})D.$$

Other forms tried, including $L(S, D) = 1-e^{-\alpha_{SS}S^2-\alpha_{SD}SD}$ and $G(S, D) = \beta_{SS}L(S, D)\left(\frac{S}{S+D}\right)S+ \beta_{SD}L(S, D)\left(\frac{D}{S+D}\right)D + \beta_{DD}D^2$, yielded qualitatively similar results.

7 Appendix C: Detailed proofs

Proof of Proposition 4:

Proof. Period t values are shown with no subscript, and period $t+1$ values are marked with a $'$, e.g. $S_t \equiv S, S_{t+1} \equiv S'$. Applying the Implicit Function Theorem to equation 10, we get

$$\frac{\partial X}{\partial S} = -\frac{[\frac{\partial L(S',D')}{\partial S'}(1 - L(S,D) - \frac{\partial L(S,D)}{\partial S}S) + \frac{\partial L(S',D')}{\partial D'}\frac{\partial G(S,D)}{\partial S}]}{[\frac{\partial L(S',D')}{\partial S'}\frac{\partial S'}{\partial X} + \frac{\partial L(S',D')}{\partial D'}\frac{\partial D'}{\partial X}]} < 0$$
$$\frac{\partial X}{\partial D} = -\frac{\frac{\partial L(S',D')}{\partial S'}(-S\frac{\partial L(S,D)}{\partial D}) + \frac{\partial L(S',D')}{\partial D'}(1 - \delta + \frac{\partial G(S,D)}{\partial D})}{\frac{\partial L(S',D')}{\partial S'}\frac{\partial S'}{\partial X} + \frac{\partial L(S',D')}{\partial D'}\frac{\partial D'}{\partial X}} \lesseqgtr 0$$

$\frac{\partial X}{\partial D} < 0$ if

$$(S,D): \quad \underbrace{\frac{\partial L(S',D')}{\partial S'}S}_{\substack{t+1 \text{ marginal collision probability from} \\ t+1 \text{ satellites} \cdot \text{present satellite stock}}} < \underbrace{\frac{\frac{\partial L(S',D')}{\partial D'}}{\frac{\partial L(S,D)}{\partial D}}}_{\substack{\text{growth rate of} \\ \text{marginal collision rate} \\ \text{due to debris}}} \cdot \underbrace{\left(1 - \delta + \frac{\partial G(S,D)}{\partial D}\right)}_{\substack{\text{net growth in debris} \\ \text{due to debris}}} \tag{39}$$

and $\frac{\partial X}{\partial D} > 0$ otherwise.

□

Proof of Proposition 5:

Proof. Period t values are shown with no subscript, and period $t+1$ values are marked with a $'$, e.g. $S_t \equiv S, S_{t+1} \equiv S'$. All present-period variables are assumed to be at equilibrium values. Applying the Implicit Function Theorem to equation 10, we get

$$\frac{\partial X}{\partial r_s} = \frac{1}{L_S(S',D') + mL_D(S',D')} > 0$$
$$\frac{\partial D'}{\partial r_s} = \frac{1}{L_D(S',D')} > 0$$

$$\frac{\partial X}{\partial m} = -\frac{L_D(S',D')X}{L_S(S',D') + mL_D(S',D')} < 0$$
$$\frac{\partial D'}{\partial m} = X + m\frac{\partial X}{\partial m}$$
$$\frac{\partial D'}{\partial m} > 0 \implies 1 > \frac{mL_D(S',D')}{L_S(S',D') + mL_D(S',D')},$$

which is true as long as $L_S > 0$.

$$\frac{\partial X}{\partial \delta} = \frac{L_D(S', D')D}{L_S(S', D') + mL_D(S', D')} > 0$$
$$\frac{\partial D'}{\partial \delta} = -D + m\frac{\partial X}{\partial m}$$
$$\frac{\partial D'}{\partial \delta} < 0 \implies 1 > \frac{mL_D(S', D')}{L_S(S', D') + mL_D(S', D')},$$

which is also true as long as $L_S > 0$. □

Proof of Proposition 6:

Proof. The open access steady states are defined by equations 31, 32, and 33. Equation 31 implicitly determines the number of satellites as a function of the amount of debris, the excess return on a satellite, and the collision rate function,

$$L(S, D) = r_s - r \implies S = S(r_s - r, D). \tag{40}$$

Since we assumed $L(S, D)$ is increasing in each argument, $S(r_s - r, D)$ is decreasing in D. This implicit function allows us to reduce equations 31, 32, and 33 to a single equation in debris,

$$\mathscr{Y}(D) = -\delta D + G(S(r_s - r, D), D) + m(r_s - r)S(r_s - r, D) = 0. \tag{41}$$

$-\delta D$ and $m(r_s - r)S(r_s - r, D)$ are both monotonically decreasing in D. Since $G(S, D)$ is increasing in both arguments but $S(r_s - r, D)$ is decreasing in D, $G(S(r_s - r, D), D)$ may be increasing or decreasing in D over an arbitrary positive interval. In the limiting cases where $D \to 0$ or $D \to \infty$, $G(S(r_s - r, D), D) \to G(S(r_s - r, 0), 0) > 0$ and $G(S(r_s - r, D), D) \to G(0, D) > 0$. If $G(S, D) = L(S, D)$, $G(S(r_s - r, D), D)$ would be constant for all D because $S(r_s - r, D)$ is such that $L(S, D) = r_s - r$ for all S, D where this is possible. If $G(S, D)$ is locally more convex than $L(S, D)$, then $G(S(r_s - r, D), D)$ will be first decreasing and then increasing (over the local interval) as D increases, and vice versa if $G(S, D)$ is locally more concave than $L(S, D)$ over a local interval. So,

$$\delta D = G(S(r_s - r, D), D) + m(r_s - r)S(r_s - r, D)$$

may have zero, one, or more than one interior solutions. If $L(S, D)$ is globally strictly concave and $G(S, D)$ is globally weakly convex, there can be up to 2 solutions. □

Proof of Proposition 7:

Proof. We use the reduction from Proposition 6 to simplify the proof. The open access steady states are solutions to equation 41, and the sign of $\frac{\partial \mathscr{Y}}{\partial D}$ at the solutions allows us to classify the stability of the system. Applying the Implicit Function Theorem to equation 31 to calculate $S_D(r_s - r, D)$, then differentiating $\mathscr{Y}(D)$ in a neighborhood of an arbitrary solution D^*,

$$\frac{\partial \mathscr{Y}}{\partial D}(D^*) = -\delta - \frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}(G_S(S^*, D^*) + m(r_s - r)) + G_D(S^*, D^*), \tag{42}$$

where $S^* \equiv S(r_s - r, D^*)$. The first two terms of $\frac{\partial \mathscr{Y}}{\partial D}$ are negative and the last term is positive. Since $\frac{\partial \mathscr{Y}}{\partial D}(D^*)$ may be positive or negative, the generic solution considered may be stable or unstable depending on the physical and economic parameters.

Analysis of $\frac{\partial^2 \mathscr{Y}}{\partial D^2}(D^*)$ shows a necessary condition for $\frac{\partial \mathscr{Y}}{\partial D}(D^*)$ to be increasing in D, again using the Implicit Function Theorem to calculate $S_{DD}(r_s - r, D)$:

$$\begin{aligned}\frac{\partial^2 \mathscr{Y}}{\partial D^2}(D^*) > 0 &\implies S_D G_S D + S_D^2 G_{SS} + S_{DD}(G_S - m(r_s - r)) > 0 \\ &\implies \frac{L_D L_S(G_{SD} L_S - G_{SS} L_D)}{L_{DD} L_S^2 + L_{SD} L_D^2} > m(r_s - r) - G_S,\end{aligned}$$

which is not necessarily satisfied by every open access steady state without further restrictions on the physical and economic parameters. □

Proof of Proposition 8:

Proof. Consider an arbitrary open access steady state (S', D'). Our approach will be to first characterize the set of points which can reach the open access steady state in one period, then show that other points in the action region must overshoot at least one state variable in approaching the equilibrium isoquant.

The set of initial conditions which can reach that steady state are one iteration of the physical dynamics away from a line segment intersecting the steady state. Given an arbitrary point (S, D), (S', D') can be written as

$$\begin{bmatrix} S' \\ D' \end{bmatrix} = \begin{bmatrix} S \\ D \end{bmatrix} + \begin{bmatrix} -L(S, D)S \\ -\delta D + G(S, D) \end{bmatrix} + \begin{bmatrix} X \\ mX \end{bmatrix}. \tag{43}$$

The second term on the right hand side is the effect of the physical dynamics independent of launches. The third term on the right hand side is a line segment with magnitude $X(1 + m^2)^{1/2}$ and slope m. $X > 0$ implies that the sum of the first two terms is in the action region. Under open access, X is determined so that

$L(S', D') = r_s - r$. Since the magnitude of X only changes the length of the line segment from $[S(1-L(S,D)), D(1-\delta)+G(S,D)]^T$ but not its slope, any initial condition for which firms launch satellites and reach an open access steady state in one step must be one iteration of the physical dynamics away from a line segment intersecting the steady state in question. The case where $X = 0$ and the system reaches an open access steady state is less interesting, since in this case one iteration of the physical dynamics places the system exactly at the steady state in question (which is by definition on a line segment with slope m intersecting itself).

Now consider a different initial condition, (S_a, D_a), which still leads to the interior of the action region after one iteration of the physical dynamics, but does not end up on a line segment with slope m which intersects an open access steady state. Since it leads to the interior of the action region, $X > 0$ holds and will be such that $L(S'_a, D'_a) = r_s - r$. Now, $L(S,D)$ and $G(S,D)$ are both monotonic in each argument, and $L(S'_a, D'_a) = L(S', D') = r_s - r$ since $X > 0$ implies next period aggregates are on the equilibrium isoquant. If $S'_a = S$, the monotonicity of $L(S,D)$ implies that $D'_a = D$ and vice versa. If $S'_a < S'$ and $D'_a < D$ (or both inequalities were reversed), either (S', D') or (S'_a, D'_a) would not be an equilibrium. So, either $S'_a > S'$ or $D'_a > D'$ must be true. If $S'_a > S'$, then $D'_a < D'$ must be true, and vice versa. So any initial condition which leads to the action region must do one of three things: it must lead directly to an open access steady state, or it must overshoot the steady state satellite level, or it must overshoot the steady state debris level. If it overshot both variables, it would no longer be on the equilibrium collision rate isoquant. So, the sets in the action region with which each type of overshooting is associated must be disjoint. □

Proof of Proposition 9:

Proof. We suppress function arguments to reduce notation; all functions are evaluated at an arbitrary open access steady state. Applying the Implicit Function Theorem to equation 41, and applying it again to equation 31 to calculate $\frac{\partial S}{\partial r_s}$ where necessary, we get

$$\begin{aligned}
\frac{\partial D}{\partial r_s} &= \frac{\frac{G_S}{L_S} + m(S(r_s - r, D) + \frac{r_s - r}{L_S})}{\delta + \frac{L_D}{L_S}(G_S + m(r_s - r)) - G_D} \lesseqgtr 0, \\
\frac{\partial D}{\partial m} &= \frac{(r_s - r)S(D)}{\delta + \frac{L_D}{L_S}(G_S + m(r_s - r)) - G_D} \lesseqgtr 0, \\
\frac{\partial D}{\partial \delta} &= \frac{D}{-\delta - \frac{L_D}{L_S}(G_S + m(r_s - r)) + G_D} \lesseqgtr 0.
\end{aligned}$$

An open access steady state is locally stable if $-\delta - \frac{L_D}{L_S}(G_S + m(r_s - r)) + G_D < 0$.

If a steady state is locally stable, then $\frac{\partial D}{\partial r_s} > 0$, $\frac{\partial D}{\partial m} > 0$, and $\frac{\partial D}{\partial \delta} < 0$. The inequalities are reversed when a steady state is locally unstable. $\square$

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