Economic Principles of Space Traffic Control

Akhil Rao*
University of Colorado Boulder†

Latest draft available here

October 8, 2018

Abstract

Open access to Earth's orbits presents a unique regulatory challenge. Technical solutions to space traffic control tend to emphasize launch restrictions or public funding of debris removal technology development and use, but often ignore that current and prospective orbit users dissipate rents under open access. In this paper, I derive economic principles governing the choice of space traffic control policies. I show that policies which target satellite ownership, such as satellite taxes or permits, achieve greater expected social welfare than policies which target satellite launches, such as launch taxes or permits. Price or quantity policies can achieve equal expected social welfare due to the symmetry of uncertainty between regulators and firms. I also show that under open access a commonly-proposed engineering approach to space debris reduction, active debris removal, can only reduce the risk of satellite-destroying collisions if it is financed by satellite owners. My results show that attempts to control orbital debris growth and collision risk through launch fees, debris removal subsidies, or purely technical solutions may be ineffective or backfire.

*I am grateful to Dan Kaffine, Jon Hughes, Martin Boileau, Miles Kimball, Alessandro Peri, participants at the University of Colorado Environmental and Resource Economics, Macroeconomics, and Applied Microeconomics Seminars, and participants at the International Institute of Space Law's 2018 Space Law Workshop for their comments and feedback. Funding for this research was generously provided by Center for Advancement of Teaching and Research in Social Science and the Reuben A Zeubrow Fellowship in Economics. I would also like to thank Francesca Letizia of the European Space Agency, Teri Grimwood of the Union of Concerned Scientists, and Debra Shoots of the NASA Orbital Debris Program Office for sharing data on collision risk, active satellites, and debris growth. All errors are my own.

†Department of Economics, 256 UCB, 80309-0256; akhil.rao@colorado.edu

1 Introduction

Open access to common-pool resources tends to cause resource overuse or stock collapse. Open access orbit use has led to the accumulation of orbital debris, from nonoperational satellites to nuts, bolts, and propellant fuel particulates. Collisions between orbiting bodies can shatter satellites into thousands of dangerous high-velocity fragments, some of which may be too small to track. Runaway debris growth, known as Kessler Syndrome, threatens to render high-value orbits unusable for decades or centuries. As technology makes satellites cheaper to launch and more reliable, firms are planning to launch thousands of satellites into already-congested orbits. The need for policies to manage orbital congestion is more pressing than ever. Unfortunately, engineers, economists, and policymakers know little about how space traffic should be managed and debris removal technologies should be employed. In this paper I answer two fundamental questions of space traffic control. First, what do optimal space traffic control policies look like? Second, how should active debris removal be financed? The key insights of my paper are that space traffic control policies should target satellites in orbit rather than satellite launches, and that satellite owners must pay for debris removal for it to reduce equilibrium collision risk.

I derive economic principles of space traffic control policy in the first dynamic model of satellite launch and ownership with physical uncertainty over collisions and positive feedbacks in debris growth. I highlight the key policy design constraints imposed by open access and show how the use of active debris removal technologies will affect equilibrium collision risk and debris growth. I show that despite uncertainty over the risk of catastrophic collisions, the traditional "prices vs. quantities" question is moot. Price or quantity policies can achieve first-best outcomes because both regulators and firms are equally uncertain about the collision risk. The key design issue is whether the regulator's policy targets satellites in orbit (for example, a satellite tax) or the act of launching satellites (for example, a launch tax). In the setting I study, regulating satellites in orbit achieves higher expected social welfare than regulating the act of launching satellites. Regulating satellite launches instead of satellites in orbit creates rents to satellite ownership and induces suboptimal spikes in equilibrium collision risk just before the policy takes effect. Satellite launch controls are also limited in their ability to induce deorbits, and optimal satellite launch controls have unfavorable dynamic properties. Contrary to predictions from non-economic models of orbit use, active debris removal may reduce the debris stock without affecting equilibrium collision risk. If satellite owners receive debris removal for free, more launchers will enter to take advantage of the cleared space. For active debris removal to reduce equilibrium collision risk, satellite owners must bear the cost of removal.

Prior analyses have quantified the costs and benefits of mitigating and reducing debris and collision risk (Liou and Johnson, 2008, 2009b; Bradley and Wein, 2009; Ansdell, 2010; Schaub et al., 2015; Macauley, 2015), noted that open access and the common-pool nature of orbits make rational actors ignore their effects on other orbit users (Merges and Reynolds, 2010; Weeden and Chow, 2012; Adilov, Alexander, and Cunningham, 2015; Salter, 2015; Rao and Rondina, 2018), and considered necessary legal and institutional features that an orbit use management policy framework ought to have (Weeden, 2010; Weeden and Chow, 2012; Akers, 2012).[1] I build on these analyses by formally modeling open access incentives with a realistic dynamic structure which reveals feedbacks between the environment and orbit-users. This structure allows me to identify issues with launch taxes and publicly-provided debris removal not visible in earlier studies which did not include maximizing behavior or realistic dynamics. My results show that ignoring these issues can result in welfare losses as rational orbit-users attempt to capture the rents created by launch taxes or dissipate the rents created by debris removal.[2]

I contribute to the literature on orbit use in three ways: I present the first economic analysis of orbit use management policy under open access; I explicitly incorporate uncertainty and dynamic feedbacks in orbit use and study their economic effects; and I present the first economic analysis of the effects of active debris removal on orbit use, accounting for profit-maximization and open access. My modeling framework identifies previously-unknown issues in orbit use management and suggests qualitative features of optimal orbit use management policy. Though I do not pursue it here, my framework can also be augmented with high-fidelity engineering models of orbit use and economic data to give regulators quantitative guidance on policy design and the magnitude of optimal or second-best instruments.

The remainder of the paper is organized as follows. In section 2 I describe institutional details of orbit use and present the basic definitions and modeling framework. In section 3 I analyze orbit management policies and the use of debris removal technologies, and present my key results: Proposition 5, that stock controls achieve greater expected social welfare than flow

[1] The legal issues of debris removal are non-trivial. International space law gives satellite operators ownership of their debris even after the satellite's lifetime, forcing potential salvage operations to negotiate with each individual satellite or fragment owner for rights to remove debris. For small fragments, attributing ownership and negotiating removal may be infeasible. A substantial legal and engineering literature has considered these issues and potential solutions, for example Carroll (2009); Merges and Reynolds (2010); Ansdell (2010). Many of these scholars have suggested amending existing legal frameworks to allow salvage and debris removal bounties, so as to incentivize negotiations between debris owners and would-be debris removers. Yet the march of technology continues despite legal uncertainty, and debris removal technologies are being developed and tested, for example Pearson, Carroll, and Levin (2010).

[2] To be clear, the need for orbit use management policy is not uniform across all orbits. Formal orbit allocation procedures exist in the geosynchronous (GEO) belt (Macauley, 1998; Jehn, Agapov, and Hernandez, 2005). But in low-Earth orbit (LEO), no such procedures exist. Orbital management must be done indirectly through spectrum management by national authorities (such as the Federal Communications Commission in the US), or directly through non-binding guidelines from international agencies (such as the Inter-Agency Debris Committee).

controls, and Proposition 6, that satellite owners must pay for debris removal if the technology is to reduce equilibrium collision risk. I show proofs of these and a few other economically important results in the main text (the rest are in the Appendix, section 8). Finally, I conclude in section 4 with discussion of the results and thoughts on the future of commercial orbit use.

2 Essentials of Orbit Use

In this section I discuss the history and current status of space traffic control policies. Readers interested in going directly to the modeling approach may skip to section 2.2. Readers interested in learning more of the institutional details of orbit use may go to the Appendix, section 5.

2.1 Defining "space traffic control"

One of the central challenges of space traffic control is how to define "space traffic control". Nicholas Johnson, a scientist at NASA, has proposed an aim of space traffic control: "...the goal of space traffic management is to minimize the potential for (radio frequency) or physical interference at any time" Johnson (2004). The radio frequency interference problem is relatively tractable and being handled by existing institutions (Jones et al., 2010). The physical interference problem, essentially collision avoidance, is more difficult from technical and legal perspectives. In GEO, space traffic control is "position control": since satellites in GEO have very low speeds relative to each other, traffic control is as simple as spacing satellites far enough apart that they are unlikely to collide or cause radio frequency interference. In the current regulatory regime, the International Telecommunications Union assigns frequency blocks and geostationary "slots" to national authorities. These authorities are then free to assign their frequencies and slots to entities within their jurisdiction as they see fit, and are also responsible for enforcing responsible spectrum use. In the United States, this is handled by the FCC.[3]

Space traffic control in LEO is harder than in GEO. Satellites in LEO are constantly in motion with respect to each other and have little or no control over their trajectories. Notions like "keep-out zones" are impractical since satellites may only occasionally or accidentally pass through them, and concepts like "rules of the road" raise the question of how a road is to be defined in LEO. Figure 1 shows the orbits of 56 cataloged satellites with mean altitudes of 700-710 kilometers, and makes the inaptness of road, sea, and air analogies clear. The growth in LEO use has motivated calls for broader notions of space traffic control which encompass non-GEO regimes. There are currently no international regulatory agencies which

[3] Readers interested in more detail about the history and institutions of space traffic control are referred to Johnson (2004); Jones et al. (2010). Technical proposals for mass removal are discussed in Klinkrad and Johnson (2009), Weeden (2010) discusses the legal challenges, and Tkatchova (2018) examines the potential for markets in debris removal.

coordinate launches and satellite placements to manage debris growth and collision risk; the extent of management policies currently is a patchwork of national regulations and non-binding international guidelines. Table 1 shows the breakdown of currently-operational satellites by location of launch site to emphasize the international dimension of the problem. Figure 2 shows the growth in orbit use from active satellites and debris, as well as the increase in competition to provide commercial launch services.

[Figure 1 about here.]

[Table 1 about here.]

For this paper, I define space traffic control as policies or technologies intended to manage the probability of collisions between active satellites and other bodies. This definition encompasses satellite path as well as debris growth management. Any space traffic control policy, including command-and-control regulations, can be characterized as a price or quantity control, such as a tax or a quota. If the effect of a policy is to raise the cost or limit the availability of satellite launch, I label it as a "flow" control. If the effect is to raise the cost of operating a satellite or constrain the allowed number of satellites in orbit, I label it as a "stock" control. The existing patchwork of policies includes both flow controls intended to manage launch capacity and prevent launches from interfering with air traffic, and stock controls intended to manage spectrum congestion. While most existing literature on space traffic control focuses on controlling the trajectories of objects in orbit, I focus on controlling the number of objects in orbit. Brief consideration will show that the former implies the latter. I treat debris removal separately because the technology is not yet commercially available, so analysis of a world without debris removal is more immediately relevant to policy design.

[Figure 2 about here.]

2.2 A simple model of orbital mechanics

In this section I describe the laws of motion for orbital stocks, the type of uncertainty most relevant to the economics of managing collision risk and debris growth (symmetric physical uncertainty), and the functional forms I use for simulations. Following analytical debris modeling studies such as Rossi et al. (1998) and Bradley and Wein (2009), I consider the evolution of orbital stocks in an arbitrary spherical shell around the Earth, referred to as the "shell of interest". More detailed physical models of Earth orbit use multiple shells. I ignore such features in this paper for tractability. I consider two types of fictitious agents: a social planner who launches and owns all satellites in orbit to motivate optimal satellite launch and debris removal plans, and a global regulator who manages all satellites launched or in orbit to motivate policy choice.

Let S_t denote the number of active satellites in orbit in period t, D_t the number of debris objects in orbit in t, X_t the number of launches in t, and ℓ_t the proportion of satellites which will be lost in collisions at the end of period t (the collision rate). δ is the proportion of debris objects which deorbit at the end of t (the decay rate), and $G(S_t, D_t, \ell_t)$ is the number of new debris fragments generated due to all collisions between satellites and debris. I assume that the collision rate is nonnegative and bounded below by 0 and above by 1[4]. No satellites can be destroyed when there are none in orbit ($S_t = 0 \implies \ell_t \equiv 0$). As $S_t \to \infty$ or $D_t \to \infty$, $\ell_t \to 1$ due to physical crowding (unless there are no satellites in orbit). I ignore the fact that active satellites may be deorbited when their useful lifetime is over, as it does not impact the economics of open access satellite launching. I consider the ability of different types of control policies to induce deorbits in Proposition 3. The effect of finite lifetimes is examined in Rao and Rondina (2018).

The number of active satellites in orbit in t is the number of launches in the previous period plus the number of satellites which survived the previous period. The amount of debris in orbit is the amount from the previous period which did not decay, plus the number of new fragments created in collisions, plus the amount of debris in the shell created by new launches. Formally,

$$S_{t+1} = S_t(1 - \ell_t) + X_t \tag{1}$$

$$D_{t+1} = D_t(1 - \delta) + G(S_t, D_t, \ell_t) + mX_t. \tag{2}$$

I assume that the number of new fragments is nonnegative, increasing in each argument, and zero when there are no objects in orbit ($G(0,0,0) = 0$). δ is the rate of orbital decay for debris, and m is the amount of launch debris created by launching new satellites. To allow the possibility of Kessler Syndrome, I also assume that the growth in new fragments due to debris interactions alone ($G(0,D,0)$) will eventually be greater than δ.

The most important source of uncertainty in orbit management is uncertainty over the proportion of satellites lost to collisions in a given period, ℓ_t. However, the growth in debris objects is also uncertain, so why is uncertainty in $G()$ not treated similarly? The answer is that actual statistical uncertainty in the position and interactions between objects in orbit is not economically relevant to orbit management. Uncertainty in orbit is of economic interest only insofar as it affects active satellites. Consider a counterfactual world where active satellites could not be affected by debris. In this case the uncertainty in debris growth would be irrelevant

[4]Firms try to avoid collisions by maneuvering their satellites when possible; the collision rate in this model should be thought of as the rate of collisions which could not be avoided, with easily avoided collisions optimized away. Collisions which could have been avoided but were not due to human error are included in this. Implicitly I am assuming that firms operate their satellites as imperfect cost-minimizers.

for orbit management, because the regulator's interest in orbit management is in controlling the value generated by active satellites. Thus, uncertainty in this model is represented by ℓ_t not because debris growth is known perfectly, but because uncertainty over orbital stocks only matters to the extent to which it affects active satellites. Definition 1 describes the revelation of ℓ_t.

Definition 1. *(Symmetric physical uncertainty) The collision rate in period t is revealed to all agents after any debris removal decisions are made but before any launch decisions are made for the period.*

"Symmetric physical uncertainty" means that launching firms and the regulator all know how many satellites will be lost in period t before acting, but not which satellites. On the other hand, satellite owners engaging in debris removal actions don't know how many satellites will be lost until after their removal action. The timing reflects three features of orbit use: (1) while conjunction alerts may be issued to affected operators up to a few days before an anticipated collision, longer-term forecasts of the collision environment are inherently probabilistic; (2) satellite owners who wish to remove debris will attempt to do so before the collisions are unavoidable; and (3) firms choosing whether or not to launch satellites can anticipate satellite owners' removal actions, in part because conjunction alerts are issued publicly to all satellite operators near the affected region so as to better coordinate avoidance maneuvers. This timing does not drive the main results; qualitatively similar results for orbit use are derived in the deterministic setting of Rao and Rondina (2018).

The symmetry between firms and the regulator is practically plausible: firms and regulatory agencies all have access to the same types of information about the position of orbital bodies[5], and can run similar calculations to predict the motion of orbital bodies from given position data. The US Department of Defense makes orbital object data fine enough to perform high-fidelity conjunction analysis on specific satellites available for nominal fees, while aggregate patterns can be modeled using data the Department of Defense makes publicly available. The European Space Agency makes similar data publicly available, albeit at lower fidelity for academic and hobbyist use. Most of these analyses are probabilistic in nature. Since satellites in the model are all identical, the identity of the satellites lost doesn't matter, and the probability any specific satellite is lost is the same as the aggregate rate.

I assume that ℓ_t has a conditional density $\phi(\ell_t|S_t, D_t)$. With physical uncertainty, only the density of ℓ_t is determined by S_t and D_t, so I explicitly include the draw of ℓ_t as an argument

[5] State actors, particularly national security agencies, may have different information than other agents, breaking the symmetry. As long as the regulator(s) are equally ignorant of this information as firms, the symmetry holds. In some cases the regulator may actually have an informational advantage over firms. This suggests a question not pursued here: what are the limits to a regulator's ability to manage orbital congestion without revealing state secrets?

of $G()$. The expected value at the end of period t of a function $f(\ell_{t+1})$ is

$$E_t[f(\ell_{t+1})] = \int_0^1 f(\ell_{t+1})\phi(\ell_{t+1}|S_{t+1}, D_{t+1})d\ell_{t+1}.$$

I also assume that the distribution of the collision rate is "increasing" in the number of satellites and amount of debris, in the sense that an increase in either satellites or debris results in a new distribution which first-order stochastically dominates the old one. Assumption 1 states this precisely.

Assumption 1. *(The collision rate is increasing in satellites and debris) An increase in either satellites or debris results in a new distribution which first-order stochastically dominates the old one, that is,*

$$\int_0^k \ell\ \phi(\ell|S+\varepsilon_S, D+\varepsilon_D)d\ell \geq \int_0^k \ell\ \phi(\ell|S,D)d\ell \quad \forall \varepsilon_S, \varepsilon_D \geq 0,\ \forall k \in (0,1),$$

with strict inequality for some $\varepsilon_S > 0$ *or* $\varepsilon_D > 0$.

To reduce notational burden, I suppress the conditioning variables where it is clear from context, though I sometimes make them explicit in proofs. For brevity, I refer to $E_t[\ell_{t+1}]$ as the "collision risk".

2.2.1 Functional forms for the collision risk and number of new fragments

For simulations, I use functional forms for the collision risk and new fragment formation based on engineering model in Bradley and Wein (2009):

$$E[\ell|S,D] = \min\{\alpha_{SS}S^2 + \alpha_{SD}SD, 1\}, \tag{3}$$

$$G(S,D,\ell) = \begin{cases} \beta_{SS}\left(\frac{S}{S+D}\right)\ell S + \beta_{SD}\left(\frac{D}{S+D}\right)\ell S + \beta_{DD}\alpha_{DD}D^2 \text{ if } S+D>0 \\ 0 \text{ if } S+D=0 \end{cases} \tag{4}$$

which satisfy all the properties described above. α_{SS}, α_{SD}, and α_{DD} are positive constants which can be derived from an ideal gas model and descriptions of the shapes and sizes of the subscripted object types. They are often referred to as "intrinsic collision probabilities" in engineering studies. β_{SS}, β_{SD}, and β_{DD} are positive constants describing the mean "effective" (that is, adjusted for size and time spent in the shell of interest) number of fragments created in collisions between the subscripted object types. These can be calculated from descriptions of the material compositions of the objects colliding, their relative velocities, and masses. They are often referred to as "fragmentation parameters" in engineering studies. These forms are used to generate figures and simulations, but not for analytical results.

Economically, the expected collision risk can be thought of as a matching function which matches active satellites to debris and other active satellites. The form in equation 3 implies that matching between active satellites and debris or other active satellites exhibits "thick market effects": one more active satellite or unit of debris increases the ease with which all active satellites are matched with other orbital bodies. The economic intuition of the expected collision risk function is discussed in more detail in Rao and Rondina (2018).

2.2.2 Kessler Syndrome

Kessler Syndrome is a central concern in orbit use management. If open access can prevent Kessler Syndrome, regulating orbit use is not as important from an environmental perspective. Even though orbit use will be inefficient it will not cause irreversible environmental damage. On the other hand, if open access can cause Kessler Syndrome, orbit use management is more urgent.

In this section I formally define Kessler Syndrome and establish some properties of the debris threshold beyond which it occurs. Open access debris levels are increasing in the excess return on a satellite while the Kessler threshold is constant, implying that sustained increases in the return on a satellite can cause Kessler Syndrome under open access. Though the Kessler threshold is defined purely in terms of the system's physics, the occurrence of Kessler Syndrome depends critically on the economics of orbit use.

Assumption 2. *(Debris growth) The growth in new fragments due to debris is larger than the decay rate for all levels of the debris stock greater than some level $\bar{D} > 0$,*

$$\bar{D} : G_D(0, D, \ell) > \delta \ \forall D > \bar{D} \ \ \forall \ell.$$

Due to assumption 2 and $G(S, D, \ell)$ being increasing in all arguments, there is a unique threshold $D^\kappa \geq \bar{D}$ above which Kessler Syndrome occurs. Past this threshold, the number of new fragments created by collisions between debris exceeds the amount which decays in a single period. For regimes where this condition doesn't hold at any level of debris, Kessler Syndrome is impossible. Such regimes are likely to be at extremely low altitudes, possibly sub-orbital. For all of the simulations shown in this paper, Kessler Syndrome is possible.

Definition 2. *(Kessler Syndrome) The Kessler region is the set of debris levels for which cessation of launch activity and immediate deorbit of all active satellites cannot prevent continued debris growth, that is,*

$$D^\kappa : G(0, D^\kappa, \ell) > \delta D^\kappa \ \ \forall \ell.$$

Kessler Syndrome has occurred when the debris stock enters the Kessler region.

Without active debris removal technologies, Kessler Syndrome is an absorbing state. Once Kessler Syndrome occurs in the model, the debris stock grows without bound. In reality, the fragments would eventually pulverize each other into small fragments and either find stable orbits or decay back to the Earth, but this process could take centuries or millennia (Kessler and Cour-Palais (1978)).

2.3 The economics of open access and optimal orbit use without debris removal

A firm which owns a satellite collects a return of π every period that the satellite survives. A fraction ℓ_t of the orbiting satellites are destroyed in collisions every period. Since they are identical, the probability that an individual satellite survives the period is $(1-\ell_t)$. The value of a satellite in period t is

$$Q(S_t, D_t, \ell_t, X_t) = \pi + \beta[(1-\ell_t)E_t[Q(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] + \ell_t E_t[V_i(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})]] \tag{5}$$

A firm which does not own a satellite in period t faces the decision to pay a fixed cost F and launch a satellite which will reach orbit and start generating revenues in period $t+1$, or to wait and decide again whether or not to launch in period $t+1$. Once the firm decides to launch, it must wait one period before the satellite will reach orbit and begin producing returns. Assuming potential launchers are risk-neutral profit maximizers, the value of potential launcher i at period t is

$$\begin{aligned} V_i(S_t, D_t, \ell_t, X_t) = \max_{x_{it}\in\{0,1\}} &\{(1-x_{it})\beta E_t[V_i(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] \\ &+ x_{it}[\beta E_t[Q(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] - F]\} \\ S_{t+1} &= S_t(1-\ell_t) + X_t \\ D_{t+1} &= D_t(1-\delta) + G(S_t, D_t, \ell_t) + mX_t \\ \ell_{t+1} &\sim \phi(\ell|S_{t+1}, D_{t+1}) \end{aligned} \tag{6}$$

Under open access, firms launch until profits are zero:

$$X_t > 0: \ \beta E_t[Q(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] = F. \tag{7}$$

The value of a satellite is then

$$Q(S_t, D_t, \ell_t, X_t) = \pi + (1 - E_t[\ell_t])F, \tag{8}$$

and the equilibrium collision risk is

$$E_t[\ell_{t+1}] = r_s - r. \tag{9}$$

The fleet planner maximizes the expected net present value of the entire fleet. Their problem is

$$W(S_t, D_t, \ell_t) = \max_{X_t \geq 0} \{\pi S_t - F X_t + \beta E_t[W(S_{t+1}, D_{t+1}, \ell_{t+1})]\} \tag{10}$$

$$\text{s.t. } S_{t+1} = S_t(1 - \ell_t) + X_t \tag{11}$$

$$D_{t+1} = D_t(1-\delta) + G(S_t, D_t, \ell_t) + m X_t. \tag{12}$$

The planner launches so that the loss rate is equated to the rate of excess return net of the marginal external cost (ξ_{t+1}), that is,

$$E_t[\ell_{t+1}] = r_s - r - \frac{E_t[\xi(S_{t+1}, D_{t+1})]}{F}. \tag{13}$$

where $E_t[\xi(S_{t+1}, D_{t+1})]$ is the marginal external cost of a satellite launch. For the results in this paper, it suffices to assume that the marginal external cost is weakly positive for all S_{t+1} and D_{t+1} along the optimal path, and strictly positive for some values of S_{t+1} and D_{t+1}. Readers interested in the properties of the marginal external cost of satellite launches are referred to Rao and Rondina (2018), where $\xi(S_{t+1}, D_{t+1})$ is derived explicitly in a deterministic setting and shown to be positive under economically and physically intuitive conditions. Figure 3 illustrates the differences between open access and optimal policies in the deterministic setting.

[Table 3 about here.]

2.4 Debris removal technologies

Orbital debris is dangerous to active satellites in part because debris objects cannot be maneuvered and often do not transmit their location to ground stations. Active satellites, on the other hand, tend to do both, making collision avoidance maneuvering easier. Active satellites also tend to have some guidance and control systems which allow them to be deorbited remotely, if necessary. Debris objects tend not to have such systems, because they are fragments of a satellite, non-responsive to ground operator commands, or out of fuel and incapable of further maneuvers. Active debris removal technologies are those which can interact with debris objects and deorbit them. They are contrasted with passive removal, which involves measures like setting a satellite on a path which will result in its deorbit in a specified timeframe.

Active debris removal technologies are being developed, but have not yet been commercially deployed. Some of these technologies involve specialized removal satellites which use the

Earth's magnetic field for propulsion and deploy nets, harpoons, or tethers (for example, Pearson, Carroll, and Levin (2010)) to either deorbit debris or recycle the materials for in-space manufacturing. Ground-based lasers are another candidate technology to deorbit debris.

I assume no new satellites are required to implement removal, which can be interpreted in two ways: that the removal technology is ground-based; or that the satellites required are already in orbit and can never be destroyed or lost. Including the requirement that new satellites be used for removal complicates the model in interesting and relevant ways that are beyond my scope here. I also assume that only satellite owners can purchase debris removal.

With the ability to remove debris from orbit, satellite owners can remove clearly-dangerous pieces of debris before they impact their satellites. The remaining collisions will be caused by errors in debris risk assessments, satellite trajectory forecasts, and collisions which were deemed too costly to avoid. To reflect this in the model, I adjust the timing of when ℓ_t is revealed when debris removal technologies are present. Satellite owners purchase R_t total units of removal before ℓ_t is revealed, with the aim of changing the distribution of ℓ_t until the marginal private benefit of removal equals the marginal private cost. After removal has been purchased, ℓ_t is drawn from a distribution conditioned on S_t and $D_t - R_t$ (instead of just S_t and D_t) and revealed to all satellite owners and prospective launchers. The launchers then decide whether or not to launch.[6]

With debris removal before collisions, the laws of motion and distribution of the collision rate become

$$S_{t+1} = S_t(1-\ell_t) + X_t \tag{14}$$

$$D_{t+1} = (D_t - R_t)(1-\delta) + G(S_t, D_t - R_t, \ell_t) + mX_t \tag{15}$$

$$\ell_t \sim \phi(\ell_t | S_t, D_t - R_t). \tag{16}$$

Expectations before removal in t are indicated by $\tilde{E}_t[\cdot]$ and treat ℓ_t as a random variable, while expectations after removal in t are indicated by $E_t[\cdot]$ and treat ℓ_t as known. The expected collision risk before removal is effected is

$$\tilde{E}_t[\ell_t] = \int_0^1 \ell_t \phi(\ell_t | S_t, D_t - R_t) d\ell_t. \tag{17}$$

Potential launchers in t have the same expectations as before: they are aware of ℓ_t, and treat

[6] In reality, the timing of satellite launches and debris removals will not be this clearly separated. However, potential launchers will be able to anticipate satellite owners' debris removal demands, and where possible structure their launches to take advantage of these efforts.

ℓ_{t+1} as uncertain. Formally,

$$E_t[\ell_{t+1}] = \int_0^1 \ell_{t+1}\phi(\ell_{t+1}|S_{t+1}, D_{t+1} - R_{t+1})d\ell_{t+1} = \tilde{E}_{t+1}[\ell_{t+1}]. \tag{18}$$

Though $E_t[\ell_{t+1}] = \tilde{E}_{t+1}[\ell_{t+1}]$, I use separate notation so that the subscript on the expectation operator indicates the period in which the agent forms the expectation, and the tilde above the expectation operator indicates whether the expectation is formed before or after is drawn and revealed. $E_t[\ell_{t+1}]$ is an expectation formed in t after ℓ_t is drawn and revealed, $\tilde{E}_t[\ell_t]$ is an expectation formed in t before ℓ_t is drawn and revealed.

3 Space Traffic Control

3.1 Policy without active debris removal

Space traffic control policies restrict either the number of satellites launched to or the number of satellites in an orbit in a given window of time. As described earlier, I refer to policies restricting the number of launches in a given period as *flow controls*, and policies which restrict the number of satellites in orbit in a given period as *stock controls*. Stock controls entail an explicit or implicit payment made every period that the satellite is in orbit. The payment gives the satellite owner the right to keep their satellite in orbit that period. Flow controls entail a payment made once when the satellite is launched. The payment gives the satellite launcher the right to launch in that period. Table 2 gives some examples of each type of control policy. Both types of controls are currently in place around the world - the FAA's launch permit system is a flow control for launches from the United States, while the ITU's minimum spacing requirements for satellites in GEO are a stock control for GEO use.[7] Existing controls tend to be implemented as quantities, as in the two examples given, but could also be implemented as prices, for example,a launch or satellite tax.

[Table 2 about here.]

Quantity restrictions imply price restrictions and vice versa. In many settings, either mode can generate equivalent social welfare. Weitzman (1974) establishes that the equivalence can break down in the presence of regulatory uncertainty over the firm's marginal cost of production. Whether a price or quantity instrument should be preferred in such settings depends on the relative slopes of the marginal benefit and marginal cost curves. This is not the case for orbits, where the main source of uncertainty comes from the motion of physical objects

[7]These requirements tend to focus on launch capacity and spectrum interference, rather than the risk of collisions and debris growth. The framework developed here applies to orbit use management policies regardless of their intent.

which are in principle observable by all actors. Unlike the regulatory problems considered in Weitzman (1974) and Newell and Pizer (2003), the firm has no additional information about the motion of orbital bodies for the regulator to harness through instrument design.

The distinction between stock and flow controls is relevant to a broad class of economic management problems. To encourage renewable energy generation, a regulator may weigh investment (stock) vs production (flow) tax credits (Aldy, Gerarden, and Sweeney, 2018). To manage public infrastructure a regulator may weigh investment in damage abatement (flow) vs quality restoration (stock)(Keohane, Van Roy, and Zeckhauser, 2007).[8]

In the absence of informational or administrative constraints on the regulator, the preferred instrument is that which most directly targets the externality-generating activity (Sandmo, 1978). In the renewable energy case, production tax credits can encourage renewable energy generation more effectively than investment tax credits.[9] In orbit, stock controls dominate flow controls because the collision risk externality is driven by the number of objects in orbit rather than the number of objects launched in a period.

Stock and flow controls can often be made equivalent in the sense that one can be capitalized or annuitized to the same present value cost as the other. However, they have different effects on the incentive to launch or own a satellite. Imposing a fee at launch increases the cost of entering the orbital commons, penalizing entrants while increasing the rents accruing to incumbents in orbit. Imposing a recurring fee while the satellite is in orbit reduces the rents of satellite ownership without restricting entry, treating entrants and incumbents equally. These differing incentives can lead to welfare differences between stock and flow modes of orbit control. To show how stock and flow controls affect the decision to launch a satellite, consider two cases with price-based controls. In the first, a stock control is levied on satellite owners. In the second, a flow control is levied on satellite launchers. I assume the regulator can commit to future policies, so that $t+1$ values are known to firms with certainty.[10]

[8] Keohane, Van Roy, and Zeckhauser (2007) consider the use of stock and flow controls to manage the quality of a resource, but their use of "stock control" is slightly different due to the setting considered. In their setting, "stock controls" refer to policies which restore the stock of a deteriorating resource. Here, the term refers to limiting the stock of a commodity which deteriorates the resource. Keohane, Van Roy, and Zeckhauser (2007)'s use of "flow controls" is closer to the use of the term here: they consider abating the flow of pollutants into the environment, and I consider controlling the flow of satellites into orbit.

[9] Provided capacity is not a binding constraint, production effort is costly, and the production function is not characterized by decreasing returns to scale, as described in Aldy, Gerarden, and Sweeney (2018) and Parish and McLaren (1982).

[10] Stock and flow controls both require forward guidance, since announced or anticipated $t+1$ values affect the launch rate in t. But whereas flow controls require forward guidance regarding the entire time path of control values, stock controls only require forward guidance about the next-period control value. Even without commitment, the regulator faces no incentive to deviate from a previously-announced stock control rule. This may not be the case for flow controls. Because anticipated changes in the flow control rule can cause launching firms to "bunch" and

The decision to launch under a stock control: Let the price that a satellite owner pays in t be p_t^s. Firms deciding whether to launch or not in period t will account for their anticipated regulatory burden as they drive the profits of launching a satellite down to zero. The marginal benefit from owning a satellite in $t+1$ must therefore equal not only the opportunity cost of the launch, but also the direct regulatory cost of the stock control. Formally,

$$X_t : \beta E_t[Q(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] = F \tag{19}$$

$$E_t[Q(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] = \pi - p_{t+1}^s + (1 - E_t[\ell_{t+1}])F \tag{20}$$

$$\implies \pi = rF + E_t[\ell_{t+1}]F + p_{t+1}^s. \tag{21}$$

The decision to launch under a flow control: Let the price that a satellite launcher pays in t be p_t^f. Firms deciding whether to launch or not in period t will account for the regulatory burden of launching. Since they know that future launchers will face a similar regulatory burden, they will consider how the future flow control price will affect the open access value of a satellite. The marginal benefit of owning a satellite in $t+1$ must therefore equal the opportunity cost of the launch, which includes the flow control price they pay and the forgone interest. However, the marginal benefit of owning a satellite in $t+1$ now includes not only the direct revenues the satellite generates but also the additional expected value from the flow control levied on $t+1$ launchers. Formally,

$$X_t : \beta E_t[Q(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] = F + p_t^f \tag{22}$$

$$E_t[Q(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] = \pi + (1 - E_t[\ell_{t+1}])F + (1 - E_t[\ell_{t+1}])p_{t+1}^f \tag{23}$$

$$\implies \pi + (1 - E_t[\ell_{t+1}])p_{t+1}^f = rF + E_t[\ell_{t+1}]F + (1+r)p_t^f. \tag{24}$$

Figure 4 illustrates equations 21 and 24. Although imposing either type of control can reduce the equilibrium number of launches, flow controls raise the private marginal benefit of launching along with the private marginal cost. Stock controls, on the other hand, affect only the marginal cost of launching. This is the core intuition for why stock controls are preferable to flow controls for managing orbital congestion.

[Figure 4 about here.]

Leakage issues and legal hurdles: Both types of controls face leakage issues. Flow controls implemented by regional launch providers may suffer "launch leakage", while stock controls implemented by regional regulatory agencies may suffer "mission control leakage".

attempt to launch either just before a price increase or just after a price decrease, the regulator has an incentive to make flow control policy changes a surprise. Such surprises would change private expectations of the control policy path. In environmental economics, Newell, Pizer, and Zhang (2005) consider the tradeoffs between commitment and discretion in stabilizing quantity-policy prices. This tradeoff is analyzed in more depth in the monetary policy literature; Svensson (2003) provides a comprehensive discussion.

Similar leakage issues have been studied extensively in the environmental and public economics literatures, for example Fowlie (2009); Fischer and Fox (2012); Böhringer, Rosendahl, and Storrøsten (2017). Though these issues are relevant to effective policy implementation, analyzing them is beyond the scope of this paper. The legal hurdles to implementing stock controls may also be higher than those for flow controls, since they require a legal framework in which the right to exclude agents from an orbit can be held and enforced. Such a framework would have to be globally agreed-upon and potentially self-enforcing. I do not consider the prospects of such an agreement in this paper, although similar issues have been studied extensively in economics generally and environmental economics specifically, for example Telser (1980); Barrett (2005, 2013).

3.1.1 Using stock and flow space traffic control policies

In this section, I formally describe some properties of stock and flow controls and how they should be used to manage space traffic. The first property is price-quantity equivalence: under symmetric physical uncertainty, a stock or flow control can be implemented as a price or quantity and achieve equivalent expected social welfare. This allows me to consider price or quantity implementations interchangeably. I then show how stock and flow controls should be used to limit launches, and consider the implications of these details for optimal control values. I follow this by showing how the launch rate responds to the initiation of a stock or flow control, and how a regulator could use those controls to induce firms to deorbit already-orbiting satellites and stop launching new ones. These properties are used in the following section to establish that regulating orbit use through stock controls achieves higher expected social welfare than using flow controls.

Proposition 1. *(Price-quantity equivalence) Under symmetric physical uncertainty, price and quantity implementations of stock controls are equivalent, as are price and quantity implementations of flow controls.*

Proof. I show the result for stock controls first, and then for flow controls.

Stock controls: I refer to price-based stock controls as satellite taxes, and quantity-based stock controls as satellite permit quotas. Let the launch rate under a satellite tax be $\tilde{X}_t$, and the permit price under a permit quota be $\tilde{p}_{t+1}$.

Under a satellite tax, the number of satellites launched will be

$$\tilde{X}_t : \pi = rF + E_t[\ell_{t+1}]F + p_{t+1} \tag{25}$$

Under a binding satellite permit quota, firms will purchase permits and launch satellites until the price of a permit is

$$\tilde{p}_{t+1} : \pi = rF + E_t[\ell_{t+1}]F + \tilde{p}_{t+1} \tag{26}$$

For a given state vector (S_t, D_t, ℓ_t) and a chosen price p_{t+1}, the monotonicity of $E_t[\ell_{t+1}]$ ensures that equation 25 determines a unique value of $\tilde{X}_t$. For the same state vector and $X_t = \tilde{X}_t$, the monotonicity of $E_t[\ell_{t+1}]$ ensures that $\tilde{p}_{t+1} = p_{t+1}$ solves 26.

Flow controls: I refer to price-based flow controls as launch taxes, and quantity-based stock controls as launch permit quotas. Let the launch rate in t under a launch tax be $\tilde{X}_t$, and the permit price in $t+1$ under a permit quota be $\tilde{p}_{t+1}$.

Under a launch tax, the number of satellites launched will be

$$\tilde{X}_t : \pi = rF + E_t[\ell_{t+1}]F + (1+r)p_t - (1 - E_t[\ell_{t+1}])p_{t+1} \tag{27}$$

Under a binding launch permit quota, firms will purchase permits and launch satellites until the price of a permit is

$$\tilde{p}_{t+1} : \pi = rF + E_t[\ell_{t+1}]F + (1+r)p_t - (1 - E_t[\ell_{t+1}])\tilde{p}_{t+1} \tag{28}$$

For a given state vector (S_t, D_t, ℓ_t) and a chosen price p_{t+1}, the monotonicity of $E_t[\ell_{t+1}]$ ensures that equation 27 determines a unique value of $\tilde{X}_t$. For the same state vector and $X_t = \tilde{X}_t$, the monotonicity of $E_t[\ell_{t+1}]$ ensures that $\tilde{p}_{t+1} = p_{t+1}$ solves 28. $\square$

With access to commitment, a regulator using a flow control sets either the future number of permits or their price (X_{t+1} or p_{t+1}) in order to influence the launch rate today (X_t). Raising p_{t+1} in t raises the marginal benefit of launching a satellite today, but lowers it tomorrow. The use of flow controls requires the regulator to trade off the future launch disincentive of raising p_{t+1} against the current launch incentive it creates. The regulator's true instrument with a flow control is not the price of the control itself, but the *change* in price between periods. Rather than a price mapping to a quantity, here it is a (real) change in price which maps to a quantity and vice versa. The regulator can set any initial flow control price so long as they commit to a path of control prices based on equation 24. A similar penalty-rebate structure appears in the mining flow control studied in Briggs (2011), where incentivizing mine owners to mine less t requires a lower Pigouvian tax in period $t+1$.

Note that stock control prices must be positive to reduce launches in any given period, while flow control prices need not be positive to do the same. Along positive price paths the flow control is an entry restriction while along negative price paths it is an entry subsidy.

Current restrictions deter current entry, but future restrictions deter future entry and boost the rents accruing to incumbents, incentivizing current entry. Current subsidies encourage current entry, but future subsidies encourage future entry and reduce the rents accruing to incumbents, incentivizing firms to delay entry. In either case, the regulator is able to use the change in flow control prices to rearrange satellite launches over time.[11]

The need to commit to a flow control path makes terminal conditions economically relevant to their use. If the regulator plans to use the flow control for only a limited duration, after which the orbits will be under open access again, the flow control price path will decrease over time until it is zero in the period where open access is restored.[12] A flow control which attempts to ensure optimality with no planned phase-out will be forced to follow an exploding price path, positive or negative, as the regulator attempts to balance present and future incentives and disincentives without causing launchers to "bunch" suboptimally in any period while the control is active. This property of the price path is formally established in section 6.2 of the Appendix. The credibility of such a price path is doubtful, but beyond my scope here.

Limiting launches with stock and flow controls: While stock controls are straightforward - raise the price to reduce launches - flow controls are subtler. To limit launches in t, the flow control price in $t+1$ should be lowered instead of raised. The intuition for this can be seen in Figure 4 and in equation 24, where the price of a flow control in $t+1$ enters the launch decision in t as a marginal benefit rather than a marginal cost. This has implications for the design of optimal controls: an optimal stock control equates the $t+1$ control price with the expected marginal external cost in $t+1$, while an optimal flow control makes the expected real difference in t and $t+1$ control prices equal to the negative of the expected $t+1$ marginal external cost.

Lemma 1. *(Launch response to stock and flow controls) The open access launch rate is*

- *decreasing in the future price of a stock control;*
- *decreasing in the current price and increasing in the future price of a flow control.*

[11] Technically, the flow control structure creates a problem if the loss rate drawn in t is large enough that the expected loss rate in $t+1$ is one. If this happens, there is no future launch control price which can satisfy equation 24. Lemma 4 in section 6.2 of the Appendix shows this formally. If the regulator wishes to control the launch rate in periods where the expected future loss rate is one, they must break their earlier commitment and adjust p_t until the launch rate is where they want it to be. Since I assume the regulator cannot break their commitment, in this case there is simply no time-consistent flow control which can affect the launch rate. This holds for both price and quantity implementations. That said, if potential launchers expect their satellite to be destroyed after one period, they will only launch in the unrealistic edge case where one period of returns from a satellite exceeds the cost of launching. It is likely that potential launchers would rather not launch in that case, making the point is moot.

[12] Why might a regulator want to do this? Open access launching tends to overshoot the open access steady state, potentially ending up in the Kessler region. A regulator who wished to prevent this without committing to optimality may therefore impose a flow control until the risk of overshooting is sufficiently reduced.

Proof. See Appendix section 8. $\square$

Corollary 1. *The shift in marginal cost of owning a satellite due to an increase in the flow control price is greater than the prior shift in marginal benefit due to the entry restriction.*

Proof.

$$r > 0 \implies \left|\frac{\partial X_t}{\partial_t p}\right| > \left|\frac{\partial X_t}{\partial p_{t+1}}\right| \tag{29}$$

$$\implies 1 + r > 1 - E[\ell_{t+1}], \tag{30}$$

which is true because $\ell_{t+1} \in [0,1]$ by definition. $\square$

Committing in t to raising the flow control price in $t+1$ raises the marginal benefit of owning a satellite before $t+1$, when the new price comes into effect and raises the marginal cost of launching a satellite. This increases the number of launches in t and reduces the number in $t+1$. On the other hand, committing in t to lowering the flow control price in $t+1$ reduces the marginal benefit of owning a satellite in $t+1$, when the new price comes into effect and lowers the marginal cause of launching a satellite. This reduces the number of launches in t and increases the number in $t+1$. This “launch bunching” is absent in stock controls.

Optimal control policies: Making a stock control optimal is simple: set the price equal to the expected marginal external cost of another satellite. Letting p_t^s be the value of the stock control in period t,

$$p_{t+1}^s = E_t[\xi(S_{t+1}, D_{t+1})] \tag{31}$$

will make launchers behave as the planner would command.

Making a flow control optimal is more complicated. The value of the control in the previous period must be taken into account to balance intertemporal launch incentives. The expected survival rate must be accounted for as well, as it determines the expected rent due to entry restriction the firm will realize. Formally, letting p_t^f be the value of the flow control in period t,

$$p_{t+1}^f = \frac{(1+r)p_t^f - E_t[\xi(S_{t+1}, D_{t+1})]}{1 - E_t[\ell_{t+1}]} \tag{32}$$

is required. Perhaps counterintuitively, the expected marginal external cost of another satellite must be *subtracted* from the future value of the stock control. This is because the future flow control price represents a benefit to current launchers, rather than a cost, as seen in Figure 4. Figure 7 shows examples of optimal stock and flow control policies.

Initiating control: Along an interior equilibrium path, stock and flow controls can both be optimal. This raises the questions of whether the equivalence holds when a control is first put into place, and whether boundaries (periods when a control is implemented from an open access status quo, or when a control is used to shut down all launches) present any challenges to either control type. Proposition 2 shows that the equivalence does not hold at initiation boundaries. Figure 5 illustrates Proposition 2.

Proposition 2. *(Smoothness at boundaries) Stock controls can be initiated without letting the launch rate exceed the open access launch rate. Flow controls cannot be initiated without forcing the launch rate to exceed the open access launch rate.*

Proof. See Appendix section 8. □

[Figure 5 about here.]

As described in Lemma 1 and Figure 4, positive flow control prices first shift the marginal benefit of owning a satellite before the control is implemented upwards, then shift the marginal cost of owning a satellite after the control is implemented upwards. As described in Corollary 1, the increase in marginal cost is necessarily greater than the increase in marginal benefit, so if the flow control price is kept stationary after the increase, there will be fewer launches per period than before. However, more than the equivalent open access number of firms will launch just before the flow control price is raised to capture its rents.

Price and quantity stock controls are equally easy to use to halt all launching. As a price, the control value is simply raised until no firm wants to launch. As a quantity, the control value is simply frozen at whatever number of satellites in orbit us desired. Using flow controls for this purpose is trickier, with quantities being more intuitive than prices. A quantity flow control can be used to halt all launching by setting the number of allowed launches to zero. Though expectations of a launch shutdown may induce firms to launch earlier, the mechanics are described in Lemma 1 and bunching can be mostly avoided with careful attention to the entire path of allowed launch quantities (bunching before the period when control is implemented is unavoidable). Using a price flow control is not as simple as raising the price once, however, since (a) in the period before the price increase firms will want to launch to capture the rents from restricted entry the following period, and (b) if the difference in the flow control price between periods is constant launching may resume. To avoid bunching and maintain shutdown, the regulator must instead *lower* the flow control price in the period when launch shutdown is desired, and then commit to an ever-decreasing sequence of prices which will eventually go to negative infinity (the reasoning is described in Lemma 3). The credibility of such price paths is questionable.

Inducing satellite owners to deorbit: Satellite owners often have the option to deorbit their satellite if it becomes too expensive to operate[13]. In this section only, I include the deorbit option for satellite owners to consider whether stock and flow control policies can induce deorbits. The firm's net payoff from deorbiting their satellite is $V^d \lesseqgtr 0$. V^d includes any liquidation revenues (for example, from selling mission control equipment) or costs (for example, costs of damage to people or property during the deorbit). Firms decide whether or not to deorbit after ℓ_t is revealed. A firm which decides to deorbit doesn't claim the revenues from being in orbit that period. Formally,

$$Q_t = \max\{\pi + (1-\ell_t)F, V^d\} \tag{33}$$

Satellites that are in the process of being deorbited may still collide with each other or be struck by debris. Denoting the number of satellites deorbited as Z_t, the laws of motion with deorbit are

$$S_{t+1} = (S_t - Z_t)(1-\ell_t) + X_t \tag{34}$$
$$D_{t+1} = D_t(1-\delta) + G(S_t, D_t, \ell_t) + mX_t \tag{35}$$
$$\ell_t \sim \phi(\ell_t | S_t, D_t). \tag{36}$$

The satellites which are deorbited but still destroyed in collisions ($Z_t \ell_t$) are included in D_{t+1}. Firms choose to deorbit if the payoff from deorbit exceeds the payoff from remaining in orbit:

$$\text{Deorbit if } V^d > \pi + (1-\ell_t)F \tag{37}$$
$$\ell_t > 1 + \frac{\pi - V^d}{F}. \tag{38}$$

$V^d < \pi$ is a no-arbitrage condition: it ensures that firms can't pump money out of an orbit by repeatedly launching and deorbiting satellites. It is also a necessary condition for no possible loss draw to induce deorbits.[14] It implies that flow controls can't force firms to deorbit with positive flow control prices, as described in the following result.

Proposition 3. *(Controlling the rate of deorbit) Stock controls with positive prices can make satellite owners deorbit their satellites and induce net deorbits. Flow controls with positive*

[13] In the early 2000s, lack of profitability nearly induced the operators of the Iridium constellation to deorbit their satellites. Iridium SSC ultimately went bankrupt, but was able to find a consortium of buyers who kept the constellation in orbit. Modern cubesats are often launched without sufficient guidance and control capabilities to initiate deorbit. Their trajectories are typically planned so that they will naturally deorbit within a few years of their launch.

[14] In this setting, where firms own only one satellite each, it would be economically strange if a firm were to throw away the potential for future revenues by deorbiting their only satellite. When firms are modeled with multiple satellites, they may decide to deorbit one satellite to preserve others. However, the no-arbitrage condition here would still apply, since if the value of .

prices cannot make satellite owners deorbit their satellites or induce net deorbits.

Proof. See Appendix section 8. □

Intuitively, making it costlier for firms to launch new satellites cannot make already-orbiting satellites less valuable. This is why flow controls are unable to induce deorbits, at least with positive prices. Flow controls with negative prices may or may not be able to induce deorbits, depending on parameter values and the number of new entrants induced.

3.1.2 Risks and policy choice

In this section, I consider how the choice of stock or flow control mode will affect the equilibrium collision risk and the probability of Kessler Syndrome. I establish that stock controls generate weakly higher expected fleet values than flow controls over arbitrary horizons. Due to the smoothness properties described in Proposition 2, both collision and Kessler risks are increased when a flow control is initiated but not when a stock control is initiated.

Proposition 4. *(New stock controls reduce risk and debris, a new flow controls increase them) The equilibrium expected collision risk, the expected future debris stock, and the probability of Kessler Syndrome will*

- *decrease when a generic stock control is introduced;*
- *increase when a generic flow control is introduced.*

Proof. The effect of introducing a control on the equilibrium collision risk: Suppose a control is scheduled to be introduced at date t. The equilibrium collision rate under open access in period $t-2$ is

$$\hat{X}_{t-2} : E_{t-2}[\ell_{t-1}] = r_s - r. \tag{39}$$

In general, the equilibrium expected future collision rate is an increasing function of the current launch rate. Proposition 2 establishes that $X_{t-1} < \hat{X}_{t-1}$ (the equivalent uncontrolled open access launch rate in $t-1$) if the control scheduled to be introduced in t is a stock control. Similarly, Proposition 2 establishes that $X_{t-1} > \hat{X}_{t-1}$ if the control scheduled to be introduced in t is a flow control. Thus, introducing a generic stock control must reduce the equilibrium expected future collision rate, while introducing a generic flow control must increase it.

The effect of introducing a control on the expected future debris stock: The debris stock in t is an increasing function of both the collision rate and launch rate in $t-1$:

$$D_t = (1-\delta)D_{t-1} + G(S_{t-1}, D_{t-1}, \ell_{t-1}) + mX_{t-1}. \tag{40}$$

Since the launch rate decreases when a stock control is introduced, D_t is mechanically reduced due to the reduction in launch debris (mX_{t-1}). Similarly, when a flow control is introduced, the increased launch rate increases D_t through launch debris. Even without launch debris, the same conclusion holds in the following period because the expected collision risk is an increasing function of the launch rate. Formally, suppose $m = 0$:

$$E_{t-1}[D_{t+1}] = (1-\delta)D_t + E_{t-1}[G(S_t, D_t, \ell_t)] \tag{41}$$

Because the expected number of new fragments is linear in probabilities,

$$E_{t-1}[D_{t+1}] = (1-\delta)D_t + G(S_t, D_t, E_{t-1}[\ell_t]) \tag{42}$$

$$\implies \frac{\partial E_{t-1}[D_{t+1}]}{\partial p_t} = \frac{\partial G(S_t, D_t, E_{t-1}[\ell_t])}{\partial p_t} \tag{43}$$

$$= \frac{\partial G(S_t, D_t, E_{t-1}[\ell_t])}{\partial E_{t-1}[\ell_t]} \frac{\partial E_{t-1}[\ell_t]}{\partial X_{t-1}}. \tag{44}$$

Both terms on the right-hand side of the final line are positive: the expected number of new fragments formed in collisions is increasing in the expected number of collisions, and the expected number of collisions is increasing in the number of satellites launched the previous period.

The effect of introducing a control on the probability of Kessler Syndrome: The probability of Kessler Syndrome occurring in t, given information in $t-1$, is

$$Pr_t(D_t(\ell_{t-1}) > D^\kappa). \tag{45}$$

We have already established that for any ℓ_{t-1}, D_t will decrease if a stock control is implemented in t, and increase if a flow control is implemented in t. D^κ is a function of the physical parameters of the orbit, and is unaffected by economic controls. Consequently, introducing a stock control must reduce the probability of Kessler Syndrome, while introducing a flow control must increase the probability of Kessler Syndrome. □

The result in Proposition 4 is one of the main reasons why a regulator should prefer stock controls to flow controls. If the imposition of a control raises the equilibrium collision risk, it is possible that it may also cause Kessler Syndrome. The economic intuition for this effect is simple. Flow controls generate rents for firms who already own satellites. Imposing a flow control therefore creates an incentive for marginal launchers to become satellite owners before the flow control is imposed. One way around this would be to levy a flow control with no prior notice. I do not consider this possibility, as it would force firms to form expectations over the regulator's possible actions. Stock controls sidestep this issue by focusing on satellite owners. Forward-looking launchers internalize their expected costs due to the control, and can be appropriately disincentivized against launching without distorting the incentives of current satellite owners.

The relative advantage of stocks vs. flows: The question of ultimate interest to a regulator is likely one of policy choice: "which type of instrument is better, and why?" The results so far - particularly Proposition 4 - suggest that stock controls should be preferred to flow controls along generic paths. Proposition 5 compiles the results so far to answer the policy choice question along optimal paths. Since stock controls can be initiated without losing control of the launch rate and induce deorbits when necessary, they can achieve first-best outcomes in every state of the world. Flow controls cannot. Even if interior launch rates are optimal forever and no deorbits are ever required, flow controls will achieve less social welfare than stock controls when they are put into place.

Proposition 5. *(The relative advantage of stocks vs. flows) The expected social welfare under an optimal stock control strictly exceeds the expected social welfare under an optimal flow control for an arbitrary horizon where a control must be initiated, used to stop all launches, or used to force net deorbits.*

Proof. The fleet welfare from both controls can be equal along interior equilibrium paths. However, when the flow control is initiated, Proposition 2 shows that it will cause the launch rate to exceed the uncontrolled open access launch rate whereas a stock control would not. Proposition 4 shows that the launch bunching from initiating a flow control will also cause the risk of Kessler Syndrome to increase. In those periods, stock controls will achieve strictly greater expected social welfare than flow controls.

Proposition 3 shows that flow controls may not be able to induce net deorbits (never with positive prices and only possibly with negative prices), while stock controls can always do so. Therefore, for arbitrary paths with positive prices where the regulator must either initiate control, shut down orbital access, or induce net deorbits, stock controls achieve strictly greater expected social welfare than flow controls. □

Proposition 5 is fairly straightforward, and may even understate the advantages of stock controls over flow controls. From a computational perspective, optimal flow controls are much harder to implement than optimal stock controls because they require attention to the entire control time path. Lemma 3 in the Appendix shows that price-based flow controls must have an exploding price path to balance the launch incentives and disincentives described in Lemma 1 and Figure 4. One solution to this may be to use a quantity flow control, such as a launch permit quota system. However, the regulator must still commit to a time path of quantity policies when the flow control is implemented, cannot prevent launch bunching before the policy goes into effect, and cannot induce deorbits. Stock controls face none of these issues. A one-period-forward forecast of the marginal external cost is sufficient, which would have been required anyway under a flow control. The regulator faces no commitment issues and can precisely control the number of satellites in orbit at any given time.

3.1.3 Optimal space traffic control policies

Before finishing my discussion of stock and flow controls, I illustrate optimal control policy functions by simulation. For clarity and computational tractability, I use deterministic simulations where $E_t[\ell_{t+1}] = L(S_{t+1}, D_{t+1})$. Figure 7 shows an example of optimal stock and flow control policies as a satellite tax and a launch tax. Figure 6 shows the underlying satellite stocks, debris stocks, and launch rates used to compute Figure 7.

The magnitudes of the tax policies should not be taken literally; the underlying model has not been calibrated to either economic returns or physical processes. The point of the figures is to show the qualitative properties of optimal stock and flow controls. While both types of tax vary with the marginal external cost of launching a satellite ($E_t[\xi(S_{t+1}, D_{t+1})]$), only the satellite tax varies positively with the marginal external cost. This is a convenient feature for applying stock controls: the behavior of an optimal stock control is more intuitive than that of an optimal flow control. The reasoning behind this behavior is described in in Lemma 1 and Figure 4.

[Figure 6 about here.]

[Figure 7 about here.]

3.2 Active debris removal and open access

I now turn to the effects of active debris removal technologies on orbit use. My main result, Proposition 6, shows that while active debris removal can mechanically reduce the debris stock no matter how it is financed, it can only reduce the equilibrium risk of satellite-destroying collisions to the extent that satellite owners pay for debris removal. I show this in two steps. First, I show that exogenously provided removal which is free to satellite owners will reduce the debris stock but increase the satellite stock. The increase in the satellite stock will exactly offset the decrease in risk from debris removal, leaving the equilibrium collision risk unchanged. Then, I consider a case where exogenous debris removal involves a mandatory fee paid by satellite owners. I show that as the fee goes to zero, the collision risk returns to the original open access level.

Lastly, I show how endogenously chosen debris removal purchased by cooperative satellite owners reduces the debris stock, collision risk, and risk of Kessler Syndrome, while also allowing more firms to launch satellites. These results depend on some auxiliary properties of cooperative debris removal and open access launching with debris removal, shown in the Appendix, section 6.7. Though the jointly-optimal launch and removal plan is analytically complicated, I simulate the fleet planner's launch and removal plans and compare them to the launch and removal plans under open access and cooperative removal. The simulations show that cooperative decentralized removal plans are identical to the planner's removal plans, though the launch plans differ. The differences in the launch plans are particularly interesting: the fleet planner launches more intensely than firms under open access do.

3.2.1 An economic model of active debris removal

Satellite owners purchase R_{it} units of debris removal from a competitive debris removal sector. The price of a unit of removal is c_t. The amount of debris removed is nonnegative and cannot exceed the total amount of debris in orbit. Since firms are identical, each satellite owner will choose the same level of removal, making the total amount of debris removed $R_t = S_t R_{it} \leq D_t$. The maximum amount that an individual satellite owner could choose to remove is D_t/S_t. To consider the best-case outcomes of debris removal, I focus on cooperative removal plans between satellite owners. I establish an economically intuitive necessary and sufficient condition for cooperation to be locally self-enforcing in section 6.3 of the Appendix. The condition is stated below in Assumption 3.

Assumption 3. *(Making cooperation locally self-enforcing) For any non-zero cooperative removal plan, the change in the equilibrium collision risk before debris removal is greater than the ratio of the removal price to the launch cost, that is,*

$$\left.\frac{\partial \tilde{E}[\ell_t|S_t,\bar{D}-R_t]}{\partial \bar{D}}\right|_{\bar{D}=D_t} > \frac{c_t}{F} \quad \forall R_t \in [0,\bar{D}].$$

While the validity of Assumption 3 will need to be evaluated empirically for specific technologies and orbital regimes, I assume it always holds in the analysis below.[15]

The value of a satellite after debris has been removed and ℓ_t has been drawn is

$$Q_i(S_t,D_t,\ell_t,X_t) = \pi + \beta[(1-\ell_t)\tilde{Q}_i(S_{t+1},D_{t+1}) + \ell_t E_t[V_i(S_{t+1},D_{t+1},\ell_{t+1},X_{t+1})]]. \quad (46)$$

[15]For example, cooperation may be enforced by a grim trigger mechanism under which any deviation by any firm results in no debris removal at all.

The value of a satellite owner who purchases debris removal before the loss is

$$\begin{aligned}
\tilde{Q}_i(S_t, D_t) &= \max_{0 \le R_{it} \le D_t/S_t} \{-c_t R_{it} + \tilde{E}_t[Q_i(S_t, D_t, \ell_t, X_t)]\} \\
\text{s.t. } \ell_t &\sim \phi(\ell_t | S_t, D_t - R_t) \\
Q_i(S_t, D_t, \ell_t, X_t) &= \pi + \beta[(1-\ell_t)\tilde{Q}_i(S_{t+1}, D_{t+1}) + \ell_t E_t[V_i(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})]] \\
S_{t+1} &= S_t(1-\ell_t) + X_t \\
D_{t+1} &= (D_t - R_t)(1-\delta) + G(S_t, D_t - R_t, \ell_t) + mX_t.
\end{aligned} \tag{47}$$

The value of a launcher is

$$V_i(S_t, D_t, \ell_t, X_t) = \max_{x_{it} \in \{0,1\}} \{(1-x_{it})\beta E_t[V_i(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})] + x_{it}[\beta \tilde{Q}_i(S_{t+1}, D_{t+1}) - F]\} \tag{48}$$

$$\begin{aligned}
\text{s.t. } \tilde{Q}_i(S_t, D_t) &= \max_{0 \le R_{it} \le D_t/S_t} \{-c_t R_{it} + \tilde{E}_t[Q_i(S_t, D_t, \ell_t, X_t)]\} \\
\ell_t &\sim \phi(\ell_t | S_t, D_t - R_t) \\
Q_i(S_t, D_t, \ell_t, X_t) &= \pi + \beta[(1-\ell_t)\tilde{Q}_i(S_{t+1}, D_{t+1}) + \ell_t E_t[V_i(S_{t+1}, D_{t+1}, \ell_{t+1}, X_{t+1})]] \\
S_{t+1} &= S_t(1-\ell_t) + X_t \\
D_{t+1} &= (D_t - R_t)(1-\delta) + G(S_t, D_t - R_t, \ell_t) + mX_t.
\end{aligned}$$

Under a generic launch plan, the decision to remove debris is dynamic. Removal today will impact the amount of debris tomorrow through the number of satellite destructions and the number of debris-debris collisions. Under open access, the value of a satellite tomorrow will always be driven down to the current value of the launch cost, so the future benefits of removal will never accrue to today's satellite owners. This makes the removal decision under open access static: the only benefit of debris removal internalized by satellite owners today is the way that it changes the probability that their satellite is destroyed. Even though the cost of removal is linear, nonlinearity in the coupling between the debris stock and the collision rate can yield an interior solution to the removal decision.

Open access launching: Under open access, firms will launch satellites until the value of launching is zero:

$$\forall t, X_t : \; V_i(S_t, D_t, \ell_t, X_t) = 0 \tag{49}$$

$$\implies \beta \tilde{Q}(S_{t+1}, D_{t+1}) = F \tag{50}$$

$$\implies Q_i(S_t, D_t, \ell_t, X_t) = \pi + (1-\ell_t)F. \tag{51}$$

Taking R_t as fixed, and assuming that launchers plan to choose R_{it+1} optimally when they are satellite owners, the flow condition determining the launch rate is

$$\pi = rF + \tilde{E}_{t+1}[\ell_{t+1}]F + R_{it+1}c_{t+1}. \tag{52}$$

This can be rewritten to yield the equilibrium collision risk,

$$\tilde{E}_{t+1}[\ell_{t+1}] = r_s - r - \frac{c_{t+1}}{F}R_{it+1}. \tag{53}$$

Equation 53 states that the equilibrium collision risk will be equal to the excess return on a satellite ($r_s - r$) minus the rate of total removal costs the launchers will face when they become satellite owners ($\frac{c_{t+1}}{F}R_{it+1}$). If there were no removal technology, $R_t = 0\ \forall t$, and the equilibrium collision rate would be equal to the excess return on a satellite. In any period t, the decisions to launch and to remove debris are undertaken by different firms. Potential launchers make the launch decisions, while current satellite owners make the removal decisions. Once they become satellite owners, launchers will face the removal decision. While open access makes satellite owners myopic, satellite launchers remain forward-looking.

Cooperative private debris removal: Profit maximizing cooperative satellite owners will demand debris removal until their marginal benefit from removal equals its marginal cost. Under open access to orbit, the first-order condition for an interior solution to the maximization problem in system of equations 47 is

$$R_{it} : c_t = \frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t} S_t F, \tag{54}$$

with the second-order condition

$$R_{it} : -\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S_t^2 F < 0. \tag{55}$$

Intuitively, open access removes any potential future benefit or cost from debris removal. Satellite owners will not get to reap any benefits from increasing Q_{it+1} because today's potential launchers will enter and capture them. Equation 54 therefore states that under open access launching, satellite owners will purchase debris removal until the price of a unit of removal (c_t) is equal to the static private marginal benefit ($\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t} S_t F$). That benefit has three pieces: the value of their satellite next period, F; the number of owners who will make the same removal decision, S_t; and the change in the probability that their satellite is destroyed at the end of the period, $\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}$.

Under open access firms launch until zero profits, while satellite owners remove debris until marginal benefits equal marginal costs. But current launchers are future satellite owners.

If they could not coordinate as launchers, how can they do so as satellite owners? The answer is property rights. International space law gives satellite launchers ownership of any objects they put into space, even after their useful life is over. As a result, satellite owners must either purchase or exercise rights to specific pieces of debris in order to remove them. This allows satellite owners to coordinate debris removal. I assume they do so in a cooperative and efficient manner to focus on the best-case scenario for active debris removal. I ignore both the complications of decentralized bargaining between many parties and the difficulties of attributing ownership to specific small pieces of debris. Transaction and information costs associated with debris removal are relevant to policy design and implementation, but beyond my scope here.

3.2.2 Exogenous debris removal for free and for a mandatory fee

To develop intuition for how debris removal can reduce the equilibrium collision risk, consider a setting where $\bar{R}$ units of debris are removed from orbit every period by a regulator. Such policies are advocated for by some in the space debris literature, for example Bradley and Wein (2009) and Akers (2012). If the removal is costly to satellite owners, it is because the regulator forces them to pay a fixed fee of $\bar{c}$ per unit removed. Denote the equilibrium collision risk with exogenous removal for a mandatory fee as $E_t^R[\ell_{t+1}]$, and the equilibrium collision risk with no removal as $E_t[\ell_{t+1}]$. The open access equilibrium condition for forward-looking launchers is then

$$\pi = rF + \tilde{E}_{t+1}^R[\ell_{t+1}]F + \bar{R}\bar{c}, \tag{56}$$

while in the absence of removal, firms would launch until

$$\pi = rF + \tilde{E}_{t+1}[\ell_{t+1}]F. \tag{57}$$

Inspecting the two equations above reveals that, as the mandatory removal fee approaches zero, the equilibrium collision risk with removal approaches the equilibrium collision risk without removal. Proposition 6 shows this formally.

Proposition 6. *(Satellite owners must pay for collision risk reduction) Any debris removal technology will reduce the equilibrium collision risk if and only if:*

1. *some amount of debris is removed, and*
2. *satellite owners pay for the removal.*

Proof. Let the equilibrium collision risk with debris removal be $\tilde{E}_{t+1}^R[\ell_{t+1}]$. The amount of debris removed per satellite owner is $\bar{R}$, and the per-unit cost to satellite owners is $\bar{c}$. From equation 56,

$$\tilde{E}_{t+1}^R[\ell_{t+1}] = r_s - r - \frac{\bar{R}\bar{c}}{F}. \tag{58}$$

From equation 9, the equilibrium collision risk without debris removal is[16]

$$\tilde{E}_{t+1}[\ell_{t+1}] = r_s - r. \tag{59}$$

If no debris is removed, $\bar{R} = 0$. If some debris is removed but satellite owners pay nothing for it, $\bar{c} = 0$. In either case, $\tilde{E}^R_{t+1}[\ell_{t+1}] = \tilde{E}_{t+1}[\ell_{t+1}]$. If and only if some debris is removed ($\bar{R} > 0$) and satellite owners pay something for it ($\bar{c} > 0$), $\tilde{E}^R_{t+1}[\ell_{t+1}] < \tilde{E}_{t+1}[\ell_{t+1}]$.

More generally, for any positive amount of debris removal, the equilibrium collision risk reduction is increasing in the amount that satellite owners pay for debris removal:

$$\forall \bar{R} > 0, \quad \tilde{E}_{t+1}[\ell_{t+1}] - \tilde{E}^R_{t+1}[\ell_{t+1}] = \frac{\bar{R}\bar{c}}{F}, \tag{60}$$

$$\frac{\partial(\tilde{E}_{t+1}[\ell_{t+1}] - \tilde{E}^R_{t+1}[\ell_{t+1}])}{\partial \bar{c}} = \frac{\bar{R}}{F} > 0. \tag{61}$$

As firms pay less and less for debris removal, the equilibrium collision risk with debris removal smoothly approaches the equilibrium collision risk without debris removal:

$$\lim_{\bar{c}\to 0} \tilde{E}^R_{t+1}[\ell_{t+1}] = \lim_{\bar{c}\to 0}(\pi - rF - \bar{R}\bar{c}) = \pi - rF = \tilde{E}_{t+1}[\ell_{t+1}]. \tag{62}$$

□

Since the debris stock will be lower due to removal, the launch rate will be higher with free exogenous removal than it would under open access with no removal. Economically, free removal clears up space for new launchers to enter the orbit. This case highlights the mechanism through which active debris removal can reduce the equilibrium collision risk: not by mechanically reducing the amount of debris in orbit, but by reducing the excess return of a satellite. This mechanism also acts in the case with endogenous debris removal, as shown in Proposition 7. Despite this mechanism, the launch rate may be larger with debris removal than without. While the reduction of excess return on a satellite will lower the launch rate, the reduction in debris will increase the launch rate.

This example also highlights the main reason why active debris removal can reduce collision risk: not because it removes debris, but because it approximates a stock control. This suggests that controls on debris removal could be more effective than flow controls on satellites at reducing collision risk. Figure 8 illustrates the differences between exogenous debris removal for free and for a mandatory fee. When satellite owners choose how much debris to remove, however, this type of approach must account for current satellite owners' and launchers'

[16]While the expectation in equation 9 looks slightly different from the expectation in equation 59, $\tilde{E}_{t+1}[\ell_{t+1}]$ is the same integral as $E_t[\ell_{t+1}]$. The difference in notation is described in equation 18.

responses to the price of removal. These responses are discussed in Propositions 5 and 13.

[Figure 8 about here.]

3.2.3 Endogenous debris removal financed by satellite owners

Figure 9 illustrates the effects of introducing active debris removal paid for by cooperative satellite owners. Unlike when removal is provided exogenously, endogenous removal can induce more firms to launch satellites before the technology becomes available. Despite the cost of cooperating with others and paying for removal, lower expected collision costs due to debris removal and lower individual contributions due to additional firms paying for removal makes it optimal for potential launchers to enter the orbit.

[Figure 9 about here.]

The introduction of debris removal technologies affects equilibrium orbital stocks as well as open access launch incentives. I explore the properties of cooperative private debris removal demands and open access launching further in the Appendix, section 6.7. Two of these - the uniqueness of the cooperatively-optimal post-removal debris stock and the potential for a "dynamic virtuous cycle" of debris removal - are driven by the incentives of satellite owners given open access. The third result describes intuitive physical and economic conditions under which the demand for satellite ownership by satellite launchers will be decreasing in the launch cost. Violations of these conditions may be plausible depending on the values of physical parameters.

3.2.4 Cooperative removal and open access risks

Ultimately, policymakers considering active debris removal technologies will want to know how debris removal will affect collision and Kessler Syndrome risks. In this section, I examine how active debris removal which is costly to satellite owners will change the equilibrium collision risk, the equilibrium future debris stock, and the equilibrium future probability of Kessler Syndrome. The results of this section, Propositions 7 and 8, establish that the use of active debris removal can reduce both equilibrium collision risk and the risk of Kessler Syndrome. While the risk of Kessler Syndrome can be mechanically reduced by removing debris no matter how removal is financed, reducing the equilibrium collision risk requires satellite owners to finance debris removal.

Proposition 7 extends Proposition 6 by considering the time path of collision risk when debris removal is introduced and when debris removal is ongoing. The intuition is similar to that of Proposition 6. Proposition 8 relies on some auxiliary properties of debris removal, shown in section 6.7 of the Appendix. The key intuition is that the cooperatively-optimal level of post-removal debris is a constant. As a result, even if there is an increase in the number of satellites due to debris removal, the risk of Kessler Syndrome will be reduced because firms will continuously purchase removal to keep the debris stock at its new, lower level.

Costly debris removal services can reduce the equilibrium collision risk: The removal of debris, all else equal, should reduce the collision rate. Whether it reduces the equilibrium collision risk depends on how potential satellite launchers respond to this reduction in risk. If debris removal spurs enough new entry, debris removal may result in higher collision rates. The logic seems plausible: if debris removal should reduce the collision rate, then more firms would be able to take advantage of the cleaner orbit and should therefore launch.

This logic would be correct but for an important detail: firms which launch satellites at t will become firms which own satellites at $t+1$. As they decide whether to launch or not, forward-looking firms account for their expected debris removal expenses as satellite owners. If the firm anticipates wanting to purchase debris removal services once its satellite is on orbit, and these services are costly, the introduction of the technology must reduce the excess return from launching the satellite. Since open access equates the expected collision risk with the excess return, the reduction in excess return also reduces the expected collision risk. However, if debris removal is introduced as a free service which potential launchers anticipate not paying for, the equilibrium collision risk will remain at the earlier open access level.

Proposition 7. *(ADR can reduce collision risk) The introduction of costly debris removal services in period t will reduce the equilibrium collision risk in t if and only if*

1. *it is costly to remove debris in t, and*
2. *it is privately optimal for cooperative satellite owners to remove some amount of debris in t.*

Ongoing active debris removal will reduce the equilibrium collision risk if and only if individual cooperative debris removal expenditures increase from period t to $t+1$. Formally,

$$E_{t-1}[\ell_t] - E_t[\ell_{t+1}] > 0 \iff c_{t+1}R_{it+1} > c_t R_{it}.$$

Proof. The proof of this result is similar to the proof of Proposition 6, so I omit it from the main text. See Appendix section 8. □

At first, the collision risk will decrease because of the expenditure that current launchers anticipate making once they are satellite owners. The fact that debris removal directly removes orbital debris is incidental to this risk reduction. As in the exogenous case, open access drives new launchers to take advantage of the newly-cleared space by launching satellites until the risk is the same as it was before removal because available. Subsidies for debris removal to satellite owners may increase the equilibrium amount of debris removed, but would not affect the equilibrium collision risk. Open access will still dissipate rents from orbit use; subsidized debris removal would only tilt the combination of new satellites and debris which equilibriates the system toward new satellites. Ongoing debris removal can only keep the collision rate below the no-ADR open access collision rate if and only if it is costly to potential launchers. If potential launchers anticipate that the cost will reduce, or that it will not be optimal to purchase removal as satellite owners, the collision rate will return to the no-ADR open access level.

Cooperative costly debris removal will reduce the equilibrium probability of Kessler syndrome: Since preventing Kessler Syndrome is one of the key motivations for developing active debris removal technologies, it is natural to wonder if debris removal will achieve this goal. Since Kessler Syndrome is caused by the amount of debris exceeding a threshold, debris removal in t will reduce the probability of Kessler Syndrome in $t+1$ if it is certain to reduce the $t+1$ debris stock. More precisely, the change in probability of Kessler Syndrome in $t+1$ is equal to the probability that the change in the $t+1$ debris stock due to removal in t is positive, plus the product of the change in expected collision risk due to removal and the original probability of Kessler Syndrome. Since the change in the $t+1$ debris stock due to removal is negative with probability one, debris removal will reduce future probability of Kessler Syndrome.

Proposition 8. *(Debris removal will reduce the future probability of Kessler Syndrome) Debris removal in t will reduce the probability of Kessler Syndrome in* $t+1$.

Proof. Kessler Syndrome will occur in $t+1$ when

$$D_{t+1} - D^{\kappa} > 0, \tag{63}$$

where D^{κ} is the Kessler threshold. Suppose that Kessler Syndrome has not already occurred ($D_t - D^{\kappa} < 0$). Under the probability density for satellite-destroying collisions in t ($\phi(\ell_t|S_t, D_t - R_t)$), the probability in t of Kessler Syndrome in $t+1$ is

$$Pr_t(D_{t+1} - D^{\kappa} > 0|S_t, D_t - R_t) = \int_0^1 \mathbb{1}(D_{t+1}(\ell_t) - D^{\kappa} > 0)\phi(\ell_t|S_t, D_t - R_t)d\ell_t. \tag{64}$$

The change in this probability due to an increase in R_t is

$$\frac{dPr_t(D_{t+1}(\ell_t)-D^\kappa>0|S_t,D_t-R_t)}{dR_t} = \frac{\partial}{\partial R_t}\int_0^1 \mathbb{1}(D_{t+1}(\ell_t)-D^\kappa>0)\phi(\ell_t|S_t,D_t-R_t)d\ell_t \tag{65}$$

$$= \int_0^1 \mathbb{1}(\frac{\partial D_{t+1}(\ell_t)}{\partial R_t}>0)\phi(\ell_t|S_t,D_t-R_t)d\ell_t + \int_0^1 \mathbb{1}(D_{t+1}(\ell_t)-D^\kappa>0)\frac{\partial\phi(\ell_t|S_t,D_t-R_t)}{\partial R_t}d\ell_t \tag{66}$$

$$= Pr_t\left(\frac{\partial D_{t+1}(\ell_t)}{\partial R_t}>0|S_t,D_t-R_t\right) - \left.\frac{\partial Pr_t(D_{t+1}(\ell_t)-D^\kappa>0|S_t,\tilde{D}-R_t)}{\partial\tilde{D}}\right|_{\tilde{D}=D_t}. \tag{67}$$

The first term in equation 67 is zero because debris removal reduces the future debris stock for any draw of the collision rate, and the form of the second term in equation 67 follows from Lemma 6. To see that the first term is zero, define the open access launch rate as an implicit function $X_t = X(S_t,D_t-R_t,\ell_t)$ defined by equation 52. Then, differentiate D_{t+1} with respect to R_t:

$$D_{t+1}(\ell_t) = (D_t-R_t)(1-\delta)+G(S_t,D_t-R_t,\ell_t)+mX_t$$
$$\implies \frac{\partial D_{t+1}(\ell_t)}{\partial R_t} = -\left[1-\delta+\frac{\partial G(S_t,D_t-R_t,\ell_t)}{\partial D_t}+m\frac{\partial X_t}{\partial D_t}\right].$$

From Proposition 11 and applying the Implicit Function Theorem to equation 52,

$$\frac{\partial X_t}{\partial D_t} = -\frac{\frac{\partial R_{it+1}}{\partial D_{t+1}}(1-\delta+\frac{\partial G(S_t,D_t-R_t,\ell_t)}{\partial D_t})(1-\frac{\partial R_t}{\partial D_t})}{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}F+m\left(\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}F+\frac{\partial R_{it+1}}{\partial D_{t+1}}c_{t+1}\right)+\frac{\partial R_{it+1}}{\partial S_{t+1}}c_{t+1}}$$
$$= 0 \text{ whenever } R_{it}>0 \because \frac{\partial R_t}{\partial D_t}=1 \text{ from Proposition 11}.$$

Therefore,

$$\frac{\partial D_{t+1}(\ell_t)}{\partial R_t} = -\left[1-\delta+\frac{\partial G(S_t,D_t-R_t,\ell_t)}{\partial D_t}\right]<0 \;\; \forall \ell_t\in[0,1].$$

Since the statement holds for all possible realizations of ℓ_t,

$$Pr_t\left(\frac{\partial D_{t+1}(\ell_t)}{\partial R_t}>0|S_t,D_t-R_t\right)=0.$$

The change in the probability of Kessler Syndrome due to a change in debris removal is then

$$\frac{dPr_t(D_{t+1}(\ell_t) - D^\kappa > 0|S_t, D_t - R_t)}{dR_t} = -\left.\frac{\partial Pr_t(D_{t+1}(\ell_t) - D^\kappa > 0|S_t, \tilde{D} - R_t)}{\partial \tilde{D}}\right|_{\tilde{D}=D_t}. \quad (68)$$

The right hand side of equation 68 is the negative of the change in the probability of Kessler Syndrome from the shift in the distribution of collision rates which a marginal amount of debris would cause. It is not precisely the same as the effect of another unit of debris, since the debris argument of $D_{t+1}(\ell_t)$ is held constant while the debris argument of $\phi(\ell_t|S_t, D_t - R_t)$ is increased slightly. By Assumption 1, increasing the amount of debris in orbit will shift the conditional density of the collision rate toward 1. The fact that $\mathbb{1}(D_{t+1}(\ell_t) - D^\kappa > 0)$ is at least weakly increasing in ℓ_t, combined with Lemma 8, the change in probability must be at least weakly positive. So, debris removal must reduce the probability of Kessler Syndrome:

$$\left.\frac{\partial Pr_t(D_{t+1}(\ell_t) - D^\kappa > 0|S_t, \tilde{D} - R_t)}{\partial \tilde{D}}\right|_{\tilde{D}=D_t} \geq 0 \quad (69)$$

$$\implies \frac{dPr_t(D_{t+1}(\ell_t) - D^\kappa > 0|S_t, D_t - R_t)}{dR_t} \leq 0. \quad (70)$$

□

Overall, debris removal technologies financed by satellite owners will make orbits safer, reducing both equilibrium collision risk and the risk of Kessler Syndrome. These gains in safety come from satellite owners financing the debris removal. Subsidized or publicly provided debris removal cannot reduce the equilibrium collision risk. Though it may reduce the equilibrium debris stock, subsidized or publicly provided debris removal may not reduce the equilibrium risk of Kessler Syndrome unless the agency providing debris removal commits to preventing the debris stock from exceeding a fixed level. Cooperative removal was shown to achieve this in Proposition 11. The value of debris removal technologies depends critically on the economic institutions under which the technologies are used.

3.2.5 Optimal removal and launch plans

Finally, I consider the jointly-optimal debris removal and satellite launch plans to compare with the cooperative removal and open access launch plans. The main point of this section is to show that even with cooperative debris removal financed by satellite owners, open access launching is not socially optimal. Proposition 10 establishes that a constrained planner cannot improve on the cooperative removal plan given open access, so this section is an exercise in determining how large a distortion open access launching creates. The unconstrained fleet planner coordinates removals and launches, taking advantage of the fact that they will be able to remove any unwanted debris before the collision rate is drawn. Their problem at the start of period t is

$$\begin{aligned}
\tilde{W}(S_t, D_t) &= \max_{R_t \in [0, D_t]} \{-c_t R_t + \tilde{E}_t[W(S_t, D_t - R_t, \ell_t)]\} \qquad (71)\\
\text{s.t. } W(S_t, D_t - R_t, \ell_t) &= \max_{X_t \geq 0} \{\pi S_t - F X_t + \beta \tilde{W}(S_{t+1}, D_{t+1})\}\\
\ell_t &\sim \phi(\ell_t | S_t, D_t - R_t)\\
S_{t+1} &= S_t(1 - \ell_t) + X_t\\
D_{t+1} &= (D_t - R_t)(1 - \delta) + G(S_t, D_t - R_t, \ell_t) + m X_t.
\end{aligned}$$

The planner faces the same timing of information as firms do: at the beginning of a period, before ℓ_t has been revealed, they choose how much debris they will remove. Based on their removal decision, the draw of ℓ_t is revealed. Then they decide how much they will launch. The program in system of equations 71 shows this decision-making process at the beginning of a period. Their jointly-optimal removal and launch plans must equate the social marginal costs and benefits of removing debris before ℓ_t is known and of launching satellites once ℓ_t is known. Formally,

$$R_t^* : c_t = -\left\{ \tilde{E}_t \left[\frac{\partial W(S_t, D_t - R_t, \ell_t)}{\partial D_t} \right] + \left. \frac{\partial \tilde{E}_t[W(S_t, D_t - R_t, \ell_t | S_t, D - R_t)]}{\partial D} \right|_{D = D_t} \right\} \qquad (72)$$

$$X_t^* : \frac{F}{\beta} = \frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial S_{t+1}} + m \frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial D_{t+1}} \qquad (73)$$

An optimal removal plan exists if the sum of the objects inside the curly brackets on the right side of equation 72 is positive. I assume that the marginal post-removal value of debris is negative ($\frac{\partial W(S_t, D_t - R_t, \ell_t)}{\partial D_t} < 0$) along optimal paths, making both terms on the right hand side individually negative. The negativity of the second term follows from Lemma 8. This is sufficient to make the right side of equation 72 positive.

An optimal launch plan exists if the sum of the objects on the right side of equation 73 is positive. I assume that along optimal paths, the marginal pre-removal value from another satellite is positive and the marginal pre-removal value from another piece of launch debris is negative (a loss). I also assume that the pre-removal gain from another satellite is larger than the pre-removal loss from another piece of launch debris ($\frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial S_{t+1}} \geq 0$, $\frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial D_{t+1}} < 0$, $\frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial S_{t+1}} > -m \frac{\partial \tilde{W}(S_{t+1}, D_{t+1})}{\partial D_{t+1}}$).

To compare the qualitative properties of optimal removal and launch plans with cooperative removal and open access launch plans, I simulate cases of the cooperative removal and open access launch plans (system of equations 48) and the optimal removal and launch plan (system of equations 71). Figure 10 shows these plans and the associated value functions.

[Figure 10 about here.]

Comparing Figure 10 with Figure 3 shows that the open access launch plan with debris removal is similar to the plan without debris removal given cooperative debris removal. With removal, however, there is a jump in the launch rate just as it becomes optimal for cooperative satellite owners to begin removing debris. This jump is shown in the time paths in Figure 9. This is because debris removal by incumbent satellite owners allows new firms to enter the orbit. Since the planner keeps the debris stock at a constant level as soon as the fleet value justifies it, they ignore debris while launching. More formally, controlling both satellite launches and debris removal allow the optimal policies to be piecewise-concave in satellites and debris.

The cooperative debris removal plan and the planner's removal plan are both corner solutions once debris removal starts[17]. The planner, however, begins debris removal with fewer satellites than the cooperative firms. Intuitively, the planner starts removing debris once the fleet is valuable enough to justify removal, while cooperative satellite owners start removing debris once there are enough owners sharing the removal costs to justify removal.

The discontinuity in the open access launch plan, its dependence on the debris stock, and the later start in the cooperative debris removal plan all reduce the open access-cooperative fleet value relative to the fleet planner's. The value loss from open access launching and cooperative debris removal follows the launch plan deviation and is intensified along the removal plan deviation. The gap is maximized just before open access launchers, anticipating removal, begin to launch again. At that point, the planner would have stopped launching and have begun removing debris while cooperative satellite owners would still be waiting for more contributors.

4 Conclusion

In this paper I showed how principles of economics should guide our stewardship of orbital resources. I established the equivalence of price and quantity instruments for orbital management and showed why space traffic controls should target satellite ownership rather than satellite launches. I considered the impacts of using active debris removal technology, and showed why, to reduce equilibrium collision risk, satellite owners must pay for debris removal.

[17] See Appendix section 6.5 for more details on nonconvexities and corner solutions in debris removal.

Along the way I derived practically-useful results about orbit use management under physical uncertainty with and without active debris removal. These include how to use stock and flow space traffic controls, the fact that debris removal can induce more launches no matter how it is financed, and the possibility that the open access launch rate may be increasing in the launch cost. I also examined the effects of indirect orbit control through spectrum regulation or mandatory satellite insurance. These policies approximate stock controls and are potential avenues by which regulators can induce first-best orbit use.

Knowing these details will help regulators manage orbit use effectively. However, questions will only grow as humans develop a larger presence in space. Commercial satellite operators are increasingly using many small satellites arrayed in constellations to deliver services. How should satellite constellations be regulated? International agreements will be necessary to regulate orbit use and minimize leakages, but different nations have different interests. What kinds of international orbit use management agreements are incentive-compatible? Militaries are among the most prominent orbit users, and have objectives which may conflict with commercial operators or each other. How can strategic orbit use by militaries be efficiently managed without compromising national and international security objectives? These are all important directions for future research.

Satellites are important to the modern world. We depend on satellite telecommunications to reach remote parts of the planet, enabling telemedicine and timely rescue efforts. We depend on GPS for navigation, and will rely on it more as automated transportation infrastructure develops. We depend on satellite imagery to determine the extent of natural disasters and optimize our responses to them, which climate change will only make more necessary. Economists are well-positioned to apply and develop the lessons of this paper, preventing a tragedy of the orbital commons and enabling a new wave of economic growth.

References

Adilov, Nodir, Peter J. Alexander, and Brendan M. Cunningham. 2015. "Earth Orbit Debris: An Economic Model." *Environmental and Resource Economics* 60:81–98.

Akers, Agatha. 2012. "To Infinity and Beyond: Orbital Space Debris and How to Clean It Up." *University of La Verne Law Review* 33.

Aldy, Joseph E, Todd D Gerarden, and Richard L Sweeney. 2018. "Investment versus Output Subsidies: Implications of Alternative Incentives for Wind Energy." Working Paper 24378, National Bureau of Economic Research. URL http://www.nber.org/papers/w24378.

Ansdell, Megan. 2010. "Active Space Debris Removal: Needs, Implications, and Recommendations for Today's Geopolitical Environment." *Journal of Public & International Affairs* 21:7–22.

Barrett, Scott. 2005. *The Theory of International Environmental Agreements*. Elsevier B.V., 1458–1512.

———. 2013. "Climate treaties and approaching catastrophes." *Journal of Environmental Economics and Management* 66:235–250.

Böhringer, Christoph, Knut Einar Rosendahl, and Halvor Briseid Storrøsten. 2017. "Robust policies to mitigate carbon leakage." *Journal of Public Economics* 149:35–46.

Bradley, Andrew M. and Lawrence M. Wein. 2009. "Space debris: Assessing risk and responsibility." *Advances in Space Research* 43:1372–1390.

Briggs, R.J. 2011. "Prices vs. quantities in a dynamic problem: Externalities from resource extraction." *Resource and Energy Economics* 33:843–854.

Carroll, J. 2009. "Bounties for orbital debris threat reduction." NASA-DARPA International Conference on Orbital Debris Removal.

Chow, C-S and John N Tsitsiklis. 1991. "An optimal one-way multigrid algorithm for discrete-time stochastic control." *IEEE transactions on automatic control* 36 (8):898–914.

Dvorsky, George. 2018. "California Startup Accused of Launching Unauthorized Satellites Into Orbit: Report." *Gizmodo* .

Fischer, Carolyn and Alan K. Fox. 2012. "Comparing policies to combat emissions leakage: Border carbon adjustments versus rebates." *Journal of Environmental Economics and Management* 64:199–216.

Fowlie, Meredith L. 2009. "Incomplete Environmental Regulation, Imperfect Competition, and Emissions Leakage." *American Economic Journal: Economic Policy* 1:72–112.

Jehn, R., V. Agapov, and C. Hernandez. 2005. "The situation in the geostationary ring." *Advances in Space Research* 35:1318–1327.

Johnson, Nicholas L. 2004. "Space traffic management concepts and practices." *Acta Astronautica* 55 (3-9):803–809.

Jones, Noelle, Jan Skora, Calvin Monson, Robert W Jones, Stuart Jack, and Kurt Eby. 2010. "Study on the Global Practices for Assigning Satellite Licences and Other Elements." Tech. rep.

Keohane, Nathaniel, Benjamin Van Roy, and Richard Zeckhauser. 2007. "Managing the quality of a resource with stock and flow controls." *Journal of Public Economics* 91:541–569.

Kessler, Donald J and Burton G Cour-Palais. 1978. "Collision Frequency of Artificial Satellites: The Creation of a Debris Belt." *Journal of Geophysical Research* 83:2637–2646.

Klinkrad, H. and N. L. Johnson. 2009. "Space Debris Environent Remediation Concepts." In *Fifth European Conference on Space Debris*, *ESA Special Publication*, vol. 672. 50.

Letizia, F., S. Lemmens, and H. Krag. 2018. "Application of a debris index for global evaluation of mitigation strategies." 69th International Astronautical Congress.

Letizia, Francesca, Camilla Colombo, Hugh Lewis, and Holger Krag. 2017. "Extending the ECOB space debris index with fragmentation risk estimation." .

Liou, J.C. 2006. "A statistical analysis of the future debris environment." *Acta Astronautica* 62:264–271.

Liou, J.C. and N.L. Johnson. 2008. "Instability of the present LEO satellite populations." *Advances in Space Research* 41:1046–1053.

———. 2009a. "Characterization of the catalogued Fengyun-1C fragments and their long-term effect on the LEO environment." *Advances in Space Research* 43:1407–1415.

———. 2009b. "A sensitivity study of the effectiveness of active debris removal in LEO." *Acta Astronautica* 64:236–243.

Macauley, Molly K. 1998. "Allocation of Orbit and Spectrum Resources for Regional Communications: What's at Stake?" *Journal of Law and Economics* 41:737–764.

———. 2015. "The economics of space debris: Estimating the costs and benefits of debris mitigation." *Acta Astronautica* 115:160–164.

Merges, Robert P. and Glenn H. Reynolds. 2010. "Rules of the Road for Space?: Satellite Collisions and the Inadequacy of Current Space Law." *The Environmental Law Reporter (ELR) News&Analysis* 40.

NASA Orbital Debris Program Office. 2017. *Orbital Debris Quarterly News* 21. From https://www.orbitaldebris.jsc.nasa.gov/quarterly-news/pdfs/odqnv21i2.pdf.

Newell, Richard, William Pizer, and Jiangfeng Zhang. 2005. "Managing permit markets to stabilize prices." *Environmental and Resource Economics* 31 (2):133–157.

Newell, Richard G. and William A. Pizer. 2003. "Regulating stock externalities under uncertainty." *Journal of Environmental Economics and Management* 45:416–432.

Parish, Ross M and Keith Robert McLaren. 1982. "Relative cost-effectiveness of input and output subsidies." *Australian Journal of Agricultural Economics* 26 (1):1–13.

Pearson, J., J. Carroll, and E. Levin. 2010. "Active Debris Removal: EDDE, the ElectroDynamic Debris Eliminator." *Space Manufacturing* 14.

Rao, Akhil and Giacomo Rondina. 2018. "Cost in Space: Debris and Collision Risk in the Orbital Commons." Latest draft available at https://akhilrao.github.io/assets/working_papers/Cost_in_Space.pdf.

Rossi, A., L. Anselmo, A. Cordelli, P. Farinella, and C. Pardini. 1998. "Modelling the evolution of the space debris population." *Planetary Space Science* 46:1583–1596.

Salter, Alexander William. 2015. "Space Debris: A Law and Economics Analysis of the Orbital Commons." *Mercatus Working Paper* .

Salter, Alexander William and Peter T Leeson. 2014. "Celestial Anarchy." *Cato Journal* 34.

Sandmo, Agnar. 1978. "Direct versus indirect Pigovian taxation." *European Economic Review* 7 (4):337–349.

Schaub, Hanspeter, Lee E.Z. Jasper, Paul V. Anderson, and Darren S. McKnight. 2015. "Cost and risk assessment for spacecraft operation decisions caused by the space debris environment." *Acta Astronautica* 113:66–79.

Senechal, Thierry. 2007. "Orbital Debris: Drafting, Negotiating, Implementing a Convention." *Masters Thesis* .

Shimabukuro, Alessandro. 2014. "No deal in space: A bargaining model analysis of U.S. resistance to space arms control." *Space Policy* 30:13–22.

Svensson, Lars EO. 2003. "What is wrong with Taylor rules? Using judgment in monetary policy through targeting rules." *Journal of Economic Literature* 41 (2):426–477.

Sweeney, James L. 1993. "Economic Theory of Depletable Resources: an Introduction." Elsevier, 759 – 854. URL http://www.sciencedirect.com/science/article/pii/S1573443905800041.

Telser, L.G. 1980. "A Theory of Self-Enforcing Agreements." *Journal of Business* 53:27–44.

Tkatchova, Stella. 2018. *Space Debris Mitigation*. Berlin, Heidelberg: Springer Berlin Heidelberg, 93–105. URL https://doi.org/10.1007/978-3-662-55669-6_6.

Union of Concerned Scientists. 2018. "UCS Satellite Database." From UCS at https://www.ucsusa.org/nuclear-weapons/space-weapons/satellite-database.

Weeden, Brian C. 2010. "Overview of the Legal and Policy Challenges of Orbital Debris Removal." Tech. Rep. IAC-10.A6.2.10.

Weeden, Brian C and Tiffany Chow. 2012. "Taking a common-pool resources approach to space sustainability: A framework and potential policies." *Space Policy* 28:166–172.

Weitzman, Martin L. 1974. "Prices vs. Quantities." *Review of Economic Studies* 41 (4):477–491.

5 Appendix A: Institutional details of orbit use

5.1 International laws regarding space traffic control

Orbits are inherently global resources, and space law is fragmented across nations and documents. Space law spans domestic policies, international treaties, bilateral agreements, and guidelines. Not all agreements are signed by all spacefaring nations, and many are non-binding. Most of the agreements are vague and suffer from enforcement problems. Four of the most relevant international agreements relating to orbit management are the 1967 Outer Space Treaty, the 1972 Liability Convention, the 1975 Registration Convention, and the 2007 COPUOS Guidelines.[18]

1967 Outer Space Treaty The Outer Space Treaty[19] established the legal framework for peaceful uses of outer space. Article 2 of the Treaty designates outer and orbital space as common pool resources, to be used "for the benefit of all" humankind. The only explicit restrictions are on military uses and claims of national sovereignty; the state of resource use is left ambiguous. The Treaty does not mention debris, only stating that nations should avoid causing (undefined) "harmful contamination" of outer space.

1972 Liability Convention The Liability Convention[20] established the framework for tort law of space activities. However, the Convention focused more on damage to terrestrial objects from re-entry than on damages to orbital objects which occur in space. "Damage" in this Convention is defined only in relation to realized outcomes for people and property, rather than potential outcomes caused by the environment. Additionally, the Convention places liability for such damages on the launching state rather than the launching entity. This has motivated nations like the US to require satellite owners insure their satellites, with the federal government indemnifying losses beyond a certain amount. The EU has different insurance requirements, with a similar motivation. There is no liability attached to producing debris in orbit, only to attributable damages. Liability extends to damage to people or property caused by re-entry. Such attribution is difficult in space, where damages may be caused by difficult-to-detect fragments of unknown origin.

1975 Registration Convention The Registration Convention[21] requires nations to register space objects launched from or by that nation with the UN Secretary-General. The responsibility for ensuring compliance lies with the launching state, with the UN being responsible for integrating all the registrations and publishing a publicly available international registry of

[18] A more detailed analysis of these laws can be found in Akers (2012).

[19] The "Treaty on Principles Governing the Activities of States in the Exploration and Use of Outer Space, Including the Moon and Other Celestial Bodies."

[20] The "Convention on International Liability for Damage Caused by Space Objects."

[21] The "Convention on Registration of Objects Launched into Outer Space."

objects in orbit. The Convention only requires basic orbital information to be provided: orbital parameters to ascertain the object's initial path, and the general function. It does not require more detailed information, such as orbit changes or satellite positions, or even continuous updates. The Convention does not offer a deadline by which a launched object must be registered, or specify a penalty or enforcement mechanism for noncompliance.

2007 COPUOS Guidelines The COPUOS Guidelines[22] are seven nonbinding guidelines for mitigating artificial space debris. This was the first international treaty to recognize the problem of orbital debris, though it is unenforceable and only focused on technical mitigation practices rather than economic control measures. Senechal (2007) discusses some features that an enforceable international space debris convention should possess; the COPUOS guidelines are a step in this direction, but they do not contain the kinds of clear definitions and enforceable provisions required.

In general, the legal doctrine of *nemo dat quod non habet* ("no one gives what he doesn't have") means that the Outer Space Treaty prevents states from issuing rights over orbital paths. Since they lack sovereignty in space, states do not have rights to give[23]. States retain their authority over launches within their borders, and satellites operated by firms within their borders.

5.2 US institutions regarding space traffic control

International law places responsibility for objects launched to space on the nations from which the objects are launched and the nations in which the launching entities are registered. This means that understanding the legal status of space objects also requires some background on national laws. A majority of currently-operational satellites were launched from the United States, Russia, and China. In this section, I focus on institutions in the United States.

In the US, the Federal Aviation Administration's Office of Commercial Space Launch handles issues related to launches and reentry, including issuing launch permits. The Department of Commerce currently regulates remote sensing satellite systems, with a focus on controlling the resolution and coverage of images that are sold. The DOC is set to take control of regulating

[22] The "Committee on the Peaceful Uses of Outer Space (COPUOS) 2007 Nonbinding Guidelines for Space Debris Mitigation."

[23] Salter and Leeson (2014) argue that this poses no difficulty to efficient decentralized orbit use management, as privately enforced property rights have arisen in other common resource settings on Earth. Weeden and Chow (2012) offer suggestions guided by Ostrom's principles of commons management for developing decentralized protocols for orbit use. An interesting question, not pursued in detail here, is the degree to which physical dynamics allow cooperative mechanisms to operate. Section 6.3 in the Appendix examines the stability of cooperation with debris removal plans.

on-orbit activities by US entities over the next two years. The FCC regulates satellite activities by US telecom entities through radio spectrum controls. Traffic management operations so far have therefore been limited to controlling launches or operations by entities primarily based within a country's national borders - for example, the FCC can use its control of spectrum rights to deny service to poorly-behaving telecom operators who want to provide service to North America, but have no leverage over providers who are interested in serving China. The patchwork of laws around the world has already led some firms to evade regulation in their home area by launching from another (for example, Dvorsky (2018)). Space Policy Directive-3, a presidential memorandum issued in June of 2018, directs federal agencies to cooperate in developing a framework for a national space traffic management policy centered on a new object tracking infrastructure to better predict the aggregate collision rate and coordinate collision avoidance maneuvers.

5.3 The militarization of space

Military use of space accounts for over 10% of known active satellites in orbit Union of Concerned Scientists (2018). Although I focus on commercial orbit use with no reference to military use, some understanding of military incentives is necessary to place military use in the appropriate context. Sweeney (1993) describes some of generic logic for the effects of national security concerns on nationally optimal depletable resource extraction, whereby the socially optimal extraction rate may exceed the privately optimal extraction rate. Similarly, national security concerns may drive a national fleet planner to launch more than their national space sector would.[24]

Military uses of space have been among the largest sources of new debris fragments, particularly anti-satellite missile testing Liou and Johnson (2009a); Bradley and Wein (2009). Anti-satellite missile uses can trigger Kessler Syndrome. The space traffic control policies described here are not designed to control anti-satellite missile use. International agreements around space traffic control in the future are likely to be shaped in large part by the military stances of major space-faring nations, most notably the United States, Russia, and China. The current lack of binding international space traffic agreements is partly due to a lack of science and consensus around how such controls should be designed and why, but also partly due to military-related incentives facing the governments of space-faring nations which would be party to such agreements.

[24] The depletable resource case may be more apt than the renewable resource one for some orbital regimes, particularly higher-altitude ones such as GEO or high-Earth orbits. Natural rates of orbital decay in these regimes can be on the order of millennia - long enough that Kessler Syndrome without removal technologies can render them effectively unusable (the orbital volume "completely extracted") for economic purposes.

Article 4 of the Outer Space Treaty declares that state parties "undertake not to place in orbit around the Earth any objects carrying nuclear weapons or any other kinds of weapons of mass destruction, install such weapons on celestial bodies, or station such weapons in outer space in any manner." It also forbids establishing military installations, conducting weapons tests, or any other non-peaceful activities on the Moon and other celestial bodies. Despite these provisions, the Outer Space Treaty does not explicitly prohibit using near-Earth space for reconnaissance, terrestrial warfare coordination, or even outright conflict so long as "weapons of mass destruction" are not used in orbit. The ongoing militarization of space has therefore involved these uses, with the US government being the largest such user of orbital space. The US government has not yet supported international treaty efforts to limit the militarization of space. Shimabukuro (2014) offers an explanation for the lack of more international regulation on space militarization in light of rising tensions between the US and China. The core of Shimabukuro's explanation is that the US benefits from high ambiguity over acceptable uses of outer space given the US's high level of military dependence on space systems. This allows the US to induce its adversaries to spend ever-increasing amounts on developing comparable space capabilities, potentially collapsing their warfighting capabilities without a battle.

6 Appendix B: Additional economic properties of open access orbit use, space traffic control policies, and active debris removal

6.1 Open access launch response to collisions

Lemma 2. *(Open access launch response to collision risk draws) The open access launch rate in t can be non-monotonic in the realized collision risk in t.*

Proof. From equation 9,

$$\mathscr{F} = r_s - r - E_t[\ell_{t+1}] = 0. \tag{74}$$

Applying the Implicit Function Theorem to $\mathscr{F}$,

$$\frac{\partial X_t}{\partial \ell_t} = -\frac{\partial \mathscr{F}/\partial \ell_t}{\partial \mathscr{F}/\partial X_t} \tag{75}$$

$$= \frac{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} S_t + \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}} \frac{\partial G(S_t, D_t, \ell_t)}{\partial \ell_t}}{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} + m \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}} \lesseqgtr 0 \tag{76}$$

□

Figure 11 illustrates three cases of Lemma 2: one where the open access launch rate is first decreasing and then increasing in the satellite-destroying collision risk draw, another where it is uniformly decreasing in the collision risk draw, and a third where it is uniformly increasing in the collision risk draw. There are two competing effects of collisions driving this behavior: collisions generate debris, but collisions also remove other satellites from orbit.

In the first case, the effect of additional debris dominates the launch decision when the collision risk draw is low, and the effect of fewer satellites dominates when the collision risk draw is high. In the second case shown in Figure 11, the new-debris effect dominates for all draws. In the third case shown in Figure 11, the fewer-satellites effect dominates for all draws. The third case is the least-realistic, as it implies that the number of fragments from a satellite destruction is tiny compared to the number of fragments created by a debris-debris collision. Debris modeling studies such as Liou (2006); Letizia et al. (2017) find the opposite: satellite destructions contribute much more debris to the orbital environment than collisions between debris fragments. The first and second cases are plausible under realistic relative orders of magnitude between the number of fragments created by satellite-satellite, satellite-debris, and debris-debris collisions.

[Figure 11 about here.]

6.2 Two details for flow control implementations

Lemma 3. *(A feature of flow controls with no end date) If the discount rate is positive, the price of a time-consistent infinite-horizon flow control is an exploding process.*

Proof. From equation 24, we can write the price in period $t+1$ as a function of the price in period t as

$$p_{t+1} = \frac{1+r}{1-E_t[\ell_{t+1}]} p_t - \frac{\pi - rF - E_t[\ell_{t+1}]F}{1-E_t[\ell_{t+1}]}. \tag{77}$$

Differentiating with respect to p_t,

$$\frac{\partial p_{t+1}}{\partial p_t} = \frac{1+r}{1-E_t[\ell_{t+1}]} > 1 \;\; \forall r > 0. \tag{78}$$

This holds along a generic equilibrium path as shown above, or along an optimal path. Along an optimal path, the numerator of the second term on the right-hand side would be replaced with the marginal external cost of a satellite in period $t+1$. Since the first term on the right-hand side is unchanged, the flow control process still has a unit or greater-than-unit root. □

Lemma 3 is likely to be practically relevant to designing a flow control policy, even though in theory such a control could be made optimal.[25]

[25] Though it seems unlikely to me, perhaps there exists or could exist a regulatory body with the credibility to commit to an exploding price path for the foreseeable future; they would find this result irrelevant.

Lemma 4. *(A limitation of flow controls) If the expected future loss rate is one, there is no value of the future flow control price which can affect the current launch rate.*

Proof. Rewriting equation 24 with $E_t[\ell_{t+1}] = 1$, the launch rate will satisfy

$$X_t : \pi = rF + E_t[\ell_{t+1}]F + (1+r)p_t. \tag{79}$$

Since equation 79 no longer contains p_{t+1}, the regulator cannot use it to affect X_t. □

I present this result for completeness, though it is likely a moot point. The destruction of all active satellites in an orbit would likely trigger Kessler Syndrome there. The regulator and the space industry would then have bigger problems than whether or not a flow control in line with the pre-committed path exists. This may be relevant for low orbits where debris decays sufficiently quickly. If such destruction occurred in those orbits intentionally, for example missile tests or conflict, the regulator and space industry would again have bigger problems than deviation from prior commitments.

6.3 Incentives to cooperate in debris removal

Suppose all satellite owners agree to cooperate and individually purchase R^*_{it} units of debris removal. Owner i considers deviating and reducing her removal demands by $\varepsilon \in (0, R^*_{it}]$. Her payoff from not deviating by ε is

$$\underbrace{\pi - c_t R^*_{it} + (1 - \tilde{E}_t[\ell_t | S_t, D_t - R^*_t])F}_{\text{Expected value of cooperating}} - \underbrace{[\pi - c_t(R^*_{it} - \varepsilon) + (1 - \tilde{E}_t[\ell_t | S_t, D_t - (R^*_t - \varepsilon)])F]}_{\text{Expected value of deviating by } \varepsilon}$$

$$= -\varepsilon c_t + [\tilde{E}_t[\ell_t | S_t, D_t + \varepsilon - R^*_t] - \tilde{E}_t[\ell_t | S_t, D_t - R^*_t]].$$

Cooperation is a strictly dominant Nash equilibrium if and only if

$$\tilde{E}_t[\ell_t | S_t, D_t + \varepsilon - R^*_t] - \tilde{E}_t[\ell_t | S_t, D_t - R^*_t] > \varepsilon \frac{c_t}{F} \quad \forall \varepsilon \in (0, R^*_{it}]. \tag{80}$$

Proposition 9 establishes an intuitive necessary and sufficient condition for cooperation to strictly dominate small deviations. If the change in the expected loss rate before removal is greater than the ratio of the cost of removal in t to the cost of launching a satellite in t, then a tiny deviation will cost a satellite owner more expected value through a lower survival rate than it will yield in a removal expenditure savings.

Proposition 9. *(Local stability of cooperation) Cooperation with any non-zero debris removal plan strictly dominates small deviations if the change in the equilibrium collision risk from another unit of debris is greater than the ratio of the removal price to the launch cost,*

$$\left. \frac{\partial \tilde{E}[\ell_t | S_t, \bar{D} - R_t]}{\partial \bar{D}} \right|_{\bar{D} = D_t} > \frac{c_t}{F}.$$

Proof. Cooperation with a non-zero debris removal plan is robust to all deviations ε for which

$$\tilde{E}_t[\ell_t|S_t, D_t + \varepsilon - R_t^*] - \tilde{E}_t[\ell_t|S_t, D_t - R_t^*] > \varepsilon \frac{c_t}{F}.$$

Cooperation with a non-zero debris removal plan strictly dominates small deviations if

$$\lim_{\varepsilon \to 0} \frac{\tilde{E}_t[\ell_t|S_t, D_t + \varepsilon - R_t] - \tilde{E}_t[\ell_t|S_t, D_t - R_t]}{\varepsilon} > \frac{c_t}{F} \quad \forall R_t > 0$$
$$\implies \frac{\partial \tilde{E}[\ell_t|S_t, \bar{D} - R_t]}{\partial \bar{D}}\bigg|_{\bar{D}=D_t} > \frac{c_t}{F} \quad \forall R_t > 0.$$

□

The following proposition establishes that the debris removal solution described in equation 54 is in fact the cooperative private debris removal solution.

Proposition 10. *(A cooperative private removal plan) The debris removal solution described by equation 54 is maximizes the value of the currently-orbiting satellite fleet, given open access in that period.*

Proof. Given open access launch rates, the value of a satellite already in orbit is

$$Q_i(S, D) = \pi - cR_i + (1 - \tilde{E}[\ell])F.$$

Given open access launch rates, the value of all satellites already in orbit is

$$\begin{aligned} W(S, D) &= \int_0^S Q_i di \\ &= \pi S - cR + (1 - \tilde{E}[\ell])FS. \end{aligned}$$

Equation 54 is the first-order condition for the firm's problem,

$$Q_i(S, D) = \max_{0 \leq R_i \leq D/S} \{\pi - cR_i + (1 - \tilde{E}[\ell])F\}.$$

A constrained planner who maximizes the value of the currently-orbiting satellite fleet, taking open access to orbit as given, solves

$$\begin{aligned} W(S, D) &= \max_{0 \leq R \leq D} \{\pi S - cR + (1 - \tilde{E}[\ell])SF\} \\ &= \max_{\{0 \leq R_i \leq D/S\}_i} S\{\pi - cR_i + (1 - \tilde{E}[\ell])F\} \\ &= \max_{\{0 \leq R_i \leq D/S\}_i} SQ_i(S, D). \end{aligned}$$

The constrained planner's objective function is an individual satellite owner's objective scaled by the current size of the fleet, which the constrained planner takes as given. The individual removal solution given by equation 54 therefore characterizes a cooperative debris removal solution, where each firm behaves as an open-access-constrained social planner would command. $\square$

6.4 Competitive debris removal pricing

The profits of the cleanup industry, which supplies active debris removal, are

$$I_t(R_t) = \underbrace{c_t R_t}_{\substack{\text{Cleaning} \\ \text{revenues}}} - \underbrace{\gamma R_t^2}_{\substack{\text{Cleaning} \\ \text{costs}}} . \tag{81}$$

If the cleanup industry is competitive and a positive amount of debris is removed, debris will be removed from orbit until industry profits are zero,

$$R_t^s : I_t(R_t^s) = 0 \tag{82}$$

$$\implies R_t^s(c_t - \gamma R_t^s) = 0 \tag{83}$$

$$\implies R_t^s = \frac{c_t}{\gamma}, \tag{84}$$

subject to the constraint that $R_t^s \leq D_t$. Tkatchova (2018) examines the potential for debris removal markets.

Combining equation 54 with equation 84 and the market clearing condition $R_t^s = R_t$, the aggregate amount of debris removed from orbit will be

$$R_t = \frac{\partial E_t[\ell_t | S_t, D_t - R_t]}{\partial D_t} \frac{F}{\gamma} S_t. \tag{85}$$

6.5 Nonconvexities and corner solutions

For brevity, I write $E_t[\ell_{t+1}]$ as $L(S_t, D_t - S_t R_{it})$ in this subsection and use S and D subscripts to indicate the respective partial derivatives. Since these results are intratemporal in nature, I also drop time subscripts.

Upper bounds on damages, nonconvex stock decay rates, and complementarities between stocks in damage production each imply nonconvexities in the marginal benefits of abatement. The static private marginal benefits from abatement reflect two of these features:

1. at most all satellites can be destroyed in collisions, implying the upper bound on the rate of satellite-destroying collisions;
2. the marginal effect of debris on the number of satellite-destroying collisions depends on the number of satellites in orbit, which makes collisional complementarity or substitutability

between satellites and debris possible.

When the satellite and debris couplings in the collision rate depend on each other, that is, $L_{SD} \neq 0$, changes in the satellite stock can change the returns to scale for debris removal. The dynamic benefits of debris abatement also include the effect of fragment growth from collisions between debris. This effect implies that the net marginal rate of debris decay $(\delta - G_D(S, D-R))$ can be negative.

The marginal benefit of removal is the private value of reducing the probability of a satellite-destroying collision. Debris removal has diminishing marginal benefits if and only if the collision rate is strictly convex in debris. The upper bound on $L(S,D)$ implies that debris removal will have increasing marginal benefits when the risk of a collision gets high enough. Figure 12 shows two examples of this, one with a negative exponential collision rate (globally concave) and another with a sigmoid collision rate (first convex and later concave).

[Figure 12 about here.]

For any positive initial level of debris and satellites (S,D), removal must be nonnegative and no more than all of the debris can be removed. When all satellite owners are identical, the maximum that any one can remove is D/S. This closes the feasible set. Any intermediate amount can also be removed, making the feasible set convex.

The nonconvexity of marginal removal benefits complicates analysis of the optimal amount of removal. There are two cases: the collision rate is globally concave, or the collision rate is convex over some nonnegative interval.

1. If the collision rate is globally concave, there can be no interior solution to the satellite owner's removal problem. Global concavity implies increasing marginal benefits of debris removal, so satellite owners will choose either to remove all debris or none of it.
2. If the collision rate is convex over some nonnegative interval, an interior solution is possible but not guaranteed. For example, suppose the collision rate is convex initially and concave near the end, as in the sigmoid case in Figure 12. Either the right-most intersection of marginal benefits and marginal costs is optimal (where equation 54 and inequality 55 hold) or else zero removal is optimal.

Determining which corner is optimal when the collision rate is globally concave case is straightforward. If the profits of full removal are greater than the profits of zero removal, full removal is optimal; if not, zero removal is optimal.

With local convexities, the problem is more complicated. One approach is as follows. First, select all solutions to the removal first-order condition (equation 54) which satisfy the second-order condition (inequality 55), and include them in a set with zero removal and full removal. This is the set of candidate solutions. Calculate the profits of each candidate solution, and select the one with the highest profits. This procedure is computationally tractable over a closed and convex support as long as the collision rate function is reasonably well-behaved. Figure 13 illustrates how nonconvexity of the collision rate affects profits and the optimal level of removal.

[Figure 13 about here.]

6.6 Cost and congestion shifts in cooperative removal demands from new satellites

For brevity, I write $E_t[\ell_{t+1}]$ as $L(S_t, D_t - S_t R_{it})$ in this subsection and use S and D subscripts to indicate the respective partial derivatives. Since these results are intratemporal in nature, I also drop time subscripts.

To see the cost and congestion shifts in the cooperative private demand for removal, suppose there were two types of satellites, Kinds (k) and Unkinds (u). Kinds make sure their satellites can never collide with others and purchase debris removal, while Unkinds allow their satellites a non-zero chance of colliding with another satellite and never purchase debris removal. The private marginal benefit of debris removal for a Kind named i is

$$MB_i = L_D(u, D - kR_i)kF. \tag{86}$$

The congestion shift is the effect of another Unkind entering, while the cost shift is the effect of another Kind entering. Formally,

$$\text{Cost shift: } \frac{\partial MB_i}{\partial k} = -L_{DD}(u, D - kR_i)kR_iF + L_D(u, D - kR_i)F \tag{87}$$

$$\text{Congestion shift: } \frac{\partial MB_i}{\partial u} = L_{DS}(u, D - kR_i)kF \tag{88}$$

Adding them and normalizing by a function of the collisions rate's convexity in debris,

$$\frac{\frac{\partial MB_i}{\partial k} + \frac{\partial MB_i}{\partial u}}{L_{DD}k^2F} = \frac{(-L_{DD}kR_iF + L_DF) + (L_{DS}kF + L_DF)}{L_{DD}k^2F} \tag{89}$$

$$= \frac{L_D}{L_{DD}k^2} + \frac{L_{DS} - L_{DD}R_i}{L_{DD}k} = \frac{\partial R_i}{\partial S}. \tag{90}$$

The congestion shift may be positive or negative. It is the effect of increasing the number of satellites on the marginal collision risk from a unit of debris. If the collision rate were decoupled from the satellite stock, the congestion shift would disappear. If a marginal Unkind would increase the effect of a marginal unit of debris, the congestion shift will be positive. Reducing the amount of debris would greatly reduce the threat posed by the marginal satellite. If a marginal Unkind would decrease the effect of a marginal unit of debris, the congestion shift will be negative. This could be the case if the Unkind was well-shielded from debris but a threat to other satellites. Reducing the amount of debris would not change the risk of the marginal Unkind by much then.

The cost shift is the effect of increasing the number of customers in the market for debris removal on the marginal collision threat from a unit of debris. There are two pieces to this. First, the debris removed by each Kind reduces the collision risk for all owners. As long as the collision rate is increasing in debris, reducing debris is always a good thing for everyone. This will tend to make the cost shift positive. Second, the debris removed by each Kind changes the marginal benefit of the next Kind's removal. Since the collision rate must be locally convex in debris at an interior solution, this effect will tend to make the cost shift negative. If the collision rate is sufficiently locally convex, this effect can make the cost shift negative in total. Generic satellites are both Kinds and Unkinds.

6.7 Comparative statics of cooperative debris removal and open access launching

I show three results about the demand for debris removal in this section.

First, there is a unique cooperatively-optimal post-removal level of debris for any given level of the satellite stock. This is a consequence of the linear cost (to satellite owners) of debris removal and the monotonicity of the expected collision risk in debris. Due to the linearity, cooperative satellite owners will pursue a most-rapid approach path to the optimal post-removal level of debris in every period. Were the cost nonlinear, the most-rapid approach path would no longer be optimal but the optimal level of debris would remain unique due to monotonicity.

Second, if satellites and debris are "strong enough" complements in producing collision risk, increasing the number of satellite owners in orbit will reduce the optimal post-removal level of debris. This spillover effect in debris removal suggests that a "dynamic virtuous cycle" of active debris removal may be possible: removal in one period can spur entry in the next, which in turn spurs more removal in the following period. Although the functional forms I use rule this effect out, those forms are simplified from a statistical mechanics approximation of orbital interactions. A higher-fidelity model may allow this possibility. A static analog of this effect can be seen in Figure 9.

Third, the open access launch rate may be *increasing* in the launch cost. Though this result seems counterintuitive, it is a natural consequence of three features of open access orbit use:

1. open access drives the value of a satellite down to the launch cost;
2. the amount of removal is increasing in the launch cost;
3. new entry can reduce the individual expenditure required from cooperative firms to achieve the optimal post-removal level of debris.

The cooperative cost-savings from new entry exceeding the effect of new entry on collision risk is necessary and sufficient for the open access launch rate to be increasing in the launch cost.

Together, these results suggest that the use of debris removal can result in interesting and counterintuitive dynamics in orbit use. Though these results are relevant to understanding the effects of debris removal technologies on orbit use, I omit their proofs from this section. Interested readers may find the proofs in the Appendix, section 8.

Cooperative private debris removal:

Lemma 5. *(Law of cooperative private debris removal demand) The cooperative private debris removal demand is*

1. *weakly decreasing in the price of removing a unit of debris, and*
2. *weakly increasing in the cost of launching a satellite.*

Proof. I consider corner solutions first, then interior solutions. I characterize how interior solutions change in response to a change in the removal price, then show that increases in the price can only induce the firm to reduce their removal demands even at corners. I refer to the non-optimized value of a satellite as $Q_i(R_i)$.

The full removal corner: The first part of the proposition is trivially true at the full removal corner, since the amount of debris removal purchased cannot increase at this corner. So it must either stay the same, or decrease, in response to an increase in the price of removal. For the second part, suppose a firm initially finds full removal optimal. Reducing the amount of debris by a positive amount ε in response to a change in launch cost removed is optimal if and only if, at the new launch cost,

$$\begin{aligned}
Q_i(D/S-\varepsilon)-Q_i(D/S)>0 \ \ \forall\varepsilon\in\left(0,\frac{D}{S}\right] \\
\implies \pi+F-c\frac{D}{S}+c\varepsilon-\tilde{E}[\ell|S,D-S(D/S-\varepsilon)]F-\pi-F+c\frac{D}{S}+\tilde{E}[\ell|S,0]F>0 \\
\implies c\varepsilon-(\tilde{E}[\ell|S,\varepsilon]-\tilde{E}[\ell|S,0])F>0 \\
\implies \frac{\tilde{E}[\ell|S,\varepsilon]-\tilde{E}[\ell|S,0]}{\varepsilon}<\frac{c}{F} \ \ \forall\varepsilon\in\left(0,\frac{D}{S}\right].
\end{aligned}$$

If full removal was optimal to begin with, then an increase in the launch cost cannot make it optimal to switch strategies. The above inequality also shows how an increase in the cost of removal can induce a firm to reduce the amount of removal purchased.

The zero removal corner: Consider the profits from increasing the amount of removal from zero to ε in response to a change in the launch cost or removal price. The change is privately optimal if and only if, at the new cost or price,

$$\begin{aligned}
Q_i(\varepsilon)-Q_i(0)>0 \ \ \forall\varepsilon\in\left(0,\frac{D}{S}\right] \\
\implies \pi+F-c\varepsilon-\tilde{E}[\ell|S,D-S\varepsilon]F-\pi-F+\tilde{E}[\ell|S,D]F>0 \\
\implies -c\varepsilon-[\tilde{E}[\ell|S,D-S\varepsilon]-\tilde{E}[\ell|S,D]]F>0 \\
\frac{\tilde{E}[\ell|S,D-S\varepsilon]-\tilde{E}[\ell|S,D]}{\varepsilon}>\frac{c}{F} \ \ \forall\varepsilon\in\left(0,\frac{D}{S}\right].
\end{aligned}$$

If zero removal was optimal to begin with, then an increase in the price of removal cannot make it optimal to switch strategies. An increase in the cost of launching a satellite, however, may induce a firm to begin removing debris.

For interior solutions: From equation 54,

$$R_it:\mathscr{H}=c-\frac{\partial\tilde{E}[\ell]}{\partial D}SF=0. \tag{91}$$

Applying the Implicit Function Theorem to $\mathscr{H}$,

$$\begin{aligned}
\frac{\partial R_i}{\partial c} &= -\frac{\partial\mathscr{H}/\partial c}{\partial\mathscr{H}/\partial R_i} \\
&= -\frac{1}{\frac{\partial^2\tilde{E}[\ell]}{\partial D^2}S^2F}<0.
\end{aligned}$$

Strict negativity follows from the second order condition (inequality 55). If there are multiple solutions and the removal price increase causes firms to jump from interior one solution to another, they must jump to a solution with less removal.

Similarly, from applying the Implicit Function Theorem to $\mathscr{H}$,

$$\begin{aligned}\frac{\partial R_i}{\partial F} &= -\frac{\partial \mathscr{H}/\partial D}{\partial \mathscr{H}/\partial R_i}\\ &= \frac{\frac{\partial \tilde{E}[\ell]}{\partial D}S}{\frac{\partial^2 \tilde{E}[\ell]}{\partial D^2}S^2F} > 0.\end{aligned}$$

Strict positivity follows from the second order condition (inequality 55). If there are multiple solutions and the launch cost increase causes firms to jump from interior one solution to another, they must jump to a solution with more removal. □

The intuition for this result is simple. Satellite owners pay for debris removal. When the price of removal rises, the demand for removal falls. Under open access the continuation value of a satellite is the cost of launching. So, the demand for debris removal increases when satellites become more valuable. Figure 14 illustrates Lemma 5.

[Figure 14 about here.]

What about changes in the satellite and debris stocks? Increases in the debris stock increase the cost of achieving any given level of reductions, but may also increase the marginal benefit of removal if the collision rate is locally concave. Increases in the satellite stock may increase the marginal benefit of removal if the collision rate is locally jointly concave, but also increase the number of firms in the market for removal and give existing firms an incentive to reduce their expenditures. I examine this question in Propositions 11 and 12.

Proposition 11. *(Cooperative demand for debris removal and the state of the orbit) The optimal post-removal debris level is independent of the pre-removal debris level, but depends on the number of satellites in orbit.*

Proof. Proof. I focus on interior solutions, but the result follows for corner solutions as well due to the monotonicity of $\tilde{E}_t[\ell_t]$ in S_t and D_t.

From equation 54,

$$R_{it} : \mathscr{H} = c_t - \frac{\partial \tilde{E}_t[\ell_t|S_t, D_t - R_t]}{\partial D_t} S_t F = 0.$$

Applying the Implicit Function Theorem to $\mathscr{H}$,

$$\begin{aligned}\frac{\partial R_{it}}{\partial D_t} &= -\frac{\partial \mathscr{H}/\partial D_t}{\partial \mathscr{H}/\partial R_{it}}\\ &= \frac{1}{S_t} > 0.\end{aligned}$$

The total quantity of debris removed is

$$R_t = \int_0^{S_t} R_{it} di = S_t R_{it}.$$

Suppose that a positive amount of removal is optimal before and after the change in debris. Differentiating R_t with respect to D_t and using the earlier results for individual removal demands,

$$\frac{\partial R_t}{\partial D_t} = S_t \frac{\partial R_{it}}{\partial D_t} = 1.$$

The monotonicity of $\tilde{E}_t[\ell_t]$ in S_t and D_t also implies that, for any S_t, there is a unique $D_t - R_t$ such that

$$c_t = \frac{\partial \tilde{E}_t[\ell_t | S_t, D_t - R_t]}{\partial D_t} S_t F.$$

□

□

Proposition 11 shows that when it is optimal to remove debris, firms will increase their removal efforts in response to increases in debris. The uniqueness of the optimal post-removal debris stock requires aggregate removal demanded to match changes in the debris stock. This is analogous to the uniqueness of optimal escapement policies in fisheries management. Figure 15 illustrates this behavior.

[Figure 15 about here.]

Proposition 12. *Additional satellite owners decrease the cooperative individual debris removal demand unless satellites and debris are "strong enough" complements in collision risk production.*

Proof. I show the result for interior solutions. A similar condition also holds for corner solutions.

The total quantity of debris removed is

$$R_t = \int_0^{S_t} R_{it} di = S_t R_{it}.$$

Differentiating R_t with respect to S_t,

$$\frac{\partial R_t}{\partial S_t} = R_{it} + S_t \frac{\partial R_{it}}{\partial S_t}.$$

R_{it} and S_t are both nonnegative by definition. It follows that

$$\frac{\partial R_t}{\partial S_t} > 0 \iff \frac{R_{it}}{S_t} > -\frac{\partial R_{it}}{\partial S_t}.$$

This is always true when individual removal demands increase in response to additional satellite owners ($\frac{\partial R_{it}}{\partial S_t} > 0$). The following steps establish the complementarity condition for interior solutions.

From equation 54,

$$R_{it} : \mathscr{H} = c_t - \frac{\partial \tilde{E}_t[\ell_t | S_t, D_t - R_t]}{\partial D_t} S_t F = 0.$$

Applying the Implicit Function Theorem to $\mathscr{H}$,

$$\begin{aligned}
\frac{\partial R_{it}}{\partial S_t} &= -\frac{\partial \mathscr{H} / \partial S_t}{\partial \mathscr{H} / \partial R_{it}} \\
&= \frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S_t^2} + \frac{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t} - \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} R_{it}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S_t} \lesseqgtr 0.
\end{aligned}$$

So, increases in the amount of debris must increase the privately optimal amount of removal at all interior solutions, while increases in the number of satellites will have ambiguous effects. The privately optimal demand for removal will be increasing in the number of satellites if and only if

$$\begin{aligned}
\frac{\partial R_{it}}{\partial S_t} > 0 &\iff \frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S^2} + \frac{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t} - \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} R_i}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S} > 0 \\
&\iff \frac{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S} > -\frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} S^2} + \frac{R_i}{S} \\
&\iff \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t} < -\frac{\frac{\partial \tilde{E}_t[\ell_t]}{\partial D_t}}{S} + \frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t^2} R_i.
\end{aligned}$$

The right hand side of the final line is strictly negative at an interior optimum from the second-order condition, inequality 55. So $\frac{\partial^2 \tilde{E}_t[\ell_t]}{\partial D_t \partial S_t}$ must be "sufficiently" negative for the presence of new satellites to increase privately optimal removal. Economically, this means that satellites and debris are "strong enough" substitutes.

□

Proposition 12 shows that firms may increase or decrease their removal demands in response to more firms entering the orbit. At the full removal corner, they will always reduce their demands. This is a cooperative cost-sharing effect: it remains optimal to remove all debris, but the contribution required of each firm decreases when more firms enter. At an interior solution, their response to more satellites depends on two effects: a congestion effect and a cooperation effect. Their net effect depends on the collision rate's convexity in debris and satellites, particularly whether satellites and debris are "strong enough" complements in producing collision risk. If

- the cooperation and congestion effects are collectively positive, then the presence of more satellites increases the marginal benefit of debris removal (positive spillover effects);
- the cooperation and congestion effects are collectively negative, then the presence of more satellites decreases the marginal benefit of debris removal (negative spillover effects).

These shifts are shown algebraically in the Appendix, section 6.6. Conditional on a positive amount of removal being optimal, increases in debris are matched by increases in removal. Increases in the number of satellites can result in aggregate removal decreases if the individual demand reduction is large enough. Given the functional forms I assume for simulations, the shifts are always collectively negative. I remain agnostic about the "correct" functional form to assume.

Any increase in debris is matched by a commensurate increase in aggregate removal. Since satellites are identical, owners collectively agree on the optimal level of debris. The total quantity of debris removed can not be decreasing in the number of satellites unless individual owners reduce their removal demands in response to more satellites in orbit. Proposition 12 shows this is a necessary but not sufficient condition.

Proposition 11 also offers some insight into the effects of launches on privately optimal removal. With no launch debris, the effect of a marginal launch on privately optimal removal is only the effect of a new satellite on privately optimal removal. With launch debris, the effect of a launch is a combination of the effect of a new satellite and the effect of new debris.

Open access launching: The option - or, in the cooperative case studied here, the obligation - to remove space debris alters the incentives of open access satellite launchers. While they will still launch until expected profits are zero, the expected collision risk is no longer the only object which equilibriates their launching behavior. In addition to expected collision risk, the expenditure they expect to incur removing debris as satellite owners will also adjust to equilibriate the launch rate. Though they will be price takers when their satellites reach orbit, they can anticipate the number of satellite owners who will contribute to debris removal. This acts in the opposite direction as the expected collision risk: while more satellites in orbit increases risk, more firms with satellites in orbit decreases each individual firm's debris removal expense. Since debris removal also reduces collision risk, the net effect of introducing debris removal financed by satellite owners may be more launches than would otherwise occur.

Indeed, this is precisely what occurs in the cases simulated here.

In addition to this perhaps-counterintuitive effect, it is plausible that an increase in the cost of launching a satellite could increase the launch rate. This is not as pathological a case as it may seem at first. Since open access drives the value of a satellite down to the launch cost, and the cooperatively-optimal amount of debris removal which satellite owners will pay for is increasing in the launch cost, and increase in the launch cost under open access could increase the value of owning a satellite by more than it increases the cost of launching it, at least locally near an existing equilibrium. This is not a violation of the law of demand for satellite ownership; rather, it is a violation of the "all else equal" clause. Assumption 4 describes a necessary and sufficient condition to rule this case out.

Assumption 4. *(New launches reduce the expected profits of satellite ownership) The change in individual removal expenses from a marginal satellite launch is smaller in magnitude than the sum of the change in expected future collision costs from a marginal satellite launch and the change in individual removal expenses from a marginal piece of launch debris. Formally,*

$$\left|\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}F + m\left(\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}F + \frac{\partial R_{it+1}}{\partial D_{t+1}}c_{t+1}\right)\right| > \left|\frac{\partial R_{it+1}}{\partial S_{t+1}}c_{t+1}\right|.$$

If this assumption is violated, then launches increase the profitability of owning a satellite through the debris removal expenditure channel described above. It is also possible that increases in the cost to satellite owners of removing a unit of debris could increase the launch rate Assumption 5 describes an additional condition necessary for increases in the price of debris removal to reduce the launch rate. An increase in the price of debris removal will reduce the cooperatively-optimal amount of debris removal satellite owners purchase, potentially reducing the total debris removal expenditure and increasing the profits of owning a satellite. As in the case of launch rates being increasing in launch costs, this is not a violation of the law of demand for satellite ownership; it is a violation of the "all else equal" clause.

Assumption 5. *(Removal expenditure is increasing in the removal cost) The cooperative private debris removal expenditure is increasing in the price of removing a unit of debris. Formally,*

$$\frac{\partial}{\partial c_{t+1}}(R_{it+1}c_{t+1}) = R_{it+1} + \frac{\partial R_{it+1}}{\partial c_{t+1}}c_{t+1} > 0.$$

Assumption 5 states that the amount of debris removed (R_{it+1}) is larger than the reduction in removal due to a price increase ($\frac{\partial R_{it+1}}{\partial D_{t+1}}c_{t+1}$, which is weakly negative from Proposition 5). This is likely to hold whenever the change in individual removal demands from a change in removal cost is small, for example,if removal demand is in the interior before and after the change. It is unlikely to hold if the opposite is true, for example,if the change in removal cost causes individual removal demands to jump from the full removal corner to the zero removal corner at a time when there are few satellites and many debris fragments. Though future cooperative private debris removal demands are an anticipated cost to current satellite launchers, those same launchers may find their willingness to launch increasing in the cost of removal if it reduces the burden of cooperating and purchasing removal.

Proposition 13. *(Private demand for satellite ownership) The open access launch rate is*

1. *strictly decreasing in the cost of launching a satellite if and only if new launches reduce the expected profits of satellite ownership; and*
2. *strictly decreasing in the price of removing a unit of debris in $t+1$ only if new launches reduce the expected profits of satellite ownership AND the cooperative private expenditure on debris removal is increasing in the cost of removal.*

Proof. From equation 52,

$$X_t : \mathscr{F} = \pi - rF - E_t[\ell_{t+1}]F - R_{it+1}c_{t+1} = 0.$$

Applying the Implicit Function Theorem to $\mathscr{F}$ and assuming R_{it+1} is chosen optimally (as described in Proposition 5),

$$\begin{aligned}\frac{\partial X_t}{\partial F} &= -\frac{\partial \mathscr{F}/\partial F}{\partial \mathscr{F} \partial X_t} \\ &= -\frac{r + E_t[\ell_{t+1}] + \frac{\partial R_{it+1}}{\partial F}c_{t+1}}{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}F + m\left(\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}F + \frac{\partial R_{it+1}}{\partial D_{t+1}}c_{t+1}\right) + \frac{\partial R_{it+1}}{\partial S_{t+1}}c_{t+1}},\end{aligned}$$

which is negative for all parameter values when Assumption 4 holds.

Similar manipulations yield

$$\begin{aligned}\frac{\partial X_t}{\partial c_{t+1}} &= -\frac{\partial \mathscr{F}/\partial c_{t+1}}{\partial \mathscr{F} \partial X_t} \\ &= -\frac{R_{it+1} + \frac{\partial R_{it+1}}{\partial c_{t+1}}c_{t+1}}{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}F + m\left(\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}F + \frac{\partial R_{it+1}}{\partial D_{t+1}}c_{t+1}\right) + \frac{\partial R_{it+1}}{\partial S_{t+1}}c_{t+1}}.\end{aligned}$$

$\frac{\partial X_t}{\partial c_{t+1}}$ is negative only if Assumptions 4 and 5 hold. □

Given Assumption 4, Assumption 5 is necessary and sufficient for $\frac{\partial X_t}{\partial c_{t+1}}$ to be negative. A simultaneous violation of Assumptions 4 and 5 would indicate that the private marginal cost of orbit use was decreasing in the costs of access and debris removal - a counterintuitive situation, but not an *a priori* impossible one.

7 Appendix C: Model extensions

7.1 Spectrum use management

So far I have assumed that there is no spectrum congestion from satellites. In practice, radio frequency interference is one of the major concerns of space traffic control. However, policy to manage spectrum use is not a focus of this paper because it is generally handled well by existing institutions Johnson (2004). The effect of optimally managed spectrum congestion on the expected collision risk in a deterministic setting is shown in the appendix of Rao and Rondina (2018). In this section, I adapt the result to this paper's setting and show that permits or fees for spectrum use can approximate stock controls.

Spectrum congestion degrades the quality of the signals to and from satellites. This makes the per-period output from a satellite decreasing in the number of orbital spectrum users. For simplicity, suppose that all satellites in orbit use enough spectrum to have some congestion impact. The per-period return function is then $\pi = \pi(S), \pi'(S) < 0$, and the one-period rate of return on a satellite is $\pi(S)/F = r_s(S)$. Assuming spectrum is optimally managed, firms will account for their marginal impact on spectrum congestion when they launch their satellite. The open access equilibrium condition, equation 9, becomes

$$E_t[\ell_{t+1}] = r_s(S_{t+1}) - r + r_s'(S_{t+1}). \tag{92}$$

Satellite owners would internalize the final term, $r'(S_{t+1})$, through a permit or fee system. Though spectrum permits may be purchased before the satellite is launched, their continued use is contingent on the firm abiding by non-interference protocols and any other stipulations by the appropriate regulatory body. Similarly, an optimal fee for spectrum use would adjust to reflect the marginal spectrum congestion from another broadcasting satellite. In general, regulated spectrum use will adjust the equilibrium collision rate to be

$$E_t[\ell_{t+1}] = r_s(S_{t+1}) - r - q_{t+1}, \tag{93}$$

where q_{t+1} is the spectrum use fee or permit price. Note that equation 93 is similar to equation 21,

$$\begin{aligned} \pi &= rF + E_t[\ell_{t+1}]F + p^s_{t+1} \\ \implies E_t[\ell_{t+1}] &= r_s - r - \frac{p^s_{t+1}}{F}. \end{aligned} \tag{94}$$

This suggests another avenue for controlling the equilibrium collision rate. By setting the price of spectrum use, q_{t+1}, equal to the sum of marginal spectrum and collision risk congestion costs, $r_s'(S_{t+1}) + E_t[\xi(S_{t+1}, D_{t+1})]$, a spectrum regulator can implement an optimal stock control. More generally, this would be an optimal space traffic control in the sense of Johnson (2004), as it would account for both radio frequency and physical interference.

7.2 Mandatory satellite insurance

Can insurance markets correct the orbital congestion externality in the absence of active debris removal? Suppose that satellites were required to be fully insured against loss once they reached orbit and the satellite insurance sector was perfectly competitive. The insurance payment will act as a stock control, so the only question remaining is how the insurance industry will price the product. Denote the price of insurance in period t by p_t, and the profits of the insurance sector by I_t.

$$Q(S_t, D_t, \ell_t, p_t) = \pi - \underbrace{p_t}_{\substack{\text{Insurance}\\\text{premium}}} + (1-\ell_t)F + \underbrace{\ell_t F}_{\substack{\text{Insurance}\\\text{payout}}} = \pi - p_t + F \tag{95}$$

$$I(S_t, \ell_t) = \underbrace{p_t S_t}_{\substack{\text{Inflow of}\\\text{premium payments}}} - \underbrace{\ell_t S_t F}_{\substack{\text{Outflow of}\\\text{reimbursements}}} . \tag{96}$$

Competitive insurance pricing With competitive insurance pricing, satellite insurance will be actuarially fair. Plugging this price into the open access equilibrium condition, we can solve for the loss rate under mandatory insurance:

$$p_t : I(S_t, \ell_t) = 0 \implies p_t = \ell_t F \tag{97}$$

$$\pi = rF + E_t[\ell_{t+1}]F + p_{t+1} \tag{98}$$

$$\implies E_t[\ell_{t+1}] = r_s - r. \tag{99}$$

Proposition 14. *(Competitive insurance won't change collision risk) The equilibrium collision risk given mandatory satellite insurance with actuarially fair pricing is the same as the equilibrium collision risk given uninsured open access.*

Proof. From equation 99, the equilibrium expected loss rate with actuarially fair insurance is

$$E_t[\ell_{t+1}] = r_s - r.$$

From equation 49, the equilibrium expected loss rate with no insurance is

$$E_t[\ell_{t+1}] = r_s - r.$$

□

Regulated insurance pricing As in the case of spectrum management policies, mandatory satellite insurance premiums approximate a stock control. This suggests another avenue by which a regulator could induce optimal orbit use without assigning property rights over orbits or levying an explicit satellite tax.

Suppose the regulator was able to give insurers a per-satellite penalty or subsidy of τ_t to ensure insurance would be priced at the marginal external cost ($p_{t+1} = E_t[\xi(S_{t+1}, D_{t+1})]$) while still allowing free entry into the insurance sector. When τ_t is positive the regulator would be issuing an underwriting subsidy, and when τ_t is negative the regulator would be issuing an underwriting penalty. The insurance sector's profit is then

$$I(S_t, \ell_t) = (E_{t-1}[\xi(S_t, D_t)] - \ell_t F + \tau_t) S_t \tag{100}$$

$$\tau_t : I(S_t, \ell_t) = 0 \implies \tau_t = \ell_t F - E_{t-1}[\xi(S_t, D_t)]. \tag{101}$$

Equation 101 shows that the socially optimal mandatory insurance pricing can be achieved by an incentive which imposes the difference between the actuarial cost of satellite insurance and the marginal external cost on the insurer. The insurer then passes the marginal external cost on to the satellite owner. Depending on the magnitude of the risk and the marginal external cost, this may be a net subsidy or tax on the insurer.

8 Appendix D: Proofs and technical details

8.1 Proofs not shown in the main text

Lemma 1 (Launch response to stock and flow controls): *The open access launch rate is*

- *decreasing in the future price of a stock control;*
- *decreasing in the current price and increasing in the future price of a flow control.*

Proof. Stock controls: From equation 21, we can write

$$\mathscr{I} = \pi - rF - E_t[\ell_{t+1}]F - p_{t+1} = 0. \tag{102}$$

Applying the Implicit Function Theorem, we get that

$$\frac{\partial X_t}{\partial p_{t+1}} = -\frac{\partial \mathscr{I}/\partial p_{t+1}}{\partial \mathscr{I}/\partial X_t} \tag{103}$$

$$= -\frac{-1}{-\frac{\partial E[\ell_{t+1}]}{\partial X_t}} \tag{104}$$

$$= -\frac{1}{\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial X_t} + \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial X_t}} \tag{105}$$

$$= -\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} + m\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}} < 0. \tag{106}$$

Flow controls: From equation 24, we can write

$$\mathscr{G} = \pi - rF - E_t[\ell_{t+1}]F - (1+r)p_t + E_t[(1-\ell_{t+1})p_{t+1}] = 0. \tag{107}$$

Applying the Implicit Function Theorem, we get that

$$\frac{\partial X_t}{\partial p_t} = -\frac{\partial \mathscr{G}/\partial p_t}{\partial \mathscr{G}/\partial X_t} \tag{108}$$

$$= -\frac{-(1+r)}{-\frac{\partial E_t[\ell_{t+1}]}{\partial X_t}F - \frac{\partial E_t[\ell_{t+1}]}{\partial X_t}p_{t+1}} \tag{109}$$

$$= -\frac{1+r}{[\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial X_t} + \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial X_t}](F+p_{t+1})} \tag{110}$$

$$= -\frac{1+r}{[\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} + m\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}](F+p_{t+1})} < 0. \tag{111}$$

Similarly, we can obtain

$$\frac{\partial X_t}{\partial p_{t+1}} = -\frac{\partial \mathscr{G}/\partial p_{t+1}}{\partial \mathscr{G}/\partial X_t} \tag{112}$$

$$= -\frac{1-E_t[\ell_{t+1}]}{-\frac{\partial E_t[\ell_{t+1}]}{\partial X_t}F - \frac{\partial E_t[\ell_{t+1}]}{\partial X_t}p_{t+1}} \tag{113}$$

$$= \frac{1-E_t[\ell_{t+1}]}{[\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}}\frac{\partial S_{t+1}}{\partial X_t} + \frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}\frac{\partial D_{t+1}}{\partial X_t}](F+p_{t+1})} \tag{114}$$

$$= \frac{1-E_t[\ell_{t+1}]}{[\frac{\partial E_t[\ell_{t+1}]}{\partial S_{t+1}} + m\frac{\partial E_t[\ell_{t+1}]}{\partial D_{t+1}}](F+p_{t+1})} > 0. \tag{115}$$

$\square$

Proposition 2 (Smoothness at boundaries): *Stock controls can be initiated without letting the launch rate exceed the open access launch rate. Flow controls cannot be initiated without forcing the launch rate to exceed the open access launch rate.*

Proof. In both cases, I suppose that there is open access before the control is initiated.
Initiating a stock control: Suppose a stock control is scheduled to take effect at t, that is, satellite owners in t begin paying p_t. In $t-1$, firms would launch with this fact in mind:

$$X_{t-1} : \pi = rF + E_{t-1}[\ell_t]F + p_t. \tag{116}$$

Let the open access launch rate in $t-1$ with no stock control in t be $\hat{X}_{t-1} : \pi = rF + E_{t-1}[\ell_t]F$. Lemma 1 implies that for all $p_t > 0$, $\hat{X}_{t-1} > X_{t-1}$.

Initiating a flow control: Suppose a flow control is scheduled to be implemented at t, that is, satellite launchers in t begin paying p_t to launch. In $t-1$, firms would launch with this fact in mind:

$$X_{t-1} : \pi = rF + E_{t-1}[\ell_t]F - (1 - E_{t-1}[\ell_t])p_t. \tag{117}$$

Let the open access launch rate in $t-1$ with no flow control implemented in t be $\hat{X}_{t-1} : \pi = rF + E_{t-1}[\ell_t]F$. Lemma 1 implies that for all $p_t > 0$, $\hat{X}_{t-1} < X_{t-1}$.

□

Proposition 3 (Controlling the rate of deorbit): *Stock controls with positive prices can make satellite owners deorbit their satellites and induce net deorbits. Flow controls with positive prices cannot make satellite owners deorbit their satellites or induce net deorbits.*

Proof. A satellite owner facing a stock control in period t will deorbit if

$$p_t > \pi + (1 - E_t[\ell_{t+1}])F - V^d. \tag{118}$$

The regulator can induce firms to deorbit in t by raising p_t high enough in $t-1$. A potential launcher in $t-1$ will not launch if

$$p_t > \pi + (1 - E_{t-1}[\ell_t])F. \tag{119}$$

By raising p_t high enough, the regulator can both discourage further launches and induce existing satellite owners to deorbit their satellites.

A satellite owner facing a flow control in period t will deorbit if

$$p_t : \pi + (1-\ell_t)\beta E_t[Q_{t+1}] < V^d, \text{ where } X_t : \beta E_t[Q_{t+1}] = F + p_t \tag{120}$$

$$\implies (1-\ell_t)(F + p_t) < V^d - \pi, \tag{121}$$

which cannot be satisfied by positive p_t, given $V^d < \pi$. A potential launcher in t will not launch if

$$p_{t+1}: \ \pi + (1 - E_t[\ell_{t+1}])p_{t+1} < rF + E_t[\ell_{t+1}]F + (1+r)p_t. \tag{122}$$

If $\pi < rF + E_t[\ell_{t+1}]F + (1+r)p_t$, equation 122 will not be satisfied for any positive p_{t+1}. Although equation 122 can be satisfied if p_{t+1} is sufficiently negative, this would require the regulator to commit to a path of ever-decreasing negative prices as long as they wished to prevent launches (as described earlier and in Lemma 3). Regardless, the regulator cannot induce net deorbits (no new arrivals and some deorbits) in $t+1$ with a positive p_{t+1}. □

Proposition 7 (ADR can reduce collision risk):

Proof. I show the result first for the introduction of debris removal services, then for the ongoing use of debris removal services.

The introduction of ADR: Suppose an active debris removal service will become available at date t. To clarify whether removal is an option or not, I explicitly include the conditioning variables in the loss rate, that is, $E_t[\ell_{t+1}]$ is written as $E_t[\ell_{t+1}|S_{t+1}, D_{t+1} - R_{t+1}]$ when removal is an option.

Under open access, firms without satellites at $t-2$ will launch until

$$E_{t-2}[\ell_{t-1}|S_{t-1}, D_{t-1}] = r_s - r. \tag{123}$$

At $t-1$, launchers will expect to be able to remove debris once their satellites reach orbit. They will launch until

$$E_{t-1}[\ell_t|S_t, D_t - R_t] = r_s - r - \frac{c_t}{F}R_{it}. \tag{124}$$

Comparing $E_{t-2}[\ell_{t-1}|S_{t-1}, D_{t-1}]$ and $E_{t-1}[\ell_t|S_t, D_t - R_t]$ yields the necessary and sufficient conditions:

$$E_{t-2}[\ell_{t-1}|S_{t-1}, D_{t-1}] - E_{t-1}[\ell_t|S_t, D_t - R_t] > 0 \iff c_t R_{it} > 0. \tag{125}$$

Ongoing use of ADR: Under open access, the equilibrium collision risk in $t+1$ after debris removal in t is

$$E_t[\ell_{t+1}] = r_s - r - \frac{c_{t+1}}{F}R_{it+1}.$$

Similarly, the equilibrium collision risk in t after debris removal in $t-1$ is

$$E_{t-1}[\ell_t] = r_s - r - \frac{c_t}{F}R_{it}.$$

Subtracting one equilibrium risk from the other yields the necessary and sufficient condition for ongoing debris removal to continue to reduce the collision risk:

$$E_{t-1}[\ell_t] - E_t[\ell_{t+1}] > 0 \iff r_s - r - \frac{c_t}{F} R_{it} - (r_s - r - \frac{c_{t+1}}{F} R_{it+1}) > 0$$
$$\iff c_{t+1} R_{it+1} > c_t R_{it}.$$

□

8.2 Technical assumptions and lemmas

Assumption 6. *Let S be a vector of state variables, and I_k be a vector of same size as S with 1 in the k^{th} position and 0 in all other positions. $\phi(\ell|S)$ is a conditional density which satisfies the following properties:*

1. *The derivative of $\phi(\ell|S)$ with respect to the k^{th} argument of S,*

$$\frac{\partial \phi(\ell|S)}{\partial S_k} = \lim_{h \to 0} \frac{\phi(\ell|S + I_k h) - \phi(\ell|S)}{h} \equiv \phi_S(\ell|S),$$

   exists and is bounded $\forall \ell$, with $\phi_S(\ell|S) \neq 0$ for some ℓ, $\forall S$.

2. *Let S^1 and S^2 be two vectors which are identical except for the k^{th} entry, where $I_k S^1 < I_k S^2$. $\forall S^1, S^2$, and $\forall A \in [0,1]$, define $\bar{\ell}_1 \equiv \bar{\ell}(A, S^1), \bar{\ell}_2 \equiv \bar{\ell}(A, S^2)$ such that*

$$\int_0^{\bar{\ell}_1} \phi_S(\ell|S^1) d\ell = \int_0^{\bar{\ell}_2} \phi_S(\ell|S^2) d\ell = A.$$

   Then $\phi(\ell|S)$ satisfies a Lipschitz condition

$$\left|\left| \int_{\bar{\ell}_1}^{\bar{\ell}_2} \phi(\ell|S^2) d\ell \right|\right| < ||S^2 - S^1||,$$

The first condition places a lower bound on the change in collision probability from new satellite placements and ensures some smoothness for the changes in the density across the support.

The second condition places an upper bound on changes to the density $\phi(\ell|S,D)$ over the space of (S,D). The idea is that the additional area under the new density required to achieve a target area under the old density is bounded by the change in S or D. Physically, this requires that new satellites or debris will not be placed in orbits that will cause a drastic change in the collision probability. Rather, the change in collision probability from new launches should be bounded and proportional to the number of new satellites placed in orbit. Together, the physical implication of these two conditions is that new satellites or debris will cause some changes to the collision probability, but that those changes will be bounded across the possible outcomes. This is economically reasonable for satellites - a violation of this implies that firms are deliberately placing their satellites in risky orbits. This may be less reasonable for debris, since the orbits of debris objects resulting from collisions are uncontrolled and difficult to predict. These conditions facilitate the proofs of the lemmas below, but are not crucial to the main results of the paper.

Note that the proofs of the lemmas below often assume uniformly bounded functions. While no such property is proven for the value functions studied, realistic economically sensible parameter choices should guarantee the existence of uniform bounds on the value functions.

Lemma 6. *(Measurable functions under changes in distribution) Let ℓ be a random variable with a conditional density $\phi(\ell|S)$ defined on the compact interval $[a,b]$ and with range $[r(a),r(b)]$. Let $f(\cdot):[r(a),r(b)]\to[f(a),f(b)]$ be a measurable function of ℓ. Then*

$$\int_a^b f(\ell)\frac{\partial\phi(\ell|S)}{\partial S}d\ell=\frac{\partial E[f(\ell)|S]}{\partial S}$$

Proof.

$$\begin{aligned}\int_a^b f(\ell)\frac{\partial\phi(\ell|S)}{\partial S}d\ell &= \int_a^b f(\ell)\lim_{h\to 0}\frac{\phi(\ell|S+h)-\phi(\ell|S)}{h}d\ell\\ &= \lim_{h\to 0}\frac{1}{h}\left(\int_a^b f(\ell)\phi(\ell|S+h)d\ell-\int_a^b f(\ell)\phi(\ell|S)d\ell\right)\\ &= \lim_{h\to 0}\frac{1}{h}\left(E[f(\ell)|S+h]-E[f(\ell)|S]\right)\\ &= \frac{\partial E[f(\ell)|S]}{\partial S}.\end{aligned}$$

□

Lemma 7. $\frac{\partial E[f(x)|S]}{\partial S}=0$ *$\forall S$ and $\forall f(x)$ which do not depend on ℓ, the argument of $\phi(\ell|S)$.*

Proof. From Assumption 6 and Lemma 6,

$$\begin{aligned}\frac{\partial E[f(x)|S]}{\partial S} &= \int_0^1 f(x)\left[\lim_{h\to 0}\frac{\phi(\ell|S+h)-\phi(\ell|S)}{h}\right]d\ell\\ &= f(x)\lim_{h\to 0}\frac{1}{h}\left[\int_0^1\phi(\ell|S+h)d\ell-\int_0^1\phi(\ell|S)d\ell\right]\\ &= f(x)\lim_{h\to 0}\frac{1}{h}[1-1]=0.\end{aligned}$$

□

Lemma 8. *If $f(\ell)$ is a nonnegative and uniformly bounded function, then under assumption 1*

1. $\frac{\partial E[f(\ell)|S]}{\partial S} = 0$ *if* $\frac{\partial f(\ell)}{\partial \ell} = 0 \ \forall \ell$
2. $\frac{\partial E[f(\ell)|S]}{\partial S} < 0$ *if* $\frac{\partial f(\ell)}{\partial \ell} < 0 \ \forall \ell$
3. $\frac{\partial E[f(\ell)|S]}{\partial S} > 0$ *if* $\frac{\partial f(\ell)}{\partial \ell} > 0 \ \forall \ell$

Proof. For simplicity, the proof is written for a scalar-valued S. Extending the argument to vector-valued S is possible but not particularly informative.

The first statement, $\frac{\partial E[f(\ell)|S]}{\partial S} = 0$ if $\frac{\partial f(\ell)}{\partial \ell} = 0$, follows directly from Lemma 7 and the assumption that $f(\ell)$ is constant $\forall \ell$.

To show that $\frac{\partial E[f(\ell)|S]}{\partial S} < 0$ if $\frac{\partial f(\ell)}{\partial \ell} < 0 \ \forall \ell$, without any loss of generality let $S^2 > S^1$. Pick $\bar{\ell}_1, \bar{\ell}_2 : \int_0^{\bar{\ell}_1} \phi_S(\ell|S^1)d\ell = \int_0^{\bar{\ell}_2} \phi_S(\ell|S^2)d\ell = A \in (0,1)$. Note that when $A = 0, \bar{\ell}_1 = \bar{\ell}_2 = 0$, and when $A = 1, \bar{\ell}_1 = \bar{\ell}_2 = 1, \ \ \forall S^1, S^2$. Assumption 1 implies that $\forall A \in (0,1)$, $\bar{\ell}_2 > \bar{\ell}_1$. Since $\frac{\partial f(\ell)}{\partial \ell} < 0 \ \forall \ell$,

$$
\begin{aligned}
& \int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell > \int_0^{\bar{\ell}_2} f(\ell)\phi_S(\ell|S^2)d\ell \\
\implies & \int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell - \int_0^{\bar{\ell}_2} f(\ell)\phi_S(\ell|S^2)d\ell > 0 \\
\implies & \int_0^{\bar{\ell}_2} f(\ell)\phi_S(\ell|S^2)d\ell - \int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell < 0 \\
\implies & \lim_{S^2 \to S^1} \frac{1}{S^2 - S^1}\left[\int_0^{\bar{\ell}_2} f(\ell)\phi_S(\ell|S^2)d\ell - \int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell\right] \\
& = \lim_{S^2 \to S^1} \frac{1}{S^2 - S^1}\left[\int_0^{\bar{\ell}_1} f(\ell)\{\phi_S(\ell|S^2)d\ell - \phi_S(\ell|S^1)\}d\ell + \int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\phi(\ell|S^2)d\ell\right] \\
& = \int_0^{\bar{\ell}_1} f(\ell) \lim_{S^2 \to S^1}\left\{\frac{\phi_S(\ell|S^2)d\ell - \phi_S(\ell|S^1)}{S^2 - S^1}\right\}d\ell + \lim_{S^2 \to S^1}\int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\frac{\phi(\ell|S^2)}{S^2 - S^1}d\ell \\
& = \int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell + \lim_{S^2 \to S^1}\int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\frac{\phi(\ell|S^2)}{S^2 - S^1}d\ell < 0
\end{aligned}
$$

By assumption 6 and $f(\ell) \geq 0$, $S^2 > S^1$, $\phi(\ell|S^2) \geq 0$,

$$\int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\frac{\phi(\ell|S^2)}{S^2 - S^1}d\ell \geq 0.$$

Now, taking the limit as A goes to 1, we get

$$\lim_{A \to 1}\int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell + \lim_{S^2 \to S^1}\lim_{A \to 1}\int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\frac{\phi(\ell|S^2)}{S^2 - S^1}d\ell < 0,$$

where

$$\lim_{A \to 1}\int_{\bar{\ell}_1}^{\bar{\ell}_2} f(\ell)\frac{\phi(\ell|S^2)}{S^2 - S^1}d\ell = O(\bar{\ell}_2 - \bar{\ell}_1)$$

is a nonnegative remainder term is bounded by $\bar{\ell}_2 - \bar{\ell}_1$. This leaves us with

$$\lim_{A\to 1}\left[\int_0^{\bar{\ell}_1} f(\ell)\phi_S(\ell|S^1)d\ell\right] + \lim_{S^2\to S^1} O(\bar{\ell}_2 - \bar{\ell}_1) < 0.$$

When $A = 1, \bar{\ell}_2 = \bar{\ell}_1 = 1$ and the remainder is exactly $0 \quad \forall S^1, S^2$. Since S^1 was chosen arbitrarily, we can therefore say that

$$\int_0^1 f(\ell)\phi_S(\ell|S)d\ell \equiv \frac{\partial E[f(\ell)|S]}{\partial S} < 0 \text{ if } \frac{\partial f(\ell)}{\partial \ell} < 0 \ \ \forall \ell.$$

Repeating the argument above with $f(\ell)$ strictly increasing instead of decreasing yields the third statement, $\frac{\partial E[f(\ell)|S]}{\partial S} > 0$ if $\frac{\partial f(\ell)}{\partial \ell} > 0 \ \ \forall \ell$. □

9 Appendix E: Computational framework

9.1 Stochastic and deterministic models

The dynamics of the satellite and debris stocks make the process for ℓ_t dependent and heterogeneously distributed over time. For exposition and to illustrate specific points (most often transition dynamics), I simulate deterministic versions of the models described in this paper. The basic deterministic orbit use model is analyzed in detail in Rao and Rondina (2018). Using deterministic models simplifies making "apples-to-apples" comparisons between different policies without collision rate draws complicating matters. Making the collision risk process independent and identically distributed would remove the economically interesting physics of the problem - open access becomes socially optimal because launch and removal choices have no impact on the collision risk. In the deterministic model, I use the expected collision risk without drawing from a distribution. In the stochastic model, I draw the collision risk from a binomial distribution with floor(S_t) many trials and mean equal to $\min\{\alpha_{SS}S^2 + \alpha_{SD}SD, 1\}$. Formally,

$$\text{Deterministic model collision risk: } \ell_t = \min\{\alpha_{SS}S^2 + \alpha_{SD}SD, 1\}, \tag{126}$$

$$\text{Stochastic model collision risk: } \ell_t \sim Bin(\text{floor}(S_t), \min\{aS_t^2 + bS_tD_t, 1\}). \tag{127}$$

Figure 16 compares sequences of launch rates, satellite and debris stocks, and expected collision risks under open access in the deterministic and stochastic models. The deterministic model behaves as expected - it is the mean of the stochastic model. The expected collision risk in each model is identical, which is unsurprising given that open access and the planner both target the expected collision risk.

[Figure 16 about here.]

Although the time paths are similar, there are some interesting properties of the stochastic values and policies (such as Lemma 2) which are not captured by the deterministic ones.

9.2 Value and policy functions

In general, the algorithms I use to compute decentralized solutions under open access are simpler than those used to compute the planner's solutions. This is because open access simplifies the continuation value to the cost of launching a satellite. I use `R` for all simulations, parallelizing where possible.

I compute optimal value functions by alternating between value and policy function iteration with a version of the one-way multigrid approach described in Chow and Tsitsiklis (1991). The multigrid approach involves computing the value function first on a coarse grid, then progressively refining the grid for further computation while using the previous results as initial guesses. I use linear interpolation to fill in new elements of the initial guess when moving to a finer grid. For simplicity, I use the same number of grid points, with the same limits, over S and D. Algorithm 1 describes how I compute the policy and value functions for a given grid $(grid_S, grid_D, grid_\ell)$ and given initial guess $guess(S, D, \ell)$.

Algorithm 1: Value function iteration with policy evaluation

Set

$$W_0(S,D,\ell) = guess(S,D,\ell)$$

for all $(S,D,\ell) \in (grid_S, grid_D, grid_\ell)$

Set $i = 1$ and $\delta = 100$ (some large value to begin).

while $\delta > \varepsilon$ **do**

  At each grid point in $(grid_S, grid_D, grid_\ell)$, use a numerical global optimizer to obtain

$$X^* = \text{argmax}_X \{\pi S - FX + \beta \hat{W}_{i-1}(S', D', E[\ell'|S', D'])\},$$
$$W_i(S,D,\ell) = \pi S - FX^* + \beta \hat{W}_{i-1}(S', D', E[\ell'|S', D']),$$

  where $\hat{W}_{i-1}(S', D', E[\ell'|S', D'])$ is computed using linear interpolation from $W_{i-1}(S,D,\ell)$, $S' = S(1-\ell) + X$, $D' = D(1-\delta) + G(S,D,\ell) + mX$, and $E[\ell'|S', D'] = \min\{\alpha_{SS} S'^2 + \alpha_{SD} S'D', 1\}$.

  $\delta \leftarrow ||W_i(S,D,\ell) - W_{i-1}(S,D,\ell)||_\infty$.

  if $\delta < \bar{\delta}$: **then**

    Evaluate the policy. Compute $W_i^T(S,D,\ell) = \sum_{t=1}^{T-1} \beta^{t-1}(\pi S - FX^*) + W_i(S,D,\ell)$ by backwards induction, using the laws of motion for S and D and the form of $E[\ell'|S', D']$ and sufficiently large T. Set $W_i(S,D,\ell) = W_i^T(S,D,\ell)$ and return to step (a).

  end

  i ← i+1

end

My use of $\hat{W}_{i-1}(S', D', E[\ell'|S', D'])$ implies that the value function is linear in ℓ. If it is not, then the continuation value I interpolate is a lower bound due to concavity of the value function and Jensen's inequality. An alternative approach, which would be more computationally intensive, would be to numerically compute the expectation using the distribution function of the binomial evaluated at each grid point in grid_ℓ. However, as the grid gets successively finer the approximation error due to Jensen's inequality shrinks to zero. Since each value function iteration until the final grid size is only used to provide a guess for the next-finer grid, the final approximation error can be made arbitrarily small by choosing a sufficiently fine final grid. There is no such approximation error for the deterministic model since ℓ is $E[\ell'|S', D']$ there.

Once the algorithm converges for a given grid, I add more points to the grid. I linearly interpolate the final value of $W_i(S,D,\ell)$ from the coarser grid onto the finer grid to obtain the new value of $guess(S,D,\ell)$. I then repeat the algorithm until convergence. I use the same size grid for all state variables. I begin with an initial grid of 2 points for each state variable (4 points in the deterministic model, 8 points in the stochastic model) and continue until a grid of 128 points for each state variable (16384 points in the deterministic model, $2,097,152$ points in the stochastic model). I use $\varepsilon = 1e^{-8}$ for the final grid, and $\varepsilon = h^{0.35} \cdot 1e^{-8}$ for the intermediate grids (where h is the number of points in a single dimension of the grid). This saves computation time for grids where the converged value function will only be used as an initial guess.[26] I set $\bar{\bar{\delta}} = 1$ for all grids, and use a value for T between 10 and 100 depending on the grid size. These values were chosen by experimentation for different parameterizations to balance speed and accuracy.

The numerical global optimization can take some time. I use this approach because the first-order condition to the planner's Bellman equation can have multiple solutions (maxima and minima), and this approach is about as fast as storing each solution, evaluating the second-order condition at each solution, and comparing the value function level at each solution. This approach also makes it easier to code additional choice variables, such as the amount of debris removal, by adjusting the per-period return function and laws of motion. With debris removal, I use the numerical global optimizer to find the pair (X^*, R^*) which maximizes the value function. This approach allows the code to scale easily to multiple orbital shells or debris types for future work. However, for the problem with debris removal, I only compute solutions to the deterministic model.

Computing the open access equilibrium is much simpler than computing the planner's solution. Algorithm 2 describes how I compute the open access equilibrium with debris removal. As in the case of finding the planner's solution, I use a numerical global optimizer on the objective function rather than a rootfinder on the first-order condition to avoid issues of multiple optima and facilitate future extensions. To compute the open access equilibrium launch rate without debris removal, I skip the first step of the algorithm 2 and set $R, R' = 0$ in the laws of motion and expected collision risk.

[26] I use $||\cdot||_\infty$ for the sup norm.

Algorithm 2: Open access launch and removal plans

1 At each point on the final grid used in the planner's solution, use a numerical global optimizer to obtain

$$R_i^o = \text{argmax}_{R_i}\{\pi - cR_i + E[\ell|S, D - SR_i]F\},$$

and set $R^o = SR_i^o$.

2 Use a numerical rootfinder to find the X^o which solves

$$E[\ell'|S', D' - R'] = r_s - r,$$

where $E[\ell'|S', D' - R']$ is computed using linear interpolation from $S' = S(1-\ell) + X$, $D' = (D-R)(1-\delta) + G(S, D-R, \ell) + mX$, $E[\ell|S, D-R] = \min\{\alpha_{SS}S^2 + \alpha_{SD}S(D-R), 1\}$, and the mapping R^o from the prior step.

3 Approximate $W_i^\infty(S,D,\ell) = \sum_{t=1}^{\infty} \beta^{t-1}(\pi S - FX^*) + W_i(S,D,\ell)$ as $W_i^T(S,D,\ell) = \sum_{t=1}^{T-1} \beta^{t-1}(\pi S - FX^*) + W_i(S,D,\ell)$. Compute $W_i^T(S,D,\ell)$ by backwards induction, using the laws of motion for S and D and the form of $E[\ell'|S', D']$ and sufficiently large T.

To compute deterministic value and policy functions, the above algorithms can be modified to use only two state variables (S,D) instead of three (S,D,ℓ). The deterministic value and policy functions are much faster to compute than the ones with stochastic collision rates. I show deterministic value and policy functions in the paper for ease of exposition.

9.3 Time paths for the stochastic model

For all of the time path simulations, I simulate the path for a large enough T that finite-horizon effects can be ignored along most of the path. I then truncate the sequences at a t far enough into the sequence to observe convergence to a steady state.[27]

As with the value functions, the nature of open access simplifies computation of open access time paths. However, the addition of debris removal slightly complicates the open access time path computation. Algorithm 3 describes how I compute the open access launch path without debris removal. It is fast even for large T (fractions of a second for $T = 1000$).

[27] This convergence is not guaranteed; see Rao and Rondina (2018) for more details on how the collisions between satellites and debris make periodic or aperiodic oscillations around an open access steady state possible and economically plausible. I set parameter values so that this is not an issue in any simulations in this paper.

Algorithm 3: Stochastic open access launch time path

for t *in* $1,\ldots,T-1$ **do**

Draw ℓ_t from $Bin(\text{floor}(S_t), \min\{aS_t^2 + bS_tD_t, 1\})$

Use a numerical rootfinder to find the X_t^o which solves

$$E_t[\ell_{t+1}|S_{t+1}, D_{t+1}] = r_s - r,$$

using the laws of motion for S_t, D_t, and the form of $E[\ell|S,D]$.

Update S_{t+1} and D_{t+1} using their laws of motion.

end

Set $X_T^o = 0$.

Simulating time paths for the planner, or even open access time paths with removal, is slightly more complicated due to the nature of the stochastic process for collisions. As mentioned above, the dynamics of the satellite and debris stocks make the process for ℓ_t dependent and heterogeneously distributed over time. Open access time paths with debris removal, and the planner's time paths generally, depend on future values of choice variables (R_{t+1} for open access, and (X_{t+1}, R_{t+1}) for the planner), which in turn depend on future draws. Simulating these stochastic processes directly is computationally challenging even in the open access case. Instead, I simulate the analogous deterministic processes. Algorithm 4 describes how I compute the deterministic time path of open access launch rates and cooperative removal.

Algorithm 4: Deterministic open access launch time path with cooperative endogenous debris removal

Set $T = 100$ (some large value). Initialize $\{X_t^1, R_t^1\}_0^T$.

Set $i = 1$ and $\delta = 100$ (some large value to begin).

while $\delta < \varepsilon$ **do**

  Compute the launch rate sequence:

  for t *in* $1, \dots, T-1$ **do**

    Use a numerical rootfinder to find the X_t^{i+1} which solves

$$E_t[\ell_{t+1}|S_{t+1}, D_{t+1} - R_{t+1}^i] = r_s - r,$$

    using the laws of motion for S_t, D_t, and the form of $E[\ell|S,D]$.

    Update S_{t+1} and D_{t+1} using their laws of motion.

  end

  Set $X_T^{i+1} = 0$.

  Compute the removal rate sequence:

  Use a numerical global optimizer to find the $\{R_t^{i+1}\}_0^T$ which maximizes

$$W_i^T(S,D,\ell) = \sum_{t=1}^{T} \beta^{t-1}(\pi S_t - F X_t^{i+1} - c_t R_t^{i+1})$$

  using the laws of motion for S_t, D_t, and the form of $E[\ell|S,D]$.

  $\delta \leftarrow \frac{1}{2}||X_t^i - X_t^{i+1}||_\infty + \frac{1}{2}||R_t^i - R_t^{i+1}||_\infty$

  i $\leftarrow$ i+1

end

1.

10 Tables and figures

Figures in the main text

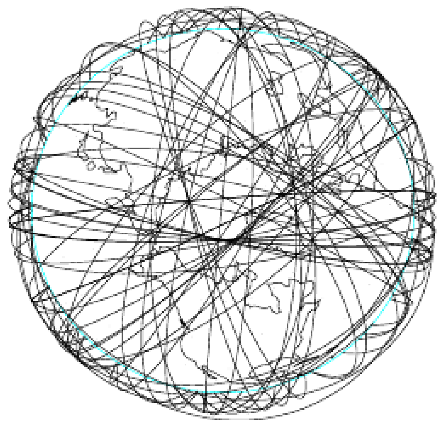

Figure 1: *Orbits of 56 cataloged satellites with mean altitudes between 700-710km. Source: Johnson (2004).*

Table 1: Currently-operational satellites by origin, orbit class, and orbit type as of April 30, 2018

Breakdown of operating satellites					**Total**
by Country of origin	United States: 859	Russia: 146	China: 250	Other: 631	1,886
by Orbit Class	LEO: 1,186	MEO: 112	Elliptical: 40	GEO: 548	1,886
Breakdown of US satellites by Owner Type	Civil: 20	Commercial: 495	Government: 178	Military: 166	859

Source: Union of Concerned Scientists (2018).

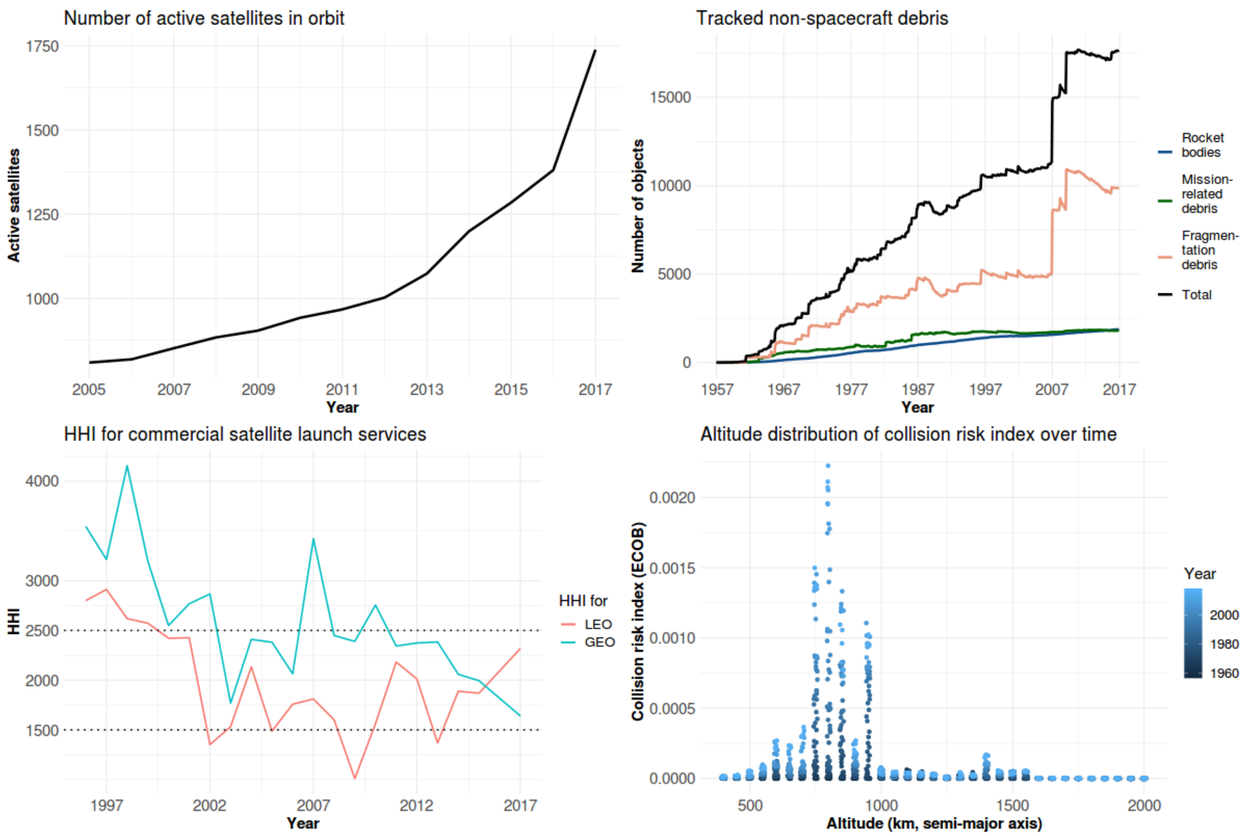

Figure 2: *Trends in orbit use.*

Upper left panel: Number of active satellites in orbit per year since 2005.

Upper right panel: Monthly tracked non-spacecraft debris. These do not include derelict satellites which were not deorbited.

Lower left panel: Herfindahl-Hirschman Index for commercial launch services to low-Earth orbit and geostationary orbit.

Lower right panel: Evolution over time of the spatial distribution of ECOB collision risk index in low-Earth orbit. The large spike between 500-1000km is driven by a combination of commercial activity and China's 2007 anti-satellite missile test.

Sources: Union of Concerned Scientists (2018), NASA Orbital Debris Program Office (2017), and Letizia, Lemmens, and Krag (2018).

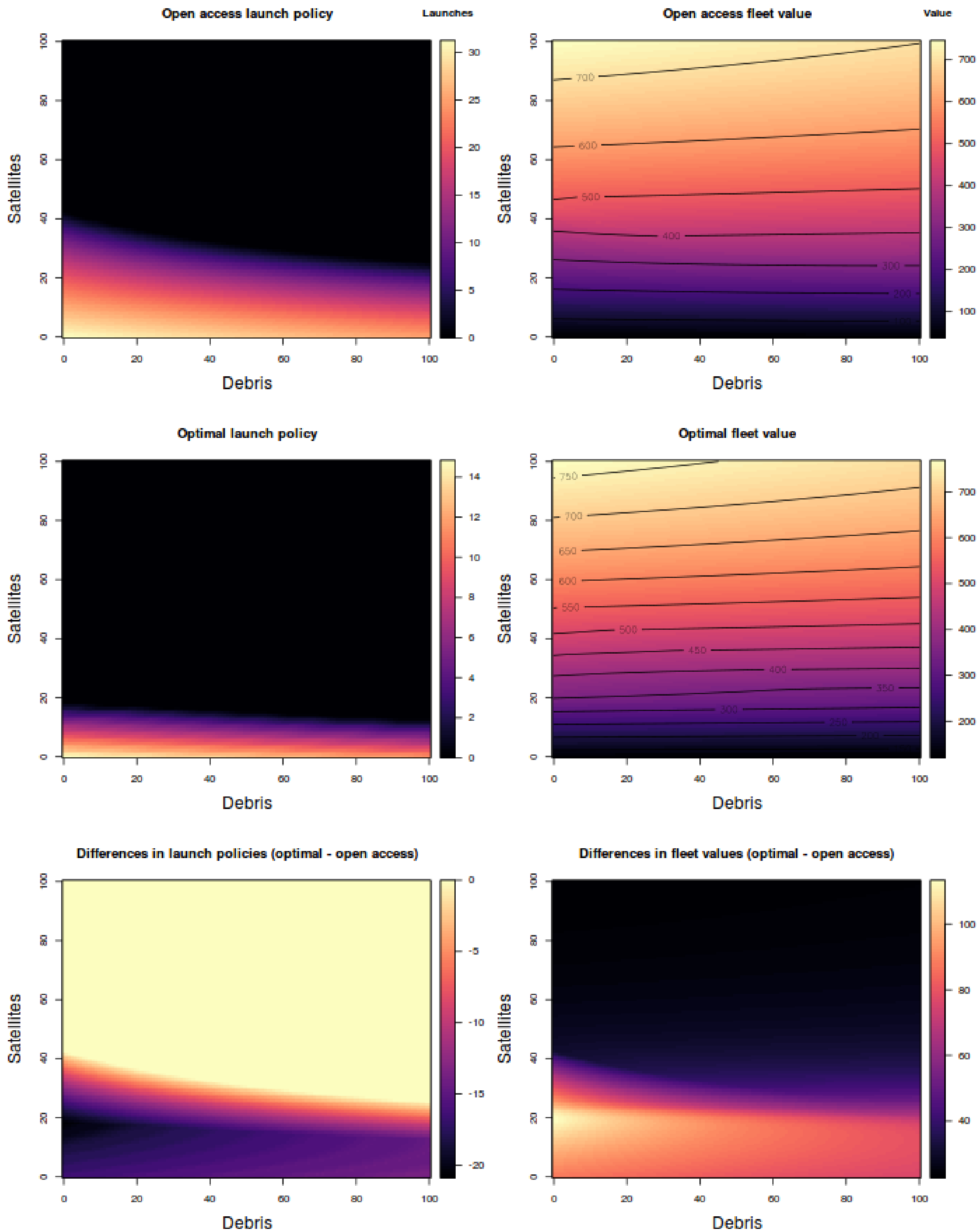

Figure 3: *An example of the gap between open access and optimal launch policies, with the corresponding gap in fleet values.*
The planner launches fewer satellites in every state than open access firms would. The value gap is maximized when (a) there is no debris and (b) the planner would stop launching satellites but open access firms do not.

Table 2: Examples of different types of orbit management policies

	Quantity control	**Price control**
Flow control	Launch permits	Launch taxes
Stock control	Orbit use leases	Satellite taxes

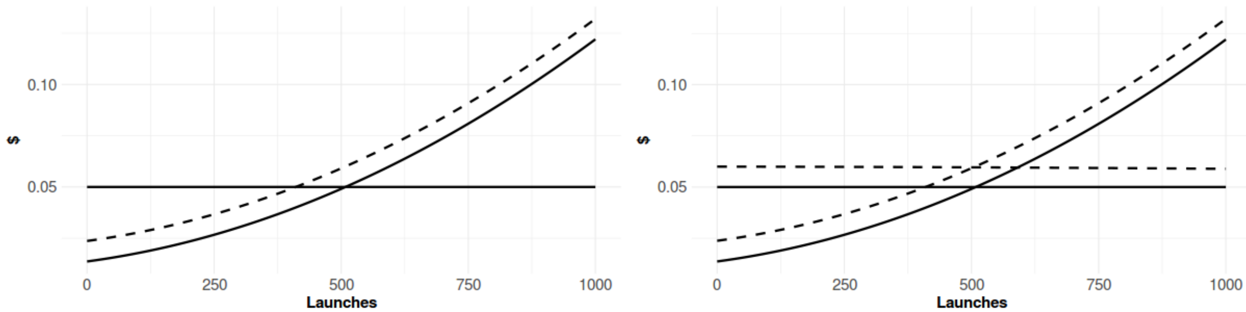

Figure 4: *Private marginal benefits and costs of launching a satellite under stock (left) and flow (right) controls.*
Left panel: The horizontal solid line is the marginal benefit of launching a satellite, while the upward-sloping solid line is the marginal cost. The dashed line indicates the effect of imposing a stock control: the marginal cost is increased, lowering the equilibrium number of satellites launched.
Right panel: The horizontal solid line is the marginal benefit of launching a satellite, while the upward-sloping solid line is the marginal cost. The dashed lines indicate the effects of imposing a flow control: the period t control raises the marginal cost of launching in t, but the entry restriction of the period $t+1$ control raises the marginal benefit of launching in t. Corollary 1 establishes that a constant price will increase the marginal cost by more than it increases the marginal benefit, but the net effect size may be very small.

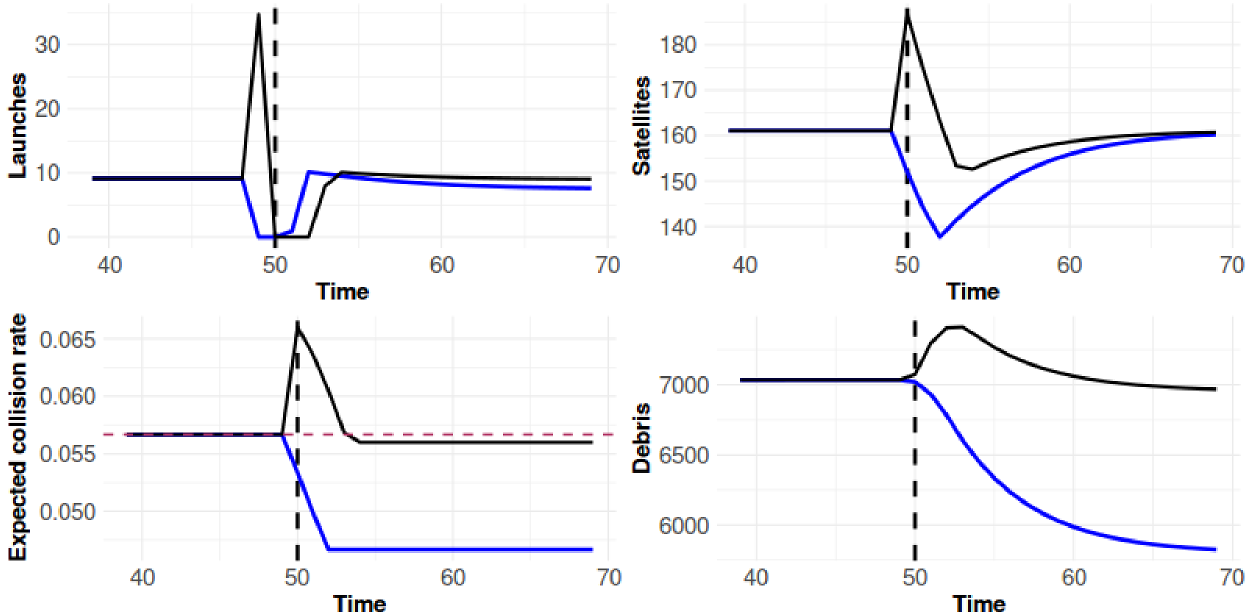

Figure 5: *The effects of introducing a generic constant stock (blue line) or flow control (black line).* The purple dashed line shows the equilibrium collision risk under open access. Introducing a stock control smoothly reduces the expected collision risk and debris stock, while introducing a flow control forces both to jump above the open access levels before they are reduced. The assumption of constancy is made for exposition, and is not important to the result.

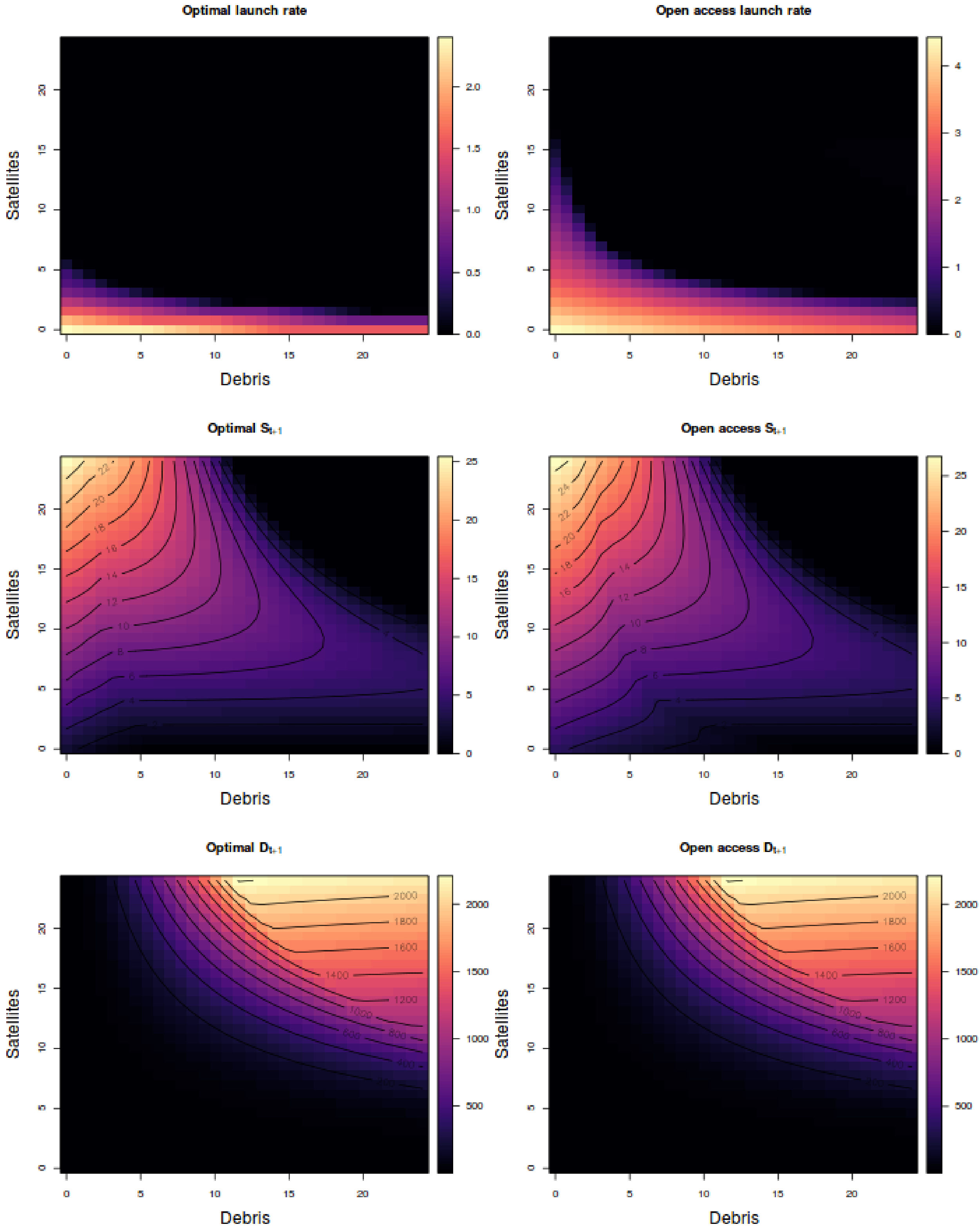

Figure 6: *Optimal and open access stocks and launch rates.*
Left column: Optimal launch rate (X_t), next-period satellite stock (S_{t+1}), and next-period debris stock (D_{t+1}).
Right column: Optimal launch rate (X_t), next-period satellite stock (S_{t+1}), and next-period debris stock (D_{t+1}).
The per-period return on a satellite is normalized to 1, the discount factor is set to 0.95, and the launch cost is set to 10. The open access next-period satellite stock is small but not zero in the upper right of the figure, while the optimal next-period satellite stock is zero there.

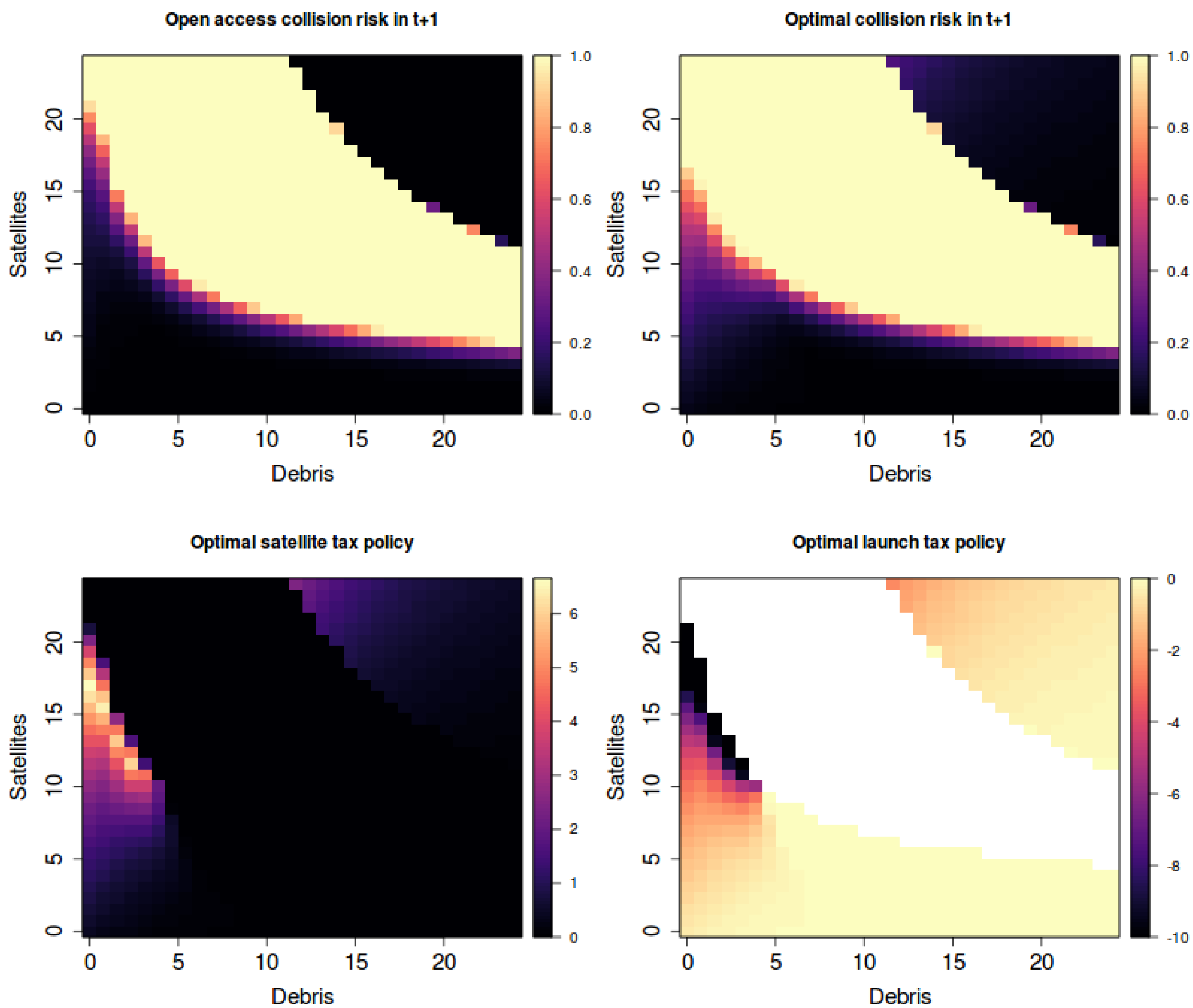

Figure 7: *Optimal space traffic control policies.*
Upper panels: The collision risk in $t+1$ ($E_t[\ell_{t+1}]$) under the optimal launch plan (left) and open access launch plan (right).
Lower left panel: An optimal satellite tax (stock control) in $t+1$.
Lower right panel: An optimal launch tax (flow control) in $t+1$. The tax in period t is normalized to 0.
The tax rates should be read as multiples of the per-period satellite return (normalized to 1). The white areas in the launch tax are where the collision risk is 1 and the tax is undefined; see Lemma 4 for an explanation of this feature. The collision risk jumps from 1 to 0 in the upper right section of the figures because there are no satellite left to be destroyed; see Figure 6 for the underlying satellite and debris stocks and launch rates. The marginal external cost is computed as $E_t[\xi(S_{t+1}, D_{t+1})] = E_t[\ell_{t+1}|\text{open access}] - E_t[\ell_{t+1}|\text{optimal}]$, following equations 9 and 13. The tax rates are then computed according to equations 31 and 32. The jump in the tax rates in the upper right is due to the slight gap in the satellite stocks described in Figure 6.

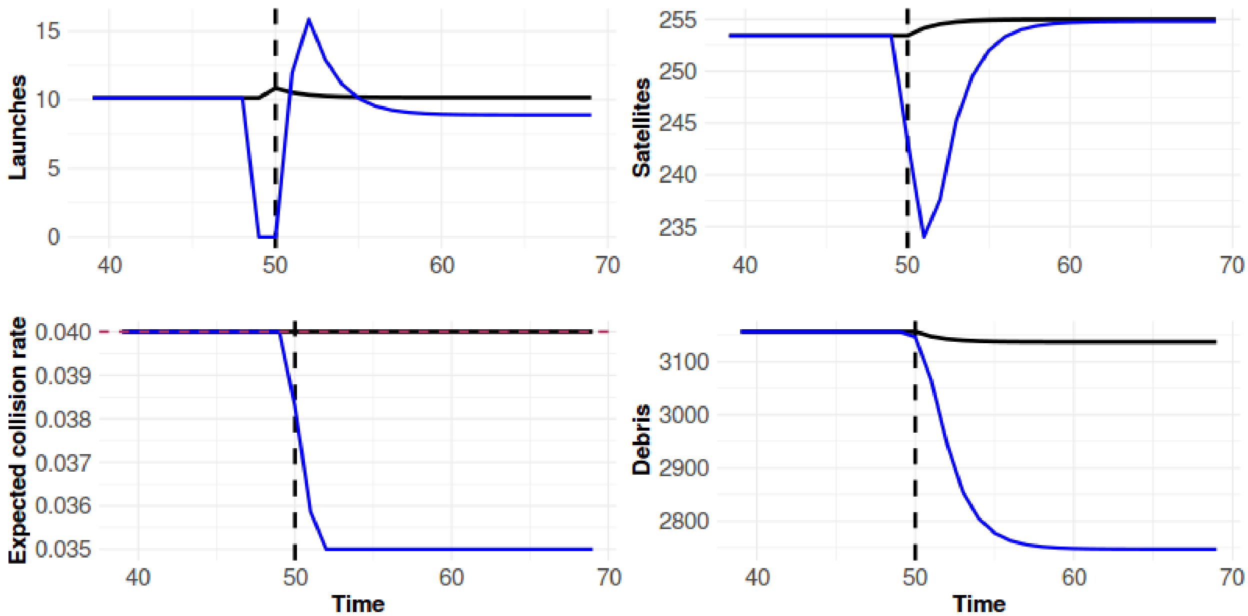

Figure 8: *The effects of exogenous removal for free (black line) or a mandatory fee (blue line). When debris removal is provided to satellites owners for free, potential launchers respond by launching more satellites - even though the debris stock falls, the equilibrium collision risk remains unchanged. The equilibrium collision risk will fall when active debris removal is an option if and only if it is costly to satellite owners. In the case of costly debris removal, the launch rate falls to zero until the expected collision risk is no longer above the new equilibrium level. The dashed red line shows the equilibrium collision risk under open access.*

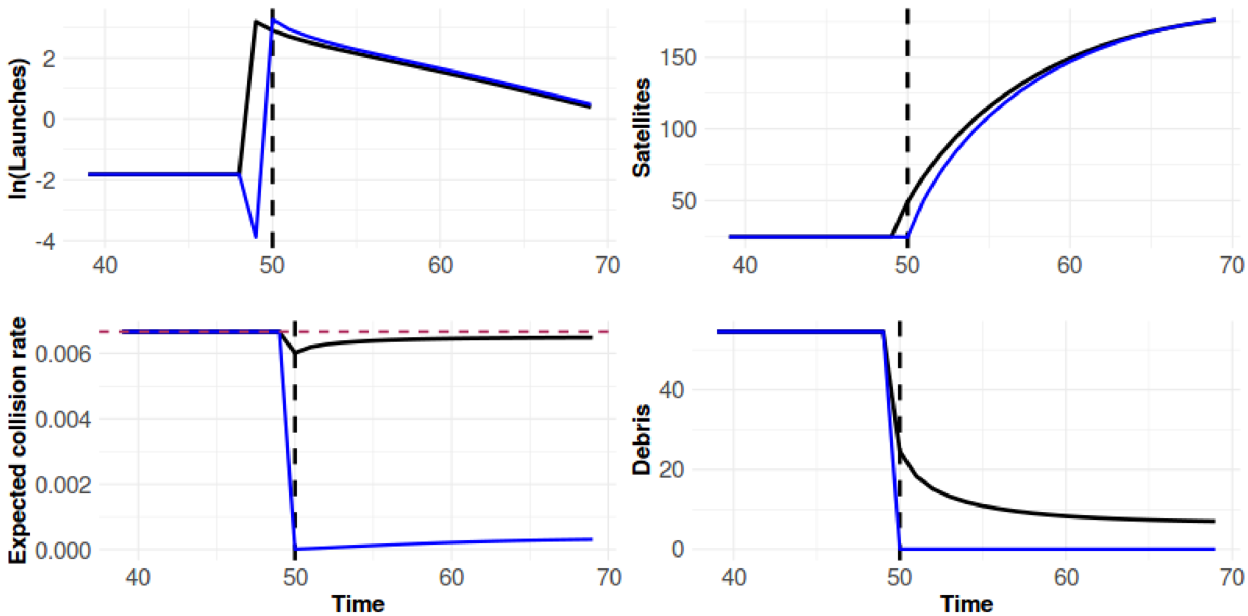

Figure 9: *The effects of endogenously chosen cooperative debris removal (blue line) and exogenous removal for a mandatory fee (black line). The exogenous removal path in the exogenous case is set equal to endogenous removal path. Endogenous removal reduces both the equilibrium collision risk and the debris stock more effectively than exogenous removal, even if the same removal schedule is used. The endogenous removal schedule and launch response involves completely cleaning the orbit initially, and keeping the orbit relatively clean after. The same removal schedule provided exogenously induces firms to launch earlier than they would if they chose the schedule. The dashed red line shows the equilibrium collision risk under open access.*

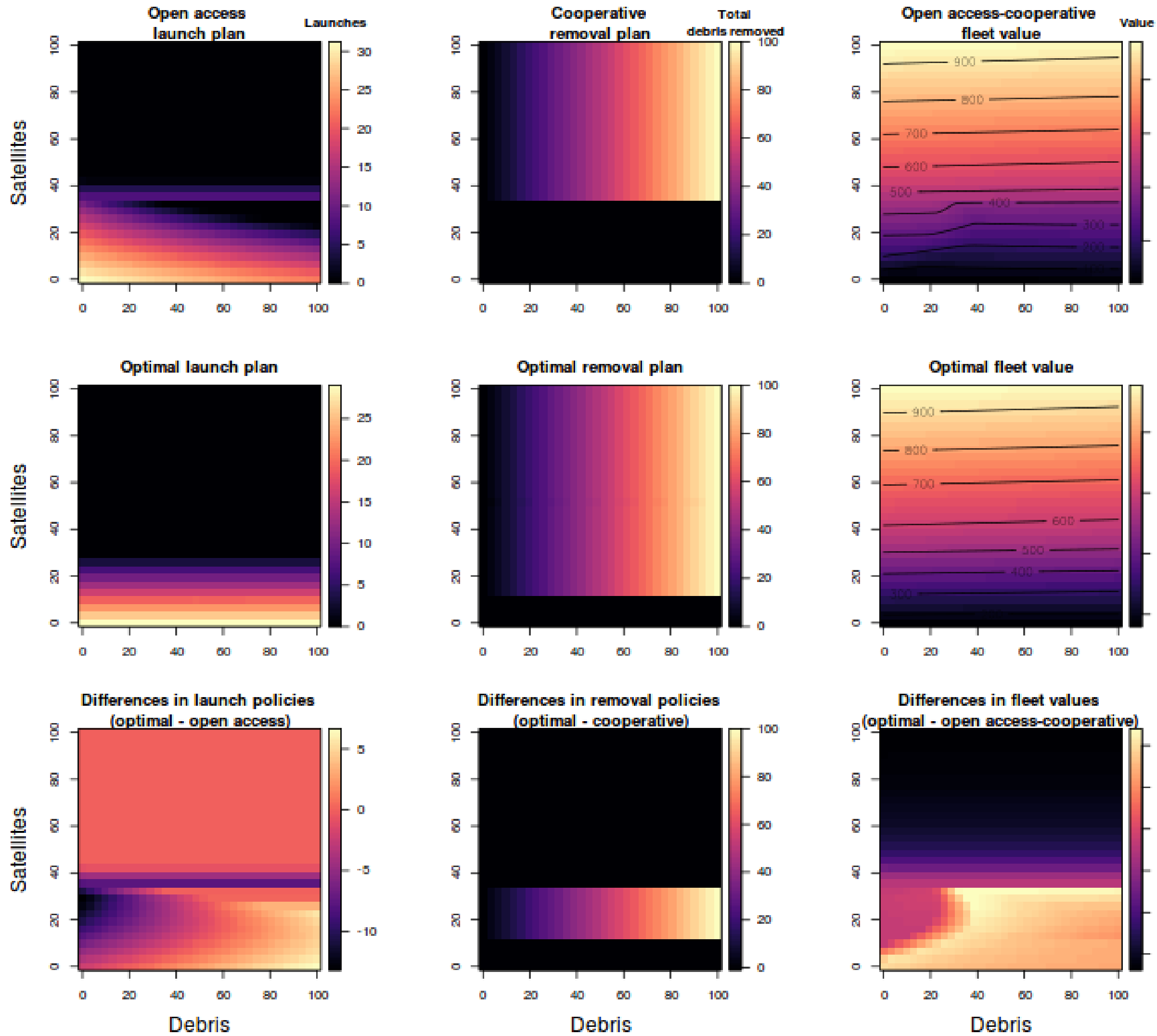

Figure 10: *Comparing optimal and open access-cooperative launch and removal plans.*
Upper row: The open access launch plan (left), cooperative removal plan (middle), and resulting fleet value (right). The jump in the launch plan just above 10 reflects open access launches taking advantage of debris removal beginning, as shown in the time paths in Figure 9.
Middle row: The optimal launch plan (left), optimal removal plan(middle), and resulting fleet value (right).
Bottom row: The gap between optimal plans/values and open access-cooperative plans/values. The gap between optimal and open access-cooperative fleet values is maximized when (a) the planner would begin removing debris but cooperative satellite owners have not, and (b) just before open access launchers begin to launch again (anticipating removal) and the planner has stopped.

Figures in the appendices

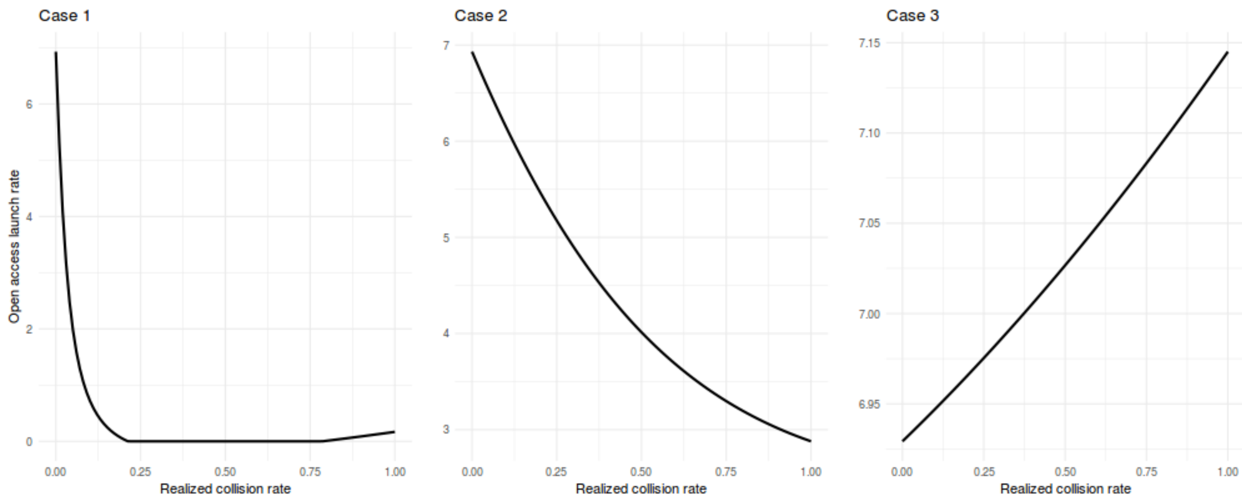

Figure 11: *Three ways the open access launch rate may respond to collision risk draws.*
Left panel: The open access launch rate is decreasing then increasing in the collision risk draw. The effect of additional debris dominates the launch decision when the collision risk draw is low, and the effect of fewer satellites dominates when the collision risk draw is high.
Middle panel: The open access launch rate is uniformly decreasing in the collision risk draw. The new-debris effect dominates for all draws.
Right panel: The open access launch rate is uniformly increasing in the collision risk draw. The fewer-satellites effect dominates for all draws.
The left panel and middle panel cases are more plausible than the right panel case under realistic relative orders of magnitude between the number of fragments created by satellite-satellite, satellite-debris, and debris-debris collisions.

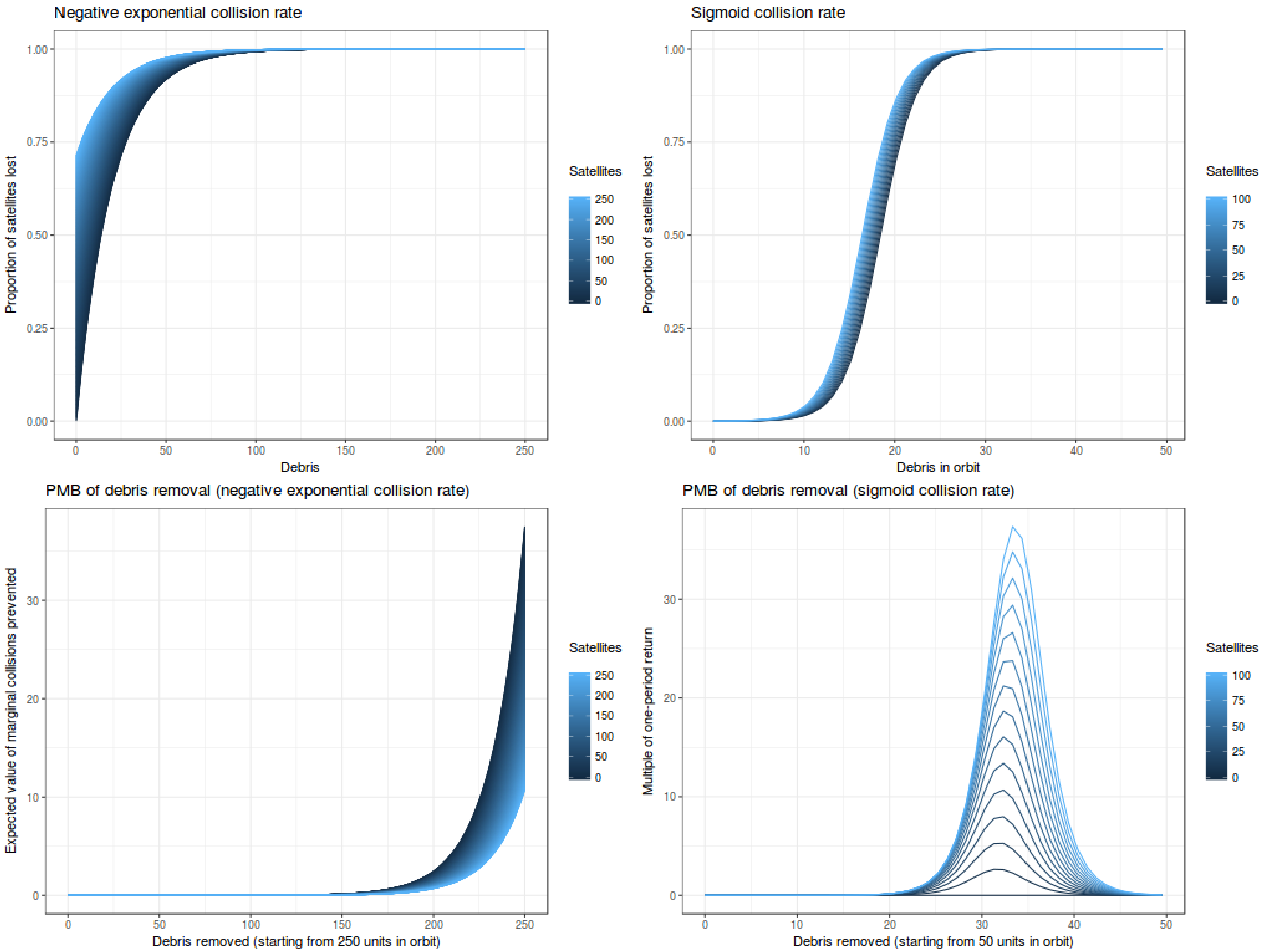

Figure 12: *Two collision rate functions and the private marginal benefit of debris removal.*
Upper row: Collision risks given different levels of debris removal.
Lower row: Private marginal benefits of debris removal.
Left column: Negative exponential collision rate (globally concave).
Right column: Sigmoid collision rate (convex then concave).
Darker colors correspond to fewer satellites. More satellites may reduce or increase the marginal benefits of debris removal, depending on whether satellites and debris are complements or substitutes in collision production.
Not shown: More initial debris in orbit shifts the removal benefit curves to the right. This makes the optimal removal amount increase until a jump to zero removal.

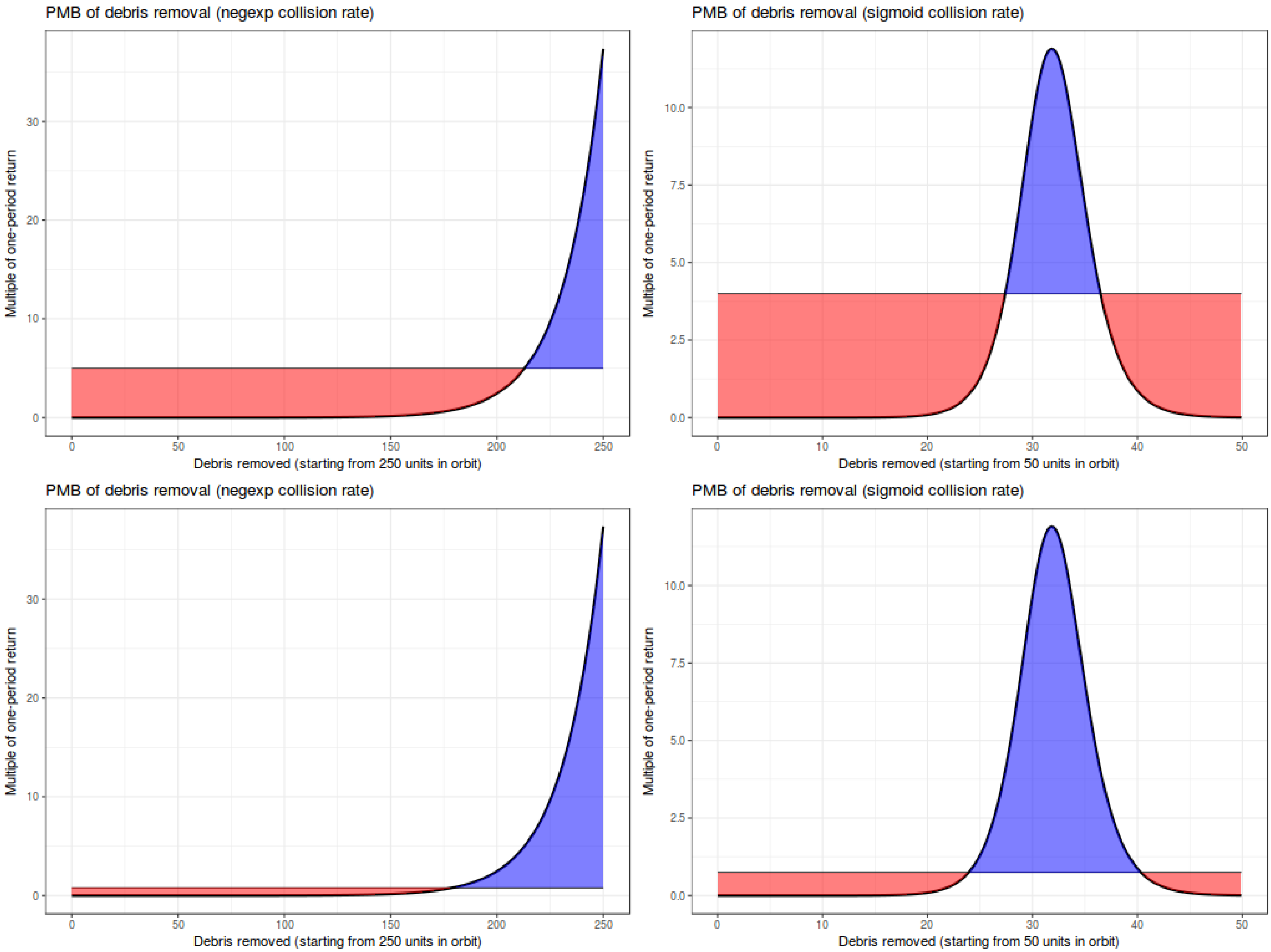

Figure 13: *Nonconvexity and privately optimal removal.*
Upper row: High cost scenario where zero removal is cooperatively optimal.
Lower row: Low cost scenario where some removal is cooperatively optimal.
Left column: Negative exponential collision rate (globally concave), where the optimal removal demand is always on a corner.
Right column: Sigmoid collision rate (convex then concave), where the optimal removal demand may be in the interior.
The thin horizontal line is the marginal cost of removal. The thicker curve is the marginal benefit of removal. Red regions are losses, blue regions are profits. In the upper row, zero removal is optimal. In the lower left panel, full removal is optimal. In the lower right panel, removal of about 40 units is optimal. Because the collision risk is bounded in $[0,1]$, it cannot be strictly convex globally over S and D.

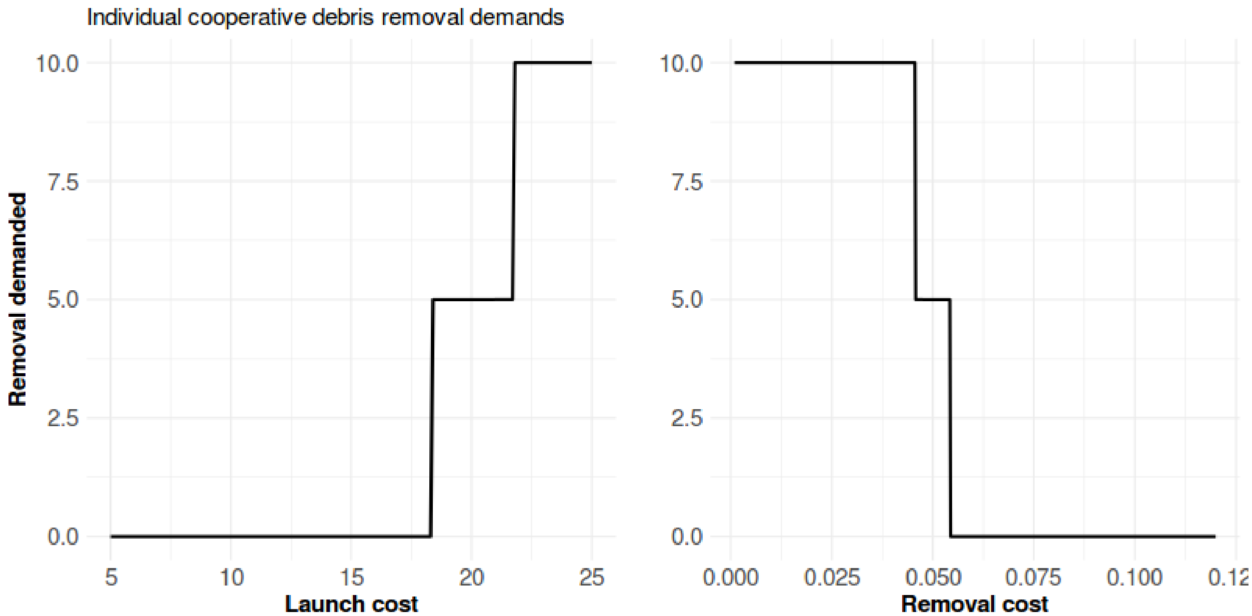

Figure 14: *The effects of changes in satellite launch and debris removal costs on individual cooperative debris removal demands.*
Increases in the cost of launching a satellite increase the open-access value of satellites in orbit, increasing the amount of debris removal demanded. As expected, increases in the cost of debris removal decrease the amount demanded. Costs are stated in multiples of the one-period return generated by a satellite in orbit.

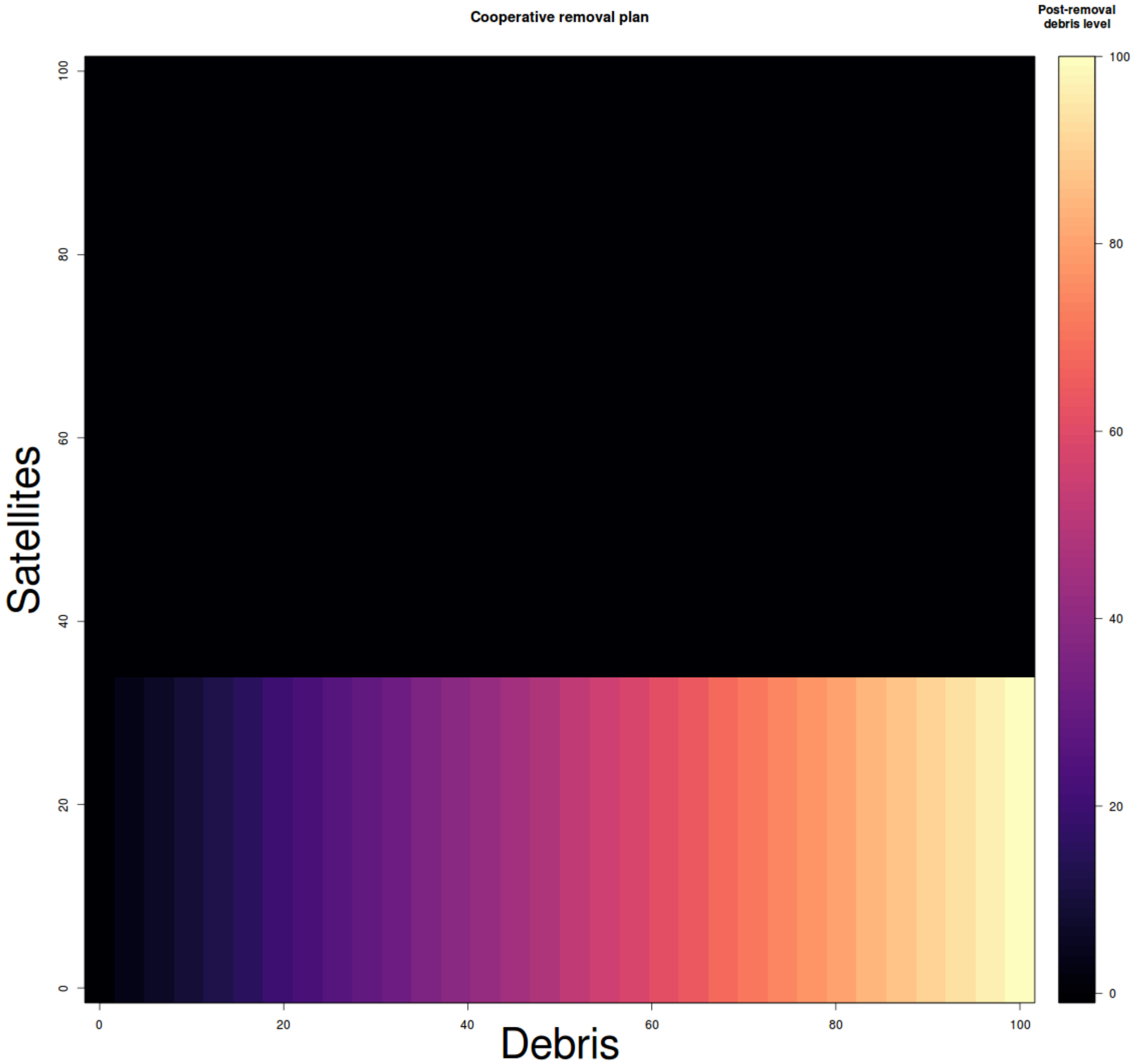

Figure 15: *The effects of changes in the number of firms and debris in orbit on the post-removal level of debris.*
The color scale represents the amount of debris left in orbit after removal. The cooperatively optimal post-removal level of debris does not depend on the amount of debris initially in orbit, but on the number of firms who are available to share the cost of removal. Once there are enough firms to begin removal the post-removal debris level is constant (full removal).

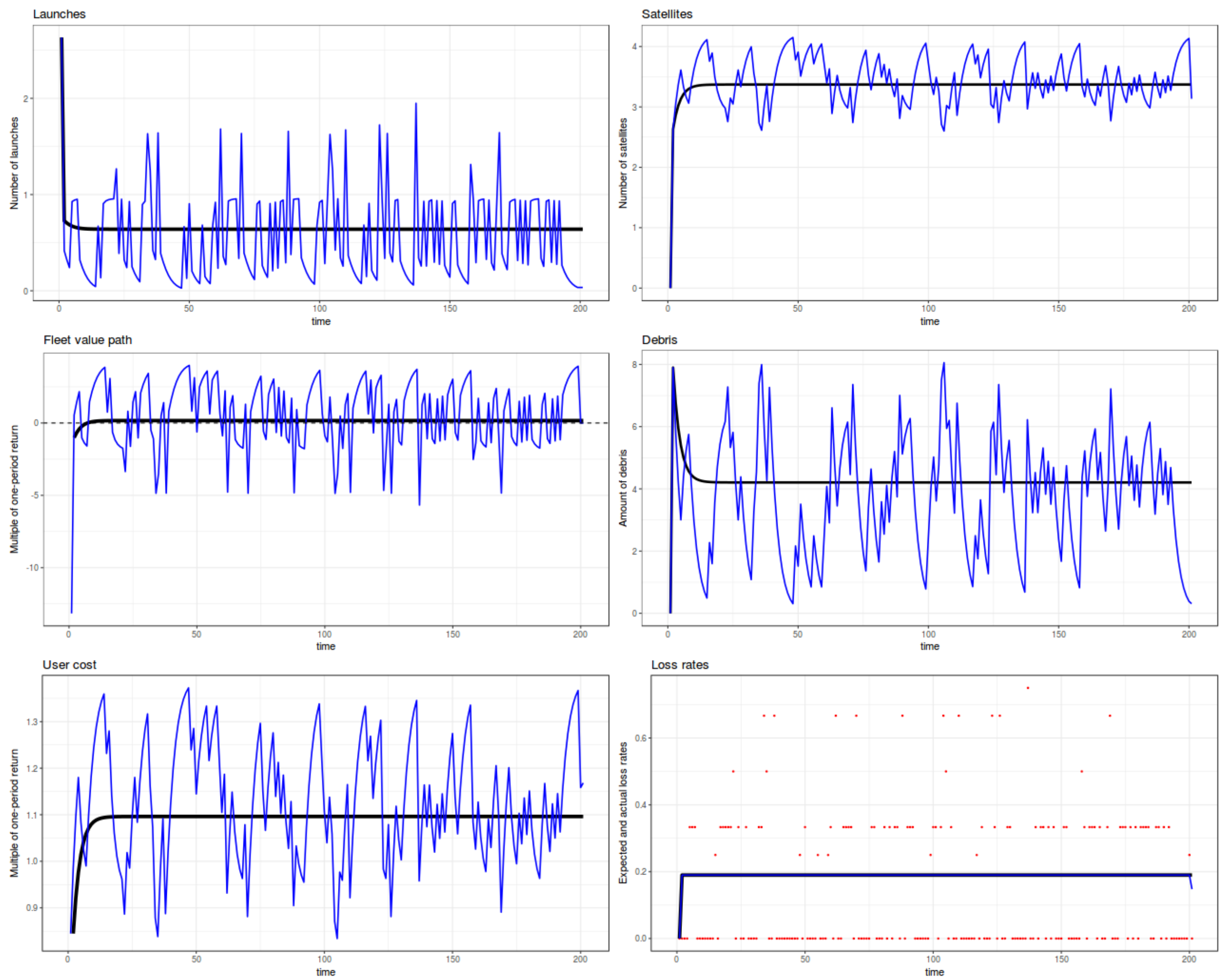

Figure 16: *Time paths under the stochastic (blue line) and deterministic (black line) models.* The red dots in the "collision rates" panel are the draws of ℓ_t.