Functions worksheet 1

A multi-plant linear cost-minimization problem

Robocorp operates three robot manufacturing plants, in Los Angeles, Detroit, and Pittsburgh. You are responsible for optimizing Robocorp's operations. You know that the marginal cost functions for each plant are $MC_L(R) = a_L + b_L R, MC_D(R) = a_D + b_D R$, and $MC_P(R) = a_P$, where $a_L < a_D < a_P$ and $b_L > b_D$.

(a) Draw all three marginal cost functions on the same graph with R on the x-axis. Label the y-intercepts.

(b) Mark on the x-axis where $MC_L = MC_D$ and $MC_D = MC_P$. Label these points as A and B.

(c) Solve $MC_L(A) = MC_D(A)$ and $MC_D(B) = MC_P(B)$ for A and B.

(d) Highlight the minimum marginal cost curve,

$$MC(R) = \min\{MC_L(R), MC_D(R), MC_P(R)\}$$

Robocorp's econometrician estimates the marginal cost functions and tells you that $a_L = 0.1, a_D = 1, a_P = 3$ and $b_L = 2, b_D = 0.2$.

(a) Re-label your graph using the parameter values (don't erase the existing labels, write below or next to them)

The econometrician forecasts demand and tells you to expect 9970 units of demand. Your goal is to allocate production to meet this demand while minimizing production costs. Your plan is to always produce at the cheapest plant[1].

(a) Under your plan, you will produce at Los Angeles until the marginal cost of producing in Los Angeles is equal to the marginal cost of producing at Pittsburgh (call this amount Q_L^*). Solve for $Q_L^* : MC_L(Q_L^*) = MC_P(Q_L^*)$.

(b) You will produce at Detroit until the marginal cost of producing in Detroit is equal to the marginal cost of producing at Pittsburgh (call this amount Q_D^*). Solve for $Q_D^* : MC_D(Q_D^*) = MC_P(Q_D^*)$.

(c) You will produce the remaining units at Pittsburgh. Calculate $Q_P^* = 9970 - Q_L^* - Q_D^*$.

[1] You know this is a good plan because it will equalize marginal costs across all plants, which you remember from your microeconomics class is the relevant cost-minimization condition for this situation.