ECON 1078-003
Problem set # 5

March 16, 2018

Due Monday 4/2 at the start of class. Please write clearly, draw a box around your final answer, and submit your answers in the order listed below. If there are no real solutions, write that a real solution does not exist.

This problem set will be graded out of 5 points.

Book Problems:

Section 4.9: Numbers 7, 8, 9

Section 4.10: Numbers 2, 4, 5

Word Problems:

1. Statisticians and econometricians often want to estimate parameters of linear models from data that they have found or collected. The "least-squares" approach is a popular way to do this. Least-squares methods involve finding parameter values which minimize a quadratic function of the model, using the data to determine numerical values for the variables. In this problem, we will use a least-squares method to estimate a parameter of a model.

Suppose you are an economist interested in a linear model, $Y = X\beta + \epsilon$, where (X, Y) are observed data points; Y is the variable you want to predict; X is a variable you think can explain Y; β is the parameter you want to estimate which describes the relationship between X and Y; and ϵ is a "statistical error" which accounts for things other than X which affect Y.[1] To estimate β you are going to try to minimize the squared error, $\epsilon(\beta)^2$.

(a) You observe a data point, (X, Y). Write the error ϵ as function of X, Y, and β, and call it $\epsilon(\beta)$.

[1] For example, maybe you're a health economist studying the effect of cigarette taxes on smoking rates. Y could be a measure of the number of people in an area who smoke, and X could be the amount of a cigarette tax in the area.

(b) Write the squared error function, $\epsilon(\beta)^2$. Can the squared error be minimized, i.e. is it a cup or a cap?

(c) Using what we've learned about optimizing quadratic functions, find the β which minimizes $\epsilon(\beta)^2$.

(d) Suppose $Y = 1$ and $X = 0.5$. Plot $\epsilon(\beta)$ and $\epsilon(\beta)^2$ on the same graph. Your graph should have β on the horizontal axis, and the value of the functions on the vertical axis.

(e) Now suppose you get two data points, (X_1, Y_1) and (X_2, Y_2), giving you two equations, $Y_1 = X_1\beta + \epsilon_1$ and $Y_2 = X_2\beta + \epsilon_2$ (notice that β is common to both equations). Find the β which minimizes the "sum of squared errors" function, $\epsilon_1(\beta)^2 + \epsilon_2(\beta)^2$.

2. A function $f(x) : D \to R$ is called "homogeneous of degree m" if $f(tx) = t^m f(x)$ for any $t > 0$.

(a) Prove that $f(x) = ax^2$ is homogeneous of degree 2. (Hint: Show that $f(tx) = t^m f(x)$ for any $t > 0$.)

(b) Suppose that $f(x)$ is homogeneous of degree k. Show that $f(x) = 0 \implies f(rx) = 0$ for any $r > 0$.

(c) What is the degree of homogeneity of $Y(k) = Ak^b$, $b > 0$?