ECON 1078-002
Problem set # 8

April 28, 2018

Due Wednesday 5/2 before 11:59pm. All questions refer to the 4th edition of the textbook. Please write clearly, draw a box around your final answer, and submit your answers in the order listed below. This problem set is worth 10 points.

3.1: 2
3.2: 1, 3(a)
3.7: 3

Word problem:

Two firms, 1 and 2, are competing in the market for widgets. x_1 is firm 1's widget production, and x_2 is firm 2's widget production. The inverse demand function (the price) is

$$P(x_1 + x_2) = A - B(x_1 + x_2),$$

where A and B are positive constants. The firms have identical cost functions,

$$c(x_i) = \alpha x_i,$$

where α is a positive constant such that $A > \alpha$.

1. Suppose x_2 is just a constant. Find the x_1 which maximizes Firm 1's profits,

$$P(x_1 + x_2)x_1 - c(x_1).$$

(Your answer should be a function $x_1(x_2)$.)

2. Suppose x_1 is just a constant. Find the x_2 which maximizes Firm 2's profits,

$$P(x_1 + x_2)x_2 - c(x_2).$$

(Your answer should be a function $x_2(x_1)$.)

3. Plot $x_1(x_2)$ and $x_2(x_1)$ with x_1 on the horizontal axis and x_2 on the vertical axis. Use algebra to solve the system of equations $x_1(x_2)$ and $x_2(x_1)$ for x_1^* and x_2^*. Label your solution on the graph.

4. Now suppose Firm 1 buys Firm 2, creating a monopoly in the market for widgets. Find the production level, x, which maximizes the monopoly's profits,

$$\max_x P(x)x - c(x).$$

(Your answer should be an equation for x. Call the solution $\hat{x}$.)

5. Use your solutions from 3. and 4. to prove the following results:

(a) the monopoly produces less total output than the duopoly (i.e., that $\hat{x} < x_1^* + x_2^*$), and

(b) the price is higher under the monopoly than under the duopoly (i.e., that $P(\hat{x}) > P(x_1^* + x_2^*)$).