

# Cost in Space: Debris and Collision Risk in the Orbital Commons

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## Abstract

As Earth's orbits fill with satellites and debris, the probability of collisions between orbiting bodies increases. Runaway debris growth, known as Kessler Syndrome, may cause Earth's orbits to become unusable for millennia. We present the first economic model of Earth orbit use which accounts for the risk of satellite-destroying collisions and Kessler Syndrome under rational expectations in the short and long runs. Orbital decay and profit maximization can prevent Kessler Syndrome even in the absence of cleanup technologies, but open access will result in inefficiently high levels of launches, debris, and collision risk. Steady-state debris levels and the equilibrium collision probability are increasing in the excess return on a satellite, and sustained increases in the excess return will lead open access to cause Kessler Syndrome. Short-run "rebound" effects can also make open-access debris levels increase as the rate of orbital decay increases and as launches generate less debris. These results suggest that careful attention to economic incentives, particularly the returns to satellite ownership, is necessary to ensure orbital sustainability.

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# 1 Introduction

Earth's orbits are the world's largest common-pool resource, and increasingly necessary for services that power the modern world. As society launches more satellites, the risk of collisions between orbiting objects increases. Such collisions can destroy satellites and produce orbital debris, further increasing the risk of future collisions. Collision risk and debris accumulation threaten active satellites and the future of human activity in space. How will open access affect orbital debris accumulation, satellite collision risk, and occurrence of Kessler Syndrome? These questions have been explored very little in economics, and have not yet been addressed in the physics and engineering or law and policy literatures.<sup>1</sup> In this paper, we examine the consequences of open access to orbit in the first long-run dynamic economic model of satellite launch, show that the equilibrium collision probability is determined by the excess return on a satellite, and consider the economics of debris accumulation.

Satellites produce debris over their lifecycle. Launching satellites produces orbital debris (spent rocket stages, separation bolts), satellites can produce some debris while in orbit (paint chips, lost tools, etc.), and satellites which are not deorbited or shifted to disposal orbits at the end of their life become debris.<sup>2</sup> Objects in orbit move at velocities higher than 5 km/s, so debris as small as 10 cm in diameter can be hazardous to active satellites.

Though debris does deorbit naturally, this process can be very slow, especially at higher altitudes. Debris as low as 900 km above the Earth's surface can take centuries to deorbit, while debris at 36000 km can take hundreds of years (Weeden (2010)). Compounding the problem is the fact that collisions between debris objects can generate even more debris, also moving at high velocities and capable of damaging operational satellites.<sup>3</sup> It is possible for debris accumulation to cause a cascading series of collisions between orbital objects, resulting in an expanding field of debris which can render an orbital region unusable and impassable for thousands of years. Engineers and physicists call this phenomenon "collisional cascading" or "Kessler Syndrome" (Kessler and Cour-Palais (1978)). Kessler Syndrome can cause large economic losses, directly from damage to active satellites and indirectly from limiting access to space (Bradley and Wein (2009), Schaub et al. (2015)). Proposed commercial uses of space, like low-Earth orbit (LEO) broadband internet constellations with global coverage, asteroid mining, and space-based solar power, could become infeasible if Kessler Syndrome occurs. Existing estimates of debris growth indicate that the risk of Kessler Syndrome is highest in

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<sup>1</sup>Wienzierl (2018) highlights a number of issues in the development of a space economy, from space debris to coordination problems and market design, which economists have the tools to address.

<sup>2</sup>Reusable rockets can significantly lower the cost of launching and the amount of launch debris generated. SpaceX and Blue Origin are currently the only launch providers who offer such vehicles.

<sup>3</sup>The risk of debris striking another satellite is generally orders of magnitude larger than the risk of debris deorbiting and harming consumers directly.

LEO, placing imaging and future telecommunications satellites at risk and potentially reducing access to higher orbits ([Kessler et al. \(2010\)](#)). Currently, there are more than 1,000 operating satellites in orbit, up to 600,000 pieces of debris large enough to cause satellite loss, and millions of smaller particles that can degrade satellite performance ([Ailor et al. \(2010\)](#)). Approximately 49 percent of active satellites are in LEO, 41 percent are in geostationary orbit (GEO), and the remainder are in elliptical or other orbits [Union of Concerned Scientists \(2017\)](#).

Existing legal frameworks for orbit use, such as the Outer Space Treaty, complicate the process of establishing of explicit orbital property rights and hinder cleanup efforts. For example, Article 2 of the OST forbids national appropriation or claims of sovereignty over outer space, which could be interpreted as prohibiting national authorities from unilaterally establishing orbital property rights ([Gorove, 1969](#)).<sup>4</sup> As launch and satellite costs decrease and more firms enter markets for satellite services, the debris problem is likely to get worse ([Selk \(2017\)](#)).

The congestibility of orbital resources has been recognized in economics as early as Sandler and Schulze's 1981 paper, "*The Economics of Outer Space*" ([Sandler and Schulze \(1981\)](#)). In it the authors present a series of models to analyze issues which may face a future space economy, including a static optimization program to manage orbital spectrum and positions as club goods. Their program accounts for congestion due to radio frequency interference and collision risk, but ignores debris accumulation. Despite this early recognition of economic externalities in orbit, economists have paid relatively little attention to orbital management, likely in part due to the lack of development in commercial space markets. More recent economic analyses have considered the static economic costs of inefficient electromagnetic spectrum and position use ([Macauley, 1998, 2015](#)), the inefficiency of open access and voluntary debris mitigation under monopolistically competitive behavior in static or two-period settings ([Adilov, Alexander, and Cunningham, 2015](#)), the economic incentives to prevent complete orbital unusability in a relatively-uncoupled physical environment ([Adilov, Alexander, and Cunningham, 2018](#)), the steady-state inefficiency of open access in terms of the value of a individual satellites ([Rouillon, 2019](#)), and the use of ex-post takeback schemes to incentivize ex-ante satellite quality in a stationary state of low-Earth orbit ([Grzelka and Wagner, 2019](#)). To our knowledge, we are the first to bridge the short-term and long-term physical and economic dynamics of perfectly-competitive open-access and socially-optimal orbit use in a highly-coupled environment which allows for Kessler Syndrome to be caused unintentionally over

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<sup>4</sup>[Weeden \(2010\)](#) and [Weeden and Chow \(2012\)](#) are skeptical of what decentralized bargaining can achieve in this setting, given the difficulties in solving other global coordination problems, like climate change, and the number of current and potential orbit users and their incentives. [Salter and Leeson \(2014\)](#) and [Salter \(2015\)](#) take more optimistic views of what commercial users can achieve under celestial anarchy, the former based on the idea of self-enforcing property rights and the latter based on the Coase Theorem, technological advancement, and insurance markets.

multiple periods. Our analysis generalizes those presented in [Adilov, Alexander, and Cunningham \(2015, 2018\)](#) and [Rouillon \(2019\)](#) in the physical dimensions by allowing non-stationary dynamics and runaway debris growth, and makes the sources of external effects clear by explicitly considering the economics of general forms of couplings between physical state variables. While our model is most similar to that of [Rouillon \(2019\)](#), it is not always directly comparable due to differences in model assumptions and the types of planner we consider.

Orbital congestion has been studied more actively in physics and engineering, beginning with Kessler and Cour-Palais' analysis of the creation of a debris belt ([Kessler and Cour-Palais \(1978\)](#)). This community has focused on two areas: how orbital congestion might evolve in particular orbits as the numbers of satellites and debris fragments increase, and how satellite systems and trajectories should be designed to be more robust to spectral congestion and physical collision risk. Papers in the former literature have relied on launch rate models estimated from historical trends, and have abstracted away from optimizing or forward-looking behavior by the agents launching the satellites. Gordon's observation on early models of fisheries ([Gordon \(1954\)](#)) comes to mind:

On the whole, biologists tend to treat the fisherman as an exogenous element in their analytical model, and the behavior of fishermen is not made into an integrated element of a general and systematic 'bionomic' theory... the 'overfishing problem' has its roots in the economic organization of the industry.

Similarly, with the exception of the economic analyses cited above, current models of Earth orbit use applied by major space agencies and regulatory bodies do not explicitly account for the economic incentives involved in launching satellites.

First, satellites are assets, and collision risk reduces their expected present value. This gives firms an incentive to avoid launching if the collision risk becomes too high, potentially stabilizing orbit use by deterring entry. However, because orbits are an open access commons, firms will launch until the collision risk makes expected profits zero, rather than stopping when expected profits are maximized.<sup>5</sup> Entry until zero profits would be socially optimal if not for the presence of endogenous collision risk (i.e. collision risk that is physically coupled with the decision to launch). Second, debris is an accumulating stock pollutant which only matters because it increases collision risk. This gives firms an incentive to prevent debris accumulation, but only to the extent to which they are directly affected by it now and in the future. Since open access makes the expected net present value of owning a satellite zero, firms' incentives to protect the future value of their satellites is reduced. The problem of debris accumulation is

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<sup>5</sup>The geosynchronous belt is an exception; though it is still a commons, it is not under open access. The International Telecommunication Union auctions orbital slots which national authorities then allocate. See [Macauley and Portney \(1984\)](#) and [Jones et al. \(2010\)](#) for more discussion of these mechanisms.

made uglier by the potential for Kessler Syndrome, but it is not primarily a pollution problem.<sup>6</sup>

The feedback from collision risk to profits causes the equilibrium launch rate to decrease as collision risk increases. This can make the equilibrium launch rate also decrease as the debris stock increases, but the lack of feedback from future satellite operators to present satellite operators means that the equilibrium launch rate may not be responsive enough to debris accumulation to prevent Kessler Syndrome. The appropriate property rights or system of charges could make the collision risk level efficient and prevent runaway debris growth.

We build on prior economic analysis of orbital debris (such as (Adilov, Alexander, and Cunningham, 2015; Macauley, 2015)) by extending the setting from two periods to arbitrary finite and infinite horizons and by allowing more general analytical forms of satellite and debris accumulation. We build on the analysis presented in Rouillon (2019) by considering non-stationary dynamics and by “opening the black box” of physical linkages somewhat between variables. We contribute to the literature on common-pool resource management in the presence of environmental risk by incorporating a capital stock that is affected by congestion of the commons and a pollution stock which increases congestion. We do not consider spectrum use explicitly, though we comment on the effects of spectrum congestion in section 5.1.

Open access is characterized by too many launches and collisions and too much debris relative to the socially optimal plan, in the short-run and the long-run. While open-access steady states cannot cause Kessler Syndrome and firms always face incentives to limit satellite-destroying collisions, open access may cause Kessler Syndrome while approaching a steady state. Intuitively, open access dissipate the rents firms would be able to earn by adjusting their launch rates to manage longer-term satellite-debris dynamics, leaving firms with no incentive to account for those dynamics. Like bioeconomic collapse in fisheries, Kessler Syndrome becomes more likely under open access as the cost of launching satellites falls, or more generally as the excess return on a satellite rises. Regions with higher decay rates or lower incidence of launch debris may have higher long-run debris levels due to a combination of short-run rebound effects and local instability of long-run equilibria. Additionally, increases in the rate of excess return earned by a satellite will tend to locally destabilize open-access steady states with higher levels of debris. These results suggest that purely technical solutions aiming to limit debris growth by targeting launch debris or debris decay rates may ultimately increase

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<sup>6</sup>Haveman (1973) describes the similarities and differences between open access, congestion, and pollution externalities in more detail. The key distinction between congestion and pollution is that congestion is a reciprocal externality stemming from crowding costs not being reflected in marginal use decisions, while pollution is a one-sided externality borne by agents who are different from the ones creating the pollution. Pollution is part of the problems of orbit use insofar as debris is an intertemporal externality imposed by current users on future users, but debris also imposes costs on current users, so the distinction is not clean-cut. Since debris is irrelevant in the absence of collision risk, it seems reasonable to argue that congestion, not pollution, is the heart of the problems of social cost in orbit.

the amount of orbital debris or destabilize existing equilibria. Market-based instruments or a new legal paradigm for orbit use may be essential for orbital sustainability.

## **2 Social cost in orbit**

### **2.1 Orbits as an environmental resource**

Orbits share characteristics of renewable and nonrenewable resources. On any timescale relevant for human decision making, the Earth will continue to exert its gravitational pull. Bodies in its gravity well will eventually either exit the well or fall to the surface. In low-Earth orbits, the decay is fast enough (on the order of hours to years) that the gravity well and paths within it are renewable on human timescales. In higher orbits, such as the geosynchronous belt, the decay is slow enough (on the order of millions of years) that the well and its paths are nonrenewable on human timescales. Between the two lie a continuum of possible paths through the well, each with their own rates of renewal. Elliptical paths through the well, such as Molniya orbits, span multiple regions of renewability.<sup>7</sup>

Fishing offers useful intuition for the economics of orbit use. In open-access fisheries, fish are harvested until the expected profits of harvesting another fish are zero. This may drive the stock below the minimum level required for biological renewal. While fishing may become uneconomical at or past that level, once the threshold is crossed the natural dynamics of the stock will drive the fish population to zero anyway. The stock level at which fishing becomes uneconomical may also be above the minimum biological level, in which case open access will result in sustainable, if suboptimal, harvest rates. The lack of property rights over the fish stock induces myopic fishing — though the fish stock evolves over time, fishers make a sequence of static decisions about harvest levels. Since future profits will be zero anyway due to open access, there is no incentive to conserve today for higher profits tomorrow.

As in an open-access fishery, firms under open access launch satellites until expected profits are zero, and may fill the orbits past the maximum level at which natural decay can prevent net debris growth. While satellite launching may become uneconomical at or past that level, the natural dynamics of orbital mechanics will cause further collisions and debris growth if the number of objects in orbit is not reduced. If owning a satellite is not uneconomical before this occurs, it is likely to become uneconomical eventually.

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<sup>7</sup>A Molniya orbit has a low perigee over the Southern Hemisphere and a high apogee over the Northern Hemisphere. Molniya orbits require less power to cover regions in the Northern Hemisphere (e.g. former Soviet Union countries) than geosynchronous orbits, due to the low incidence angles of rays from the Northern Hemisphere to geosynchronous positions.

The neoclassical growth model with pollution is another helpful reference point. Satellites are capital assets which produce a constant per-period net profit, but incur investment costs to build and launch. Satellite launching and collisions generate debris. The collision probability is similar to the capital depreciation rate in how it enters the law of motion for satellites. Debris is a stock of residuals from production (launch debris) and “depreciation” (collision fragments). If debris did not cause collisions, it would be irrelevant and only the satellite stock would need management. If the collision probability were uncoupled from even the satellite stock, then this model would reduce to the neoclassical growth model with an irrelevant pollution stock. Open access would then be efficient.

The coupling between the satellite stock and the collision probability ( $L(S)$ ) implies congestion in orbit under open access, since firms will not internalize the cost of the risks their satellites pose to others.<sup>8</sup> Adding a coupling between the debris stock and the collision probability ( $L(S, D)$ ) adds to the congestion.

When a firm launches a satellite, its entry to the orbit adds launch debris to the orbit. While its satellite is in orbit, the firm contributes to congestion in the orbit. If the satellite is lost in a collision, its removal reduces the risk to survivors, but the new fragments from its destruction increase the risk to survivors later. Firms ignore the congestion they cause through the debris created by their launch, the addition of their satellite to the orbit, and the new fragments created by their satellite’s eventual destruction. The debris effect of launch is “dynamic” congestion imposed by a firm’s entry on itself and others. The risk generated by a satellite’s presence is “steady state” congestion imposed by a firm on others. The debris effects of destruction are also dynamic congestion imposed by a firm’s exit on others. As long as launches create debris, marginal satellites in orbit are bundled with debris creation, so satellites and launch debris are also complements in producing social value. The essential tradeoff of the planner’s problem is in balancing the lifetime value created by satellites and launch debris against the present and future congestion created by each.

## 2.2 A simple model of orbital mechanics

An orbital region is a set of closed paths around a central body. When the paths are chosen to form a closed spherical shell around the central body, the orbital region is called an orbital shell. Paths which span shells are possible and useful for some applications, but highly elliptical orbits (such as Molniya orbits) are the exception rather than the rule. In what follows, we

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<sup>8</sup>This is a mirror of the difference between a competitive firm which doesn’t internalize the effect of its entry on the price and a monopolist who internalizes the price effect of marginal units of production. As in other natural resource settings, an orbital monopolist acts as a “satellite conservationist” by restricting and resequencing launches to preserve the fleet.



assume all bodies are in an orbital shell. This “shell of interest” approach is frequently used in debris modeling, e.g. Rossi et al. (1998) and Bradley and Wein (2009), though higher fidelity models use large numbers of small regions to track individual objects, e.g. Liou et al. (2004), Liou and Johnson (2008), and Liou and Johnson (2009). We abstract from the composition of orbital stocks, and assume that all satellites and debris are identical. Our model is best applied to individual orbital shells, e.g. the region between 600-650km above mean sea level, as we do not explicitly account for variation in physical and economic parameters with altitude. Unless otherwise specified, we use the term “orbit” interchangeably with “orbital shell”.

The number of active satellites in orbit in period  $t + 1$  is the number of launches in the previous period ( $X_t$ ) plus the number of satellites which survived the previous period ( $S_t(1 - L(S_t, D_t))$ ). The amount of debris in orbit in  $t + 1$  is the amount from the previous period which did not decay ( $D_t(1 - \delta)$ ), plus the number of new fragments created in collisions ( $G(S_t, D_t)$ ), plus the amount of debris in the shell created by new launches ( $mX_t$ ).<sup>9</sup> The laws of motion for the satellite and debris stocks formalize this:

$$S_{t+1} = S_t(1 - L(S_t, D_t)) + X_t \quad (1)$$

$$D_{t+1} = D_t(1 - \delta) + G(S_t, D_t) + mX_t. \quad (2)$$

$L(S_t, D_t)$  is the proportion of orbiting active satellites which are destroyed in collisions, and  $G(S_t, D_t)$  is the number of new fragments created by collisions between orbiting bodies. Since we assume active satellites are identical,  $L(S_t, D_t)$  is also the probability an individual satellite is destroyed in a collision. We assume that the collision probability is nonnegative, increasing in each argument, and bounded below by 0 and above by 1.<sup>10</sup> No satellites can be destroyed when there are none in orbit ( $L(0, D_t) = 0 \forall D_t$ ). In a model with uncertainty over the number of losses,  $L(S_t, D_t)$  would be the expected number of losses in period  $t$ .

To fix concepts, a form for  $L(S_t, D_t)$  is helpful. We can model the probability that objects of type  $j$  are struck by objects of type  $k$  as

$$p_{jk}(k_t) = 1 - e^{-\alpha_{jk}k_t}, \quad (3)$$

where  $\alpha_{jk} > 0$  is a physical parameter reflecting the relative mean sizes, speeds, and

<sup>9</sup>Satellites’ orbits also decay and satellites eventually cease being productive, features we abstract from in the main model. We extend the model to include this in section 5.2 of the Appendix. This abstraction makes some results a little clearer but does not qualitatively affect them. Rouillon (2019) includes these features in the main model and derives qualitatively similar results, albeit in a continuous-time setting.

<sup>10</sup>Firms try to avoid collisions by maneuvering their satellites when possible; the collision probability in this model should be thought of as the probability of collisions which could not be avoided, with easily avoided collisions optimized away. Collisions which could have been avoided but were not due to human error are included in this. Implicitly we are assuming that firms operate their satellites as imperfect cost-minimizers.



inclinations of the object types. The probability a satellite is destroyed is the sum of the probabilities it is struck by debris and by other satellites, adjusted for the probability it is struck by both. For satellite-satellite and satellite-debris collisions, equation 3 gives us

$$L(S, D) = p_{SS}(S) + p_{SD}(D) - p_{SS}(S)p_{SD}(D) \quad (4)$$

$$\begin{aligned} &= (1 - e^{-\alpha_{SS}S}) + (1 - e^{-\alpha_{SD}D}) - (1 - e^{-\alpha_{SS}S})(1 - e^{-\alpha_{SD}D}) \\ \implies L(S, D) &= 1 - e^{-\alpha_{SS}S - \alpha_{SD}D}. \end{aligned} \quad (5)$$

The form in equation 5 is convenient as it allows us to solve explicitly for the open access launch rate and is easy to manipulate, though it does not satisfy our assumption that  $L(0, D) \equiv 0$ . We use this form where it facilitates exposition or computation, but our analytical results do not assume a specific functional form for the collision probability. A similar form has been used in engineering studies of the orbital debris environment (Letizia et al., 2017; Letizia, Lemmens, and Krag, 2018). We derive this form from a model of random collisions in an orbit in Appendix 7.

We assume that the number of new fragments  $G(S_t, D_t)$  is nonnegative, increasing in each argument, and zero when there are no objects in orbit ( $G(0, 0) = 0$ ). To derive results about the occurrence of Kessler Syndrome in section 3.2, we also assume that the growth in new fragments due to debris alone will eventually be greater than  $\delta$ , formalized in Assumption 1.  $\delta$  is the rate of orbital decay for debris, and  $m$  is the amount of launch debris created by new satellites.

**Assumption 1.** (*Debris growth*) *When the satellite stock is zero, the growth in new fragments due to debris is larger than the decay rate for all levels of the debris stock greater than some level  $\bar{D} > 0$ ,*

$$\bar{D} : G_D(0, D) \geq \delta \quad \forall D > \bar{D}.$$

This motivates our definition of Kessler Syndrome as a state where the number of new fragments created by collisions between debris exceeds the amount which decays in a single period. By Assumption 1 and  $G(S, D)$  being increasing in both arguments, there is a threshold  $D^K \geq \bar{D}$  above which Kessler Syndrome occurs.

**Definition 1.** (*Kessler Syndrome*) *The Kessler region is the set of debris levels for which the growth in new fragments in collisions between debris exceeds the decay rate, even with*

cessation of launch activity and immediate deorbit of all active satellites, i.e.

$$\begin{aligned} & \{D : G_D(0, D) > \delta\} \\ \implies & \{D : D_{t+1} - D_t > 0 | S_t = 0, D_t = D, X_t = 0\} \end{aligned}$$

*Kessler Syndrome has occurred when the debris stock enters the Kessler region. The Kessler threshold is  $D^K = \min\{D : G_D(0, D) > \delta\}$ .*

For orbits where Assumption 1 doesn't hold at any level of debris, Kessler Syndrome is impossible. Our assumption that the new fragments function is monotonically increasing in both arguments, and the lack of debris removal technologies, imply that Kessler Syndrome is an absorbing state.<sup>11</sup>

### 2.3 Satellites as productive assets

Active satellites provide services to various types of entities: individuals, firms, governments, research agencies, and others. These services tend to be information services like mobile broadband, images of the Earth, and GPS. We ignore such differentiation, though it is an important feature of orbit use, to focus on the dynamics of collisions and debris. [Adilov, Alexander, and Cunningham \(2015\)](#) account for product differentiation in a two-period setting.

We assume that satellites are identical, infinitely lived unless destroyed in a collision, and produce a single unit of output per period. The net payoff from this output is  $\pi > 0$ , and each satellite costs  $F$  to build and launch.<sup>12</sup> The market for satellite output is perfectly competitive, so that the price per unit of output is the same as its social marginal benefit. The one-period rate of return on a satellite is  $\pi/F \equiv r_s > 0$ . We discuss the effects of relaxing the assumption that satellites are infinitely lived in section 5.2. We assume that costs and returns are constant over time, and comment on the effect of the rate of return on a satellite varying over time in section 5.3.

We consider two scenarios: open access by profit-maximizing firms which can own up to one satellite at a time — or equivalently that all firms are choosing whether or not to launch identical constellations of satellites at any given time, changing the units of  $S_t$  from individual satellites to constellations — and launching controlled by a fleet planner who owns

<sup>11</sup>Note that our definition of the Kessler threshold implies that the number of new fragments in debris-only collisions exceeds the number of fragments which have decayed, i.e.  $G_D(S, D) > \delta \implies G(S, D) > \delta D \forall S$ .

<sup>12</sup>This is a simplification to focus on the margin of launch decisions. Operational costs like managing receiver stations on the ground or monitoring the satellite to perform stationkeeping are incurred each period, and may push  $\pi \leq 0$ . We assume that the firms have correctly forecasted that  $\pi > 0$  before deciding to launch. This also allows us to abstract from the margins of decisions to sell or deorbit satellites, or to purchase already-orbiting satellites.

all satellites.<sup>13</sup> A firm which owns a satellite collects a revenue of  $\pi$  every period the satellite continues to operate. The probability that an individual satellite is destroyed in period  $t$  is  $L(S_t, D_t)$ . The discount factor used by all agents is  $\beta = (1 + r)^{-1}$ , where  $r > 0$  is the discount rate. The value of a satellite,  $Q(S_t, D_t, X_t)$ , is the sum of present profits and the expected discounted value of its remaining lifetime profits, formalized below:

$$Q(S_t, D_t, X_t) = \pi + \beta(1 - L(S_t, D_t))Q(S_{t+1}, D_{t+1}, X_{t+1}), \quad (6)$$

where  $X_t = \int_0^\infty x_{it} di$  is the aggregate launch rate based on each potential launcher's entry decision  $x_{it} \in \{0, 1\}$ . We assume for simplicity that firms cannot choose to deorbit satellites.

A firm which does not own a satellite in period  $t$  decides whether to pay a cost  $F > \pi$  to launch a satellite which will reach orbit and start generating revenues in period  $t + 1$ , or to wait and decide again in period  $t + 1$ .<sup>14</sup> Assuming potential launchers are risk-neutral profit maximizers, the value of potential launcher  $i$  at the beginning of period  $t$  is

$$\begin{aligned} V_i(S_t, D_t, X_t) &= \max_{x_{it} \in \{0, 1\}} \{ (1 - x_{it})\beta V_i(S_{t+1}, D_{t+1}, X_{t+1}) + x_{it}[\beta Q(S_{t+1}, D_{t+1}, X_{t+1}) - F] \} \quad (7) \\ \text{s.t. } S_{t+1} &= S_t(1 - L(S_t, D_t)) + X_t \\ D_{t+1} &= D_t(1 - \delta) + G(S_t, D_t) + mX_t \end{aligned}$$

<sup>13</sup>Our assumption that each firm is identified with a single satellite can be equivalent to assuming that either the marginal satellite in equilibrium is launched by such a firm, or that firms may own multiple satellites but that they all ignore their inframarginal satellites in launching. In either case we could obtain the same equilibrium condition given by equation 9 below. We maintain our stricter interpretation for two reasons. First, we wish to avoid the complications of solving the dynamic satellite inventory management problem facing a constellation owner outside the steady state, with the attendant assumptions about when and how they choose to refresh their constellation. See [Adilov et al. \(2019\)](#) for a model of GEO satellite inventory management which abstracts from the orbital mechanics we focus on here. Versions of the orbital warehousing issues examined in [Adilov et al. \(2019\)](#) are also relevant to LEO constellation management, but with additional dynamic complications through the collision risk and debris growth functions. Similarly, when firms have the ability to coordinate groups of satellites, their satellites face different risks from those under their control than from those which are not. This raises the possibility of strategic collision risk dynamics in orbit use — the relevant “measure-zero” assumption which would allow us to ignore such possibilities is that firms are relatively small in the physical space the satellites occupy, not the output market in which they compete. When a particular shell is dominated by a few firms which own hundreds or thousands of satellites there, it seems difficult to argue they are small relative to neighbors which own one or a handful of satellites. Second, if all satellites are identical and we are focused on a single orbital shell, then implying that firms may operate at different scales appeals to an unspecified dimension of firm heterogeneity which may or may not be relevant to the equilibrium condition. For example, if some firms face financing constraints but others do not (or are vertically integrated with launch providers while others are not), it seems plausible that the unconstrained/integrated firms may wish to take advantage of their competitors' constraints in some way. Ruling out such cases or tracing out their effects on the open-access equilibrium is beyond our scope here. The constellation management problem is eminently important to model, but given the complications we leave it to future research.

<sup>14</sup>Whether the lag reflects time-to-build or time-to-launch, there is a difference between the timescale of physical interactions in orbit and the timescale of launch decisions. The former occurs continuously, while the latter does not. We include the lag to capture this feature.

The launch Bellman has an  $i$  subscript to indicate that firms may or may not choose to launch. Since satellites are identical, there is no  $i$  subscript on the value of a satellite.

## 2.4 Open access and the equilibrium collision probability

Under open access, firms launch satellites until the value of launching is 0,

$$X_t \geq 0 : V_i(S_t, D_t, X_t) = 0 \quad \forall t \quad (8)$$

$$\implies \beta Q(S_{t+1}, D_{t+1}, X_{t+1}) = F. \quad (9)$$

Equation 9 is the zero profit condition which defines the open-access equilibrium. The resulting value of a satellite is the period profit plus the expected value of its survival under open access,

$$Q(S_t, D_t, X_t) = \pi + (1 - L(S_t, D_t))F. \quad (10)$$

The open access condition and the satellite value can be rewritten to show that in equilibrium the flow of benefits generated by a satellite is equated with the flow of opportunity costs and expected collision costs (the marginal private costs of satellite ownership):

$$F = \beta Q(S_{t+1}, D_{t+1}, X_{t+1}) \quad \forall t \quad (11)$$

$$\implies F = \beta [\pi + (1 - L(S_{t+1}, D_{t+1}))F] \quad (12)$$

$$\implies F \left( \frac{1 - \beta}{\beta} \right) = \pi - L(S_{t+1}, D_{t+1})F \quad (13)$$

$$\implies \pi = rF + L(S_{t+1}, D_{t+1})F. \quad (14)$$

The Implicit Function Theorem and our assumptions on the derivatives of the collision probability and new fragment function allow us to obtain comparative statics on the launch rate from equation 14 without imposing a specific functional form for  $L(S, D)$ . We provide general analytical results in section 2.10, and a specific example in this section to illustrate the intuition. Let  $L(S, D) = 1 - e^{-\alpha_{SS}S - \alpha_{SD}D}$ , where  $\alpha_{SS}, \alpha_{SD} \geq 0$ . Define the following quantities:

$$(\text{Log rate of excess return:}) R = -\log(1 + r - r_s) \quad (15)$$

$$(\text{Launch contribution to collision probability:}) a^{-1} = \alpha_{SS} + \alpha_{SD}m \quad (16)$$

$$(\text{Carryover satellite stock:}) s_t = S_t(1 - L(S_t, D_t)) \quad (17)$$

$$(\text{Carryover debris stock:}) d_t = D_t(1 - \delta) + G(S_t, D_t), \quad (18)$$

The forms of the first two terms come from  $L(S, D)$ . The first term is the log rate of excess return, and is only defined when  $r_s - r < 1$ . Since the open-access equilibrium collision

probability is equal to the excess return on a satellite, this restriction imposes that some satellite survives each period.<sup>15</sup> The second term is the direct effect of a new launch on the collision probability: each new launch adds 1 satellite to orbit with collision parameter  $\alpha_{SS}$ , and  $m$  units of debris with collision parameter  $\alpha_{SD}$ . Modern launches typically carry more than one satellite; a launch delivering  $k$  satellites would contribute  $ka^{-1}$  to the collision probability, though we do not model ridesharing on launches. The third and fourth terms are the present stock's contributions to the future stock.

Using the form for  $L(S, D)$  with equation 14, we can solve for the interior equilibrium launch rate as

$$\bar{X}(S_t, D_t) = a[R - \alpha_{SS}\mathcal{J}_t - \alpha_{SD}\mathcal{D}_t]. \quad (19)$$

Increases in the excess return cause more firms to want to own a satellite, and hence to launch. On the other hand, increases in  $\alpha_{SS}$  and  $\alpha_{SD}$  (described more generally as “collision probability couplings” in section 2.6) increases the persistence of prior stocks and reduces the scope for firms to launch new satellites. Increasing the amount of launch debris,  $m$ , increases the risk that the newly launched satellites pose to themselves, and reduces the equilibrium launch rate. Since the launch rate cannot be negative, the open access launch policy is

$$X_t = \begin{cases} \bar{X}(S_t, D_t) & \text{if } R > \alpha_{SS}\mathcal{J}_t + \alpha_{SD}\mathcal{D}_t \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

As  $\alpha_{SS} + \alpha_{SD}m \rightarrow 0$ , launches become decoupled from the collision probability and debris evolution. If this happens, the launch rate goes to infinity when  $r_s > r$ . This is an extreme and unrealistic case, but it highlights the role of collisions in the model: without the risk of a collision, a satellite is a perfectly safe asset that provides a higher rate of return than the risk-free rate. In reality, the risk of a collision is just one of many factors to be weighed in the expected cost of launching a satellite.

## 2.5 The effect of another satellite on the expected number of survivors

In this section we suppress function arguments for generality as to the types of couplings involved. We discuss the couplings and their effects in the following section.

The expected number of survivors from adding another satellite,  $\frac{\partial}{\partial S}(1 - L)S = 1 - L - \frac{\partial L}{\partial S}S$ , may be positive or negative. The sign of this quantity is related to a result in Rouillon (2019)

<sup>15</sup>If  $r_s - r \geq 1$  the model would become uninteresting, as the returns to satellite ownership would be so large as to induce firms to launch even knowing that their satellite would only survive a single period. This would be equivalent to  $\pi \geq (1 + r)F$ , allowing firms to pump money out of the orbit by borrowing  $F$  at rate  $r$  and earning  $\pi$  at no risk.

about the orbital maximum carrying capacity. They define the orbital maximum carrying capacity as the maximum expected number of active satellites that the orbit can sustain in the long term. They then show that both open access and optimal policies may involve exceeding the orbital maximum carrying capacity if the cost of launching another satellite falls low enough. We formalize the assumption that the current stock is below the orbital maximum carrying capacity, i.e. such that the expected number of survivors is nonnegative, below.

**Assumption 2.** *(New satellites do not reduce the expected number of survivors) The marginal number of survivors is nonnegative,*

$$1 - L - \frac{\partial L}{\partial S} S \geq 0,$$

*implying that a new launch will not reduce the expected number of satellites.*

This assumption does not drive any of our results. We formalize it for clarity when discussing the marginal external cost in section 2.9.

## 2.6 Three types of physical couplings

We use the term “physical coupling” here to refer to ways that variables in the laws of motion for satellite and debris stocks are connected to each other. These couplings give some structure to the laws of motion, and drive the physical dynamics of orbit use. There are three economically relevant physical couplings between objects in orbit. If all of these couplings were turned off, there would be no problem of excess congestion or interesting economic dynamics in orbit.

The couplings also highlight the role of debris in the economics of orbit use. Without debris, the only congestion externality can be a “steady state” one created by the present stock of satellites being too high. With debris and some of the couplings described below, the congestion externality can be “dynamic” in the sense that the debris stock makes present congestion a function of past launch rates.

**The collision probability couplings** The collision probability couplings refer to the arguments of the collision probability function,  $L(\cdot)$ . When the couplings are turned off, the collision probability is exogenous:  $L(\cdot) = L$ . It may vary over time, but since collision probability is exogenous with the couplings turned off, there is no congestion externality. Even if open access causes Kessler Syndrome due to other couplings, the open access launch rate is efficient in the sense that the planner would produce the same outcome. With the collision probability couplings turned off, there are no consequences to excess satellite levels or debris growth.

If the collision probability is coupled with the satellite stock ( $L(\cdot) = L(S)$ ), then there can be a “steady state” congestion externality. The strength of this coupling can be measured by  $\frac{\partial L}{\partial S}$ , with  $\frac{\partial L}{\partial S} = 0$  when there is no coupling. Despite any other couplings and their effects on debris growth, the only source of inefficiency is that open access results in too many satellites being launched relative to the optimal plan. Though the other couplings may produce interesting dynamics in debris accumulation, those dynamics are irrelevant to socially optimal orbit use. Both the open access and socially optimal launch paths will then involve single-period jumps to the steady state. The optimality of the most rapid approach path (MRAP) is driven by the linearity of the objective function and the lack of persistent consequences due to satellite launches. Though the state equations make the setting dynamic, firms and the planner face a sequence of static decisions. This situation is explored further in section 2.7.

If the collision probability is coupled with the debris stock ( $L(\cdot) = L(D)$ ) but not the satellite stock, there can be a “dynamic” congestion externality. The strength of this coupling can be measured by  $\frac{\partial L}{\partial D}$ , with  $\frac{\partial L}{\partial D} = 0$  when there is no coupling. The presence of other couplings will influence the nature of this externality, but with at least a coupling between debris and collision probability there can be persistent consequences for specific launch histories due to debris accumulation. Depending on the other couplings, the planner may no longer find the MRAP to the steady state optimal, although firms will continue to take the MRAP. As a result of this coupling and others, firms may end up overshooting the steady state, particularly if there are initially low levels of satellites and debris. The “fully coupled” case of collision probability, which is the main focus of this paper, links collision probability to both the satellite and debris stocks ( $L(\cdot) = L(S, D)$ ), allowing for dynamic congestion where the MRAP is not optimal. If the collision probability is completely uncoupled from the satellite stock (including indirectly through the debris stock), then Assumption 2 always holds by definition of  $L$ .

**The launch debris coupling** The launch debris coupling refers to the amount of launch debris created, i.e. the parameter  $m$  in the debris law of motion. The strength of this coupling can be measured by  $m$ , with higher values of  $m$  implying a stronger coupling. When  $m = 0$ , launches are uncoupled from the debris stock. When collision probability is coupled to the debris stock, this coupling strengthens the externality of launch, since when  $m > 0$  launches create debris independent of any collision effects. However, when collision probability is coupled to the debris stock, the launch debris coupling can “stabilize” open access orbit use by forcing firms to internalize some of the persistent effects of their launches on orbital congestion.

**The new fragment formation couplings** The new fragment formation coupling refers to the arguments of the new fragment function,  $G(\cdot)$ . We assume that with any coupling,  $G(\cdot)$  is a strictly increasing function of its arguments. When the coupling is turned off, the number of



new fragments created is exogenous and there are no positive feedbacks between debris. With the collision probability coupling and the launch debris coupling activated, current congestion can still depend on past launch rates, but the persistence may be muted by of the lack of the new fragment formation coupling. If the new fragment function is coupled to the satellite stock,  $G(\cdot) = G(S)$ , then there can be persistent consequences of excess satellite levels. When the new fragment function is coupled to the debris stock,  $G(\cdot) = G(D)$ , there can be multiple open-access steady states.<sup>16</sup> This multiplicity will exist independent of the collision probability and launch debris couplings. When the collision probability is only coupled to the satellite stock or uncoupled ( $L(\cdot) = L(S)$  or  $L(\cdot) = L$ ), the multiplicity is only in debris levels. When the collision probability is fully coupled ( $L(\cdot) = L(S, D)$ ) and the new fragment function is coupled with only the debris stock, multiple equilibria in both satellites and debris can exist: one with low debris and high satellites, and one with high debris and low satellites. As in the previous case, only the low debris equilibrium is stable. As with the collision probability couplings, the strengths of these couplings can be measured by  $\frac{\partial G}{\partial S}$  and  $\frac{\partial G}{\partial D}$ , with  $\frac{\partial G}{\partial S} = 0$  or  $\frac{\partial G}{\partial D} = 0$  indicating no coupling of the given type.

## 2.7 Steady state congestion: collisions without debris

Suppose the collision probability does not depend on the debris stock, i.e.  $L_t = L(S_t)$ . This could be because there is no debris, or because satellites are perfectly shielded against debris. In this case there are no interesting dynamics in the model: open access jumps to the steady state in a single period, as does the planner.

The lack of persistent consequences for specific launch or collision histories means that the congestion problem is effectively static. Present decisions do not affect future decisions. In this case, marginal satellites create a negative congestion externality if they increase the collision probability, i.e. cause a first-order stochastically dominant shift in the distribution of collisions. Satellites are always a good in this case (the marginal value of a satellite to the fleet is always positive). The linearity of the expected social welfare function in collision probabilities suggests that the planner will be indifferent to or like any shift in the collision distribution which does not increase the average collision probability.

## 2.8 Dynamic congestion: collisions with debris

When the collision probability depends on the debris stock, i.e.  $L_t = L(S_t, D_t)$ , debris creates persistent consequences for specific launch and collision histories. In this case open access

<sup>16</sup>When  $L(S, D)$  is strictly concave in both arguments and  $G(S, D)$  is strictly increasing in both arguments, there can be two open-access steady states: one with low debris and one with high debris. Of the two, only the former will be stable, while the latter will either return to the former or else explode to infinite debris. We discuss the multiplicity issue in more detail in Proposition 6.

can have interesting dynamics, and the planner’s optimal satellite accumulation path will in general not be a jump to the steady state. Firms under open access still face a linear objective function and attempt to take a single-period jump to the steady state, but the delayed effect of period  $t$  launches on period  $t + 2$  debris means the system dynamics may not allow such a jump.

Both firms and the planner will launch more satellites when the skies are clear. While the planner will progressively decrease their launch rate to reach the optimal steady state, firms may overshoot the stable region of the state space and end up on a trajectory which results in Kessler Syndrome. This happens because firms set their launch rates in response to debris accumulation one period ahead, but no farther. Open access removes any incentive for firms to care about longer-horizon effects. Even if firms do not launch into Kessler Syndrome, they may still consistently miss the steady state. This can happen because firms will launch at rates which make the future satellite and debris stocks create collision risk greater than the excess rate of return on a satellite. When that happens, firms will stop launching, allowing the stocks to fall again.

## 2.9 Marginal external cost and the optimal launch plan

In this section, we use letter subscripts on functions to indicate partial derivatives with respect to a given argument, e.g.  $L_S(S, D) \equiv \frac{\partial L(S, D)}{\partial S}$ .

The fleet planner owns all of the satellites in orbit, and controls all launches to maximize the expected net present value of the satellite fleet. This is the “sole owner” benchmark used in other environmental and natural resource settings.<sup>17</sup> In doing so, the planner equates the per-period marginal benefit of another satellite with its per-period social marginal cost. The per-period social marginal cost in this setting is the sum of the opportunity cost of the investment and the collision probability ( $r + L(S_{t+1}, D_{t+1})$ ), and the effect of the marginal satellite on future collisions and debris growth ( $\xi(S_{t+1}, D_{t+1})/F$ ). The first two terms are private marginal costs internalized by firms, and the final term is a marginal external cost not internalized by firms.

---

<sup>17</sup>For example, [Gordon \(1954\)](#) appeals to the sole owner’s management of a fishery in showing that the competitive equilibrium is inefficient, and [Scott \(1955\)](#) appeals to the sole owner’s management to show cases where short-run competitive equilibria can manage fisheries efficiently.

The fleet planner's problem is

$$W(S_t, D_t) = \max_{X_t \geq 0} \{ \pi S_t - F X_t + \beta W(S_{t+1}, D_{t+1}) \} \quad (21)$$

$$\text{s.t. } S_{t+1} = S(1 - L(S_t, D_t)) + X_t \quad (22)$$

$$D_{t+1} = D(1 - \delta) + G(S_t, D_t) + m X_t. \quad (23)$$

The planner's optimality condition, expressed in terms of the flow of marginal benefits and costs, is

$$X_t^* \geq 0 : \pi = rF + L(S_{t+1}, D_{t+1})F + \xi(S_{t+1}, D_{t+1}). \quad (24)$$

The above flow condition allows us to determine the socially optimal collision probability. We derive equation 24 in the Appendix, section 7.

**Proposition 1.** (*Optimal collision probability*) *The planner launches so that the collision probability is equated to the rate of excess return net of the rate of marginal external cost, i.e.*

$$L(S_{t+1}, D_{t+1}) = r_s - r - \frac{\xi(S_{t+1}, D_{t+1})}{F}. \quad (25)$$

*Proof.* Rearranging equation 24 and dividing by  $F$  yields equation 25.  $\square$

From the planner's problem, the marginal external cost is

$$\begin{aligned} \xi(S_{t+1}, D_{t+1}) = & \underbrace{S_{t+1} L_S(S_{t+1}, D_{t+1}) F}_{\substack{\text{"congestion" channel:} \\ \text{cost of more satellites colliding} \\ \text{with each other due to crowding}}} + \underbrace{\frac{\pi - rF - L(S_t, D_t)F - S_t L_S(S_t, D_t)F}{\beta(1 - \delta + G_D(S_{t+1}, D_{t+1}) + mL_D(S_{t+1}, D_{t+1})S_{t+1})}}_{\substack{\text{"pollution" channel:} \\ \text{cost of more satellites colliding} \\ \text{with debris due to debris growth}}} \\ & - \underbrace{\frac{\beta G_S(S_{t+1}, D_{t+1}) + m(1 - L(S_{t+1}, D_{t+1}) - S_{t+1} L_S(S_{t+1}, D_{t+1}))}{\beta(1 - \delta + G_D(S_{t+1}, D_{t+1}) + mL_D(S_{t+1}, D_{t+1})S_{t+1})} S_{t+1} L_D(S_{t+1}, D_{t+1}) F}_{\substack{\text{"reduction" channel:} \\ \text{value gained from debris-source removal due to collisions and} \\ \text{reduction in replacement launch fragments}}} \end{aligned} \quad (26)$$

The first term of  $\xi(S_{t+1}, D_{t+1})$ , the congestion channel, represents the cost of additional satellite collision probability due to satellite crowding. As long as the collision probability is coupled with the satellite stock and new satellites weakly increase the number of satellite-destroying collisions ( $L_S(S, D) \geq 0$ ), this term is nonnegative. The second term, the pollution channel, is the current value of the profit lost per additional fragment caused by other fragments, net of the cost of congestion. As long as additional satellites in period  $t$  produced profits net of the opportunity cost, risk, and congestion ( $\pi - rF - L(S_t, D_t)F - S_t L_S(S_t, D_t)F \geq 0$ ), this term is nonnegative. The third term shows the gain in value from reducing the number of satellites

in orbit, per additional fragment caused by other fragments. If Assumption 2 holds, this term is also nonnegative. Intuitively, removing satellites currently in orbit removes a source of fragments to be destroyed in collisions, while reducing the number of satellites to maintain in the future reduces the number of launch fragments that will be produced.

As long as the profits from a satellite net of opportunity cost, risk, and congestion in period  $t$  outweigh the cost of new fragment creation due to replacement satellites reaching orbit in  $t + 1$ , the cost of the pollution channel will outweigh the benefit of the reduction channel, making the marginal external cost positive. This implies that the planner will launch fewer satellites than open access firms. Assumption 3 formalizes this, with time subscripts suppressed and  $'$  used to indicate variables at  $t + 1$ .

**Assumption 3.** *Along optimal paths, the cost of the pollution channel outweighs the benefit of the reduction channel, i.e.*

$$\pi - rF - L(S, D)F - SL_S(S, D)F > [\beta G_S(S', D') + m(1 - L(S', D') - S' L_S(S', D'))] S' L_D(S', D') F.$$

The potential for a negative marginal external cost here reflects two facts. First, debris is a complement in production to satellites. The stronger the launch debris coupling (greater  $m$ ), the greater this effect is. Second, the only way to reduce the number of satellites in orbit in this setting is to allow them to be destroyed. The combination of these two facts creates the potential for collisions which reduce the number of satellites in orbit to be good from the planner's perspective.

Assumption 3 ensures that the marginal external cost is always positive. One physical intuition for this assumption is that  $m$  and  $G_S$  are kept small by careful engineering. Launch integrators and vehicle providers have incentives to invest in reducing  $m$ , as a strong launch coupling increases the chance that debris attributable to a specific launch may cause damage to people on the ground or to other satellites in orbit. The Liability Convention states that damage to objects in orbit is under fault-based liability — the entity causing damages is liable for them. While attribution can be tricky in orbit, launch debris is more straightforward to attribute than most types of debris. Additionally, a strong launch debris coupling creates the possibility that a satellite will face some probability of destruction from its own launch debris. Similarly, due to liability concerns, satellite owners have incentives to invest in reducing  $G_S$ , as a strong coupling between satellites and new fragment production increases their expected liability as long as there is some non-zero probability of fragment attribution.

Figure 1 shows how the satellite-debris dynamics shift private and social marginal cost

curves over time. Increases in satellites and debris shift both costs up, with both marginal cost curves shifting up more under open access than under the optimal plan. This offers another intuition for the congestion wedge in orbit use: firms are unable to control how much marginal costs shift as well as the planner would.

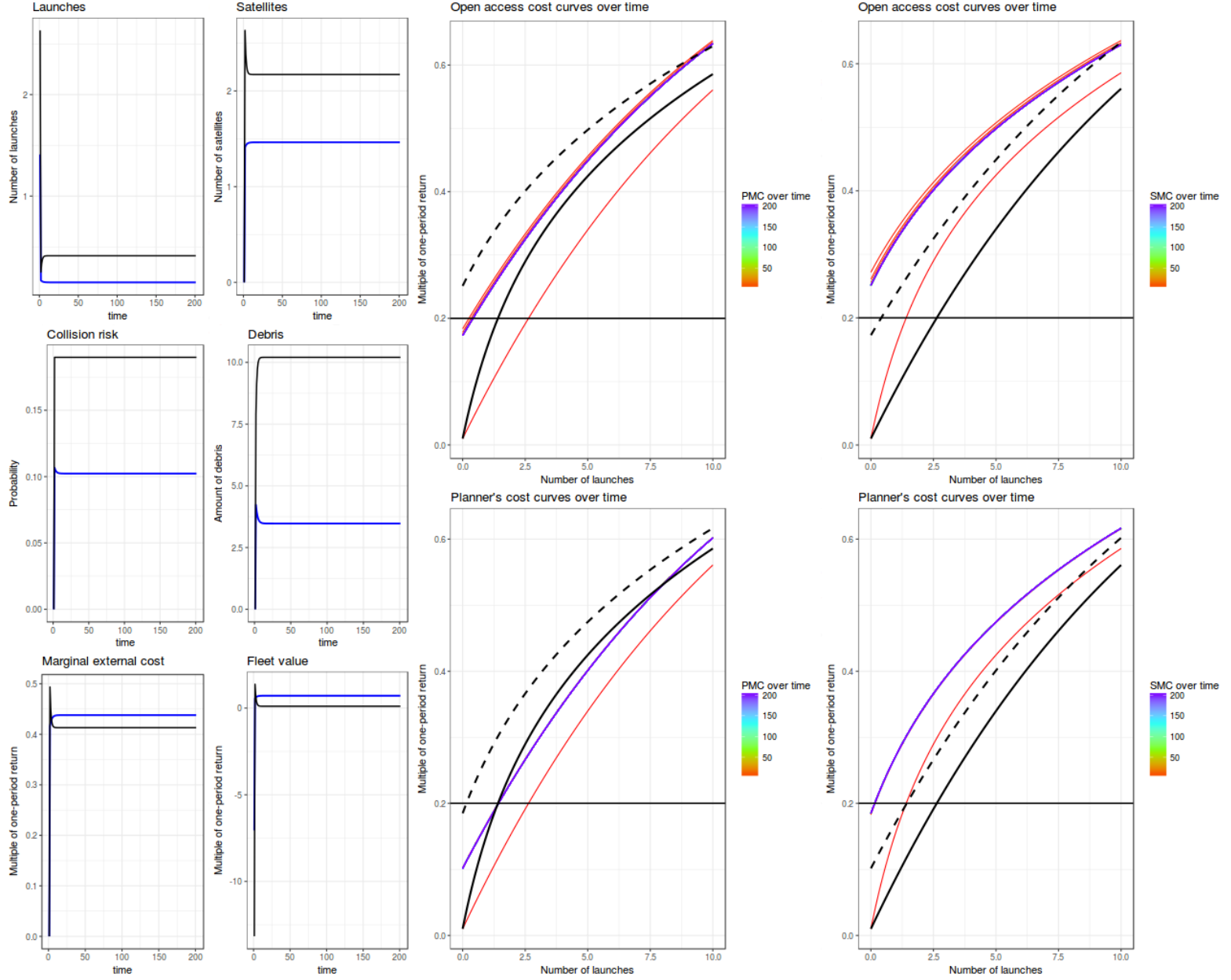


Figure 1: Left: time paths under open access (black) and the optimal plan (blue). Right: shifts in private and social marginal cost curves over time under each plan (open access above and the optimal plan below). The middle column of cost curves show how the private marginal cost shifts each period in color, with the initial social marginal cost shown in solid black and the final social marginal cost shown in dashed black. The right column of cost curves show how the social marginal cost shifts each period in color, with the initial private marginal cost shown in solid black and the final private marginal cost shown in dashed black. The marginal benefit of another satellite is the horizontal black line.

## 2.10 Properties of open-access equilibria

In this section we explore the properties of open-access equilibria. The main results are that the equilibrium collision probability is determined by the excess return on a satellite, that the collision probability and debris stocks are too large under open access relative to the optimal plan, and that exogenous increases in the debris decay rate or decreases in the amount of launch debris may increase the equilibrium debris stock due to rebound effects. The first two properties establish the nature of open access launching as well as the existence and magnitude of the congestion externality. The third property is relevant to policy proposals which seek to limit debris growth through technical changes, without addressing the economic incentives involved in launching satellites.

**Proposition 2.** (*Equilibrium collision probability*) *In an open-access equilibrium, firms launch satellites until the collision probability in the next period is equal to the rate of excess return on a satellite over the safe asset.*

*Proof.* Rearranging the open-access equilibrium flow condition (equation 14) and dividing by  $F$ , the equilibrium collision probability can be written as

$$L(S_{t+1}, D_{t+1}) = r_s - r. \quad (27)$$

□

In the case where the economic parameters  $\pi, F, \beta$  are time-varying, Proposition 2 implies that the equilibrium collision probability in period  $t$  under open access will be determined by the values of the economic parameters in period  $t - 1$ , i.e. the period when the newest satellites in orbit were launched. In this case, the equilibrium collision probability will not be a constant, but will still be determined by the economic fundamentals of launching satellites. Section 5.3 explores this in more detail.

Proposition 2 implies that the equilibrium collision probability is increasing in the profitability of a satellite, decreasing in the cost of launching a satellite, and increasing in the discount factor. Proposition 2 does not imply that the open access equilibrium must be a steady state. Any type of dynamics in the satellite and debris stocks which keep the next-period collision probability equal to the excess return on a satellite can be an equilibrium path.

**Corollary 1.** *There is a continuum of open-access equilibria, defined by the collision probability isoquant which is equal to the excess return on a satellite.*

Corollary 1 follows from the form of the equilibrium condition. Every point on the isoquant where the collision probability is equal to the excess return on a satellite is an open-access equilibrium. The initial conditions and physical dynamics determine which equilibrium is reached from a non-equilibrium state.



**Corollary 2.** *When it is profit-maximizing to launch, firms under open access pursue the most rapid approach path to the equilibrium isoquant. The equilibrium need not be a steady state.*

Corollary 2 follows from the fact that open-access equilibria require a launch rate that equates the rate of excess return on a satellite with the next-period collision probability. In a sense, this is an assumption of the model and not a result: if firms equilibrate the collision probability in every period, then they are necessarily following paths directly from the initial conditions to the equilibrium set.

The implications of this type of approach path are more interesting. Unless the equilibrium reached is also a steady state, MRAPs from an initial condition to the equilibrium set are not MRAPs to steady states, and result in overshooting in at least one state variable. This is explored in more detail in Proposition 8.

**Corollary 3.** *Along the open access equilibrium path, the collision probability is*

1. *increasing in the per-period satellite return,*
2. *decreasing in the launch cost, and*
3. *increasing in the discount factor.*

Corollary 3 shows that policies which increase the cost or reduce the profitability of operating a satellite will reduce the equilibrium collision probability. If firms become more patient, they will be willing to tolerate a higher equilibrium collision probability. Proposition 3 shows the existence of the externality: equilibrium collision probability is too high.

**Proposition 3.** *(Externality) The open access equilibrium results in higher collision probability than the optimal plan.*

*Proof.* We denote objects along the open access path with hats, and objects along the optimal path with stars, e.g.  $\hat{S}_t$  is the satellite stock in period  $t$  under open access and  $S_t^*$  is the satellite stock in period  $t$  under the optimal plan. The equilibrium collision probability is

$$L(\hat{S}_{t+1}, \hat{D}_{t+1}) = r_s - r, \quad (28)$$

while the socially optimal collision probability is

$$L(S_{t+1}^*, D_{t+1}^*) = r_s - r - \frac{\xi(S_{t+1}^*, D_{t+1}^*)}{F}. \quad (29)$$

Given assumption 3  $\xi(S_{t+1}^*, D_{t+1}^*)$  is positive, and  $L(S_{t+1}^*, D_{t+1}^*) < L(\hat{S}_{t+1}, \hat{D}_{t+1})$ .  $\square$

It is useful to know how the launch rate and debris stock respond to changes in the model parameters. Particularly policy-relevant parameters include the launch cost, the launch debris,

and the decay rate. Pigouvian launch taxes have been suggested to reduce orbital debris, and would manifest as increases in the launch cost, while Pigouvian satellite taxes would manifest as reductions in the per-period profit from a satellite. Command-and-control policies to reduce the amount of launch debris have also been proposed, as have increasing the use of reusable launch vehicles reduce the amount of launch debris. Decay rates are typically decreasing as altitude increases and policies which reduce the lifespan of orbital debris can be modeled as increasing the decay rate.<sup>18</sup> Proposition 4 considers the effects of changes to the collision probability, satellite stock, and debris stock on the equilibrium launch rate. Proposition 5 considers the effects of changes in the launch costs, launch debris, and decay rates on the equilibrium launch rate and debris stock.

**Proposition 4.** *(Economic incentives to reduce congestion) Along the open access equilibrium path, the launch rate is*

1. *decreasing in the current satellite stock, and*
2. *decreasing in the current debris stock if and only if the marginal number of survivors is nonnegative.*

The proof, shown in Appendix C, follows from applying the Implicit Function Theorem to equation 14, shown in the appendix. The key inequality from the proof determines how the launch rate varies with the debris stock (period  $t$  values are shown with no subscript, and period  $t + 1$  values are marked with a ', e.g.  $S_t \equiv S, S_{t+1} \equiv S'$ ):

$$(S, D) : \underbrace{\frac{\partial L(S', D')}{\partial S'} S}_{\substack{t+1 \text{ marginal collision probability from} \\ t+1 \text{ satellites} \cdot \text{present satellite stock}}} < \underbrace{\frac{\frac{\partial L(S', D')}{\partial D'}}{\frac{\partial L(S, D)}{\partial D}}}_{\substack{\text{growth rate of} \\ \text{marginal collision probability} \\ \text{due to debris}}} \cdot \underbrace{\left(1 - \delta + \frac{\partial G(S, D)}{\partial D}\right)}_{\substack{\text{net growth in debris} \\ \text{due to debris}}} \quad (30)$$

Inequality 30 defines a set of values for  $(S, D)$  within which the launch rate is decreasing in the debris stock and outside of which the launch rate is increasing in the debris stock. The condition states that the marginal collision probability in period  $t + 1$  from satellites in that period across the stock of satellites in  $t$  is less than the product of the growth rate of the marginal collision probability from debris and the net growth in debris due to debris. If  $1 - L - L_S S \geq 0$ , as we have assumed, then  $\frac{\partial X}{\partial D}$  is nonpositive. Intuitively, inequality 30 states that the launch rate should be decreasing in the debris stock whenever the increase in the collision

<sup>18</sup>Current FCC policy mandates that inactive satellites must be deorbited within 25 years of launch, which could be captured in the decay rate parameter of this model. Similar policies are encouraged by the IADC and other space agencies.

probability due to debris and debris growth exceeds the increase in collision probability from marginal satellites. This is the essence of “careful placement” of new satellites.

**Proposition 5.** (*Short-run rebound effects*) *Along the open access equilibrium path, the current launch rate is*

1. *increasing in the return on a satellite,*
2. *decreasing in the amount of launch debris, and*
3. *increasing in the debris decay rate.*

*The next-period debris stock is*

1. *increasing in the return on a satellite,*
2. *increasing in the amount of launch debris, and*
3. *decreasing in the debris decay rate.*

The proof is shown in Appendix C. The potential for policies intended to reduce future debris stocks to have perverse effects arises from a combination of Propositions 4 and 5. Policies which reduce amount of time before inactive satellites must deorbit (i.e. increase the debris decay rate) can lead to better steady states, shown in Proposition 9, but the convergence may be non-monotonic. The initial reduction in debris spurs an increase in the launch rate, which may be too large to reach the steady state right away. Figure 2 shows an example of how a non-monotonic steady state transition caused by an increase in the decay rate can lead to higher short-run levels of debris.

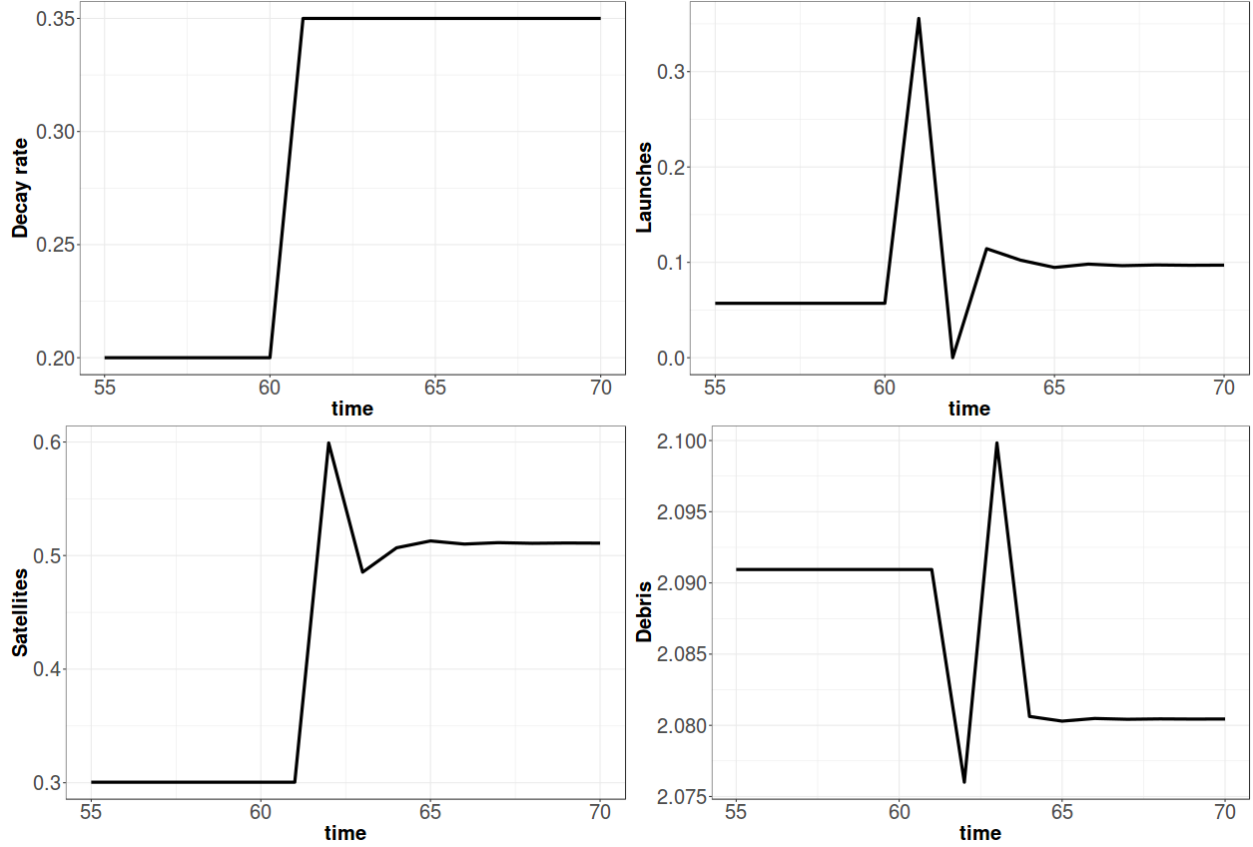


Figure 2: An example of an increase in the rate of debris decay,  $\delta$ , causing non-monotonic steady state convergence. Though the debris stock initially falls, the launch response is large enough that the debris stock rises above its previous level before reaching the new steady state.

These effects stem from the substitutability of the rate of return on a satellite and the collision risk in the equilibrium condition (equation 14). Since firms will enter the commons until the expected profits are zero, reducing the risk along one margin (e.g. less launch debris) can result in firms increasing risk along another margin (e.g. more launches). Since the launch debris and decay rate effects are (a) the opposite of what a purely physical model which ignores open access would predict, and (b) they occur between steady states, we refer to them as short-run rebound effects. Proposition 9 in section 3.1 explores the long-run effects of the same parameter changes on debris levels. Figure 6 illustrates numerically how the statics results in Proposition 5 interact with the stability results in Proposition 7 across thousands of time series simulations.

Economic controls which increase the cost of launching will have the expected effects: fewer launches and less debris. The results in Proposition 5 do not account for the fact that command-and-control policies which decrease the amount of launch debris or increase the decay rate may also increase the cost of launching a satellite. Such feedbacks could reduce or

remove the perverse effects. The takeaway is that in order for policies targeting launch debris generation and debris decay rates to be effective at reducing the equilibrium debris stock, they must affect the incentive to launch a satellite in the first place.

## 2.11 Action and inaction

Under open access, firms will not launch satellites if the excess return is insufficient to cover the congestion cost. Holding the launch rate constant, collisions and debris growth may drive an orbit from the action region, where some find it economical to launch, to the inaction region, where it is uneconomical for any to launch. Launch shutdown in some periods may be socially optimal. As long as Kessler Syndrome doesn't occur, the physical dynamics of the system will eventually bring satellite and debris stocks back to the action region. Figure 3 shows an example of action and inaction regions in the open access and optimal launch policies, and figure 5 shows the associated fleet values<sup>19</sup>.

**Definition 2.** (*Action region*) *The action region is the set of satellite and debris levels for which open access results in launches, i.e.*

$$(S_t, D_t) : L(S_{t+1}, D_{t+1}) \leq r_s - r.$$

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<sup>19</sup>Figure 15 in the appendix shows time paths where open access bounces between the regions and the planner does not.

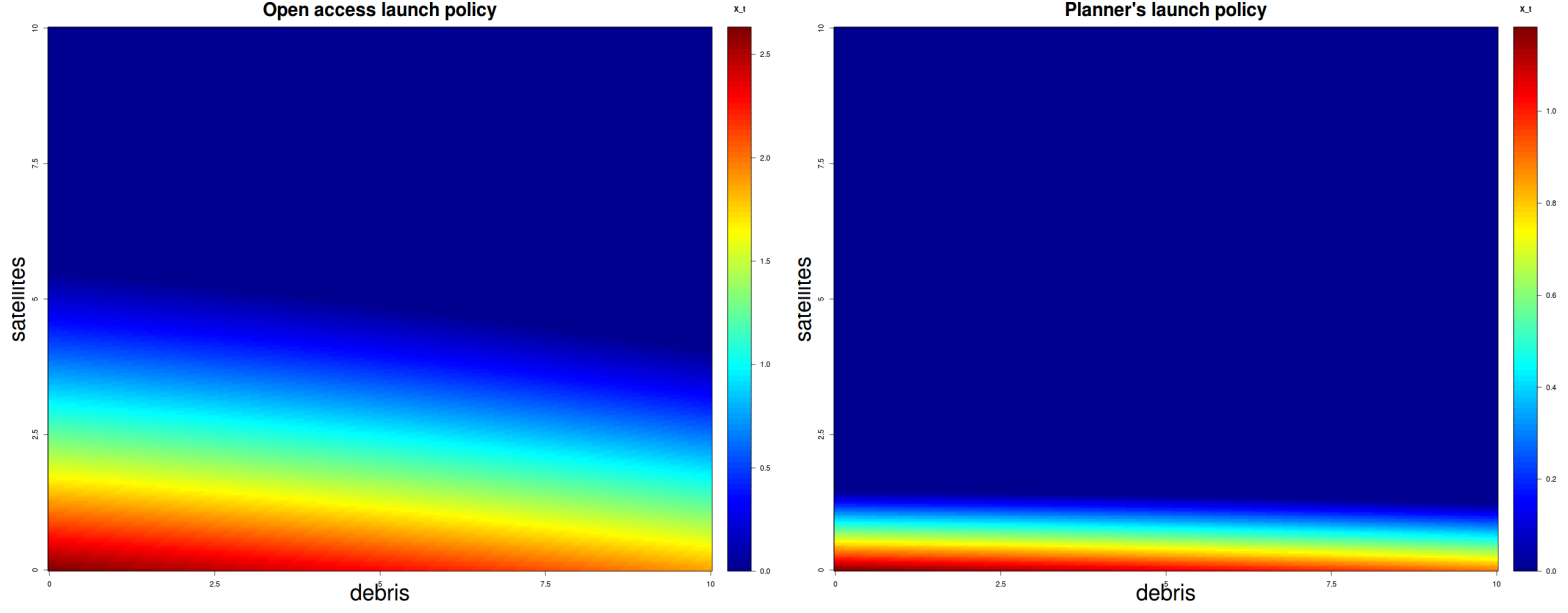


Figure 3: An example of the action and inaction regions under open access (left) and the optimal plan (right). Open access does not restrict launches in response to satellite or debris stocks as quickly as the planner or as severely. Colors represent launch rates: colors closer to blue are lower rates, and colors closer to red are higher rates.

The complement of the action region is the inaction region. Under open access, the inaction region is where the collision probability is greater than the excess return on a satellite. In the inaction region, aggregate dynamics are driven entirely by the collision probability, decay rate, and new fragment production (the “physical dynamics”). The laws of motion become

$$S_{t+1} = S_t(1 - L(S_t, D_t)) \quad (31)$$

$$D_{t+1} = D_t(1 - \delta) + G(S_t, D_t). \quad (32)$$

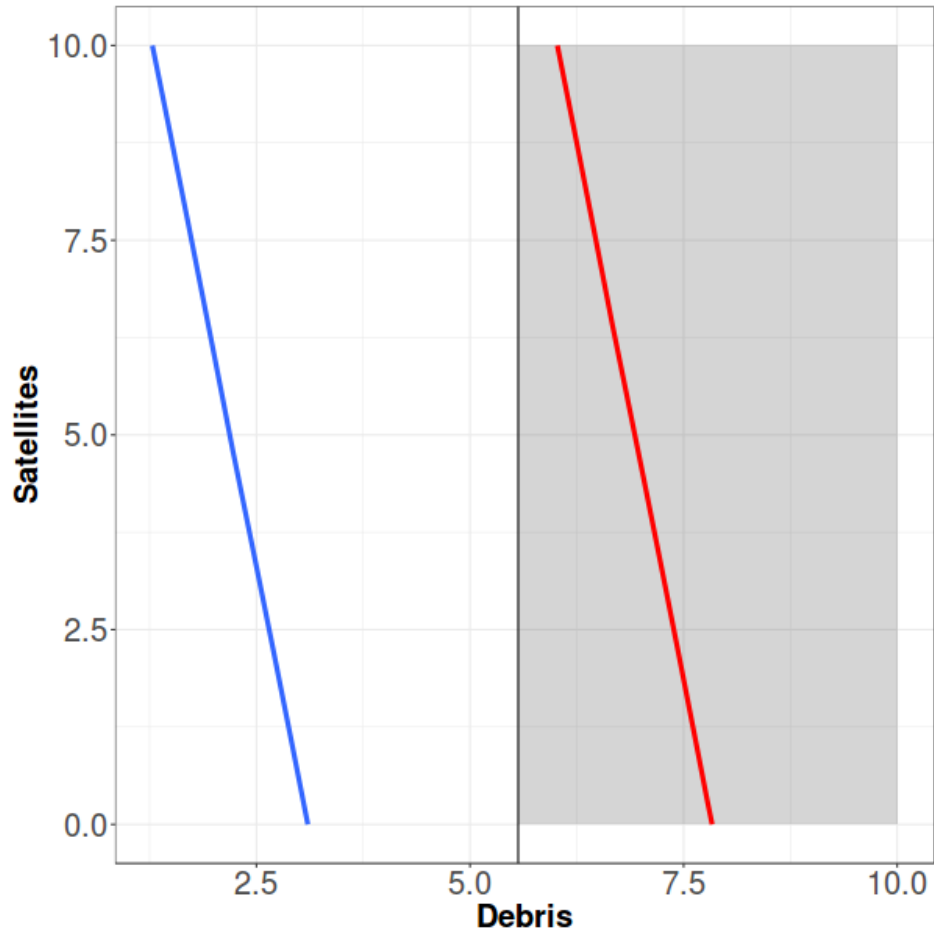


Figure 4: An example where a decrease in launch costs pushes the equilibrium isoquant into the Kessler region (the shaded area to the right of the dark gray line). The blue line is the original equilibrium isoquant, and the red line is the new equilibrium isoquant.

One possible steady state under these laws is to have no satellites or debris. Before that happens the stocks would again reach the action region, and the zero objects steady state will not be reached. The other possibility is that, as the satellite stock is driven to zero, the debris stock reaches the threshold at which Kessler Syndrome occurs. Though open access reduces launches in response to exogenous increases in the steady state debris level, the point on the equilibrium isoquant reached by a MRAP from the initial conditions may be in a place where the physical dynamics lead to the Kessler region. Figure 4 shows an example where a decrease in launch costs pushes the equilibrium isoquant into the Kessler region.



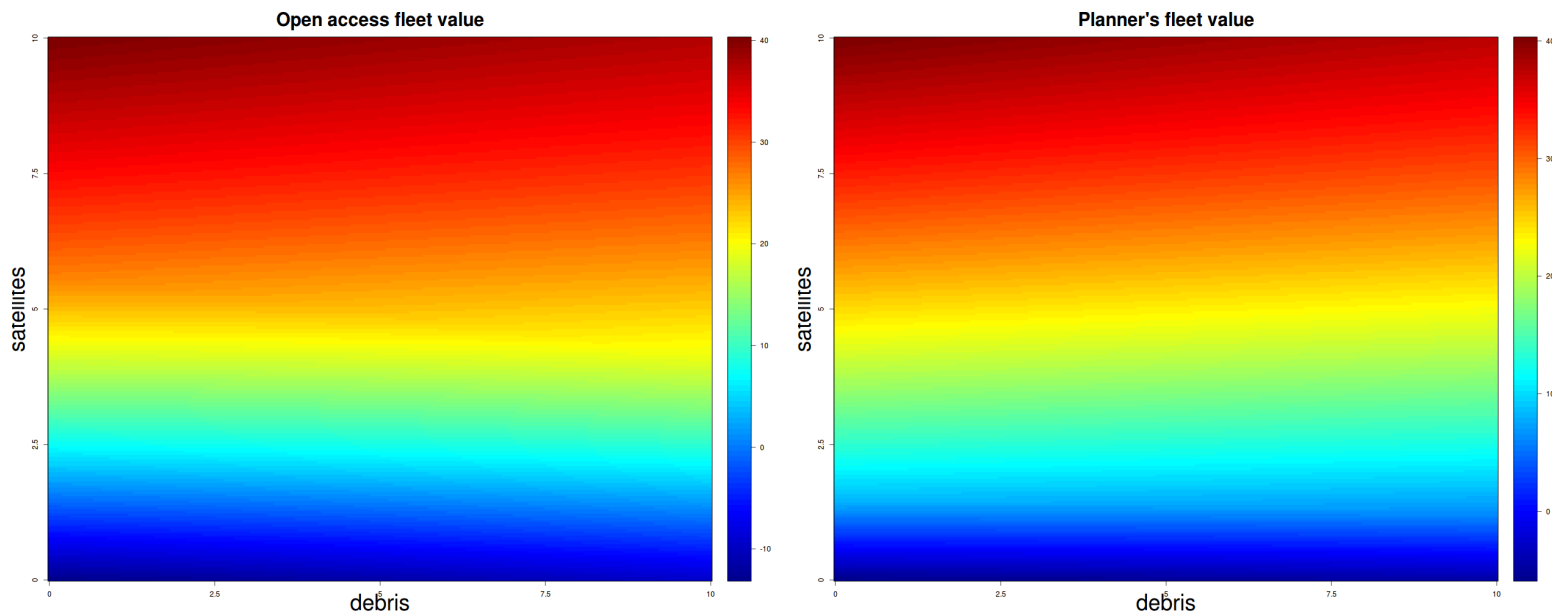


Figure 5: An example of the fleet value functions under open access (left) and the optimal plan (right). The planner's relatively muted launch policy creates a smaller region of negative profits and a larger region of positive profits. Losses under the open access plan are steeper, larger fleets less valuable, and higher debris stocks more costly than under the planner's policy.

### 3 The economic dynamics of orbital mechanics

Having described the nature of open access equilibria, we now turn to their dynamic properties. Two issues of first-order policy concern are the long-run properties of open access equilibria, and the conditions under which open access will prevent or cause Kessler Syndrome. In this section, we focus on understanding these issues by analyzing open-access steady states and by formally characterizing Kessler Syndrome and its relation to open access equilibria. We also consider the effects of other dynamic issues affecting orbit use, namely time-varying economic parameters and space weather. Our main results are that there can be multiple steady states of varying stability, and that increases in the excess return on a satellite shift the equilibrium set outward and bring it close to the region of the state space where Kessler Syndrome occurs. Allowing rates of return to vary over time does not change these properties, though it highlights the role of launch costs as a source of satellite value and suggest that Pigouvian launch fees will need to account for this in order to be efficient. Space weather does not change the properties of the equilibrium set, though it shifts its location and in doing so may cause Kessler Syndrome.

### 3.1 Properties of open-access steady states

The distinction between a “steady state” and an “equilibrium” is relevant here. The laws of motion for satellites and debris may be stationary under constant non-equilibrium launch rates, a case frequently analyzed in the physics and engineering literature. Another possibility is for firms to launch until zero profits in every period, while the satellite and debris stocks vary over time. As long as the collision probability in each such period is equal to the excess return on a satellite, the firms are in an open access equilibrium. What we are interested in here are states where both conditions are true: the physical aggregates are stationary over time and firms are earning zero economic profits. We refer to these as “open-access steady states”.

In an open-access steady state, the collision probability must be equal to the excess return on a satellite ( $L(S, D) = L(S', D') = r_s - r$ ) along with the usual conditions that the aggregate variables be stationary:

$$(S, D) : L(S, D) = r_s - r, \quad (33)$$

$$\begin{aligned} S' = S &\implies S = (1 - L(S, D))S + X \\ &\implies X = L(S, D)S, \end{aligned} \quad (34)$$

$$\begin{aligned} D' = D &\implies D = (1 - \delta)D + G(S, D) + mX \\ &\implies \delta D = G(S, D) + mX. \end{aligned} \quad (35)$$

Equations 33, 34, and 35 define the open-access steady states. If there is no launch debris ( $m = 0$ ), then steady states only require balancing natural debris decay ( $\delta D$ ) against debris growth due to objects already in orbit ( $G(S, D)$ ). As  $m$  increases, the effect of replacement satellites (reflected in  $mX$ ) on the steady state level of debris also increases, requiring a lower rate of debris growth due to objects already in orbit.

**Proposition 6.** (*Multiplicity*) *There can exist multiple open-access steady states.*

The proof is shown in Appendix C, with the precise number of open-access steady states depending on the shapes of  $L(S, D)$  and  $G(S, D)$ . For example, if  $L(S, D)$  is globally strictly concave and  $G(S, D)$  is globally increasing and asymptotically strictly convex, there can be up to 2 open-access steady states. The exact open-access steady state reached is determined by the physical and economic dynamics. Not all open-access steady states are locally stable. When there are multiple stable open-access steady states, the one reached will depend on the initial conditions. Shocks to the debris stock, like weapons tests by national authorities, could move the system around in the basin of the same steady state or to a different one.

**Proposition 7.** (*Local stability*) *Open-access steady states can be locally unstable.*

The proof is shown in Appendix C. The key equation from the proof is

$$\frac{\partial \mathcal{Y}}{\partial D}(D^*) = -\delta - \frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}(G_S(S^*, D^*) + m(r_s - r)) + G_D(S^*, D^*), \quad (36)$$

where  $S^* \equiv S(r_s - r, D^*)$ . The stability of an open-access steady state depends on three factors, shown on the right hand side of equation 36. The first is the decay rate ( $\delta$ ), which can ensure stability if it is high enough. Physically, higher decay rates indicate a region with greater renewability. The second is the equilibrium effect of new satellites on debris growth. The effect is increasing in the strength of the collision probability debris coupling ( $L_D$ ), the new fragment satellite coupling ( $G_S$ ), and the launch debris coupling ( $m$ ). These are all launch byproducts which deter future launches through debris creation and reduce the number of satellites which can be sustained. The effect is decreasing in the strength of the collision probability debris coupling ( $L_S$ ), which determines the effect of new satellites on the collision probability through their presence in orbit. The third is the strength of the new fragment debris coupling, which creates the potential for local instability if it is large enough. Physically, it measures the strength of the positive feedback between debris. The same coupling creates the possibility of Kessler Syndrome.

We can use the above expression for  $\frac{\partial \mathcal{Y}}{\partial D}(D^*)$  to obtain a necessary condition for stability:

$$\frac{\partial \mathcal{Y}}{\partial D}(D^*) < 0 \implies -\delta - \frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}(G_S(S^*, D^*) + m(r_s - r)) + G_D(S^*, D^*) < 0 \quad (37)$$

$$\frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}(G_S(S^*, D^*) + m(r_s - r)) > G_D(S^*, D^*) - \delta. \quad (38)$$

The right hand side of the above inequality is the condition used to define the threshold  $\bar{D}$  in Assumption 1, above which Kessler Syndrome can occur. This suggests a connection between the unstable open-access steady states and Kessler Syndrome: for pairs  $(S^*, D^*)$  where  $G_D(S^*, D^*) - \delta > 0$ , we may see the onset of instability precede the occurrence of Kessler Syndrome.

If we could argue that  $\frac{\partial \mathcal{Y}}{\partial D}(D^*)$  was increasing in  $D$ , then we could conjecture that steady states with higher amounts of debris are less likely to be stable. Analysis of  $\frac{\partial^2 \mathcal{Y}}{\partial D^2}(D^*)$  shows a necessary condition for  $\frac{\partial \mathcal{Y}}{\partial D}(D^*)$  to be increasing in  $D$ . Again using the Implicit Function Theorem to calculate  $S_{DD}(r_s - r, D)$ :

$$\frac{\partial^2 \mathcal{Y}}{\partial D^2}(D^*) > 0 \implies S_D G_S D + S_D^2 G_{SS} + S_{DD}(G_S - m(r_s - r)) > 0 \quad (39)$$

$$\implies \frac{L_D L_S (G_{SD} L_S - G_{SS} L_D)}{L_{DD} L_S^2 + L_{SD} L_D^2} > m(r_s - r) - G_S, \quad (40)$$

which is not necessarily satisfied by every open-access steady state without further restrictions on the physical and economic parameters. However, suppose that  $L(S, D)$  is strictly concave in both arguments, and the  $G(S, D)$  is strictly convex in at least  $S$ . Then, as  $S$  and  $D$  increase, the left-hand side of the inequality will tend to zero and the right-hand side will tend to negative infinity, rendering the inequality true. So strict concavity of  $L(S, D)$  and strict convexity of  $G(S, D)$  would support our conjecture.

Open-access steady states with less debris are more likely to be locally stable if the rate of excess return ( $r_s - r$ ) is not too high, or if the coupling between new fragments and active satellites ( $G_S$ ) is large enough. Their stability depends linearly on the convexity of the new fragment satellite coupling ( $G_{SD}$  and  $G_{SS}$ ), and nonlinearly on the strength and convexity of the collision probability couplings ( $L_S, L_D, L_{SD}$ , and  $L_{DD}$ ). Short-run dynamics of the type described in Propositions 4 and 5 can interact with local instability to prevent the system from reaching a stable steady state, as shown in figure 6.

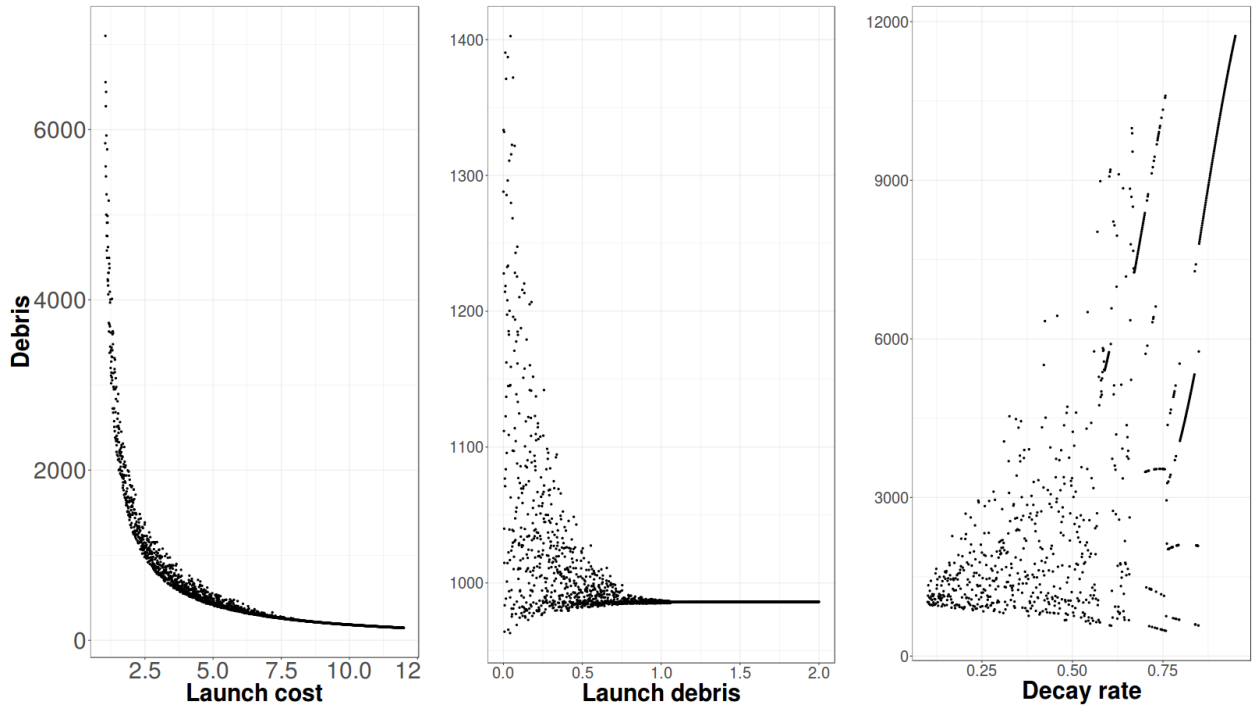


Figure 6: Each point is a long-run ( $t = 1000$ ) debris level for different values of  $F$ ,  $m$ , and  $\delta$  under open access. The spreads indicate the presence and amplitudes of cycles in debris stocks.

**Proposition 8. (Overshooting)** *For each open-access steady state, paths which do not converge to the steady state in one step will either initially overshoot the steady-state debris level or the steady-state satellite level.*

*Proof.* (Sketch) The full proof is shown in Appendix C. The intuition is that there is a limited

set of points which can reach the open-access steady state in one period, and that all other points in the action region are on paths which overshoot at least one state variable in approaching an steady state via the equilibrium isoquant. The monotonicity of  $L(S, D)$  implies that the two sets of paths are disjoint, so that the initial overshooting happens in only one state variable while the other undershoots.  $\square$

Proposition 8 highlights two properties of open access. First, open access attempts to immediately equilibrate the collision probability by moving to the isoquant where it is equal to the excess return, even if the point on the isoquant that can be reached in one period is not a steady state. This is analogous to a driver speeding to reach their destination and forgetting to decelerate as they approach. Second, this “uninternalized acceleration” problem is more severe at low levels of debris than at higher levels, provided the debris level is below the set leading directly to the steady state. In the driving analogy, this is the effect of starting location on the uninternalized acceleration: when the driver is farther away, they must accelerate more and go faster to reach their destination in the same amount of time, meaning that forgetting to decelerate has higher consequences. Economically, open access induces firms to ignore longer-term environmental consequences of their scramble to take advantage of cleaner orbits. Overshooting can cause Kessler Syndrome if the portion of the equilibrium isoquant reached is in the basin of the Kessler region. Figure 7 shows a case where open access and the planner reach steady states.

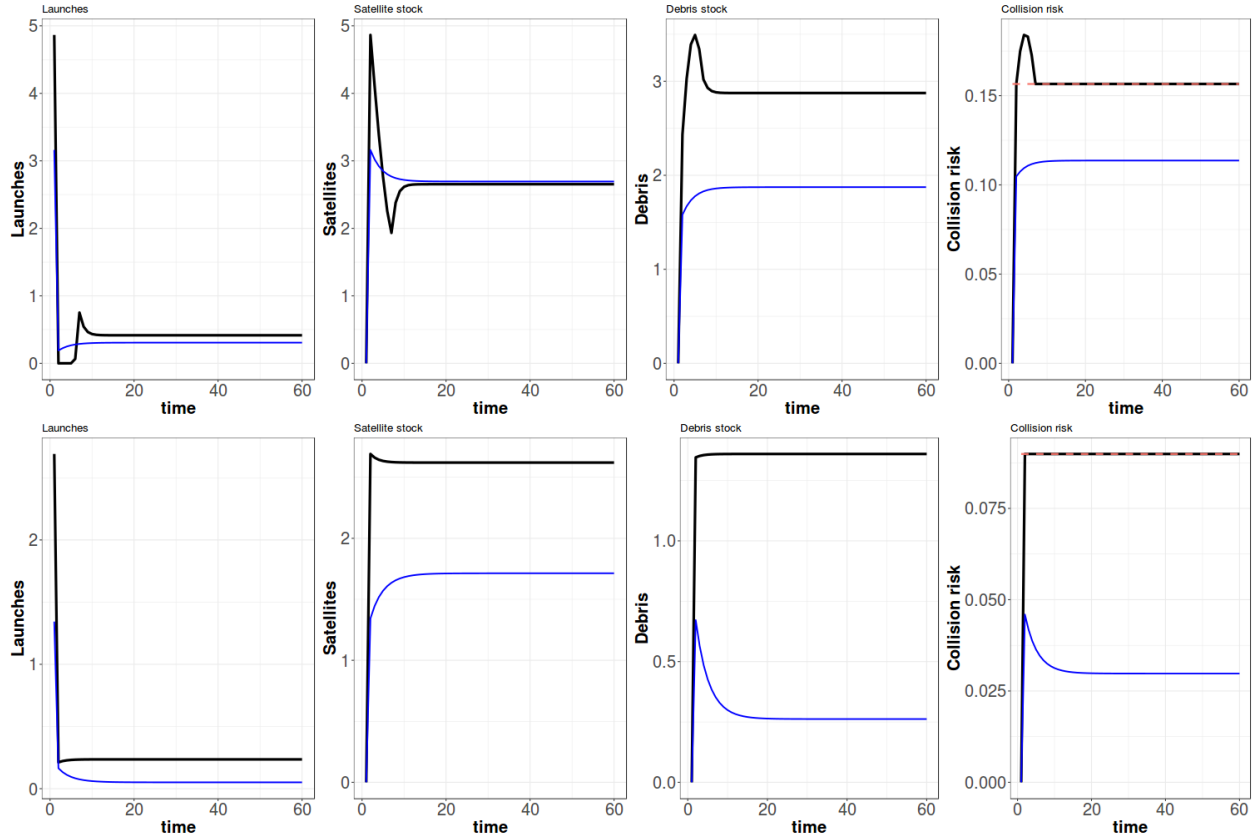


Figure 7: Examples paths which reach the steady state under open access (black) and the optimal plan (blue). Open access is not as responsive as the planner to delayed debris accumulation, which can result in firms under open access launching into the inaction region (left).

The overshooting becomes more severe as the initial condition and physical dynamics bring firms to points in the action region which are farther from the line segment leading to the open-access steady state. To see this, note that all open access launch plans are parallel lines in the satellite-debris space. Points farther from the line segment leading to the open-access steady state in question will therefore end up farther from that steady state. Given the potential multiplicity of open-access steady states, however, these points may lead to a different open-access steady state. Since they are all on the same isoquant, reaching one of many steady states implies that other steady states have been overshoot. This result suggests some scope for orbit use stabilization policies which simply impose a fixed cap on the number of launches per period, since slowing the rate at which firms approach the equilibrium isoquant can reduce the amount of overshooting. Figures 8 and 9 show examples of overshooting in the open access and planner's phase diagrams.

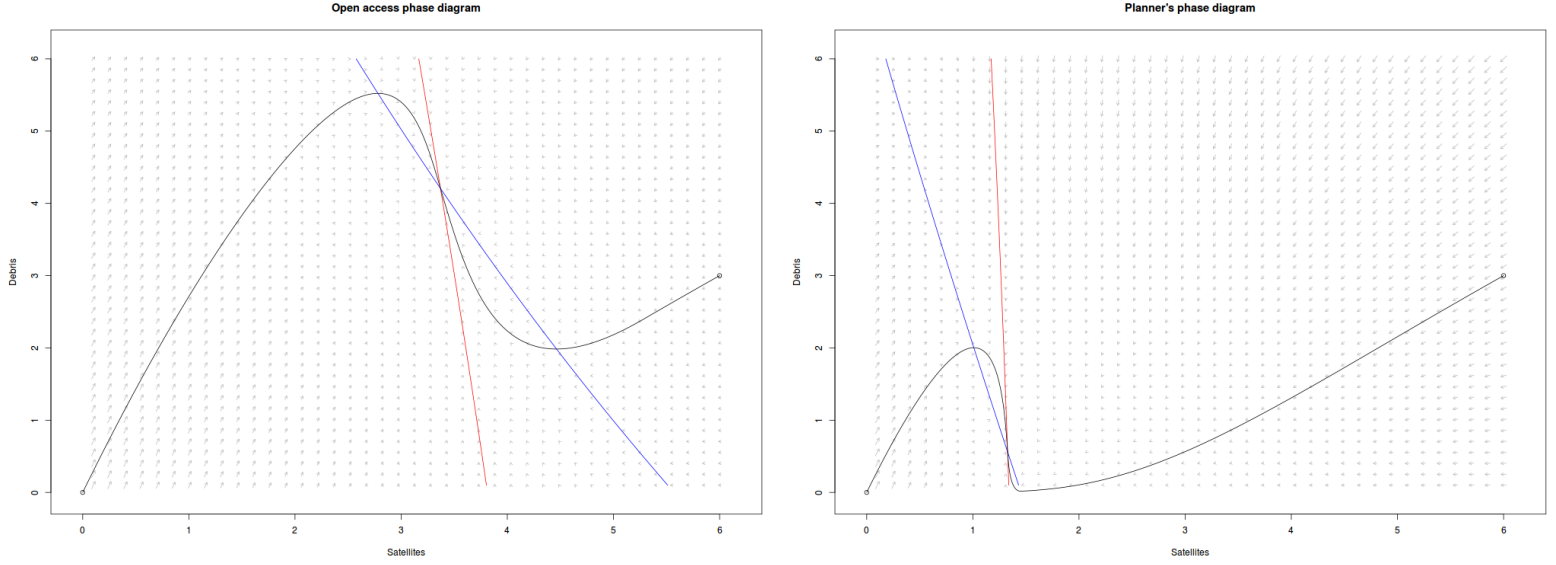


Figure 8: Left: open access phase diagram of the satellite-debris system with trajectories. Right: optimal plan phase diagram of the satellite-debris system. The blue line is the debris nullcline, the red line is the satellite nullcline, and the black lines are trajectories from  $(0,0)$  and  $(6,3)$ .

In figure 8, both open access and the planner overshoot when starting from no satellites and no debris.

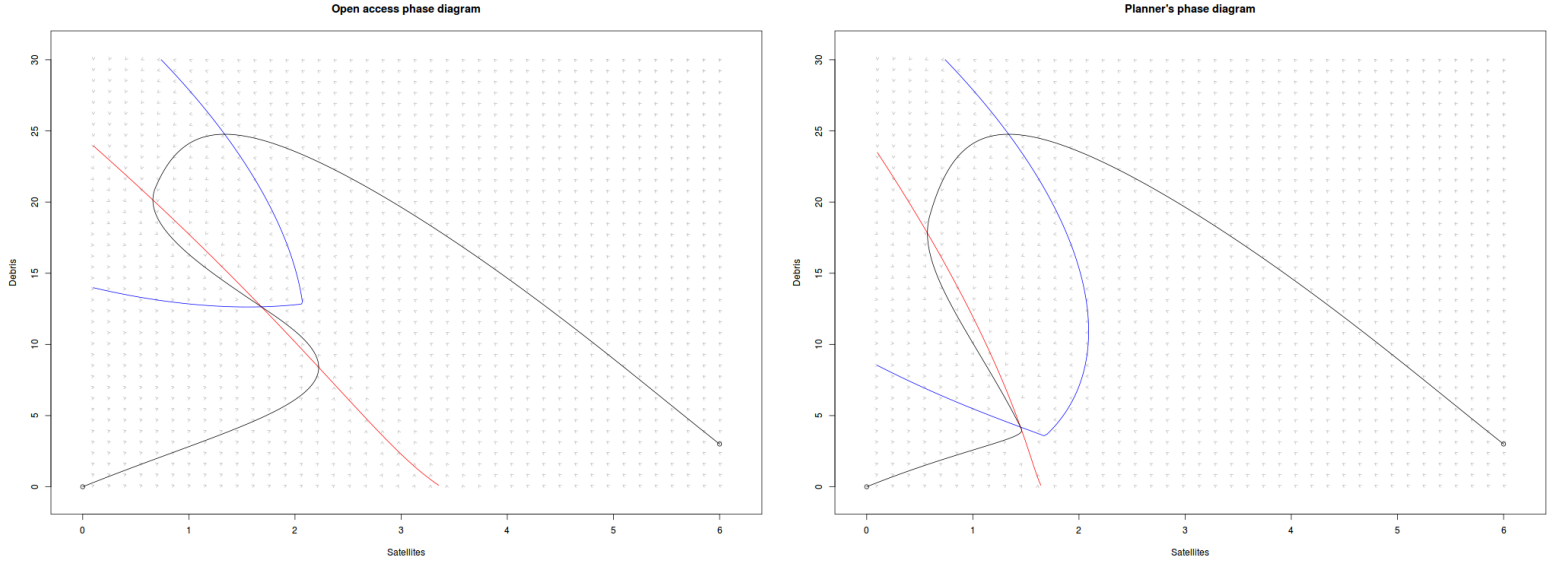


Figure 9: Left: open access phase diagram of the satellite-debris system with trajectories. Right: optimal plan phase diagram of the satellite-debris system. The blue line is the debris nullcline, the red line is the satellite nullcline, and the black lines are trajectories from  $(0,0)$  and  $(6,3)$ . The parameterization is the same as figure 8 except for the numbers of new fragments created in collisions (stronger new fragment couplings with satellites and debris in this figure).



In figure 9, open access overshoots when starting from no satellites and no debris, but the planner doesn't.

**Proposition 9.** *(Long-run debris levels) Open access steady state debris levels are*

1. *increasing in the return on a satellite and the amount of launch debris if and only if the open-access steady state is locally stable, and*
2. *decreasing in the decay rate if and only if the open-access steady state is locally stable.*

The proof is shown in Appendix C. When the open-access steady state is locally stable, long-run debris levels have the expected signs under small changes in return rates, decay rates, and launch debris levels. These signs are reversed when the open-access steady state is locally unstable. Unstable steady states will not be where the system ends up in the long-run, so a thought experiment starting from one of these states is on questionable footing. Instead, consider a steady state near the boundary of stability. For such steady states, the effects of technology shocks affecting the return on a satellite or the amount of launch debris produced, or sunspots affecting the decay rate (see section 11), can be ambiguous. If a combination of shocks pushes an open-access steady state over the boundary into instability - for example, an increase in debris due to military activities coinciding with sunspot activity - the system will move to a different steady state with more debris. These types of long-run effects can matter if the short-run dynamics preclude moving smoothly between nearby stable steady states. In such cases the short-run dynamics and the number and stability of open-access steady states will determine the long-run behavior of the system.

## 3.2 Kessler Syndrome

The occurrence of Kessler Syndrome is a key concern in managing orbit use. If open access can prevent Kessler Syndrome, regulating orbit use is not as important from an environmental perspective. Even though orbit use will be inefficient it won't cause irreversible environmental damage, though from an economic perspective there may be substantial gains from regulation.. On the other hand, if open access can cause Kessler Syndrome, orbit use management becomes more urgent. If policymakers delay too long, it may become too late.

In this section we establish some properties of the debris threshold at which Kessler Syndrome occurs. Our main result, Proposition 10, is that open access debris levels are increasing in the excess return on a satellite while the Kessler threshold is constant, implying that sustained increases in the return on a satellite can cause Kessler Syndrome under open access. Though the Kessler threshold is defined purely in terms of the system's physics, the occurrence of Kessler Syndrome depends critically on the economics of orbit use. Figure 10 shows an example of an open access path which causes Kessler Syndrome when, given identical physics and economics, the planner would avoid it.

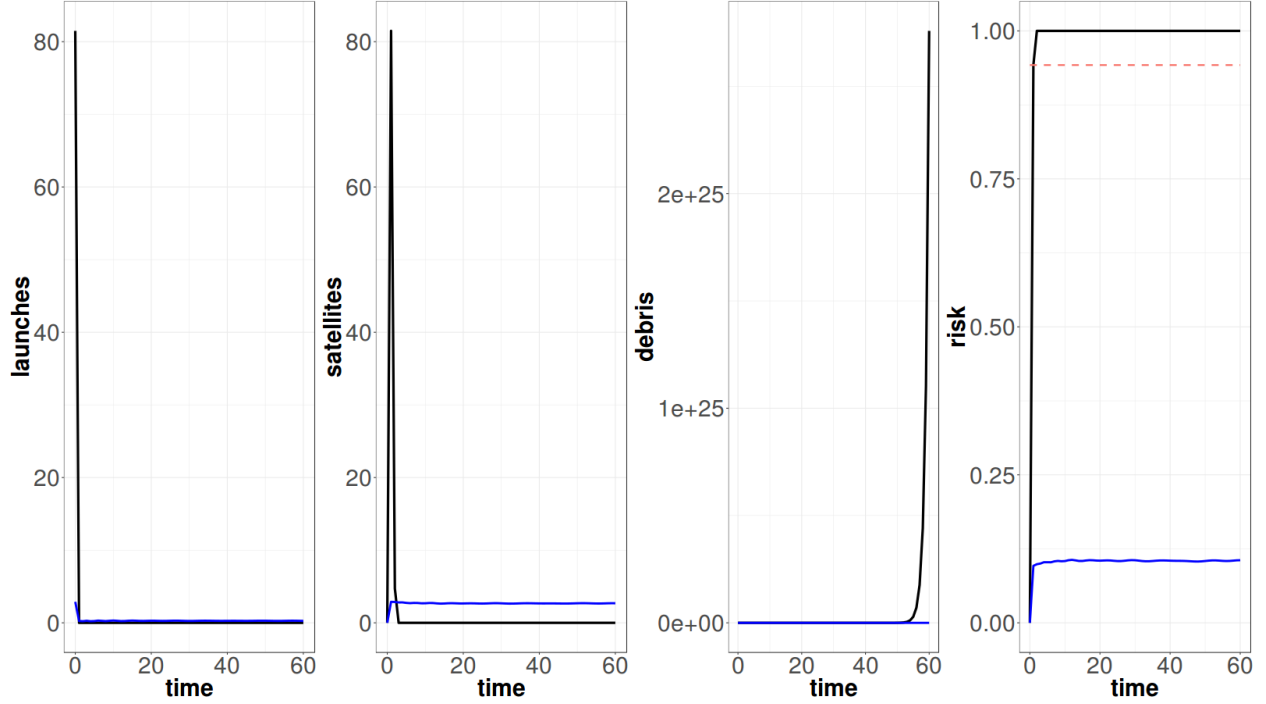


Figure 10: An example of open access causing Kessler Syndrome (black) when the planner would avoid it (blue).

The Kessler threshold  $D^K$  is entirely determined by the physical properties of the orbit. As the open-access steady state debris level rises, say due to a change in the decay rate, the open access launch rate decreases (Proposition 9). The open-access steady state debris level is by definition less than or equal to the Kessler threshold, so the open access launch rate is decreasing as steady state debris levels approach the threshold. However, this does not imply that open access will never cause Kessler Syndrome; open access launch policies may overshoot the steady state due to debris growth in period  $t + 1$  (Proposition 8). While steady states are inconsistent with Kessler Syndrome, open access equilibria need not be.

**Corollary 4.** *The Kessler threshold is increasing in the decay rate.*

This is an intuitive property of the physical system: as its absorptive capacity grows, the level at which its capacity is overwhelmed also grows.

**Proposition 10.** *(Kessler Syndrome and open access) If there are sustained increases in the return on a satellite, open access will eventually cause Kessler Syndrome.*

*Proof.* The result follows from Proposition 5 and the fact that the Kessler threshold is independent of the economics of satellite use. Since  $D^K$  is constant with changes in  $r_s$ , and the open access debris level  $\hat{D}_{t+1}$  is increasing in  $r_s$ , sustained increases in  $r_s$  will eventually push  $\hat{D}_{t+1}$  over  $D^K$ .  $\square$

Proposition 10 shows that profit motives are not necessarily sufficient to prevent Kessler Syndrome, and may in fact cause it. While the open-access steady state will avoid the Kessler region by definition, open access equilibria need not. Because of this, when the excess return on a satellite rises, Kessler Syndrome may occur in the transition from the original steady state to the new one.<sup>20</sup>

### 3.3 The risk to LEO

LEO is one of the fastest growing segments in the satellite industry today, particularly for smaller satellites with fewer guidance and control systems (Brodkin (2017), Selk (2017), Dvorsky (2018)). LEO users typically have shorter planning horizons than GEO users and face lower costs. Propositions 4, 5, 9, and 10 reinforce the conclusions from physical models of debris growth: LEO is at the highest risk of Kessler Syndrome (Liou and Johnson (2008), Kessler et al. (2010)). Our results show that this is not just a feature of the physics given current use patterns, but of the economics underlying those use patterns. While higher decay rates in LEO may protect it against Kessler Syndrome, Proposition 10 shows that the economic properties of LEO - particularly the low cost of access - work in the opposite direction. Attempts by national authorities to unilaterally prevent satellites from being launched face a leakage problem similar to the one faced by national authorities attempting to unilaterally control pollution emitted by mobile capital - firms can leave the regulated area and launch from an unregulated one.<sup>21</sup>

Larger collision or fragmentation parameters are known to increase the chance of Kessler Syndrome (e.g., Rossi et al. (1998), Kessler and Anz-Meador (2001), Liou (2006)). Adilov, Alexander, and Cunningham (2015) have shown that lower launch costs will increase the equilibrium amount of debris. We reaffirm these findings, and show that lower launch costs will also increase the equilibrium collision probability. An increase in the collision and fragmentation parameters could be caused by cost-minimizing satellite launchers opting to launch cheaper satellites with fewer guidance and control systems and less shielding. A reduction in launch costs could occur independent of changes in satellite characteristics, for example as more firms enter the launch market and drive launch prices down.

These issues are not as pronounced in GEO in part because property rights have been

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<sup>20</sup>Though we express this result elsewhere in the paper in terms of “chance” or “likelihood”, these are only informal uses for expository purposes. There is no probability measure defined anywhere in the model, so no sense in which one outcome can be “more likely” than another.

<sup>21</sup>Dvorsky (2018) documents what may be the first instance of launch leakage: a California startup denied launch permission by the FAA went outside the FAA’s jurisdiction and purchased a launch from an Indian launch provider instead. The FAA denied permission on the grounds that the startup’s satellites were too small to be effectively tracked and would increase the risk of unavoidable collisions and debris growth.

defined for GEO slots. Our model suggests that other characteristics of GEO, in particular the high cost of access and the low natural decay rate, create additional economic incentives to reduce debris accumulation and collision risk there.

## 4 Conclusion

There are potentially serious problems associated with open access to Earth's orbits. In this paper we present the first long-run economic model of orbit use, explore the nature of the congestion externality associated with launching satellites, and study policy implications of this externality. We highlight three key messages for readers concerned for the future of Earth's orbits.

First, too many firms will launch satellites because they won't internalize the risk they impose on other orbit users. Though profit maximizing satellite owners have incentives to reduce launches as risk of a collision grows, they do not respond to debris growth or the risk of collisions optimally. Second, rebound effects can result in higher debris levels when the rate of natural renewal is high and when the debris created by a launch is low. Combined with the low cost of access, these effects suggest that lower regions of LEO may end up more congested than higher regions. Third, Kessler Syndrome is more likely under open access as the excess return on a satellite rises. As launch costs fall and new commercial satellite applications become viable, open access will be more likely to cause Kessler Syndrome despite launch reductions in response to orbital congestion.

Economists tend to focus on property rights, corrective taxes, or other market-based mechanisms to solve externality problems. While these mechanisms may prevent Kessler Syndrome and ensure efficient orbit use, more study is needed to understand how orbital management policies should be designed in light of the unique physical features of this resource. Orbits are a global commons, and will likely require global policy solutions. These solutions may be enforced by states, arise as self-enforcing agreements between private actors, or some combination of the two. The problems of social cost in orbit, of interacting congestion and pollution with endogenous regime change, are similar to the problems of climate change. The fact that economic policy solutions to climate change have proven difficult to build consensus around and implement suggests enacting policy solutions to orbital externalities may not be easy either.

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## 5 Appendix A: Extensions to the baseline model

### 5.1 Spectrum congestion or price effects

Satellite applications generally require transmissions to and from the Earth. These transmissions may be the satellite's main output or incidental to its operation. In both cases, satellite operators must secure spectrum use rights from the appropriate national authorities for their broadcast and receiving locations. How will spectrum management affect collisions and debris growth?

Spectrum congestion degrades signal quality, making the per-period output of a satellite decreasing in the number of satellites in orbit, i.e.  $\pi = \pi(S), \pi'(S) < 0$ . If satellites are launched only when they have appropriate spectrum rights and spectrum use is optimally managed, then firms will be forced to account for their marginal effects on spectrum congestion in their decision to launch or not. The equilibrium condition becomes

$$L(S_{t+1}, D_{t+1}) = r_s(S_{t+1}) + r'_s(S_{t+1}) - r, \quad (41)$$

where  $r'_s(S_{t+1}) = \pi'(S_{t+1})/F < 0$  by assumption. The equilibrium set would no longer be a collision probability isoquant, although it would still be a surface in the state space. Even if it isn't managed optimally, spectrum congestion will reduce the equilibrium collision probability by reducing the rate of excess return on a satellite.<sup>22</sup>

Open access orbit use will still be inefficient. Although spectrum congestion can reduce the chance of Kessler Syndrome, efficient spectrum management will not incorporate the marginal external cost of collisions and debris growth,  $\xi(S_{t+1}, D_{t+1})$ . Collision and debris growth management policies could be implemented through spectrum pricing. The rest of the analysis in this paper still goes through when spectrum congestion is considered, though some proofs become more complicated since the equilibrium set is no longer a collision probability isoquant.

Mathematically, this modification would also apply to price effects induced by additional firms entering a particular orbit. Such effects may be relevant for orbits where the dominant satellite application does not face terrestrial competition, such as satellite imaging in sun-synchronous orbits. In orbits where the price of the service provided by satellites is pinned down by terrestrial applications, such as LEO or GEO internet or television service provided to urban areas, either the baseline model where  $r_s$  is constant or the interpretation of  $r'_s(S)$  as spectrum congestion is appropriate. Both spectrum and price effects may be relevant for

<sup>22</sup>If spectrum use were also under open access then the marginal congestion effect ( $r'_s(S_{t+1})$ ) would not be in the equilibrium condition, but the equilibrium set would still not be a collision probability isoquant.

applications which require substantial spectrum use and face little terrestrial competition, such as satellite telecom service to remote areas. We put the spectrum congestion interpretation first since all satellites require some spectrum and could interfere with each other regardless of application, whereas price effects are application-specific. Price effects, however, can be orbit-agnostic in the sense that satellite systems in different physical orbits may have price effects on each other if they compete in the same market.

## 5.2 The effects of limited satellite lifespans

Satellites do not, in general, produce returns forever until destroyed in a collision. Over 1967-2015, planned satellite lifetimes ranged from 3 months to 20 years, with longer lifetimes being more representative of larger and more expensive GEO satellites.<sup>23</sup> How would finite lifetimes affect the collision probability and debris growth problems?

Suppose satellite lifetimes are finite and exogenously distributed with mean  $\mu^{-1}$ . Satellites live at least one time period in this setting, so that  $\mu^{-1} > 1$ . The probability that a satellite will exogenously “die” in any given period is then  $\mu$ . The value of a single satellite becomes

$$Q(S_t, D_t, X_t) = \pi + \beta(1 - \mu)(1 - L(S_t, D_t))Q(S_{t+1}, D_{t+1}, X_{t+1}), \quad (42)$$

and the equilibrium condition becomes

$$\beta Q(S_{t+1}, D_{t+1}, X_{t+1}) = F \quad (43)$$

$$\implies L(S_{t+1}, D_{t+1}) = \frac{r_s - r - \mu}{1 - \mu}, \quad (44)$$

which is lower than the equilibrium collision probability when satellites are infinitely lived. Intuitively, the fact that the satellite will stop generating returns at some point reduces its expected present value, and with it the incentive to launch. All else equal, shorter lifetimes reduce the equilibrium collision probability. The rest of the analysis in this paper goes through with minor modifications. Note that equation 44 is the non-stationary discrete-time analog of the physico-economic equilibrium condition in Definition 2 of Rouillon (2019).

Satellites built for GEO tend to be longer lived than satellites launched for LEO. If shorter lifetimes tend to reduce satellite costs, then the downward shift in the collision probability isoquant from the shorter lifetimes will be balanced against the upward shift caused by higher rates of return. The net effect may be higher or lower equilibrium rates of collisions.

<sup>23</sup>These numbers are taken from the Union of Concerned Scientists’ publicly available data on satellites. The data are available at <https://www.ucsusa.org/nuclear-weapons/space-weapons/satellite-database>.



The distinction between exogenous and endogenous lifetimes is relevant here. The above analysis hinges on satellite lifetimes being exogenously set. This is not the case in reality. Satellite lifetimes are determined by cost minimization concerns, technological constraints, expectations of component failures, and expectations of future technological change. Incorporating all these features to realistically model the choice of satellite life along with launch decisions and their effects on orbital stock dynamics is beyond our scope in this paper, though it is an interesting area for future research. The assumption that the end-of-life is random simplifies the analysis, but does not change any conclusions over imposing a pre-specified end date in this model.<sup>24</sup>

### 5.3 The effects of changes in satellite returns over time

Though it simplifies the analysis, the rate of return on a satellite is not constant over time. How would changes over time in these economic parameters affect our conclusions? For simplicity, suppose that costs, returns, and discount rate vary exogenously and are known in advance.<sup>25</sup> The open access equilibrium condition is then

$$\pi_{t+1} = (1 + r_t)F_t - (1 - L(S_{t+1}, D_{t+1}))F_{t+1} \quad (45)$$

Equation 45 can be rewritten as

$$L(S_{t+1}, D_{t+1}) = 1 + \frac{\pi_{t+1} - (1 + r_t)F_t}{F_{t+1}} \quad (46)$$

$$\implies L(S_{t+1}, D_{t+1}) = 1 + r_{s,t+1} - (1 + r_t) \frac{F_t}{F_{t+1}} \quad (47)$$

$$\implies L(S_{t+1}, D_{t+1}) = \underbrace{\left( r_{s,t+1} - r \frac{F_t}{F_{t+1}} \right)}_{\text{excess return from satellite ownership}} + \underbrace{\left( 1 - \frac{F_t}{F_{t+1}} \right)}_{\text{capital gains from changes in satellite launch cost}}. \quad (48)$$

If  $\pi_{t+1} > (1 + r)F_t$ , then the one period return on a satellite is greater than the gross return on the launch cost from the safe asset, and the collision probability will be 1. Ignoring that corner case, the equilibrium collision probability is decreasing in the current cost of launching a satellite, but increasing in the future cost of launching a satellite. All else equal, the collision probability in  $t + 1$  will be lower when  $F_{t+1}$  increases. This highlights the role of the launch cost under open access: if firms enter until zero profits in each period, future increases in the cost deter firms from entering in the future, increasing the value of satellites already in orbit by

<sup>24</sup>This simplification could matter in a model where firms own multiple satellites and have to plan replacements.

<sup>25</sup>Uncertainty over costs, returns, and discount rates doesn't change the qualitative results, though it introduces expectations over the changes. Endogeneity in the changes, for example due to investment in R&D or marketing, may have more significant consequences which are beyond the scope of this paper.

the amount of the cost increase. Alternately, the equilibrium collision risk can be decomposed into two components: one representing the excess returns of satellite ownership, and the other representing the capital gains from changes in the cost of launching a satellite under open access.

When the costs and returns are time-varying, the equilibrium set is still a collision probability isoquant, though the isoquant selected may vary over time. These changes do not affect the physical dynamics or the Kessler threshold, though they may affect how close the selected equilibrium is to the threshold. If the parameters vary so that the ratio  $\frac{\pi_{t+1} - (1+r)F_t}{F_{t+1}}$  is stationary, then the equilibrium set will stay on the same isoquant.

## 5.4 The effects of space weather

Sunspots have two effects: first, changes in radiation pressure force satellites in higher orbits to spend more fuel on stationkeeping; second, the Earth's atmosphere expands in response to the solar activity, increasing drag and debris decay in all but the highest orbits. The latter effect can be particularly significant for active satellites in LEO. Formally, suppose sunspots cause decay rates to vary periodically with mean  $\bar{\delta}$ , making the laws of motion

$$\begin{aligned} S_{t+1} &= S_t(1 - L(S_t, D_t)) + X_t \\ D_{t+1} &= D_t(1 - \delta_t) + G(S_t, D_t) + mX_t \end{aligned}$$

If the variations are exogenous and publicly known, there are no changes to the firm or planner's Bellman equations. Figure 11 shows example time paths with and without sunspot activity.

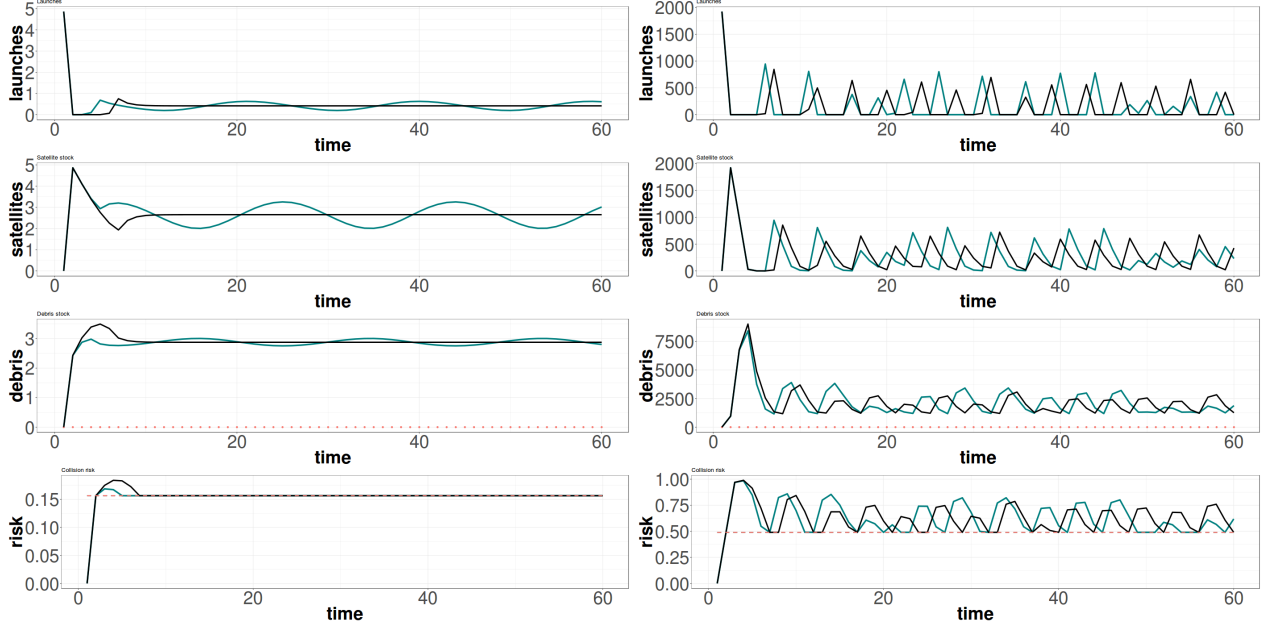


Figure 11: Open access equilibrium paths with no sunspots (black line) and with sunspots driven by  $\delta_t = \bar{\delta} + \omega_1 \sin(\omega_2 t)$  (teal line).

The results in Proposition 5 describe how open-access equilibrium outcomes will respond to increases in the decay rate caused by space weather, holding costs constant. As the sunspot activity increases, firms will have incentives to take advantage of the increased renewal by launching more satellites. The net effect may be an increase or decrease the equilibrium amount of debris. As the activity decreases, firms will reduce the launch rate, again with ambiguous effects on the equilibrium amount of debris. These effects will be offset by the extent to which sunspot activity increases the cost of operating a satellite, e.g. by increasing the necessary fuel expenditures on stationkeeping.

Increases in the decay rate have the benefit of shifting the Kessler threshold up (Proposition 4). If the net increase in debris due to the launch response is not too large, then increased sunspot activity may stabilize orbit use. On the other hand, if the rebound effect for decay rates is strong enough, sunspot activity may push the equilibrium debris level into the basin of the Kessler region. Even when the rebound effect is not very strong, a decrease in the decay rate at the end of a period of extra sunspot activity may make the original open-access steady state locally unstable given the increase in debris due to the rebound effect.

## 5.5 Open access equilibrium between multiple shells

Our model has so far focused on a single orbital shell in isolation. While we can expand the “shell of interest” to cover a large region, in practice this is unlikely to yield accurate results over heterogeneous regions and use cases. Suppose we move up one level of generality

to consider two orbital shells,  $H$  and  $L$ , with access costs  $F^H > F^L$ , and returns are equal at  $\pi$ . What would open access imply for the collision risks between the shells? Letting  $H$  superscripts index the higher orbit and  $L$  subscripts index the lower one, the new laws of motion are

$$\begin{aligned} S_{t+1}^H &= S_t^H(1 - L^H(S_t^H, D_t^H)) + X_t^H \\ D_{t+1}^H &= D_t^H(1 - \delta^{HE} - \delta^{HL}) + G(S_t^H, D_t^H) + \mu_L X_t^H + \delta^{LH} D_t^L \\ S_{t+1}^L &= S_t^L(1 - L^L(S_t^L, D_t^L)) + X_t^L \\ D_{t+1}^L &= D_t^L(1 - \delta^{LE} - \delta^{LH}) + G(S_t^L, D_t^L) + \mu_L X_t^L + \delta^{HL} D_t^H \end{aligned}$$

with the debris transport matrix

$$\mathbf{D} = \begin{array}{c} H \\ L \\ E \end{array} \left\| \begin{array}{ccc} H & L & E \\ 1 - \delta^{HL} - \delta^{HE} & \delta^{HL} & \delta^{HE} \\ \delta^{LH} & 1 - \delta^{LH} - \delta^{LE} & \delta^{LE} \\ 0 & 0 & 1 \end{array} \right\|,$$

where  $E$  represents the Earth (which is a sink for all orbiting objects). We impose that all coefficients are bounded in  $[0, 1]$  and sum to 1 across rows, with row labels indexing the source of debris and column labels indexing the destination.

The Bellman equations for the launch decision are now

$$Q_t^H = \pi + \beta(1 - L_t^H)Q_{t+1}^H \quad (49)$$

$$Q_t^L = \pi + \beta(1 - L_t^L)Q_{t+1}^L \quad (50)$$

$$V_{it} = \max_{x_{it} \in \{0, L, H\}} \{ \mathbb{1}(x_{it} = 0)\beta V_{t+1} + \mathbb{1}(x_{it} = H)[\beta Q_{t+1}^H - F^H] \quad (51)$$

$$+ \mathbb{1}(x_{it} = L)[\beta Q_{t+1}^L - F^L] \}, \quad (52)$$

where we suppress function arguments and use time subscripts for brevity (though these are still infinite-horizon Bellman equations). In an open-access equilibrium, the returns from

owning a satellite in either shell should be zero, i.e.

$$(X_t^H, X_t^L) : V_{it} = 0 \implies \beta Q_{t+1}^H - F^H = 0, \beta Q_{t+1}^L - F^L = 0 \quad (53)$$

$$\implies \beta[\pi + (1 - L_{t+1}^H)F^H] - F^H = 0, \beta[\pi + (1 - L_{t+1}^L)F^L] - F^L = 0 \quad (54)$$

$$\implies \pi = \left(\frac{1-\beta}{\beta}\right)F^H + L_{t+1}^H F^H, \pi = \left(\frac{1-\beta}{\beta}\right)F^L + L_{t+1}^L F^L \quad (55)$$

$$\implies rF^H + L_{t+1}^H F^H = rF^L + L_{t+1}^L F^L \quad (56)$$

$$\implies \frac{F^L}{F^H} = \frac{r + L_{t+1}^H}{r + L_{t+1}^L} \quad (57)$$

If the lower orbit is cheaper to access, then the higher orbit will have lower equilibrium collision risk, i.e.

$$F^L < F^H \implies L_{t+1}^H < L_{t+1}^L. \quad (58)$$

If the returns earned by satellites in either shell are not identical, but rather  $\pi^H$  and  $\pi^L$  with  $\pi^H = \pi^L + \gamma$ , then we have

$$\frac{F^L}{F^H} = \frac{r + L_{t+1}^H - \frac{\gamma}{F^H}}{r + L_{t+1}^L} \quad (59)$$

Now if the lower orbit is again cheaper to access as we assumed above, then  $F^L < F^H$ , then  $L_{t+1}^H - \frac{\gamma}{F^H} < L_{t+1}^L$ . This gives us two possibilities:

- If the higher orbit generates greater returns, either orbit may be riskier in equilibrium, depending on parameter values. Formally,  $\gamma > 0 \implies L_{t+1}^H \leq L_{t+1}^L$ .
- If the lower orbit generates greater returns, then the higher orbit will again have lower equilibrium collision risk. Formally,  $\gamma < 0 \implies L_{t+1}^H < L_{t+1}^L$ .

The physical and economic dynamics in the multi-shell setting are likely far richer than the dynamics of the single-shell setting. Further research will be needed to understand such generalizations in detail.



## 6 Appendix B: Additional figures

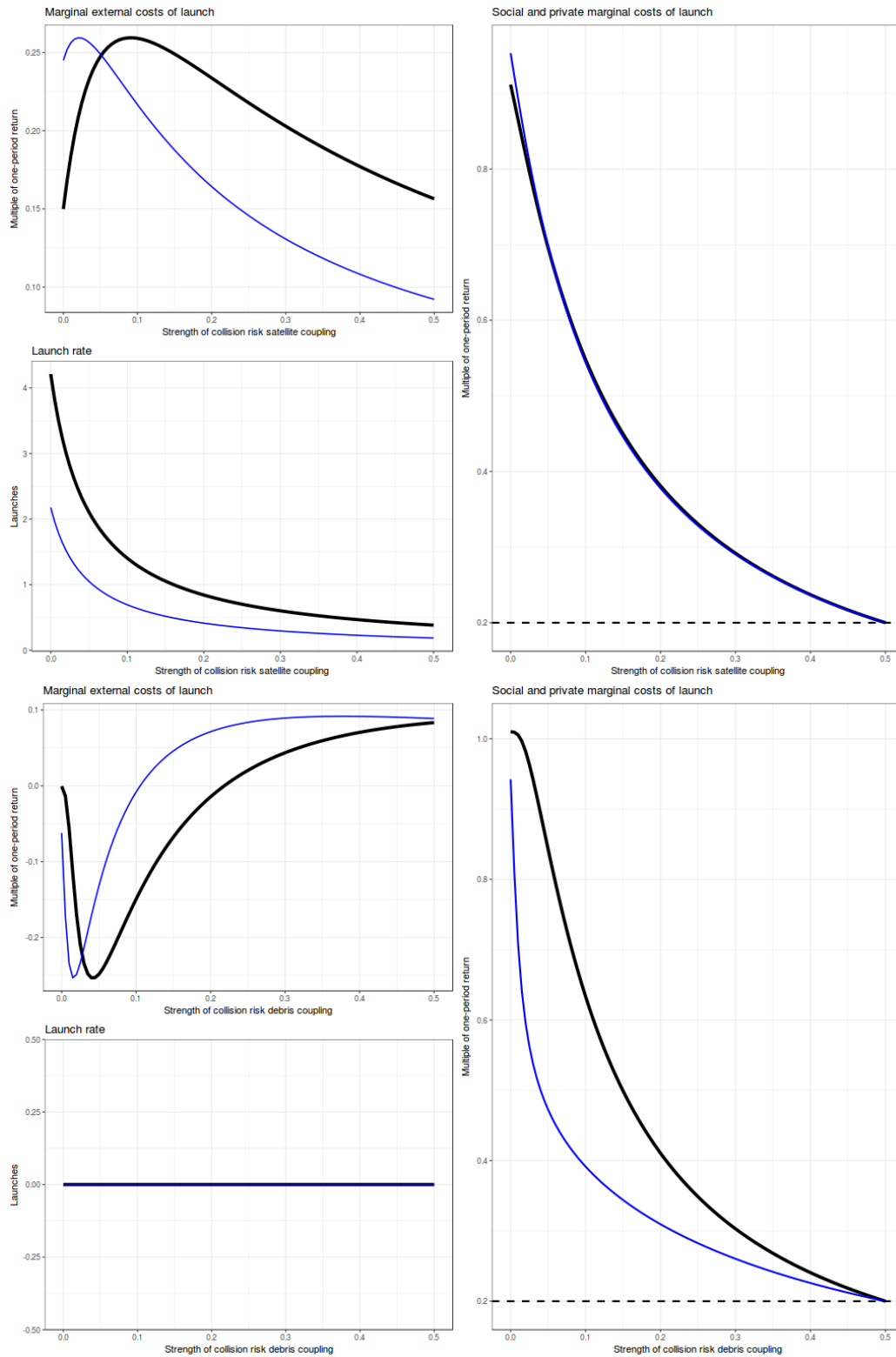


Figure 12: Left: open access (black) and planner (blue) MEC and launch rate; private marginal cost (blue) and social marginal cost (blue) over  $L(S)$  coupling. Right: open access (black) and planner (blue) MEC and launch rate; private marginal cost (blue) and social marginal cost (blue) over  $L(D)$  coupling

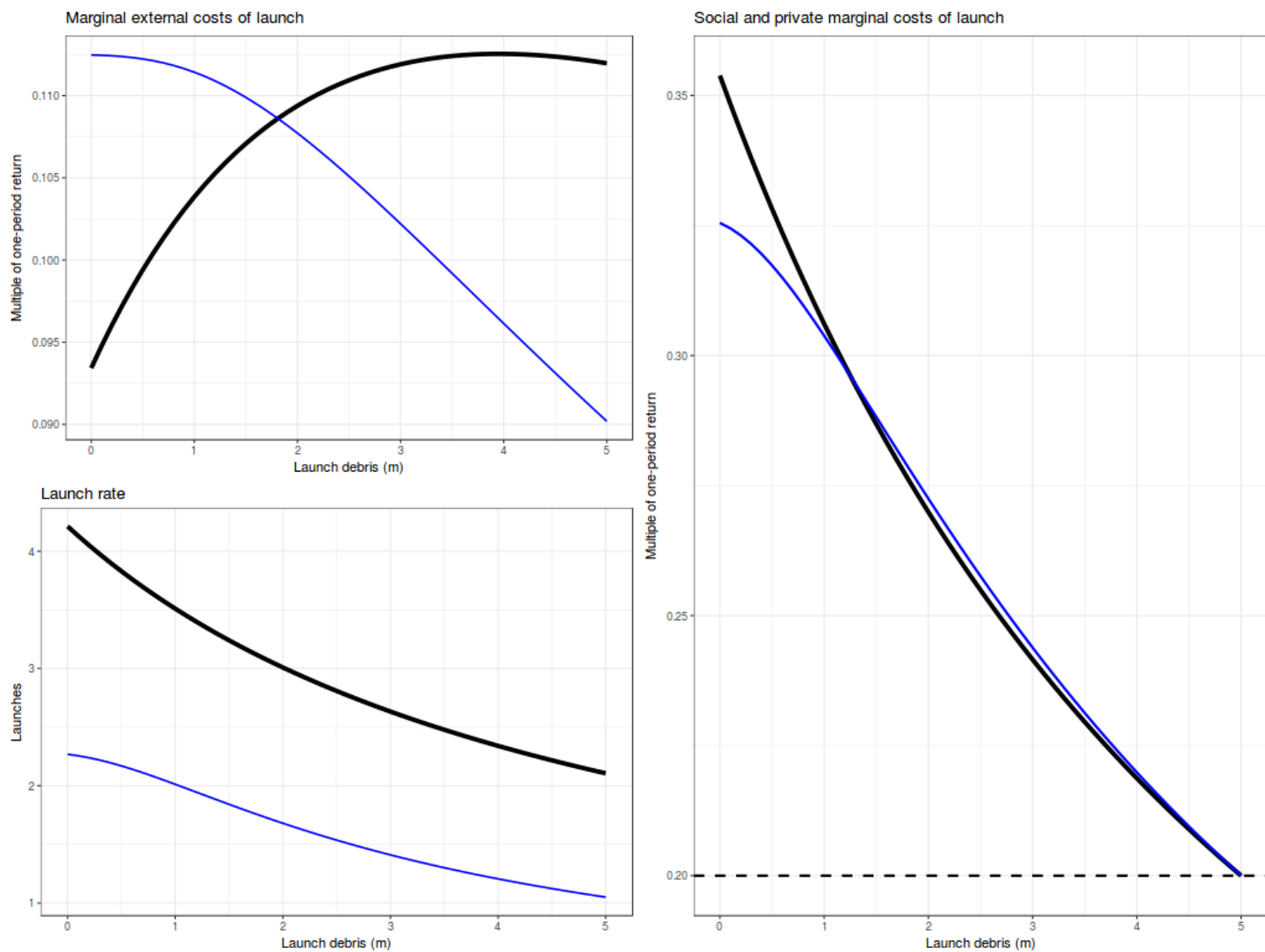


Figure 13: open access (black) and planner (blue) MEC and launch rate; private marginal cost (blue) and social marginal cost (blue) over launch debris coupling.



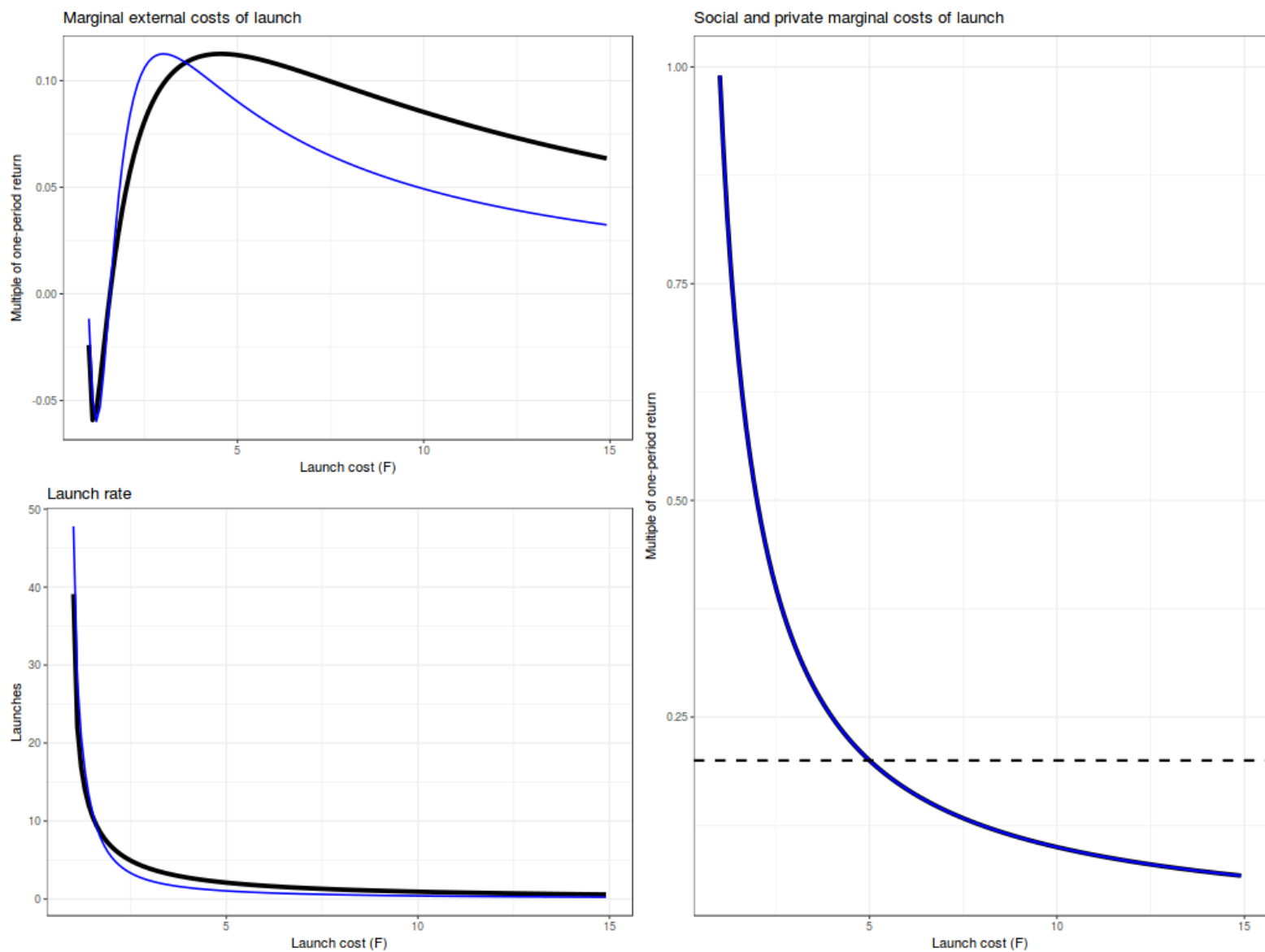


Figure 14: Open access (black) and planner (blue) MEC and launch rate; private marginal cost (blue) and social marginal cost (blue) over launch cost.

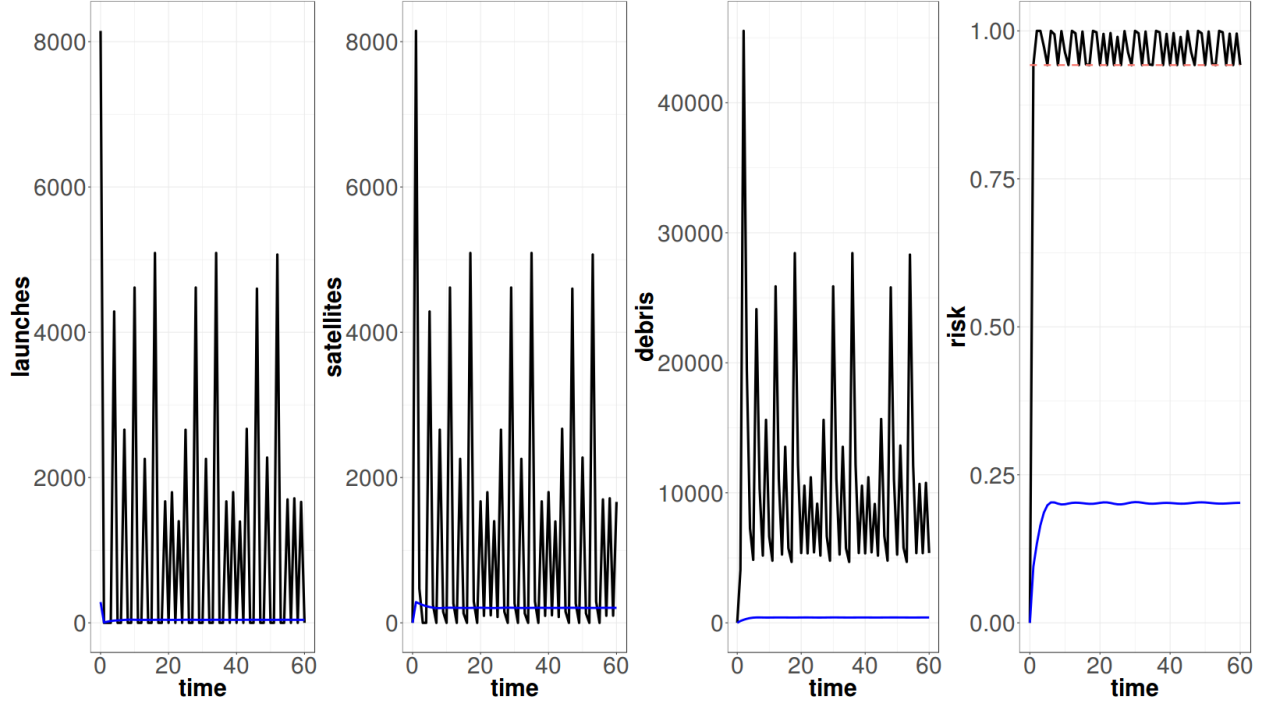


Figure 15: An example of open access time paths which bounce between action and inaction regions (black). The planner (blue) does not find this optimal.

Figures were generated using

$$L(S, D) = 1 - e^{-\alpha_{SS}S - \alpha_{SD}D},$$

$$G(S, D) = \beta_{SS}L(S, D) \left( \frac{S}{S+D} \right) S + \beta_{SD}L(S, D) \left( \frac{D}{S+D} \right) D + \beta_{DD}(1 - e^{-\alpha_{DD}D})D.$$

Other forms tried, including  $L(S, D) = 1 - e^{-\alpha_{SS}S^2 - \alpha_{SD}SD}$  and  $G(S, D) = \beta_{SS}L(S, D) \left( \frac{S}{S+D} \right) S + \beta_{SD}L(S, D) \left( \frac{D}{S+D} \right) D + \beta_{DD}D^2$ , yielded qualitatively similar results.

## 7 Appendix C: Detailed proofs

**Deriving the marginal external cost** Period  $t$  values are shown with no subscript, and period  $t + 1$  values are marked with a  $'$ , e.g.  $S_t \equiv S, S_{t+1} \equiv S'$ . The fleet planner's problem is

$$W(S, D) = \max_{X \geq 0} \{ \pi S - FX + \beta W(S', D') \} \quad (60)$$

$$\text{s.t. } S' = S(1 - L(S, D)) + X \quad (61)$$

$$D' = D(1 - \delta) + G(S, D) + mX. \quad (62)$$

The fleet planner's launch plan will satisfy

$$X^* : \beta[W_S(S', D') + mW_D(S', D')] = F, \quad (63)$$

that is, the planner will launch until the marginal value to the fleet of a new satellite plus the marginal value to the fleet of its launch debris is equal to the launch cost. Note that while the per-period objective function is linear in the control variable, the steady-state objective function is not due to nonlinearity created by the collision probability function — letting  $X = L(S, D)S$ ,  $\sum_{t=\tau}^{\infty} \beta^{\tau-t} (\pi S - FX) = (1 - \beta)^{-1} S(\pi - FL(S, D))$ , which is not linear in  $S$  unless  $L(S, D)$  is uncoupled from  $S$  and  $D$  (i.e.  $L(S, D) \equiv L \forall (S, D)$ ). More generally, the fleet pre-value function is similarly nonlinear in the control variable as launches in  $t$  impose a linear cost in  $t$  but have nonlinear effects in periods beyond  $t$ . Thus, the optimal policy is not of the bang-bang type when the collision probability function is coupled with at least the satellite stock and through it, the launch rate.

Applying the envelope condition, we have the following expressions for the fleet's marginal value of another satellite and another piece of debris:

$$W_S(S, D) = \pi + \beta[W_S(S', D')(1 - L(S, D) - SL_S(S, D)) + W_D(S', D')G_S(S, D)] \quad (64)$$

$$W_D(S, D) = \beta[W_D(S', D')(1 - \delta + G_D(S, D)) + W_S(S', D')(-SL_D(S, D))] \quad (65)$$

Rewriting equation 63, we have

$$W_S(S', D') = \left[ \frac{F}{\beta} - mW_D(S', D') \right] \quad (66)$$

Plugging equation 66 into equations 64 and 65,

$$W_S(S, D) = \pi + F(1 - L(S, D) - SL_S(S, D)) - \beta W_D(S', D')[m(1 - L(S, D) - SL_S(S, D)) - G_S(S, D)] \quad (67)$$

$$W_D(S, D) = (-SL_D(S, D))F + \beta W_D(S', D')[1 - \delta + G_D(S, D) - m(-SL_D(S, D))] \quad (68)$$

Define the following quantities:

$$\begin{aligned}\alpha_1(S, D) &= \pi + (1 - L(S, D) - SL_S(S, D))F \\ \alpha_2(S, D) &= -SL_D(S, D)F \\ \Gamma_1(S, D) &= G_S(S, D) - m(1 - L(S, D) - SL_S(S, D)) \\ \Gamma_2(S, D) &= 1 - \delta + G_D(S, D) + mSL_D(S, D).\end{aligned}$$

These allow us to rewrite equations 67 and 68 as

$$W_S(S, D) = \alpha_1(S, D) + \beta\Gamma_1(S, D)W_D(S', D') \quad (69)$$

$$W_D(S, D) = \alpha_2(S, D) + \beta\Gamma_2(S, D)W_D(S', D'). \quad (70)$$

As long as  $\delta < 1$ ,  $\Gamma_2(S, D) \neq 0 \forall (S, D)$ , allowing us to rewrite equation 70 as

$$W_D(S', D') = \frac{W_D(S, D) - \alpha_2(S, D)}{\beta\Gamma_2(S, D)}. \quad (71)$$

Plugging equation 71 into equation 69, we get

$$\begin{aligned}W_S(S, D) &= \alpha_1(S, D) + \beta\Gamma_1(S, D) \frac{W_D(S, D) - \alpha_2(S, D)}{\beta\Gamma_2(S, D)} \\ &= \alpha_1(S, D) + \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} (W_D(S, D) - \alpha_2(S, D)) \\ \implies W_S(S, D) &= \alpha_1(S, D) - \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} \alpha_2(S, D) + \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} W_D(S, D)\end{aligned} \quad (72)$$

Iterating equation 66 one period backwards and plugging it into equation 69, we get

$$\begin{aligned}\frac{F}{\beta} - mW_D(S, D) &= \alpha_1(S, D) - \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} \alpha_2(S, D) + \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} W_D(S, D) \\ \implies W_D(S, D) &= \left[ \frac{F}{\beta} - \alpha_1(S, D) - \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} \alpha_2(S, D) \right] \left[ \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} + m \right]^{-1}.\end{aligned} \quad (73)$$

Substituting in the forms for  $\alpha_1(S, D)$ ,  $\alpha_2(S, D)$ ,  $\Gamma_1(S, D)$ , and  $\Gamma_2(S, D)$ , combining equations 70 and 73 yields an Euler equation for the optimal launch rate,

$$\begin{aligned}\frac{1}{\beta} &= \alpha_2(S, D) \left[ \frac{\Gamma_1(S, D) + m\Gamma_2(S, D)}{\Gamma_2(S, D)(F - \beta\alpha_1(S, D)) + \beta\Gamma_1(S, D)\alpha_2(S, D)} \right] + \\ &\Gamma_2(S, D) \left[ \frac{\Gamma_2(S', D')(F - \beta\alpha_1(S', D')) + \beta\Gamma_1(S', D')\alpha_2(S', D')}{\Gamma_2(S, D)(F - \beta\alpha_1(S, D)) + \beta\Gamma_1(S, D)\alpha_2(S, D)} \right]\end{aligned} \quad (74)$$

This Euler equation can be rearranged and compared with the open access equilibrium condition (equation 14) to obtain the planner's optimality condition (equation 24) and the marginal external cost (equation 26).

#### Proof of Proposition 4:

*Proof.* Period  $t$  values are shown with no subscript, and period  $t + 1$  values are marked with a  $'$ , e.g.  $S_t \equiv S, S_{t+1} \equiv S'$ . Applying the Implicit Function Theorem to equation 14, we get

$$\begin{aligned}\frac{\partial X}{\partial S} &= -\frac{[\frac{\partial L(S', D')}{\partial S'}(1 - L(S, D)) - \frac{\partial L(S, D)}{\partial S}S] + \frac{\partial L(S', D')}{\partial D'}\frac{\partial G(S, D)}{\partial S}}{[\frac{\partial L(S', D')}{\partial S'}\frac{\partial S'}{\partial X} + \frac{\partial L(S', D')}{\partial D'}\frac{\partial D'}{\partial X}]} < 0 \\ \frac{\partial X}{\partial D} &= -\frac{\frac{\partial L(S', D')}{\partial S'}(-S\frac{\partial L(S, D)}{\partial D}) + \frac{\partial L(S', D')}{\partial D'}(1 - \delta + \frac{\partial G(S, D)}{\partial D})}{\frac{\partial L(S', D')}{\partial S'}\frac{\partial S'}{\partial X} + \frac{\partial L(S', D')}{\partial D'}\frac{\partial D'}{\partial X}} \leq 0\end{aligned}$$

$$\frac{\partial X}{\partial D} < 0 \text{ if}$$

$$(S, D) : \underbrace{\frac{\partial L(S', D')}{\partial S'}S}_{\substack{t+1 \text{ marginal collision probability from} \\ t+1 \text{ satellites} \cdot \text{present satellite stock}}} < \underbrace{\frac{\frac{\partial L(S', D')}{\partial D'}}{\frac{\partial L(S, D)}{\partial D}}}_{\substack{\text{growth rate of} \\ \text{marginal collision probability} \\ \text{due to debris}}} \cdot \underbrace{\left(1 - \delta + \frac{\partial G(S, D)}{\partial D}\right)}_{\substack{\text{net growth in debris} \\ \text{due to debris}}} \quad (75)$$

and  $\frac{\partial X}{\partial D} > 0$  otherwise.

□

#### Proof of Proposition 5:

*Proof.* Period  $t$  values are shown with no subscript, and period  $t + 1$  values are marked with a  $'$ , e.g.  $S_t \equiv S, S_{t+1} \equiv S'$ . All present-period variables are assumed to be at equilibrium values. Applying the Implicit Function Theorem to equation 14, we get

$$\begin{aligned}\frac{\partial X}{\partial r_s} &= \frac{1}{L_S(S', D') + mL_D(S', D')} > 0 \\ \frac{\partial D'}{\partial r_s} &= \frac{1}{L_D(S', D')} > 0\end{aligned}$$

$$\begin{aligned}\frac{\partial X}{\partial m} &= -\frac{L_D(S', D')X}{L_S(S', D') + mL_D(S', D')} < 0 \\ \frac{\partial D'}{\partial m} &= X + m\frac{\partial X}{\partial m} \\ \frac{\partial D'}{\partial m} > 0 &\implies 1 > \frac{mL_D(S', D')}{L_S(S', D') + mL_D(S', D')},\end{aligned}$$

which is true as long as  $L_S > 0$ .

$$\begin{aligned}\frac{\partial X}{\partial \delta} &= \frac{L_D(S', D') D}{L_S(S', D') + m L_D(S', D')} > 0 \\ \frac{\partial D'}{\partial \delta} &= -D + m \frac{\partial X}{\partial m} \\ \frac{\partial D'}{\partial \delta} < 0 &\implies 1 > \frac{m L_D(S', D')}{L_S(S', D') + m L_D(S', D')},\end{aligned}$$

which is also true as long as  $L_S > 0$ . □

### Proof of Proposition 6:

*Proof.* The open-access steady states are defined by equations 33, 34, and 35. Equation 33 implicitly determines the number of satellites as a function of the amount of debris, the excess return on a satellite, and the collision probability function,

$$L(S, D) = r_s - r \implies S = S(r_s - r, D). \quad (76)$$

Since we assumed  $L(S, D)$  is increasing in each argument,  $S(r_s - r, D)$  is decreasing in  $D$ . This implicit function allows us to reduce equations 33, 34, and 35 to a single equation in debris,

$$\mathcal{Y}(D) = -\delta D + G(S(r_s - r, D), D) + m(r_s - r)S(r_s - r, D) = 0. \quad (77)$$

$-\delta D$  and  $m(r_s - r)S(r_s - r, D)$  are both monotonically decreasing in  $D$ . Since  $G(S, D)$  is increasing in both arguments but  $S(r_s - r, D)$  is decreasing in  $D$ ,  $G(S(r_s - r, D), D)$  may be increasing or decreasing in  $D$  over an arbitrary positive interval. In the limiting cases where  $D \rightarrow 0$  or  $D \rightarrow \infty$ ,  $G(S(r_s - r, D), D) \rightarrow G(S(r_s - r, 0), 0) > 0$  and  $G(S(r_s - r, D), D) \rightarrow G(0, D) > 0$ . If  $G(S, D) = L(S, D)$ ,  $G(S(r_s - r, D), D)$  would be constant for all  $D$  because  $S(r_s - r, D)$  is such that  $L(S, D) = r_s - r$  for all  $S, D$  where this is possible. If  $G(S, D)$  is locally more convex than  $L(S, D)$ , then  $G(S(r_s - r, D), D)$  will be first decreasing and then increasing (over the local interval) as  $D$  increases, and vice versa if  $G(S, D)$  is locally more concave than  $L(S, D)$  over a local interval. So,

$$\delta D = G(S(r_s - r, D), D) + m(r_s - r)S(r_s - r, D)$$

may have zero, one, or more than one interior solutions. □

### Proof of Proposition 7:

*Proof.* We use the reduction from Proposition 6 to simplify the proof. The open-access steady states are solutions to equation 77, and the sign of  $\frac{\partial \mathcal{Y}}{\partial D}$  at the solutions allows us to classify the stability of the system. Applying the Implicit Function Theorem to equation 33 to calculate

$S_D(r_s - r, D)$ , then differentiating  $\mathcal{Y}(D)$  in a neighborhood of an arbitrary solution  $D^*$ ,

$$\frac{\partial \mathcal{Y}}{\partial D}(D^*) = -\delta - \frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}(G_S(S^*, D^*) + m(r_s - r)) + G_D(S^*, D^*), \quad (78)$$

where  $S^* \equiv S(r_s - r, D^*)$ . The first two terms of  $\frac{\partial \mathcal{Y}}{\partial D}$  are negative and the last term is positive. Since  $\frac{\partial \mathcal{Y}}{\partial D}(D^*)$  may be positive or negative, the generic solution considered may be stable or unstable depending on the physical and economic parameters.  $\square$

### Proof of Proposition 8:

*Proof.* Consider an arbitrary open-access steady state  $(S', D')$ . Our approach will be to first characterize the set of points which can reach the open-access steady state in one period, then show that other points in the action region must overshoot one and only one state variable in approaching the equilibrium isoquant.

The set of initial conditions which can reach that steady state are one iteration of the physical dynamics away from a line segment intersecting the steady state. Given an arbitrary point  $(S, D)$ ,  $(S', D')$  can be written as

$$\begin{bmatrix} S' \\ D' \end{bmatrix} = \begin{bmatrix} S \\ D \end{bmatrix} + \begin{bmatrix} -L(S, D)S \\ -\delta D + G(S, D) \end{bmatrix} + \begin{bmatrix} X \\ mX \end{bmatrix}. \quad (79)$$

The second term on the right hand side is the translation applied to the initial condition by the physical dynamics, before any launches. The third term on the right hand side is a line segment with magnitude  $(X^2 + (mX)^2)^{1/2} = X(1 + m^2)^{1/2}$  and slope  $m$ .  $X > 0$  implies that the sum of the first two terms is in the action region. Under open access,  $X$  is determined so that  $L(S', D') = r_s - r$ . Since varying  $X$  only changes the length of the line segment from  $[S(1 - L(S, D)), D(1 - \delta) + G(S, D)]^T$  but not its slope, any initial condition for which firms launch satellites and reach an open-access steady state in one step must be one iteration of the physical dynamics away from a line segment intersecting the steady state in question. The case where  $X = 0$  and the system reaches an open-access steady state is less interesting, since in this case one iteration of the physical dynamics places the system exactly at the steady state in question.

Now consider a different initial condition,  $(S_a, D_a)$ , which still leads to the interior of the action region after one iteration of the physical dynamics, but not on the ray with slope  $m$  intersecting an open-access steady state. Since it leads to the interior of the action region,  $X > 0$  will hold and be such that  $L(S'_a, D'_a) = r_s - r$ . Now,  $L(S, D)$  and  $G(S, D)$  are both monotonic in each argument, and  $L(S'_a, D'_a) = L(S', D') = r_s - r$  since  $X > 0$  implies next period aggregates are on the equilibrium isoquant. If  $S'_a = S$ , the monotonicity of  $L(S, D)$  implies that  $D'_a = D$

and vice versa. If  $S'_a < S'$  and  $D'_a < D$  (or both inequalities were reversed), either  $(S', D')$  or  $(S'_a, D'_a)$  would not be an equilibrium. So, either  $S'_a > S'$  or  $D'_a > D'$  must be true. If  $S'_a > S'$ , then  $D'_a < D'$  must be true, and vice versa. So any initial condition which leads to the action region must do one of three things: it must lead directly to an open-access steady state, or it must overshoot the steady state satellite level, or it must overshoot the steady state debris level. If it overshoot both variables, it would no longer be on the equilibrium collision probability isoquant. So, the sets in the action region with which each type of overshooting is associated must be disjoint, indicating that open access launching which does not immediately reach the steady state will overshoot in only one variable in the first period.  $\square$

### Proof of Proposition 9:

*Proof.* We suppress function arguments to reduce notation; all functions are evaluated at an arbitrary open-access steady state. Applying the Implicit Function Theorem to equation 77, and applying it again to equation 33 to calculate  $\frac{\partial S}{\partial r_s}$  where necessary, we get

$$\begin{aligned}\frac{\partial D}{\partial r_s} &= \frac{\frac{G_S}{L_S} + m(S + \frac{r_s - r}{L_S})}{\delta + \frac{L_D}{L_S}(G_S + m(r_s - r)) - G_D} \leq 0, \\ \frac{\partial D}{\partial m} &= \frac{(r_s - r)S}{\delta + \frac{L_D}{L_S}(G_S + m(r_s - r)) - G_D} \leq 0, \\ \frac{\partial D}{\partial \delta} &= \frac{D}{-\delta - \frac{L_D}{L_S}(G_S + m(r_s - r)) + G_D} \leq 0.\end{aligned}$$

An open-access steady state is locally stable if  $-\delta - \frac{L_D}{L_S}(G_S + m(r_s - r)) + G_D < 0$ . So if an open-access steady state is locally stable, then  $\frac{\partial D}{\partial r_s} > 0$ ,  $\frac{\partial D}{\partial m} > 0$ , and  $\frac{\partial D}{\partial \delta} < 0$ . The inequalities are reversed when a steady state is locally unstable.  $\square$

**Deriving the probability of a satellite colliding with another object in orbit** To derive the functional form we use for exposition and simulations (equation 5), we map the setting of satellites and debris colliding in an orbital shell to a setting where balls are randomly dropped in bins, i.e. satellites and debris are mapped to balls and their positions in the orbital shell are mapped to bins. The probability of a specific satellite being struck by another object is then equivalent to the probability that a randomly-dropped ball ends up in a bin containing the specific ball we are focusing on.

Suppose we have  $b$  equally-sized bins and  $n + 1$  balls in total, where  $b \geq n + 1$ . Without loss of generality, we label the ball we are interested in as  $i$ . We will first place  $i$  into an arbitrary bin, and then drop the remaining  $N$  balls into the  $b$  bins with equal probability over bins. The probability a ball is dropped into a given bin is  $\frac{1}{b}$ , and the probability a ball is not dropped into



a given bin is then  $\frac{b-1}{b} = 1 - \frac{1}{b}$ . As we drop the remaining  $n$  balls, the probability that none of the balls is dropped in the same bin containing  $j$  is

$$Pr(\text{no collision with } i) = \left(1 - \frac{1}{b}\right)^n \quad (80)$$

Consequently, the probability that any of the  $n$  balls are dropped into  $i$ 's bin is

$$Pr(\text{collision with } i) = 1 - \left(1 - \frac{1}{b}\right)^n. \quad (81)$$

Now suppose we are interested in the probability that members of a collection of  $j$  balls,  $1 \leq j < b$ , end up in a bin with one of the remaining  $n+1-j$  balls. The probability that any of the remaining balls end up in a bin with any of the  $j$  balls we are interested in is then

$$Pr(\text{collision with } i) = 1 - \left(1 - \frac{j}{b}\right)^{n+1-j}. \quad (82)$$

As the number of bins and balls grow large ( $\lim_{b,n \rightarrow \infty}$ ), we obtain

$$Pr(\text{collision with } i) = 1 - e^{-j}. \quad (83)$$

Though neither the number of objects in orbits nor the possible positions they could occupy is infinite, the negative natural exponential form is likely a reasonable approximation. If we suppose that we have two types of balls  $j$  and  $k$  of different sizes and bins the size of the smallest type of ball<sup>26</sup>, we get that the probability a ball of type  $k$  is dropped into in a bin with a ball of type  $j$  as

$$Pr(k\text{-}j \text{ collision}) = 1 - \left(1 - \frac{\alpha_{jk}k}{b}\right)^{n+1-k} \quad (84)$$

$$\implies \lim_{b,n \rightarrow \infty} Pr(k\text{-}j \text{ collision}) = 1 - e^{-\alpha_{jk}k}, \quad (85)$$

which is the form in equation 3, where  $\alpha_{jk}$  is a nonnegative parameter indexing the relative sizes of objects  $j$  and  $k$ . To reiterate, though this form was empirically validated in [Letizia et al. \(2017\)](#) and [Letizia, Lemmens, and Krag \(2018\)](#) and we use it in exposition and simulations, we do not use it for any analytical results and are not taking a stance on the correct form for the physical function  $L(S, D)$ . We simply offer this derivation for interested readers who are curious as to where the functional form we use in numerical simulations came from.

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<sup>26</sup>We are stretching the physical analogy to dropping balls into bins, but the point is that the probability a ball of a given type is dropped into a bin is not identical across types.