Functions worksheet 2

1. Calculate $w(x) = 10x - 3$ at $x = 0, 5, 9$. Plot these points on a graph and sketch $w(x)$ over $x \in [0, 9]$. Highlight $y(x) = \max\{0, w(x)\}$. Mark the value of x where $w(x) = 0$, and label it as x_0. (Hint: Solve for $x_0 : w(x_0) = 0$.)

2. Suppose $w(x) = 10x - 3$. Plot $w(x)$ over $x \in [0, 9]$, and highlight $y(x) = \max\{c, w(x)\}$ where $c \in (0, 3)$. Mark the value of x where $w(x) = c$, and label it as x_c. Is $x_c > x_0$? (Hint: Solve for $x_c : w(x_c) = 0$, then check if $x_c > x_0$ is true.)

3. Suppose $p(x) = \frac{1}{3}$, and $w(x) = 10x - 3$. Calculate $Q_x = p(x)w(x)\ \forall x \in \{0, 5, 9\}$. What is $Q = Q_0 + Q_5 + Q_9$?

4. Calculate $y(x) = 0.5x(1 - \frac{x}{10})$ at $x = 0, 2.5, 5, 7.5, 10$. Plot those points on a graph and sketch $y(x)$ over $x \in [0, 10]$.

5. Calculate $y = rx(1 - \frac{x}{K})$ at $x = 0, \frac{K}{4}, \frac{K}{2}, \frac{3K}{4}, K$, where $r, K > 0$. Plot those points on a graph and sketch the function over $x \in [0, K]$.

6. The **marginal cost** is the change in cost from producing (or consuming) one more unit. Formally, if the total cost of producing (or consuming) x units is $C(x)$, the marginal cost of the xth unit is $C'(x) = C(x+1) - C(x)$. Prove that if the total cost function is linear in x (i.e., of the form $C(x) = ax + b$), then the marginal cost function is constant over x (i.e., of the form $C'(x) = a$).

7. Prove that if the total cost function is quadratic in x (i.e., of the form $C(x) = ax^2 + bx + c$) then the marginal cost function is linear in x (i.e., of the form $C'(x) = ax + b$).

8. Consider the function $C(\alpha) = x + (1 - x)\alpha$, where $x \in (0, 1)$.

(a) Solve $C(\alpha) = t$ for α.

(b) Use your solution to calculate α when $x = 0.55, t = 0.82$.

(c) Using the α you found in (b), plot $(1 - x)\alpha$ over $x \in [0, 1]$.

(d) Use your solution from (a) to give a necessary condition for $\alpha > 0$.