

# Cost in Space: Debris and Collision Risk in the Orbital Commons

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## Abstract

As Earth's orbital space fills with satellites and debris, debris-producing collisions between orbiting bodies become more likely. Runaway debris growth, known as Kessler Syndrome, may render Earth's orbits unusable for millennia. We present the first dynamic physico-economic model of Earth orbit use under rational expectations with endogenous collision risk and Kessler Syndrome. As long as satellites can be destroyed in collisions with debris and other satellites, there is a continuum of open-access equilibria determined by the excess return on a satellite. Under reasonable physical assumptions, all are inefficient. When autocatalytic debris production is possible this continuum allows for multiple steady states, which can be destabilized by increases in the excess return on a satellite. Further, open-access launch rate paths are likely to overshoot steady states. Overshooting can cause short-run rebound effects when physical or economic parameters change, for example when environmental processes increase the debris decay rate or the costs of launching a satellite decrease. Kessler Syndrome is inefficient, and open access is increasingly likely to cause it as the excess return on a satellite increases.

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# 1 Introduction

Earth's orbits are the world's largest common-pool resource, and increasingly necessary for services that power the modern world. As humans launch more satellites, the risk of collisions between orbiting objects increases. Such collisions can destroy satellites and produce orbital debris, further increasing the risk of future collisions. Collision risk and debris accumulation threaten active satellites and the future of human activity in space. The worst-case scenario is runaway debris growth, known as Kessler Syndrome, wherein the production of debris due to collisions between orbiting bodies becomes self-sustaining and irreversible. In such a scenario, valuable regions of orbital space may become unusable and impassable for decades or centuries.

How will open access to orbit affect orbital debris accumulation, satellite collision risk, and occurrence of Kessler Syndrome? How do short-run economic dynamics transition to long-run outcomes in orbit? These questions have been explored very little in economics, and have not yet been addressed in the physics and engineering or law and policy literatures.<sup>1</sup> In this paper, we examine the consequences of open access to orbit in the first physically-general dynamic economic model of satellite launching.

We show the equilibrium collision probability is determined by the excess rate of return on a satellite, allowing for a continuum of open-access equilibria in the short run. These equilibria are all inefficient, producing too much collision risk and orbital debris. We then consider how changes in the excess rate of return on a satellite or short-run dynamics in the absence of parameter changes can lead to Kessler Syndrome under open access. We also establish that short-run open-access equilibria are unlikely to smoothly approach long-run steady states (which may not be unique), how physical parameter changes (due to technological advances or environmental processes) can cause short-run "rebound" effects, and the connection between Kessler Syndrome and local stability of open-access steady states.

Satellites produce debris over their lifecycle. Launching satellites produces orbital debris (spent rocket stages, separation bolts), satellites can produce some debris while in orbit (paint chips, lost tools, etc.), and satellites which are not deorbited or shifted to disposal orbits at the end of their life become debris.<sup>2</sup> Objects in orbit move at velocities higher than 5 km/s, so debris as small as 10 cm in diameter can be hazardous to active satellites. Satellites struck by debris can shatter into thousands of equally-hazardous debris fragments. Though debris does

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<sup>1</sup>Wienzierl (2018) highlights a number of issues in the development of a space economy, from space debris to coordination problems and market design, which economists have the tools to address.

<sup>2</sup>Reusable rockets can significantly lower the cost of launching and the amount of launch debris generated. SpaceX and Blue Origin are currently the only launch providers who offer such vehicles.

deorbit naturally, this process can be very slow, especially at higher altitudes. Debris as low as 900 km above the Earth’s surface can take centuries to deorbit, while debris at 36000 km can take even longer (Weeden, 2010). Compounding the problem is the fact that collisions between debris objects can generate even more debris, also moving at high velocities and capable of damaging operational satellites.<sup>3</sup> Debris accumulation can also cause a cascading series of collisions between orbital objects, creating an expanding field of debris which can render an orbital region unusable and impassable for thousands of years. Engineers and physicists call this phenomenon “collisional cascading” or “Kessler Syndrome” (Kessler and Cour-Palais, 1978). Kessler Syndrome can cause large economic losses, directly from damage to active satellites and indirectly from limiting access to space (Bradley and Wein, 2009; Schaub et al., 2015). Existing estimates of debris growth indicate that the risk of Kessler Syndrome is highest in low-Earth orbit (LEO), where it threatens imaging and future telecommunications satellites and can reduce access to higher orbits (Kessler et al., 2010). Currently, there are more than 1,000 operating satellites in orbit, up to 600,000 pieces of debris large enough to cause satellite loss, and millions of smaller particles that can degrade satellite performance (Ailor et al., 2010). Approximately 49 percent of currently-active satellites are in LEO, a share which is projected to grow in the near future (Union of Concerned Scientists, 2017). Existing legal frameworks for orbit use such as the Outer Space Treaty complicate the process of establishing of explicit orbital property rights and hinder debris cleanup efforts. For example, Article 2 of the OST forbids national appropriation or claims of sovereignty over outer space, which is often interpreted as prohibiting national authorities from unilaterally establishing orbital property rights (Gorove, 1969).<sup>4</sup>

To our knowledge, this paper is the first to bridge the short-term and long-term physical and economic dynamics of perfectly-competitive open-access and socially-optimal orbit use in a physically-general environment which allows for Kessler Syndrome to be caused over multiple periods. While Sandler and Schulze (1981) account for collision risk when studying geostationary belt position allocation, debris accumulation and general orbital regions are not considered. Our analysis generalizes those presented in Macauley (2015); Adilov, Alexander, and Cunningham (2015, 2018), Grzelka and Wagner (2019), and Rouillon (2019) in the physical dimensions by allowing non-stationary dynamics and runaway debris growth, and makes

<sup>3</sup>The risk of debris striking another satellite is generally orders of magnitude larger than the risk of debris deorbiting and harming consumers directly.

<sup>4</sup>Weeden (2010) and Weeden and Chow (2012) are skeptical of what decentralized bargaining can achieve in this setting, given the difficulties in solving other global coordination problems, like climate change, and the number of current and potential orbit users and their incentives. Salter and Leeson (2014) and Salter (2015) take more optimistic views of what commercial users can achieve under celestial anarchy, the former based on the idea of self-enforcing property rights and the latter based on the Coase Theorem, technological advancement, and insurance markets. It is worth noting that the international nature of the problem offers selfish actors opportunities for “launch leakage”: leaving stricter regulatory jurisdictions in favor of less-strict regions, which has happened at least once already (Selk, 2017).

the sources of external effects clear by explicitly considering the economics of general forms of couplings between physical state variables. While our model is most similar to that of [Rouillon \(2019\)](#), it is not always directly comparable due to differences in physical assumptions and the types of planner we consider. Unlike [Rao \(2018\)](#); [Grzelka and Wagner \(2019\)](#) and [Rao, Burgess, and Kaffine \(2020\)](#), we focus primarily on economic dynamics rather than policy design.

Though prior work has established that open-access launch rates exceed the socially-optimal launch rate and result in excess collision risk and debris production ([Adilov, Alexander, and Cunningham, 2015](#); [Rouillon, 2019](#)), our framework yields the novel insight that there can be a continuum of open-access equilibria in the short-run due to couplings between satellites and debris in the collision risk function. This continuum makes it likely the approach path from short-run open-access equilibria to an open-access steady state is non-monotonic. In certain cases these approach paths can cause Kessler Syndrome, in which case steady states will never be reached. Our results contrast with those in [Adilov, Alexander, and Cunningham \(2018\)](#), where open access is found to never cause Kessler Syndrome. The contrast is due to our differing definitions of Kessler Syndrome and degrees of physical generality. [Adilov, Alexander, and Cunningham \(2018\)](#) consider a definition of Kessler Syndrome where satellites are destroyed with probability one (“unusable orbits”) and disallow collisions between debris objects. We define Kessler Syndrome as states where the debris stock diverges to infinity (“runaway debris growth”) and allow collisions between debris objects. Once runaway debris growth occurs in our model, the collision probability will eventually reach one no matter what firms were targeting. While [Rao \(2018\)](#) establishes the superiority of policy instruments which target satellites in orbit over those targeting satellite launches, we show that targeting launches can reduce inefficiencies due to overshooting of open-access steady states. Finally, by incorporating the physics of orbit use directly in our economic model and allowing non-stationary dynamics, we are able to derive qualitatively new insights into the effects of exogenous physical and technological parameter changes on open-access launch behavior. These insights include a mechanism by which natural exogenous physical parameter variation can interact with open access to cause Kessler Syndrome. We contribute to the literature on common-pool resource management in the presence of environmental risk by incorporating a capital stock that is affected by congestion of the commons and a pollution stock which increases congestion.

The organization of this paper is as follows. In section 2 we describe the physical model we use to study the orbital environment. In section 3 we describe the economic institutions we use to study orbit use — open access by risk-neutral profit-maximizing firms and global coordination by a planner who owns all satellites in orbit and controls all launch activity. In this section we also derive the open-access equilibrium and the planner’s solution. In section

4 we derive and discuss the main results of this paper: in a physically-general setting, there are a continuum of equilibria, all inefficient, where the collision probability is determined by the excess return on a satellite and a multiplicity of steady states; open-access launch rate paths will tend to overshoot steady states; Kessler Syndrome is inefficient, and open access is increasingly likely to cause it as the excess return increases. We illustrate key results with numerical simulations. We conclude in section 5. Appendix A contains all proofs and functional form derivations. Appendix B contains some interesting physical and economic extensions: spectrum congestion and endogenous prices, exogenous time-varying economic returns, and open-access equilibrium between multiple orbital shells.

## 2 Physical model

Consider a spherical shell around the Earth, say the region between 600-650km above mean sea level. Active satellites and debris move through the orbital shell (“orbit”) in elliptical paths. Influences from the Earth and other celestial bodies perturb their motion and may cause their paths to intersect, upon which the colliding bodies shatter into more debris. The stock of active satellites in the orbit is periodically replenished by new launches, which may bring with them more debris. While active satellites expend fuel to counteract the perturbations they face, debris do not and eventually fall back to Earth and burn up in the atmosphere. If the supply of satellites is high enough for long enough, the production of debris from collisions may become self-sustaining.<sup>5</sup>

The number of active satellites in orbit in period  $t + 1$  ( $S_{t+1}$ ) is the number of launches in the previous period ( $X_t$ ) plus the number of satellites which survived the previous period ( $S_t(1 - L(S_t, D_t))$ ). The amount of debris in orbit in  $t + 1$  ( $D_{t+1}$ ) is the amount from the previous period which did not decay ( $D_t(1 - \delta)$ ), plus the number of new fragments created in collisions ( $G(S_t, D_t)$ ), plus the amount of debris in the shell created by new launches ( $mX_t$ ).<sup>6</sup> The laws

<sup>5</sup>Paths which span shells are possible and useful for some applications, but highly elliptical orbits (such as Molniya orbits) are the exception rather than the rule. A Molniya orbit has a low perigee over the Southern Hemisphere and a high apogee over the Northern Hemisphere. Molniya orbits require less power to cover regions in the Northern Hemisphere (e.g. former Soviet Union countries) than geosynchronous orbits, due to the low incidence angles of rays from the Northern Hemisphere to geosynchronous positions. This “shell of interest” approach is frequently used in debris modeling, e.g. Rossi et al. (1998) and Bradley and Wein (2009), though higher fidelity models use large numbers of small regions to track individual objects, e.g. Liou et al. (2004), Liou and Johnson (2008), and Liou and Johnson (2009). We abstract from the composition of orbital stocks, and assume that all satellites and debris are identical.

<sup>6</sup>Satellites’ orbits also decay and satellites eventually cease being productive, features we abstract from in the main model. We extend the model to include this in section B.2 of the Appendix. This abstraction makes some results a little clearer but does not qualitatively affect them. Rouillon (2019) includes these features in the main model and derives qualitatively similar results, albeit in a continuous-time setting.

of motion for the satellite and debris stocks formalize this:

$$S_{t+1} = S_t(1 - L(S_t, D_t)) + X_t \quad (1)$$

$$D_{t+1} = D_t(1 - \delta) + G(S_t, D_t) + mX_t. \quad (2)$$

$L(S_t, D_t)$  is the proportion of orbiting active satellites which are destroyed in collisions, and  $G(S_t, D_t)$  is the number of new fragments created by collisions between orbiting bodies. Since we assume active satellites are identical,  $L(S_t, D_t)$  is also the probability an individual satellite is destroyed in a collision. We assume that the collision probability is twice continuously differentiable, nonnegative, increasing in each argument, and bounded below by 0 and above by 1.<sup>7</sup>  $\delta$  is the rate of orbital decay for debris, and  $m$  is the amount of launch debris created by new satellites.

We assume the new fragment function  $G$  is twice continuously differentiable, nonnegative, increasing in each argument, zero when there are no objects in orbit ( $G(0, 0) = 0$ ), and unbounded above ( $\lim_{S \rightarrow \infty} G(S, D) = \lim_{D \rightarrow \infty} G(S, D) = \infty$ ). We also assume the effect of the first satellite or debris fragment on new fragment formation is negligible, i.e.  $G_S(0, D) = G_D(S, 0) = 0$  (letting subscripts denote partial derivatives). To derive results about the occurrence of Kessler Syndrome, we also assume that the growth in new fragments due to debris alone will eventually be greater than  $\delta$ , i.e. the fragment autocatalysis rate (defined below) will eventually be positive.

**Definition 1.** (*Fragment autocatalysis rate*) *The fragment autocatalysis rate is the rate of fragment growth caused by collisions between debris objects, net of natural decay processes. Formally,*

$$G_D(S, D) - \delta.$$

Positive fragment autocatalysis rates are a necessary condition for unstable fixed points of long-run orbit use (described precisely in Proposition 3), and are associated with Kessler Syndrome. Figure 5 illustrates this connection.

**Definition 2.** (*Kessler Syndrome*) *The Kessler region is the set of states (satellite and debris levels) such that the debris stock will tend to infinity, i.e.*

$$\kappa \equiv \{(S, D) : \lim_{t \rightarrow \infty} D_{t+1} = \infty \mid S_0 = S, D_0 = D, X_t = X(S_t, D_t)\}$$

<sup>7</sup>Satellite operators try to avoid collisions by maneuvering their satellites when possible; the collision probability in this model should be thought of as the probability of collisions which could not be avoided, with easily avoided collisions optimized away. Collisions which could have been avoided but were not due to human error are included in this. Implicitly we are assuming operators are imperfect cost-minimizers. Even when operators can maneuver their satellites and are aware of impending collisions, coordination can be challenging and plagued by technical glitches (Brodin, 2017).

*Kessler Syndrome has occurred when the orbit has entered the Kessler region.*

Our definition of Kessler Syndrome is deliberately vague on the nature of the launch policy function, so that it is not restricted to open access, optimal management, or any other type of orbital management institution. An exact characterization of the Kessler region is not feasible given the generality with which we treat the physical functions  $G$  and  $L$ . However, we are able to define the *Kessler threshold*,  $D^K$ , which is the maximum sustainable debris level:

$$D^K = \max_D \kappa \quad (3)$$

We characterize  $D^K$  analytically, and compute the Kessler region for specific functional forms and parameterizations.

For orbits where the fragment autocatalysis rate is negative for all satellite and debris levels, Kessler Syndrome is impossible. Our assumption that the new fragments function  $G$  is monotonically increasing in both arguments, and the lack of debris removal technologies, imply that Kessler Syndrome is an absorbing state. Intuitively, the Kessler threshold is increasing in the decay rate, making lower-altitude orbits (where drag from the Earth’s atmosphere makes debris decay faster than higher orbits) less vulnerable to Kessler Syndrome than higher-altitude orbits.

## 2.1 Three types of physical couplings

We use “physical coupling” to refer to ways that variables in the laws of motion for satellite and debris stocks are connected to each other. These couplings give structure to the laws of motion and drive the physical dynamics of orbit use. There are five economically relevant physical couplings between objects in orbit: the collision probability satellite coupling and collision probability debris coupling (the first type); the new fragment formation satellite coupling and new fragment formation debris coupling (the second type); and the launch debris coupling (the third type). If all five couplings were turned off, there would be no problems of excess congestion or debris accumulation. The strength of a coupling between  $X$  and  $f(\cdot)$  is the absolute value of  $\frac{\partial f}{\partial X}$ , with  $\frac{\partial f}{\partial X} = 0$  when there is no coupling. In line with physical intuitions about orbital objects, we assume any non-zero coupling is positive.

**The collision probability couplings** The collision probability couplings refer to the arguments of the collision probability function,  $L$ . When these couplings are turned off, the collision probability is exogenous:  $L(\cdot) = L$ . Since the collision probability is exogenous in this case, there is no congestion externality even when the probability varies over time. Even if open access causes Kessler Syndrome due to other couplings, the open access launch rate is

efficient in the sense that the planner would produce the same outcome.

If the collision probability is coupled with the satellite stock ( $L(\cdot) = L(S)$ ), then there can be a “steady state” congestion externality. Despite any other couplings and their effects on debris growth, the only source of inefficiency is that open access results in too many satellites in orbit relative to the optimal plan. This describes a world where debris is harmless but active satellites aren’t perfectly coordinated to avoid collisions with each other. If the collision probability is coupled with the debris stock ( $L(\cdot) = L(D)$ ) but not the satellite stock, there can be a “dynamic” congestion externality. Due to debris accumulation and the coupling, there can be persistent consequences for specific launch histories. This describes a world where active satellites are perfectly coordinated to avoid collisions with each other, but debris are hazardous and not always successfully avoided. The “fully coupled” case links collision probability to both the satellite and debris stocks ( $L(\cdot) = L(S, D)$ ). In this world, active satellites face collision risk from debris and each other. [Adilov, Alexander, and Cunningham \(2015, 2018\)](#) focus on a collision probability function with only a debris coupling. [Rouillon \(2019\)](#) focuses on the steady state and abstracts from the details of the collision probability couplings. The existence of both collision probability couplings allows us to define the marginal rate of technical substitution between satellites and debris in producing collision probability,  $\frac{L_D}{L_S}$ , which we show in Proposition 3 to be important in determining the stability of open-access steady states.

**The launch debris coupling** The launch debris coupling refers to the amount of launch debris created, i.e. the parameter  $m$  in the debris law of motion ( $\frac{\partial D_{t+1}}{\partial X_t} = m$ ). When collision probability is coupled with the debris stock, the launch debris coupling reduces the equilibrium and optimal number of launches per period, since launches themselves produce debris. Thus, the launch debris coupling can “stabilize” open access orbit use by forcing operators to internalize some of the persistent effects of their launches on orbital congestion.

**The new fragment formation couplings** The new fragment formation couplings refer to the arguments of the new fragment function,  $G(\cdot)$ . If the new fragment function is coupled to the satellite stock,  $G(\cdot) = G(S)$  (a world where only active satellites fragment upon collision), then there can be persistent consequences of excess satellite levels. When the new fragment function is coupled to the debris stock,  $G(\cdot) = G(D)$  (a world where only debris fragment upon collision), there can be multiple open-access steady states and Kessler Syndrome becomes possible. This multiplicity will exist independent of the collision probability and launch debris couplings. When the collision probability is only coupled to the satellite stock or uncoupled ( $L(\cdot) = L(S)$  or  $L(\cdot) = L$ ), the multiplicity is only in debris levels. When the collision probability is fully coupled ( $L(\cdot) = L(S, D)$ ) and the new fragment function is coupled with only the debris stock, multiple equilibria in both satellites and debris can exist: one with low debris and high satellites, and one with high debris and low satellites. In both cases, only



the low debris equilibrium is stable. [Adilov, Alexander, and Cunningham \(2015, 2018\)](#) focus on new fragment formation functions coupled only to the satellite stock.

For simulations and figures, we use the following functional forms:

$$L(S, D) = 1 - e^{-\alpha_{SS}S - \alpha_{SD}D} \quad (4)$$

$$G(S, D) = \beta_{SS}(1 - e^{-\alpha_{SS}S})S + \beta_{SD}(1 - e^{-\alpha_{SD}D})S + \beta_{DD}(1 - e^{-\alpha_{DD}D})D, \quad (5)$$

where  $\alpha_{SS}, \alpha_{SD}, \alpha_{DD}, \beta_{SS}, \beta_{SD}, \beta_{DD}$  are all positive physical parameters. We derive these forms and give some physical intuitions about the parameters in Appendix A.

### 3 Economic model

Active satellites provide services to individuals, firms, governments, research agencies, and other entities. They tend to be information services like mobile broadband, images of the Earth, and positioning/timing. To focus on the dynamics of collisions and debris, we ignore such differentiation. [Adilov, Alexander, and Cunningham \(2015\)](#) account for product differentiation in a two-period setting.

Satellites are identical, infinitely lived unless destroyed in a collision, and produce a single unit of output per period. The net payoff from this output is  $\pi > 0$ , and each satellite costs  $F$  to plan, build, and launch.<sup>8</sup> The market for satellite output is perfectly competitive, so the price per unit of output is the same as its social marginal benefit. The one-period rate of return on a satellite is  $\pi/F \equiv r_s > 0$ . Costs and returns are constant over time.<sup>9</sup>

We consider two scenarios: open access by profit-maximizing firms which can own up to one satellite at a time — or equivalently that all firms are choosing whether or not to launch identical constellations of satellites at any given time, changing the units of  $S_t$  from individual satellites to constellations — and launching controlled by a fleet planner who owns all satellites.<sup>10</sup> A firm which owns a satellite collects revenue of  $\pi$  every period the satellite continues

<sup>8</sup>This is a simplification to focus on the margin of launch decisions. Operational costs like managing receiver stations on the ground or monitoring the satellite to perform stationkeeping are incurred each period, and may push  $\pi \leq 0$ . We assume that the firms have correctly forecasted  $\pi > 0$  before deciding to launch. This also allows us to abstract from the margins of decisions to sell or deorbit satellites, or to purchase already-orbiting satellites.

<sup>9</sup>We relax these assumptions in Appendix B, modeling the effects of finite satellite lifetimes in Section B.2 and of time-varying rates of return in Section B.3.

<sup>10</sup>Our assumption that each firm is identified with a single satellite can be equivalent to assuming that either the marginal satellite in an equilibrium is launched by such a firm, or firms may own multiple satellites but ignore their inframarginal satellites in launching. In either case we could obtain the same equilibrium condition given by equation 9 below. [Rouillon \(2019\)](#) uses the latter interpretation and arrives at an equivalent equilibrium condition (accounting for the differences in our model environments). We maintain our stricter interpretation to avoid the complications of

to operate. An individual satellite is destroyed in period  $t$  with probability  $L(S_t, D_t)$ . All agents discount the future at rate  $r > 0$ . The value of a satellite,  $Q(S_t, D_t, X_t)$ , is the sum of present profits and the expected discounted value of its remaining lifetime profits:

$$Q(S_t, D_t, X_t) = \pi + \beta(1 - L(S_t, D_t))Q(S_{t+1}, D_{t+1}, X_{t+1}), \quad (6)$$

where  $X_t = \int_0^\infty x_{it} di$  is the aggregate launch rate based on each potential launcher's entry decision  $x_{it} \in \{0, 1\}$ , and  $\beta = (1 + r)^{-1}$ . Firms cannot choose to deorbit satellites. Up to  $\bar{X}$  satellites may be launched in one period.

A firm which does not own a satellite in period  $t$  decides whether to pay  $F$  to plan, build, and launch a satellite which will reach orbit and start generating revenues in period  $t + 1$ , or to wait and decide again in period  $t + 1$ .<sup>11</sup> All potential launchers are risk-neutral profit maximizers, and the value of potential launcher  $i$  at the beginning of period  $t$  is

$$\begin{aligned} V_i(S_t, D_t, X_t) &= \max_{x_{it} \in \{0, 1\}} \{ (1 - x_{it})\beta V_i(S_{t+1}, D_{t+1}, X_{t+1}) + x_{it}[\beta Q(S_{t+1}, D_{t+1}, X_{t+1}) - F] \} \\ \text{s.t. } S_{t+1} &= S_t(1 - L(S_t, D_t)) + X_t \\ D_{t+1} &= D_t(1 - \delta) + G(S_t, D_t) + mX_t \end{aligned} \quad (7)$$

While firms may choose to launch or not, all satellites earn *ex-ante* identical returns. We assume  $\pi < F$  to avoid the case where satellites are profitable to launch even if they will be destroyed as soon as they reach orbit, and  $\pi > rF$  so satellites are worth launching in the first place. The excess return on a satellite is  $r_s - r$ .

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solving the dynamic satellite inventory management problem facing a constellation owner outside the steady state, with the attendant assumptions about when and how they choose to refresh their constellation. See [Adilov et al. \(2019\)](#) for a model of GEO satellite inventory management. Versions of the orbital warehousing issues examined in [Adilov et al. \(2019\)](#) are also relevant to LEO constellation management, with additional dynamic complications through the collision risk and debris growth functions. Similarly, when firms have the ability to coordinate groups of satellites, their satellites face different risks from those under their control than from those which are not. This raises the possibility of strategic collision risk dynamics in orbit use — the relevant “measure-zero” assumption which would allow us to ignore such possibilities is that firms are relatively small in the physical space the satellites occupy, not the output market in which they compete. When a particular shell is dominated by a few firms which own hundreds or thousands of satellites there, it seems difficult to argue they are small relative to neighbors which own one or a handful of satellites. The constellation management problem is eminently important to study, but given the complications we leave it to future research.

<sup>11</sup>There is a difference between the timescale of physical interactions in orbit and the timescale of launch decisions. The former occurs continuously, while the latter does not. We include the lag to capture this feature.

### 3.1 Open access and the equilibrium collision probability

Under open access, firms launch satellites until launching another satellite provides zero profits,

$$X_t \geq 0 : V_i(S_t, D_t, X_t) = 0 \quad \forall t \quad (8)$$

$$\implies \beta Q(S_{t+1}, D_{t+1}, X_{t+1}) = F. \quad (9)$$

Equation 9 is the zero profit condition which defines the open-access equilibrium. The resulting value of a satellite is the period profit plus the expected value of its survival under open access,

$$Q(S_t, D_t, X_t) = \pi + (1 - L(S_t, D_t))F. \quad (10)$$

The open access condition and the satellite value can be rewritten to show that in an interior equilibrium the flow of benefits generated by a satellite is equated with the flow of opportunity costs and expected collision costs (the marginal private costs of satellite ownership):

$$F = \beta Q(S_{t+1}, D_{t+1}, X_{t+1}) \quad \forall t \quad (11)$$

$$\implies \pi = rF + L(S_{t+1}, D_{t+1})F \quad (12)$$

$$\implies \hat{X}_t : \pi = rF + L(S_{t+1}, D_{t+1})F. \quad (13)$$

Given the upper bound on launches, the open-access launch rate is then

$$X_t = \begin{cases} \bar{X} & \text{if } \pi > rF + L(S_{t+1}, D_{t+1})F \\ \hat{X}_t \in [0, \bar{X}] & \text{if } \pi = rF + L(S_{t+1}, D_{t+1})F \\ 0 & \text{if } \pi < rF + L(S_{t+1}, D_{t+1})F, \end{cases} \quad (14)$$

where the inequalities for launching 0 and  $\bar{X}$  satellites are conditioned on  $X_t = 0$  and  $X_t = \bar{X}$ , respectively. Using equation 12, we obtain the equilibrium collision probability as a function of economic parameters.

### 3.2 Marginal external cost and the optimal launch plan

In this section we use letter subscripts on functions to indicate partial derivatives with respect to a given argument, e.g.  $L_S(S, D) \equiv \frac{\partial L(S, D)}{\partial S}$ .

The net present value of an orbit is the value of the satellites currently in the orbit and those which will eventually be in the orbit, net of all costs:

$$\bar{W}(S_t, D_t, X_t) = S_t Q(S_t, D_t, X_t) + \beta \sum_{\tau=t}^{\infty} \beta^{\tau-t-1} X_{\tau} (\beta Q(S_{\tau+1}, D_{\tau+1}, X_{\tau+1}) - F) \quad (15)$$

$$\begin{aligned} &= \pi S_t - F X_t + [\beta S_{t+1} Q(S_{t+1}, D_{t+1}, X_{t+1}) + \\ &\quad \beta \sum_{\tau=t+1}^{\infty} \beta^{\tau-t-1} X_{\tau} (\beta Q(S_{\tau+1}, D_{\tau+1}, X_{\tau+1}) - F)] \end{aligned} \quad (16)$$

$$= \pi S_t - F X_t + \beta \bar{W}(S_{t+1}, D_{t+1}, X_{t+1}) \quad (17)$$

The fleet planner has exclusive rights in perpetuity to the orbit and all satellites in it. The planner therefore controls launches to maximize the net present value of the satellite fleet,  $\bar{W}(S_t, D_t, X_t)$ , equating the per-period marginal benefit of another satellite with its per-period social marginal cost. The per-period social marginal cost is the sum of the opportunity cost of the investment and the collision probability  $(r + L(S_{t+1}, D_{t+1}))$ , and the effect of the marginal satellite on the fleet through future collisions and debris growth  $(\xi(S_{t+1}, D_{t+1})/F)$ . Firms which do not own exclusive orbital rights internalize only the opportunity cost of funds and the risk to their satellite, ignoring the costs the satellite imposes on the rest of the fleet.

The fleet planner solves

$$W(S_t, D_t) = \max_{X_t \geq 0} \{ \pi S_t - F X_t + \beta W(S_{t+1}, D_{t+1}) \} \quad (18)$$

$$\text{s.t. } S_{t+1} = S_t(1 - L(S_t, D_t)) + X_t \quad (19)$$

$$D_{t+1} = D_t(1 - \delta) + G(S_t, D_t) + m X_t. \quad (20)$$

The planner's optimal launch rate, which equates the flow of marginal benefits and costs, is

$$X_t^* : \pi = rF + L(S_{t+1}, D_{t+1})F + \xi(S_{t+1}, D_{t+1}). \quad (21)$$

Given the upper bound on launches, the planner's launch policy is then

$$X_t = \begin{cases} \bar{X} & \text{if } \pi > rF + L(S_{t+1}, D_{t+1})F + \xi(S_{t+1}, D_{t+1}) \\ X_t^* \in [0, \bar{X}] & \text{if } \pi = rF + L(S_{t+1}, D_{t+1})F + \xi(S_{t+1}, D_{t+1}) \\ 0 & \text{if } \pi < rF + L(S_{t+1}, D_{t+1})F + \xi(S_{t+1}, D_{t+1}), \end{cases} \quad (22)$$

where the inequalities for launching 0 and  $\bar{X}$  satellites are conditioned on  $X = 0$  and  $X = \bar{X}$ , respectively. We derive equation 21 and  $\xi(S, D)$  in the Appendix.

From the planner's problem, the marginal external cost is

$$\begin{aligned}
\xi(S_{t+1}, D_{t+1}) = & \underbrace{S_{t+1}L_S(S_{t+1}, D_{t+1})F}_{\substack{\text{"congestion" channel:} \\ \text{cost of more satellites colliding} \\ \text{with each other due to crowding}}} + \underbrace{\frac{\pi - rF - L(S_t, D_t)F - S_tL_S(S_t, D_t)F}{\beta(1 - \delta + G_D(S_{t+1}, D_{t+1}) + mL_D(S_{t+1}, D_{t+1})S_{t+1})}}_{\substack{\text{"pollution" channel:} \\ \text{cost of more satellites colliding} \\ \text{with debris due to debris growth}}} \\
& - \underbrace{\frac{\beta G_S(S_{t+1}, D_{t+1}) + m(1 - L(S_{t+1}, D_{t+1}) - S_{t+1}L_S(S_{t+1}, D_{t+1}))}{\beta(1 - \delta + G_D(S_{t+1}, D_{t+1}) + mL_D(S_{t+1}, D_{t+1})S_{t+1})} S_{t+1}L_D(S_{t+1}, D_{t+1})F}_{\substack{\text{"reduction" channel:} \\ \text{value gained from debris-source removal due to collisions and} \\ \text{reduction in replacement launches}}}
\end{aligned} \tag{23}$$

The first term of  $\xi(S_{t+1}, D_{t+1})$ , the congestion channel, represents the cost of additional satellite collision probability due to satellite crowding. As long as the collision probability is coupled with the satellite stock and new satellites weakly increase the probability of satellite-destroying collisions ( $L_S(S, D) \geq 0$ ), this term is nonnegative.

The second term, the pollution channel, is the value of the profit from the current fleet in orbit, net of congestion costs, lost per future fragment created by current fragments. As long as additional satellites in period  $t$  produced profits net of the opportunity cost, risk, and congestion ( $\pi - rF - L(S_t, D_t)F - S_tL_S(S_t, D_t)F \geq 0$ ), this term is nonnegative. Dividing  $\xi(S_{t+1}, D_{t+1})$  by  $F$  reveals the marginal external cost to be increasing in the excess return on a satellite through the pollution channel.

The third term shows the gain in value from reducing the number of satellites in orbit, per future fragment caused by current fragments. Removing satellites currently in orbit removes a source of fragments to be destroyed in collisions, while reducing the number of satellites to maintain in the future reduces the number of launch fragments produced. As long as the profits from a satellite net of opportunity cost, risk, and congestion in period  $t$  outweigh the cost of new fragment creation due to replacement satellites reaching orbit in  $t + 1$ , the cost of the pollution channel will outweigh the benefit of the reduction channel. This will make the marginal external cost positive. A positive marginal external cost implies the planner will launch fewer satellites than open access firms.

The potential for a negative marginal external cost here reflects two facts. First, debris is a complement in production to satellites. Stronger launch debris couplings imply greater complementarity. Second, the only way to reduce the number of satellites in orbit in this setting is to allow them to be destroyed. The combination of these two facts creates the potential for collisions which reduce the number of satellites in orbit to be good from the planner's perspective.

We assume the marginal external cost is always positive along interior paths, which has been shown in similar settings e.g. [Adilov, Alexander, and Cunningham \(2015\)](#); [Rouillon \(2019\)](#). If the marginal external cost is positive, then the rate of fragment growth from new satellite launches and destructions is smaller than the rate of excess return on a satellite. Our expression for the marginal external cost of a satellite launch differs from those in related works ([Adilov, Alexander, and Cunningham, 2015](#); [Rouillon, 2019](#)) because we allow for non-stationary dynamics over multiple periods. When the marginal external cost is positive the equilibrium collision probability is too high, and there are some states in which open-access firms will launch though the planner would not.

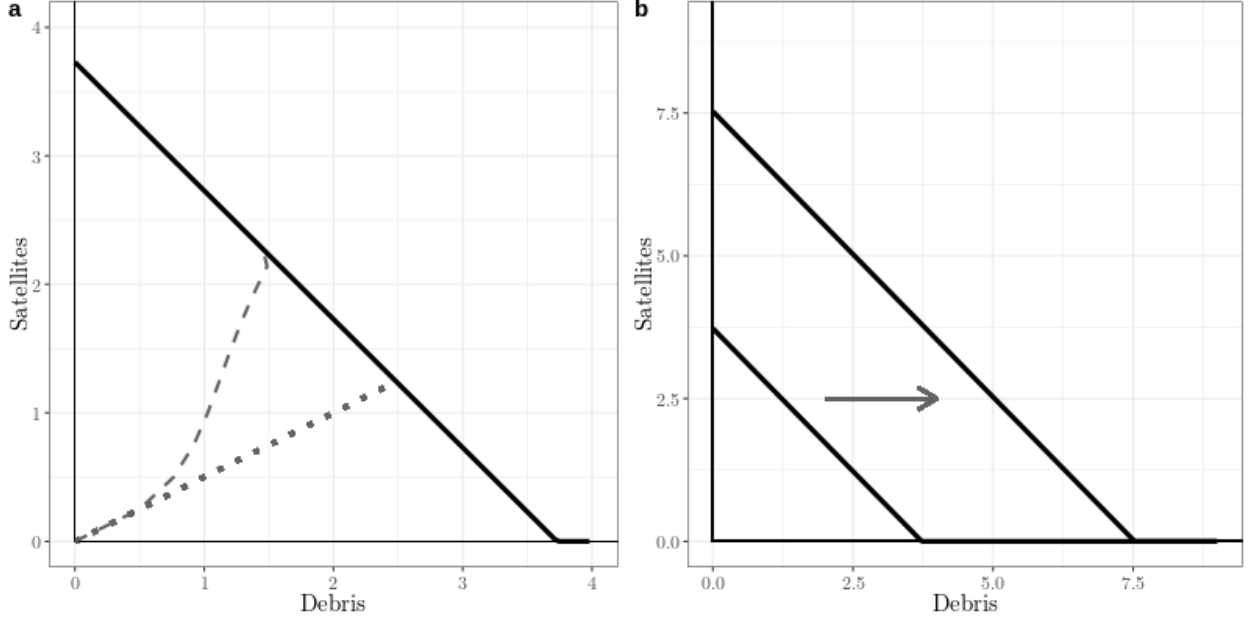
## 4 Main results and discussion

The general physical form of the collision risk allows us to not only identify the open-access equilibrium collision probability with the excess return on a satellite, but also the continuum of equilibria and their comparative statics.

**Proposition 1** (Equilibrium collision probability). *There is a continuum of interior open-access equilibria, each with collision probability in  $t + 1$  equal to the excess return on a satellite. Increases in the excess return on a satellite cause the collision probability in  $t + 1$ , launch rate in  $t$ , and debris stock in  $t + 1$  to increase.*

In the case where the economic parameters  $\pi, F, \beta$  are time-varying, Proposition 1 implies that the equilibrium collision probability in period  $t$  under open access will be determined by the values of the economic parameters in period  $t - 1$ , i.e. the period when the newest satellites in orbit were launched. In this case, the equilibrium collision probability will not be constant over time, but will still be determined by the economic fundamentals of launching satellites.

Proposition 1 implies that the equilibrium collision probability is increasing in the profitability of a satellite and increasing in the discount factor. Increases in the excess return on a satellite shift the equilibrium collision probability isoquant outwards — if satellite operators become less patient, or if satellites become more lucrative, the equilibrium collision probability will increase. Since every point on the excess-return isoquant of the collision probability function is an open-access equilibrium, which equilibrium is reached in any period will depend on the initial conditions and physical dynamics. Figure 1 illustrates these results.



**Figure 1:** Panel a shows with two approach paths from the same initial condition: one which reaches the equilibrium isoquant immediately (dotted straight line), the other with a binding launch constraint ( $\bar{X}$ ) which takes multiple periods to reach the equilibrium isoquant (dashed curving line). Panel b shows the equilibrium isoquant shifting outwards when the excess return on a satellite increases.

In addition to a continuum of open-access equilibria, there can multiple open-access steady states. removing period  $t$  subscripts and letting  $'$  superscripts denote period  $t + 1$  variables, open-access steady states are defined by the collision probability equaling the excess return on a satellite ( $L(S, D) = L(S', D') = r_s - r$ ) along with the usual stationarity conditions on the state variables:

$$(S, D) : L(S, D) = r_s - r, \quad (24)$$

$$\begin{aligned} S' = S &\implies S = (1 - L(S, D))S + X \\ &\implies X = L(S, D)S, \end{aligned} \quad (25)$$

$$\begin{aligned} D' = D &\implies D = (1 - \delta)D + G(S, D) + mX \\ &\implies \delta D = G(S, D) + mX. \end{aligned} \quad (26)$$

**Proposition 2 (Multiplicity).** *Given a positive excess return on a satellite and a fully-coupled collision probability function, multiple open-access steady states can exist if the new fragment debris coupling exists.*

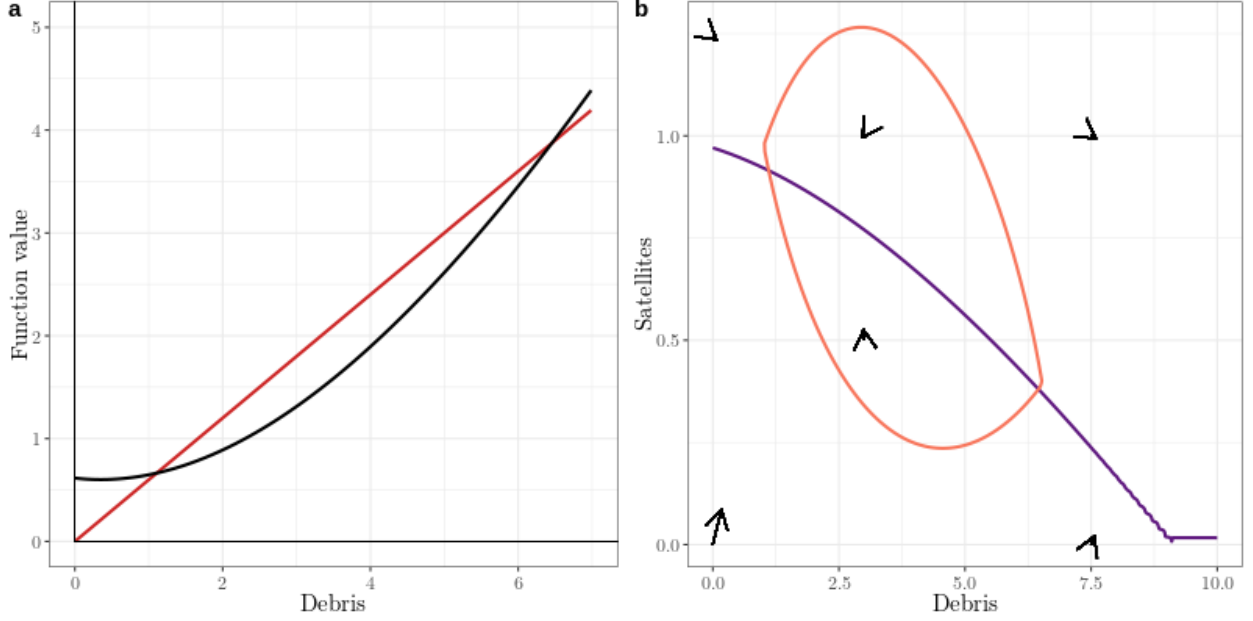
The key equation from the proof is

$$\mathcal{Y}(D) = -\delta D + G(\hat{S}, D) + m(r_s - r)\hat{S}. \quad (27)$$

This multiplicity is possible in part because open access makes the equilibrium satellite stock a decreasing function of the debris stock. Increases in the steady-state debris stock initially cause the number of new fragments created to fall as open-access launchers respond by launching fewer satellites. Eventually, the fragment-reducing effect of fewer satellites is dominated by the fragment-producing effect of additional debris through the new fragment debris coupling. The number of open-access steady states will depend on the shapes of  $L$  and  $G$ . For example, if  $L$  is strictly increasing and  $G$  is strictly convex, there can be up to two open-access solutions to equation 32.

When there are only two solutions to equation 32, say if  $L$  is strictly increasing and  $G$  is strictly convex, then the higher-debris solution is a repelling fixed point. The debris level associated with this fixed point is then the Kessler threshold defined earlier in equation 3,  $D^K$ .  $D^K$  was defined as the maximum sustainable debris level for this reason — if there is a second repelling fixed point, any debris level beyond the repelling fixed point will eventually diverge to infinity. Proposition 3 connects the stability of the fixed points to the fragment autocatalysis rate and the marginal rate of technical substitution of satellites for debris in producing collision probability.





**Figure 2:** Panel a shows equation 32 for a parameterization with two steady states. The red line shows  $\delta D$ , while the black line shows  $G(\hat{S}, D) + m(r_s - r)\hat{S}$ . Panel b shows the phase diagram of the same parameterization, with the purple line showing the satellite nullcline and the pink line showing the debris nullcline. Only the lower-debris steady state is stable.

**Proposition 3** (Local stability). *Given a positive excess return on a satellite, new fragment satellite coupling, and launch debris coupling, open-access steady states will be locally stable if and only if the fragment autocatalysis rate is small enough, or the marginal rate of technical substitution of satellites for debris in collision probability is high enough.*

The key equation from the proof is

$$\frac{\partial \mathcal{V}}{\partial D}(D^*) = (G_D(S^*, D^*) - \delta) - \frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}(G_S(S^*, D^*) + m(r_s - r)), \quad (28)$$

where  $S^* \equiv S(r_s - r, D^*)$ . The stability of an open-access steady state depends on two factors, shown on the right hand side of equation 28. The first term is the fragment autocatalysis rate,  $G_D - \delta$ , which can ensure stability if it is close enough to negative infinity. This has a clear physical intuition: higher decay rates indicate regions with greater natural renewability, while stronger new fragment debris couplings indicate stronger positive feedbacks between debris stocks.

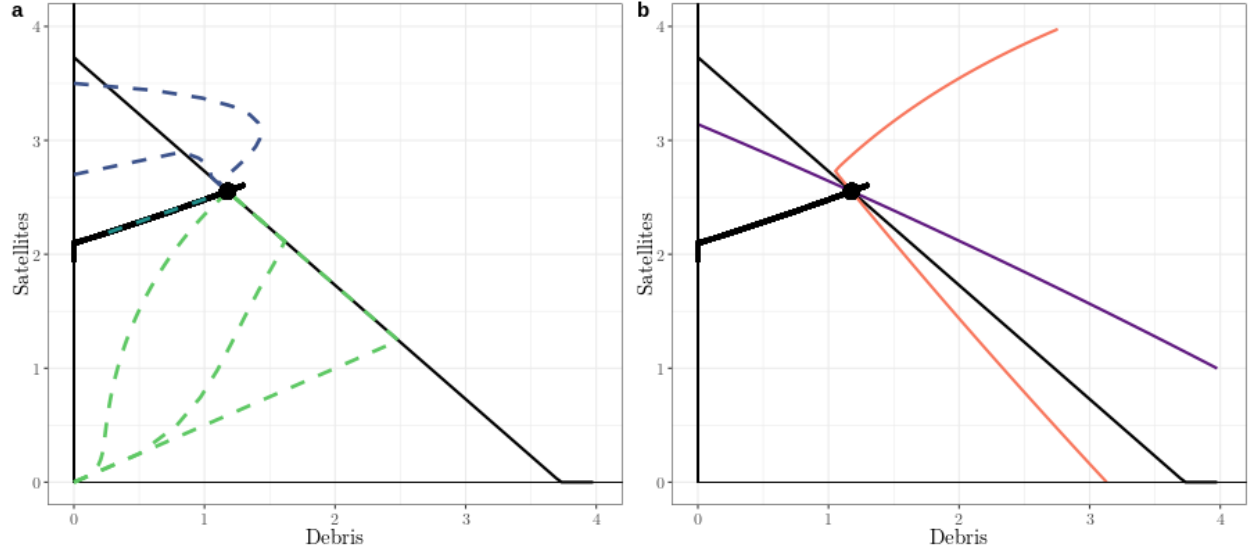
The second term is the equilibrium effect of new satellites on debris growth. This term is increasing in the MRTS of satellites for debris in collision probability, as a larger MRTS indicates the collision probability function is much more sensitive to debris fragments than satellites. A larger MRTS incentivizes profit-maximizing satellite launchers to be more re-

sponsive to debris stocks than they would be with a smaller MRTS, thus stabilizing the steady state. The effect of the MRTS is scaled by the strength of the new fragment satellite coupling, the launch debris coupling, and the excess return on a satellite. The new fragment satellite and launch debris couplings are both byproducts of putting satellites into orbit, and force satellite operators to account for the risks they face in orbit or impose on others. Figures 2 and ?? illustrates Propositions 2 and 3.

Provided there is a stable open-access steady state to approach, it is important to understand the open-access approach paths. If open access monotonically approaches stable steady states, then parameter changes which shift the steady state may not cause risky spikes in the debris stock. Such parameter changes include technological advances (e.g. better-shielded satellites), policy guidelines (e.g. encouragement to use frangibolts instead of exploding bolts for booster separation), environmental processes (e.g. sunspot activity which cause  $\delta$  to vary), or economic changes (e.g. increases in the excess return on a satellite).

Proposition 4 shows it is unlikely monotonic approach paths occur when the launch rate is unconstrained. Initial conditions which result in smooth approach paths to a steady state form a set of (Lebesgue) measure zero. The majority of initial conditions instead cause the state variables to overshoot the steady state.

**Proposition 4 (Overshooting).** *Given a new fragment formation function which is strictly convex in both arguments, a launch constraint which doesn't bind, and a non-atomic probability distribution over states with positive launch rates, open-access paths are more likely than not to overshoot the stable open-access steady state.*



**Figure 3:** In both panels, the thin black line shows the equilibrium isoquant. The thick black point on the equilibrium isoquant is the unique and stable open-access steady state. The thick black line connecting the steady state to the y-axis is the surface of points which can converge to the steady state in one step (the “one-step surface”). Panel a illustrates Proposition 4 with sample paths. The teal dashed line along the thick black line shows a path converging to the steady state in one step. The green dashed lines show paths which overshoot in debris, all of which converge to the steady state along the equilibrium isoquant. The purple dashed lines show paths which overshoot at least in satellites. When possible these paths too follow the equilibrium isoquant to the steady state, though the physical dynamics force two of the three paths to approach the steady state from outside the action region. Panel b shows the equilibrium isoquant, the one-step surface, and the satellite (purple) and debris nullclines (pink).

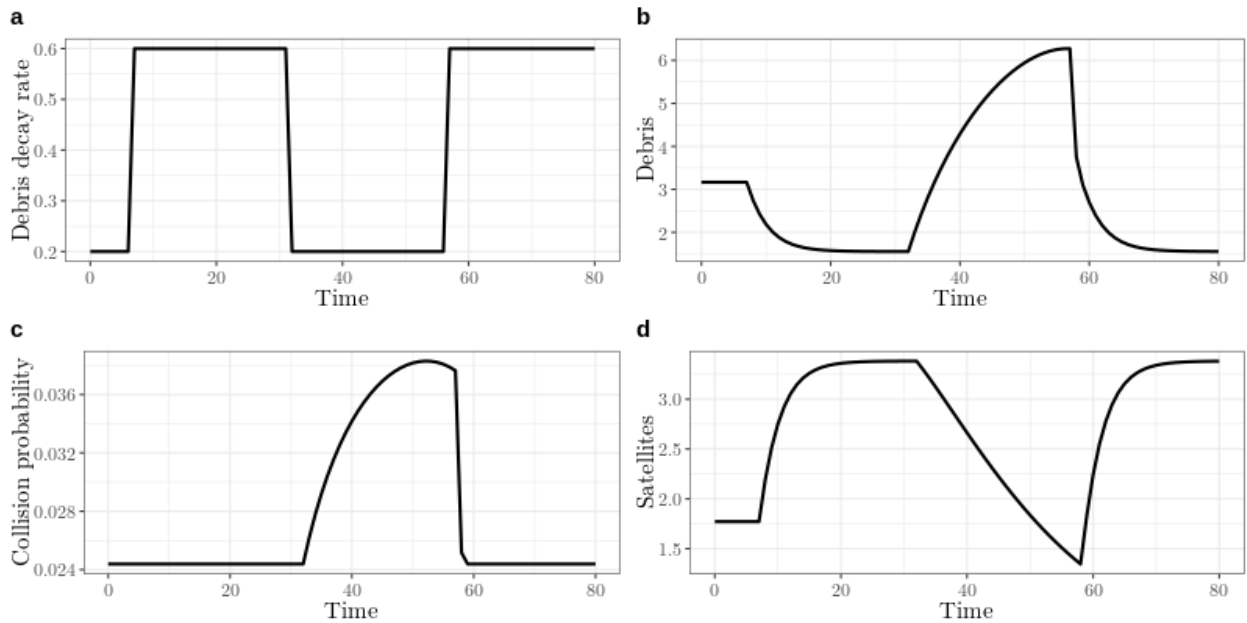
Proposition 4 highlights two properties of open access, illustrated in figure 3. First, open access attempts to immediately equilibrate the collision probability by moving to the isoquant where it is equal to the excess return, even if the point on the isoquant that can be reached in one period is not a steady state. This is analogous to a driver speeding to reach their destination and forgetting to decelerate as they approach. This same property implies that, when the physical dynamics permit, open-access firms will approach the steady state along the equilibrium isoquant.

Second, the overshooting becomes more severe as the initial condition and physical dynamics bring firms to points on the equilibrium isoquant which are farther from the surface leading to the open-access steady state. In the driving analogy, this is the effect of starting location on the uninternalized acceleration: when the driver is farther away, they must accelerate more and go faster to reach their destination in the same amount of time. In cases where the fragment autocatalysis rate becomes positive, this leads to overshooting in both state variables.

This suggests some scope for orbit use stabilization policies which impose a fixed cap

on the number of launches per period, since slowing the rate at which firms approach the equilibrium isoquant can reduce the amount of overshooting. This can be seen in figure 3. For example, an unconstrained path from the  $(0,0)$  initial condition (the straight line from  $(0,0)$ ) immediately reaches the equilibrium isoquant by overshooting the steady-state debris level. The overshooting is reduced when the launch rate is constrained, with the most tightly-constrained path showing no overshooting.

Proposition 4 suggests parameter changes which lower the open-access steady-state debris level may still cause short-run rebound effects. Sunspot activity is an environmental process which can cause such a shift. Sunspots cause the Sun to differentially heat the Earth's atmosphere, causing it to expand and contract over time and changing the radiation pressure exerted on satellites. These changes increase the debris decay rate, and force satellites to expend more fuel to remain in their intended orbit. Figure 4 shows an example of debris rebound effects under periodic decay rate variation similar to sunspot activity.



**Figure 4:** Panel a shows the debris decay rate varying over time. Panels b-d show equilibrium debris stock, equilibrium collision probability, and equilibrium satellite stock responding to the changes in the decay rate. The initial increase in the decay rate reduces the debris stock and allows more satellites to be sustained. When the decay rate falls, the debris stock grows rapidly and the satellite stock falls. This growth in debris causes the collision probability to rise above the equilibrium level, which stops launch activity. The growth in debris is halted only by the decay rate once again increasing. Once the collision probability is back in equilibrium, new satellites are again launched to take advantage of the higher decay rate.

Given this overshooting, it is natural to wonder whether open access can lead to Kessler Syndrome. While the occurrence of Kessler Syndrome is inconsistent with being at a steady

state, there are three possible cases where open access can cause Kessler Syndrome:

1. the initial condition is inside the Kessler region;
2. the fragment autocatalysis rate or rate of excess return is high enough no stable steady state exists.

The first case can occur due to non-economic factors, such as changes in environmental parameters (e.g. sunspots) or military activity in space (e.g. anti-satellite missile tests which create debris). The second can occur due to economic or non-economic factors, with examples of the former including increases in the profitability of a satellite. Figure 5 illustrates this example. In both cases Kessler Syndrome is inefficient, given reasonable conditions on economic parameters. As long as Kessler Syndrome is possible, open access is more likely to cause Kessler Syndrome as the excess return on a satellite increases.

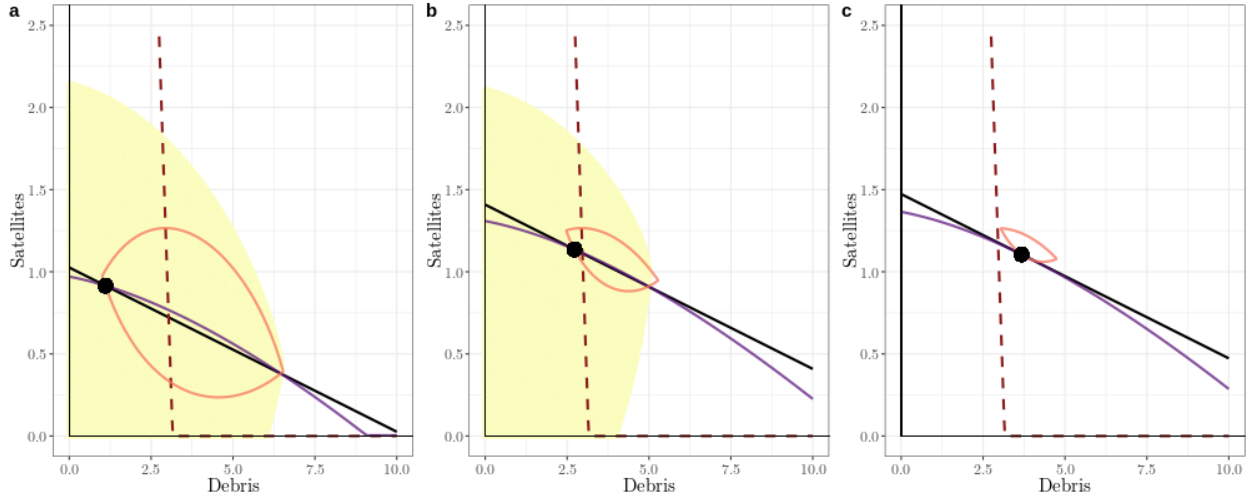
**Proposition 5** (Inefficient open-access Kessler Syndrome). *When the rate of excess return is positive but less than one, the marginal external cost is positive, and the new fragment function is strictly convex in both arguments,*

1. *Kessler Syndrome is not socially optimal, and*
2. *open access is weakly more likely to cause Kessler Syndrome as the excess return on a satellite increases.*

The proof of the suboptimality of Kessler Syndrome is based on the transversality condition of the sequence version of the planner's problem. Because Kessler Syndrome involves paths where the debris stock diverges to infinity, eventually it will force the satellite stock go to zero and remain there forever after. For a planner with a sufficiently low discount rate, this is clearly not optimal: they could reduce their launch rate by just enough to avoid Kessler Syndrome, and collect profits from the fleet in perpetuity and a cost of slightly lower profits in the short term. But what if the planner's discount rate were very high?

We assumed  $\pi < F$  early on to avoid the unrealistic case where even an instant of profitability outweighed the cost of launching, and  $r_s > r \iff \pi > rF$  to ensure satellites are assets worth investing in at all. Combining the two, we have  $rF < \pi < F \iff r < r_s < 1$ . This implies the discount rate cannot be greater than one for Kessler Syndrome to be inefficient. If  $r \geq 1$ , then either satellites are not worth investing in at all ( $r_s < 1 \leq r$ ) or satellites are such productive assets ( $1 \leq r < r_s$ ) even an instant of returns outweigh certain destruction as soon as the satellite reaches orbit. In the former case, both the planner and open-access firms launch no satellites and Kessler Syndrome does not occur. In the latter case, open-access firms cause Kessler Syndrome and, if the marginal external cost is nonpositive, this is optimal. Both cases are ruled out by requiring  $0 < r_s - r < 1$  and  $\xi(S_{t+1}, D_{t+1}) > 0$ .

Figure 5 illustrates the second part of Proposition 5. The debris level associated with the unstable steady state is  $D^K$ , the maximum sustainable debris level. As the rate of excess return on a satellite increases, the basin of attraction for the stable open-access steady state (the yellow shaded region) shrinks and  $D^K$  moves inward while the stable open-access steady state debris level moves outward, squeezing the two fixed points closer to each other. Eventually, the stable steady state crosses the boundary where the fragment autocatalysis rate becomes positive (the red dashed line). When this occurs, only a single (repelling) fixed point remains and the basin of attraction for the once-stable open-access steady state disappears — all initial conditions will lead to Kessler Syndrome.



**Figure 5:** An illustration of how increases in the rate of excess return on a satellite can cause Kessler Syndrome. The shaded yellow region shows the basin of attraction of the stable open-access steady state, and the dashed red line shows the boundary where the fragment autocatalysis rate become positive. The solid black line shows the equilibrium isoquant, and the purple and pink lines show the satellite and debris nullclines. The black dot shows the stable (in panels a and b) or only (in panel c) open-access steady state. From panel a to c the rate of excess return steadily increases, shrinking the basin of attraction of the stable open-access steady state or equivalently increasing the size of the Kessler region. When the stable open-access steady state crosses the fragment autocatalysis rate boundary, it loses stability and its basin of attraction vanishes.

## 4.1 The risk to LEO

LEO is one of the fastest growing segments in the satellite industry today, particularly for smaller satellites with fewer guidance and control systems (France-Presse (2019), Selk (2017), Dvorsky (2018)). LEO users typically have shorter planning horizons than GEO users and face lower costs. Propositions 4 and 5 reinforce the conclusions from physical models of debris growth: LEO is at the highest risk of Kessler Syndrome (Liou and Johnson, 2008; Kessler et al., 2010). Our results show that this is not just a feature of the physics given current use patterns, but of the economics underlying those use patterns. While higher decay rates in LEO

may protect it against Kessler Syndrome, the economic and other physical properties of LEO — particularly the low cost of access and the natural variability in debris decay rates caused by sunspot activity and atmospheric effects — can work in the opposite direction. Attempts by national authorities to unilaterally regulate satellite launches can face leakage problems similar to other settings where pollution is produced by mobile capital.<sup>12</sup>

Larger collision or fragmentation parameters are known to increase the chance of Kessler Syndrome (e.g., [Rossi et al. \(1998\)](#); [Kessler and Anz-Meador \(2001\)](#); [Liou \(2006\)](#)). [Adilov, Alexander, and Cunningham \(2015\)](#) showed that lower launch costs will increase the equilibrium amount of debris. We reaffirm these findings, and show that lower launch costs will also increase the equilibrium collision probability. An increase in the collision and fragmentation parameters could be caused by cost-minimizing satellite launchers opting to launch cheaper satellites with fewer guidance and control systems and less shielding. A reduction in launch costs could occur independent of changes in satellite characteristics, for example as more firms enter the launch market and drive launch prices down.

These issues are not as pronounced in GEO in part because use rights are defined for GEO slots. Our model suggests that other characteristics of GEO, in particular the high cost of access and the low natural decay rate, create additional economic incentives to reduce debris accumulation and collision risk there.

## 5 Conclusion

In this paper we present the first dynamic physico-economic model of orbit use under rational expectations with endogenous collision risk and Kessler Syndrome. We show how both economic and physical parameters drive equilibrium short- and long-run orbit-use patterns, derive the marginal external cost of a satellite, explore the multiplicity, instability, and convergence dynamics of open-access steady states, and examine the relationship between open-access behavior and Kessler Syndrome. We highlight three key messages for readers concerned for the future of Earth’s orbits.

First, too many firms will launch satellites because they won’t internalize the risk they impose on other orbit users. Though profit maximizing satellite owners have incentives to reduce launches as risk of a collision grows, they do not respond to debris growth or collision risk

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<sup>12</sup>[Dvorsky \(2018\)](#) documents what may be the first instance of launch leakage: a California startup denied launch permission by the FAA went outside the FAA’s jurisdiction and purchased a launch from an Indian launch provider instead. The FAA denied permission on the grounds that the startup’s satellites were too small to be effectively tracked and would increase the risk of unavoidable collisions and debris growth. The leakage earned the startup a fine from the FCC.

optimally. This inefficiency is independent of whether Kessler Syndrome is possible or not, though under reasonable conditions on economic parameters Kessler Syndrome is not optimal. Second, rebound effects can result in higher debris levels when the rate of debris decay increases. This suggests sunspot activity may drive debris and collision risk cycles in LEO even if satellite operators are forward-looking and rational. Third, under open access Kessler Syndrome is more likely as the excess return on a satellite rises, even if firms will respond to orbital congestion by launching fewer satellites. As launch costs fall and new commercial satellite applications become viable, LEO is thus increasingly (and inefficiently) likely to experience Kessler Syndrome.

Economists tend to focus on property rights, corrective taxes, or other market-based mechanisms to solve externality problems. While these mechanisms can likely ensure efficient orbit use and prevent Kessler Syndrome, more study is needed to understand how orbital management policies should be designed in light of the unique physical features of this resource. Whether they are enforced by states, structured as self-enforcing agreements between private actors, or some combination of the two, orbits are a global commons and will likely require global policy solutions.



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# Appendices

## A Proofs and derivations

### A.1 Proofs

**Proposition 1** (Equilibrium collision probability). *There is a continuum of interior open-access equilibria, each with collision probability in  $t + 1$  equal to the excess return on a satellite. Increases in the excess return on a satellite cause the collision probability in  $t + 1$ , launch rate in  $t$ , and debris stock in  $t + 1$  to increase.*

*Proof.* Taking equation 12 and dividing by  $F$ , the equilibrium collision probability in  $t + 1$  can be written as

$$L(S_{t+1}, D_{t+1}) = r_s - r, \quad (29)$$

where  $r_s - r$  is the excess return on a satellite. By inspection, increases in the excess return will also increase the equilibrium collision probability.

Since  $L$  is continuous and increasing in both arguments, there is a continuum of points  $(S, D)$  such that equation 12 holds — the level set of  $L$  equal to  $r_s - r$ .

To see the effect of increases in the excess return on the launch rate in  $t$  and the debris stock in  $t + 1$ , we apply the Implicit Function Theorem to equation 12:

$$\begin{aligned} \frac{\partial X_t}{\partial(r_s - r)} &= \frac{1}{L_S(S_{t+1}, D_{t+1}) + mL_D(S_{t+1}, D_{t+1})} > 0 \\ \frac{\partial D_{t+1}}{\partial(r_s - r)} &= \frac{1}{L_D(S_{t+1}, D_{t+1})} > 0, \end{aligned}$$

where variable-subscripts indicate partial derivatives, i.e.  $L_S \equiv \frac{\partial L}{\partial S}$ , and  $L_S, L_D > 0$  by assumption.

□

**Proposition 2** (Multiplicity). *Given a positive excess return on a satellite and a fully-coupled collision probability function, multiple open-access steady states can exist if the new fragment debris coupling exists.*

*Proof.* The open-access steady states are defined by equations 24, 25, and 26. Equation 24 implicitly determines the number of satellites as a function of the amount of debris, the excess return on a satellite, and the collision rate function,

$$L(S, D) = r_s - r \implies S = S(r_s - r, D). \quad (30)$$

Since  $L$  is increasing in each argument,  $S(r_s - r, D)$  is decreasing in  $D$ . Since  $S$  must be non-negative, there exists  $D^S : S(r_s - r, D^S) = 0$ . So we have

$$\hat{S} = \begin{cases} S(r_s - r, D) & \text{if } D \in [0, D^S) \\ 0 & \text{if } D \geq D^S \end{cases} \quad (31)$$

Using  $\hat{S}$  we can reduce equations 24, 25, and 26 to a single equation in debris,

$$\mathcal{Y}(D) = -\delta D + G(\hat{S}, D) + m(r_s - r)\hat{S}, \quad (32)$$

with

$$\{D^* \geq 0 : \delta D^* = G(\hat{S}, D^*) + m(r_s - r)\hat{S}\} \quad (33)$$

being the open-access steady states.

$\delta D$  is monotonically increasing in  $D$  with  $\delta D = 0$  when  $D = 0$ , and  $m(r_s - r)\hat{S}$  is monotonically decreasing in  $D$  with  $\hat{S} > 0$  when  $D = 0$ , but  $\hat{G} \equiv G(\hat{S}, D)$  is nonmonotone in  $D$ . To see this, note

$$\frac{d\hat{G}}{dD}(\hat{S}, D) = \underbrace{\frac{\partial G}{\partial S}}_{\geq 0} \underbrace{\frac{\partial \hat{S}}{\partial D}}_{\leq 0} + \underbrace{\frac{\partial G}{\partial D}}_{\geq 0}, \text{ with} \quad (34)$$

$$\frac{d\hat{G}}{dD}(\hat{S}, 0) = \frac{\partial G}{\partial S} \frac{\partial \hat{S}}{\partial D} < 0 \text{ and} \quad (35)$$

$$\frac{d\hat{G}}{dD}(0, D^S) = \frac{\partial G}{\partial D} > 0, \quad (36)$$

where  $\frac{\partial \hat{S}}{\partial D} = -\frac{L_D}{L_S} \leq 0$  by application of the Implicit Function Theorem on equation 12.

If  $G_D > 0$ , then  $\hat{G}$  is nonmonotone in  $D$  and the existence or uniqueness of  $D^*$  cannot be guaranteed. If  $G_D$  is large enough,  $D^*$  will not exist; if  $G_D$  is not too small, multiple  $D^*$  will exist. If  $G_D = 0$ , then the existence of  $D^*$  also ensures its uniqueness. If  $G_D$  is strictly convex in both arguments, at most two  $D^*$  can exist.  $\square$

**Proposition 3** (Local stability). *Given a positive excess return on a satellite, new fragment satellite coupling, and launch debris coupling, open-access steady states will be locally stable if and only if the fragment autocatalysis rate is small enough, or the marginal rate of technical substitution of satellites for debris in collision probability is high enough.*

*Proof.* We use the reduction from Proposition 2 to simplify the proof. The open access steady states are solutions to equation 32, and the sign of  $\frac{\partial \mathcal{Y}}{\partial D}$  at the solutions allows us to classify the

stability of the system. Applying the Implicit Function Theorem to equation 24 to calculate  $S_D$ , then differentiating  $\mathcal{Y}$  in the neighborhood of an arbitrary solution  $D^*$ ,

$$\frac{\partial \mathcal{Y}}{\partial D}(D^*) = (G_D(S^*, D^*) - \delta) - \frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}(G_S(S^*, D^*) + m(r_s - r)), \quad (37)$$

where  $S^* \equiv S(r_s - r, D^*)$ .  $(G_D(S^*, D^*) - \delta)$  is the fragment autocatalysis rate and  $\frac{L_D(S^*, D^*)}{L_S(S^*, D^*)}$  is the MRTS of satellites for debris in collision probability. Both  $G_S(S^*, D^*)$  and  $m(r_s - r)$  are positive by assumption. So  $\frac{\partial \mathcal{Y}}{\partial D}(D^*) < 0$  can hold if and only if the fragment autocatalysis rate is small enough, or the MRTS of satellites for debris in collision probability is large enough.  $\square$

**Proposition 4** (Overshooting). *Given a new fragment formation function which is strictly convex in both arguments, a launch constraint which doesn't bind, and a non-atomic probability distribution over states with positive launch rates, open-access paths are more likely than not to overshoot the stable open-access steady state.*

*Proof.* We first define the following sets and functions, where  $S, D \geq 0$  is assumed:

- The action region: the set of states with positive open-access launch rates,

$$A = \{(S, D) : r_s - r - L(S', D') \geq 0\}, \quad (38)$$

where

$$\begin{aligned} S' &= S(1 - L(S, D)) + X \\ D' &= D(1 - \delta) + G(S, D) + mX, \end{aligned}$$

- The equilibrium isoquant:

$$E = \{(S, D) : r_s - r - L(S, D) = 0\}. \quad (39)$$

- The stable open-access steady state: following the reduction to equation 32 from Proposition 2,

$$E_s = \{(\hat{S}, D) : \mathcal{Y}(D) = -\delta D + G(\hat{S}, D) + m(r_s - r)\hat{S} = 0, \hat{S} : L(\hat{S}, D) = r_s - r, \mathcal{Y}'(D) < 0\}. \quad (40)$$

- The one-step manifold: the set of states from which one period of launching will reach an open-access steady state,

$$A_1 = \{(S, D) : (S + X, D + mX) \in E_s, X \in (0, \bar{X}]\}, \quad (41)$$

where  $m$  is the same as in the debris law of motion.

- The physical dynamics: the effect of orbital mechanics on the satellite and debris stocks in a period,

$$P_{SD}(S, D) = (S(1 - L(S, D)), D(1 - \delta) + G(S, D)). \quad (42)$$

Since the launch constraint does not bind, any point in  $A$  will reach a point in  $E$ . Since  $E_s$  contains at most one element given the strict convexity of  $G$ ,  $E_s \subset E$ . Given that  $L$  is increasing in both arguments, points in  $E \setminus E_s$  must therefore have either larger  $S$  and smaller  $D$  than  $E_s$ , or vice versa. Consequently, reaching points in  $E \setminus E_s$  constitutes overshooting  $E_s$  in one state variable and undershooting in the other.

By definition,  $A_1 \subseteq A$ . Since  $E_s$  contains at most one element,  $A_1$  is a single line segment, so  $A_1 \subset A$ . The Lebesgue measure on  $A$  of  $A_1$  is therefore zero.

From the definition of  $P_{SD}$ , it is bijective on  $\mathbb{R}_+^2$ . Since  $P_{SD}$  is a bijection, the Lebesgue measure of  $P_{SD}^{-1}(A_1)$  (the pre-image of  $A_1$  under  $P_{SD}$ ) is the same as the Lebesgue measure of  $A_1$ .

Bringing it all together:

1. Any point in  $A$  will reach  $E$  if the launch constraint doesn't bind.
2.  $E_s \subset E$ , and reaching any point in  $E$  constitutes overshooting any element of  $E_s$  in one variable.
3. Any point in  $A_1$  can reach  $E_s$  in a single period of launching.  $P_{SD}^{-1}(A_1)$  are the points which can reach  $E_s$  in a single period of physical dynamics and launching.
4.  $A_1$  is a single line segment in  $A$ , and therefore Lebesgue measure zero over  $A$ .
5. Since  $P_{SD}$  is a bijection, the Lebesgue measure of  $P_{SD}^{-1}(A_1)$  is also zero.
6. This result holds for any non-atomic probability measure over  $\mathbb{R}_+^2$  since  $A_1 \subset A$  and  $P_{SD}$  is a bijection.

□

**Proposition 5** (Inefficient open-access Kessler Syndrome). *When the rate of excess return is positive but less than one, the marginal external cost is positive, and the new fragment function is strictly convex in both arguments,*

1. *Kessler Syndrome is not socially optimal, and*
2. *open access is weakly more likely to cause Kessler Syndrome as the excess return on a satellite increases.*

*Proof.* To see Kessler Syndrome is never socially optimal, consider the sequence problem underlying the planner's functional equation (equation 19):

$$\max_{\{X_t\}_{t=0}^{\infty}} \bar{W}(S_t, D_t, X_t) \quad (43)$$

$$S_{t+1} = S_t(1 - L(S_t, D_t)) + X_t \quad (44)$$

$$D_{t+1} = D_t(1 - \delta) + G(S_t, D_t) + mX_t \quad (45)$$

$$X_t \in [0, \bar{X}] \quad \forall t \quad (46)$$

$$S_0 = s_0, D_0 = d_0 \quad (47)$$

$$\lim_{t \rightarrow \infty} \beta^t \bar{W}_S(S_t, D_t, X_t) S_{t+1} = 0 \quad (48)$$

$$\lim_{t \rightarrow \infty} \beta^t \bar{W}_D(S_t, D_t, X_t) D_{t+1} = 0, \quad (49)$$

where  $\bar{W}(S_t, D_t, X_t)$  is defined in equation 16 as the net present value of an orbit. The transversality condition for the debris stock, equation 49, rules out paths which cause Kessler Syndrome. Thus Kessler Syndrome is never socially optimal.

To see that open access is more likely to cause Kessler Syndrome as the excess return on a satellite increases, we return to the single-equation reduction from Proposition 2, equation 32:

$$\mathcal{Y}(D) = -\delta D + G(\hat{S}, D) + m(r_s - r)\hat{S}. \quad (50)$$

If the new fragment formation function  $G$  is strictly convex in both arguments, there are up to two solutions to the above equation.<sup>13</sup> Let the smaller solution be  $D^*$ , and the larger solution be  $D^\kappa$ , i.e.  $D^* < D^\kappa$ .<sup>14</sup> Only  $D^*$  is an attracting fixed point, while  $D^\kappa$  is a repelling fixed point. This  $D^\kappa$  is the same as the one defined in equation 3 as the Kessler threshold. The basin of attraction for  $D^*$  is thus at most  $[0, D^\kappa)$ .<sup>15</sup>

We define the probability that open access avoids Kessler Syndrome as the normalized Lebesgue measure of states within a ball of fixed radius,  $R > 0$ , around a stable open-access steady state which cause the debris stock to diverge to infinity:

$$P = \frac{1}{2R} \int_{D^*-R}^{D^*+R} \mathbb{1}_{\{D \in [0, D^\kappa)\}} dD. \quad (51)$$

This is equivalent to a uniform distribution over states within the ball, though the proof goes through for any probability measure which assigns positive probability to all states in the

<sup>13</sup>Strict convexity of  $G$  is a sufficient but not necessary condition for this result. However, it is a reasonable physical assumption and simplifies the proof.

<sup>14</sup>If  $D^* = D^\kappa$ , the fixed point is repelling. We ignore this case since it is less interesting — it is a knife-edge case which guarantees open access causes Kessler Syndrome.

<sup>15</sup>We say “at most” because, as figure 5 illustrates, in two dimensions the basin depends on both  $S$  and  $D$ .

ball. The probability that open access causes Kessler Syndrome is  $1 - P$ .

Next, consider how open-access steady state debris levels change as the excess return on a satellite changes. We suppress function arguments to reduce notation; all functions are evaluated at an arbitrary open-access steady state. Applying the Implicit Function Theorem to equation 32, and applying it again to equation 24 to calculate  $\frac{\partial S}{\partial(r_s - r)}$ , we get:

$$\frac{\partial D}{\partial(r_s - r)} = -\frac{\frac{G_S}{L_S} + m(S(r_s - r, D) + \frac{r_s - r}{L_S})}{(G_D - \delta) - \frac{L_D}{L_S}(G_S + m(r_s - r))} \leq 0$$

Given our assumptions, the numerator is always positive. From Proposition 3, an open-access steady state is locally stable if and only if the denominator is negative. So if the steady state is stable, increases in the excess rate of return on a satellite will cause the debris level to increase. If the steady state is unstable, increases in the excess rate of return on a satellite will cause the debris level to decrease. So we have

$$\frac{\partial D^*}{\partial(r_s - r)} > 0, \quad \frac{\partial D^\kappa}{\partial(r_s - r)} < 0.$$

Differentiating  $P$  with respect to  $r_s - r$ , we obtain

$$\frac{\partial P}{\partial(r_s - r)} = \frac{1}{2R} \left( \mathbb{1}_{\{D^* - R \in [0, D^\kappa]\}} \frac{\partial D^*}{\partial(r_s - r)} - \mathbb{1}_{\{D^* + R \in [0, D^\kappa]\}} \frac{\partial D^*}{\partial(r_s - r)} + \int_{D^* - R}^{D^* + R} \frac{\partial \mathbb{1}_{\{D \in [0, D^\kappa]\}}}{\partial(r_s - r)} dD \right) \quad (52)$$

For any fixed choice of  $R$ , we have two cases following an increase in  $r_s - r$ : either  $D^* + R < D^\kappa$  still, or  $D^* + R \geq D^\kappa$ .

1. If  $D^* + R < D^\kappa$  still: Equation 52 becomes

$$\frac{1}{2R} \int_{D^* - R}^{D^* + R} \frac{\partial \mathbb{1}_{\{D \in [0, D^\kappa]\}}}{\partial(r_s - r)} dD = 0 \quad (53)$$

since  $D \in [D^* - R, D^* + R]$  and  $D^* + R < D^\kappa$  by assumption.

2. If  $D^* + R \geq D^\kappa$ : Equation 52 becomes

$$-\frac{1}{2R} \frac{\partial D^*}{\partial(r_s - r)} + \frac{1}{2R} \int_{D^* - R}^{D^* + R} \frac{\partial \mathbb{1}_{\{D \in [0, D^\kappa]\}}}{\partial(r_s - r)} dD < 0 \quad (54)$$

since  $\frac{\partial D^*}{\partial(r_s - r)} > 0$  and  $\frac{\partial D^\kappa}{\partial(r_s - r)} < 0$ .

The first case is trivial in the sense that for a larger  $R$  or a larger change in  $r_s - r$  the inequality will become strict. Since  $\frac{\partial P}{\partial(r_s - r)} \leq 0$  in any case, we have the desired result:  $\frac{\partial(1-P)}{\partial(r_s - r)} \geq 0$ , i.e. increases in the excess return on a satellite will weakly increase the probability of Kessler Syndrome.



□

## A.2 Derivations

### A.2.1 Optimal launch policy and marginal external cost

Period  $t$  values are shown with no subscript, and period  $t + 1$  values are marked with a  $'$ , e.g.  $S_t \equiv S, S_{t+1} \equiv S'$ . The fleet planner's problem is

$$W(S, D) = \max_{X \geq 0} \{ \pi S - FX + \beta W(S', D') \} \quad (55)$$

$$\text{s.t. } S' = S(1 - L(S, D)) + X \quad (56)$$

$$D' = D(1 - \delta) + G(S, D) + mX. \quad (57)$$

The fleet planner's launch plan will satisfy

$$X^* : \beta[W_S(S', D') + mW_D(S', D')] = F, \quad (58)$$

that is, the planner will launch until the marginal value to the fleet of a new satellite plus the marginal value to the fleet of its launch debris is equal to the launch cost. Note that while the per-period objective function is linear in the control variable, the fleet pre-value function is not, i.e. launches in  $t$  impose a linear cost in  $t$  but have nonlinear effects in periods beyond  $t$ . This can be seen from the form of the fleet pre-value function in equation 17: changes in  $X_t$  have nonlinear effects on the fleet value in period  $t + 1$  through the appearance of  $S_{t+1}$  and  $D_{t+1}$  in  $S_{t+2} = L(S_{t+1}, D_{t+1}) + X_{t+2}$ . In order to obtain linearity of the fleet pre-value function, we must impose that the collision probability function is linear in all arguments influenced by the launch rate. Such linearity conflicts with our assumption that the collision probability is bounded in  $[0, 1]$  and twice continuously differentiable over the real line. Thus, given our assumptions, the optimal policy is not of the bang-bang type.

Applying the envelope condition, we have the following expressions for the fleet's marginal value of another satellite and another piece of debris:

$$W_S(S, D) = \pi + \beta[W_S(S', D')(1 - L(S, D) - SL_S(S, D)) + W_D(S', D')G_S(S, D)] \quad (59)$$

$$W_D(S, D) = \beta[W_D(S', D')(1 - \delta + G_D(S, D)) + W_S(S', D')(-SL_D(S, D))] \quad (60)$$

Rewriting equation 58, we have

$$W_S(S', D') = \left[ \frac{F}{\beta} - mW_D(S', D') \right] \quad (61)$$

Plugging equation 61 into equations 59 and 60,

$$W_S(S, D) = \pi + F(1 - L(S, D) - SL_S(S, D)) - \beta W_D(S', D')[m(1 - L(S, D) - SL_S(S, D)) - G_S(S, D)] \quad (62)$$

$$W_D(S, D) = (-SL_D(S, D))F + \beta W_D(S', D')[1 - \delta + G_D(S, D) - m(-SL_D(S, D))] \quad (63)$$

Define the following quantities:

$$\alpha_1(S, D) = \pi + (1 - L(S, D) - SL_S(S, D))F$$

$$\alpha_2(S, D) = -SL_D(S, D)F$$

$$\Gamma_1(S, D) = G_S(S, D) - m(1 - L(S, D) - SL_S(S, D))$$

$$\Gamma_2(S, D) = 1 - \delta + G_D(S, D) + mSL_D(S, D).$$

These allow us to rewrite equations 62 and 63 as

$$W_S(S, D) = \alpha_1(S, D) + \beta \Gamma_1(S, D) W_D(S', D') \quad (64)$$

$$W_D(S, D) = \alpha_2(S, D) + \beta \Gamma_2(S, D) W_D(S', D'). \quad (65)$$

As long as  $\delta < 1$ ,  $\Gamma_2(S, D) \neq 0 \forall (S, D)$ , allowing us to rewrite equation 65 as

$$W_D(S', D') = \frac{W_D(S, D) - \alpha_2(S, D)}{\beta \Gamma_2(S, D)}. \quad (66)$$

Plugging equation 66 into equation 64, we get

$$\begin{aligned} W_S(S, D) &= \alpha_1(S, D) + \beta \Gamma_1(S, D) \frac{W_D(S, D) - \alpha_2(S, D)}{\beta \Gamma_2(S, D)} \\ &= \alpha_1(S, D) + \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} (W_D(S, D) - \alpha_2(S, D)) \\ \implies W_S(S, D) &= \alpha_1(S, D) - \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} \alpha_2(S, D) + \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} W_D(S, D) \end{aligned} \quad (67)$$

Iterating equation 61 one period backwards and plugging it into equation 64, we get

$$\begin{aligned} \frac{F}{\beta} - mW_D(S, D) &= \alpha_1(S, D) - \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} \alpha_2(S, D) + \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} W_D(S, D) \\ \implies W_D(S, D) &= \left[ \frac{F}{\beta} - \alpha_1(S, D) - \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} \alpha_2(S, D) \right] \left[ \frac{\Gamma_1(S, D)}{\Gamma_2(S, D)} + m \right]^{-1}. \end{aligned} \quad (68)$$

Combining equations 65 and 68 yields an Euler equation for the optimal launch rate,

$$\frac{1}{\beta} = \alpha_2(S, D) \left[ \frac{\Gamma_1(S, D) + m\Gamma_2(S, D)}{\Gamma_2(S, D)(F - \beta\alpha_1(S, D)) + \beta\Gamma_1(S, D)\alpha_2(S, D)} \right] + \Gamma_2(S, D) \left[ \frac{\Gamma_2(S', D')(F - \beta\alpha_1(S', D')) + \beta\Gamma_1(S', D')\alpha_2(S', D')}{\Gamma_2(S, D)(F - \beta\alpha_1(S, D)) + \beta\Gamma_1(S, D)\alpha_2(S, D)} \right] \quad (69)$$

Substituting in the forms for  $\alpha_1(S, D)$ ,  $\alpha_2(S, D)$ ,  $\Gamma_1(S, D)$ , and  $\Gamma_2(S, D)$ , we obtain the planner's optimality condition (equation 21) and the marginal external cost (equation 23).

### A.2.2 Forms for the collision probability function and new fragment formation function

For numerical simulations, we model the probability that objects of type  $j$  are struck by objects of type  $k$  as

$$p_{jk}(k_t) = 1 - e^{-\alpha_{jk}k_t}, \quad (70)$$

where  $\alpha_{jk} > 0$  is a physical parameter reflecting the relative mean sizes, speeds, and inclinations of the object types. The probability a satellite is destroyed is the sum of the probabilities it is struck by debris and by other satellites, adjusted for the probability it is struck by both. For satellite-satellite and satellite-debris collisions, equation 70 gives us

$$L(S, D) = p_{SS}(S) + p_{SD}(D) - p_{SS}(S)p_{SD}(D) \quad (71)$$

$$= (1 - e^{-\alpha_{SS}S}) + (1 - e^{-\alpha_{SD}D}) - (1 - e^{-\alpha_{SS}S})(1 - e^{-\alpha_{SD}D}) \\ \implies L(S, D) = 1 - e^{-\alpha_{SS}S - \alpha_{SD}D}. \quad (72)$$

We write the new fragment formation function as

$$G(S, D) = F_{SD}p_{SD}(D) + F_{SS}p_{SS}(S) + F_{DD}p_{DD}(D), \quad (73)$$

where  $F_{jk}$  models the number of fragments produced in a collision between objects of type  $j$  and  $k$ . Letting  $F_{SS} = \beta_{SS}S$ ,  $F_{SD} = \beta_{SD}S$ , and  $F_{DD} = \beta_{DD}D$  where  $\beta_{jk} > 0$  is a physical parameter reflecting the physical compositions and masses of the colliding objects, and using the forms in equation 70, we obtain

$$G(S, D) = \beta_S S(1 - e^{-\alpha_{SS}S}) + \beta_{SD}(1 - e^{-\alpha_{SD}D})S + \beta_{DD}(1 - e^{-\alpha_{DD}D})D. \quad (74)$$

The form in equation 72 is convenient as it allows us to solve explicitly for the open access launch rate and is easy to manipulate. Similar forms have been used in engineering studies of the orbital debris environment (Bradley and Wein, 2009; Letizia et al., 2017; Letizia, Lem-

mens, and Krag, 2018).

To derive equation 72, we map the setting of satellites and debris colliding in an orbital shell to a setting where balls are randomly dropped in bins, i.e. satellites and debris are mapped to balls and their positions in the orbital shell are mapped to bins. The probability of a specific satellite being struck by another object is then equivalent to the probability that a randomly-dropped ball ends up in a bin containing the specific ball we are focusing on.

Suppose we have  $b$  equally-sized bins and  $n + 1$  balls in total, where  $b \geq n + 1$ . Without loss of generality, we label the ball we are interested in as  $i$ . We will first place  $i$  into an arbitrary bin, and then drop the remaining  $N$  balls into the  $b$  bins with equal probability over bins. The probability a ball is dropped into a given bin is  $\frac{1}{b}$ , and the probability a ball is not dropped into a given bin is then  $\frac{b-1}{b} = 1 - \frac{1}{b}$ . As we drop the remaining  $n$  balls, the probability that none of the balls is dropped in the same bin containing  $j$  is

$$Pr(\text{no collision with } i) = \left(1 - \frac{1}{b}\right)^n \quad (75)$$

Consequently, the probability that any of the  $n$  balls are dropped into  $i$ 's bin is

$$Pr(\text{collision with } i) = 1 - \left(1 - \frac{1}{b}\right)^n. \quad (76)$$

Now suppose we are interested in the probability that members of a collection of  $j$  balls,  $1 \leq j < b$ , end up in a bin with one of the remaining  $n + 1 - j$  balls. The probability that any of the remaining balls end up in a bin with any of the  $j$  balls we are interested in is then

$$Pr(\text{collision with } i) = 1 - \left(1 - \frac{j}{b}\right)^{n+1-j}. \quad (77)$$

As the number of bins and balls grow large ( $\lim_{b,n \rightarrow \infty}$ ), we obtain

$$Pr(\text{collision with } i) = 1 - e^{-j}. \quad (78)$$

Though neither the number of objects in orbits nor the possible positions they could occupy is infinite, the negative natural exponential form is likely a reasonable approximation. If we suppose that we have two types of balls  $j$  and  $k$  of different sizes and bins the size of the smallest type of ball<sup>16</sup>, we get that the probability a ball of type  $k$  is dropped into in a bin with

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<sup>16</sup>We are stretching the physical analogy to dropping balls into bins, but the point is that the probability a ball of a given type is dropped into a bin is not identical across types.

a ball of type  $j$  as

$$Pr(k-j \text{ collision}) = 1 - \left(1 - \frac{\alpha_{jk}k}{b}\right)^{n+1-k} \quad (79)$$

$$\implies \lim_{b,n \rightarrow \infty} Pr(k-j \text{ collision}) = 1 - e^{-\alpha_{jk}k}, \quad (80)$$

which is the form in equation 70, where  $\alpha_{jk}$  is a nonnegative parameter indexing the relative sizes of objects  $j$  and  $k$ . Equation 74 follows from the form of  $L(S, D)$ .

## B Physical and economic extensions

### B.1 Spectrum congestion or price effects

Satellite applications generally require transmissions to and from the Earth. These transmissions may be the satellite's main output or incidental to its operation. In both cases, satellite operators must secure spectrum use rights from the appropriate national authorities for their broadcast and receiving locations. How will spectrum management affect collisions and debris growth?

Spectrum congestion degrades signal quality, making the per-period output of a satellite decreasing in the number of satellites in orbit, i.e.  $\pi = \pi(S), \pi'(S) < 0$ . If satellites are launched only when they have appropriate spectrum rights and spectrum use is optimally managed, then firms will be forced to account for their marginal effects on spectrum congestion in their decision to launch or not. The equilibrium condition becomes

$$L(S_{t+1}, D_{t+1}) = r_s(S_{t+1}) + r'_s(S_{t+1}) - r, \quad (81)$$

where  $r'_s(S_{t+1}) = \pi'(S_{t+1})/F < 0$  by assumption. The equilibrium set would no longer be a collision probability isoquant, although it would still be a surface in the state space. Even if it isn't managed optimally, spectrum congestion will reduce the equilibrium collision probability by reducing the rate of excess return on a satellite.<sup>17</sup>

Open access orbit use will still be inefficient. Although spectrum congestion can reduce the chance of Kessler Syndrome, efficient spectrum management will not incorporate the marginal external cost of collisions and debris growth,  $\xi(S_{t+1}, D_{t+1})$ . Collision and debris growth management policies could be implemented through spectrum pricing. The rest of the analysis in this paper still goes through when spectrum congestion is considered, though some proofs be-

<sup>17</sup>If spectrum use were also under open access then the marginal congestion effect ( $r'_s(S_{t+1})$ ) would not be in the equilibrium condition, but the equilibrium set would still not be a collision probability isoquant.

come more complicated since the equilibrium set is no longer a collision probability isoquant.

Mathematically, this modification would also apply to price effects induced by additional firms entering a particular orbit. Such effects may be relevant for orbits where the dominant satellite application does not face terrestrial competition, such as satellite imaging in sun-synchronous orbits. In orbits where the price of the service provided by satellites is pinned down by terrestrial applications, such as LEO or GEO internet or television service provided to urban areas, either the baseline model where  $r_s$  is constant or the interpretation of  $r'_s(S)$  as spectrum congestion is appropriate. Both spectrum and price effects may be relevant for applications which require substantial spectrum use and face little terrestrial competition, such as satellite telecom service to remote areas. We put the spectrum congestion interpretation first since all satellites require some spectrum and could interfere with each other regardless of application, whereas price effects are application-specific. Price effects, however, can be orbit-agnostic in the sense that satellite systems in different physical orbits may have price effects on each other if they compete in the same market.

## B.2 The effects of limited satellite lifespans

Satellites do not, in general, produce returns forever until destroyed in a collision. Over 1967-2015, planned satellite lifetimes ranged from 3 months to 20 years, with longer lifetimes being more representative of larger and more expensive GEO satellites.<sup>18</sup> How would finite lifetimes affect the collision probability and debris growth problems?

Suppose satellite lifetimes are finite and exogenously distributed with mean  $\mu^{-1}$ . Satellites live at least one time period in this setting, so that  $\mu^{-1} > 1$ . The probability that a satellite will exogenously “die” in any given period is then  $\mu$ . The value of a single satellite becomes

$$Q(S_t, D_t, X_t) = \pi + \beta(1 - \mu)(1 - L(S_t, D_t))Q(S_{t+1}, D_{t+1}, X_{t+1}), \quad (82)$$

and the equilibrium condition becomes

$$\beta Q(S_{t+1}, D_{t+1}, X_{t+1}) = F \quad (83)$$

$$\implies L(S_{t+1}, D_{t+1}) = \frac{r_s - r - \mu}{1 - \mu}, \quad (84)$$

which is lower than the equilibrium collision probability when satellites are infinitely lived. Intuitively, the fact that the satellite will stop generating returns at some point reduces its expected present value, and with it the incentive to launch. All else equal, shorter lifetimes reduce

<sup>18</sup>These numbers are taken from the Union of Concerned Scientists’ publicly available data on satellites. The data are available at <https://www.ucsusa.org/nuclear-weapons/space-weapons/satellite-database>.

the equilibrium collision probability. The rest of the analysis in this paper goes through with minor modifications. Note that equation 84 is the non-stationary discrete-time analog of the physico-economic equilibrium condition in Definition 2 of [Rouillon \(2019\)](#).

Satellites built for GEO tend to be longer lived than satellites launched for LEO. If shorter lifetimes tend to reduce satellite costs, then the downward shift in the collision probability isoquant from the shorter lifetimes will be balanced against the upward shift caused by higher rates of return. The net effect may be higher or lower equilibrium rates of collisions.

The distinction between exogenous and endogenous lifetimes is relevant here. The above analysis hinges on satellite lifetimes being exogenously set. This is not the case in reality. Satellite lifetimes are determined by cost minimization concerns, technological constraints, expectations of component failures, and expectations of future technological change. Incorporating all these features to realistically model the choice of satellite life along with launch decisions and their effects on orbital stock dynamics is beyond our scope in this paper, though it is an interesting area for future research. The assumption that the end-of-life is random simplifies the analysis, but does not change any conclusions over imposing a pre-specified end date in this model.<sup>19</sup>

### B.3 The effects of changes in satellite returns over time

Though it simplifies the analysis, the rate of return on a satellite is not constant over time. How would changes over time in these economic parameters affect our conclusions? For simplicity, suppose that costs, returns, and discount rate vary exogenously and are known in advance.<sup>20</sup> The open access equilibrium condition is then

$$\pi_{t+1} = (1 + r_t)F_t - (1 - L(S_{t+1}, D_{t+1}))F_{t+1} \quad (85)$$

<sup>19</sup>This simplification could matter in a model where firms own multiple satellites and have to plan replacements.

<sup>20</sup>Uncertainty over costs, returns, and discount rates doesn't change the qualitative results, though it introduces expectations over the changes. Endogeneity in the changes, for example due to investment in R&D or marketing, may have more significant consequences which are beyond the scope of this paper.

Equation 85 can be rewritten as

$$L(S_{t+1}, D_{t+1}) = 1 + \frac{\pi_{t+1} - (1 + r_t)F_t}{F_{t+1}} \quad (86)$$

$$\implies L(S_{t+1}, D_{t+1}) = 1 + r_{s,t+1} - (1 + r_t) \frac{F_t}{F_{t+1}} \quad (87)$$

$$\implies L(S_{t+1}, D_{t+1}) = \underbrace{\left( r_{s,t+1} - r \frac{F_t}{F_{t+1}} \right)}_{\text{excess return from satellite ownership}} + \underbrace{\left( 1 - \frac{F_t}{F_{t+1}} \right)}_{\text{capital gains from changes in satellite launch cost}}. \quad (88)$$

If  $\pi_{t+1} > (1 + r)F_t$ , then the one period return on a satellite is greater than the gross return on the launch cost from the safe asset, and the collision probability will be 1. Ignoring that corner case, the equilibrium collision probability is decreasing in the current cost of launching a satellite, but increasing in the future cost of launching a satellite. All else equal, the collision probability in  $t + 1$  will be lower when  $F_{t+1}$  increases. This highlights the role of the launch cost under open access: if firms enter until zero profits in each period, future increases in the cost deter firms from entering in the future, increasing the value of satellites already in orbit by the amount of the cost increase. Alternately, the equilibrium collision risk can be decomposed into two components: one representing the excess returns of satellite ownership, and the other representing the capital gains from changes in the cost of launching a satellite under open access.

When the costs and returns are time-varying, the equilibrium set is still a collision probability isoquant, though the isoquant selected may vary over time. These changes do not affect the physical dynamics or the Kessler threshold, though they may affect how close the selected equilibrium is to the threshold. If the parameters vary so that the ratio  $\frac{\pi_{t+1} - (1+r)F_t}{F_{t+1}}$  is stationary, then the equilibrium set will stay on the same isoquant.

## B.4 Open access equilibrium between multiple shells

We have so far focused on a single orbital shell in isolation. While we can expand the “shell of interest” to cover a large region, in practice this is unlikely to yield accurate results over heterogeneous regions and use cases. Consider two orbital shells,  $H$  and  $L$ , with access costs  $F^H > F^L$  and equal returns  $\pi$ . What would open access imply for the collision risks between the shells? Letting  $H$  superscripts index the higher orbit and  $L$  superscripts index the lower



one, the new laws of motion are

$$\begin{aligned}
S_{t+1}^H &= S_t^H(1 - L^H(S_t^H, D_t^H)) + X_t^H \\
D_{t+1}^H &= D_t^H(1 - \delta^{HE} - \delta^{HL}) + G(S_t^H, D_t^H) + \mu_L X_t^H + \delta^{LH} D_t^L \\
S_{t+1}^L &= S_t^L(1 - L^L(S_t^L, D_t^L)) + X_t^L \\
D_{t+1}^L &= D_t^L(1 - \delta^{LE} - \delta^{LH}) + G(S_t^L, D_t^L) + \mu_L X_t^L + \delta^{HL} D_t^H
\end{aligned}$$

with the debris transport matrix

$$\mathbf{D} = \begin{matrix} & \begin{matrix} H & L & E \end{matrix} \\ \begin{matrix} H \\ L \\ E \end{matrix} & \left\| \begin{array}{ccc} 1 - \delta^{HL} - \delta^{HE} & \delta^{HL} & \delta^{HE} \\ \delta^{LH} & 1 - \delta^{LH} - \delta^{LE} & \delta^{LE} \\ 0 & 0 & 1 \end{array} \right\| \end{matrix},$$

where  $E$  represents the Earth. All transport coefficients are bounded in  $[0, 1]$  and sum to 1 across rows, with row labels indexing the source of debris and column labels indexing the destination.

The Bellman equations for the launch decision are now

$$Q_t^H = \pi + \beta(1 - L_t^H)Q_{t+1}^H \quad (89)$$

$$Q_t^L = \pi + \beta(1 - L_t^L)Q_{t+1}^L \quad (90)$$

$$V_{it} = \max_{x_{it} \in \{0, L, H\}} \{ \mathbb{1}(x_{it} = 0)\beta V_{t+1} + \mathbb{1}(x_{it} = H)[\beta Q_{t+1}^H - F^H] \quad (91)$$

$$+ \mathbb{1}(x_{it} = L)[\beta Q_{t+1}^L - F^L] \}, \quad (92)$$

where we suppress function arguments and use time subscripts for brevity (though these are still infinite-horizon Bellman equations). In an open-access equilibrium, the returns from owning a satellite in either shell should be zero, i.e.

$$(X_t^H, X_t^L) : V_{it} = 0 \implies \beta Q_{t+1}^H - F^H = 0, \beta Q_{t+1}^L - F^L = 0 \quad (93)$$

$$\implies \beta[\pi + (1 - L_{t+1}^H)F^H] - F^H = 0, \beta[\pi + (1 - L_{t+1}^L)F^L] - F^L = 0 \quad (94)$$

$$\implies \pi = \left( \frac{1 - \beta}{\beta} \right) F^H + L_{t+1}^H F^H, \pi = \left( \frac{1 - \beta}{\beta} \right) F^L + L_{t+1}^L F^L \quad (95)$$

$$\implies rF^H + L_{t+1}^H F^H = rF^L + L_{t+1}^L F^L \quad (96)$$

$$\implies \frac{F^L}{F^H} = \frac{r + L_{t+1}^H}{r + L_{t+1}^L} \quad (97)$$

In this case the cheaper orbit will have higher equilibrium collision risk, e.g. if the lower

orbit is cheaper to access then the higher orbit will have lower equilibrium collision risk:

$$F^L < F^H \implies L_{t+1}^H < L_{t+1}^L. \quad (98)$$

If the returns earned by satellites in either shell are not identical, but rather  $\pi^H$  and  $\pi^L$  with  $\pi^H = \pi^L + \gamma$ , then we have

$$\frac{F^L}{F^H} = \frac{r + L_{t+1}^H - \frac{\gamma}{F^H}}{r + L_{t+1}^L} \quad (99)$$

If the lower orbit is cheaper to access,  $F^L < F^H \implies L_{t+1}^H - \frac{\gamma}{F^H} < L_{t+1}^L$ . This gives us two possibilities:

- If the higher orbit generates greater returns, either orbit may be riskier in equilibrium depending on how big the return premium to the higher orbit is:  $\gamma > 0 \implies L_{t+1}^H \leq L_{t+1}^L$ .
- If the lower orbit generates greater returns, then the higher orbit will again have lower equilibrium collision risk:  $\gamma < 0 \implies L_{t+1}^H < L_{t+1}^L$ .

The physical and economic dynamics in the multi-shell setting are likely richer than those of the single-shell setting.