Econometrica, Vol. 29, No. 4 (October 1961)

THE THEORY AND MEASUREMENT OF PRIVATE AND SOCIAL COST OF HIGHWAY CONGESTION

BY A. A. WALTERS[1]

This paper attempts to apply the theory of marginal cost pricing to the services of the highways of the U.S.A. Empirical evidence suggests that "efficient prices" are generally much higher than present levels. This implies that gasoline taxes should be increased and special tolls charged in congested areas.

1. INTRODUCTION AND SUMMARY

THE MAIN purpose of this paper is to estimate an efficient system of taxation of highway transport. The data consist of figures of traffic flow and velocity collected by highway engineers. The engineers' results are translated into a form suitable for answering questions about the efficient allocation of vehicles.

For roads where traffic is much below capacity flow, the private unit cost curve for individual motorists rises as traffic flow increases. Marginal social cost is equal to private unit cost multiplied by (1 + elasticity of unit cost). For heavily congested highways—typically bottleneck situations—the increase in the number of entrants eventually increases the density to such high levels that traffic flow is reduced. High demand for road services has the effect of *reducing* traffic flow. In Section 3 we examine the case of a *network* of moderately congested highways. Equilibrium traffic flows are derived as functions of the parameters of the demand and unit-cost curves.

Section 4 discusses the applicability of marginal cost pricing on the highways. It is argued that, for efficient allocation, marginal cost pricing is a *minimum* level of charge in the majority of cases. Desirable traffic flow is always less than capacity flow, and the desirable number of vehicles is always less than the number required for capacity flow. For a network of highways, we present a formal solution for efficient tolls (or taxes) which equate marginal private cost to marginal social cost. To apply marginal cost pricing, we suggest a mixture of gasoline taxes, urban mileage taxes, and special tolls. The suggestion that urban motor taxation should be increased

[1] I have benefited from discussions with Professor Gilbert Walker, Mr. Ezra Bennathan and Dr. John Wise. I am especially indebted to Mr. W. M. Gorman who found an important error in an early draft. This paper is adapted from Discussion Paper B.1. (Dec., 1959) of the Faculty of Commerce, University of Birmingham. Further details of sources and methods are set out in that paper. Gillian Ridgway gave valuable assistance in collecting and appraising the data. A shortened version of this paper was read at the Seminar on Traffic Engineering (1960) organised by the Institute of Municipal Engineers.

is the opposite conclusion to that reached by Meyer, Peck, Stenason and Zwick [**16**]. We argue that their solution would be inefficient.

Section 5 reviews and analyses the empirical evidence. The estimation of parameters of demand functions is bedevilled by identification problems, and no useful results can be elicited from existing data. In estimating the elasticities of the private unit cost curve, there are few difficulties of identification. From studies of flows and velocities on good urban highways we conclude that only a very small proportion of urban traffic will enjoy a cost elasticity of less than 0.2. In order to make private cost approach social cost we suggest that a *minimum* fuel tax of 33 cents a gallon must be raised. This should be supplemented by mileage taxes levied by special mileometers in the more congested areas. The limited evidence reviewed here suggests that these should be in the region of 4–8 cents a mile for off-peak daytime traffic. For peak traffic we guess a figure of 10–15 cents a mile would be appropriate.

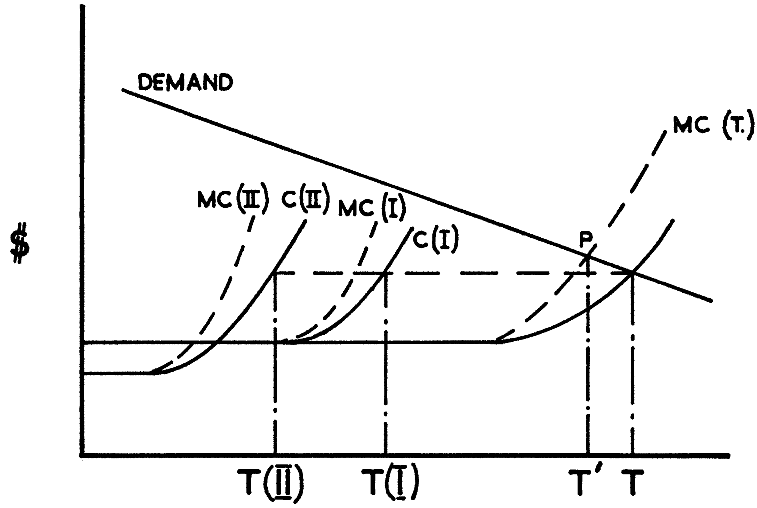
The magnitude and incidence of the taxes and tolls suggested here are very different from those entertained by both statesmen and transport economists. Thought and research on the highway problem should be redirected.

2. THE ANALYSIS FOR A SINGLE ROUTE

a. *The Classical Case*

The classical economists' account of highway congestion is to be found in the first edition of Pigou's *Wealth and Welfare* [**21**]. Pigou's case has been described in detail in Knight [**13**], Walters [**26**], and Beckman, McGuire, and Winsten [**1**], so we shall here present a slightly more general case. There is, however, one important assumption which we shall carry throughout this article. We shall assume that traffic is homogeneous. All vehicles are the same and all drivers and owners are equally bad (or good). We suppose that with a given volume of traffic, each vehicle will experience exactly the same costs, speed, etc. This assumption of homogeneity is "unrealistic," but it does simplify enormously the theoretical development and the empirical tests.

Consider two roads connecting A and B and assume that the unit (private) cost of a vehicle using road I is represented by C(I) in Figure 1. The unit cost is first horizontal but, as the number of vehicles increases, some congestion occurs and unit costs rise. Similar conditions are apparent on road II. The unit cost curve or vehicles using road II is lower than the unit cost on road I for low traffic volumes, but for higher traffic volumes the unit cost on road II rise above the cost on road I. The unit cost curve of road II is shown by C(II). If we assume that the routes are perfect substitutes for one another we can draw one demand curve for the services of the two roads. If each vehicle operator takes account of his private unit cost in making decisions and if there are a large number of operators on these routes, no operator will take account of the congestion costs caused to other vehicles. The unit cost curves C(I) and C(II) in Figure 1 then measure marginal *private*



Number of Vehicles an Hour

FIGURE 1

cost. Equilibrium traffic flow can be found by summing the unit cost curves horizontally to give $C(T)$. Then OT is the total traffic flow, OT(I) on highway I and OT(II) on highway II. If we take the total traffic as given at OT, as assumed in the diagrammatic presentation of Beckman, McGuire, and Winsten, [**1**, p. 84], the distribution of traffic between roads will be efficient only if at T(I) and T(II) the marginal *social* costs, represented by MC(I) and MC(II), are equal. In general, efficiency requires a reduction in total traffic from T to T' and the distribution between roads such that the marginal social costs are both equal to PT'.[2]

b. *The Bottleneck Case*

In the previous subsection we dealt with highways where we supposed traffic flow always increased in response to an increase in demand. This is clearly not the case with all highways. Consider, for example, a stretch of road

[2] I have presented this diagramatic treatment of the Pigou problem at some length because the exposition here seems to me to be both simpler and more general than that of Beckman *et al.*, p. 85.

2,000 ft. in length, consisting of a single one-way lane, and suppose that the rest of the highway has three one-way lanes. Clearly if the demand for the services of this road is so high that there are 100 vehicles on the narrow section, traffic will be almost at a standstill. Consequently the flow achieved with a very high demand may, in fact, be *lower* than the flow achieved with a low demand.

The time of the journey through the 2,000 ft. section depends on the *density* of vehicles. As density increases so the time of the journey increases. This is illustrated in Figure 2(b). At the maximum density $D_{\max}$, the time of a trip-mile approaches infinity. When density is very low, the time of a trip-mile approaches a positive constant. Let us suppose that unit cost is directly proportional to the time of trip, so that we can draw up a demand curve relating the number of entrants to the road during the hour to the time of the trip-mile, Figure 2(a). In order to abstract from complications connected with the dynamic adjustment of density, we shall also suppose that density is the same all the way along the road. Adjustments of density are supposed to be instantaneous. As a consequence of these assumptions we can write the flow of vehicles off the road (i.e., the number of exits an hour) as the product of the density and speed, or as the product of density and the reciprocal of the time of a trip-mile. The resulting value of flow is plotted on the horizontal axis against time of trip-mile on the vertical axis in Figure 2(c).

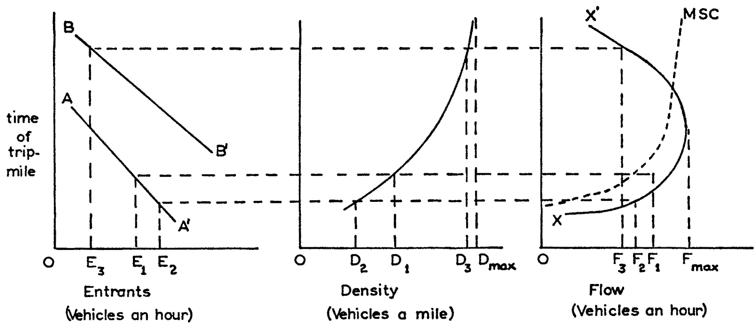


FIGURE 2(a) — Demand. FIGURE 2(b) — Density. FIGURE 2(c) — Flow.

Now we can put this apparatus to work. Suppose that the density of traffic in the bottleneck is D_1; then, if the demand is represented by the curve AA', the number of entrants will be OE_1. At the same time the outflow from the bottleneck is given (by arithmetic) at OF_1. Suppose that OF_1 exceeds OE_1 so that the density decreases. The time of trip also decreases and consequently the number of entrants increases. At low levels of the time of a mile trip, it is likely that the decrease in density more than offsets the increase in speed so that the flow is diminished. Thus density is reduced to D_2 and the number of entrants increased to E_2 and the flow of exits decreased to OF_2. This process of adjustment will then stop if $OE_2 = OF_2$, and these values represent an equilibrium flow of entrants and exits.

Consider now an increase in demand represented by BB'. Density will increase and eventually an equilibrium will be reached at, say, D_3 where $OE_3 = OF_3$. In this case the increase of density has reduced the speed so much that the flow was actually *decreased*. An increase in demand above BB' would result in a further decrease in flow. The relationship between flow and time of trip is shown by the curve XX' in Figure 2(c). We can interpret this curve as the equilibrium relation between flow and unit cost when density has been taken into account.

The level of equilibrium flow reaches a maximum at $F_{\max}$. This level should be called the "capacity" of the road. It is the maximum level of traffic that can be maintained over long periods; and $F_{\max}$ is associated with *one* demand curve. If demand increases, the level of flow falls below $F_{\max}$ and the unit cost rises. One interesting result is that when demand is higher than that consistent with $F_{\max}$, a suitable tax or toll will reduce the demand so that the same flow may be achieved at *reduced* unit cost (before tax).

The marginal social cost curve is shown by the broken line *MSC* in Figure 2(c). It rises above the unit cost curve as before; but when flow reaches $F_{\max}$, the marginal social cost is infinite. On the backward sloping part of the unit cost curve the marginal social cost is not defined since the change in output is *negative*. For our purposes we can suppose that the marginal social cost is infinite for those levels of flow which are associated with a unit cost higher than that at $F_{\max}$.[3] The theory of marginal cost pricing suggests that taxes be levied to reduce demand until traffic flow is at a level where private unit cost (with tax) is equal to marginal social cost.[4] Thus traffic flow should always be kept below capacity.

We can summarize this section by observing that the marginal social cost exceeds marginal private cost (i.e., average social cost). The relationship is:

$$\begin{aligned}\text{marginal social cost} &= \text{average cost} \times (1 + \text{elasticity of average social cost curve})\\ &= \text{marginal private cost} \times (1 + \text{elasticity of marginal private cost curve}).\end{aligned}$$

[3] The theory developed in this subsection is a hypothesis concerning *equilibrium* traffic flows. The flows that we can record on the curve in Figure 2(c) are the levels that can be maintained in the long run. It is also useful to develop a theory of non-equilibrium relationships that exist in the peak period. We can suppose that the density is not homogeneous throughout the bottleneck, and that density does not adjust rapidly as entrants increase. In the early part of peak periods the number of entrants exceeds the flow, and we get a backward sloping relationship between flow and unit cost similar to that derived for the equilibrium situation.

[4] These should be taxes on "motoring." In practice we have to be content with approximations, see Section 5.

The toll or tax which would bring marginal private cost up to the level of marginal social cost should be equal to the marginal private cost multiplied by the elasticity.[5] The toll should simultaneously satisfy the demand curve; so we can write:

$$P(X) = C(X) \times (1 + \text{elasticity of } C(X))$$

where $P(X)$ is the price at which the number of vehicles per hour is X, and $C(X)$ is unit private cost with X vehicles.

3. AN ANALYSIS OF A NETWORK OF HIGHWAYS

In the previous section we examined the case of one route in isolation. This approach is useful to analyse effects on that particular highway (bridge or tunnel) when one can ignore consequences on other roads because there are so many of them which each carry a small part of the diverted traffic, and when the private cost of diversion is much larger than that of transportation via the direct road. If, on the other hand, there are only a "few" roads to be considered and each carries a "substantial" part of the traffic, the model of Section 2 is inappropriate.[6] We have to devise another.

a. *Free Market Equilibrium in Non-Bottleneck Situations*

First consider the cost conditions. Write $x_i = \log X_i$, $c_i = \log C_i$, $p_i = \log P_i$, where X, C, and P are the quantity of traffic, unit cost of a vehicle journey, and the "price" of a vehicle journey. The subscripts refer to different highways. One simple hypothesis is that the cost on road i is related to the volume of traffic on road i:

$$c_i = d_i x_i + b_i \qquad (i = 1, \ldots, n)$$

where $d_i \geqslant 0$ and the unit cost is not influenced by the volume of traffic on other roads.[7] This does not take into account the backward sloping part of the unit cost curve, and so it can only apply to those roads that have a flow below capacity. On the demand side, we assume the form of demand curve used by Stone *et al* (1954). We restrict it by supposing that only the "prices" on roads in the network enter into the demand relation.

[5] The formula is

$$\frac{\partial K}{\partial X} = \frac{K}{X} + X\,\frac{\partial (K/X)}{\partial X} \qquad (K \text{ is } \textit{total} \text{ cost})$$

and is valid only when ∂x is nonnegative. Unit private cost $C(X)$ is then equal to $K(X)/X$.

[6] What is meant by "few" and "substantial" will vary with the problem in hand.

[7] This hypothesis has been contradicted by empirical evidence. On most roads it seems that a *linear* approximation is better than a *log* linear relation. Unfortunately a linear function would complicate the formal analysis. In view of the fact that there are no data available to test the results, we have chosen the simpler form for illustrative convenience.

Thus:

(2) $$x_i = a_{1i}p_1 + a_{2i}p_2 + \dots + a_{ni}p_n + g_i \qquad (i = 1, \dots, n) \; .$$

The elasticities are the constants a_{1i}, a_{2i}, etc. (In the analysis of a network in Beckman, McGuire, and Winsten, the "inverse" demand function shows price, p_i, as a single valued function of the *one* variable x_i. Each demand curve and supply curve is independent of values on other highways.[8] The Beckman model is much simpler than the one presented here and is much superior if it stands up to empirical tests.) In matrix notation:

(3) $$c = Dx + b$$

where c, x, and b are $n \times 1$ column vectors and D is an $n \times n$ matrix with $d_1, d_2, \dots, d_n$ in the leading diagonal and zeros for all off-diagonal elements. Also,

(4) $$x = Ap + g$$

where p and g are $n \times 1$ column vectors and A is the matrix of elasticities. The condition of private unit cost being equal to price, gives:

$$x = A(Dx + b) + g$$

or

(5) $$x = (I - AD)^{-1}(Ab + g)$$

as the equilibrium solution in terms of traffic flows. This requires that the matrix $(I - AD)$ be nonsingular.[9]

This hypothesis holds only where there are no important bottlenecks in the network. This suggests that its applicability is limited, and it would normally encourage a theorist to develop a model including bottlenecks. Such an extension is exceedingly difficult and no progress has been made here. We have already outstripped the empirical data, however, so this omission is not serious at this stage.

[8] The Beckman model distinguished between the "trip" and the "route." The "trip" is from the origin to the destination and this may be achieved by running over various roads, i.e., by various "routes." The demand function has the number of trips as the quantity variable whereas the cost function relates to the quantity of traffic on particular roads. I think this is an excellent way of organising the material. But I have not followed their lead in this paper partly because I can add nothing to their treatment in this respect and partly because it would introduce much complication into the notation.

[9] In the Beckman case, when A is also diagonal, this condition amounts to $a_{11}d_1 \neq 1$ and $a_{22}d_2 \neq 1$, etc. Since $d_1 \geqslant 0$ and, provided that road services are not an inferior good, $a_{11} \leqslant 0$, this condition will normally hold. In the case of $n = 2$ and when $a_{12} \neq 0$, the condition is

$$(a_{11}a_{22} - a_{12}a_{21})d_1 d_2 - a_{11}d_1 - a_{22}d_2 + 1 \neq 0 \; .$$

b. *Optimum Tolls or Taxes*

For a network of roads marginal cost pricing gives rather more complicated results than those we derived for a single highway. The Pigou condition is:

$$P_i = \text{marginal cost} = C_i + X_i \frac{dC}{dX}$$

which can be written as

$$P_i = C_i\left(1 + \frac{X_i}{C_i}\frac{dC_i}{dX_i}\right) = C_i(1 + d_i)$$

i.e.,

$$p_i = c_i + \log(1 + d_i)\ . \qquad (6)$$

Substituting (6) into (4),

$$x = A(c + l) + g$$

where l is a column vector of elements log $(1 + d_i)$. And, as before, we can get a solution vector x in terms of the parameters:

$$x = (I - AD)^{-1}(A(b + l) + g)\ .$$

If the toll is written as T_i ,then

$$T_i = C_i d_i$$

or, if $t_i = \log T_i$, then

$$t_i = c_i + \log d_i$$

so that

$$t = D(I - AD)^{-1}(A(b + 1) + g) + b + \log d \qquad (7)$$

where t and log d are column vectors. In principle all the values on the right hand side of (7) can be computed from observations.

As one might expect, it is impossible to apply this formula in practice because of the paucity of data. As we shall see in Section 5, the information required to estimate the A matrix is very difficult to collect. This is a pity because one suspects that this model could be usefully employed for a wide variety of practical problems.

4. WELFARE AND EFFICIENCY

a. *The Application of Marginal Cost Pricing*

The results of Sections 2 and 3 suggest that there is a good case for cutting down the volume of road traffic generally from that which would obtain in a free enterprise system with neutral taxation. On heavily congested roads, traffic should be reduced, while on the lightly travelled alternative routes, the flow should probably be increased. Some of these effects might be achieved by physical controls, by banning some kinds of vehicles on some roads, by suitably selected parking restrictions, etc. But it would be very difficult to implement measures of this kind in all cases, and they would have the undesirable effect of excluding from the road the man who would be willing to pay the additional (social) cost of his journey but who falls into the government's class of undesirable road users.

Marginal cost pricing is not so obviously as desirable a solution, however, as might appear from Pigou and Beckman *et al.* First considering road traffic generally, if the "price" were increased we would expect demand for its closest competitor to increase. Probably the closest competitor is rail transport (and air transport in the U.S.A.). Railroads are notorious for charging fares above marginal cost, except for peak periods on commuter services, when they may be below. This would suggest that, if we take the railroad pricing policy as given, we should try to distribute taxation and tolls on highways such that "prices" were generally above marginal cost. For efficient allocation, marginal cost "pricing" is probably a *minimum* level of charge in the majority of cases.

b. *Administration*

The particular administrative instruments which might be used include, in order of administrative complexity, special mileometers (similar to those used by taxis), tolls, and taxation. The special mileometers would record mileage when the "flag" is up and a charge would be levied on this mileage.[10] These would be suitable for large urban areas such as New York, London, Chicago, etc. "Flag-up" streets could be specified for certain hours of the day. Vehicles might be allowed to travel on those streets without a special mileometer provided that they bought and displayed a daily "sticker." The occasional traffic on "sticker" authority would have to be charged more than the maximum amount paid by those on mileometer authority. The supervision of the scheme would be fairly simple. Cameras could be set up to record those cars without sticker or flag, and a suitable fine could be levied for contravention.

Tolls are most suitable for particular streets, tunnels, or bridges which are very congested, and where there is a large difference between private and marginal social cost. Tolls should be restricted to these cases because they are expensive to collect over more complicated networks and because toll collection causes delays.

Since the general level of congestion in urban areas is greater than that

[10] The cost of the meters would be a serious drawback if this were applied on a very limited scale. With suitably large application, however, the cost of a mileometer could probably be reduced to about $10.

in rural areas, we might take this into account by fixing higher motor taxes for urban areas—and in particular for the very large metropolitan areas of New York, Chicago, and London. Taxes should be arranged to have their incidence on the mileage run in the urban areas. Obviously this can only be done as a rough and ready approximation. Fuel taxes are probably the most useful form of deterrent. Gasoline duties should be high in the urban areas and low in rural areas. But there is a limit to this differential since there is a danger that cheap rural fuel may give rise to "fuel-fetching" journeys. Local licence fees are rather less efficient than fuel taxes since they affect only *ownership* in the various areas. It may be useful, however, as a supplementary form of tax differential.

c. *Alternative Proposals*

The pattern of taxes suggested here is almost precisely the opposite of that proposed by Meyer, Peck, Stenason, and Zwick [**16**]. They suggest that taxes on urban roads should be reduced.[11] I believe that their analysis is a mistaken application of the marginal cost principle and that the conclusions they reach are both wrong and harmful. Marginal cost pricing is designed to make the best use of an *existing* network of highways. The costs of providing the network are irrelevant for optimal utilization.[12] But it is precisely these costs, together with maintenance costs, which are thought to be relevant to "marginal" cost pricing (see Meyer *et al.*, p. 70–73). The arithmetic of Meyer *et al.* is concerned with allocating "overhead" costs between property owners and highway users and between trucks and passenger cars.[13] (There is one element of marginal cost entering into these calculations—the additional cost of repairing the damage to the surface of the highway caused by a vehicle journey. The evidence suggests that this element is so small compared

[11] *Op. cit.*, p. 269: "If the principle of marginal cost fare-setting is adopted as a policy, it should be applied to all public transport facilities. This would mean that highway user taxes on most urban roads would have to drop substantially...." There is a suggestion, but a very faint one, of the policy advocated in this section on p. 72 where, because of the "possible disutility to passenger... highway users created by heavy truck traffic" they suggest charging trucks very high fees at rush hours or seasonal peaks. This seems to me to be a superficial tribute to the congestion problem and marginal cost pricing. If trucks, why not cars?

[12] High "efficiency tariffs" on congested roads are evidence of the need for new investment. The optimum investment programme, of course, depends on the levels of the "efficiency prices." This problem is not pursued in this paper. (See Walters, A. A., "The Consumer Surplus Criterion for Investment in Roads" (mimeographed), Discussion Paper B.2., Birmingham.)

[13] On p. 75 we read, "The portion of highway cost attributable to truck traffic will vary with different circumstances. In the first place, cost will depend upon the number and value of special structures... that must be constructed."

with marginal social costs that I have ignored it completely throughout this paper.[14]

The allocations of cost carried out in the Harvard study (Meyer *et al.*) have almost no relevance to marginal cost pricing at all. And in general they give the opposite results; on congested roads, for example, the argument would conclude that there are more vehicles to "bear the cost" so the tax should be lower than that on similar uncongested roads. It seems to me that the proper application of marginal cost pricing, as outlined above, is practically unknown among highway economists,[15] and much needs to be done to reorient research in the right direction.

Problems of the distribution of income—who would and who would not be harmed by the policy advocated—will not be considered here. The general ramifications of such a policy are reasonably clear, but the detailed analysis would be cumbersome and boring.

5. EMPIRICAL EVIDENCE

a. *Identification and the Demand Curve*

The theoretical discussion of Sections 2 and 3 and the normative proposals of Section 4 require empirical content. This involves estimating the parameters of demand and cost relations. As might be expected, the cost function is much easier to estimate than the demand function. This is partly due to the simple form of the cost function—cost is determined only by the quantity of traffic on that road. But there is, I think, another less obvious reason. The typical situation is one in which the demand curve varies, continuously almost, over time of day and periods of the year, while the cost curve remains where it is. Only occasionally is there any substantial change in cost conditions, such as when a new road is built or an existing road considerably widened. That is to say, the data available enable one to identify the cost curve, but the demand curve (or, rather, curves) remain unidentified. The demand parameters are, however, very important for estimating the effects of road improvement as well as for the more menial task of fixing efficiency tolls.

The gap left by the highway economist has been filled in by the highway engineer, but not in a satisfactory way. The engineer considers a situation where one road has been improved (suppose it has been widened) and then tries to estimate the *proportion* of existing traffic that will be diverted to the

[14] See Highway Research Board Reports 4 and 22. Much of the large sum being spent on research on highway economics in the U.S.A. is at present being devoted to this kind of problem. See Highway Revenue Act of 1956, Hearings 9075, pp. 221–409. It seems to be a very serious misallocation of resources.

[15] With the very important exception of Beckman, MacGuire, and Winsten [**1**].

new highway from other roads. He relates the proportion of traffic carried on the new road to the time ratio, i.e., the ratio of time on the new highway to time on the old route, or, less frequently, to the time difference. Points are plotted for different origins and destinations, and the resulting curve is called the "assignment curve."[16] Unfortunately there is no way of eliciting the characteristics of the demand curves from the engineers' results.[17]

b. *Cost Curves (excluding "bottlenecks")*

In all the empirical studies of road congestion the variables recorded were speed or time of journey and volume of traffic. Cost was not measured. We shall assume, however, that, for the range of speeds considered here, *unit cost is inversely proportional to speed.* Where time of journey is measured, our assumption is that unit cost is directly proportional to time of journey.[18]

The main bases for the estimates of elasticities of cost on good urban highways are data reported by Carmichael and Haley [**4**] and Rothrock [**23**]. Carmichael and Haley investigated the flow on two 36–40 ft. one-way highways with no obstruction and no parking. The number of vehicles per hour varied between 430 and 2,040 and speeds varied between 27 and 19.3 m.p.h.

[16] The typical form of this curve is logistic. See Highway Research Board Bulletins 61 and 130 [**10**].

[17] In Discussion Paper B.1 (see footnote 1), we show that the engineer's logistic implies certain restrictions on the elasticities and cross elasticities of demand. If this restriction is justified, the results suggest that cross elasticities of competing roads are low (i.e., about 3). This is not very good evidence, but it does cast *some* doubt on the assumption used in Section 2 above and in Beckman *et al.*

[18] For the fragments of evidence that tend to support this hypothesis, see Gibbons and Procter [**6**]. In their Table 3, fuel consumption in heavily congested business areas is 2.6 times the consumption on expressways (p. 11 of Lawton [**14**]). This test was carried out in 1942 in Southern California; congestion of cities has increased considerably since those days. Probably the most detailed study of fuel consumption was carried out by Bone [**3**]. In Figure 4 on p. 447 he suggests that there is a curvilinear relation between speed and fuel consumption—with gasoline consumption increasing at a decreasing rate up to about 30 m.p.h. Between 5 m.p.h. and 30 m.p.h. an approximation with a straight line gives:

$$\text{miles/gallon} = .5 \times \text{speed} + 4.1 \; .$$

This differs from proportionality by the constant factor 4.1. For dense city streets and cross city traffic, the approach to proportionality is much closer. The average time costs in stop-and-go driving on concrete pavements was more than three times the cost of travelling the open road (Moyer and Tesdall [**18**]). We have not included insurance premiums (and the cost in terms of the probability of an accident) in the estimate of running costs, although it clearly does vary with congestion and with speeds. See pp. 14, 15, 16 of Gibbons and Procter [**6**]. None of the empirical studies referred to provides powerful evidence to reject the hypothesis that costs are inversely proportional to speed for ranges of speed between 5 m.p.h. and 30 m.p.h. Because it is a very simple hypothesis we shall adopt it throughout this paper.

The Rothrock data were for two streets in Charleston, West Virginia during the war year 1944. Rothrock reports the percentage of total daily traffic passing during each hour and the mean length of time of the trip. The percentage varied between 0.6 and 7.8 while speed varied between 25 m.p.h. and 15.6 m.p.h.

The relationship between speed and volume in Carmichael and Haley and between time of trip and percentage of daily total traffic in Rothrock was not obviously curvilinear. Consequently a linear regression was calculated with the results shown in Table I. The fact that the confidence intervals of the regression are relatively narrow with such small samples is some indication

TABLE I

	No. of observations	Regression	Confidence interval (95%) for regression coefficient*
Carmichael and Haley	14	Speed (m.p.h.) on no. of vehicles	–.00529, –.00399
Rothrock (1) Washington St.	24	Time (minutes) on per cent traffic	.770, 1.084
Rothrock (2) Kanawha Blvd.	24	Time (minutes) on per cent traffic	.685, 1.043

* These are calculated as $\beta \pm t_{.05}\hat{\sigma}_{\hat{\beta}}$ where $\hat{\sigma}_{\hat{\beta}}$ is an estimate of the standard error of the regression coefficient, and $t_{.05}$, with 12 or 22 degrees of freedom, is derived from tables of the t function.

that comparatively few observations are sufficient to get an accurate estimate of the speed-volume regression on a *particular* road. One suspects, however, that non-sampling errors are rather more important than sampling errors.

The estimates of the regression coefficient ($\hat{\beta}$) and of the constant ($\hat{\alpha}$) were used to estimate the elasticities of speed and of time with respect to the two measures of volume. If the estimate of the regression equation were

$$\hat{y} = \hat{\beta}x + \hat{\alpha}$$

then the estimate of the elasticity (L) at a particular value of x, say x_0, is

$$L(x_0) = \frac{x_0}{\hat{y}_{x_0}} \left(\frac{d\hat{y}}{dx}\right)_{x=x_0}$$

where $\hat{y}_{x_0} = \hat{\beta}x_0 + \hat{\alpha}$. Therefore,

$$\hat{L}(x_0) = \frac{\hat{\beta}x_0}{\hat{\beta}x_0 + \hat{\alpha}}.$$

Elasticities evaluated at selected levels of traffic are shown in Table II.

TABLE II

Carmichael and Haley	$x_0 = 450$	$x_0 = \bar{x} = 1047$	$x_0 = 1200$
	.08	.18	.21
	$x_0 = 2\%$	$x_0 = \bar{x} = 4.17\%$	$x_0 = 5\%$
Rothrock (1)	.14	.26	.29
(2)	.21	.32	.36

These estimates of elasticities are biassed and those for low traffic flow are biassed upwards. But it is easy to show that the relative bias is small in relation to the relative standard error, and that it is unlikely to disturb any of the conclusions.[19]

For the Rothrock data, a value of 2% for X_0 (i.e., 2% of total daily traffic during that particular hour), is less than the hourly percentage of traffic travelling between 6 A.M. and 1 A.M. Only for the night hours, 1 A.M. to 6 A.M., does the percentage per hour fall below 2%; and during these hours only 4.0 and 4.1 per cent of the total traffic travelled on Rothrock (1) and (2) respectively.[20]

The road examined by Carmichael and Haley represented only the very

[19] With a linear regression of average cost on traffic volume, we get estimates $\hat{\beta}$ of β and $\hat{\alpha}$ of α from the regression equation

$$y_t = \beta x_t + \alpha + \varepsilon_t$$

where ε_t is a random variable with expectation zero and distributed normally with the variance σ^2.

The estimate of the elasticity of the average cost is found from

$$\hat{L}(x) = \left.\frac{\hat{\beta}x}{\hat{y}}\right|_{x=x_0} = \frac{\hat{\beta}x_0}{\hat{\beta}x_0 + \hat{\alpha}},$$

and this is a ratio of random variables, which gives rise to biassed estimates. In Discussion Paper B.1 we show that the bias is trivial and that the relative standard error does not exceed 3.4%.

There is another bias arising from the fact that most of the values of x were read from charts. So even if there were no errors of observation in the original estimates of x, there must be transcription errors. These are probably small enough to be insignificant in the case of the Charleston data but not for the data of Carmichael and Haley. This factor would introduce an upward bias into the estimate of β itself.

[20] It will be recalled that the Rothrock data applied to the war year 1944. Since that date traffic has expanded more rapidly than the capacity of the roads, so it is reasonable to suppose that if we calculated elasticities for 1960 they would be much higher.

best conditions of urban traffic flow. The elasticities are considerably lower than the values obtained from Rothrock's data. But a precise comparison is impossible because of the differences in the data.[21] As a rough guide, one can say that the elasticity for average traffic flow on the best urban roads corresponded to the elasticity during the very low traffic periods on good urban roads.[22]

c. *General Urban Fuel Tax*

There is, I think, sufficient evidence to suggest that only a very small proportion of urban traffic in the U.S.A. will enjoy traffic conditions which are associated with a cost elasticity of less than 0.2. From what evidence we have, I would guess that only about 5% of total urban traffic will experience these highly favourable conditions. The elasticity of 0.2 is a reasonable basis for estimating the level of a general urban fuel tax.

We require estimates of the running cost of vehicle miles. This varies greatly from passenger cars to trucks. In this paper only passenger cars are examined. Suitable adjustments can be made for trucks in a more comprehensive study. The following table is an attempt to put the figures of cost per mile of May and Michael [**15**] and Lawton [**14**] into 1959 prices and technological conditions. Fuel (gasoline) has been priced before tax at 20 cents a gallon. Again emphasizing our conservative assumptions we have valued time at $2 per hour per car, which, at 30 m.p.h. gives a value of 7 cents a mile. Since the average car occupancy in urban areas appears to be about 2 (p. 105 of "Expressway Influence on Parellel Routes," Cook County Highway Department), this supposes a value of time of about one dollar an hour—a very low figure indeed for the vast majority of car owners.[23]

[21] One suspects that the test-car method used by Carmichael and Haley may have given rise to bias towards mediocre speeds, and so reduced the regression coefficient. No evidence is available on this point.

[22] There is other information—not given in a form suitable for calculating regression coefficients and elasticities—which suggests that these values are not atypical of good urban highways, e.g., Norman [**20**], Highway Capacity Manual [**10**], and Berry [**2**]. These studies and others are reviewed in Appendix I of Discussion Paper B.1, see footnote 1.

[23] Some independent evidence on the value of time to road users could be derived from statistics of the use of tollways. Lawton [**14**], for 1947 data, found that the implied valuation was $2.28 an hour for a passenger car. Using 1940 data at 1947 prices, Lawton found another value at $1.11 an hour. These values are averages of the prices actually paid for time. Owing to the discontinuities in journey purchases we cannot be sure that the average of marginal valuations of time is higher than this value; but I should have thought that it was likely. Glaze's and Van Mieghan's [**7**] results are not markedly inconsistent with those in this table.

TABLE III
RUNNING COST PER MILE AT 30 M.P.H., 1959*

Gasoline (assumed 15 miles per gallon)	1.40¢
Oil	.21
Tyres, etc.	.35
Maintenance	1.00
Depreciation (user)	1.12
Total operating	4.08¢
Time	7.00
Total cost per mile	11.08¢

* Excluded from this estimate of running costs are licence and registration fees, insurance premiums and garage rent. Only part of depreciation and part of maintenance costs have been included. Probably gasoline expenditure is overestimated here since consumption is likely to be better than 15 mpg. This figure will give us a conservative estimate of the fuel tax effect.

Now we can compute the minimum value of the optimum gasoline tax, as a first approximation at least, from

$$T_t = C_t d_t = 11.08 \times .2 \, ,$$

finding that the desirable gasoline tax is 2.2 cents per mile. This value does not take account of the effect on unit cost that such a tax would bring about *via* a reduction in congestion. The optimum tax should be rather lower than this. But we have selected a very high speed (30 m.p.h.) for urban traffic, and it is unlikely if, after the reduction in congestion occasioned by the tax increase, very much urban traffic achieves an average speed as high as 30 m.p.h. In any case, with a tax per gallon of fuel, there is bound to be some tendency to switch to cars with a higher mileage per gallon. This will tend to offset some of the reduction in congestion.

The tax per gallon of 33 cents seems very high indeed compared with state and federal taxes of about nine or ten cents, but it is roughly comparable to the level of taxation in many European countries. In practice the main objections to this level of taxation are the administrative difficulties of maintaining a level of 33 cents a gallon in urban areas while leaving it at 10 cents or less in rural areas. I should have thought that it would be possible to hold this differential in the large urban areas such as New York, Philadelphia, Chicago, and Los Angeles. For the vast majority of the population in these areas, the distance from any rural area, where the elasticity is very low, is usually great enough to prevent gasoline "poaching." For the smaller urban areas surrounded by rural highways with low elasticities the tax differential would probably have to be lower. On the other hand, the motorist who undertakes a long cross-country journey will, of course, be able to buy gasoline at the low rate of tax. This is desirable since most of his mileage will be on (uncongested) tollways or on the freeways between urban areas.

There is no excuse for charging a toll on the vast majority of existing toll roads in the U.S.A. since there is little or no congestion. At existing traffic levels the elasticity is, in most cases, zero. (The only important exception is the New Jersey turnpike at certain times.) Even without the deterrent of tolls, it seems to me doubtful whether the traffic volume would be sufficient to produce anything more than a very low elasticity.

For truck traffic it is desirable to charge special mileage taxes. This can be done through the existing administrative organisations such as the Interstate Commerce Commission. Trucks should not be exempt from the gasoline tax but they usually cause much more social cost than is likely to be reflected in the gasoline tax. These additional congestion costs could be collected by the I.C.C.

d. *Cost on Very Congested Roads and Bottlenecks*

We now turn to consider the more congested roads, where density is so great that flow decreases as demand increases. Clearly optimum taxes or tolls will depend to a very large extent upon the elasticity of demand at peak periods. We know very little about the demand curve, however. All we can do is to trace out the cost curve and give some indication of the minimum speed which should be attained. The data which we use here are derived from three studies, one by Rothrock and Keefer [**22**] for Charleston, and two by Greenberg [**8**] for the Merritt Parkway and the Lincoln Tunnel.

Rothrock and Keefer [**22**] collected data for an urban street in Charleston, West Virginia. They recorded the number of entrants and the average trip time for various intervals of time. In Figure 4, we show the results for the 12 minute interval.[24] The curve has been fitted by free hand and we have calculated the elasticities (shown as ringed figures alongside the appropriate points). This suggests that the elasticity for the section of the cost curve rising from left to right is rather less than unity until there are about 110 vehicles. After that the elasticity rises very rapidly and becomes infinite at the capacity point of the road—in this case about 120 vehicles (i.e., 10 vehicles a minute).

To find the optimum traffic flow we require the elasticity of demand. Unfortunately, there are no data available even to guess its value. We cannot work out the ideal toll for the peak period, but we can suggest a toll per mile for traffic from 70 to 110 vehicles and this will give us a basis for making conjectures about the peak period toll. Using the estimates in Table 3 we can estimate the cost per mile (8 minutes in the figure) at about 19 cents

[24] These data are not quite suitable for our purpose since they recorded entrants and not flow. But the length of the test road was so short that the numerical divergence between flow and entrants could only be very small.

per mile when speed is 17 m.p.h. Assuming that the gasoline tax still applies and that the elasticity is 0.7 (i.e., rather lower than those appearing on the curve in Figure 4), this would give a toll of 10 cents a mile. Allowing for an assumed elasticity of demand of unity, the toll should be about $8\frac{1}{2}$ cents a mile. The toll during peak periods should obviously be much higher than this. I would guess that a toll of 15 cents a mile is about right. This toll should cut down traffic to somewhere in the region of 110–120 vehicles and speed up the flow to 12–14 m.p.h. This charge could best be levied by the special mileometer system.

Let us now examine the New York data. In the Lincoln Tunnel, a Simplex Productograph machine was set up in the North tube alongside a "short" length of roadway. Two observers were used to record the time at which vehicles passed each observer, one at the entrance and one at the exit to the zone. The data were then separated into speed classes and the average headway was calculated for each speed class. (Greenberg [**8**].) For the Merritt Parkway, five-minute time profiles were used, and average velocity and average headway were calculated.[25] In addition to flow and speed, Greenberg [**8**] and Huber [**12**] also recorded density.[26]

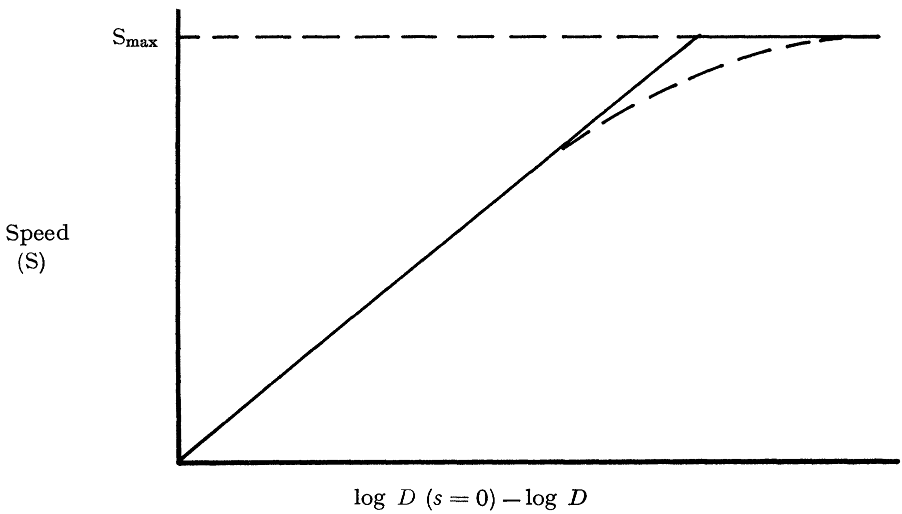


FIGURE 3

[25] Headway is the distance between the front of one vehicle and the front of the next vehicle.

[26] Again it is possible that density in the tunnel was increased by the inability to clear vehicles once they had got through the tunnel. I should not have thought that there was any strong evidence for this explanation with regard to the Lincoln Tunnel, and it is most unlikely to be the case on the Merritt Parkway.

Following the lead of Greenberg we can develop a function derived from the theory of fluid dynamics:

$$s = \beta\,(\log D(s=0) - \log D)$$

where s is speed and D is density and $D(s=0)$ is the density when traffic comes to a complete stop. This relationship is, of course, only valid (or likely to be empirically useful) for a low range of speed, as illustrated in Figure 3. Speed increases as density decreases. But as density becomes very low, speed does not continue to increase. We can suppose that there is a maximum speed ($s_{\max}$) of free running vehicles (or laid down by the law). In practice the linear function holds only for high densities and we should expect the speed to approach $s_{\max}$ quite slowly, as indicated by the broken line in the figure.

If x be the flow, we can write the flow as a product of the density and speed,

$$x = Ds\,.$$

Substituting in the linear relation

$$s = \text{constant} - \beta(\log x - \log s)\,.$$

Therefore,

$$\frac{x}{s}\frac{ds}{dx} = \frac{\beta}{\beta - s} = \text{elasticity of speed.} \tag{8}$$

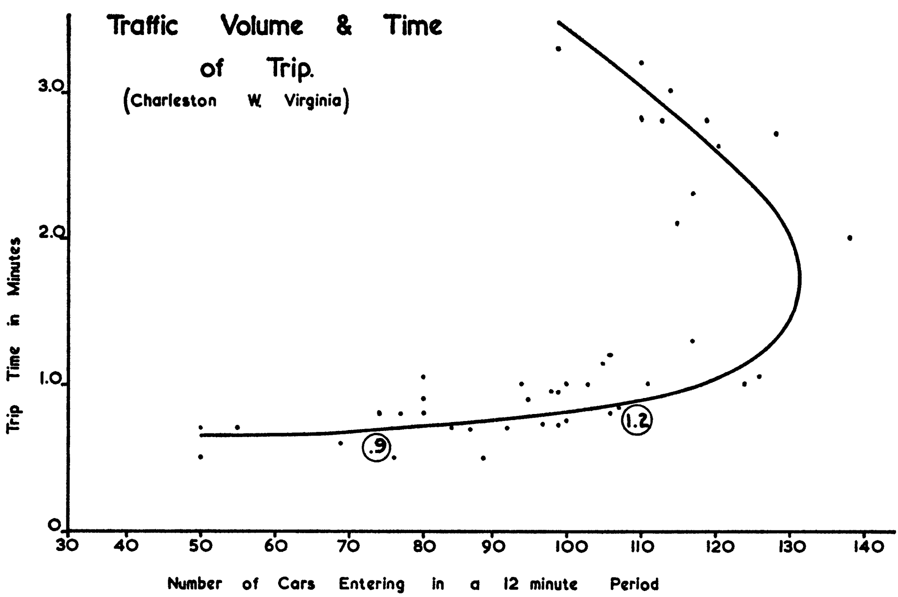


FIGURE 4

This clearly takes negative values when $s > \beta$. These speeds are above the capacity speeds of the road. Capacity speed is simply the constant β. For speeds lower than β, the elasticity is taken conventionally at infinity.

An estimate of β ($\hat{\beta}$) can be obtained by calculating the regression

$$s = \beta(\log_e h - \log_e \alpha) \tag{9}$$

where s is the speed, h the headway and α a constant. The estimate $\hat{\beta}$ can then be inserted into equation (8) to get an estimate of the elasticity of the unit cost curve. Unfortunately the data collected for the Lincoln Tunnel are strictly not suitable for calculating an estimate of (β). The headway figures were averaged for *given* intervals of speed. In equation (9) we require the average speed for given intervals of headway. The least squares estimates of β computed from the data may therefore be both biassed and not consistent. One would suspect that any fault in the estimates arising from these considerations would be so small that it would not disturb the conclusions drawn. The estimates of β are presented in Table IV. We have also inserted an approximate value of β for the Rothrock and Keefer data. This was simply interpolated from the graph, and so little reliance can be placed on this figure. On the other hand, the narrow confidence intervals for β on both the Merritt Parkway and in the Lincoln Tunnel are encouraging.

TABLE IV

ESTIMATES OF SPEED AT MAXIMUM FLOW

Facility	$\hat{\beta}$ (m.p.h.)	95% Confidence interval	Estimate of correlation between speed & headway
Lincoln Tunnel	17.00	16.07, 17.94	.995
Merritt Parkway	15.77	15.13, 16.41	.996
Street in Charleston	8	— —	—

For traffic flows less than the capacity level and for speeds greater than β, we can evaluate the optimum toll per mile for any given speed or the speed associated with any given toll. The cost C_j is assumed to be inversely proportional to the speed s, i.e.,

$$C_j = k/s_j$$

where k is a constant. The optimum toll T_j for a given speed s_j is given by

$$T_j = C_j \left(\frac{\beta}{s_j - \beta}\right) = \frac{k}{s_j} \left(\frac{\beta}{s_j - \beta}\right), \tag{10}$$

and $k = 11.08 \times 30 = 333$. Using the estimates of β, we can estimate the optimum toll for any given speed, as shown in Table V.[27]

TABLE V

Toll per Passenger Car per Mile

Speed (m.p.h.)	Lincoln Tunnel (cents per mile)	Merritt Parkway (cents per mile)
35	9.0	7.8
30	14.4	12.3
25	28.3	22.8
20	94.5	62.0

The tolls at present charged for these facilities are 50 cents for the Lincoln Tunnel and 40 cents for the Merritt Parkway. In terms of distance they amount to 35.3 and 1.07 cents a mile respectively. This toll for the Lincoln Tunnel is efficient only when traffic is travelling at about 23 m.p.h. The charge for the Merritt Parkway, on the other hand, is far below the efficient toll, and any calculations based on equation (10) would be meaningless.[28] The tolls in Table V suppose that the demand side of the relationship has been taken into account. In order to estimate tolls for *existing* traffic flows, we have to know the elasticity of demand. And clearly we should also take into account congestion and demand on alternative routes, that is to say, we should collect data to apply the more ambitious model in Section 3. Unfortunately there are no data available. Only two conclusions can be risked at this stage: (a) the toll on the Merritt Parkway and on "alternative routes" is clearly much too low, and (b) the toll on the Lincoln Tunnel is too high for low traffic volumes, but for the majority of traffic it is too low.

[27] Confidence intervals for these tolls have not been computed, since the elasticity of the cost curve, on which the estimates of tolls are based, is a ratio of random variables. But this can be simplified so that the estimate of the relative variance of the elasticity estimate is

$$\sigma^2_{\hat\beta} \frac{s}{\hat\beta(s-\hat\beta)}.$$

As one would expect, the relative standard error increases rapidly as $s \to \beta$. Similarly, we can show that an estimate of the relative bias in these estimates is

$$\sigma^2_{\hat\beta} \frac{2\hat\beta - s}{\hat\beta(s-\hat\beta)^2}.$$

And this is zero only when $s = 2\beta$ (e.g., in the case of the Merritt Parkway when $s = 34$ m.p.h.). Again as $s \to \beta$ the relative bias increases rapidly.

[28] If the general urban gasoline tax were adopted, these tolls should be reduced by the amount of this tax.

When we consider optimum tolls for periods of peak traffic, it is clear that the elasticity of demand is a key parameter and one can say very little without this knowledge. I should suggest that a general toll of about 14 cents a mile on all roads similar to the Merritt Parkway might be an efficient toll during the peak period. But this is merely a guess and little reliance should be placed on it. As for the Lincoln Tunnel, one would suggest that the various competing facilities for crossing the river should be the subject of an experiment in the use of tolls. The first aim should be to reduce the number of entrants so that maximum flow is achieved. As a second stage the authority might attempt to fix tolls consistent with our theory.

e. *Conclusions on Empirical Evidence*

From the evidence surveyed in this section, one may conclude that there is a good case for much higher gasoline taxes in urban areas of the U.S.A. The precise figure of 33 cents should perhaps not be taken too seriously, but it does indicate the order of magnitude of the ideal tax. We have indeed argued that 33 cents is something like the lower limit, that is to say, the tax in the most favourable traffic circumstances. Similarly, the tax or toll per mile for congested urban streets must be taken as indicating the order of magnitude rather than as a precise estimate. All these taxes and tolls are very large relative to existing levels. More data on flows and speeds must be collected before any more precise estimates of social costs can be attempted. I believe that these data are much more important than information about damages to road surfaces, etc., which is at present being collected by various research institutions.

Generally, urban car users will be injured by these arrangements, and the reduction in the density of traffic will have various other repercussions on property owners—especially filling stations—and on the automobile industry. It would be both difficult and tedious to try, at this stage, to trace all the reactions on the various classes of the community. There could be some simple built-in compensation in the scheme by turning over the taxes collected in urban areas to the local authorities to spend as they think fit. It is likely that local electorates would require governments to spend a large proportion of these sums on road improvements. But these reflections are outside the scope of this paper.

University of Birmingham

REFERENCES

[1] Beckman, M., C. B. McGuire, and C. B. Winsten: *Studies in the Economics of Transportation*, Cowles Commission, Yale, 1955.

[2] Berry, D. S.: "Evaluation of Techniques for Determining Over-all Travel Time," *Highway Research Board Proceedings*, 1952, pp. 429–440.

[3] Bone, A. J.: "Travel-Time and Gasoline Consumption Studies in Boston," *Highway Research Board Proceedings*, 1952, pp. 440–456.

[4] Carmichael, T. J., and C. E. Haley: "A Study of Vehicle, Roadway and Traffic Relationships by Means of Statistical Instruments," *Highway Research Board Proceedings*, 1950, pp. 282–296.

[5] Forbes, T. W.: "Speed, Headway and Volume Relationships on a Freeway," *Proceedings of the Institute of Traffic Engineers*, Institute of Transportation and Traffic Engineering, California, 1952. Reprint no. 13.

[6] Gibbons, J. W., and A. Procter: "Economic Costs of Traffic Congestion," *Highway Research Board Bulletin 86*, 1954.

[7] Glaze, C. K., and G. van Mieghem: "Washington Motor Vehicle Operating Cost Survey," *Highway Research Board Proceedings*, 1957, pp. 51–60.

[8] Greenberg, H.: "An Analysis of Traffic Flow," *Operations Research*, 1959, pp. 79–85.

[9] Grime, G.: "Traffic and Road Safety Research at the Road Research Laboratory, England," *Highway Research Board Proceedings*, 1952, pp. 466–486.

[10] Highway Research Board: *Highway Capacity Manual*, Highway Research Board, U.S. Government Printing Office, Washington, D.C., 1950.

[11] Hooper, C. J.: "The Need for Further Research on Traffic Assignment," *Highway Research Board Bulletin 61*, 1952, pp. 66–70.

[12] Huber, M. J.: "Effect of a Temporary Bridge on Parkway Performance," *Highway Research Board Bulletin 167*, 1957, pp. 63–74.

[13] Knight, F. H.: "Some Fallacies in the Interpretation of Social Cost," *Quarterly Journal of Economics*, 38, 1924, pp. 582–606.

[14] Lawton, L.: "Evaluating Highway Improvements on Mileage and Time Cost Basis," *Traffic Quarterly*, 1950, pp. 102–125.

[15] May, A. D. (Jr.), and H. L. Michael: "The Allocation of Traffic to Bypasses," *Highway Research Board Bulletin 61*, 1952, pp. 38–58.

[16] Meyer, J. R., M. J. Peck, J. Stenason, and C. Zwick: *The Economics of Competition in the Transportation Industries*, Harvard, 1959.

[17] Moskowitz, K.: "Waiting for a Gap in Traffic Stream," *Highway Research Board Proceedings*, 1954.

[18] Moyer, R. A., and G. L. Tesdall: "Tire Wear and Cost on Selected Roadway Surfaces," Iowa Engineering Experiment Station, *Bulletin No. 161*, 1945.

[19] Murray, F. J.: "Comparative Traffic Usage of Olentangy River Road and Alternative City Arterials in Columbus, Ohio," *Highway Research Board Bulletin 61*, 1952, pp. 8–17.

[20] Norman, O. K.: "Preliminary Results of Highway Capacity Studies," *Public Roads*, 1939.

[21] Pigou, A. C.: *Wealth and Welfare*, 1st edition, 1920.

[22] Rothrock, C. A., and L. E. Keefer: "Traffic Speed and Volume Measurements," *Highway Research Board Bulletin 156*, 1956, pp. 1–13.

[23] Rothrock, C. A.: "Urban Congestion Index Principles," *Highway Research Board Bulletin 86*, 1954, pp. 26–39.

[24] STONE, R., *et al.*: *The Measurement of Consumer Expenditure and Behaviour in the United Kingdom 1920–38*, Cambridge, 1954.
[25] STRICKLAND, R. I.: "Traffic Operation at Vehicle Tunnels," *Highway Research Board Proceedings*, 1954, pp. 395–404.
[26] WALTERS, A. A.: "Track Costs and Motor Taxation," *Journal of Industrial Economics*, 1954.
[27] WEBB, G. M., AND K. MOSKOWITZ: "California Freeway Capacity Study—1956," *Highway Research Board Proceedings*, 1957, pp. 587–641.