

# Midterm Report

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## Objective:

The main goal of this project is to design a Implement the basic multiply-and-accumulate (MAC) unit and to implement a 4x4 systolic array suggested on the next slides as a synchronous system controlled by a single global clock "clk" along with an asynchronous reset "rst" (toy version of Google's Tensor Processing Unit i.e. TPU).

## Introduction:

This project consists of two parts:

- 1) Mac unit design
- 2) 4X4 Systolic array TPU design

Where the mac unit is the basic cell in which the multiplication of the given two inputs 'a' and 'w' is done, the result is then propagated and gets added with a carry\_in. Below is a brief description of the mac unit.

## Mac unit:

Fig.1 depicts the actual mac unit's internal structure, where there are input ports and output ports:

### Input ports:

a (8bits)  
w(8bits)  
clock (1bit)  
reset(1bit)  
carry\_in (24bits)

### output ports:

a1(8bits)  
temp (24bits)

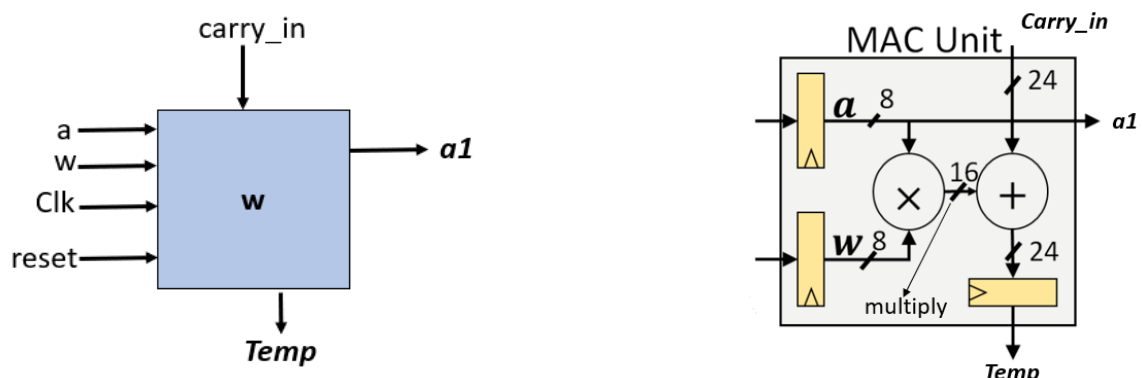


Fig.1 Depicts the Mac unit ports and internal structure of the mac

A register of 24 bits is introduced inside the design of the Mac unit, where it stores the value of multiplication and addition value of carry\_in.

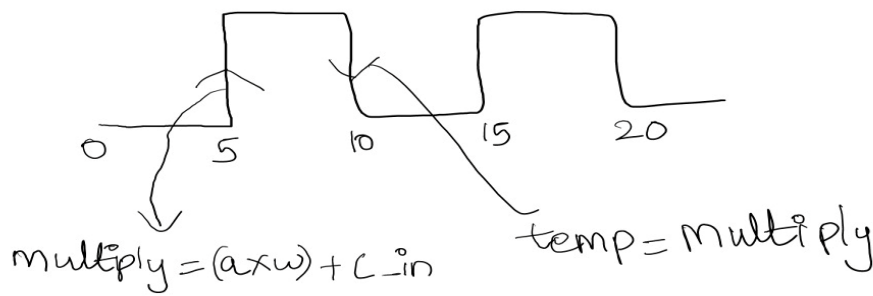


Fig.2 Depicts the functioning of the Mac with the clock

Fig.2 explains how the Mac produces the resultant product with carry-in, as we can see that,

- 1) For Posedge of the clock the multiplication and addition of carry-in is done (multiply is filled with value)
- 2) For Negedge of the clock the value of the computation done in posedge is copied in the variable Temp.

The results for the given question which are ran in iverilog are in the below Fig.3 and Fig.4:

```

D:\iverilog\bin>vvp mac_tb.vvp
    0 a= x, w= x, carry_in= x, a1= x, output_sum= x, reset=1
    5 a= 1, w= 2, carry_in= 0, a1= x, output_sum= x, reset=0
   10 a= 1, w= 2, carry_in= 0, a1= 1, output_sum= 2, reset=0
   15 a= 3, w= 4, carry_in= 0, a1= 1, output_sum= 2, reset=0
   20 a= 3, w= 4, carry_in= 0, a1= 3, output_sum= 12, reset=0
   25 a= 5, w= 6, carry_in= 0, a1= 3, output_sum= 12, reset=0
   30 a= 5, w= 6, carry_in= 0, a1= 5, output_sum= 30, reset=0
   35 a= 7, w= 8, carry_in= 0, a1= 5, output_sum= 30, reset=0
   40 a= 7, w= 8, carry_in= 0, a1= 7, output_sum= 56, reset=0
  
```

Fig .3 Output result of mac unit

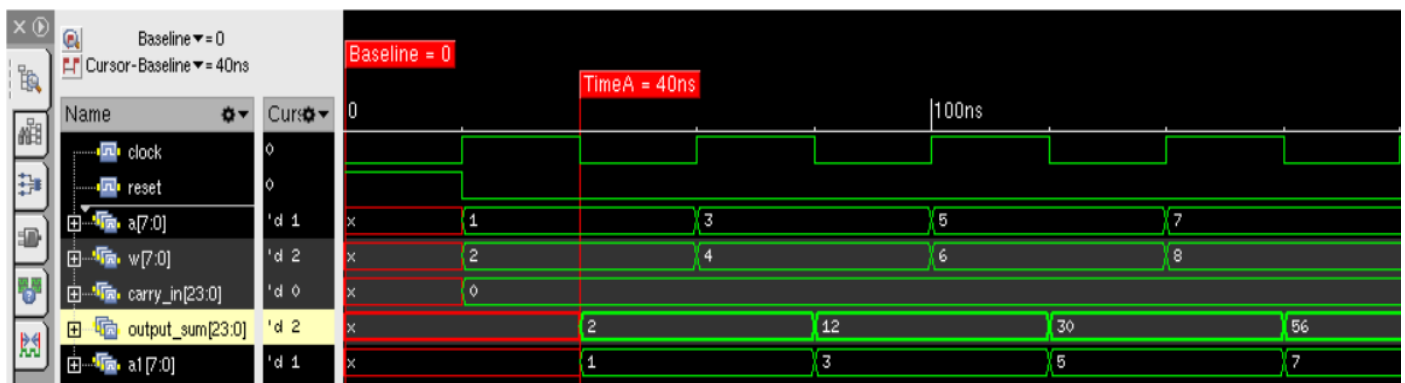


Fig.4 Output of mac unit waveform in Cadence Xcelium

Note:

The output results are updated in temp (output\_sum) register where the values are stored at the “NEGEDGE” of the clock.

## Systolic Array:

The Systolic array consists of 5 units:

- 1) 16 Mac units
- 2) Weight memory
- 3) Feature memory
- 4) Quantization unit
- 5) Activation Unit

### 16 Mac units:

The systolic array is implemented by using the mac units where there are connected to one another, the below diagram shows the design of systolic array in Fig.4,

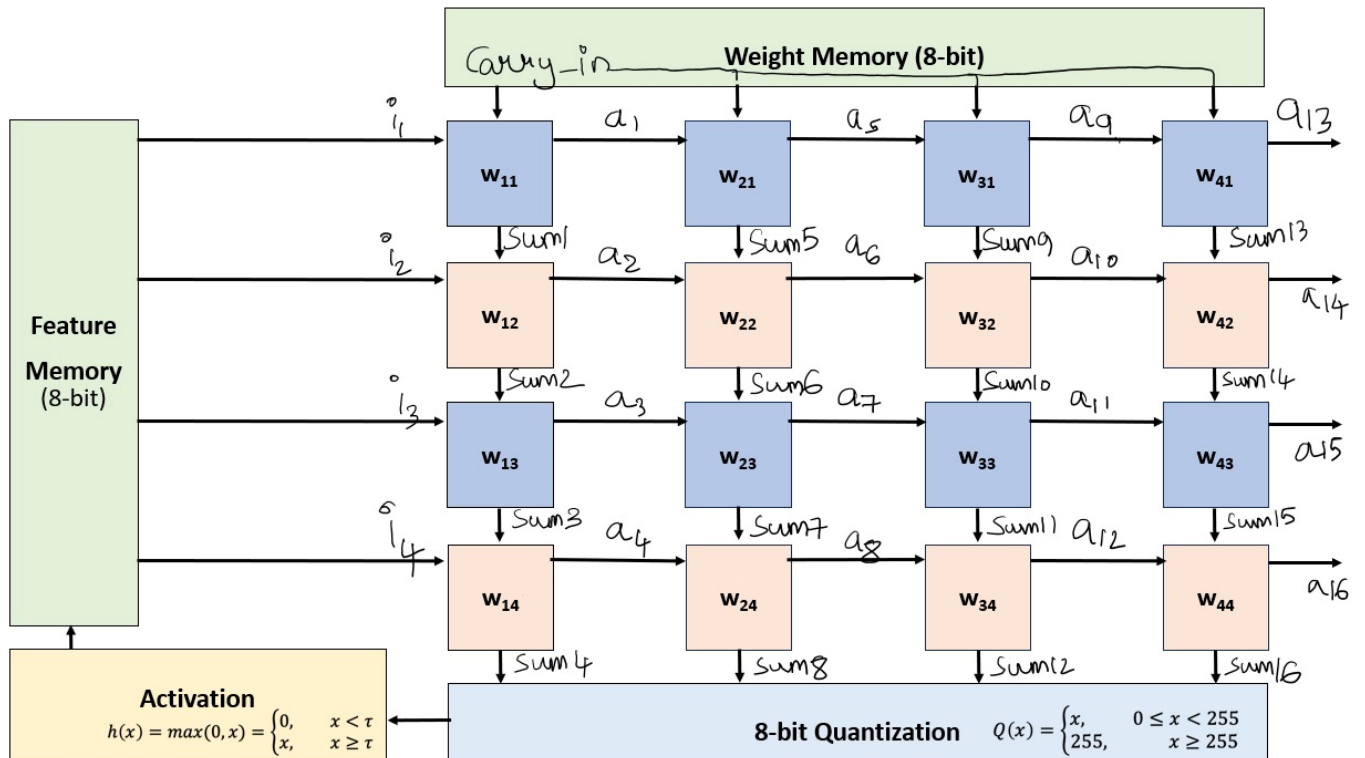


Fig 6 Design of the Systolic Array Unit

The above diagram shows the design of the systolic array units where there are:

Inputs  $i_1, i_2, i_3, i_4$  are 4 elements of the A matrix coming clock by clock basis,

Output partial sums are  $sum1, sum2, sum3, \dots, sum16$ .

The propagated inputs are  $a_1, a_2, a_3, \dots, a_{16}$ , which go to the other Mac units which go on a clock basis.

The actual design overview in cadence Xcelium is shown below :

In the left side the Red block indicates the feature memory and the orange block indicates the weight memory of the design.

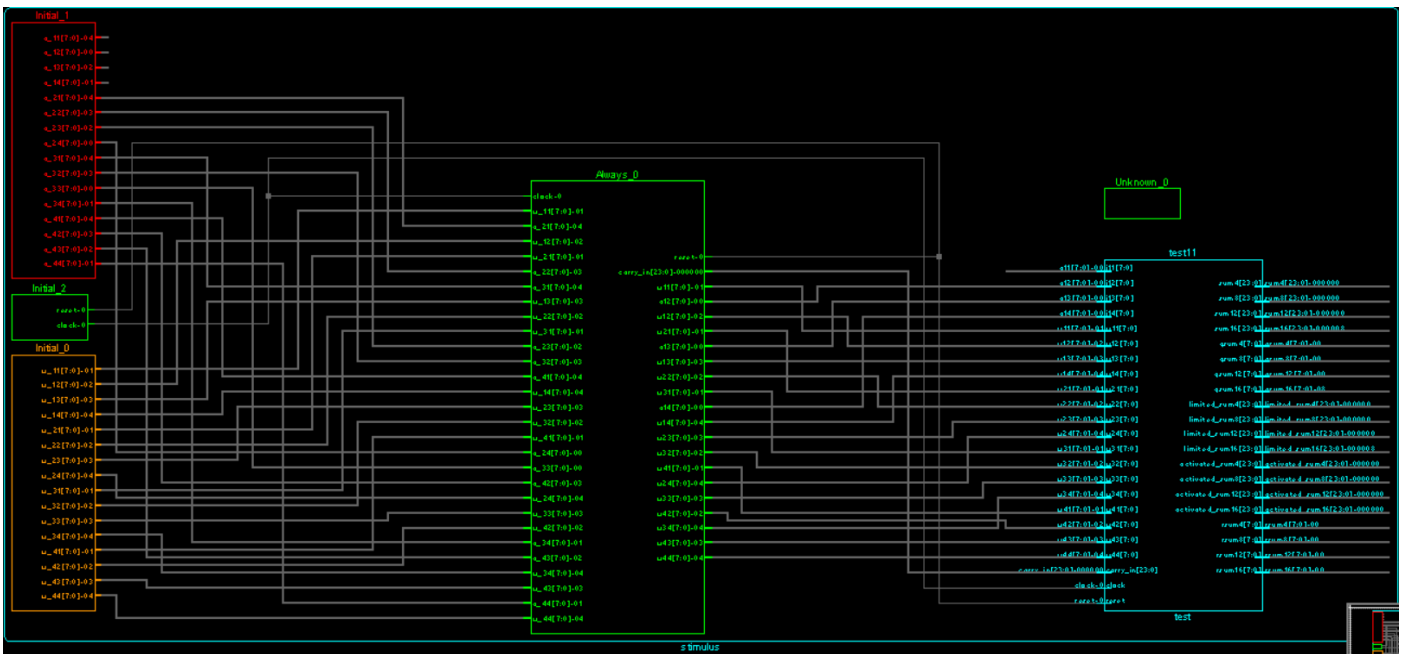


Fig 7 Overview of the design in Cadence Xcelium

### Weight memory:

The weight memory is a 8 bits registers of 16 elements starting from w11, w12, w13, w14, .....w44.

```
reg[7:0] w_11,w_12,w_13, w_14,w_21,w_22,w_23, w_24,w_31,w_32,w_33, w_34,w_41, w_42,w_43,w_44;
w_11<=1;w_12<=2;w_13<=3; w_14<=4;
w_21<=1;w_22<=2;w_23<=3; w_24<=4; // Weight memory
w_31<=1;w_32<=2;w_33<=3; w_34<=4;
w_41<=1; w_42<=2;w_43<=3;w_44<=4;
```

### Feature memory:

The feature memory is a 8 bits registers of 16 elements starting from a11, a12, a13, a14,....a44.

```
reg[7:0] a_11,a_12,a_13, a_14, a_21,a_22,a_23, a_24, a_31,a_32,a_33, a_34, a_41, a_42,a_43,a_44;
a_11<=4;a_12<=0;a_13<=2; a_14<=1;
a_21<=4;a_22<=3;a_23<=2; a_24<=0;// feature memory
a_31<=4;a_32<=3;a_33<=0; a_34<=1;
a_41<=4; a_42<=3;a_43<=2;a_44<=1;
```

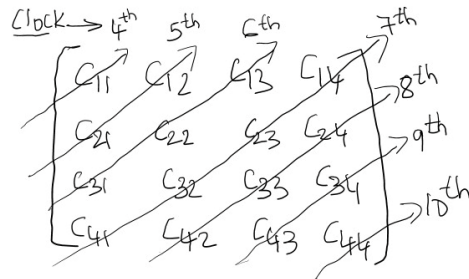
### Matrix multiplication:

Here the matrix multiplication is shown below with the A and W matrix being multiplied and the output matrix is C, which is filled on a clock basis. The below is the multiplication of the values and the output.

## matrix multiplication:

$$\begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{23} & w_{24} \\ w_{31} & w_{32} & w_{33} & w_{34} \\ w_{41} & w_{42} & w_{43} & w_{44} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$



$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & 2 & 1 \\ 4 & 3 & 2 & 0 \\ 4 & 3 & 0 & 1 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

Clock  $\rightarrow$  4<sup>th</sup> 5<sup>th</sup> 6<sup>th</sup> 7<sup>th</sup>

The output matrix gets filled by 10 clock cycles where the first element is filled for the 4<sup>th</sup> clock C11,

In 5<sup>th</sup> clock the next two element are filled C12 and C21, from the above diagram we can see how the elements are getting filled on clock basis.

Below is the table of the Computations of the 10 clock cycles and matrix filled are shown in detail with expressions:

| Clock | columns | Computation Equations   | Matrix Elements |
|-------|---------|---|-----------------|
| 1     | column1 | $(a_{11} * w_{11})$   | x               |
|       | column2 | 0   | x               |
|       | column3 | 0   | x               |
|       | column4 | 0   | x               |
|       |         |   |                 |
| 2     | column1 | $(a_{11} w_{11}) + (a_{21} w_{12})$                                     | x               |
|       | column2 | $(a_{11} w_{21})$   | x               |
|       | column3 | 0   | x               |
|       | column4 | 0   | x               |
|       |         |   |                 |
| 3     | column1 | $(a_{11} w_{11}) + (a_{21} w_{12}) + (a_{31} w_{13})$                   | x               |
|       | column2 | $(a_{11} w_{21}) + (a_{12} w_{22})$                                     | x               |
|       | column3 | $(a_{11} w_{31})$   | x               |
|       | column4 | 0   | x               |
|       |         |   |                 |
| 4     | column1 | $(a_{11} w_{11}) + (a_{21} w_{12}) + (a_{31} w_{13}) + (a_{41} w_{14})$ | c11             |
|       | column2 | $(a_{11} w_{21}) + (a_{12} w_{22}) + (a_{13} w_{23})$                   | x               |
|       | column3 | $(a_{11} w_{31}) + (a_{21} w_{32})$                                     | x               |
|       | column4 | $(a_{11} w_{41})$   | x               |
|       |         |   |                 |
| 5     | column1 | $(a_{11} w_{21}) + (a_{21} w_{22}) + (a_{31} w_{23}) + (a_{41} w_{24})$ | c12             |
|       | column2 | $(a_{12} w_{11}) + (a_{22} w_{12}) + (a_{32} w_{13}) + (a_{42} w_{14})$ | c21             |
|       | column3 | $(a_{11} w_{31}) + (a_{21} w_{32}) + (a_{31} w_{33})$                   | x               |
|       | column4 | $(a_{11} w_{41}) + (a_{21} w_{42})$                                     | x               |
|       |         |   |                 |
| 6     | column1 | $(a_{11} w_{31}) + (a_{21} w_{32}) + (a_{31} w_{33}) + (a_{41} w_{34})$ | c13             |
|       | column2 | $(a_{12} w_{21}) + (a_{22} w_{22}) + (a_{32} w_{23}) + (a_{42} w_{24})$ | c22             |
|       | column3 | $(a_{13} w_{11}) + (a_{23} w_{12}) + (a_{33} w_{13}) + (a_{43} w_{14})$ | c31             |
|       | column4 | $(a_{11} w_{41}) + (a_{21} w_{42}) + (a_{31} w_{43})$                   | x               |
|       |         |   |                 |
| 7     | column1 | $(a_{11} w_{41}) + (a_{21} w_{42}) + (a_{31} w_{43}) + (a_{41} w_{44})$ | c14             |
|       | column2 | $(a_{12} w_{31}) + (a_{22} w_{32}) + (a_{32} w_{33}) + (a_{42} w_{34})$ | c23             |
|       | column3 | $(a_{13} w_{21}) + (a_{23} w_{22}) + (a_{33} w_{23}) + (a_{43} w_{24})$ | c32             |
|       | column4 | $(a_{14} w_{11}) + (a_{24} w_{12}) + (a_{34} w_{13}) + (a_{44} w_{14})$ | c41             |
|       |         |   |                 |
| 8     | column1 | 0   | x               |
|       | column2 | $(a_{12} w_{41}) + (a_{22} w_{42}) + (a_{32} w_{43}) + (a_{42} w_{44})$ | c24             |
|       | column3 | $(a_{13} w_{31}) + (a_{23} w_{32}) + (a_{33} w_{33}) + (a_{43} w_{34})$ | c33             |
|       | column4 | $(a_{14} w_{21}) + (a_{24} w_{22}) + (a_{34} w_{23}) + (a_{44} w_{24})$ | c42             |
|       |         |   |                 |
| 9     | column1 | 0   | x               |
|       | column2 | 0   | x               |
|       | column3 | $(a_{13} w_{41}) + (a_{23} w_{42}) + (a_{33} w_{43}) + (a_{43} w_{44})$ | c34             |
|       | column4 | $(a_{14} w_{31}) + (a_{24} w_{32}) + (a_{34} w_{33}) + (a_{44} w_{34})$ | c43             |
|       |         |   |                 |
| 10    | column1 | 0   | x               |
|       | column2 | 0   | x               |
|       | column3 | 0   | x               |
|       | column4 | $(a_{14} w_{41}) + (a_{24} w_{42}) + (a_{34} w_{43}) + (a_{44} w_{44})$ | c44             |

## Results:

Here the output matrix is shown on different clock cycles starting from 40<sup>th</sup> sec which is the 4<sup>th</sup> cycle,

And completing the matrix in column wise at 100<sup>th</sup> sec which the 10<sup>th</sup> cycle.

Here the outputs sum4 is row1, sum8 is row2, sum12 is row3, sum16 is row4 of the output matrix.

```
D:\iverilog\bin>iverilog -o 4matrix_tb.vvp 4matrix_tb.v
D:\iverilog\bin>vvp 4matrix_tb.vvp
      0 sum4=  x, sum8=  x, sum12=  x, sum16=  x
     40 sum4= 40, sum8=  x, sum12=  x, sum16=  x
     50 sum4= 27, sum8= 40, sum12=  x, sum16=  x
     60 sum4= 14, sum8= 27, sum12= 40, sum16=  x
     70 sum4=  8, sum8= 14, sum12= 27, sum16= 40
     80 sum4=  0, sum8=  8, sum12= 14, sum16= 27
     90 sum4=  0, sum8=  0, sum12=  8, sum16= 14
    100 sum4=  0, sum8=  0, sum12=  0, sum16=  8
```

## Quantization unit:

The quantization unit of registers where receives the elements from the output matrix of 24bits each then, it compares if the element is greater than 255 if yes it sets the value as 255 which 8bits (11111111) in binary form.

The set value is then truncated from 24-bit to 8-bit.

Below is the logic implementation of the quantization unit:

```
// quantization unit

assign limited_sum4 = (sum4 > 255) ? 24'd255 : sum4;
assign limited_sum8 = (sum8 > 255) ? 24'd255 : sum8;
assign limited_sum12 = (sum12 > 255) ? 24'd255 : sum12;
assign limited_sum16 = (sum16 > 255) ? 24'd255 : sum16;

// Quantize the limited values

assign qsum4 = limited_sum4[7:0];
assign qsum8 = limited_sum8[7:0];
assign qsum12 = limited_sum12[7:0];
assign qsum16 = limited_sum16[7:0];
```

## Result:

```
D:\iverilog\bin>iverilog -o quant_tb.vvp quant_tb.v
D:\iverilog\bin>vvp quant_tb.vvp
      0 qsum4=  x, qsum8=  x, qsum12=  x, qsum16=  x
     40 qsum4= 40, qsum8=  x, qsum12=  x, qsum16=  x
     50 qsum4= 27, qsum8= 40, qsum12=  x, qsum16=  x
     60 qsum4= 14, qsum8= 27, qsum12= 40, qsum16=  x
     70 qsum4=  8, qsum8= 14, qsum12= 27, qsum16= 40
     80 qsum4=  0, qsum8=  8, qsum12= 14, qsum16= 27
     90 qsum4=  0, qsum8=  0, qsum12=  8, qsum16= 14
    100 qsum4=  0, qsum8=  0, qsum12=  0, qsum16=  8
```

Truncation from 24bit to 8bit of qsum4 which 40 in decimal, below shows the binary form of it:

```
D:\iverilog\bin>iverilog -o quant_tb.vvp quant_tb.v

D:\iverilog\bin>vvp quant_tb.vvp
  0 sum4=xxxxxxxxxxxxxxxxxxxxxxxx, Quantized_binary4=xxxxxxx
 40 sum4=000000000000000000101000, Quantized_binary4=00101000
 50 sum4=0000000000000000000011011, Quantized_binary4=00011011
 60 sum4=0000000000000000000001110, Quantized_binary4=00001110
 70 sum4=0000000000000000000001000, Quantized_binary4=00001000
 80 sum4=0000000000000000000000000, Quantized_binary4=00000000
```

### Activation Unit:

The activation unit is an 8-bit registers where it receives the values from the quantization unit, from here there is a value called Tau, which is used to compare if the value is then 10, the value will change to 0. If the element is greater than 10 it remains the same value.

Below is the logic implementation of the activation :

```
// **Activation Unit**
wire lt_tau4, lt_tau8, lt_tau12, lt_tau16; // signals for comparison with tau (assuming tau = 10)
wire [23:0] activated_sum4, activated_sum8, activated_sum12, activated_sum16;

assign lt_tau4 = qsum4 < 10;
assign activated_sum4 = lt_tau4 ? 0 : qsum4;

assign lt_tau8 = qsum8 < 10;
assign activated_sum8 = lt_tau8 ? 0 : qsum8;

assign lt_tau12 = qsum12 < 10;
assign activated_sum12 = lt_tau12 ? 0 : qsum12;

assign lt_tau16 = qsum16 < 10;
assign activated_sum16 = lt_tau16 ? 0 : qsum16;

// **Update final outputs with activated values**
assign rsum4 = activated_sum4;
assign rsum8 = activated_sum8;
assign rsum12 = activated_sum12;
assign rsum16 = activated_sum16;
```

Result:

8 vanishes because it is less than 10.

```
D:\iverilog\bin>iverilog -o act_tb.vvp act_tb.v

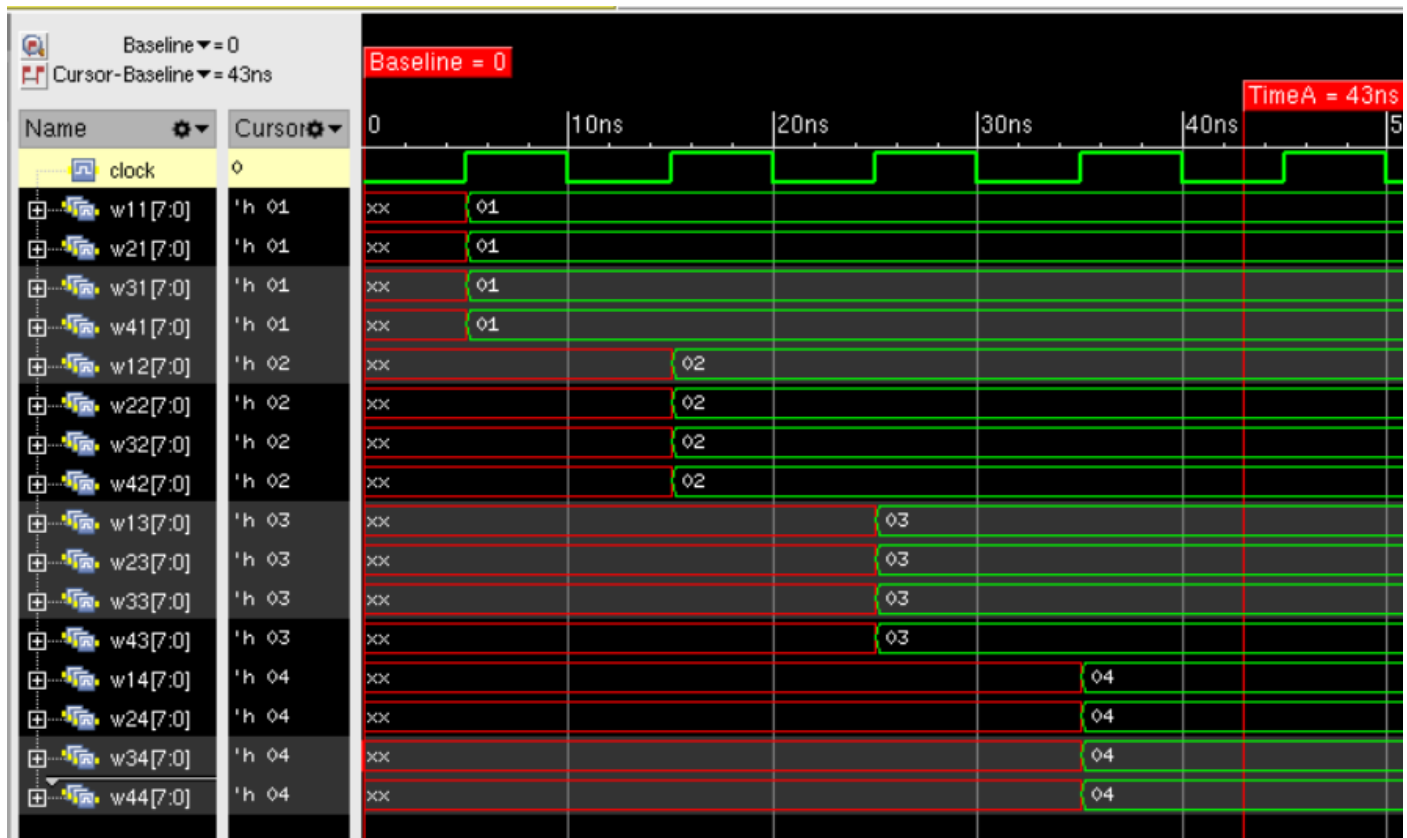
D:\iverilog\bin>vvp act_tb.vvp
  0 act4= X, act8= X, act12= X, act16= X
 40 act4= 40, act8= X, act12= X, act16= X
 50 act4= 27, act8= 40, act12= X, act16= X
 60 act4= 14, act8= 27, act12= 40, act16= X
 70 act4= 0, act8= 14, act12= 27, act16= 40
 80 act4= 0, act8= 0, act12= 14, act16= 27
 90 act4= 0, act8= 0, act12= 0, act16= 14
100 act4= 0, act8= 0, act12= 0, act16= 0
```



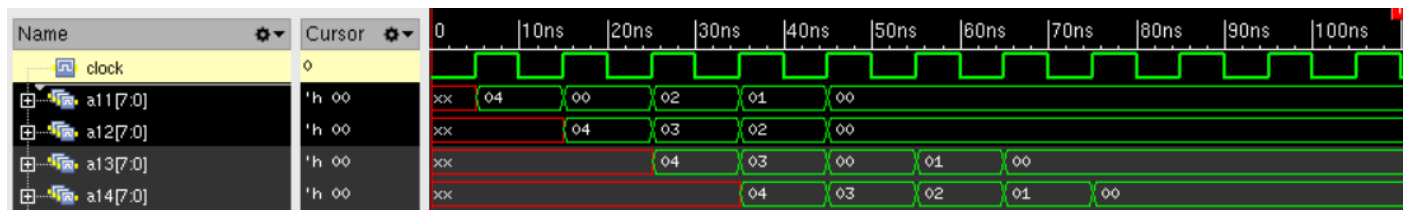
## Cadence Xcelium:

Showing four column outputs across the 4 clock cycles of initialization:

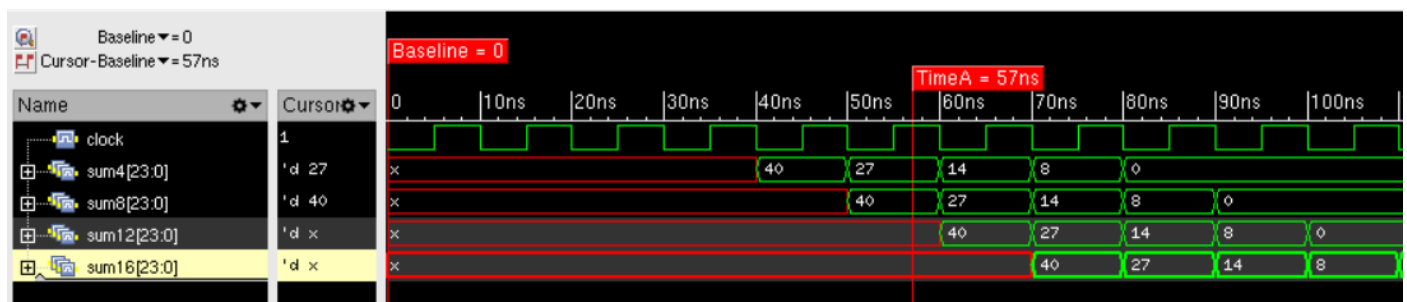
The initialization of weighted memory:



The initialization of Feature memory:



Showing four column outputs across the 10 clock cycles of computation:



Showing changes in memory (any 4 elements of the output matrix of your choice) that stores the output matrix, across the different clock cycles:

