compare

April 16, 2023

[1]:
```
import numpy as np
from numba import jit
from numpy.random import default_rng as rg
rng = rg(12345)
from tqdm import tqdm
import sys
sys.path.append('../core')
%matplotlib inline
```

[2]:
```
import torch
torch.manual_seed(0)
import torch.optim as optim
```

[3]:
```
from FLDojo import dojo
from FL import FL
from DNN_R import DNN
from display2 import*
```

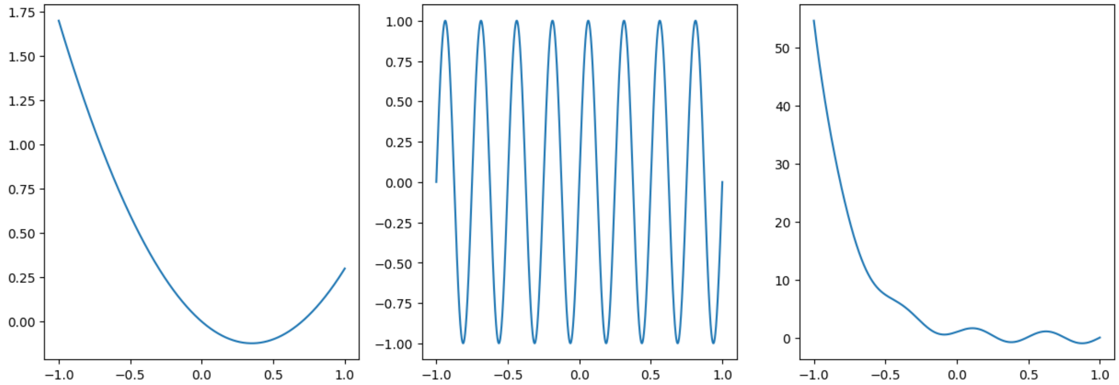
[35]:
```
Xs = np.linspace(-1,1,40000)
Ys0 = Xs**2 - 0.7*Xs
# derivative is 2x - 0.7, which has a absval max of 2.7 (which is the Lipshitz constant)
Ys1 = np.sin(Xs*8*np.pi)
# note derivative is 8 pi cos(8 pi x), which has a max of 8 pi (which is the Lipshitz constant)
Ys2 = np.sin(Xs*4*np.pi) + np.exp(-4*Xs)
print(np.max(np.abs(4*np.pi*np.cos(4*np.pi*Xs) - 4*np.exp(-4*Xs))))
# derivative has a max of ~205 (which is the Lipshitz constant)
from matplotlib import pyplot as plt
fig,axs = plt.subplots(1,3,figsize=(15,5))
axs[0].plot(Xs,Ys0)
axs[1].plot(Xs,Ys1)
axs[2].plot(Xs,Ys2)

# We will use the same network for all three functions
ks = [2.7,8*np.pi,205]
dnn_sizes = [1,102,101,1] # so # of weights is 102*1 + 102*102 + 102*1 = 10608
```

```
fl_sizes = [1,100,100] # so # of weights is  1*a + a*1 + a*a + a*1 = 10300, for␣
↪a = 100

fl_sizes2 = [1,25,25] # so # of weights is  1*a + a*1 + a*a + a*1 = 700, for a␣
↪= 25
dnn_sizes2 = [1,26,26,1] # so # of weights is 26*1 + 26*26 + 26*1 = 728
# a three layer nn can represent any multivariate function (continuous or␣
↪discontinuous) https://arxiv.org/abs/2012.03016
```

```
205.826229518
```



[48]:
```
device = torch.device("cuda:0" if torch.cuda.is_available() else "cpu")
p = np.random.permutation(len(Xs))
Xs = Xs[p]
X = torch.from_numpy(Xs).float().unsqueeze(1).to(device)
split = 0.75
train_X = X[:int(split*len(X))]
test_X = X[int(split*len(X)):]

def update_y(Ys):
    y = torch.from_numpy(Ys[p]).float().unsqueeze(1).to(device)
    train_y = y[:int(split*len(y))]
    test_y = y[int(split*len(y)):]
    return train_y, test_y
train_y,test_y = update_y(Ys0)
```

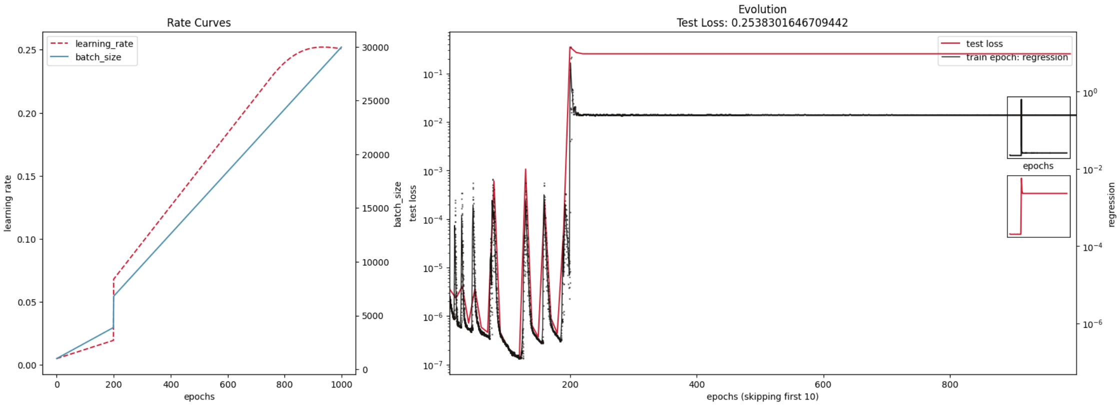
[6]:
```
D = dojo()
D.epochs=1000
D.max_batch_size=train_X.shape[0]
opt = lambda x: optim.Adam(x, lr=0.00001) # Adam better than SGD and AdamW in␣
↪quick tests.
```

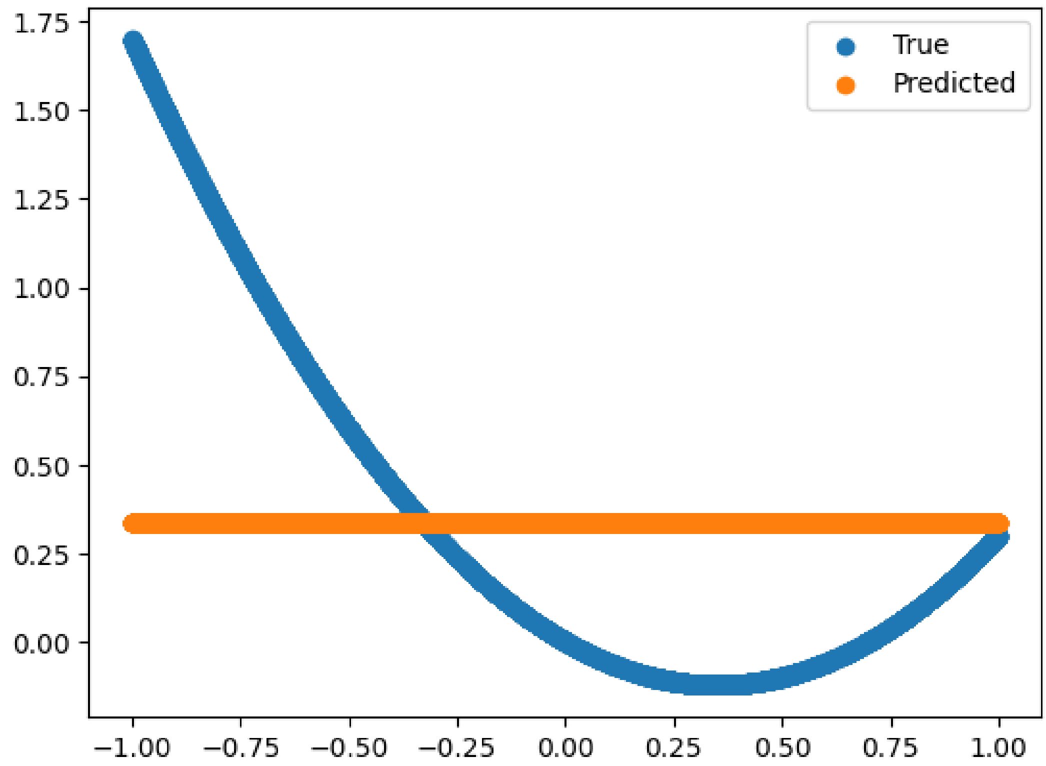
1 Function 0

1.1 DNN 0

```
[10]: act = torch.nn.ReLU()
      net = DNN(device, dnn_sizes, opttype=opt, act = act, bias=True) # expect 1.6%␣
        ↪error rate
      report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000)
      ecran(net, test_X, test_y, report, classification=False)
      plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
        ↪numpy(),label='True')
      plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
        ↪numpy(),label='Predicted')
      plt.legend()
      plt.show()
```

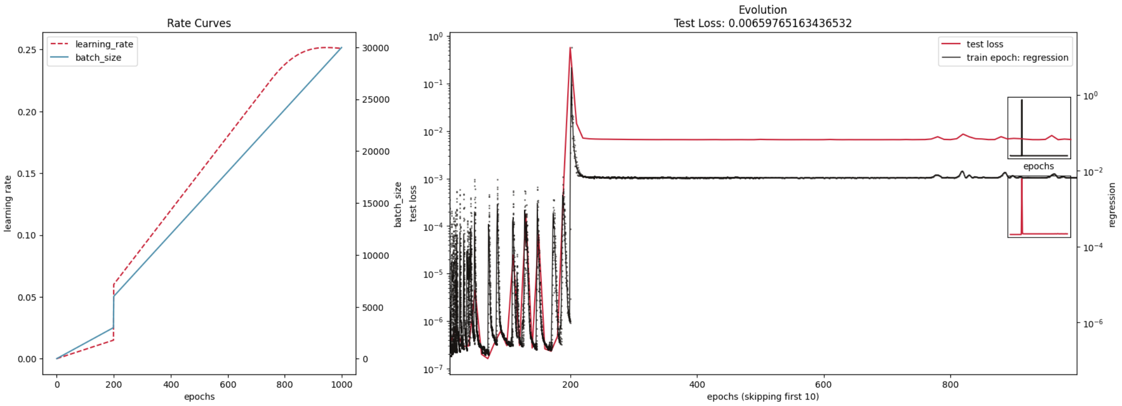
```
100%|          | 1000/1000 [00:12<00:00, 83.01it/s]
```

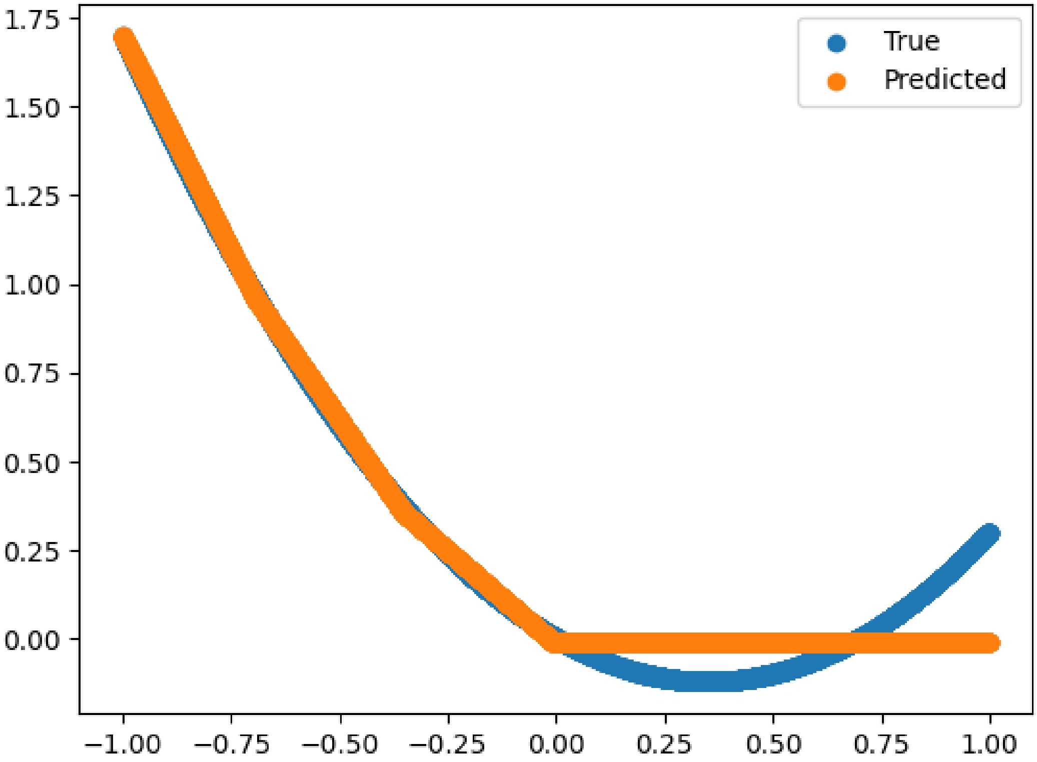




```
[13]: act = torch.nn.ReLU()
      net = DNN(device, dnn_sizes, opttype=opt, act = act, bias=True) # expect 1.6%␣
        ↪error rate
      report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=10)
      ecran(net, test_X, test_y, report, classification=False)
      plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
        ↪numpy(),label='True')
      plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
        ↪numpy(),label='Predicted')
      plt.legend()
      plt.show()
```

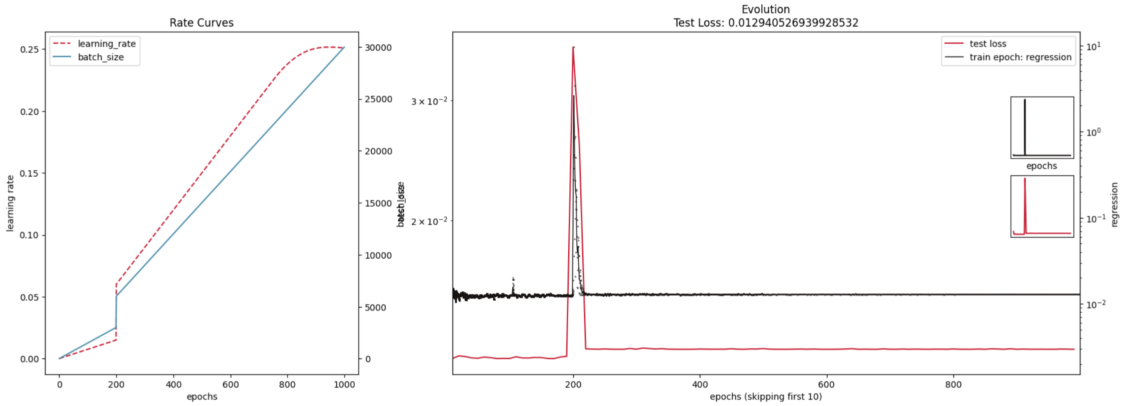
```
100%|      | 1000/1000 [00:32<00:00, 30.56it/s]
```

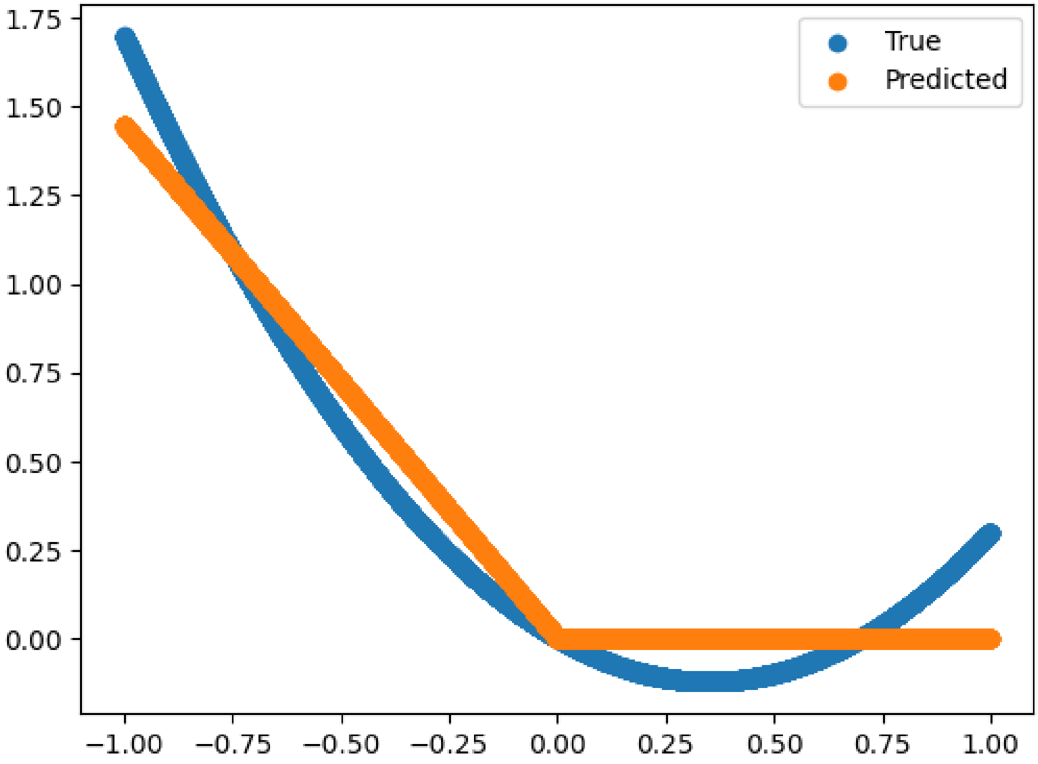




```
[12]: act = torch.nn.ReLU()
      net = DNN(device, dnn_sizes, opttype=opt, act = act, bias=False) # expect 1.6% error rate
      report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=10)
      ecran(net, test_X, test_y, report, classification=False)
      plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().numpy(),label='True')
      plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
        ↪numpy(),label='Predicted')
      plt.legend()
      plt.show()
```

```
100%|          | 1000/1000 [00:24<00:00, 40.81it/s]
```

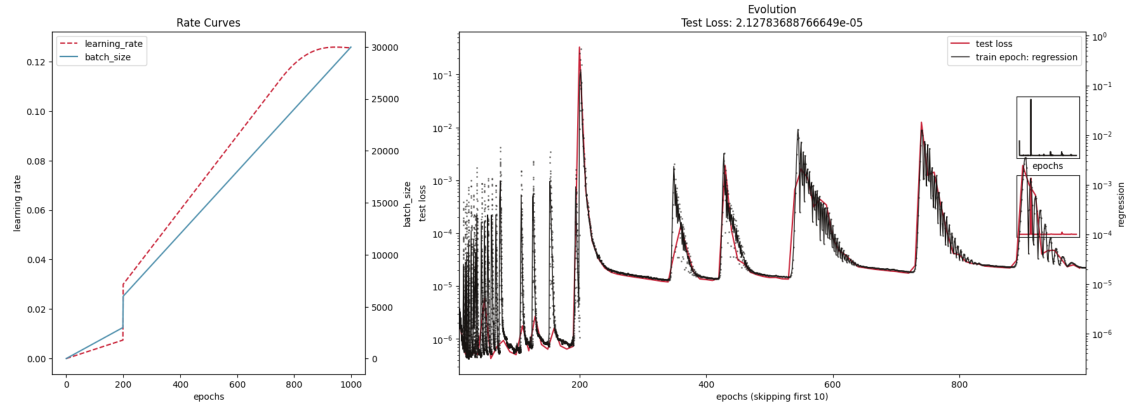


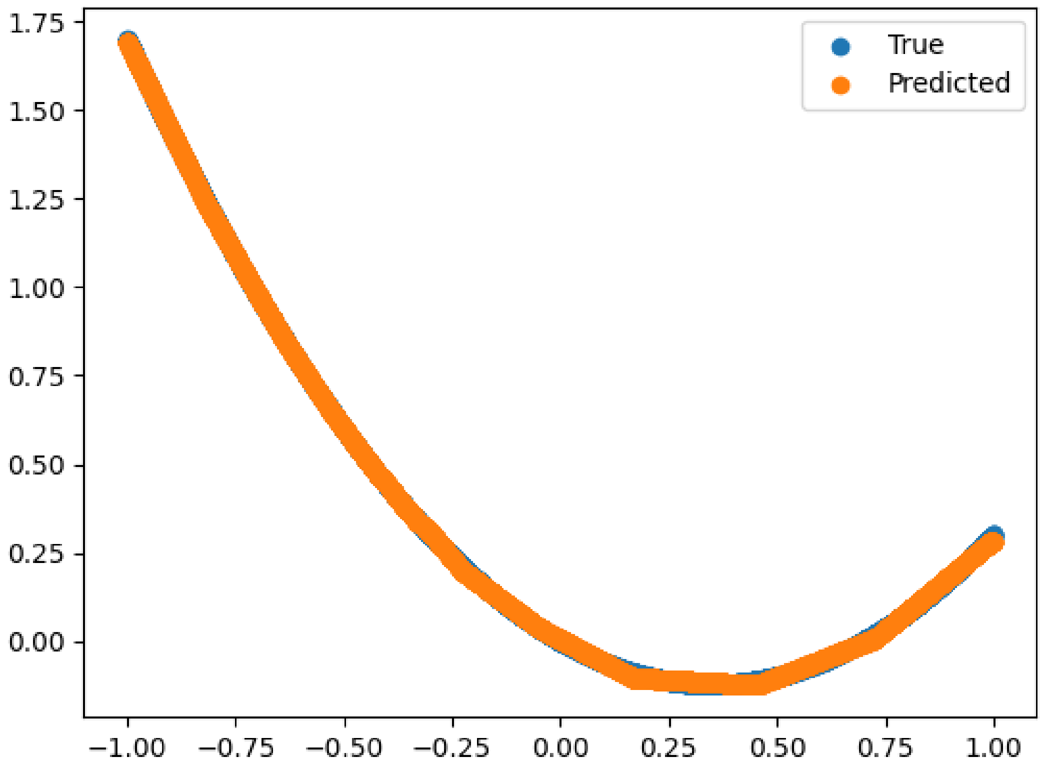


[15]:

```
act = torch.nn.ReLU()
opt2 = lambda x: optim.Adam(x, lr=0.000005)
net = DNN(device, dnn_sizes, opttype=opt2, act = act, bias=True) # expect 1.6%␣
  ↪error rate
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=10,␣
  ↪max_batch_size=100)
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
  ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
  ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 1000/1000 [00:38<00:00, 26.03it/s]
```

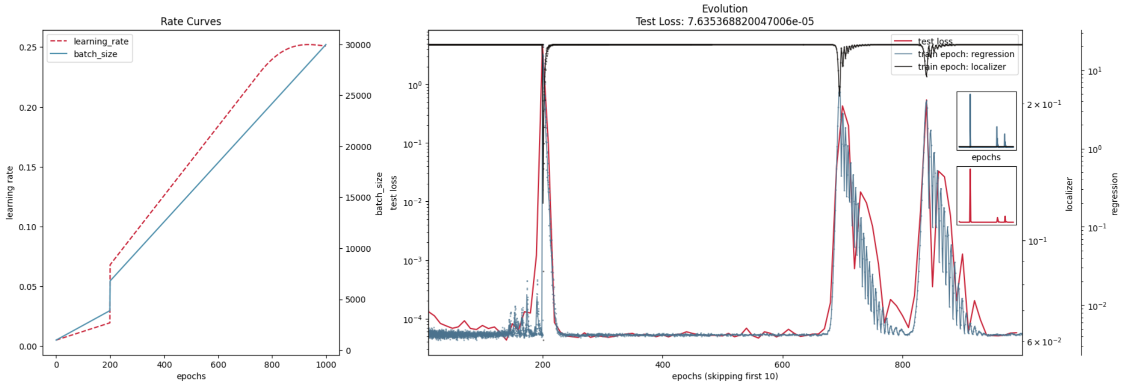


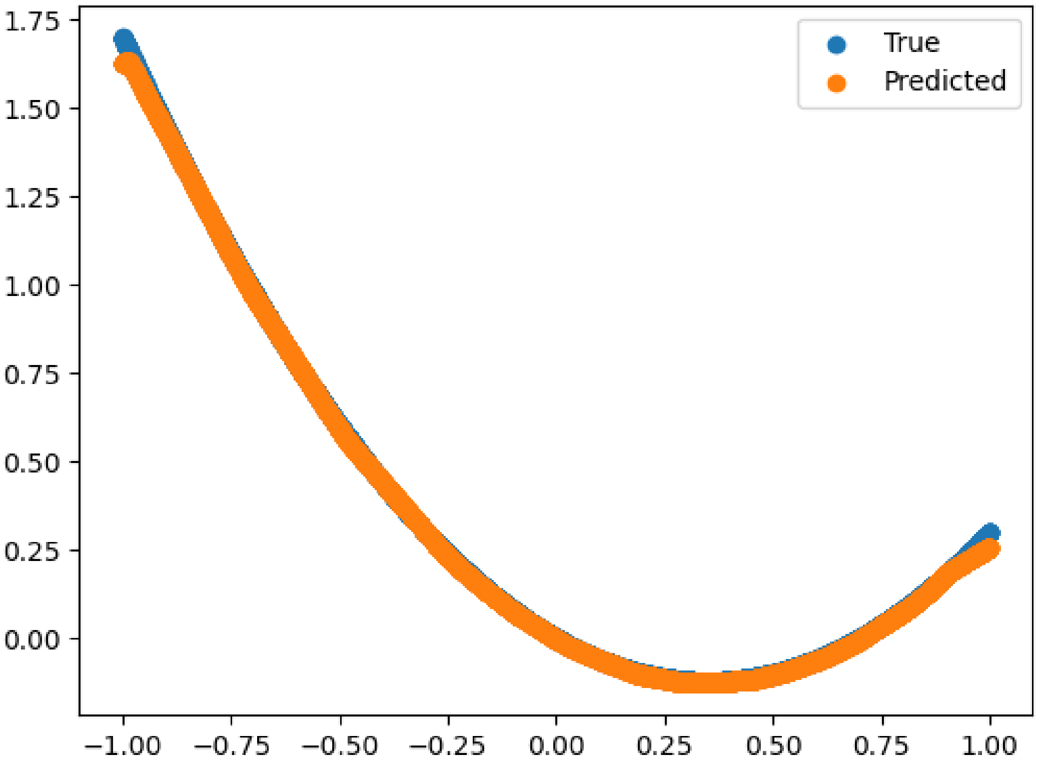


1.2 FL 0

```
[11]: delta = np.array([1]*len(fl_sizes))*0.05
k = [ks[0]]*len(fl_sizes)
act = torch.nn.ReLU()
net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
  ↪expect 1.6% error rate
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000)
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
  ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
  ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|      | 1000/1000 [00:46<00:00, 21.64it/s]
```

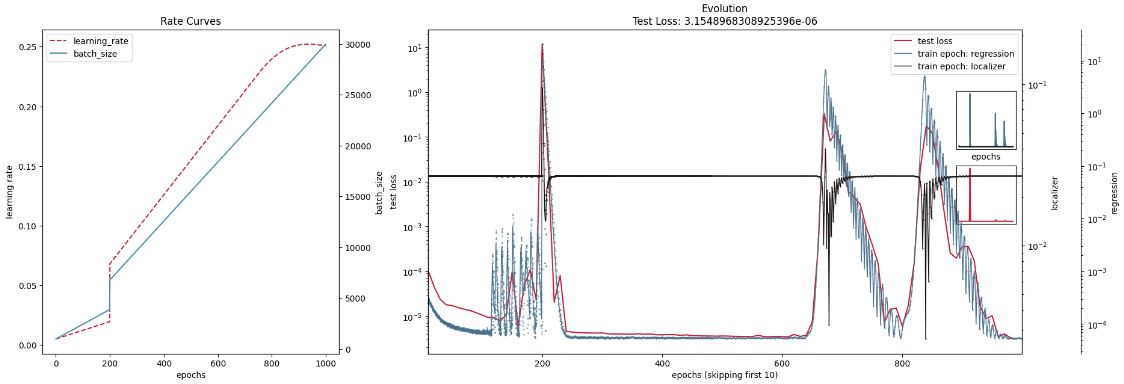


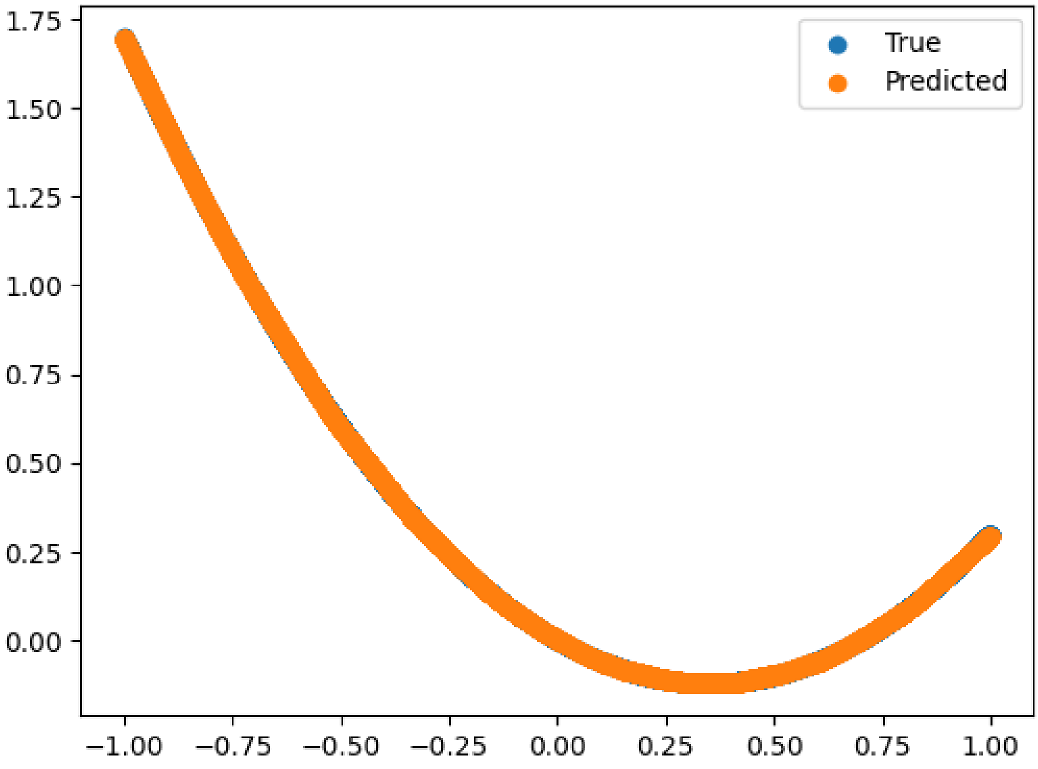


[18]:
```
delta = np.array([1]*len(fl_sizes))*0.005
k = [ks[0]]*len(fl_sizes)
act = torch.nn.ReLU()
net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) # expect 1.6% error rate
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000)
ecran(net, test_X, test_y, report, classification=False)
```

```
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
 ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
 ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 1000/1000 [00:46<00:00, 21.73it/s]
```

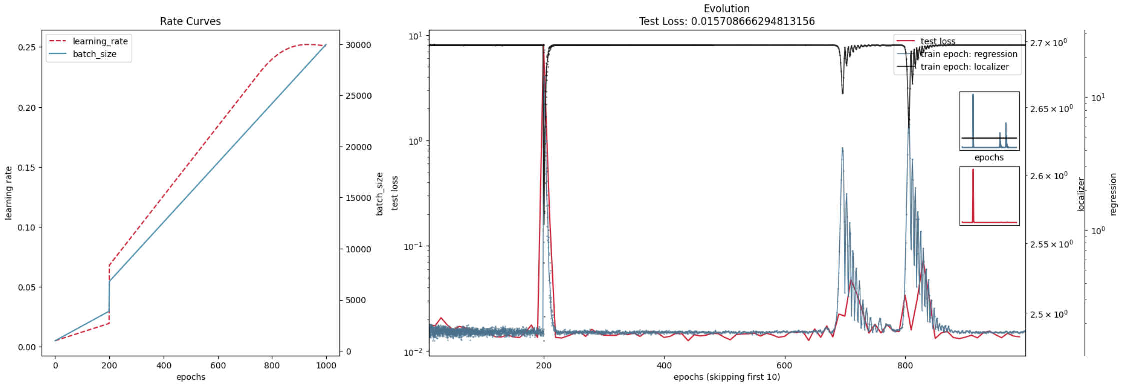


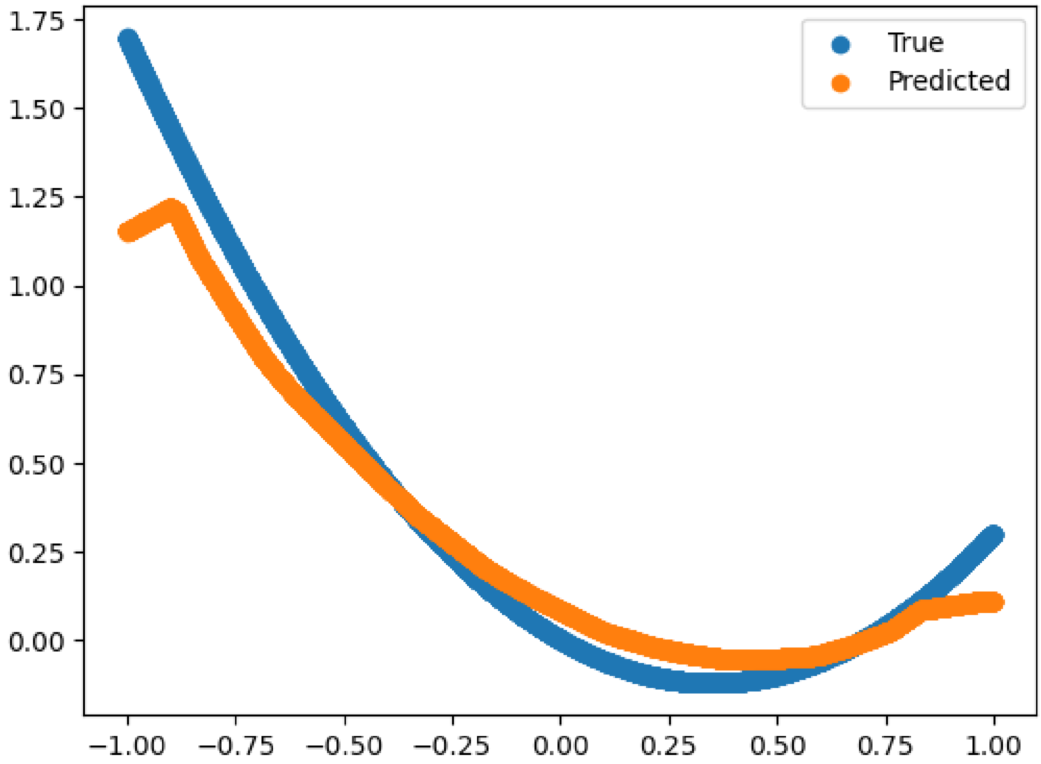


[17]:

```
delta = np.array([1]*len(fl_sizes))*0.5
k = [ks[0]]*len(fl_sizes)
act = torch.nn.ReLU()
net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
 ↪expect 1.6% error rate
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000)
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
 ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
 ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 1000/1000 [00:46<00:00, 21.41it/s]
```



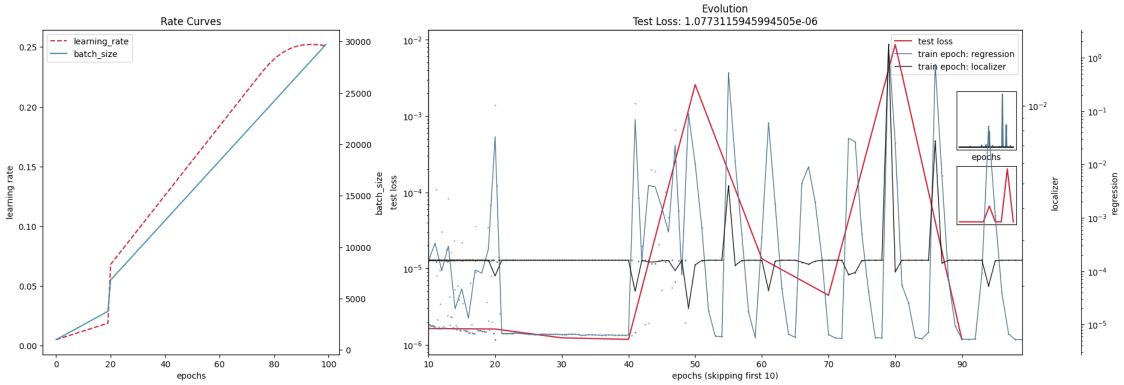


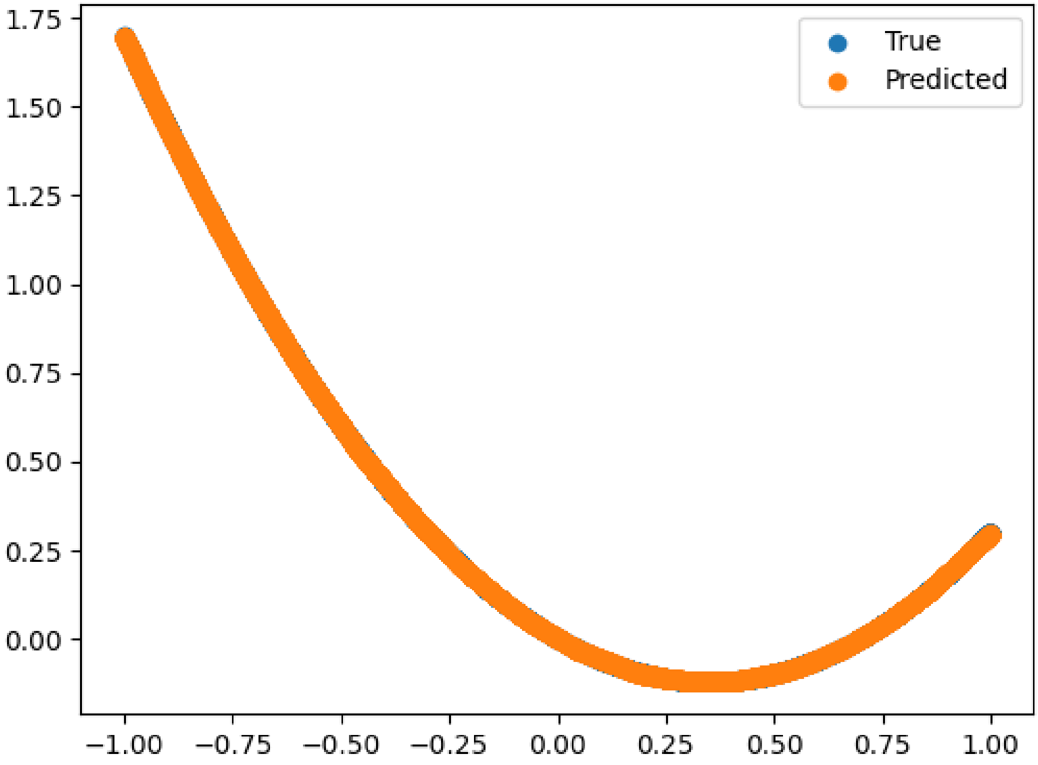
```
[ ]:
```

1.3 FL 0 Greedy

```
[49]: act = torch.nn.ReLU()
      delta = np.array([1]*len(fl_sizes))*0.00005
      k = [ks[1]]*len(fl_sizes)
      net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
        ↪expect 1.6% error rate
      D.epochs = 100
      report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000,␣
        ↪repeat_epochs=40)
      D.epochs = 1000
      ecran(net, test_X, test_y, report, classification=False)
      plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
        ↪numpy(),label='True')
      plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
        ↪numpy(),label='Predicted')
      plt.legend()
      plt.show()
```

```
100%|          | 100/100 [03:03<00:00,  1.83s/it]
```





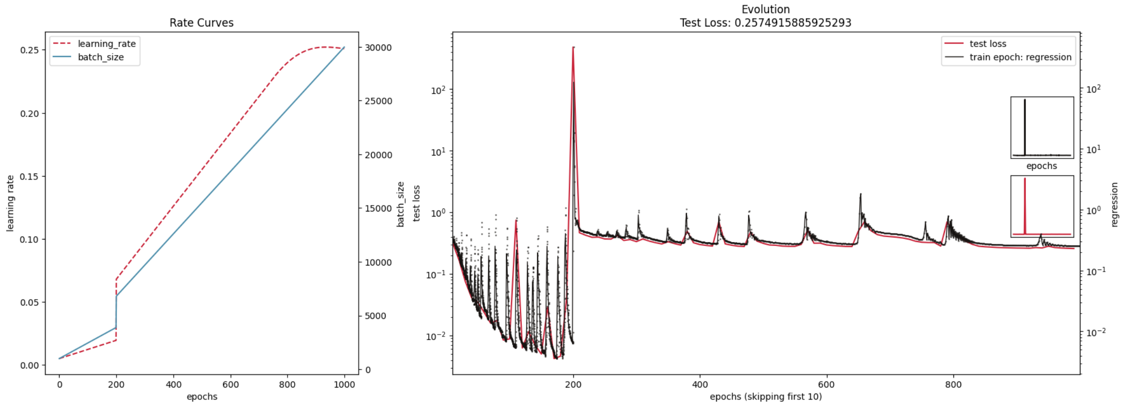
2 Function 1

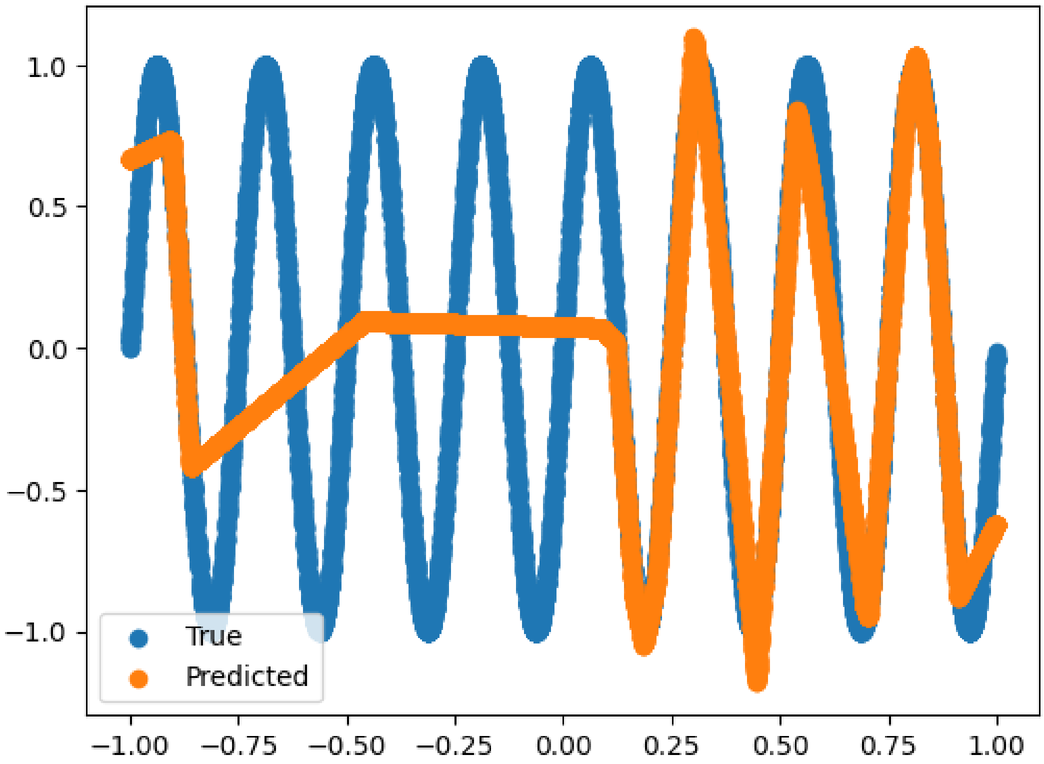
[54]:
```
train_y, test_y = update_y(Ys1)
```

2.1 DNN 1

```
[51]: act = torch.nn.ReLU()
net = DNN(device, dnn_sizes, opttype=opt, act = act, bias=True) # expect 1.6%␣
 ↪error rate
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000)
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
 ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
 ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

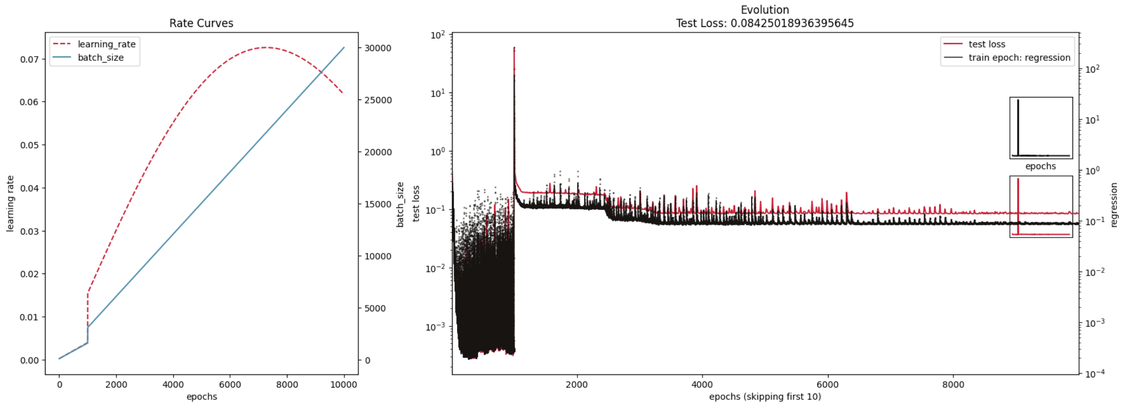
```
100%|          | 1000/1000 [00:09<00:00, 108.38it/s]
```

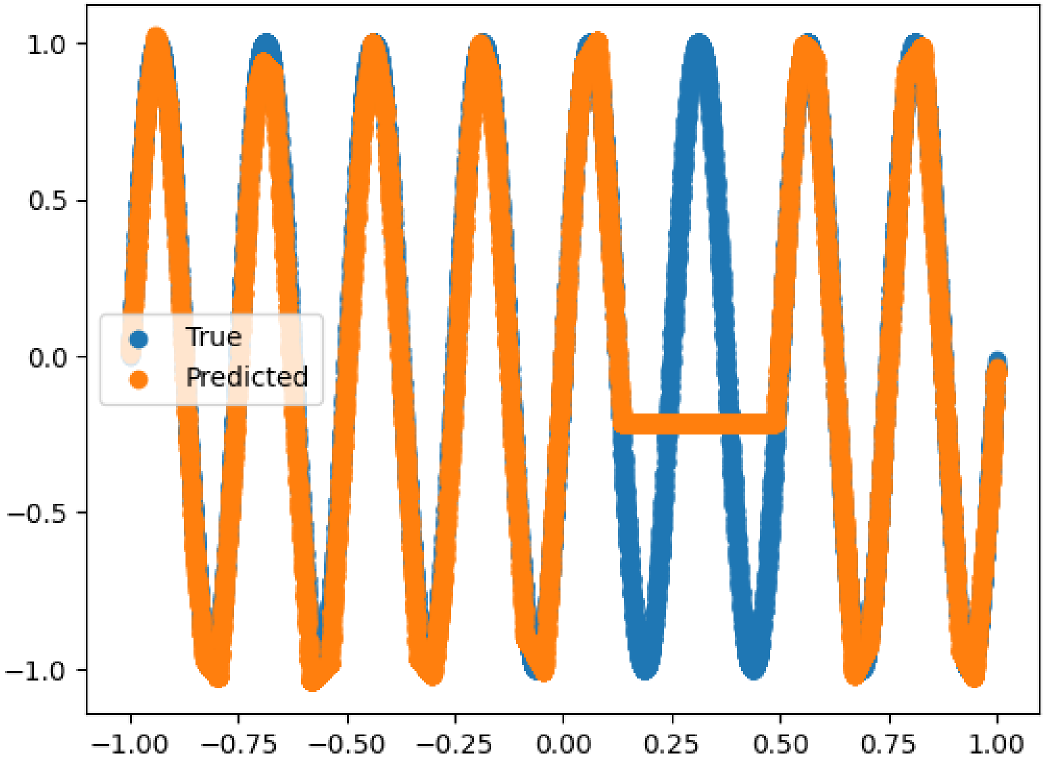




```
[55]: opt2 = lambda x: optim.Adam(x, lr=0.000005)
      act = torch.nn.ReLU()
      net = DNN(device, dnn_sizes, opttype=opt2, act = act, bias=True) # expect 1.6%␣
        ↪error rate
      D.epochs = 10000
      report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=100,␣
        ↪max_batch_size=100, splits =[0.1,0.1], final_percent_lr=0.1)
      D.epochs = 1000
      ecran(net, test_X, test_y, report, classification=False)
      plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
        ↪numpy(),label='True')
      plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
        ↪numpy(),label='Predicted')
      plt.legend()
      plt.show()
```

```
100%|          | 10000/10000 [03:03<00:00, 54.46it/s]
```



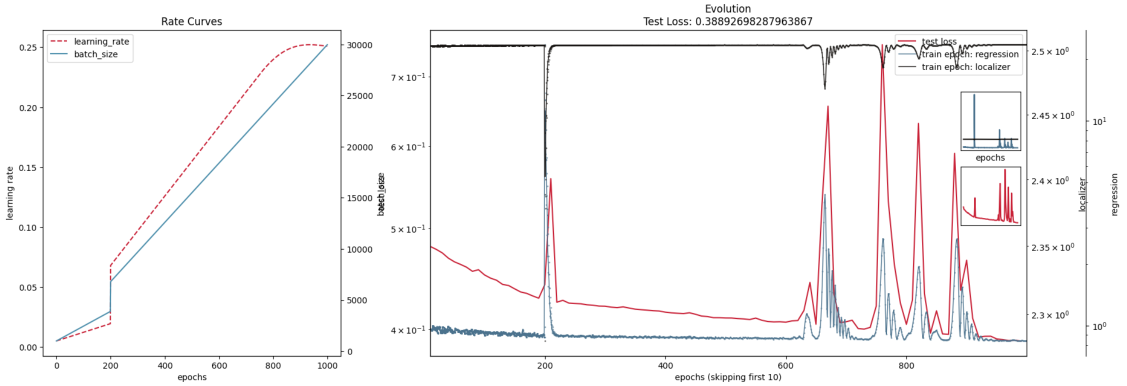


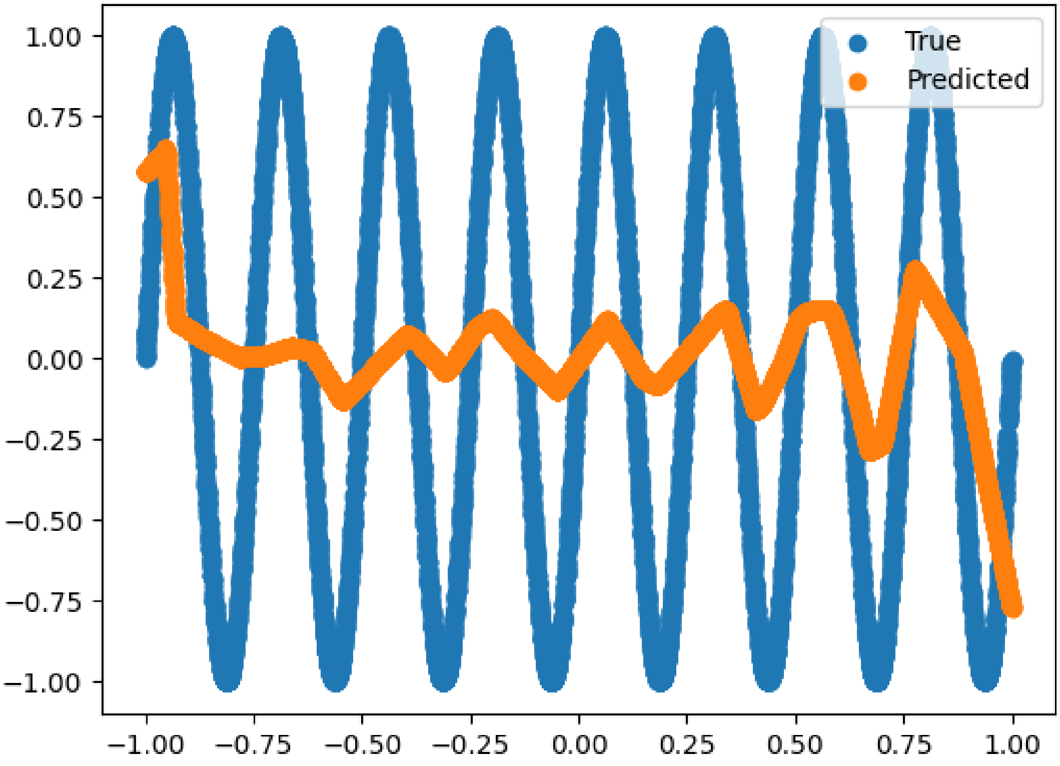
2.2 FL 1

```
[ ]: act = torch.nn.ReLU()
     delta = np.array([1]*len(fl_sizes))*0.05
     k = [ks[1]]*len(fl_sizes)
     net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
      ↪expect 1.6% error rate
```

```
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000)
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
  ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
  ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

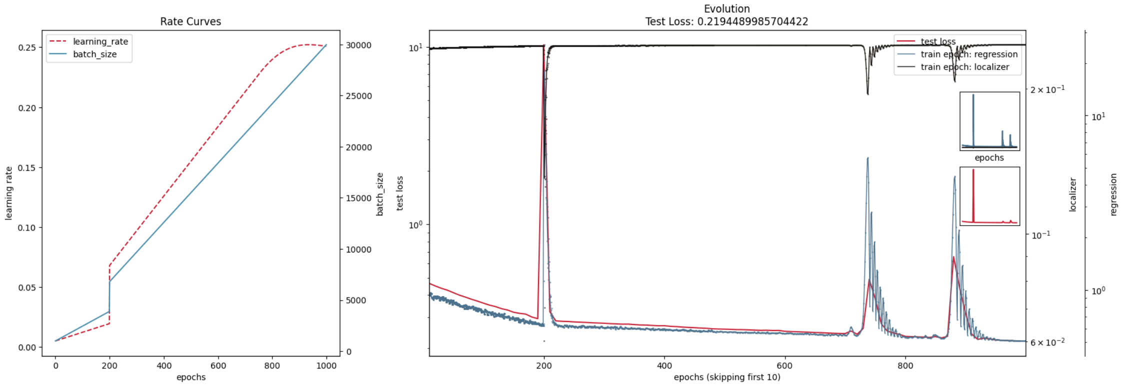
```
100%|          | 1000/1000 [00:43<00:00, 23.07it/s]
```

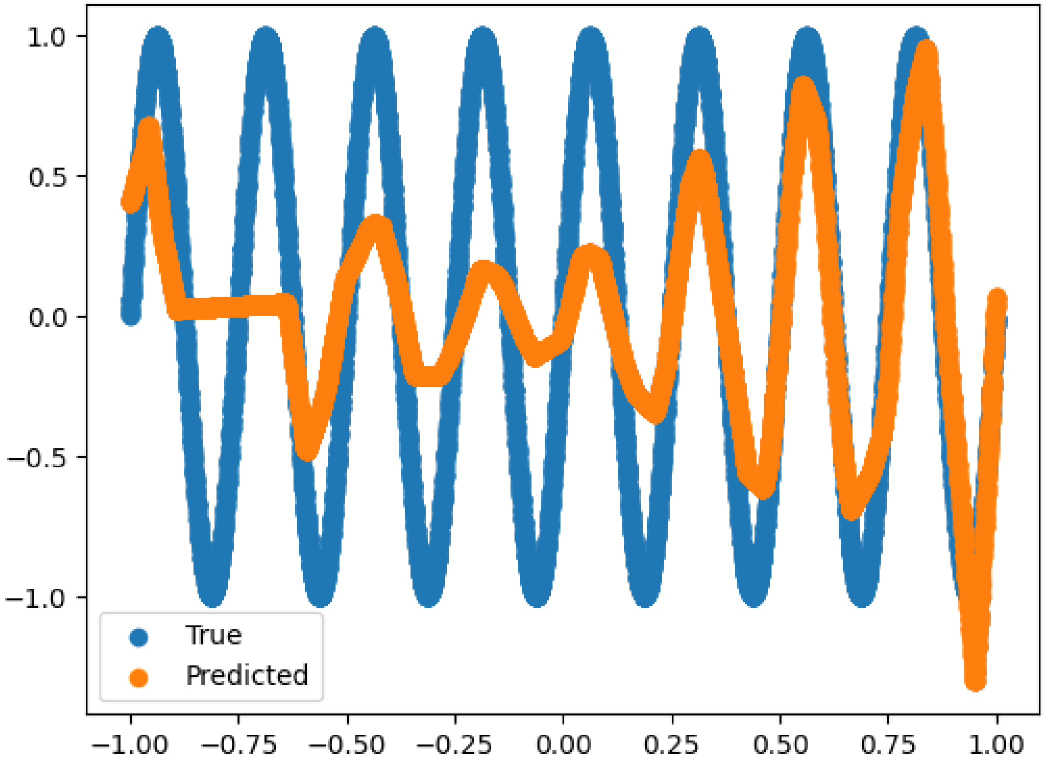




[ ]:
```
act = torch.nn.ReLU()
delta = np.array([1]*len(fl_sizes))*0.005
k = [ks[1]]*len(fl_sizes)
net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
 ↪expect 1.6% error rate
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000)
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
 ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
 ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|      | 1000/1000 [00:44<00:00, 22.23it/s]
```

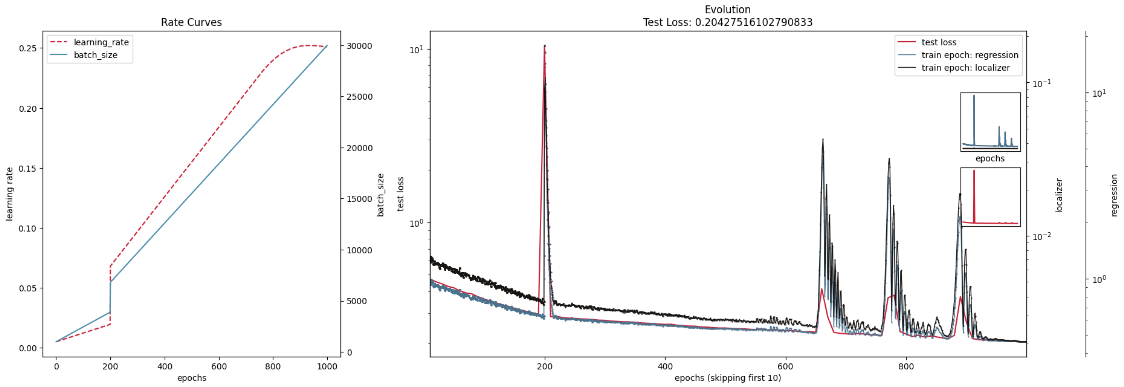




[ ]:

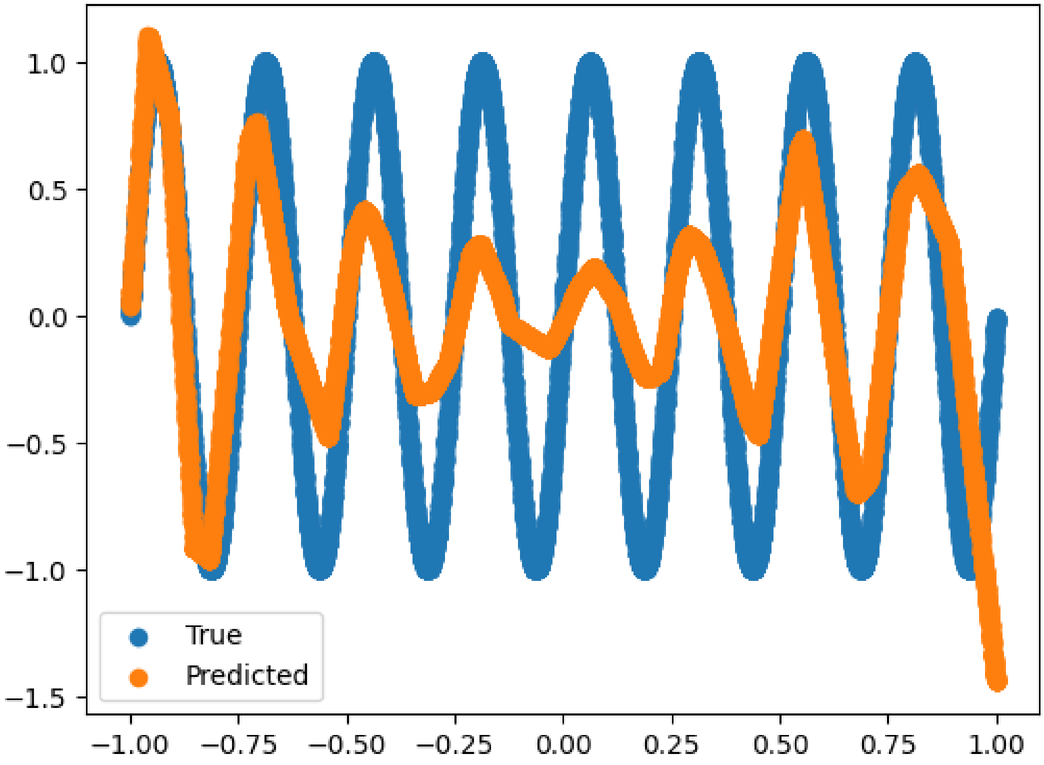
```
act = torch.nn.ReLU()
delta = np.array([1]*len(fl_sizes))*0.00005
k = [ks[1]]*len(fl_sizes)
net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
 ↪expect 1.6% error rate
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000)
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
 ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
 ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 1000/1000 [00:44<00:00, 22.27it/s]
```



Rate Curves

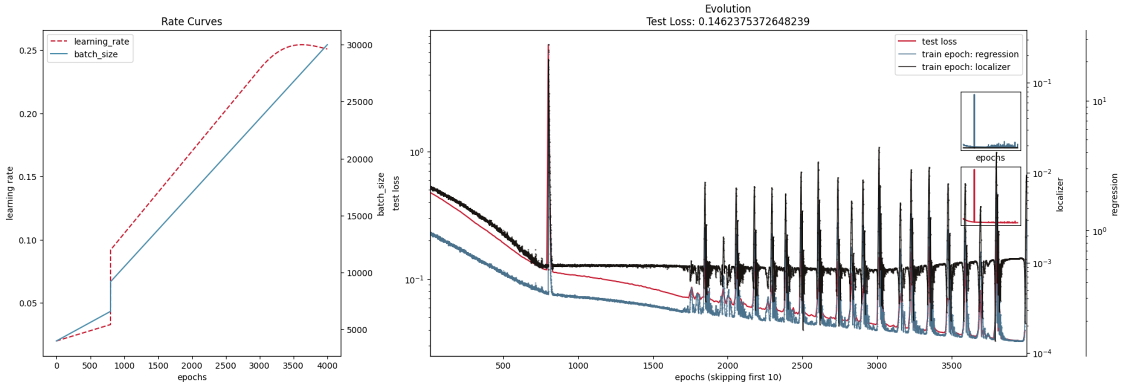
Evolution
Test Loss: 0.20427516102790833

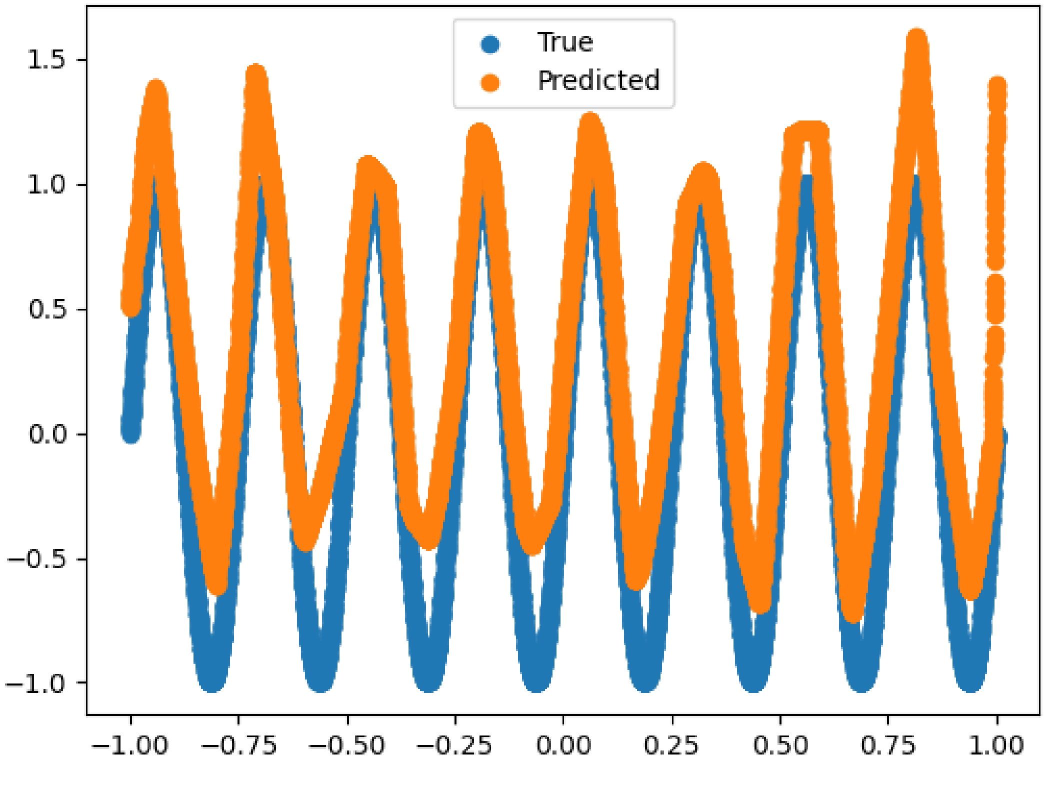


```
[ ]: act = torch.nn.ReLU()
     delta = np.array([1]*len(fl_sizes))*0.00005
     k = [ks[1]]*len(fl_sizes)
     net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
      ↪expect 1.6% error rate
     D.epochs = 4000
     report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=4000)
     D.epochs = 1000
```

```
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 4000/4000 [02:11<00:00, 30.53it/s]
```

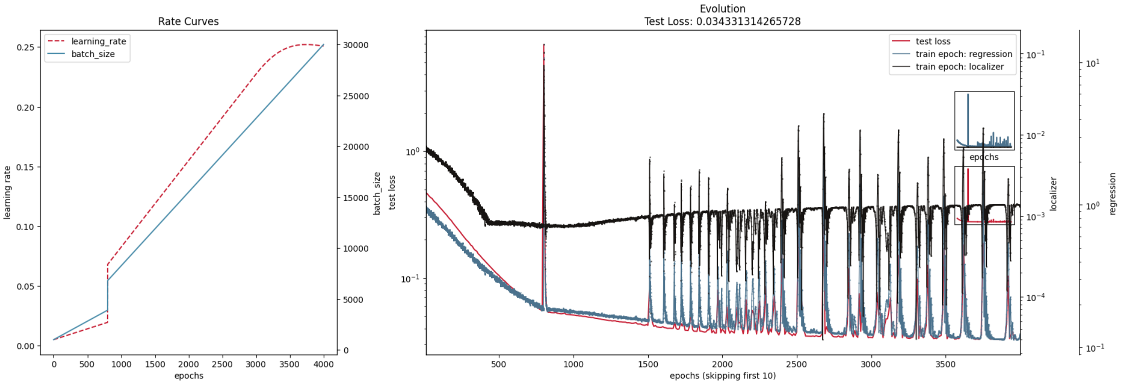


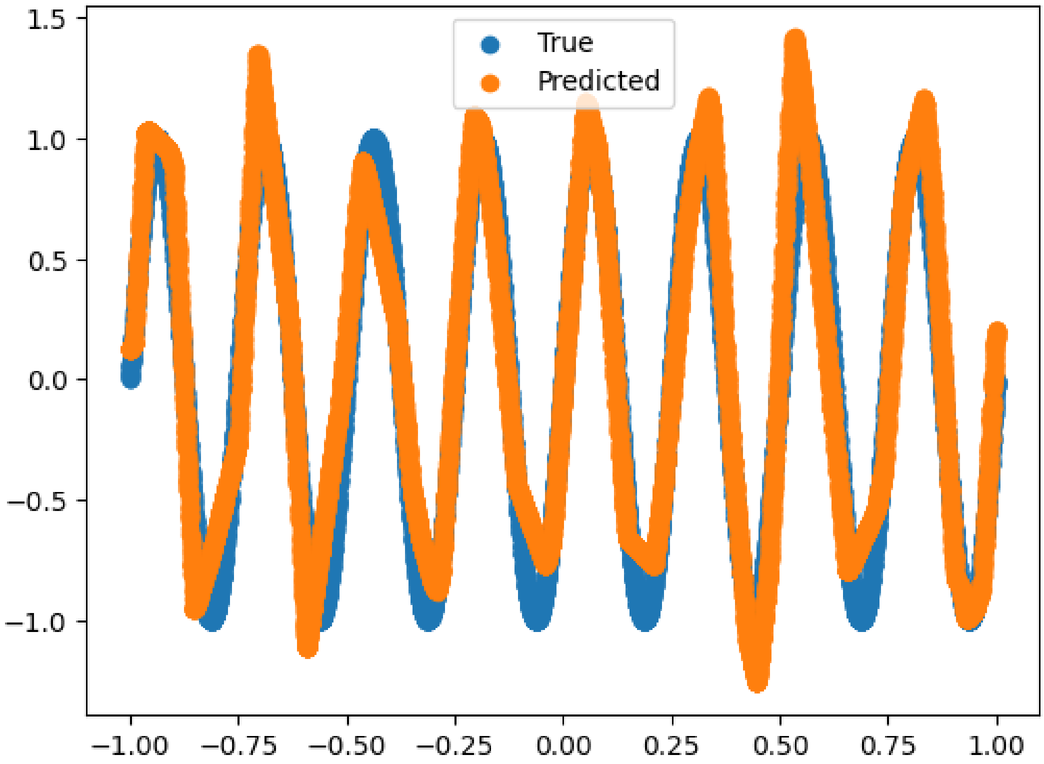


[ ]:

```
act = torch.nn.ReLU()
delta = np.array([1]*len(fl_sizes))*0.00005
k = [ks[1]]*len(fl_sizes)
net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
 ↪expect 1.6% error rate
D.epochs = 4000
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000)
D.epochs = 1000
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
 ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
 ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 4000/4000 [02:50<00:00, 23.53it/s]
```

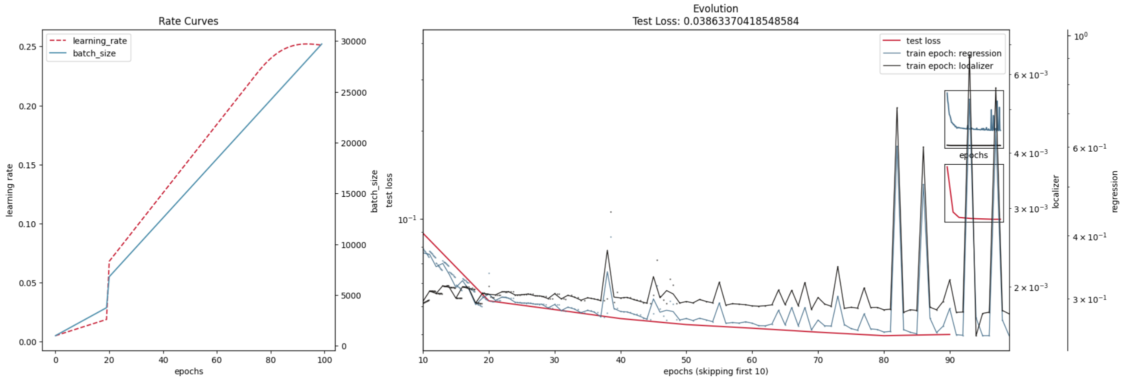


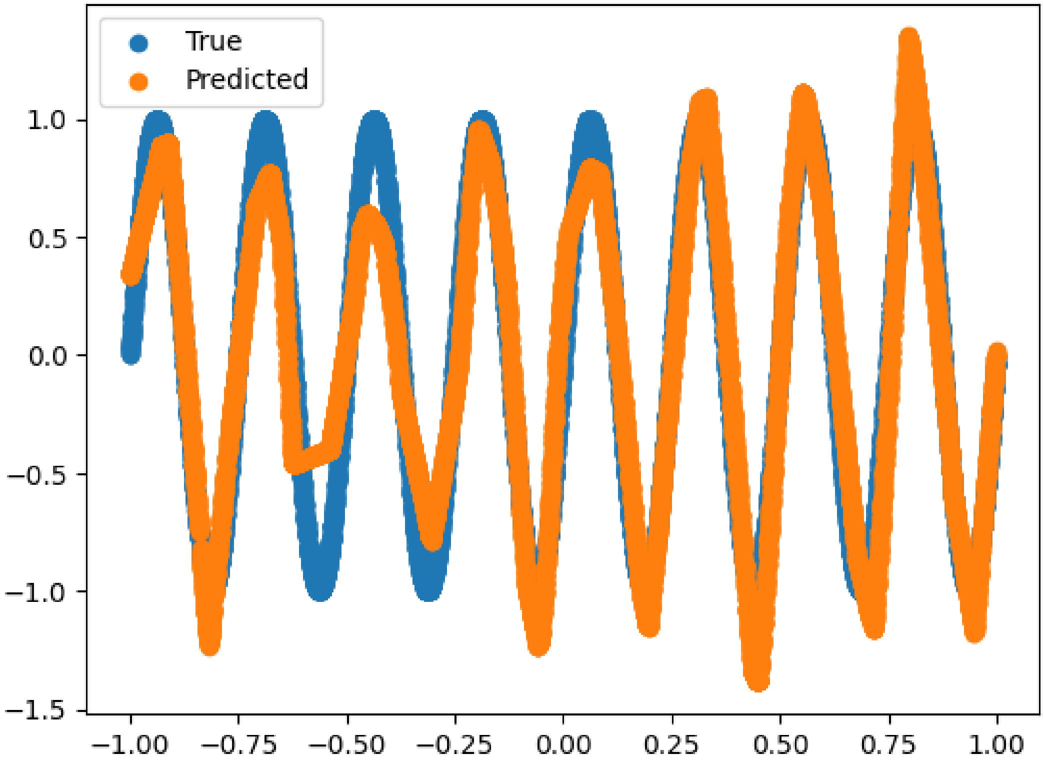


2.3 FL 1 Greedy

[56]:
```
act = torch.nn.ReLU()
delta = np.array([1]*len(fl_sizes))*0.00005
k = [ks[1]]*len(fl_sizes)
net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
↪expect 1.6% error rate
D.epochs = 100
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000,␣
↪repeat_epochs=40)
D.epochs = 1000
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 100/100 [02:57<00:00,  1.77s/it]
```





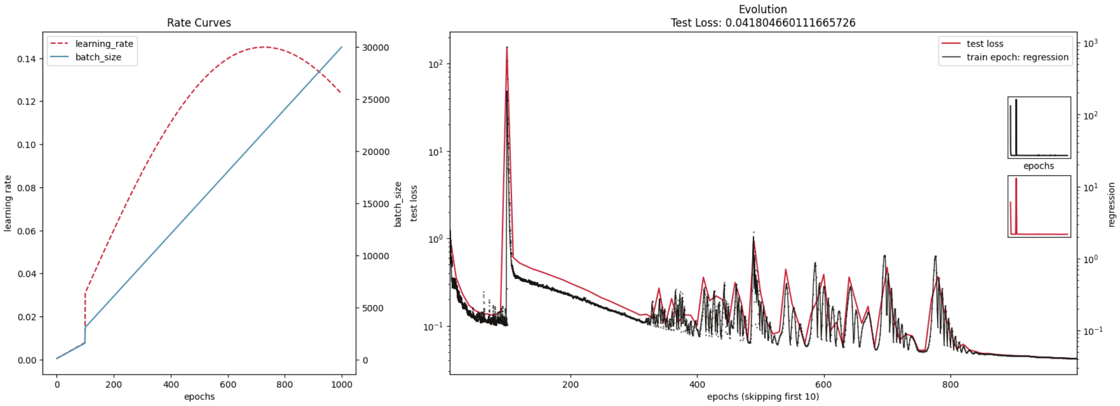
3 Function 2

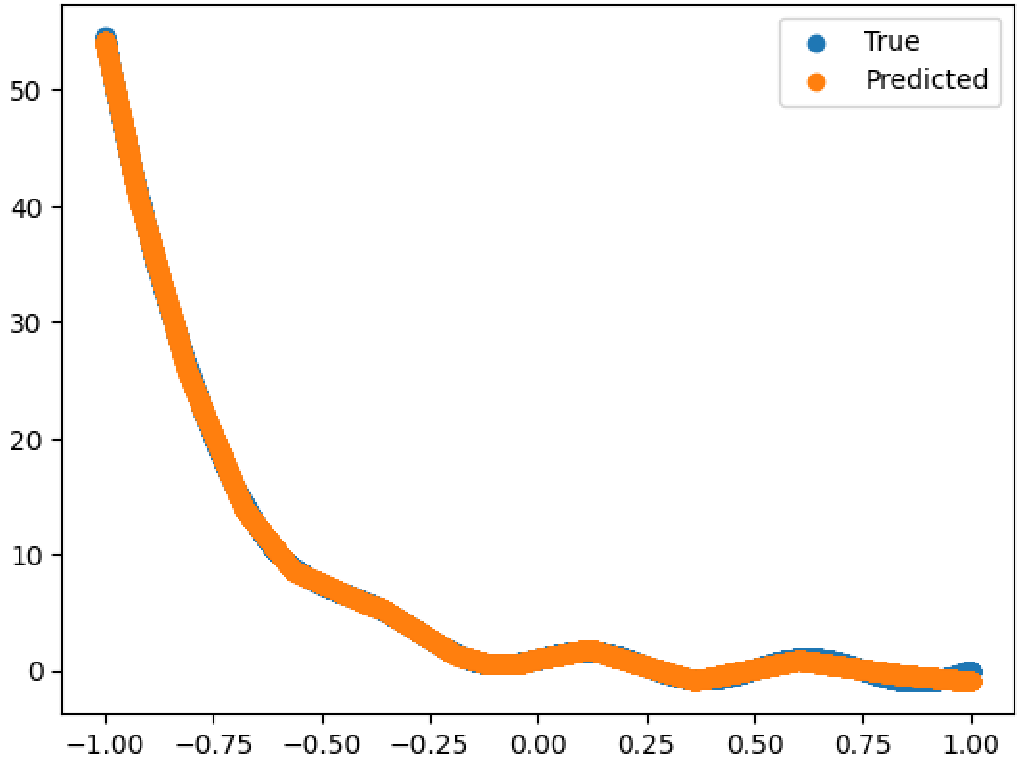
[57]:
```
train_y, test_y = update_y(Ys2)
```

3.1 DNN 2

[38]:
```python
net = DNN(device, dnn_sizes2, opttype=opt, act = act, bias=True) # expect 1.6% error rate
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=100, max_batch_size=100, splits =[0.1,0.1], final_percent_lr=0.1)
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 1000/1000 [00:20<00:00, 48.13it/s]
```

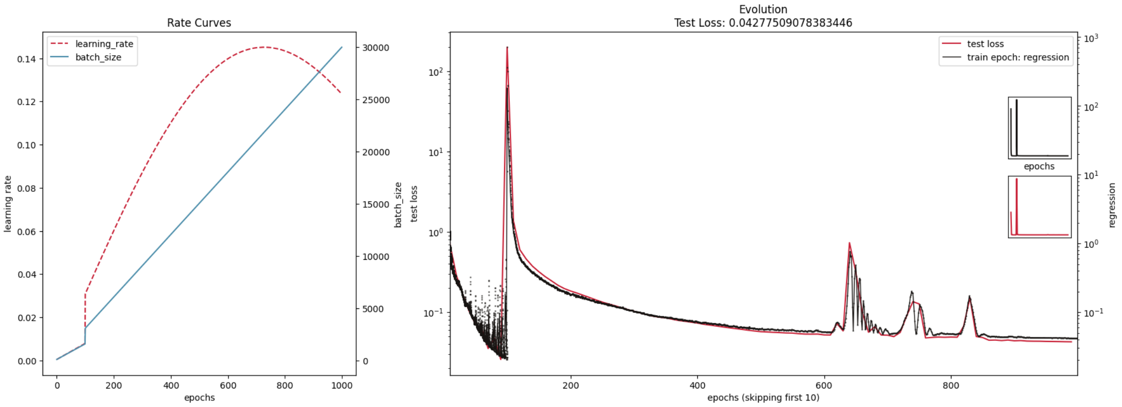


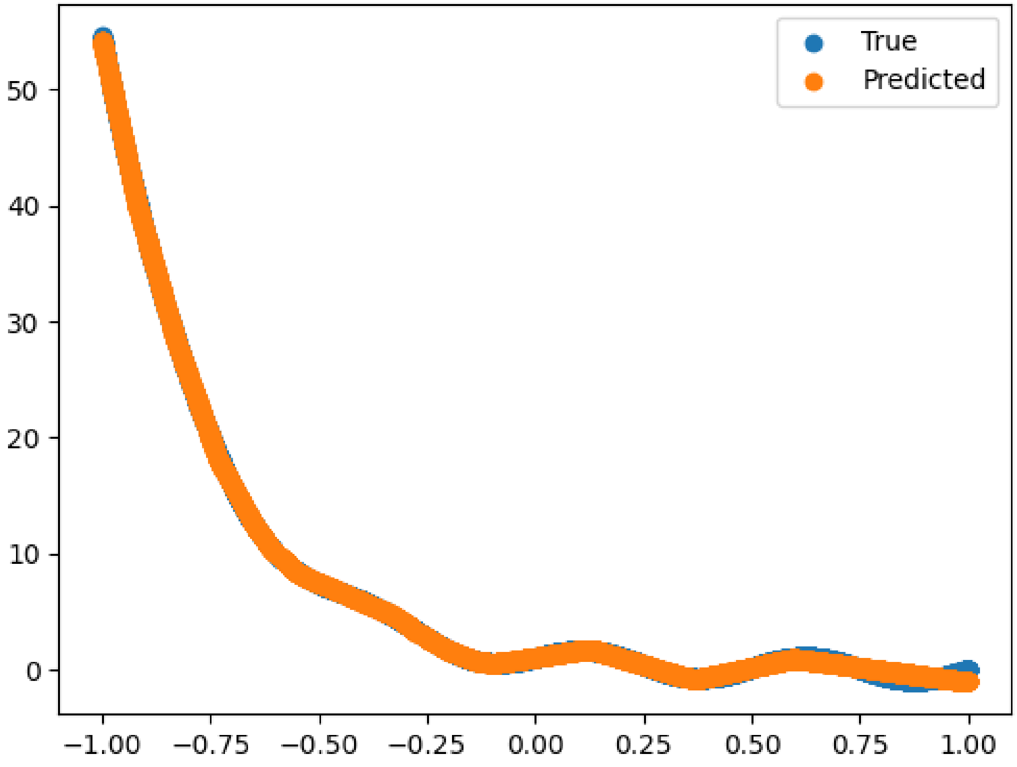


[40]:

```
net = DNN(device, dnn_sizes2, opttype=opt, act = act, bias=True) # expect 1.6% error rate
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=100, splits =[0.1,0.1], final_percent_lr=0.1)
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 1000/1000 [00:20<00:00, 48.86it/s]
```

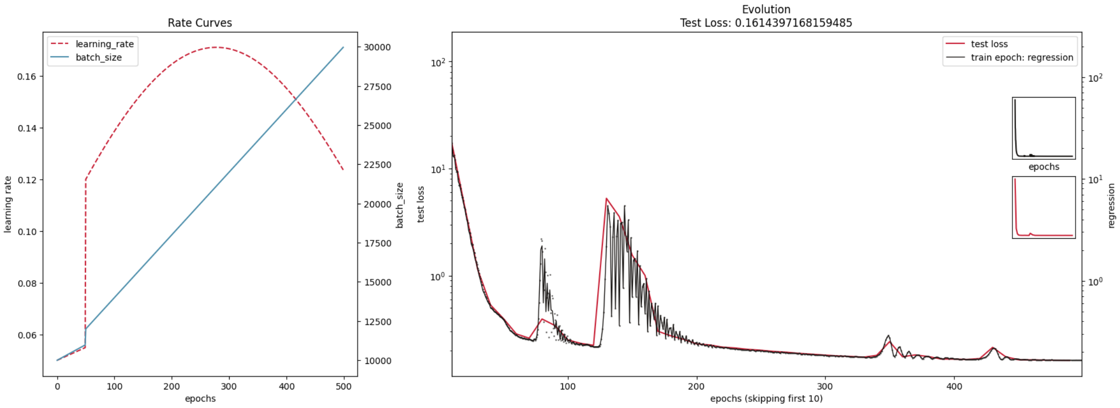


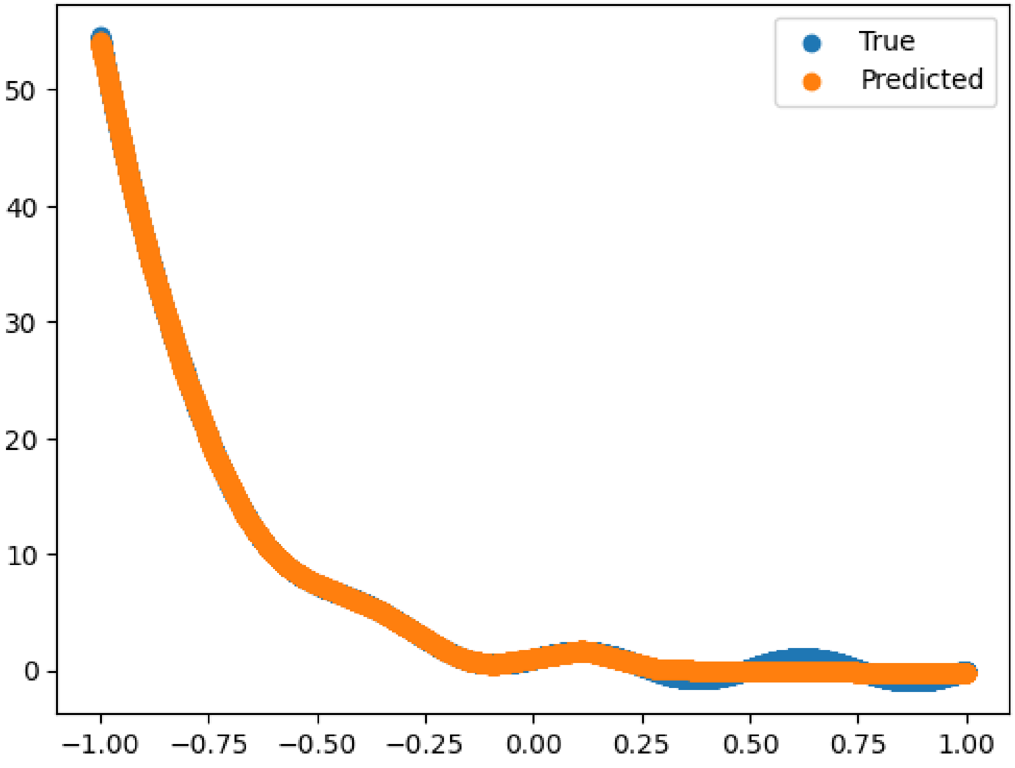


```
[45]: net = DNN(device, dnn_sizes2, opttype=opt, act = act, bias=True) # expect 1.6% error rate
      D.epochs=500
      report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=10000, splits =[0.1,0.1], final_percent_lr=0.1)
      D.epochs=1000
      ecran(net, test_X, test_y, report, classification=False)
```

```
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
 ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
 ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 500/500 [00:01<00:00, 287.94it/s]
```

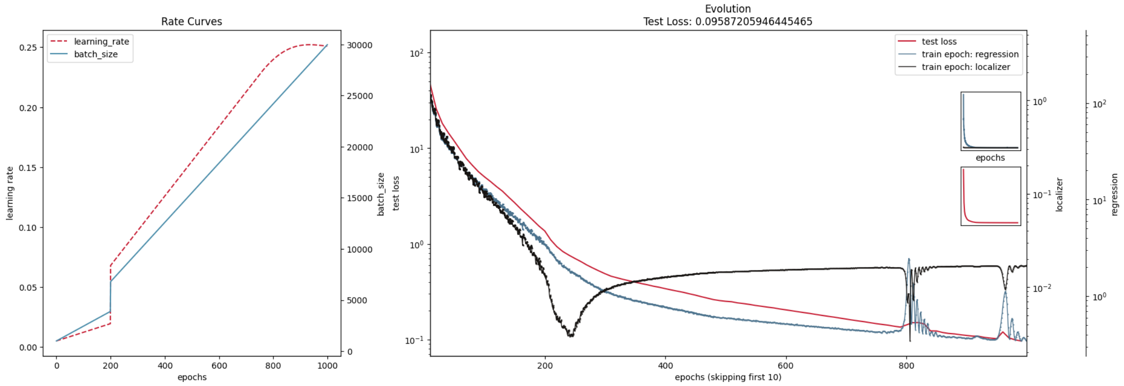


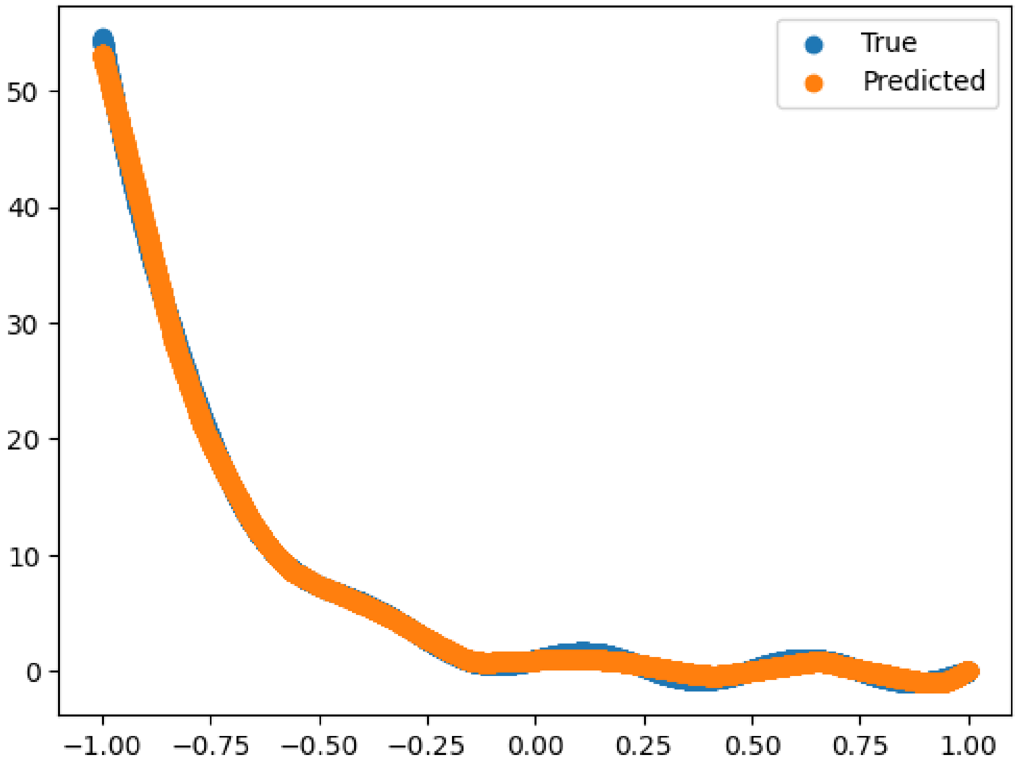


3.2 FL 2

```
[39]: act = torch.nn.ReLU()
delta = np.array([1]*len(fl_sizes))*0.00005
k = [ks[2]]*len(fl_sizes)
net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
 ↪expect 1.6% error rate
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000)
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
 ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
 ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 1000/1000 [00:44<00:00, 22.36it/s]
```

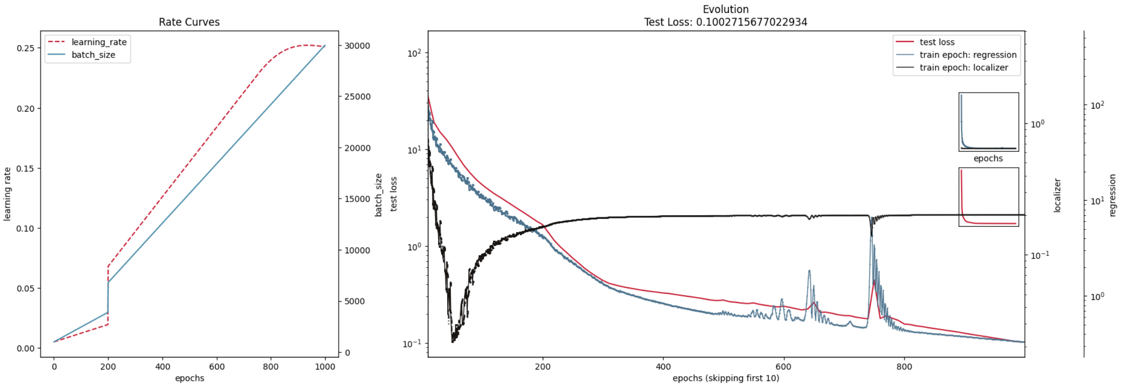


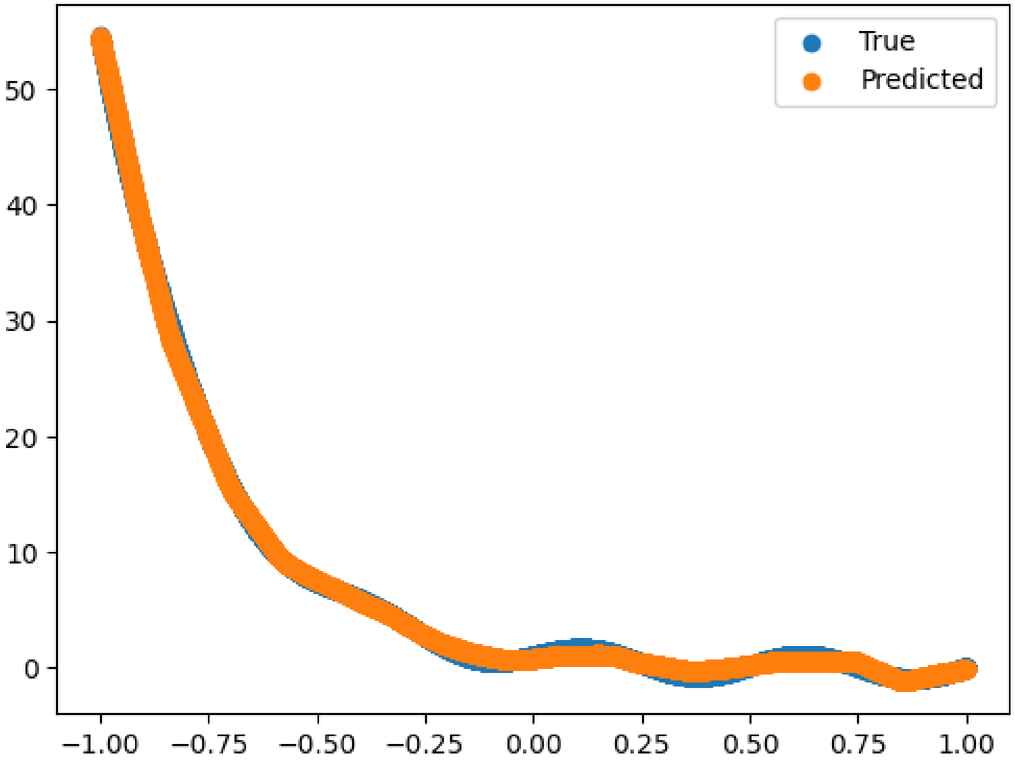


[43]:

```
act = torch.nn.ReLU()
delta = np.array([1]*len(fl_sizes))*0.0005
k = [ks[2]]*len(fl_sizes)
net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
  ↪expect 1.6% error rate
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000)
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
  ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
  ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|          | 1000/1000 [00:43<00:00, 22.98it/s]
```

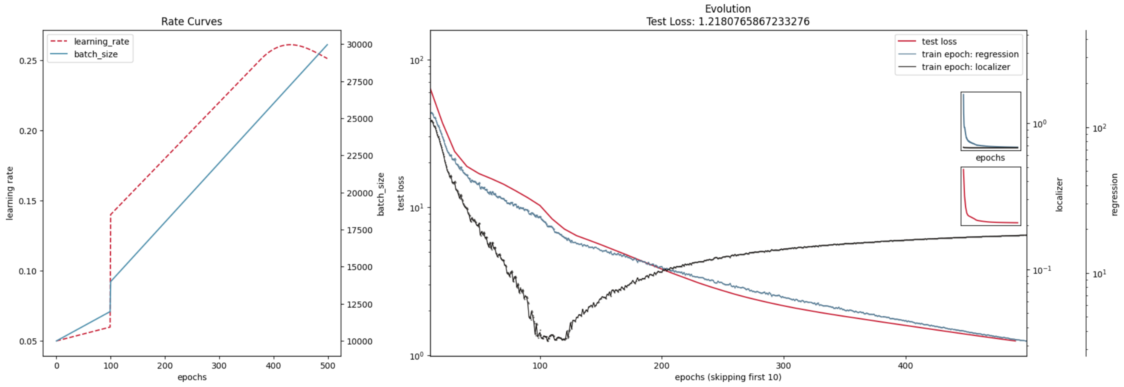


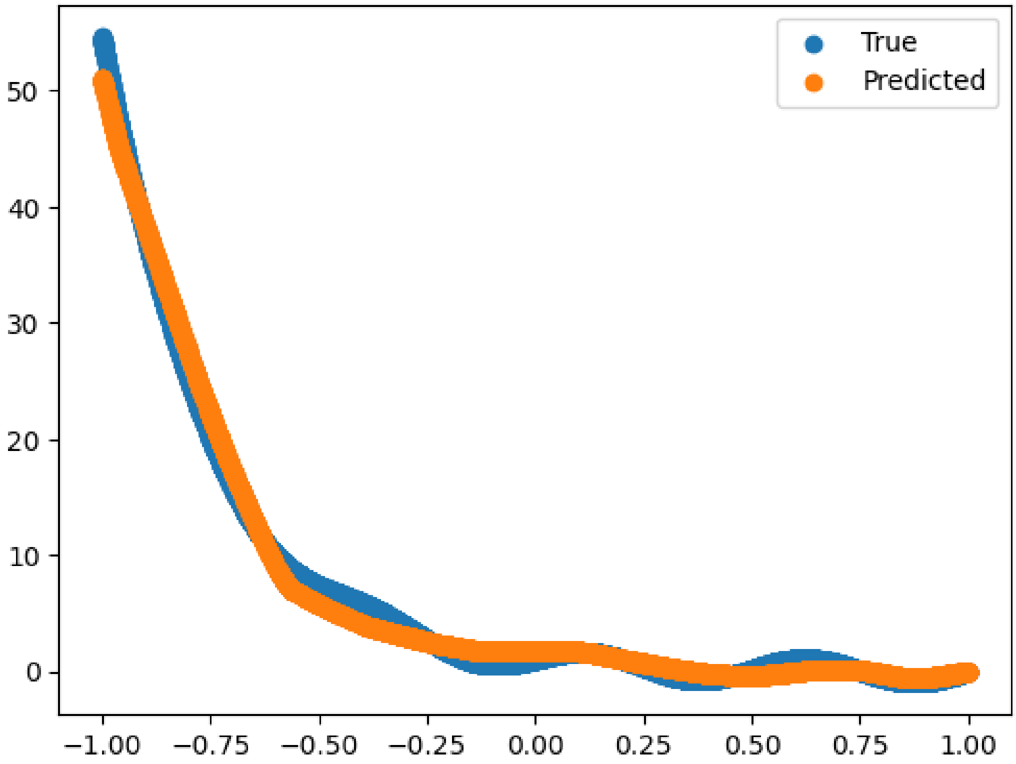


[44]:
```
act = torch.nn.ReLU()
delta = np.array([1]*len(fl_sizes))*0.0005
k = [ks[2]]*len(fl_sizes)
net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) # expect 1.6% error rate
D.epochs = 500
report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=10000)
D.epochs = 1000
```

```
ecran(net, test_X, test_y, report, classification=False)
plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
 ↪numpy(),label='True')
plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
 ↪numpy(),label='Predicted')
plt.legend()
plt.show()
```

```
100%|      | 500/500 [00:14<00:00, 35.56it/s]
```



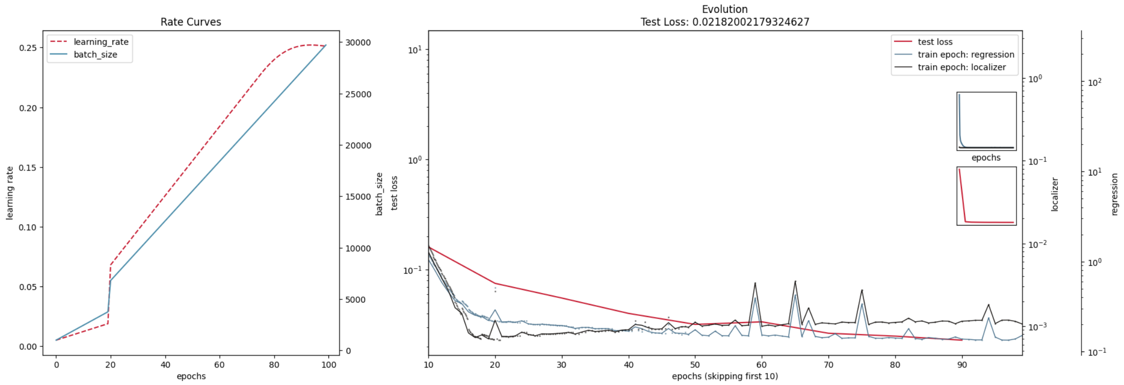


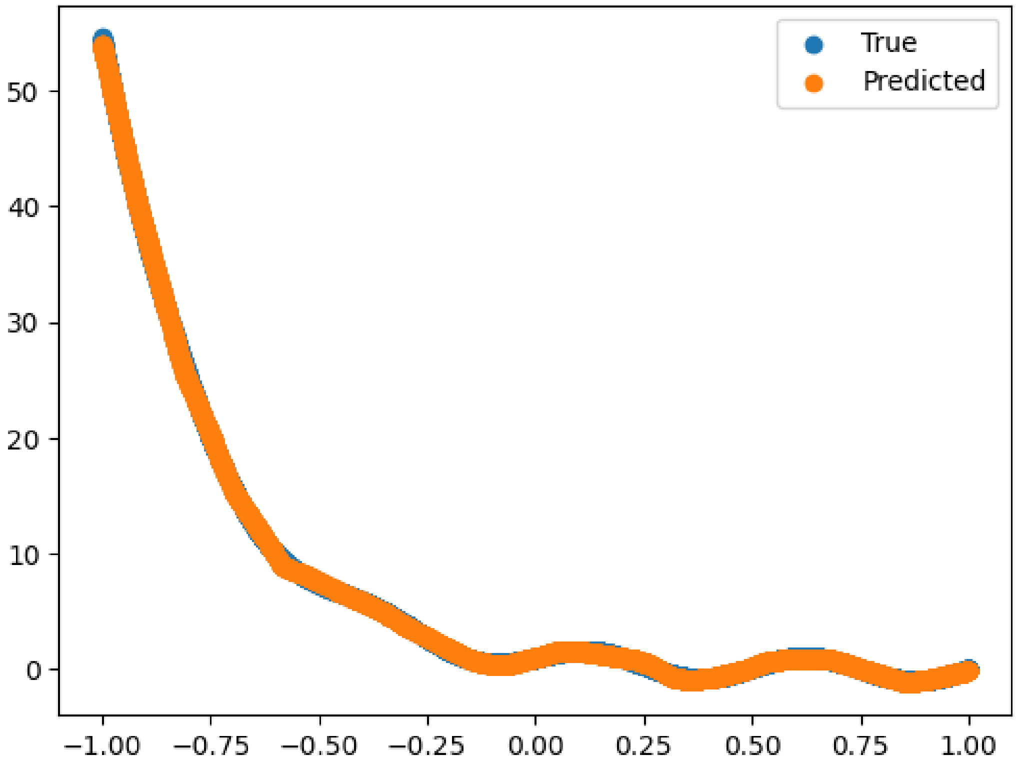
```
[ ]:
```

3.3 FL 2 Greedy

```
[58]: act = torch.nn.ReLU()
      delta = np.array([1]*len(fl_sizes))*0.00005
      k = [ks[1]]*len(fl_sizes)
      net = FL(device, fl_sizes, delta, k, opttype=opt, act = act, bias=True) #␣
        ↪expect 1.6% error rate
      D.epochs = 100
      report = D.train(net, train_X, train_y, test_X, test_y, start_batch_size=1000,␣
        ↪repeat_epochs=40)
      D.epochs = 1000
      ecran(net, test_X, test_y, report, classification=False)
      plt.scatter(test_X.detach().cpu().numpy(),test_y.detach().cpu().
        ↪numpy(),label='True')
      plt.scatter(test_X.detach().cpu().numpy(),net(test_X).detach().cpu().
        ↪numpy(),label='Predicted')
      plt.legend()
      plt.show()
```

```
100%|          | 100/100 [03:03<00:00,  1.84s/it]
```





True
Predicted
50
40
30
20
10
0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

```
[ ]:
```