MASTER OF COMPUTER APPLICATIONS

SEMESTER 1

DISCRETE MATHEMATICS AND GRAPH THEORY

Unit 7

Fundamentals of Number System

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1. INTRODUCTION

It is important to distinguish between a function's description and its execution. The function is the nebulous mathematical entity that, whether or not it is ever discussed, still exists. However, we need a means to explain the function when we do wish to discuss it. There are several methods to describe a certain function. The Rule of Four, which states that any function may be expressed in one of four different ways—algebraically (as a formula), numerically (as a table), graphically, or orally—is discussed in several calculus textbooks.

This Section examines a particular kind of connection known as a function. It is among the most important ideas in mathematics. A function may be thought of as a rule that creates new elements out of certain existing ones. The word "map" or "mapping" is one of several that are used to describe a function

1.1. Objectives

The objective of this topic is to make the students to

- *Familiarise with number system.*
- *Learn conversion from one number system to another*

1.2. RADIX OR BASE:-

The radix or base of a number system is defined as the number of different digits which can occur in each position in the number system.

1.3. RADIX POINT :-

The generalized form of a decimal point is known as radix point. In any positional number system the radix point divides the integer and fractional part.

Nr = [Integer part . Fractional part]

↑

Radix point

2. NUMBER SYSTEM

A number system provides a particular way to represent numbers. Additionally, it enables users to do mathematical operations like addition, subtraction, and division, which are crucial in digital and computerised realms. Understand the fundamentals of the number system, including conversion and application.

Every value that is saved to or retrieved from computer memory uses a certain number system, which is the method used to represent numbers in the computer system architecture.

In general a number in a system having base or radix ‘ r ’ can be written as

$$a_n\, a_{n-1}\, a_{n-2} \ldots\ldots\ldots\ldots\ldots\; a_0 \,.\, a_{-1}\;\; a_{-2} \ldots\ldots\ldots\ldots a_{-m}$$

This will be interpreted as

$$Y = a_n \times r^n + a_{n-1} \times r^{n-1} + a_{n-2} \times r^{n-2} + \cdots + a_0 \times r^0 + a_{-1} \times r^{-1} + a_{-2} \times r^{-2} + \cdots + a_{-m} \times r^{-m}$$

Where,

Y = value of the entire number

a_n = the value of the nth digit

r = radix

3. TYPES OF NUMBER SYSTEM

The four common types of Number systems are

1. Binary number system

2. Octal number system

3. Decimal number system

4. Hexadecimal number system

3.1. Binary number system:

This number system's base, or radix, is 2.

There are two distinct symbols in it.

The two utilized symbols are 0 and 1.

A binary digit is called a bit.

The position weights in the binary is given as

$\cdots$	2^3	2^2	2^1	2^0	$\cdot$	2^{-1}	2^{-2}	2^{-3}	2^{-4}	$\cdots$

$4\ bit\ binary\ word \Longrightarrow\ nibble$

$8\ bit\ binary\ word \Longrightarrow\ byte$

$16\ bit\ binary\ word \Longrightarrow\ word$

$32\ bit\ binary\ word \Longrightarrow\ double\ word$

3.2. Octal number system:

The Octal Number System is an eight-digit number system that uses the numbers 0 to 7 (i.e., 0, 1, 2, 3, 4, 5, and 7) as its foundation.

Numbers having an eight-digit base are referred regarded as being octal.

The position weights in the octal system is given as

$\cdots$	8^3	8^2	8^1	8^0	$\cdot$	8^{-1}	8^{-2}	8^{-3}	8^{-4}	$\cdots$

3.3. Decimal number system:

Numbers 0 to 9 are used in the decimal number system, which we utilise on a daily basis.

These numbers are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The base number of the decimal number system is 10, since there are ten numbers in this system.

The base of any integer that is expressed without a base is 10.

The position weights in decimal system is given as

…	10^3	10^2	10^1	10^0	.	10^{-1}	10^{-2}	10^{-3}	10^{-4}	…

Example:

(i) $7693 = 7 \times 10^3 + 6 \times 10^2 + 9 \times 10^1 + 3 \times 10^0$

$= 7 \times 1000 + 6 \times 100 + 9 \times 10 + 3 \times 1$

$= 7000 + 600 + 90 + 3$

(ii) $1936.46 = 1 \times 10^3 + 9 \times 10^2 + 3 \times 10^1 + 6 \times 10^0 + 4 \times 10^{-1} + 6 \times 10^{-2}$

$= 1000 + 900 + 30 + 6 + 0.4 + 0.06$

3.4. Hexadecimal number system:

'Hexa' and 'deci' are the two terms that make up hexadecimal, where 'Hexa' stands for six and 'deci' for ten.

The letters A through F and the numbers 0 through 9 are represented using a 16-digit system known as the hexadecimal number system.

In other words, the first nine digits or numerals are represented as numbers, while the next six digits are represented by the letters A through F.

The decimal number system, which has a base of nine, and the hexadecimal number system are quite similar. Following nine digits, the tenth digit is denoted by a symbol: 10, 11, 12, 13, 14, and 15 are denoted by A, B, C, and E, respectively.

The position weight in the hexadecimal number system is given as

…	16^3	16^2	16^1	16^0	.	16^{-1}	16^{-2}	16^{-3}	16^{-4}	…

Number System

Decimal (Base 10)	**Binary (Base 2)**	**Octal (Base 8)**	**Hexadecimal (Base 16)**

0	0000	00	0
1	0001	01	1
2	0010	02	2
3	0011	03	3
4	0100	04	4
5	0101	05	5
6	0110	06	6
7	0111	07	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

4. CONVERSION FROM ONE NUMBER SYSTEM TO ANOTHER

4.1. Conversion of decimal number to any number system

Step 1 convert the integer part by doing successive division using the radix of asked number systems.

Step 2 convert the fractional part by doing successive multiplication using radix of asked number system

4.1.1. Conversion of Decimal number into Binary number (Integer Number)

Procedure:

1. Divide the decimal no by the base 2, note the remainder.
2. Continue to divide the quotient by 2 until there is nothing left, keeping the track of the remainders from each step.
3. List the remainder values in reverse order which is number's binary equivalent.

Example: Convert 105 decimal number in to it's equivalent binary number

Solution

2	105	Reminders
2	52	1
2	26	0
2	13	0
2	6	1
2	3	0
2	1	1
	0	1

LSB

MSB

$$(105)_{10} = (1101001)_2$$

4.1.2. Conversion of Decimal number into Binary number (Fractional Number

Procedure:

1. Multiply the given fractional number by base 2.
2. Record the carry(integer value) generated in this multiplication as MSB.
3. Multiply only the fractional number of the product in step 2 by 2 and record the carry as the next bit to MSB.
4. Repeat the steps 2 and 3 up to 5 bit. The last carry will represent the LSB of equivalent binary number

Example: Convert 0.42 decimal number in to it's equivalent binary number

Solution

MSB

0.42 X 2	=	0.84	0
0.84 X 2	=	1.68	1
0.68 X 2	=	1.36	1
0.36 X 2	=	0.72	0
0.72 X 2	=	1.44	1

LSB

$$(0.42)_{10} = (0.01101)_2$$

4.1.3. Conversion of Decimal Number into Octal Number (Integer Number)

Procedure:

1. Divide the decimal no by the base 8, note the remainder.
2. Continue to divide the quotient by 8 until there is nothing left, keeping the track of the remainders from each step.
3. List the remainder values in reverse order which is number's octal equivalent.

Example: Convert 204 decimal number in to it's equivalent octal number.

Solution

			Reminders
8	204	LSB	
8	25		4
8	3		1
	0	MSB	3

$(204)_{10} = (314)_8$

4.1.4. Conversion of Decimal Number into Octal Number (Fractional Number)

Procedure:

1. Multiply the given fractional number by base 8.
2. Record the carry generated in this multiplication as MSB.
3. Multiply only the fractional number of the product in step 2 by 8 and record the carry as the next bit to MSB.
4. Repeat the steps 2 and 3 up to 5 bit. The last carry will represent the LSB of equivalent octal number

Example: Convert 0.6234 decimal number in to it's equivalent Octal number.

Solution

0.6234X8	=	4.9872	4	MSB
0.9872X8	=	7.8976	7	
0.8976X8	=	7.1808	7	
0.1808X8	=	1.4464	1	LSB

0.4464X8 = 3.5712 3

4.1.5. Conversion of Decimal Number into Hexadecimal Number (Integer Number)

Procedure:

1. Divide the decimal no by the base 16, note the remainder.
2. Continue to divide the quotient by 16 until there is nothing left, keeping the track of the remainders from each step.
3. List the remainder values in reverse order which is Number's Hex equal.

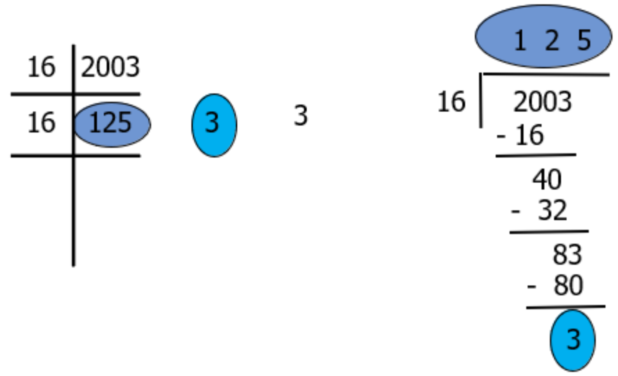
Example: Convert 2003 decimal number in to it's equivalent Hex number.

Solution

16	2003	Reminders	LSB
16	125	3	
16	7	13=D	
	0	7	MSB

$(2003)_{10} = (7D3)_{16}$

Calculation



4.1.6. Conversion of Decimal Number into Hexadecimal Number (Fractional Number)

Procedure:

1. Multiply the given fractional number by base 16.
2. Record the carry generated in this multiplication as MSB.
3. Multiply only the fractional number of the product in step 2 by 16 and record the carry as the next bit to MSB.
4. Repeat the steps 2 and 3 up to 5 bits. The last carry will represent the LSB of equivalent hex number

Example: Convert 0.122 decimal number in to it's equivalent Hex number.

Solution

0.122x16	=	1.952	1	1	MSB
0.952x16	=	15.232	15	F	
0.232x16	=	3.712	3	3	
0.712x16	=	11.392	11	B	
0.392x16	=	6.272	6	6	LSB

$$(0.122)_{10} = (0.1F3B6)_{16}$$

4.1.7. Conversion of Binary Number into Decimal Number

Procedure:

1. Write down the binary number.
2. Write down the weights for different positions.
3. Multiply each bit in the binary number with the corresponding weight to obtain product numbers to get the decimal numbers.

4. Add all the product numbers to get the decimal

Example: Convert 1011.01 binary number in to it's equivalent decimal number.

Solution

Binary No.	1	0	1	1	.	0	1
Positional Weights	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}

$$= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2}$$
$$= 8 + 0 + 2 + 1 + 0 + 0.25$$
$$= 11.25$$

4.1.8. Conversion of Octal Number into Decimal Number

Procedure:

1 Write down the octal number.

2 Write down the weights for different positions.

3 Multiply each bit in the binary number with the corresponding weight to obtain product numbers to get the decimal numbers.

4 Add all the product numbers to get the decimal equivalent.

Example: Convert 305.75 octal number in to it's equivalent decimal number.

Solution

Octal No	3	0	5	.	7	5
Positional Weights	8^2	8^1	8^0	.	8^{-1}	8^{-2}

$$= \mathbf{3 \times 8^2 + 0 \times 8^1 + 5 \times 8^0 + 7 \times 8^{-1} + 5 \times 8^{-2}}$$
$$= \mathbf{192 + 0 + 5 + 0.875 + 0.078125}$$
$$= \mathbf{197.953125}$$

4.1.9.Conversion of Hexadecimal Number into Decimal Number

Procedure:

1. Write down the hex number.
2. Write down the weights for different positions.

3. Multiply each bit in the binary number with the corresponding weight to obtain product numbers to get the decimal numbers.
4. Add all the product numbers to get the decimal equivalent

Example: Convert 586 hex number in to it's equivalent decimal number.

Solution

Octal No	5	8	6
Positional Weights	16^2	16^1	16^0

$$= \mathbf{5 \times 16^2 + 8 \times 16^1 + 6 \times 16^0}$$

$$= \mathbf{1280 + 128 + 6}$$

$$= \mathbf{1414}$$

5. BINARY ARITHMETIC

Binary arithmetic is essential part of all the digital computers and many other digital system.

5.1. Binary Addition

It is a key for binary subtraction, multiplication, division. There are four rules of binary addition.

Case	A+B	Sum	Carry
1	0+0	0	0
2	0+1	1	0
3	1+0	1	0
4	1+1	0	1

In fourth case, a binary addition is creating a sum of (1 + 1 = 10) i.e. 0 is written in the given column and a carry of 1 over to the next column.

Example – Addition

0011010 + 001100 = 00100110

```
  1 1         carry
  0 0 1 1 0 1 0   = 26₁₀
+ 0 0 0 1 1 0 0   = 12₁₀
---------------
  0 1 0 0 1 1 0   = 38₁₀
```

5.2. Binary Subtraction

Subtraction and Borrow, these two words will be used very frequently for the binary subtraction. There are four rules of binary subtraction.

Case	A-B	Subtract	Borrow
1	0-0	0	0
2	1-0	1	0

3	1-1	0	0
4	0-1	0	1

Example – Subtraction

0011010 - 001100 = 00001110

```
   1 1          borrow
 0 0 1 1 0 1 0  = 26₁₀
-0 0 0 1 1 0 0  = 12₁₀
 -------------
 0 0 0 1 1 1 0  = 14₁₀
```

5.3. Binary Multiplication

Binary multiplication is similar to decimal multiplication. It is simpler than decimal multiplication because only 0s and 1s are involved. There are four rules of binary multiplication.

Case	AxB	Multiplication
1	0x0	0
2	0x1	0
3	1x0	0
4	1x1	1

Example – Multiplication

Example:

0011010 x 001100 = 100111000

```
        0 0 1 1 0 1 0  = 26₁₀
       x0 0 0 1 1 0 0  = 12₁₀
   -------------------
        0 0 0 0 0 0 0
      0 0 0 0 0 0 0
    0 0 1 1 0 1 0
  0 0 1 1 0 1 0
   -------------------
  0 1 0 0 1 1 1 0 0 0  = 312₁₀
```

5.4. Binary Division

Binary division is similar to decimal division. It is called as the long division procedure.

Example – Division

101010 / 000110 = 000111

```
              1 1 1       = 7₁₀
000110 ) 1 0 1 0 1 0      = 42₁₀
        -1 1 0            = 6₁₀
        -------
          1 0 0 1
         -1 1 0
         --------
            1 1 0
           -1 1 0
           ------
                0
```

6. SELF-ASSESSMENT QUESTIONS

SA1: Convert the decimal number 285.68 to other number system

Solution:

Binary Number system: 100011101.10101

Octal Number system: 435.53412

Hexadecimal number system: 11D.AE147

SA2: Convert the decimal number 48.86 to other number system

Solution:

Binary Number system: 110000.11011

Octal Number system: 60.67024

Hexadecimal number system: 30.DC28F

SA3: Convert following Binary Numbers in to its equivalent Decimal Number

i. 101110.011

ii. 1101.11

iii. 1.0011

Solution:

i. 46.375

ii. 13.75

iii. 1.1875

SA4: Convert following octal Numbers in to its equivalent Decimal Number

i. 36.45

ii. 0.5637

iii. 6543.2

Solution:

i. 30.578125

ii. 0.726318359375

iii. 3427.25

7. SUMMARY

In this chapter we studied about Various Number system and method to convert from one system to another .

8. REFERENCES

- "R.P. Jain, "Modern digital Electronics", Tata McGraw Hill, 4th edition, 2009.
- Douglas Perry, "VHDL", Tata McGraw Hill, 4th edition, 2002.
- W.H. Gothmann, "Digital Electronics- An introduction to theory and practice", PHI, 2nd edition, 2006. 2012.
- https://www.tutorialspoint.com/