MASTERS OF COMPUTER APPLICATIONS

SEMESTER 1

DISCRETE MATHEMATICS AND GRAPH THEORY

Unit 4

Introduction to Set Theory

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1. INTRODUCTION

In this part, we look at the set, the basic discrete mathematical structure that serves as the foundation for all other discrete mathematical structures. Objects are grouped together using sets. The items in a set often, but not necessarily, exhibit like qualities.

For instance, a set is made up of every student presently enrolled at your school. Similar to this, a set is made up of every student presently enrolled in a discrete mathematics course at any given institution. In addition, a set that can be created by taking the components shared by the previous two collections is made up of the students registered in your school who are taking a discrete mathematics course. Such collections may be studied methodically using the language of sets. A definition of a set is now given. This definition, which does not fall within the purview of a formal theory of sets, is an intuitive definition.

1.1. Objectives:

At studying this unit, you should be able to:

- *Understand concept of Basics of sets.*
- *Apply addition principle and Venn diagram.*

2. BASIC DEFINITIONS AND EXAMPLES

2.1. Sets

A **Set** is a well-defined collection of objects.

Examples

1. Collection of natural numbers.
2. Collection of alphabets in English.
3. $A = \{1,2,3,4, \dots\}$
4. $B = \{a, b, c, \dots\}$

Non-examples

1. Collection of 5 world-renowned Mathematicians.
2. Collection of beautiful flowers in a garden.
3. Collection of easy words in the English language.

2.2. Members/ Elements

The objects in a set are called Members/ Elements.

Note:

Sets are usually denoted by capital letters $A, B, C, X, Y, Z, \dots$

Elements are usually denoted by small letters $a, b, c, x, y, z, \dots$

While defining a set, there should be no ambiguity, whether a given object belongs to the set or does not belong to the set.

2.3. Set Membership

If $\boldsymbol{a}$ is an element of a set $\boldsymbol{A}$, we say that $\boldsymbol{a}$ belongs to $\boldsymbol{A}$. The Greek symbol ∈(epsilon) is used to denote the phrase belongs to. Thus, we write $\boldsymbol{a} \in \boldsymbol{A}$.

If $\boldsymbol{b}$ is not an element of a set $\boldsymbol{A}$, we write $b \notin A$ and read $\boldsymbol{b}$ does not belong to $\boldsymbol{A}$

2.4. Cardinality

The number of elements in a set $\boldsymbol{A}$ is called cardinality of $\boldsymbol{A}$, denoted as $|\boldsymbol{A}|$ or $\boldsymbol{n}(\boldsymbol{A})$ or $\mathbf{o(A)}$.

Example: If $A = \{1,2,3\}$ then $|A| = 3$.

2.5. Representation of Sets

There are two methods of representing a set

1. Roaster form or tabular form.
2. Rule Method or Set builder form.

2.6. Roaster or tabular form

In this method, all the elements of the set are listed. The elements are separated by commas and are enclosed within flower brackets[braces].

Example:

1. $A = \{1,2,3,6,7,14,21,42\}$.
2. $B = \{a, e, i, o, u\}$.
3. $C = \{1,3,5, \dots\}$.
4. $D = \{1,2,3,4, \dots\}$

Note:

- In this form, the order in which the elements are listed, is immaterial.
- In this form, the elements are not generally repeated.

2.7. Rule Method or Set builder form

In this method, all the elements of the set posses a single common property which is not possessed by any elements outside the set.

Example:

$A = \{x : x \text{ is a natural number which divides } 42\}$.

$B = \{y : y \text{ is a vowel in the English alphabet}\}$.

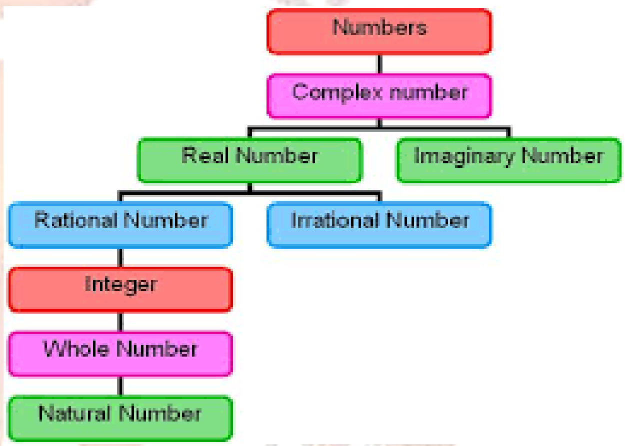
$C = \{z : z \text{ is an odd natural number}\}$.

$D = \{n : n \in \mathbb{N}\}$

Note:

In Rule method set can also be written as $\{x/P(x)\}$, where x is the element of set and $P(x)$ is the property satisfied by the element.

Some important set notation:



- $\mathbb{N}$ = Set of natural numbers.
- $\mathbb{Z}$ = Set of integers.
- $\mathbb{Q}$ = Set of rational numbers.
- $\mathbb{Q}'$ = Set of irrational numbers
- $\mathbb{R}$ = Set of real numbers.
- $\mathbb{C}$ = Set of complex numbers.
- $\mathbb{Z}^+$ = Set of positive integers.
- $\mathbb{Z}^*$ = Set of non-zero integers.
- $\mathbb{Q}^+$ = Set of positive rational numbers.
- $\mathbb{Q}^*$ = Set of non-zero rational numbers.
- $\mathbb{R}^+$ = Set of positive real numbers.
- $\mathbb{R}^*$ = Set of non-zero real numbers.

3. TYPES OF SETS

3.1. Empty Set

A set which does not contain any element is called the **empty set** or the **null set** or the **void set**. The empty set is denoted by the symbol $\emptyset$ or $\{\}$. The cardinality of an empty set is 0.

Example:

i. $A = \{x: 1 < x < 2,\ x \in \mathbb{N}\}$ is an empty sent, because there is no natural number between 1 and 2.

ii. $B = \{\ \}$.

3.2. Singleton set

A set consisting of only one element is called a singleton set.

The cardinality of a singleton set is 1.

Example:

i. $A = \{2\}$.

ii. $B = \ Set\ of\ even\ prime\ number$.

iii. $C = Director\ of\ SET$-JU

3.3. Finite Set

A set which is empty or consists of a definite number of elements is called a finite set. The cardinality of a finite set is the number of elements in the set (without repetition of elements).

Example:

i. $A = \{2\}$.

ii. $B = \emptyset$.

iii. $C = \{a, e, i, o, u\}$.

iv. The Solution set S, of the equation $x^2 - 16 = 0$.

3.4. Infinite Set

If the number of elements in a set is infinite, then the set is called infinite set. The cardinality of an infinite set is ∞.

Example:

i. $A = \{Stares\ in\ the\ Sky\}$.

ii. $\mathbb{N} = \{Set\ of\ all\ natural\ numbers\}$.

3.5. Equal Sets and Unequal Sets

Two sets A and B are said to be equal if they have exactly the same elements and we write $A = B$. Otherwise, the sets are said to be unequal and we write $A \neq B$. The cardinality of two equal sets is equal.

Example:

i. Let A be the set of prime numbers less than 6 and P the set of prime factors of 30. Then A and P are equal, since 2, 3 and 5 are the only prime factors of 30 and these are less than 6. i.e., $A = \{2,3,5\}$ and $P = \{2,3,5\}$.

ii. Let $A = \{1,2,3,4\}$ and $B = \{3,4,1,2\}$ then $A = B$.

iii. Let $A = \{1,2,3\}$ and $B = \{3,4,1,2\}$ then $A \neq B$.

Note:

A set does not change if one or more elements of the set are repeated. For example, the sets $A = \{1, 2, 3\}$ and $B = \{2, 2, 1, 3, 3\}$ are equal, since each element of A is in B and vice-versa. That is why we generally do not repeat any element in describing a Set.

3.6. Equivalence sets

The finite sets are said to equivalence set if both of them having same Number of elements. i.e., cardinality of both sets must be equal.

Example:

Let $A = \{1,2,3,4\}$ and $B = \{3,4,1,2\}$ then $A = B$.

3.7. Disjoint sets

If no single elements are common in two given sets then the sets are called Disjoint sets.

Example:

Let $A = \{1,2,3,4\}$ and $B = \{a, e, i, o, u\}$.

3.8. Sub Set

A set A is said to be a subset of a set B if every element of A is also an element of B. It is denoted by $A \subset B$. In other word, $A \subset B$ if whenever $a \in A$, then $a \in B$.

If set A contains an element which is not in set B then A is not a subset of B and is denoted by $A \not\subset B$.

The symbol ⊂ stands for "is a subset of" or "is contained in"

Example:

- Let $A = \{a,\ b,\ c,\ d,\ e,\ f\}$, $B = \{a,\ d,\ f\}$, $C = \{d,\ e,\ f,\ g,\ h\}$. Then $B \subset A$,
- while $B \not\subset C$.

Note

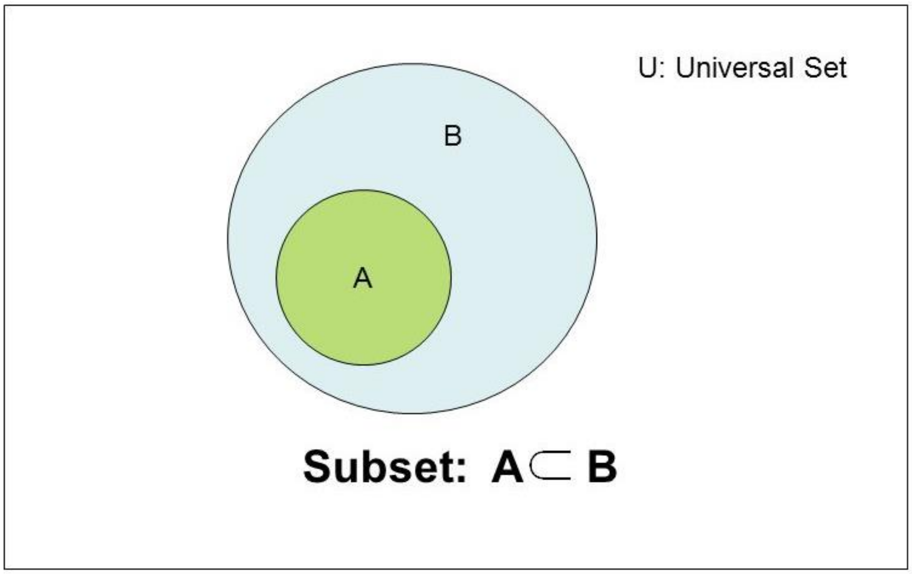
- If A is a subset of B, then B is a **superset** of A.
- The null set and the set itself are subsets of every set. These are called its **trivial subsets.**
- Any subset, other than the trivial subsets, is called a proper subset. Let A and B be two sets. If $A \subset B$ and $A \neq B$, then A is called **Proper Subset** of B. i.e., Set B should consist of at least one element which is not in set A.
- A subset which contains all the elements of the original set is called an **improper subset.** And is denoted by ⊆
- If $A \subseteq B$ and $B \subseteq A$ the $A = B$.

Example:

$A = \{1,2,3\}$ and $B = \{1,2,3,4\}$ then A is the proper subset of B .

3.9. Universal Set

A universal set is set, which has all the elements of sets under consideration. It is generally denoted by $\boldsymbol{U}$.



3.10. Power set

The collection of all subsets of a set A is called the power set of A. It is denoted by $P(A)$. If the number of elements in $A = n$, i.e., $|A| = n$, then the number of elements in $P(A) = 2^n$.

Example:

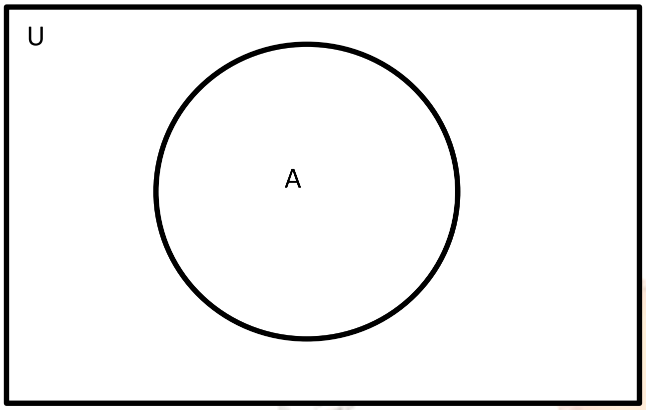
If $A = \{1,2\}$ then $P(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$. Here $|A| = 2$, thus $|P(A)| = 2^2 = 4$.

Note:

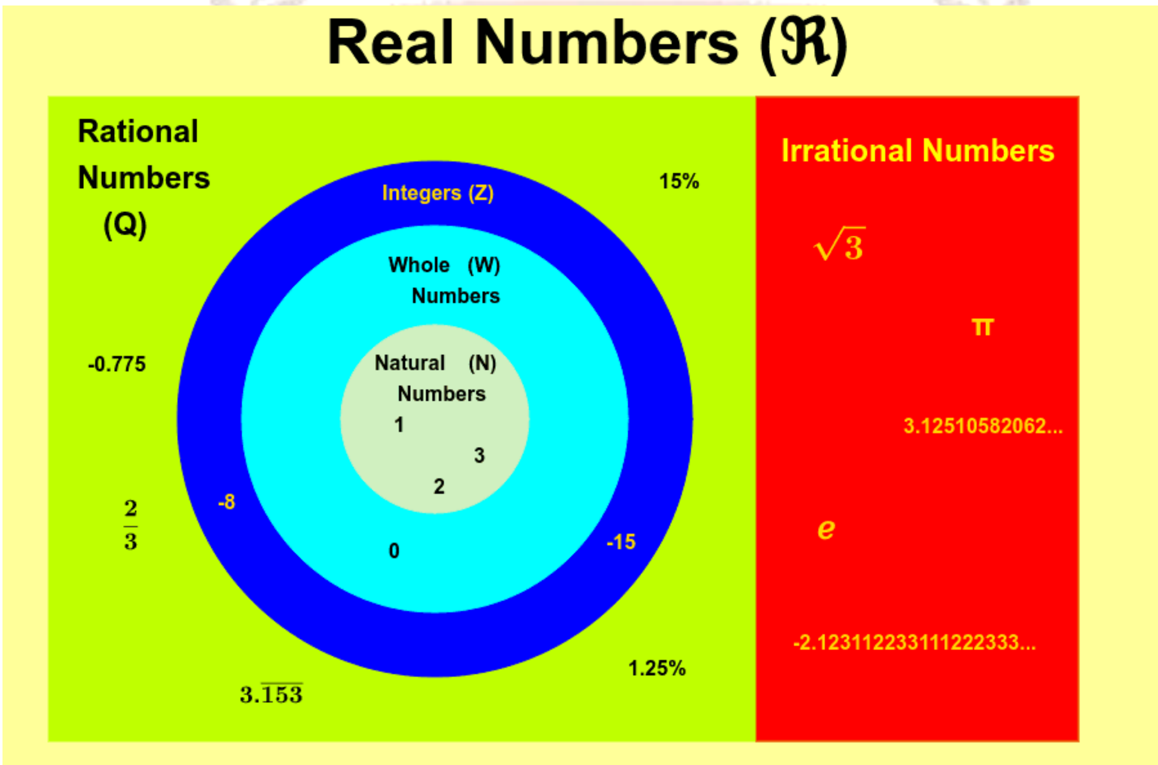
- Number of elements of Proper subsets is $2^n - 2$.

3.11. Venn Diagram

Most of the relationships between sets can be represented by means of diagrams which are known as Venn diagrams. The universal set is represented usually by a rectangle and its subsets by circles


U
A

Example:


Real Numbers (ℜ)
Rational Numbers (Q)
Integers (Z)
Whole (W) Numbers
Natural (N) Numbers
1
3
2
0
-8
-15
15%
-0.775
2/3
1.25%
3.153
Irrational Numbers
√3
π
3.12510582062...
e
-2.12311223311222333...

4. OPERATION IN SETS

4.1. Union of Sets

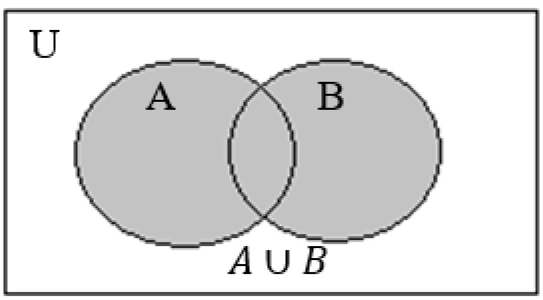
Let A and B be any two sets. The union of A and $B$is the set which consists of all the elements of either A or B or both and is denoted by $A \cup B$ and read as A union B. Symbolically we write $A \cup B = \{x: x \in A \text{ } or \text{ } x \in B\}$.

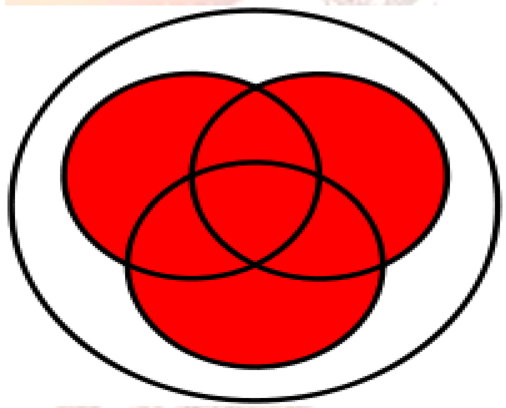
Example:

If $A = \{1,2,3\}$ & $B = \{2,3,4\}$ then $A \cup B = \{1,2,3,4\}$.

Note: $A \subseteq A \cup B$, $B \subseteq A \cup B$.

Ven Diagram:





4.2. Intersection of Sets

Let A and B be any two sets. The Intersection of A and $B$is the set which consists of all the elements belongs to both A and B and is denoted by $A \cap B$ and read as A intersection B. Symbolically we write $A \cap B = \{x: x \in A \text{ } and \text{ } x \in B\}$.

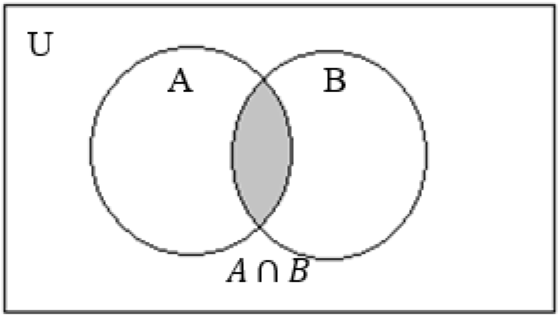
Example:

If $A = \{1,2,3\}$ & $B = \{2,3,4\}$ then $A \cap B = \{2,3\}$.

Note:

- $A \cap B \subseteq A$, $A \cap B \subseteq B$.
- Two sets A & B are said to be disjoint whenever $A \cap B$ is a null set.

Ven Diagram:



4.3. Difference of Sets or Relative complement of Sets

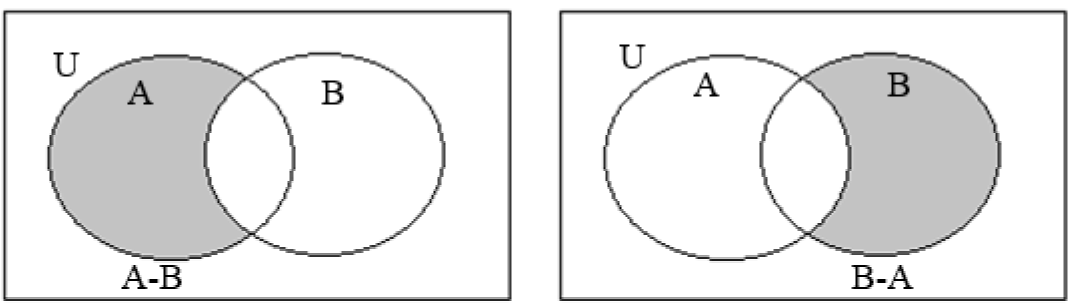
The difference of two sets A and B, denoted by $A - B$ is defined as set of elements which belong to A but not to B.

We write $A - B = \{x: x \in A \; and \; x \notin B\} \; or \; B - A = \{x: x \in B \; and \; x \notin A\}$.

Example:

If $A = \{1,2,3\}$ & $B = \{2,3,4\}$ then $A - B = \{1\}$ and $B - A = \{4\}$

Ven Diagram:



4.4. Compliment of a Set

Let U be a universal set and A be any subset of U, then the compliment of A is the set consists of the elements belongs to U but not A. It is denoted by A' or $\bar{A}$.

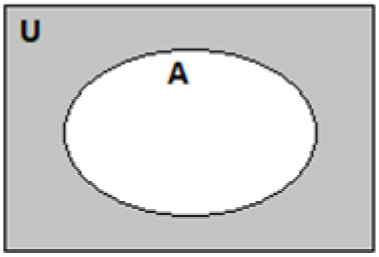
Symbolically we write $\bar{A} = U - A = \{x: x \in U \; and \; x \notin A\}$.

Example:

If $U = \{1,2,3,4,5\}$ & $A = \{2,3,4\}$ then $\bar{A} = U - A = \{1,5\}$.

Note: $\overline{U} = \emptyset$; $\emptyset = U$; $A \subset U$; $\bar{A} \subset U$; $\bar{A}$ and A are disjoint set

Ven Diagram:



4.5. Symmetric Difference/ Relative symmetric

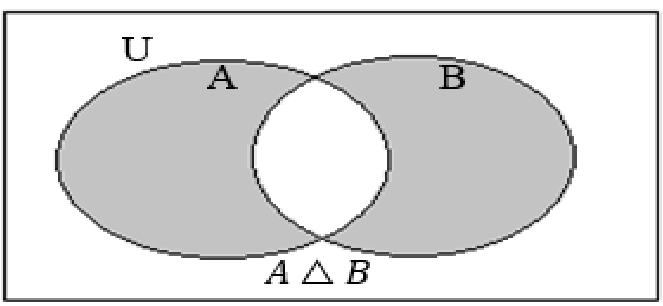
The relative complement of $A \cup B$ and $A \cap B$ is called symmetric difference of A and B and is denoted by $A \triangle B = \{x: x \in A \cup B \text{ } and \text{ } x \notin A \cap B\}$.

Example:

If $A = \{1,2,3\}$ & $B = \{2,3,4\}$ and $A \cup B = \{1,2,3,4\}$, $A \cap B = \{2,3\}$ then

$A \triangle B = \{1,4\}$.

Ven Diagram:



5. THE LAWS OF SET THEORY [ALGEBRAIC PROPERTIES OF SET OPERATION]

For any sets A, B, & C taken from a universe U

Law of double Complement:

$\overline{\overline{A}}=A$.

DeMorgan's Laws:

$\overline{(A\cup B)}=\overline{A}\cap\overline{B}$.

$\overline{(A\cap B)}=\overline{A}\cup\overline{B}$.

Commutative Laws:

$A\cup B=B\cup A$.

$A\cap B=B\cap A$.

Associative Laws:

$A\cup(B\cup C)=(A\cup B)\cup C$.

$A\cap(B\cap C)=(A\cap B)\cap C$.

Distributive Laws:

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$.

Idempotent Laws:

$A\cup A=A$.

$A\cap A=A$.

$A\cup A=A$.

$A\cap A=A$.

Identity Laws:

$A\cup\emptyset=A$.

$A\cap U=A$.

Inverse Laws:

$A\cup\overline{A}=U$.

$A\cap\overline{A}=\emptyset$.

Domination Laws:

$A\cup U=U$.

$A\cap\emptyset=\emptyset$.

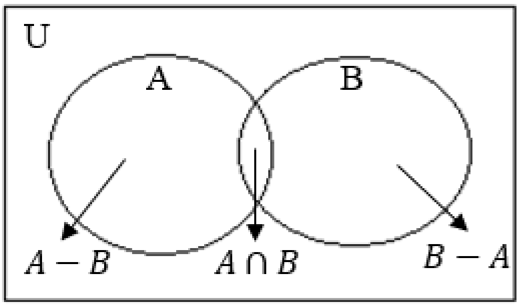
Absorption Laws:

$A\cup(A\cap B)=A$.

$A\cap(A\cup B)=A$.

5.1. Addition Principle

Statement: For any Sets A and B then Prove that $|A \cup B| = |A| + |B| - |A \cap B|$



Proof:

$$A = (A - B) \cup (A \cap B)|A| = |A - B| + |A \cap B| - - - - - - - - (1)B$$
$$= (B - A) \cup (A \cap B)|B| = |B - A| + |A \cap B| - - - - - - - - (2)A \cup B$$
$$= (A - B) \cup (A \cap B) \cup (B - A)|A \cup B|$$
$$= |A - B| + |A \cap B| + |B - A| - - - - - - - - (3$$

Adding (1) & (2) we get

$$|A| + |B| = |A - B| + |A \cap B| + |B - A| + |A \cap B||A| + |B| = |A \cup B| + |A \cap B||A \cup B|$$
$$= |A| + |B| - |A \cap B|$$

Hence the proof

Note: For any Sets A, B, and C then Prove that

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

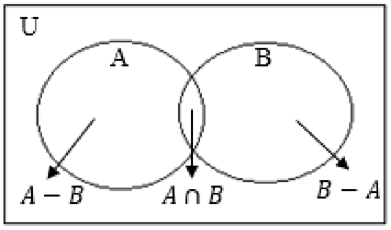
Example:

1. A computer company required 30 programmers for handling system program jobs. 40 programmers for application programming job. If the company appoints 55 programmers to carry out these jobs,

(i) How many of these perform jobs of both type?

(ii) How many handles only system programming jobs?

(iii) How many handles only application programming job?



Solution:

Given $|A \cup B| = 55, \ |A| = 30, \ |B| = 40.$

w.k.t $|A \cup B| = |A| + |B| - |A \cap B|.$

i. $|A \cap B| = |A| + |B| - |A \cup B|$

$|A \cap B| = 30 + 40 - 55$

$|A \cap B| = 15$.

ii. $|A| = |A - B| + |A \cap B|$

$|A - B| = |A| - |A \cap B|$

$|A - B| = 30 - 15$

$|A - B| = 15$.

iii. $|B| = |B - A| + |A \cap B|$

$|B - A| = |B| - |A \cap B|$

$|B - A| = 40 - 15$

$|B - A| = 25$.

6. CARTESIAN PRODUCT AND RELATION

Let A and B be any two non-empty sets. The Cartesian product (or) cross product of A and B denoted by A×B is set of all ordered pairs (a,b) where a belongs to A and b belongs to B.

$\therefore A\times B=\{(a,b)/a\in A,\ b\in B\}$

Thus a relation R from a non-empty set A to a non-empty set B is a subset of a Cartesian product A×B.

i.e., If $(a,b)\in R$, we say that a is related b and we write aRb

Note:

$|A|=m$, $|B|=n$ then total number of relations from A to B formed is given by 2^mn

When a relation is defined on A i.e., (A to A) then the relation is called binary relation.

Example

1. Let A={2,4,6,8} and B={1,2,3} relations R_1, R_2,R_3,R_4 from A to B be defined as follows

i. aR_1 b if a≤b
ii. aR_2 b if a>b
iii. aR_3 b if a divides b
iv. aR_4 b if b divides a

Solution: Given that A={2,4,6,8} and B={1,2,3}

∴A×B={(2,1),(2,2),(2,3),(4,1),(4,2),(4,3),(6,1),(6,2),(6,3),(8,1),(8,2),(8,3)}

i. R_1={(2,2),(2,3)}
ii. R_2={(2,1),(4,1),(4,2),(4,3),(6,1),(6,2),(6,3),(8,1),(8,2),(8,3)}
iii. R_3={(2,2)}
iv. R_4={(2,1),(2,2),(4,1),(4,2),(6,1),(6,2),(6,3),(8,1),(8,)}

7. SELF-ASSESSMENT QUESTIONS

1. What is the power set of the set {0, 1, 2}?
2. What is the power set of the empty set? What is the power set of the set {∅}?
3. Let A = {0, 2, 4, 6, 8}, B = {0, 1, 2, 3, 4}, and C = {0, 3, 6, 9}. What are A ∪ B ∪ C and A ∩ B ∩ C?
4. In a survey of 260 college students, the following data were obtained 64 had take mathematics course 94 had take computer science course 58 had taken business course 28 had taken both maths and business course, 26 had taken both maths and computer science course , 22 had taken both computer science and business, 14 had taken all 3 type of course,

   i. How many students were survey who had taken none of the 3-type pf course?

   ii. In the student's survey, how many had taken only computer science?

8. SUMMARY

In chapter we have discussed about Sets and types of sets. Later we have learnt about operations of sets along with Venn diagram. Also at the end we experienced application of set theory through Additional principle.

9. ANSWERS TO SELF ASSESSMENT QUESTIONS

1. The power set P({0, 1, 2}) is the set of all subsets of {0, 1, 2}. Hence,
   P({0, 1, 2}) = {∅, {0}, {1}, {2}, {0, 1}, {0, 2}, {1, 2}, {0, 1, 2}}.
   Note that the empty set and the set itself are members of this set of subsets.
2. The empty set has exactly one subset, namely, itself. Consequently, P(∅) = {∅}.
   The set {∅} has exactly two subsets, namely, ∅ and the set {∅} itself.
   Therefore, P({∅}) = {∅, {∅}}.
   If a set has n elements, then its power set has 2^n elements. We will demonstrate this fact in several ways in subsequent sections of the text.
3. The set $A \cup B \cup C$ contains those elements in at least one of A, B, and C. Hence,
   $A \cup B \cup C$ = {0, 1, 2, 3, 4, 6, 8, 9}.
   The set $A \cap B \cap C$ contains those elements in all three of A, B, and C. Thus,
   $A \cap B \cap C$ = {0}.
4. Let A represents Mathematics, B represents computer science, C represents business,

   Given $|A| = 64,\ |B| = 94,\ |C| = 58,\ |A \cap B| = 26,\ |A \cap C| = 28,\ |B \cap C| = 22,\ |A \cap B \cap C| = 14\ ,\ |U| = 260.$

   i. To find $|\overline{A \cup B \cup C}| =?$

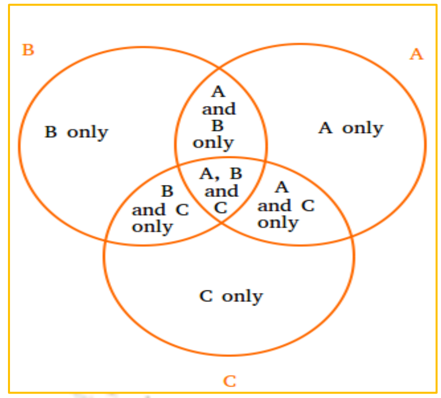
   w.k.t $|\overline{A \cup B \cup C}| = |U| - |A \cup B \cup C|$

   But w.k.t

   $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

   $|A \cup B \cup C| = 64 + 94 + 58 - 26 - 28 - 22 + 14$

   $|A \cup B \cup C| = 154$



$|\overline{A \cup B \cup C}| = |U| - |A \cup B \cup C|$

$|\overline{A \cup B \cup C}| = 260 - 154$

$|\overline{A \cup B \cup C}| = 106.$

ii. We need to find $|B - A - C| = |only\ B|$

$|only\ B| = |B| - \{|A \cap B| + |C \cap B| - |A \cap B \cap C|\}$

$|only\ B| = |B| - |A \cap B| - |C \cap B| + |A \cap B \cap C|\}$

$|only\ B| = 94 - 26 - 22 + 14\}$

$|only\ B| = 94 - 34$

$|only\ B| = 60.$

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