MASTERS OF COMPUTER APPLICATIONS

SEMESTER 1

DISCRETE MATHEMATICS AND GRAPH THEORY

Unit 3

Sequences and Series

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1. INTRODUCTION

Sequences and series are fundamental concepts in mathematics that deal with ordered lists of numbers and their summation, respectively.

A sequence is a collection of numbers arranged in a specific order, following a rule or pattern. Each number in the sequence is called a term, and these terms can be finite or infinite. Sequences can be arithmetic, where the terms follow a constant difference pattern, or geometric, where the terms follow a constant ratio pattern. Other types of sequences exist, such as harmonic sequences or Fibonacci sequences, each governed by their unique recurrence relations.

On the other hand, a series is the sum of the terms in a sequence. It represents the accumulation of all the values in a sequence, either up to a certain point (finite series) or extending to infinity (infinite series). Understanding series involves studying convergence (whether the sum approaches a finite value) and divergence (whether the sum tends toward infinity) of these infinite additions.

Sequences and series find applications across various fields, including calculus, physics, engineering, and computer science, playing a crucial role in modeling and solving real-world problems.

1.1. Objectives:

At studying this unit, you should be able to:

- *Understand Arithmetic Progression (A.P.) and Geometric Progression (G.P.) by recognizing their patterns and rules, forming the foundation for sequence comprehension.*
- *Develop the ability to calculate sums of 'n' terms in Arithmetic and Geometric Progressions, including infinite series, fostering practical problem-solving skills.*
- *Investigate the correlation between Arithmetic Mean (A.M.) and Geometric Mean (G.M.) within sequences and series, enhancing understanding of their roles and significance in mathematical contexts..*

2. SEQUENCE AND EXAMPLE

2.1. Sequence

A sequence is an organization of any objects/elements/set of digits in a particular order accompanied by some rule. In general, sequence is represented as $a_1, a_2, a_3, \dots, a_n$ where The nth term is the number at the nth position of the sequence and is denoted by a_n. The nth term is also called the general term of the sequence.

2.2. Finite and Infinite Sequence

A sequence containing finite number of terms is called a *finite sequence* otherwise Infinite sequence.

Example:

i) 1, 2, 3, 4, 5, 6 is a finite sequence

ii) 1,3,5,... is an infinite sequence

iii) Likewise {a, b, c, d, e, f, g, h} is an example of an alphabetic sequence.

iv) 1, 1, 2, 3, 5, 8,.. the sequence is generated by the recurrence relation given by $a_1 = 1, a_2 = 1, a_3 = a1 + a2 = 1 + 1 = 2$, ..., $a_n = a_{n-2} + a_{n-1}$, n > 2, This sequence is called Fibonacci sequence.

Problem: Write the first three terms of the sequences defined by $a_n = 2n + 6$

Solution

Here $a_n = 2n + 6$

Substituting $n = 1,2,3$

$$a_1 = 2(1) + 6 = 8, a_2 = 2(2) + 6 = 10, a_3 = 2(3) + 6 = 12$$

Thus, the sequence is 8, 10, 12.

Problem: Write the first five terms of the sequences defined by $a_n = \frac{1}{2}n^2$

Solution

Here $a_n = \frac{1}{2}n^2$

Substituting $n = 1,2,3,4,5$

$$a_1 = \frac{1}{2}.1^2 = \frac{1}{2}$$

$$a_2 = \frac{1}{2}.2^2 = 2$$

$$a_3 = \frac{1}{2}.3^2 = \frac{9}{2}$$

$$a_4 = \frac{1}{2}.4^2 = 8$$

$$a_5 = \frac{1}{2}.5^2 = \frac{25}{2}$$

3. SERIES AND EXAMPLE

3.1. Series

Let $a_1, a_2, \ldots, a_n$ be a given sequence. Then the expression $a_1 + a_2 + \cdots + a_n + \cdots$ is called the series associated with the given sequence.

The series is often represented in compact form with Greek symbol sigma. i.e.,

$$a_1 + a_2 + \cdots + a_n = \sum_{i=1}^{n} a_i$$

Note: The series is finite or infinite according as the given sequence is finite or infinite.

Problem: Let the sequence a_n be defined as $a_1 = 1, a_n = a_{n-1} + 2 \; for \; n \geq 2$. Find first five terms and write corresponding series.

Solution

$a1$ = 1, $a2$ = $a1$ + 2 = 1 + 2 = 3, $a3$ = $a2$ + 2 = 3 + 2 = 5, $a4$ = $a3$ + 2 = 5 + 2 = 7, $a5$ = $a4$ + 2 = 7 + 2 = 9.

Hence, the first five terms of the sequence are 1,3,5,7 and 9. The corresponding series is 1 + 3 + 5 + 7 + 9 +...

Problem: Let $a_n = 2n + 2$ write the first 4 terms in series form where n is natural number.

Solution

$$a_1 = 2(1) + 2 = 4, a_2 = 2(2) + 2 = 6, a_3 = 2(3) + 2 = 8, a_4 = 2(4) + 2 = 10$$

Thus series can be written as $2 + 6 + 8 + 10 + \cdots$

Problem: Find the nth partial sum of the series $\sum_{n=1}^{n} \frac{1}{2^n}$

Solution

$$S_n = \frac{1}{2^1} + \frac{1}{2^2} + \cdots + \frac{1}{2^n}$$

$$S_{n+1} = \frac{1}{2^1} + \frac{1}{2^2} + \cdots + \frac{1}{2^n} + \frac{1}{2^{n+1}}$$

$$S_{n+1} = S_n + \frac{1}{2^{n+1}} ------(1)$$

Also we can write S_{n+1} as

$$\begin{aligned} S_{n+1} &= \frac{1}{2^1} + \frac{1}{2^2} + \ldots + \frac{1}{2^n} + \frac{1}{2^{n+1}} \\ &= \frac{1}{2}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^n}\right] \\ &= \frac{1}{2}\left[1 + \left(\frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^n}\right)\right] \end{aligned}$$

$$S_{n+1} = \frac{1}{2}(1 + S_n) ------(2)$$

From (1) and (2) $\quad S_n + \frac{1}{2^{n+1}} = \frac{1}{2}[1 + S_n]$

$$2S_n + \frac{1}{2^n} = 1 + S_n$$

$$\therefore S_n = 1 - \frac{1}{2^n}$$

4. ARITHMETIC PROGRESSION (A.P.)

A sequence $a_1, a_2, a_3,\ldots,$ is called arithmetic sequence or arithmetic progression if $a_{n+1} = a_n + d, n \in N$ where a_1 is called the first term and the constant term d is called the common difference of the A.P.

Let us consider an A.P. (in its standard form) with first term a and common

difference d, i.e., a, $a + d$, $a + 2d$, ...

Then the nth term (*general term*) of the A.P. is $an = a + (n - 1)\, d.$

Example: 4, 8, 12, 16, 20, 24, 28…….. In the above example, the difference between the successive terms is 4.

Note: If a constant is added or subtracted or multiplied or divided to each term of an A.P., the resulting sequence is also an A.P

The sum to n terms of A.P

Here, we shall use the following notations for an arithmetic progression:

a = the first term,

l = the last term,

d = common difference,

n = the number of terms.

Sn= the sum to n terms of A.P.

Let $a, a + d, a + 2d, \ldots, a + (n - 1)\, d$ be an A.P. Then

$$l = a + (n - 1)\, d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[a + l]$$

Problem The income of a person is Rs. 3,00,000, in the first year and he receives an increase of Rs.10,000 to his income per year for the next 19 years. Find the total amount, he received in 20 years.

Solution Here, we have an A.P. with a = 3,00,000, d = 10,000, and n = 20.

Using the sum formula, we get,

$$S_{10} = \frac{20}{2}[60000 + 19 \times 1000] = 7900000$$

Hence, the person received Rs. 79,00,000 as the total amount at the end of 20 years.

5. ARITHMETIC MEAN

Given two numbers a and b. We can insert a number 'A' between them so that a, A, b is an A.P. Such a number A is called the *arithmetic mean* (A.M.) of the numbers a and b. Note that, in this case, we have

A – a = b – A, i.e., $A = \frac{a+b}{2}$

We may also interpret the A.M. between two numbers a and b as their average $A = \frac{a+b}{2}$

For example, the A.M. of two numbers 5 and 15 is 10. We have, thus

constructed an A.P. 5, 10, 15 by inserting a number 10 between 5 and 15.

Thus generally, given any two numbers a and b, we can insert as many numbers as we like between them such that the resulting sequence is an A.P.

Let $A_1, A_2, \ldots A_n$ be n numbers between a and b such that a, $A_1, A_2, \ldots A_n$ b is an A.P. Here, b is the (n + 2)th term, i.e.,

$$b = a + [(n + 2) - 1]d = a + (n + 1)\, d.$$

Which results

$$d = \frac{b-a}{n+1}$$

Problem: Insert 6 numbers between 3 and 24 such that the resulting sequence is an A.P.

Solution:

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be six numbers between 3 and 24 such that $3, A_1, A_2, A_3, A_4, A_5, A_6, 24$ are in A.P. Here, a = 3, b = 24, n = 8. Therefore, 24 = 3 + (8 –1) d, so that d = 3.

Thus A1 = a + d = 3 + 3 = 6; A2 = a + 2d = 3 + 2 × 3 = 9;

A3= a + 3d = 3 + 3 × 3 = 12; A4= a + 4d = 3 + 4 × 3 = 15;

A5 = a + 5d = 3 + 5 × 3 = 18; A6 = a + 6d = 3 + 6 × 3 = 21.

Hence, six numbers between 3 and 24 are 6, 9, 12, 15, 18 and 21.

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Problem: Insert four A.Ms between − 1 and 14.

Solution

Let A1, A2, A3, A4 be the four A.Ms between − 1 and 14.

By the definition − 1, A1, A2, A3, A4, 14 are in A.P. Let d be the common difference.

∴ A1= − 1 + ; A2 = − 1 + 2d ; A3 = − 1 + 3d, ; A4 = − 1 + 4d ; 14 = −1+5d

∴ d = 3

∴ A1= − 1 + 3 = 2 ; A2 = − 1+2 × 3 = 5 ; A3 = −1+3×3 = 8 ; A4= − 1 + 12 = 11

6. GEOMETRIC PROGRESSION (G.P.)

A sequence where every successive term possesses a fixed ratio between them is called a geometric sequence. In other words, a sequence where every term can be obtained by multiplying or dividing a particular number with the preceding number is called a geometric sequence or geometric progression GP

A sequence $a_1, a_2, a_3, \ldots, a_n \ldots$ is called geometric progression, if each term is non zero and $\frac{a_{k+1}}{a_k} = r$ (constant) for $k \geq 1$.

By letting a1 = a, we obtain a geometric progression, a, ar, ar^2, ar^3, …. where a is called the first term and r is called the common ratio of the G.P

6.1. General term of a G .P.

Let us consider a G.P. with first non-zero term ‘a’ and common ratio ‘r’. Write a few terms of it. The second term is obtained by multiplying a by r, thus $a_2 = ar$. Similarly, third term is obtained by multiplying a_2 by r. Thus, $a_3 = a_2 r = ar^2$, and so on. Therefore, the pattern suggests that the nth term of a G.P. is given by $a_n = ar^{n-1}$

Thus, a, G.P. can be written as $a, ar, ar^2, ar^3, \ldots., ar^{n-1}$ or $a, ar, ar^2, ar^3, \ldots., ar^{n-1}, \ldots$ according as G.P. is finite or infinite, respectively.

The series $a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$ or $a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} + \cdots$ are called finite or infinite geometric series, respectively.

Example of Geometric Progression: 3, 6, 12, 24, 48.

In the above example, the difference is a common ratio between each term is 2, and the first term here is 3. Finally, the formula becomes $a_n = 3 * 2^{n-1}$.

Problem: Which term of the G.P., 2,8,32, ... up to n terms is 131072?

Solution:

Let 131072 be the nth term of the given G.P. Here a = 2 and r = 4.

Therefore 131072=an=2(4)n-1 or 65536=4n-1

Which gives 48=4n-1

So that n – 1 = 8, i.e., n = 9. Hence, 131072 is the 9th term of the G.P.

Problem: In a G.P., the 3rd term is 24 and the 6th term is 192.Find the 10th term.

Solution

Here, a3= ar^2 = 24 ... (1)

And a6=ar^5=192 ... (2)

Dividing (2) by (1), we get r = 2. Substituting r = 2 in (1), we get a = 6.

Hence $a_{10}=6(2)^9=3072$

6.2. Sum to n terms of a G .P.

Let the first term of a G.P. be *a* and the common ratio be *r*. Let us denote by S*n* the sum to first *n* terms of G.P.

Then $S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$--------(1)

Case I: If *r* = 1, we have S*n* = *a* + *a* + *a* + ... + *a* (*n* terms) **=** *na*

Case II: If *r* ¹ 1, multiplying (1) by *r*,

we have $rS_n = ar + ar^2 + ar^3 + \cdots + ar^n$--------(2)

Subtracting (2) from (1), we get

$$(1 - r)S_n = a - ar^n = a(1 - r^n)$$

This gives

$$S_n = \frac{a(1 - r^n)}{1 - r}\ if\ r < 1 \quad or \quad S_n = \frac{a(r^n - 1)}{r - 1}\ if\ r > 1$$

Problem: Find the 10th and nth terms of the G.P. 5, 25,125,...

Solution.

Here a = 5 and r = 5. Thus, a10 = 5(5)10–1 = 5(5)9 = 510

and $a_n = ar^{n-1} = 5(5)^{n-1} = 5^n$

Problem: Let n be the number of terms needed. Given that a = 3, r = ½ and Sn=3069/512.

Solution

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\frac{3069}{512} = \frac{3(1 - \frac{1}{2^n})}{1 - \frac{1}{2}} = 6\left(1 - \frac{1}{2^n}\right)$$

$$\frac{3069}{3072} = 1-\frac{1}{2^n}$$

$$\frac{1}{2^n} = 1-\frac{3069}{3072} = \frac{3}{3072} = \frac{1}{1024}$$

$$2^n = 1024 = 2^{10}, \text{ which gives } n = 10.$$

Problem: Find the sum of the sequence 7, 77, 777, 7777, ... to n terms.

Solution: This is not a G.P., however, we can relate it to a G.P. by writing the terms as

$S_n = 7 + 77 + 777 + 7777 + \cdots$ to n terms

$$= \frac{7}{9}[9+99+999+9999+\ldots \text{to } n \text{ term}]$$

$$= \frac{7}{9}[(10-1)+(10^2-1)+(10^3-1)+(10^4-1)+\ldots n \text{ terms}]$$

$$= \frac{7}{9}[(10+10^2+10^3+\ldots n \text{ terms})-(1+1+1+\ldots n \text{ terms})]$$

$$= \frac{7}{9}\left[\frac{10(10^n-1)}{10-1}-n\right] = \frac{7}{9}\left[\frac{10(10^n-1)}{9}-n\right].$$

7. ARITHMETIC AND GEOMETRIC SERIES INFINITE G.P. AND ITS SUM

7.1. Arithmetic-Geometric Progression

An arithmetic-geometric progression (AGP) is a progression in which each term can be represented as the product of the terms of an arithmetic progressions (AP) and a geometric progressions (GP). Which is given by

$$a, (a+d)r, (a+2d)r^2, (a+3d)r^3, \ldots, [a+(n-1)d]r^{n-1},$$

where

a is the initial term,

d is the common difference, and

r is the common ratio.

General term of AGP: The nth term of the AGP is obtained by multiplying the corresponding terms of the arithmetic progression (AP) and the geometric progression (GP). So, in the above sequence the nth term is given by

$$t_n = [a + (n-1)d]r^{n-1}$$

Sum of terms of AGP:

The sum of the first n terms of the AGP is

$$S_n = \sum_{k=1}^{n} [a + (k-1)d]\, r^{k-1},$$

which can be solved further to obtain

$$S_n = \frac{a - [a + (n-1)d]\, r^n}{1 - r} + \frac{dr(1 - r^{n-1})}{(1 - r)^2}.$$

Sum to infinity of AGP:

If |r|<1 then the sum to infinity is given by

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}.$$

For Example: n the following series, the numerators are in AP and the denominators are in GP

$$\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \cdots$$

7.2. Infinite Geometric Series

A geometric series is the sum of a sequence wherein every successive term contains a constant ratio to its preceding term. An Infinite geometric series has an infinite number of terms and can be represented as a, ar, ar^2, ..., to ∞.

The infinite series formula if the value of r is such that −1<r<1, can be given as,

$$Sum = \frac{a}{1-r}$$

Where,

a = first term of the series

r = common ratio between two consecutive terms and $-1 < r < 1$

Note:

- The geometric series converges to a sum only if $r < 1$.
- If $r > 1$, the series does not converge and doesn't have a sum.

  For example 8, 12, 18, 27, ... is the given geometric series.

  To find the sum : 8 + 12 + 18 + 27 + ... , we find that a = 8 and r = 12/8 = 18/ 12 = 3/2

  Here $r > 1$. Thus the sum does not converge and the series has no sum.

Problem: Find the sum of the terms 1/9 + 1/27 + 1/81 + ... to ∞?

Solution: To find: Sum of the geometric series

Given:

a = 1/9, r = 1/3

Using the infinite geometric series formula,

S_n = a /(1-r)

S_n = (1/9)(1 - 1/3)

S_n = 1/6

Problem: Calculate the sum of series 1/5, 1/10, 1/20, ... if the series contains infinite terms.

Solution: To find: Sum of the geometric series

Given:

a = 1/5, r = 1/5

Using the infinite geometric series formula, Sum = a /(1-r)

= (1/5)(1 - 1/5)

= 1/4

8. GEOMETRIC MEAN (G .M.)

The geometric mean of two positive numbers a and b is the number $\sqrt{ab}$

Therefore, the geometric mean of 2 and 8 is 4.

Note

Let $G_1, G_2, \dots, G_n$ be n number between positive numbers a and b such that $a, G_1, G_2, \dots, G_n, b$ is GP. Thus b being (n+2)th term we have

$$b = ar^{n+1} \;\; or \;\; r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

Hence,

$$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Problem: Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.

Solution:

Let G1, G2, G3 be the three numbers between 1 and 256 such that, 1, G1, G2, G3, 256 is a GP

Therefore 256 = r^4 giving r = ± 4 (Taking real roots only)

For r = 4, we have G1= ar = 4, G2= ar^2 = 16, G3= ar^3 = 64

Similarly, for r = – 4, numbers are – 4,16 and – 64.

Hence, we can insert 4, 16, 64 between 1 and 256 so that the resulting sequences are in G.P.

Problem Find 5 geometric means between 576 and 9.

Solution

Let G1, G2, G3, G4, G5 be 5 G.Ms between a = 576 and b = 9

Let the common ratio be *r*

G1 = 576*r*, G2 = 576r^2, G3 = 576r^3, G4 = 576r^4, G5 = 576r^5, 9 = 576r^6

$$\Rightarrow \quad r^6 = \frac{9}{576} \quad \Rightarrow \quad r = \left(\frac{9}{576}\right)^{\frac{1}{6}} = \left(\frac{1}{64}\right)^{\frac{1}{6}}$$

$$r = \frac{1}{2}$$

$$G_1 = 576r = 576 \times \frac{1}{2} = 288 \qquad G_2 = 576r^2 = 576 \times \frac{1}{4} = 144$$

$$G_3 = 576r^3 = 576 \times \frac{1}{8} = 72 \qquad G_4 = 576r^4 = 576 \times \frac{1}{16} = 36$$

$$G_5 = 576r^5 = 576 \times \frac{1}{32} = 18$$

Hence 288, 144, 72, 36, 18 are the required G.Ms between 576 and 9.

Relationship Between A.M. and G.M.

Let A and G be A.M. and G.M. of two given positive real numbers a and b, respectively. Then

$$A = \frac{a+b}{2} \ \& \ G = \sqrt{ab}$$

Thus we have,

$$A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{a - b - 2\sqrt{ab}}{2}$$

$$= \frac{\left(\sqrt{a} - \sqrt{b}\right)^2}{2} \geq 0 - - - - (1)$$

From (1), we obtain the relationship A>=G.

Problem: If A.M. and G.M. of two positive numbers a and b are 10 and 8, respectively, find the numbers.

Solution

Given that

$$A = \frac{a+b}{2} = 10 - - - - (1)$$

And

$$GM = \sqrt{ab} = 8 - - - - (2)$$

From (1) and (2) we get

$$a + b = 20 - - - - (3)$$

$$ab = 64 - - - - (4)$$

Putting the value of a and b from (3), (4) in the identity

$$(a-b)^2 = (a+b)^2 - 4ab$$

We get

$$(a-b)^2 = 400 - 256 = 144$$

$$a - b = \pm 12 - - - - (5)$$

Solving (3) and (5), we obtain a = 4, b = 16 or a = 16, b = 4

Thus, the numbers a and b are 4, 16 or 16, 4 respectively

9. SELF-ASSESSMENT QUESTIONS

1. What is the 20th term of the sequence defined by $an = (n-1)(2-n)(3+n)$?
2. Find a_{17} *and* a_{24} in the sequences $a_n = 4n - 3$
3. Find the sum of odd integers from 1 to 2001.
4. Which term of the G.P., 2,8,32, ... up to n terms is 131072?
5. A person has 2 parents, 4 grandparents, 8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own.
6. Find the sum of n terms in the GP $\sqrt{7}, \sqrt{21}, 3\sqrt{7}$ ...
7. Find the sum of the geometric series 125, 25, 5, 1......∞

10. SUMMARY

In chapter is about mastering sequence basics (A.P. and G.P.), forming the basis for sequence understanding. Developing skills in summing 'n' terms in A.P. and G.P., including infinite series, aids practical problem-solving. Investigating the link between Arithmetic Mean (A.M.) and Geometric Mean (G.M.) in sequences enhances comprehension of their mathematical roles.

11. ANSWERS TO SELF ASSESSMENT QUESTIONS

1. Putting $n = 20$, we obtain

   $a_{20} = (20 - 1)(2 - 20)(3 + 20) = 19 \times (-18) \times (23) = -7866.$

2. Given the given equation is $a_n=4n-3$

   Substitute $n=17$ in the equation.

   $a_{17}=4(17)-3$

   $\Rightarrow a_{17}=65$

   Similarly substitute $n=24$ in the equation.

   $a_{24}=4(24)-3$

   $\Rightarrow a_{24}=93$

   Therefore, the 17th and 24th term of $a_n=4n-3$ is 65 and 93 respectively.

3. Odd integers from 1 to 2001 are 1, 3, 5, 7, ..., 2001.

   The first term of the A.P. is $a=1$ and the common difference is $d=2$

   The nth term of the A.P. is given by the equation $a_n=a+(n-1)d$

   Therefore, $a+(n-1)d=2001$

   Substitute $a=1$ and $d=2$ in the equation.

   $\Rightarrow 1+(n-1)2=2001$

   $\Rightarrow 2n-1=2001$

   $\Rightarrow n=(2001+1)/2$

   $\Rightarrow n=1001$

   The sum of first n terms of an arithmetic progression is given by the equation

$$S_n = \frac{n}{2}[2a + (n-1)\,d]\,.$$

Substitute the values of n, a and d in the equation.

$$S_n = \frac{1001}{2}[2\,(1) + (1001 - 1)\,2]$$

$$= \frac{1001}{2}[2 + (1000)\,2]$$

$$= \frac{1001}{2} \times 2002$$

$$= 1001 \times 1001$$

$$= 1002001$$

Therefore, 1002001 is the sum of the odd integers from 1 to 2001 .

4. Let 131072 be the nth term of the given G.P. Here a = 2 and r = 4.

   Therefore $131072 = a_n = 2(4)^{n-1}$ *or* $65536 = 4^{n-1}$

   This gives $4^8 = 4^{n-1}$.

   So that n – 1 = 8, i.e., n = 9. Hence, 131072 is the 9th term of the G.P.

5. Here a = 2, r = 2 and n = 10

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} = 2(2^{10} - 1) = 2046$$

   Hence, the number of ancestors preceding the person is 2046.

6.

$$r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$$

$$S_n = \frac{a\,(1 - r^n)}{1 - r}$$

$$S_n = \frac{\sqrt{7}\,[1 - (\sqrt{3})^n]}{1 - \sqrt{3}}$$

$$= \frac{\sqrt{7}\,[1 - (\sqrt{3})^n]}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}} \qquad \textbf{(By rationalizing)}$$

$$= \frac{\sqrt{7}\,(1 + \sqrt{3})\,[1 - (\sqrt{3})^n]}{1 - 3} = \frac{-\sqrt{7}\,(1 + \sqrt{3})}{2}\left[1 - (3)^{\frac{n}{2}}\right]$$

$$= \frac{\sqrt{7}\,(1 + \sqrt{3})}{2}\left[(3)^{\frac{n}{2}} - 1\right].$$

7. The series is,125+25+5+1+ …….. a =125, and = 25/125 = 1/5

   Using the infinite geometric series formula,

   Sum = a /(1-r)

   = 125/(1-1/5)

   = 125/(4/5)

   = 625/4

12. REFERENCES

1. Ralph P. Grimaldi (2019) Discrete and Combinatorial Mathematics: An Applied Introduction (5ed.) Pearson Publication.
2. Kenneth H. Rosen (2012) Discrete Mathematics and Its Applications (7 th Edition) McGraw-Hill Publication.
3. Dr. DSC (2016) Discrete Mathematical Structures (5th Edition), PRISM publication.
4. https://ncert.nic.in
5. https://testbook.com/maths/sequences-and-series