MASTER OF COMPUTER APPLICATIONS

SEMESTER 1

DISCRETE MATHEMATICS AND GRAPH THEORY

Unit 8

Basics of Logic

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1. INTRODUCTION

Mathematical statements have specific meaning thanks to the Laws of logic. The distinction between legitimate and false mathematical arguments is made using these guidelines. We start our study of discrete mathematics with an introduction to logic since one of the main objectives of this module is to educate the reader on how to comprehend and how to create proper mathematical arguments.

Beyond its significance for comprehending mathematical reasoning, logic has several uses in computer technology. These rules are used in a variety of contexts, including the design of computer circuits, the creation of computer programs, and testing the accuracy of code. Additionally, software systems have been created to automatically create certain sorts of evidence, but not all of them. These uses of logic will be covered in this chapter.

1.1 Objectives:

The objective of this topic is to make the students to

- *Understand concept of Propositional Logic.*
- *Apply various Logical connectives.*

2. PROPOSITIONAL LOGIC DEFINITIONS AND EXAMPLES

2.1. Logics:

Logic is the display that deals with the method of reasoning. It provides rules and techniques for determining whether a given argument is valid. Logical reasoning is used in mathematics used to prove the theorem. In computer science to verify the correctness of programs and to prove theorem.

2.2. Proposition or Statement:

A Proposition is a statement or declaration which is either true or false but not both.

Propositions are usually denoted by small letters such as $p,q,r,s,\ldots$

If the proposition is true, then it is denoted by 1 or T.

If the proposition is false, then it is denoted by 0 or F.

EXAMPLE:

i. p: $A\ Square\ is\ a\ quadrilateral$- True [1 or T].

ii. q: $7\ is\ an\ even\ number$- False [0 or F].

iii. r: $Do\ you\ speak\ Kannada$? It is a question not a statement.

iv. $x + y = y + x$ it is a declarative sentence but not a statement since it is true or false depending on the value of x & y.

2.3. Types of Propositions:

Simple Proposition:

The proposition which is free from logical connectives are called **simple proposition.**

Example: $\boldsymbol{p}$: $Sun\ Raises\ in\ east$

Compound Proposition:

Any statement obtained by combining two or more statement by logical connective.

Example: *Sun Raises in east and* 8 *is odd number*.

3. LOGICAL CONNECTIVES AND TRUTH TABLES

The word or phrases like not, and, or, if then, if and only if[iff], etc., are called **logical statement.**

3.1. Negation [NOT]:

If $\boldsymbol{p}$ be any proposition, then negation of $\boldsymbol{p}$ is read as $\boldsymbol{not\ p}$ and it is denoted by $\neg\boldsymbol{p}$ or $\sim\boldsymbol{p}$.

If the truth value of $\boldsymbol{p}$ is **1** then truth value of $\boldsymbol{not\ p}$ is **0** & vice versa.

Truth table:

p	$\neg p$
0	1
1	0

Example: Find the negation of the proposition "Michael's PC runs Linux".

The negation is: "Michael's PC does not run Linux".

3.2. Conjunction [AND]:

Let p and q be simple proposition, then conjunction of p & q is read as $p\ and\ q$ and is denoted by $p \wedge q$.

The truth value of $p \wedge q$ is 1 if both p & q values are 1, otherwise value of $p \wedge q$ is 0.

Truth table:

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{p \wedge q}$

0	**0**	**0**
0	**1**	**0**
1	**0**	**0**
1	**1**	**1**

3.3. Disjunction [OR]:

Let p and q be simple proposition, then disjunction of p & q is read as p *or* q and is denoted by $p \vee q$.

The truth value of $p \vee q$ is 1 if any one of p and q is 1 but 0 if both p & q are 0.

Truth table:

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

3.4. Exclusive Disjunction [Either or but not both]:

Let p and q be simple proposition, then exclusive disjunction of p & q is read as *either p or q but not both* and is denoted by $p \underline{\vee} q$.

The truth value of $p \underline{\vee} q$ is 1 if any one of value of p & q is different but 0 if both p & q are same.

Truth table:

p	q	$p \veebar q$
0	0	0
0	1	1
1	0	1
1	1	0

3.5. Conditional [If, then]:

Let p and q be simple proposition, then conditional of p & q is read as $If\ p,\ then\ q$ and is denoted by $p \rightarrow q$.

The truth value of $p \rightarrow q$ is 0 if p is 1 & q is 0 and otherwise it is 1.

Truth table:

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

3.5.1 Converse, Inverse & Contrapositive of Conditional

Let $\boldsymbol{p} \rightarrow \boldsymbol{q}$ be a conditional then,

i. $\boldsymbol{q} \rightarrow \boldsymbol{p}$ is called converse.

ii. $\neg\boldsymbol{p} \rightarrow \neg\boldsymbol{q}$ is called inverse.

iii. $\neg\boldsymbol{q} \rightarrow \neg\boldsymbol{p}$ is called contrapositive.

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$q \rightarrow p$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
0	0	1	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
1	1	0	0	1	1	1	1

Note:

- Conditional statement and contrapositive statement are logically equivalent.
- Converse statement and inverse statement are logically equivalent.

3.6. Biconditional [Iff]:

Let p and q be simple proposition, then biconditional of p & q is read as $p\ iff\ q$ [p if and only if q] and is denoted by $p \leftrightarrow q$.

The truth value of $p \leftrightarrow q$ is 1 only when both p & q are same otherwise 0.

Truth table:

p	q	$p \leftrightarrow q$
0	0	1

0	1	0
1	0	0
1	1	1

Note: The biconditional statement is also read as $If\ p\ then\ q\ and\ if\ q\ then\ p$.

3.7. Tautology [T_0]:

A compound proposition which is true [1] for all possible truth values of its components [Primitive] is called **tautology.**

3.8. Contradiction / Absurdity [F_0]:

A compound proposition which is false [0] for all possible truth value of its components is called **contradiction.**

3.9. Contingency:

A compound proposition that can be true or false i.e., neither tautology nor contradiction depending upon the ruth values of its compound is called **contingency.**

Example: *Show that for any proposition* p & q *the compound proposition* $p \rightarrow (p \vee q)$ *is a tautology and the compound proposition* $p \wedge (\neg p \wedge q)$*is a contradiction.*

p	q	$\neg p$	$p \vee q$	$\neg p \wedge q$	$p \rightarrow (p \vee q)$	$p \wedge (\neg p \wedge q)$
0	0	1	0	0	1	0
0	1	1	1	1	1	0

1	0	0	1	0	1	0
1	1	0	1	0	1	0

∴ ∀ possible truth value of compound proposition $p \to (p \vee q)$is tautology

& $p \wedge (\neg p \ \wedge q)$ is a contradiction.

3.10. Logical Equivalence:

Let u & v be two compound proposition which are said to be logically equivalent whenever u & v have same truth value.

In other words its biconditional is a tautology. It is denoted by $u \Leftrightarrow v$.

Example: Using truth table check whether the given compound proposition is logically equivalent or not.

i. $p \veebar q \Leftrightarrow (p \vee q) \wedge (\neg p \vee \neg q)$

Solution

p	q	$\sim p$	$\sim q$	(a) $p \vee q$	(b) $\sim p \vee \sim q$	(A) $a \wedge b$	(B) $p \veebar q$
0	0	1	1	0	1	0	0
0	1	1	0	1	1	1	1
1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	0

The column A & B have the same truth value and are equivalent

$\therefore p \veebar q \Leftrightarrow (p \vee q) \wedge (\neg p \vee \neg q)$

4. SELF-ASSESSMENT QUESTIONS

SA1: *Construct the truth table for the following compound Propositions* $p \vee \neg q$

Solution

p	q	$\neg q$	$p \vee \neg q$
0	0	1	1
0	1	0	0
1	0	1	1
1	1	0	1

SA2: *Construct the truth table for the following compound Propositions* $p \to \neg q$

Solution:

p	q	$\neg q$	$p \to \neg q$
0	*0*	*1*	*1*
0	*1*	*0*	*1*
1	*0*	*1*	*1*
1	*1*	*0*	*0*

SA3:. Show that the truth value of the following compound proposition are independent of the truth value of their components.

i. $\{p \wedge (p \to q)\} \to q$[homework]

ii. $(p \to q) \leftrightarrow (\neg p \vee q)$

Solution:

p	q	(a) $p \to q$	$\neg p$	(b) $\neg p \vee q$	$a \leftrightarrow b$
0	0	1	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	1	1	0	1	1

SA4: *Show that the following compound propositions are tautology* $\{(p \vee q) \wedge [(p \to r) \wedge (q \to r)]\} \to r$

Solution

p	q	r	(a) $p \vee q$	(b) $p \to r$	(c) $q \to r$	(d) $b \wedge c$	(e) $a \wedge d$	$e \to r$
0	**0**	**0**	**0**	**1**	**1**	**1**	**0**	**1**
0	**0**	**1**	**0**	**1**	**1**	**1**	**0**	**1**
0	**1**	**0**	**1**	**1**	**0**	**0**	**0**	**1**
0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1	**0**	**0**	**1**	**0**	**1**	**0**	**0**	**1**
1	**0**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1	**1**	**0**	**1**	**0**	**0**	**0**	**0**	**1**
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

SA5: Write converse, inverse & contrapositive of conditional given

p: Quadrilateral is parallelogram. q: Diagonals of quadrilateral bisects eachother.

Solution

Conditional: $p \rightarrow q$: If quadrilateral is a parallelogram, then diagonals of a quadrilateral bisects each other.

Converse: $q \rightarrow p$: If diagonals of a quadrilateral bisects each other, then quadrilateral is a parallelogram.

Inverse: $\neg p \rightarrow \neg q$: If quadrilateral is not a parallelogram, then diagonals of a quadrilateral does not bisect each other.

Contrapositive: $\neg q \rightarrow \neg p$: If diagonals of a quadrilateral does not bisect each other, then quadrilateral is not a parallelogram.

5. SUMMARY

This Second Chapter has introduced some fundamentals of logic in particular Propositional Logic, Logical Connectives. We have considered logic both as its own sub-discipline of mathematics, At the most basic level, a statement might combine simpler statements using logical connectives.

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