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Brief Paper

Task-space regulation of cooperative manipulators☆

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Abstract

In this paper a new task-space regulation scheme for a system of two cooperative manipulators tightly grasping a rigid object is proposed. The control architecture is based on individual task-space regulators for the two manipulators. In order to overcome problems arising from representation singularities, the unit quaternion is used to describe the orientation of relevant frames. To avoid steady-state internal stresses at the held object, kinetostatic filtering of the control action is introduced together with internal force feedback. Also, the performance of the control scheme is analyzed in the presence of a class of modeling uncertainties. The equilibria of the closed-loop system are then explicitly computed and asymptotic stability is proven via Lyapunov-like analysis.

Keywords: Manipulators; Robot control; Regulation; Equilibrium; Stability

1. Introduction

When a multi-arm system is employed for the manipulation of a common object, it is important to control both the variables describing the absolute motion of the system and the internal stresses at the held object. To this aim, the mappings between forces and velocities at the end effectors of the single manipulators and their counterparts at the manipulated object are to be considered (Uchiyama & Dauchez, 1993). Control schemes designed in this framework have been proposed for the control of absolute motion and internal forces (e.g. Li, Hsu & Sastry, 1989; Chiacchio, Chiaverini, Sciavicco & Siciliano, 1990; Wen & Kreutz, 1992; Hsu, 1993; Khatib, 1995). More recently, joint-space control schemes (Luecke & Lai, 1997), adaptive approaches to force/motion control (Jean & Fu, 1993; Liu & Arimoto, 1998) as well as impedance control schemes (Bonitz & Hsia, 1996) have been proposed.

A major problem of the approaches recalled above is that the task-space variables deriving from kinetostatic mappings are often unsuitable for the description of the cooperative task. Moreover, minimal representations of the orientation (e.g., three Euler angles) are usually adopted in the control laws; however, this class of representations suffers from representation singularities and does not provide a geometrically meaningful tool to describe the orientation.

As for the definition of the task-space variables, in this paper the task-oriented formulation devised by (Chiacchio, Chiaverini & Siciliano, 1996) is adopted, which fully characterizes a coordinated motion task and allows the user to specify the task in terms of meaningful variables. Based on this formulation, a kinematic control strategy has been recently proposed (Chiacchio & Chiaverini, 1996; Caccavale, Chiacchio & Chiaverini, 1999). The desired cooperative task-space trajectory is transformed into the corresponding joint motion for each manipulator via an inverse kinematics algorithm; then, an individual joint-space controller is designed for each manipulator. An alternative approach is represented by task-space control, in which a centralized control loop is closed directly on the cooperative task-space variables (Caccavale, Chiacchio & Chiaverini, 1997).

In this paper a novel force/position regulation scheme based on the same task-oriented formulation is proposed.

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Namely, the cooperative task-space desired trajectory is transformed into the corresponding motion at the two end effectors; then, a regulation loop is designed in the task space of each manipulator. Remarkably, the proposed strategy still allows the specification of the motion in the cooperative task space, while each control loop is designed in the task space of the single robot. Therefore, the proposed approach does not need the adoption of an inverse kinematics procedure, as required for the kinematic control strategy. Null internal forces at steady state are achieved by resorting to kinetostatic filtering of the task-space control actions, while force regulation is performed via internal force feedback.

A remarkable feature of the proposed scheme is the adoption the unit quaternion to represent the orientation of relevant frames; this choice allows avoiding the representation singularities and provides a geometrically meaningful way to describe the orientation as well as a powerful analysis tool.

The equilibrium of the closed-loop system is analyzed by resorting to the quaternion algebra. It is worth remarking that the use of the unit quaternion has made possible to compute and discuss the existence of the equilibria in terms of geometrically meaningful quantities (i.e., rotation matrices and equivalent rotation axes and angles). The stability analysis of the cooperative system under the proposed control law is developed and asymptotic convergence to the equilibria is proven via a Lyapunov-like argument.

Also, the behavior of the proposed scheme in the presence of imperfect gravity compensation is discussed in terms of position and internal force regulation accuracy. The performance of the regulation scheme has been experimentally verified on a setup composed by two six-dof industrial manipulators (Caccavale, Chiacchio & Chiaverini, 1998). The results — not reported here for brevity — have confirmed that good performance can be achieved in the presence of modeling errors and noisy measurements.

Finally, it is worth remarking that, although the proposed scheme has been conceived to solve regulation problems, the validity of the underlying ideas is more general, and can be extended to solve, e.g., tracking control problems.

2. Cooperative task-space formulation

In this section the task-oriented formulation developed in Chiacchio et al. (1996) is reformulated by using the unit quaternion for the description of the orientation.

Consider a system of two manipulators tightly grasping a rigid object. For each manipulator ($i = 1,2$) let n_i be the number of joints, $\mathbf{q}_i$ be the $(n_i \times 1)$ vector of joint variables, $\mathbf{p}_i$ the (3×1) vector denoting the end-effector position, $\mathbf{R}_i$ the (3×3) rotation matrix expressing the end-effector orientation, and $\mathcal{Q}_i$ the unit quaternion corresponding to $\mathbf{R}_i$ (see the appendix). Let also $\mathbf{v}_i = [\dot{\mathbf{p}}_i^{\mathrm{T}}\ \boldsymbol{\omega}_i^{\mathrm{T}}]^{\mathrm{T}}$ be the (6×1) end-effector (linear and angular) velocity vector which is related to the joint velocity vector through the manipulator Jacobian matrix $\mathbf{J}_i(\mathbf{q}_i)$. All quantities are expressed in a common base frame.

The *absolute position* of the system is given by the vector

$$\mathbf{p}_{\mathrm{a}} = \tfrac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2), \tag{1}$$

which defines the origin of the absolute frame. The rotation matrix defining the orientation of the absolute frame (i.e., the *absolute orientation*) is given by

$$\mathbf{R}_{\mathrm{a}} = \mathbf{R}_1 \mathbf{R}_k^1(\mathbf{k}_{21}^1, \vartheta_{21}/2), \tag{2}$$

where $\mathbf{k}_{21}^1$ and ϑ_{21} are the unit vector and the angle, respectively, that realize the rotation described by $\mathbf{R}_2^1 = \mathbf{R}_1^{\mathrm{T}}\mathbf{R}_2$, and the expression $\mathbf{R}_k^j(\mathbf{k}^j, \vartheta)$ denotes the rotation matrix corresponding to a rotation of ϑ about the axis $\mathbf{k}^j$. Let

$$\mathcal{Q}_k^1 = \{\eta_k, \boldsymbol{\varepsilon}_k^1\} = \left\{\cos\frac{\vartheta_{21}}{4},\ \mathbf{k}_{21}^1 \sin\frac{\vartheta_{21}}{4}\right\} \tag{3}$$

denote the unit quaternion extracted from $\mathbf{R}_k^1(\mathbf{k}_{21}^1, \vartheta_{21}/2)$; then, the absolute orientation can be expressed in terms of quaternion product (see the appendix) as follows:

$$\mathcal{Q}_{\mathrm{a}} = \{\eta_{\mathrm{a}}, \boldsymbol{\varepsilon}_{\mathrm{a}}\} = \mathcal{Q}_1 * \mathcal{Q}_k^1. \tag{4}$$

In order to fully describe a coordinated motion, the position and orientation of one manipulator relative to the other is also of concern. The *relative position* between the two end effectors is defined as the vector

$$\mathbf{p}_{\mathrm{r}} = \mathbf{p}_2 - \mathbf{p}_1. \tag{5}$$

The *relative orientation* between the two end effectors is defined with reference to the first end-effector frame in terms of the rotation matrix

$$\mathbf{R}_{\mathrm{r}}^1 = \mathbf{R}_2^1 = \mathbf{R}_1^{\mathrm{T}}\mathbf{R}_2, \tag{6}$$

representing the mutual orientation of the two end effectors. The relative orientation can be also expressed in terms of quaternion product through Eq. (A.2):

$$\mathcal{Q}_{\mathrm{r}}^1 = \{\eta_{\mathrm{r}}, \boldsymbol{\varepsilon}_{\mathrm{r}}^1\} = \mathcal{Q}_1^{-1} * \mathcal{Q}_2. \tag{7}$$

3. Modeling

3.1. Dynamics

The dynamics of the two manipulators can be written in compact form as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} - \mathbf{J}^{\mathrm{T}}(\mathbf{q})\mathbf{h}, \tag{8}$$

where the matrices are block diagonal, e.g. $\mathbf{M} = \mathrm{blockdiag}(\mathbf{M}_1, \mathbf{M}_2)$, and the vectors are stacked, e.g. $\mathbf{g} = [\mathbf{g}_1^{\mathrm{T}} \ \ \mathbf{g}_2^{\mathrm{T}}]^{\mathrm{T}}$. For each manipulator, $\mathbf{M}_i$ is the $(n_i \times n_i)$ symmetric positive-definite inertia matrix, $\mathbf{C}_i\dot{\mathbf{q}}_i$ is the $(n_i \times 1)$ vector of Coriolis and centrifugal forces, $\mathbf{F}_i$ is the $(n_i \times n_i)$ diagonal matrix of friction coefficients, $\mathbf{g}_i$ is the $(n_i \times 1)$ vector of gravitational forces, the vector $\boldsymbol{\tau}_i$ represents the joint torques and $\mathbf{h}_i$ is the (6×1) vector of generalized forces acting at the end effector of the ith manipulator. The dynamics of the object can be described by the equation

$$\mathbf{M}_{\mathrm{ext}}\dot{\mathbf{v}}_{\mathrm{ext}} + \mathbf{C}_{\mathrm{ext}}\mathbf{v}_{\mathrm{ext}} + \mathbf{g}_{\mathrm{ext}} = \mathbf{h}_{\mathrm{ext}}, \tag{9}$$

where $\mathbf{v}_{\mathrm{ext}}$ is the vector expressing the (linear and angular) velocity of a frame attached to a fixed point on the object (e.g., the center of mass), $\mathbf{M}_{\mathrm{ext}}$ is the object inertia matrix, $\mathbf{C}_{\mathrm{ext}}\mathbf{v}_{\mathrm{ext}}$ is the vector of velocity-dependent terms, $\mathbf{g}_{\mathrm{ext}}$ is the vector of gravitational forces, and $\mathbf{h}_{\mathrm{ext}}$ is the vector of external forces acting on the object.

3.2. Grasp geometry

Since the grasp is tight, each end effector can exert both a force and a moment on the object at the contact point. The mapping of the contact force vector $\mathbf{h}$ into the (6×1) external force vector $\mathbf{h}_{\mathrm{ext}}$ is

$$\mathbf{h}_{\mathrm{ext}} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{O}_3 & \mathbf{I}_3 & \mathbf{O}_3 \\ \mathbf{L}_1 & \mathbf{I}_3 & \mathbf{L}_2 & \mathbf{I}_3 \end{bmatrix}\mathbf{h} = \mathbf{W}\mathbf{h}, \tag{10}$$

where $\mathbf{W}$ is the grasp matrix, $\mathbf{O}_l$ denotes the $(l \times l)$ null matrix and $\mathbf{I}_l$ denotes the $(l \times l)$ identity matrix. The matrix $\mathbf{L}_i = \mathbf{S}(\mathbf{r}_i)$ $(i = 1{,}2)$ transforms the applied contact forces in moments at the object frame, $\mathbf{S}(\cdot)$ is the skew-symmetric matrix operator performing the cross product and $\mathbf{r}_i$ is the (3×1) vector from the ith end effector to the point fixed on the object (i.e., $\mathbf{r}_i$ is the so-called *virtual stick* in Uchiyama and Dauchez (1993)).

The matrix $\mathbf{W}$ is full row rank; then, for a given $\mathbf{h}_{\mathrm{ext}}$ the inverse solution to (10) is given by

$$\mathbf{h} = \mathbf{W}^{\dagger}\mathbf{h}_{\mathrm{ext}} + \mathbf{V}\mathbf{h}_{\mathrm{int}}, \tag{11}$$

where $\mathbf{W}^{\dagger}$ denotes a pseudoinverse of $\mathbf{W}$, $\mathbf{V}$ is a full column rank matrix spanning the null space of $\mathbf{W}$, and $\mathbf{h}_{\mathrm{int}}$ represents the vector of internal forces, i.e., the forces which do not contribute to the motion of the object, but represent mechanical stresses applied to the object. It has been recognized in Walker, Freeman and Marcus (1989) that the use in (11) of a generic pseudoinverse of $\mathbf{W}$, e.g. the Moore–Penrose pseudoinverse, may lead to internal stresses even if $\mathbf{h}_{\mathrm{int}} = \mathbf{0}$; to avoid this, $\mathbf{W}^{\dagger}$ must be properly chosen. One possible choice for the matrix $\mathbf{V}$ is proposed in Chiacchio, Chiaverini, Sciavicco and Siciliano (1991) and is not reported here for brevity.

When each virtual stick $\mathbf{r}_i$ is null, then the grasp matrix reduces to $\mathbf{W} = [\mathbf{I}_6 \ \ \mathbf{I}_6]$; this is the case when the kinematics of the ith manipulator is expressed in terms of an end-effector frame located at the tip of the corresponding virtual stick; without loss of generality, it is assumed also that the orientation of the two end-effector frames coincides (i.e., $\mathbf{R}_1^{\mathrm{T}}\mathbf{R}_2 = \mathbf{I}_3$). It can be recognized that in such a case the task-oriented formulation reviewed in Section 2 collapses into the so-called *symmetric formulation* proposed in Uchiyama and Dauchez (1993).

3.3. Closed-chain constraints

Tight grasp of a rigid object results in the differential constraint

$$\mathbf{V}^{\mathrm{T}}\mathbf{v} = \mathbf{V}^{\mathrm{T}}\mathbf{J}\dot{\mathbf{q}} = \mathbf{0}, \tag{12}$$

which represents the closed-chain constraint expressed in terms of velocities. When the direct kinematics of each manipulator is referred to the tip of the corresponding virtual stick (i.e., $\mathbf{r}_i = \mathbf{0}$), the closed-chain constraint can be expressed at position and orientation level as follows:

$$\begin{aligned} &\mathbf{p}_2 - \mathbf{p}_1 = \mathbf{0}, \\ &\mathcal{Q}_1^{-1} * \mathcal{Q}_2 = \mathcal{Q}_2 * \mathcal{Q}_1^{-1} = \mathcal{Q}_0, \end{aligned} \tag{13}$$

where $\mathcal{Q}_0 = \{1, \mathbf{0}\}$ is the unitary quaternion (see the appendix).

It is worth remarking that the above conditions represent a set of mechanical constraints, and thus they are always fulfilled during system's motion. On the other hand, the desired trajectories may violate the constraints if the trajectory generation module computes the set-points based on an inaccurate geometrical model of the grasp as well as if even small algorithmic errors arise when the desired configuration is computed on line. This may cause building up of internal stresses at the held object at steady state. Finally, it is worth remarking that during fast transients large internal forces may be experienced as well, e.g., due to imperfectly and/or partially compensated dynamics in the system or large tracking errors.

In the following, a regulation strategy is proposed which is capable of counteracting the above-mentioned effects at steady state.

4. Task-space regulation

4.1. Generation of reference trajectories

The task-oriented formulation described in Section 2 fully characterizes a coordinated motion task and, at the same time, allows the user to naturally specify the task in terms of meaningful variables. Therefore, it is convenient to assign the task for the cooperative system in terms of absolute and relative variables.

At the motion control level, the adoption of a kinematic control strategy (Chiacchio & Chiaverini, 1996; Caccavale et al., 1999) requires the presence of an inverse kinematics generating the reference variables for the joint-space individual controllers. On the other hand, a control law designed directly in the cooperative task space (Caccavale et al., 1997) does not allow the adoption of two individual controllers as building blocks of the control architecture. The goal here is to retain the advantage of the kinematic control (i.e., the adoption of individual controllers for the manipulators), and, at the same time, to avoid the use of an inverse kinematics procedure to generate the joint motion references. To the purpose, a little extra computational effort has to be spent at the trajectory generation level.

Once the desired motion for the system is assigned in terms of cooperative task-space variables (i.e., absolute and relative), these can be transformed into the corresponding ones at the end effectors of the two arms. Let $\mathbf{p}_{\mathrm{ad}}$ and $\mathbf{p}_{\mathrm{rd}}$ be the vectors of desired absolute and relative positions, respectively; then, the corresponding desired end-effector position vectors $\mathbf{p}_{1\mathrm{d}}$ and $\mathbf{p}_{2\mathrm{d}}$ can be easily derived from (1) and (5) as

$$\mathbf{p}_{1\mathrm{d}} = \mathbf{p}_{\mathrm{ad}} - \tfrac{1}{2}\mathbf{p}_{\mathrm{rd}}, \tag{14}$$

$$\mathbf{p}_{2\mathrm{d}} = \mathbf{p}_{\mathrm{ad}} + \tfrac{1}{2}\mathbf{p}_{\mathrm{rd}}. \tag{15}$$

The desired rotation matrices $\mathbf{R}_{1\mathrm{d}}$ and $\mathbf{R}_{2\mathrm{d}}$ can be derived via (2) and (6) as

$$\mathbf{R}_{1\mathrm{d}} = \mathbf{R}_{\mathrm{ad}}\mathbf{R}_k^{1\mathrm{d}^{\mathrm{T}}}(\mathbf{k}_{\mathrm{rd}}^{1\mathrm{d}}, \vartheta_{\mathrm{rd}}/2), \tag{16}$$

$$\mathbf{R}_{2\mathrm{d}} = \mathbf{R}_{1\mathrm{d}}\mathbf{R}_{\mathrm{rd}}^{1\mathrm{d}}, \tag{17}$$

where $\mathbf{R}_{\mathrm{ad}}$ and $\mathbf{R}_{\mathrm{rd}}^{1\mathrm{d}}$ are the desired absolute and relative rotation matrices, and $\mathbf{k}_{\mathrm{rd}}^{1\mathrm{d}}$ and ϑ_{rd} are the desired unit vector and rotation angle used to assign $\mathbf{R}_{\mathrm{rd}}^{1\mathrm{d}} = \mathbf{R}_k^{1\mathrm{d}}(\mathbf{k}_{\mathrm{rd}}^{1\mathrm{d}}, \vartheta_{\mathrm{rd}})$. Of course, the above two relations can be expressed in terms of quaternions as follows:

$$\mathcal{Q}_{1\mathrm{d}} = \mathcal{Q}_{\mathrm{ad}} * \left\{\cos\frac{\vartheta_{\mathrm{rd}}}{4}, -\mathbf{k}_{\mathrm{rd}}^{1\mathrm{d}}\sin\frac{\vartheta_{\mathrm{rd}}}{4}\right\}, \tag{18}$$

$$\mathcal{Q}_{2\mathrm{d}} = \mathcal{Q}_{1\mathrm{d}} * \mathcal{Q}_{\mathrm{rd}}^{1\mathrm{d}} = \mathcal{Q}_{1\mathrm{d}} * \left\{\cos\frac{\vartheta_{\mathrm{rd}}}{2}, \mathbf{k}_{\mathrm{rd}}^{1\mathrm{d}}\sin\frac{\vartheta_{\mathrm{rd}}}{2}\right\}, \tag{19}$$

where the unit quaternion $\mathcal{Q}_{\mathrm{ad}}$, representing the desired absolute orientation, can be keenly computed in terms of an equivalent angle/axis description, i.e. $\mathcal{Q}_{\mathrm{ad}} = \{\cos(\vartheta_{\mathrm{ad}}/2), \mathbf{k}_{\mathrm{ad}}\sin(\vartheta_{\mathrm{ad}}/2)\}$.

4.2. Control law

Consider a system of two spatial nonredundant manipulators ($n_1 = n_2 = 6$) moving in a singularity-free region of their workspace (i.e., the Jacobian matrix $\mathbf{J}$ is square and nonsingular).

If a regulation problem is considered (i.e., constant desired trajectories), a possible control law designed in the task space of the single manipulators is the PD regulator with gravity compensation:

$$\tau = \mathbf{J}^{\mathrm{T}}\mathbf{K}_{\mathrm{p}}\mathbf{e} - \mathbf{K}_{\mathrm{d}}\dot{\mathbf{q}} + \mathbf{g}_{\mathrm{e}}, \tag{20}$$

where $\mathbf{K}_{\mathrm{d}}$ is a positive-definite matrix, $\mathbf{K}_{\mathrm{p}} = \mathrm{diag}\{\mathbf{K}_{\mathrm{p}P}, k_{\mathrm{p}O}\mathbf{I}_3, \mathbf{K}_{\mathrm{p}P}, k_{\mathrm{p}O}\mathbf{I}_3\}$ is a block-diagonal positive-definite matrix, and $\mathbf{g}_{\mathrm{e}} = \mathbf{g} + \mathbf{J}^{\mathrm{T}}\mathbf{W}^{\dagger}\mathbf{g}_{\mathrm{ext}}$.

The first term on the right-hand side of (20) is a proportional action on the task-space error $\mathbf{e} = [\mathbf{e}_1^{\mathrm{T}}\ \mathbf{e}_2^{\mathrm{T}}]^{\mathrm{T}}$, with

$$\mathbf{e}_i = \begin{bmatrix}\tilde{\mathbf{p}}_i \\ \tilde{\boldsymbol{\varepsilon}}_i\end{bmatrix} = \begin{bmatrix}\mathbf{p}_{i\mathrm{d}} - \mathbf{p}_i \\ \mathbf{R}_i\tilde{\boldsymbol{\varepsilon}}_i^i\end{bmatrix}, \quad i = 1,2 \tag{21}$$

where $\tilde{\boldsymbol{\varepsilon}}_i^i$ is the vector part of the quaternion $\tilde{\mathcal{Q}}_i^i = \mathcal{Q}_i^{-1} * \mathcal{Q}_{i\mathrm{d}}$ and $\tilde{\boldsymbol{\varepsilon}}_i$ corresponds to $\tilde{\mathcal{Q}}_i = \mathcal{Q}_{i\mathrm{d}} * \mathcal{Q}_i^{-1}$ (see (A.2) and (A.3) in the appendix).

The equilibrium of system (8), (9), (11) under the control law (20) satisfies the conditions (the subscript ss denotes steady-state quantities)

$$\mathbf{W}\mathbf{K}_{\mathrm{p}}\mathbf{e}_{\mathrm{ss}} = \mathbf{0}, \tag{22}$$

$$\mathbf{h}_{\mathrm{int,ss}} = \mathbf{V}^{\dagger}\mathbf{K}_{\mathrm{p}}\mathbf{e}_{\mathrm{ss}}, \tag{23}$$

which reveal that internal forces arise at steady state if a task-space error is present. Such an error may be due to even small inaccuracies of the trajectory generation module, leading to reference trajectories not consistent with the geometry of the grasp (i.e., with the closed-chain constraints).

To avoid building of internal forces at steady state, it is proposed to modify the control law (20) to include kinetostatic filtering of the cooperative task-space control action

$$\tau = \mathbf{J}^{\mathrm{T}}(\mathbf{W}^{\dagger}\mathbf{W} + \mathbf{V}\boldsymbol{\Sigma}\mathbf{V}^{\dagger})\mathbf{K}_{\mathrm{p}}\mathbf{e} - \mathbf{K}_{\mathrm{d}}\dot{\mathbf{q}} + \mathbf{g}_{\mathrm{e}}. \tag{24}$$

In (24) the (6×6) diagonal matrix $\boldsymbol{\Sigma} = \mathrm{diag}\{\sigma_i\}$ weights the components of $\mathbf{K}_{\mathrm{p}}\mathbf{e}$ in each direction of the subspace of the internal forces via the constant values $0 \le \sigma_i \le 1$. In detail, if $\boldsymbol{\Sigma} = \mathbf{O}_6$ then all these components are completely cancelled from the control action, while the choice $\boldsymbol{\Sigma} = \mathbf{I}_6$ leads to the control law (20) without kinetostatic filtering. The equilibrium of system (8), (9), (11) under the control law (24), satisfies the conditions

$$\mathbf{W}\mathbf{K}_{\mathrm{p}}\mathbf{e}_{\mathrm{ss}} = \mathbf{0}, \tag{25}$$

$$\mathbf{h}_{\mathrm{int,ss}} = \boldsymbol{\Sigma}\mathbf{V}^{\dagger}\mathbf{K}_{\mathrm{p}}\mathbf{e}_{\mathrm{ss}}. \tag{26}$$

Remarkably, the former condition implies that the component of the task-space error term in the external-force space vanishes, the latter ensures that the ith component of the internal force at steady state is reduced according to σ_i. Therefore, if $\sigma_i = 0$ the control law (24) cancels the ith component of the internal force at steady state, even if the corresponding task-space set point cannot be reached due to the closed-chain constraints.

4.3. Equilibrium analysis

Eqs. (25) and (26) can be interpreted as a set of constraints on $\mathbf{e}_{ss}$. Nonlinearity of the equations does not allow drawing general conclusions about their solution. Nevertheless, when the direct kinematics of each manipulator is expressed in terms a virtual end effector coincident with the tip of the virtual stick, the equilibria can be explicitly computed. Therefore, all the results in this section are derived under this assumption.

In the case of kinematics referred to the tip of virtual sticks, Eq. (25) can be rewritten as

$$\begin{cases} \tilde{\mathbf{p}}_{1,ss} + \tilde{\mathbf{p}}_{2,ss} = \mathbf{0}, \\ \tilde{\boldsymbol{\varepsilon}}_{1,ss} + \tilde{\boldsymbol{\varepsilon}}_{2,ss} = \mathbf{0}. \end{cases} \tag{27}$$

It can be recognized that the second equation in (27) is implied either by the condition

$$\tilde{\mathcal{Q}}_{1,ss} = \tilde{\mathcal{Q}}_{2,ss}^{-1}, \tag{28}$$

or by the condition

$$\tilde{\mathcal{Q}}_{1,ss} = \tilde{\mathcal{Q}}'_{2,ss} = \{-\tilde{\eta}_{2,ss}, -\tilde{\boldsymbol{\varepsilon}}_{2,ss}\}. \tag{29}$$

The constant reference trajectories satisfy the following constraints:

$$\mathbf{p}_{2d} - \mathbf{p}_{1d} = \bar{\mathbf{p}}_{21d}, \tag{30}$$

$$\mathcal{Q}_{2d} * \mathcal{Q}_{1d}^{-1} = \bar{\mathcal{Q}}_{21d} = \{\bar{\eta}_{21d}, \bar{\boldsymbol{\varepsilon}}_{21d}\}, \tag{31}$$

where it is assumed $\bar{\eta}_{21d} \geq 0$. According to (13), it is $\bar{\mathbf{p}}_{21d} = \mathbf{0}$ and $\bar{\mathcal{Q}}_{21d} = \mathcal{Q}_0$ if the set-points are consistent with the closed-chain constraints. From (30), (31) and the constraint equations (13), it follows

$$\begin{cases} \tilde{\mathbf{p}}_{2,ss} - \tilde{\mathbf{p}}_{1,ss} = \bar{\mathbf{p}}_{21d}, \\ \tilde{\mathcal{Q}}_{2,ss} * \tilde{\mathcal{Q}}_{1,ss}^{-1} = \bar{\mathcal{Q}}_{21d}. \end{cases} \tag{32}$$

As for the position errors, Eqs. (27) and (32) provide the equilibrium solution

$$\tilde{\mathbf{p}}_{1,ss} = -\tilde{\mathbf{p}}_{2,ss} = -\tfrac{1}{2}\bar{\mathbf{p}}_{21d}. \tag{33}$$

If the reference trajectories are compatible with the constraints (i.e., $\bar{\mathbf{p}}_{21d} = \mathbf{0}$), the equilibrium solution for the position is represented by

$$\tilde{\mathbf{p}}_{1,ss} = \tilde{\mathbf{p}}_{2,ss} = \mathbf{0}, \tag{34}$$

which reveals that the position set-points are reached at steady state.

Regarding the orientation errors, Eqs. (28) and (29) must be separately combined with the second equation in (32). Let us first consider condition (28) which, combined with the second equation in (32), yields

$$\tilde{\mathcal{Q}}_{1,ss} * \tilde{\mathcal{Q}}_{1,ss} = \bar{\mathcal{Q}}_{21d}^{-1} = \{\bar{\eta}_{21d}, -\bar{\boldsymbol{\varepsilon}}_{21d}\}. \tag{35}$$

By applying Property A.1 in the appendix, it can be recognized that Eq. (35) provides two solutions for $\tilde{\mathcal{Q}}_{1,ss}$ (if $\bar{\eta}_{21d} \geq 0$)

$$\tilde{\mathcal{Q}}_{1,ss} = \{\tilde{\eta}_s, \tilde{\boldsymbol{\varepsilon}}_s\}, \qquad \tilde{\mathcal{Q}}_{1,ss} = \{-\tilde{\eta}_s, -\tilde{\boldsymbol{\varepsilon}}_s\}, \tag{36}$$

where

$$\{\tilde{\eta}_s, \tilde{\boldsymbol{\varepsilon}}_s\} = \left\{\sqrt{\frac{1+\bar{\eta}_{21d}}{2}}, -\frac{\sqrt{2}}{2\sqrt{1+\bar{\eta}_{21d}}}\bar{\boldsymbol{\varepsilon}}_{21d}\right\}. \tag{37}$$

The corresponding value of $\tilde{\mathcal{Q}}_{2,ss}$ for each solution is computed from (28)

$$\tilde{\mathcal{Q}}_{2,ss} = \tilde{\mathcal{Q}}_{1,ss}^{-1}. \tag{38}$$

Note that Eq. (36) provides two equilibria corresponding to the same relative orientation between the actual and desired frames. In the case $\bar{\mathcal{Q}}_{21d} = \mathcal{Q}_0$ (i.e., set points compatible with the constraints), the equilibria in (36) become

$$\tilde{\mathcal{Q}}_{1,ss} = \tilde{\mathcal{Q}}_{2,ss} = \mathcal{Q}_0, \qquad \tilde{\mathcal{Q}}_{1,ss} = \tilde{\mathcal{Q}}_{2,ss} = \mathcal{Q}'_0, \tag{39}$$

i.e., the desired orientation is reached at steady state. On the other hand, it can be easily recognized that condition (29) holds if and only if $\bar{\mathcal{Q}}_{21d} = \{-1, \mathbf{0}\}$, which is in contradiction with the assumption $\bar{\eta}_{21d} \geq 0$; hence, the only equilibrium solutions for the orienation errors are given by (36) or (39).

Finally, it is worth investigating the steady-state behavior of the system in terms of absolute and relative variables. Obviously, the desired absolute and relative configurations are reached if the set-points do not violate the constraints. On the other hand, when the set-points are not compatible with the closed-chain constraints it is not trivial to predict the behavior of the absolute and relative displacements. In the following, it is shown that the desired absolute configuration is always reached at steady state, even if the set-points are not compatible with the geometry of the grasp.

It can be easily recognized that the first condition in (27) implies

$$\mathbf{p}_{ad} - \mathbf{p}_{a,ss} = \mathbf{0} \tag{40}$$

and thus the absolute position set-point is reached. As for the orientation error, Eqs. (19) and (A.3) imply $\bar{\eta}_{21d} = \cos(\vartheta_{rd}/2)$ and $\bar{\boldsymbol{\varepsilon}}_{21d} = \mathbf{R}_{1d}\mathbf{k}_{rd}^{1d}\sin(\vartheta_{rd}/2)$. Hence, from (37) and (A.3) it follows that

$$\mathcal{Q}_{1,\mathrm{ss}}^{-1} * \mathcal{Q}_{1,\mathrm{d}} = \left\{ \cos\frac{\vartheta_{\mathrm{rd}}}{4}, -\mathbf{k}_{\mathrm{rd}}^{1\mathrm{d}} \sin\frac{\vartheta_{\mathrm{rd}}}{4} \right\}, \tag{41}$$

$$\mathcal{Q}_{1,\mathrm{ss}}^{-1} * \mathcal{Q}_{1,\mathrm{d}} = \left\{ -\cos\frac{\vartheta_{\mathrm{rd}}}{4}, \mathbf{k}_{\mathrm{rd}}^{1\mathrm{d}} \sin\frac{\vartheta_{\mathrm{rd}}}{4} \right\}, \tag{42}$$

which correspond to the same mutual orientation between the desired and the actual frame. Hence, the expression of the absolute orientation error can be derived via (18) and, e.g., (41)

$$\begin{aligned}\mathcal{Q}_{\mathrm{ad}} * \mathcal{Q}_{\mathrm{a,ss}}^{-1} &= \mathcal{Q}_{1\mathrm{d}} * \left\{ \cos\frac{\vartheta_{\mathrm{rd}}}{4}, \mathbf{k}_{\mathrm{rd}}^{1\mathrm{d}} \sin\frac{\vartheta_{\mathrm{rd}}}{4} \right\} * \mathcal{Q}_{\mathrm{a,ss}}^{-1} \\ &= \mathcal{Q}_{1\mathrm{d}} * (\mathcal{Q}_{1,\mathrm{ss}}^{-1} * \mathcal{Q}_{1\mathrm{d}})^{-1} * \mathcal{Q}_{1,\mathrm{ss}}^{-1} = \mathcal{Q}_0,\end{aligned} \tag{43}$$

where the equality $\mathcal{Q}_{1,\mathrm{ss}} = \mathcal{Q}_{\mathrm{a,ss}}$, coming from the closed-chain constraint (13) and the definition of the absolute orientation (4), has been exploited. Hence, the absolute orientation at steady state coincides with the desired one.

In conclusion, the proposed control law ensures null internal forces at the expense of the sole relative variables, while the desired absolute configuration is reached. This ensures always a precise positioning of the held object while avoiding object's internal stresses.

4.4. Stability analysis

Assume the matrix $\mathbf{J}$ square and nonsingular, i.e., the two manipulators are nonredundant and moving in a singularity free region of the workspace. Under this assumption a stability analysis can be devised based on the invariant set theorem (e.g., see Khalil, 1996).

Consider the scalar function with continuous first partial derivatives

$$V(\mathbf{x}) = \tfrac{1}{2}\mathbf{v}^{\mathrm{T}}\mathbf{M}_{\mathrm{e}}(\mathbf{q})\mathbf{v} + V_P + V_O, \tag{44}$$

where $\mathbf{x} = [\mathbf{e}^{\mathrm{T}}\ \mathbf{v}^{\mathrm{T}}]^{\mathrm{T}}$ belongs to the set $\Omega \subset \mathbb{R}^{24}$ of task-space errors and end-effector velocities satisfying the closed-chain constraints, and $\mathbf{M}_{\mathrm{e}} = \mathbf{J}^{-\mathrm{T}}\mathbf{M}\mathbf{J}^{-1} + \mathbf{W}^{\dagger}\mathbf{M}_{\mathrm{ext}}\mathbf{W}^{\dagger\mathrm{T}}$. In (44) V_P is defined as follows:

$$V_P = \tfrac{1}{2}\mathbf{e}_P^{\mathrm{T}}\mathbf{K}_P\mathbf{e}_P, \tag{45}$$

where $\mathbf{e}_P = [\tilde{\mathbf{p}}_1^{\mathrm{T}}\ \tilde{\mathbf{p}}_2^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{K}_P = \mathrm{diag}\{\mathbf{K}_{\mathrm{p}P}, \mathbf{K}_{\mathrm{p}P}\}$. The function V_O is chosen as

$$V_O = k_{\mathrm{p}O}((1-\tilde{\eta}_1)^2 + \|\tilde{\boldsymbol{\varepsilon}}_1^1\|^2 + (1-\tilde{\eta}_2)^2 + \|\tilde{\boldsymbol{\varepsilon}}_2^2\|^2). \tag{46}$$

Note that V_P and V_O are both quadratic, thus $V(\mathbf{x})$ is positive definite and radially unbounded in Ω.

The time derivative of (44) along the trajectories of system (8), (9), (11) under the control law (24) is given by

$$\begin{aligned}\dot{V}(\mathbf{x}) = &-\mathbf{v}^{\mathrm{T}}(\mathbf{F}_{\mathrm{e}} + \mathbf{J}^{-\mathrm{T}}\mathbf{K}_{\mathrm{d}}\mathbf{J}^{-1})\mathbf{v} + \mathbf{v}^{\mathrm{T}}\mathbf{W}^{\dagger}\mathbf{W}\mathbf{K}_{\mathrm{p}}\mathbf{e} \\ &+ \dot{V}_P + \dot{V}_O,\end{aligned} \tag{47}$$

where the identity $\mathbf{v}^{\mathrm{T}}(\dot{\mathbf{M}}_{\mathrm{e}} - 2\mathbf{C}_{\mathrm{e}})\mathbf{v} = \mathbf{0}$ and the closed-chain constraint (12) have been used, and $\mathbf{F}_{\mathrm{e}} = \mathbf{J}^{-\mathrm{T}}\mathbf{F}\mathbf{J}^{-1}$. From (21) and (A.4) it follows

$$\dot{V}_P + \dot{V}_O = -\dot{\mathbf{p}}^{\mathrm{T}}\mathbf{K}_P\mathbf{e}_P - \boldsymbol{\omega}^{\mathrm{T}}\mathbf{K}_O\mathbf{e}_O = -\mathbf{v}^{\mathrm{T}}\mathbf{K}_{\mathrm{p}}\mathbf{e}, \tag{48}$$

where $\mathbf{K}_O = \mathrm{diag}\{k_{\mathrm{p}O}\mathbf{I}_3, k_{\mathrm{p}O}\mathbf{I}_3\}$. Thus, Eq. (47) becomes

$$\begin{aligned}\dot{V}(\mathbf{x}) = &-\mathbf{v}^{\mathrm{T}}(\mathbf{F}_{\mathrm{e}} + \mathbf{J}^{-\mathrm{T}}\mathbf{K}_{\mathrm{d}}\mathbf{J}^{-1})\mathbf{v} \\ &-\mathbf{v}^{\mathrm{T}}(\mathbf{I}_{12} - \mathbf{W}^{\dagger}\mathbf{W})\mathbf{K}_{\mathrm{p}}\mathbf{e}.\end{aligned} \tag{49}$$

It can be recognized that the matrix $(\mathbf{I}_{12} - \mathbf{W}^{\dagger}\mathbf{W})$ is a projector onto the null space of the grasp matrix $\mathbf{W}$; in view of constraint (12), this implies that the second term on the right-hand side of the above equality is null, and thus Eq. (49) reduces to

$$\dot{V}(\mathbf{x}) = -\mathbf{v}^{\mathrm{T}}(\mathbf{F}_{\mathrm{e}} + \mathbf{J}^{-\mathrm{T}}\mathbf{K}_{\mathrm{d}}\mathbf{J}^{-1})\mathbf{v}, \tag{50}$$

which shows that $\dot{V}$ is negative semi-definite all over Ω.

The set R of all points $\mathbf{x} \in \Omega$ where $\dot{V}(\mathbf{x}) = 0$ is given by

$$R = \{\mathbf{x} \in \Omega\colon\ \mathbf{v} = \mathbf{0}\}. \tag{51}$$

From the equilibrium analysis in the previous subsection it follows that the largest invariant set in R is

$$M = \{\mathbf{x} \in R\colon\ \mathbf{e}_{\mathrm{ss}} \text{ satisfies (25)}\}. \tag{52}$$

Therefore, the global invariant set theorem ensures asymptotic convergence to the set M. In the cases analyzed in the previous subsection the set M reduces to the elements defined either by (33), (36), (38) or by (34), (39). In the last case, the equilibrium is unique for the position error, while Eq. (39) defines two different equilibria (although they represent the same mutual orientation). However, it can be recognized that the equilibrium represented by $\mathcal{Q}_{1,\mathrm{ss}} = \mathcal{Q}_{2,\mathrm{ss}} = \{-1, \mathbf{0}\}$ is unstable. In fact, the value of function (44) at this equilibrium is

$$V_{\mathrm{ss}} = 8k_{\mathrm{p}O}. \tag{53}$$

Consider a small perturbation around the equilibrium characterized by $\tilde{\mathbf{p}}_i = \mathbf{0}$, $\mathbf{v}_i = \mathbf{0}$, $\tilde{\eta}_i = -1 + \sigma$ $(\sigma > 0)$, and consequently $\|\tilde{\boldsymbol{\varepsilon}}_i^i\|^2 = \sigma(2-\sigma)$ $(i = 1,2)$. The perturbed Lyapunov function is

$$V_\sigma = 8k_{\mathrm{p}O}\left(1 - \frac{\sigma}{2}\right) < V_{\mathrm{ss}}. \tag{54}$$

Since $V(t)$ has been proven to be decreasing, it will never return to V_{ss}, implying that this equilibrium is unstable. Therefore, the only asymptotically stable equilibrium is $\mathbf{x} = \mathbf{0}$, with $\tilde{\eta}_i = 1$ $(i = 1,2)$.

4.5. Addition of internal force feedback

If it is desired to impose a given internal force set point, force feedback should be added. Also, addition of force feedback might improve the performance in the presence of modeling errors and disturbances. To the purpose, control law (24) can be modified as follows:

$$\tau = \mathbf{J}^{\mathrm{T}}(\mathbf{W}^{\dagger}\mathbf{W} + \mathbf{V}\boldsymbol{\Sigma}\mathbf{V}^{\dagger})\mathbf{K}_{\mathrm{p}}\mathbf{e} - \mathbf{K}_{\mathrm{d}}\dot{\mathbf{q}} + \mathbf{g}_{\mathrm{e}} + \mathbf{J}^{\mathrm{T}}\mathbf{V}\mathbf{h}_{\mathrm{int,c}}. \quad (55)$$

The new control input $\mathbf{h}_{\mathrm{int,c}}$ is defined as

$$\mathbf{h}_{\mathrm{int,c}} = \mathbf{h}_{\mathrm{int,d}} + \mathbf{K}_f(\mathbf{h}_{\mathrm{int,d}} - \mathbf{h}_{\mathrm{int}}), \quad (56)$$

where $\mathbf{K}_f$ is a positive-definite matrix and $\mathbf{h}_{\mathrm{int,d}}$ is the vector of desired internal forces. The internal force vector $\mathbf{h}_{\mathrm{int}}$ can be computed from the vector of measured end-effector forces $\mathbf{h}$ via the inverse of mapping (11).

For given set points, the equilibrium of system (8), (9), (11) under the control law (55), (56) is defined by the two equations

$$\mathbf{W}\mathbf{K}_{\mathrm{p}}\mathbf{e}_{\mathrm{ss}} = \mathbf{0}, \quad (57)$$

$$\mathbf{h}_{\mathrm{int,ss}} = \mathbf{h}_{\mathrm{int,d}} + (\mathbf{I}_6 + \mathbf{K}_f)^{-1}\boldsymbol{\Sigma}\mathbf{V}^{\dagger}\mathbf{K}_{\mathrm{p}}\mathbf{e}_{\mathrm{ss}}, \quad (58)$$

which reveal that the adoption of the force loop ensures $\mathbf{h}_{\mathrm{int,ss}} = \mathbf{h}_{\mathrm{int,d}}$ when $\boldsymbol{\Sigma} = \mathbf{O}_6$; if $\boldsymbol{\Sigma} \neq \mathbf{O}_6$ the desired value of internal force is reached along the ith component if $\sigma_i = 0$.

The equilibrium analysis for $\mathbf{e}_{\mathrm{ss}}$ is the same as that developed in the previous section. To analyze the stability of the equilibria defined by (57) and (58) the same scalar function $V(\mathbf{x})$ as in (44) can be used. It can be recognized that its time derivative along the trajectories of the system is the same as in (50); indeed, by substituting control law (55) into (47), the force feedback terms do not contribute to $\dot{V}(\mathbf{x})$ in view of the closed-chain constraint (12). Therefore, the same argument leads to establishing asymptotic convergence to the set

$$M_f = \{\mathbf{x} \in R\colon\ \mathbf{e}_{\mathrm{ss}} \text{ satisfies (57)}\}. \quad (59)$$

4.6. Imperfect gravity compensation

In many practical cases, the object's dynamic model is not accurately known and only nominal estimates of the dynamic parameters are available. Thus, if an estimate of the gravity term $\hat{\mathbf{g}}_{\mathrm{e}}$ is used, the control law (55) becomes

$$\tau = \mathbf{J}^{\mathrm{T}}(\mathbf{W}^{\dagger}\mathbf{W} + \mathbf{V}\boldsymbol{\Sigma}\mathbf{V}^{\dagger})\mathbf{K}_{\mathrm{p}}\mathbf{e} - \mathbf{K}_{\mathrm{d}}\dot{\mathbf{q}} + \hat{\mathbf{g}}_{\mathrm{e}} + \mathbf{J}^{\mathrm{T}}\mathbf{V}\mathbf{h}_{\mathrm{int,c}}, \quad (60)$$

where $\hat{\mathbf{g}}_{\mathrm{e}} = \hat{\mathbf{g}} + \mathbf{J}^{\mathrm{T}}\mathbf{W}^{\dagger}\hat{\mathbf{g}}_{\mathrm{ext}}$.

In this case the equilibria satisfy the conditions

$$\mathbf{W}\mathbf{K}_{\mathrm{p}}\mathbf{e}_{\mathrm{ss}} = \mathbf{W}(\mathbf{g}_{\mathrm{e}} - \hat{\mathbf{g}}_{\mathrm{e}}), \quad (61)$$

$$\mathbf{h}_{\mathrm{int,ss}} = \mathbf{h}_{\mathrm{int,d}} + (\mathbf{I}_6 + \mathbf{K}_f)^{-1} \times (\boldsymbol{\Sigma}\mathbf{V}^{\dagger}\mathbf{K}_{\mathrm{p}}\mathbf{e}_{\mathrm{ss}} - \mathbf{V}^{\dagger}(\mathbf{g}_{\mathrm{e}} - \hat{\mathbf{g}}_{\mathrm{e}})). \quad (62)$$

It can be easily recognized that zero steady-state errors cannot be obtained, even if the set-points are compatible with the geometry of grasp; moreover, an additional term affects the internal forces at steady state. It is worth noticing that the object's gravity force does not contribute to this additional term, and thus $\mathbf{V}^{\dagger}(\mathbf{g}_{\mathrm{e}} - \hat{\mathbf{g}}_{\mathrm{e}}) = \mathbf{0}$ if the gravity terms in the manipulators model are perfectly compensated.

The convergence to the new set of equilibria can be shown via the same Lyapunov argument as above, where the candidate function is to be modified as follows:

$$V_{\mathrm{g}}(\mathbf{x}) = \tfrac{1}{2}\mathbf{v}^{\mathrm{T}}\mathbf{M}_{\mathrm{e}}(\mathbf{q})\mathbf{v} + V_P + V_O + U_{\mathrm{g}}(\mathbf{q}) - \hat{U}_{\mathrm{g}}(\mathbf{q}), \quad (63)$$

where U_{g} and $\hat{U}_{\mathrm{g}}$ are two scalar energy functions representing the potential associated with the gravity terms (see Tomei, 1991)

$$\frac{\partial U_{\mathrm{g}}(\mathbf{q})}{\partial \mathbf{q}} = \mathbf{J}^{\mathrm{T}}\mathbf{g}_{\mathrm{e}}, \qquad \frac{\partial \hat{U}_{\mathrm{g}}(\mathbf{q})}{\partial \mathbf{q}} = \mathbf{J}^{\mathrm{T}}\hat{\mathbf{g}}_{\mathrm{e}}. \quad (64)$$

In view of the boundedness of the two energy functions, it can be recognized that V_{g} is radially unbounded and its time derivative is the same as $\dot{V}$ in (50). Hence, the invariant set theorem ensures global asymptotic convergence to the set

$$M_{\mathrm{g}} = \{\mathbf{x} \in R\colon\ \mathbf{e}_{\mathrm{ss}} \text{ satisfies (61)}\}. \quad (65)$$

5. Conclusions

In this paper a novel task-space regulation scheme for a two-manipulator system has been proposed. The regulator is based on kinetostatic filtering of the control action and on internal force feedback. The equilibrium of the system under the control law has been studied leading to a set of equilibria corresponding to null internal forces. Application of the invariant-set theorem has led to proving asymptotic stability of the equilibria. The closed-loop behavior of the proposed scheme is also analyzed in the case of imperfect gravity compensation. The performance of the regulation scheme in the presence of noise and modeling inaccuracies has been experimentally verified on an industrial cooperative setup in Caccavale et al. (1998).

Future research efforts will be devoted to extend the approach to force/position tracking control problems as well as to cooperative systems for the manipulation of elastic objects.

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Appendix. The unit quaternion

For the reader's convenience, a few basic concepts regarding the unit quaternion are summarized hereafter (see e.g. Roberson & Schwertassek, 1988; Chou, 1992 for further details).

The orientation of a rigid body in space is typically described in terms of a (3×3) rotation matrix $\mathbf{R}$ expressing the orientation of a frame attached to the body with respect to a fixed base frame. An alternative description can be obtained by resorting to a four-parameter representation in terms of a unit quaternion (viz., Euler parameters)

$$\mathcal{Q} = \{\eta, \boldsymbol{\varepsilon}\} = \left\{\cos\frac{\theta}{2}, \mathbf{k}\sin\frac{\theta}{2}\right\}, \tag{A.1}$$

where θ and $\mathbf{k}$ are, respectively, the rotation and the (3×1) unit vector of an equivalent angle/axis description of orientation. Note that the scalar part and the vector part of the unit quaternion are constrained by the equation $\eta^2 + \boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\varepsilon} = 1$. It is worth remarking that $\mathcal{Q} = \{\eta, \boldsymbol{\varepsilon}\}$ and $\mathcal{Q}' = \{-\eta, -\boldsymbol{\varepsilon}\}$ represent the same orientation.

Consider now two frames, conventionally labeled i and j. Let $\mathbf{R}_i$ and $\mathbf{R}_j$ respectively denote the rotation matrices expressing the orientation of the two frames with respect to the base frame. Then, the mutual orientation between the two frames can be described by the rotation matrix $\mathbf{R}^i_j = \mathbf{R}_i^{\mathrm{T}}\mathbf{R}_j$. The unit quaternion $\tilde{\mathcal{Q}}^i_{ji}$ describing the mutual orientation can be computed by composition (quaternion product) of the corresponding unit quaternions $\mathcal{Q}_i^{-1} = \{\eta_i, -\boldsymbol{\varepsilon}_i\}$ (i.e., the conjugate of $\mathcal{Q}_i$) and $\mathcal{Q}_j = \{\eta_j, \boldsymbol{\varepsilon}_j\}$, i.e.

$$\tilde{\mathcal{Q}}^i_{ji} = \mathcal{Q}_i^{-1} * \mathcal{Q}_j = \{\tilde{\eta}_{ji}, \tilde{\boldsymbol{\varepsilon}}^i_{ji}\}, \tag{A.2}$$

where $*$ denotes the quaternion product (composition). If the two frames coincide (i.e., $\mathbf{R}_i^{\mathrm{T}}\mathbf{R}_j = \mathbf{I}_3$) then $\tilde{\mathcal{Q}}^i_{ji} = \mathcal{Q}_0 = \{1, \mathbf{0}\}$ or $\tilde{\mathcal{Q}}^i_{ji} = \mathcal{Q}'_0 = \{-1, \mathbf{0}\}$; given a unit quaternion $\mathcal{Q}$, the unitary quaternion $\mathcal{Q}_0$ is such that $\mathcal{Q}_0 * \mathcal{Q} = \mathcal{Q} * \mathcal{Q}_0 = \mathcal{Q}$, while $\mathcal{Q}'_0 * \mathcal{Q} = \mathcal{Q} * \mathcal{Q}'_0 = \mathcal{Q}'$.

The vector part of $\tilde{\mathcal{Q}}^i_{ji}$ can be represented in a common base frame through the rotation matrix $\mathbf{R}_i$, i.e., $\tilde{\boldsymbol{\varepsilon}}_{ji} = \mathbf{R}_i\tilde{\boldsymbol{\varepsilon}}^i_{ji}$; it can be shown that the following equality holds (Chou, 1992):

$$\tilde{\mathcal{Q}}_{ji} = \mathcal{Q}_j * \mathcal{Q}_i^{-1} = \mathcal{Q}_i * \tilde{\mathcal{Q}}^i_{ji} * \mathcal{Q}_i^{-1} = \{\tilde{\eta}_{ji}, \mathbf{R}_i\tilde{\boldsymbol{\varepsilon}}^i_{ji}\}, \tag{A.3}$$

which corresponds to the rotation represented by $\mathbf{R}_j\mathbf{R}_i^{\mathrm{T}}$.

The relationship between the time derivative of the quaternion components and the relative angular velocity is established by the so-called quaternion propagation rule

$$\dot{\tilde{\eta}}_{ji} = -\tfrac{1}{2}\tilde{\boldsymbol{\varepsilon}}^{i\mathrm{T}}_{ji}\tilde{\boldsymbol{\omega}}^i_{ji}, \qquad \dot{\tilde{\boldsymbol{\varepsilon}}}^i_{ji} = \tfrac{1}{2}\mathbf{E}(\tilde{\eta}_{ji}, \tilde{\boldsymbol{\varepsilon}}^i_{ji})\tilde{\boldsymbol{\omega}}^i_{ji}, \tag{A.4}$$

where $\mathbf{E}(\cdot) = \eta\mathbf{I}_3 - \mathbf{S}(\cdot)$, $\mathbf{S}(\cdot)$ is the skew-symmetric matrix operator performing the cross product and $\tilde{\boldsymbol{\omega}}^i_{ji} = \boldsymbol{\omega}^i_j - \boldsymbol{\omega}^i_i = \mathbf{R}_i^{\mathrm{T}}(\boldsymbol{\omega}_j - \boldsymbol{\omega}_i)$ is the angular velocity of frame j relative to frame i, which has been referred to frame i.

Property A.1. *Given a unit quaternion $\mathcal{Q} \neq \mathcal{Q}'_0 = \{-1, \mathbf{0}\}$, then the equation $\mathcal{Q} = \mathcal{Q}_x * \mathcal{Q}_x$ admits only two solutions which represent the same orientation.*

Proof. The thesis follows from the explicit expression of $\mathcal{Q} = \mathcal{Q}_x * \mathcal{Q}_x$:

$$\eta_x^2 - \|\boldsymbol{\varepsilon}_x\|^2 = \eta, \qquad 2\eta_x\boldsymbol{\varepsilon}_x = \boldsymbol{\varepsilon}.$$

If $\eta \neq -1$, the above system of equations admits only two real solutions $\mathcal{Q}_{x1} = \{\eta_x, \boldsymbol{\varepsilon}_x\}$ and $\mathcal{Q}_{x2} = \{-\eta_x, -\boldsymbol{\varepsilon}_x\}$ with

$$\eta_x = \sqrt{\frac{\eta+1}{2}}, \qquad \boldsymbol{\varepsilon}_x = \frac{\sqrt{2}}{2\sqrt{\eta+1}}\boldsymbol{\varepsilon}. \qquad \square$$

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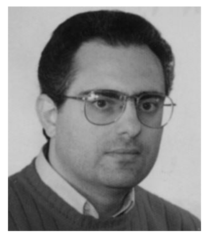

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