DYNAMIC MANIPULABILITY OF ROBOT MANIPULATORS

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The concept of dynamic manipulability measure of robot arms is proposed as a quantitative measure of their manipulating ability in positioning and orienting the end-effectors, which takes the arm dynamics into consideration. This measure is defined on the basis of the relation between the joint driving force and the acceleration of the end-effector. Some properties of the measure are established. A two-joint link mechanism is analyzed and its best posture is obtained under certain condition from the viewpoint of this measure. A numerical example is also given to illustrate the utilization of this concept for the design of robot manipulators.

INTRODUCTION

The design of robot manipulators and the determination of the working position and posture in the workspace of a manipulator have largely been done so far on the basis of experience and intuition. One of various measures used for these decisions, seems to be the easiness of changing arbitrarily the position and orientation of the end-effector at the tip of the manipulator.

The manipulability measure has been proposed as one such quantitative measure in a previous paper.[1] Properties of this measure, best postures of various robotic mechanisms from the viewpoint of this measure, and its application to manipulator control have also been discussed.[1-3] However, this concept is a kinematic one, and the arm dynamics is completely ignored. Therefore, although it has a merit that it can be applied to conceptual design of arm mechanisms and singularity avoidance control without considering complicated arm dynamics,[3-5] it may not be suitable for detailed arm design or high-speed, high-precision motion control.

In the present paper, the concept "dynamic manipulability measure" of robot arms is proposed as a quantitative measure of their ability in manipulating the end-effectors, which takes the arm dynamics into consideration. First, several properties of this measure will be made clear. Then a two-joint link mechanism, which is the simplest case of multi-articulated robot manipulators, will be analyzed from the viewpoint of the dynamic manipulability measure. A numerical example will also be given to illustrate the applicability of this measure to the arm design which takes the arm dynamics into consideration.

MANIPULABILITY MEASURE

Before getting into the discussion of dynamic manipulability, the definition of manipulability measure, which is based on the arm kinematics, and its properties will be briefly introduced in this section.[1-3]

We consider a manipulator with n degrees of freedom whose joint variables are denoted by θ_i, i=1, 2, ..., n. We assume that the position and/or orientation of the end-effector can be described by m variables r_j, j=1, 2, ..., m ($m \leq n$) with respect to a reference orthogonal coordinate frame and that the kinematic relation between θ_i and r_j is assumed to be given by

$$r = p(\theta) \tag{1}$$

where $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T \in R^n$ (n-dimensional Euclidian space), $r = [r_1, r_2, \ldots, r_m]^T \in R^m$ and the superscript ^{T} denotes the transpose. The end-effector velocity $v \in R^m$ corresponding to r, is related to joint velocity $\dot{\theta}$ by

$$v = J(\theta)\dot{\theta} \tag{2}$$

where $\dot{\theta} = d\theta/dt \in R^n$, and $J(\theta) \in R^{m \times n}$ (the set of all $m \times n$ real matrices). The matrix $J(\theta)$ is called the Jacobian[6] ($J(\theta)$ will be written as J hereafter). A scalar value w given by

$$w = \sqrt{\det J J^T} \tag{3}$$

is defined to be the **manipulability measure** at θ with respect to r.

Some properties of this manipulability measure will be given in the following.

(i) The set of all end-effector velocity v which is realizable by a joint velocity $\dot{\theta}$ such that $\|\dot{\theta}\|^2 = \dot{\theta}_1^2 + \dot{\theta}_2^2 + \cdots + \dot{\theta}_n^2 \leq 1$ ($\|\cdot\|$ is the Euclidian norm) forms an ellipsoid in R^m. This is called the **manipulability ellipsoid.** Its volume is given by $\{\pi^{m/2}/\Gamma[(m/2)+1]\}\, w$, where $\Gamma(\cdot)$ is the gamma function. Therefore, w is proportional to the volume of the manipulability ellipsoid.

(ii) When m=n, the manipulability measure is simply given by

$$w = |\det J| \tag{4}$$

(iii) Letting $f \in R^m$ denote the force (and torque) applied to an object by the end-effector and letting $\tau \in R^n$ denote the necessary joint driving force (and torque), we have $\tau = J^T f$ (see reference [6]). Hence the set of all manipulating force f which is realizable by a joint driving force τ such that $\|\tau\| \le 1$, is an ellipsoid in R^m. This is called the manipulating force ellipsoid. Its volume is given by $\{\pi^{m/2}/\Gamma[(m/2)+1]\}/w$ and is inversely proportional to the manipulability measure w. Also the principal axes of the manipulability ellipsoid and the manipulating force ellipsoid are the same and their radii in each principal axis direction are inversely proportional. This means that the direction in which a large manipulating force can be generated is the one in which the manipulability is poor and vice versa.

DYNAMIC MANIPULABILITY MEASURE

The dynamics equation of robot manipulators is generally given by

$$M(\theta)\ddot{\theta} + h(\theta, \dot{\theta}) + g(\theta) = \tau \tag{5}$$

where $\ddot{\theta} = d^2\theta/dt^2$, $M(\theta) \in R^{n \times n}$ is the nonsingular inertia matrix ($M(\theta)$ will be written as M hereafter), $h(\theta, \dot{\theta}) \in R^n$ represents the centrifugal and Coriolis forces, and $g(\theta) \in R^n$ represents the effect of gravity.

On the other hand, by differentiating Eq.(2) with respect to time, we obtain

$$\dot{v} = J\ddot{\theta} + a_r(\theta, \dot{\theta}) \tag{6}$$

$$a_r(\theta, \dot{\theta}) = \dot{J}\dot{\theta} \tag{7}$$

The term $a_r(\theta, \dot{\theta})$ in Eq.(6) can be interpreted as the virtual acceleration caused by the nonlinear relation (1) between θ and r. Introducing the new vectors $\tilde{\tau} \in R^n$ and $\tilde{\dot{v}} \in R^m$ by

$$\tilde{\tau} = \tau - h(\theta, \dot{\theta}) - g(\theta) \tag{8}$$

$$\tilde{\dot{v}} = \dot{v} - a_r(\theta, \dot{\theta}) \tag{9}$$

we obtain from Eqs.(5), (6), (8) and (9)

$$\tilde{\dot{v}} = JM^{-1}\tilde{\tau} \tag{10}$$

The basic idea here is to quantify the degree of arbitrariness of changing the acceleration $\tilde{\dot{v}}$ under some constraint on the joint driving force τ on the basis of Eq.(10), and to adopt this quantity as a measure of arm manipulability.

We first give the following definition.

[Definition] A scalar value w_d given by

$$w_d = \sqrt{\det [J(M^T M)^{-1} J^T]} \tag{11}$$

is called the **dynamic manipulability measure** at state θ with respect to manipulation vector r.

The following facts can then be established in a similar way as in the case of manipulability measure.

(i) Let the singular value decomposition [7] of JM^{-1} be

$$JM^{-1} = U \Sigma V^T \tag{12}$$

where $U \in R^{m \times m}$ and $V \in R^{n \times n}$ are orthogonal matrices and

$$\Sigma = \left[\begin{array}{cccc|c} \sigma_1 & & & 0 & \\ & \sigma_2 & & & \\ & & \ddots & & 0 \\ 0 & & & \sigma_m & \end{array}\right] \in R^{m \times n} \tag{13a}$$

$$\sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_m \ge 0 \tag{13b}$$

Then the measure w_d can be expressed as the product of the singular values $\sigma_1, \sigma_2, \ldots, \sigma_m$:

$$w = \sigma_1 \sigma_2 \cdots \sigma_m \tag{14}$$

(ii) The set of all $\tilde{\dot{v}}$ which is realizable by a joint driving force such that $\|\tilde{\tau}\| \le 1$ is an ellipsoid in R^m described by

$$\{\tilde{\dot{v}} \mid \tilde{\dot{v}}^T (MJ^+)^T MJ^+ \tilde{\dot{v}} \le 1 \text{ and } \tilde{\dot{v}} \in \mathrm{Im}(J)\} \tag{15}$$

where J^+ denotes the pseudoinverse of J and Im(J) means the range of J. The principal axes are $\sigma_1 u_1, \sigma_2 u_2, \ldots, \sigma_m u_m$, where $u_i \in R^m$ is the i-th column vector of U, i.e., $[u_1\ u_2 \cdots u_m] = U$. Except at the singular states where the volume of the ellipsoid becomes zero, i.e., at any state for which rank J= m, the ellipsoid can be defined by

$$\tilde{\dot{v}}^T (MJ^+)^T MJ^+ \tilde{\dot{v}} \le 1 \tag{16}$$

This ellipsoid will be called the **dynamic manipulability ellipsoid** and could be a good means for the analysis, design and control of robot manipulators, for cases where the arm dynamics is taken into consideration. The volume of this ellipsoid is given by

$$\{\pi^{m/2}/\Gamma[(m/2)+1]\}\, w_d \tag{17}$$

Therefore, w_d is proportional to the volume of the dynamic manipulability ellipsoid .

(iii) When m=n, i.e., when we consider nonredundant manipulators, the dynamic manipulability measure w_d reduces to

$$w_d = |\det J| / |\det M| \tag{18}$$

Furthermore, the measure has the following physical interpretation as well as that of (ii). The set of all realizable acceleration $\tilde{\dot{v}}$ such that

$$|\tilde{\tau}_i| \le 1,\ i = 1,2,\ldots,n \tag{19}$$

is a parallelepiped in R^m, and its volume is $2^m w_d$. In other words, the measure w_d is proportional to the volume of the parallelepiped. From Eq.(4) and (18), we obtain

$$w_d = w / |\det M| \tag{20}$$

Hence when $|\det M|$ is independent of θ, the dynamic manipulability measure w_d is equal to the manipulability measure w multiplied by the constant $1/|\det M|$.

So far we have implicitly assumed that the maximum driving forces at all joints are 1 irrespectively of θ, and that the weights of all accelerations of manipulation variables are the same. When these assumptions do not hold, each variable should be normalized as follows.

We regard the case where the manipulator is standing still ($\dot{\theta}=0$) as the fundamental one for considering the dynamic manipulability. Since $h(\theta, \dot{\theta})=0$ and $a_r(\theta, \dot{\theta})=0$ for this case, we have from Eqs.(8) and (9)

$$\tilde{\tau} = \tau - g(\theta) \tag{21}$$

$$\tilde{\dot{v}} = \dot{v} \tag{22}$$

Hence we take

$$|\tilde{\tau}_i| \leq \tilde{\tau}_{i0}(\theta) \tag{23}$$

as the constraint on $\tilde{\tau}$, where

$$\tilde{\tau}_{i0}(\theta) = \tau_{i0} - |g_i(\theta)| \tag{24}$$

and τ_{i0} is the maximum driving force for τ_i, which is assumed to be a constant independent of θ. As for the acceleration $\tilde{\dot{v}}$, we assume that the maximum desirable acceleration $\tilde{\dot{v}}_{j0}$ can be given by considering the set of tasks given to the manipulator. If $\tilde{\dot{v}}_{j0}$ is difficult to determine, this value may roughly be selected as the weight of relative importance among the elements of $\tilde{\dot{v}}$.

Once the values $\tilde{\tau}_{i0}(\theta)$ and $\tilde{\dot{v}}_{j0}$ are given, we can normalize the variables $\tilde{\tau}$ and $\tilde{\dot{v}}$ by

$$\hat{\tau} = [\hat{\tau}_1, \hat{\tau}_2, \ldots, \hat{\tau}_n]^T, \quad \hat{\tau}_i = \tilde{\tau}_i / \tilde{\tau}_{i0}(\theta) \tag{25}$$

$$\hat{\dot{v}} = [\hat{\dot{v}}_1, \hat{\dot{v}}_2, \ldots, \hat{\dot{v}}_m]^T, \quad \hat{\dot{v}}_j = \tilde{\dot{v}}_j / \tilde{\dot{v}}_{j0} \tag{26}$$

Then from Eq.(10) we obtain

$$\hat{\dot{v}} = \hat{J} \hat{M}^{-1} \hat{\tau} \tag{27}$$

where

$$\hat{J} = T_r J, \quad T_r = \mathrm{diag}[1/\tilde{\dot{v}}_{j0}] \tag{28}$$

$$\hat{M} = T_\tau(\theta) M, \quad T_\tau(\theta) = \mathrm{diag}[1/\tilde{\tau}_{i0}(\theta)] \tag{29}$$

and diag[$\cdot$] denotes the diagonal matrix. Since $\hat{\tau}$ and $\hat{\dot{v}}$ satisfy the original assumption, we can define the dynamic manipulability measure using $\hat{M}$ and $\hat{J}$.

Denoting the measure w_d for $\hat{J}\hat{M}^{-1}$ as $\hat{w}_d$, and that for $J\hat{M}^{-1}$ as $\hat{w}_{d\tau}$, we have

$$\hat{w}_d = [1/(\prod_{j=1}^{m} \tilde{\dot{v}}_{j0})] \hat{w}_{d\tau} \tag{30}$$

This implies that the transformation T_r has only the effect of multiplying the scalar value $[1/(\prod_{j=1}^{m} \tilde{\dot{v}}_{j0})]$, and the relative shape of w_d as a function of θ is independent of T_r. Furthermore, when m=n, we have

$$\hat{w}_d = [\prod_{i=1}^{n}(\tilde{\tau}_{i0}(\theta)/\tilde{\dot{v}}_{i0})] w_d \tag{31}$$

Hence when m=n and $\tilde{\tau}_{i0}(\theta)$ is independent of θ, the relative shape of w_d as a function of θ is independent of both T_r and T_τ. Note that $\tilde{\tau}_{i0}(\theta)$ is independent of θ for SCARA type manipulators and manipulators used in the space where the effect of gravity need not be considered.

It is of course possible to define a dynamic manipulability measure for manipulators in motion. For example, let $\hat{M}(\theta, \dot{\theta})$ and $\hat{J}(\theta, \dot{\theta})$ be $\hat{M}$ and $\hat{J}$ obtained from Eqs.(28), (29) by replacing $\tilde{\tau}_{i0}(\theta)$ and $\tilde{\dot{v}}_{j0}$ by

$$\tilde{\tau}_{i0}(\theta, \dot{\theta}) = \tau_{i0} - |h_i(\theta, \dot{\theta}) + g_i(\theta)| \tag{32}$$

and

$$\tilde{\dot{v}}_{j0}(\theta, \dot{\theta}) = \dot{v}_{j0} - |a_{rj}(\theta, \dot{\theta})| \tag{33}$$

Then we may adopt $\hat{w}_d$ calculated from $\hat{M}(\theta, \dot{\theta})$ and $\hat{J}(\theta, \dot{\theta})$ as the dynamic manipulability measure at state $(\theta, \dot{\theta})$ with respect to r.

The generalized inertia ellipsoid (GIE) has recently been proposed as a means of representing the manipulator dynamics. [8] A remark will be given on the relation between the GIE and the dynamic manipulability ellipsoid. Consider the case where the position and orientation of the arm tip expressed in the reference coordinate frame is taken as the manipulation vector. Then the GIE can be interpreted as expressing the easiness of changing the position and orientation of the end-effector in various directions for a human operator who holds the end-effector mounted at the arm tip and applies a force with a fixed magnitude. The GIE represents this easiness by the reciprocal of the square root of the generalized moment of inertia. On the other hand, the dynamic manipulability ellipsoid expresses the easiness of changing the position and orientation of the end-effector for the set of actuators which drives the manipulator joints by applying joint driving torques with a fixed magnitude. The dynamic manipulability ellipsoid represents this easiness by the magnitude of the realizable arm tip acceleration. This difference appears as the difference between Eq.(16) and the following expression for the GIE which corresponds to Eq.(6) of reference. [8]

$$u^T (J^+)^T M J^+ u \leq 1 \tag{34}$$

Note that, when m=n and rankJ=n, the pseudoinverse J^+ in Eq.(34) can be replaced by the inverse J^{-1}.

Some other indexes than w_d representing different features of the manipulability ellipsoid will also be useful for detailed evaluation of the manipulation ability of robotic mechanisms. Two examples are the condition number σ_1/σ_m and the minimum singular value σ_m (see references[9-11]). The former can be interpreted as a measure of directional uniformity of the ellipsoid, and the latter is the upper bound of the magnitude of the velocity with which the end-effector can be moved in all direction.

TWO-JOINT LINK MECHANISM

A two-joint link mechanism shown in Figure 1, which is the simplest case of multi-joint manipulators, will be analyzed from the viewpoint of dynamic manipulability. Notations used in the figure are:

- m_i : the mass of link i;
- I_i : the moment of inertia of link i;
- ℓ_i : the length of link i;
- ℓ_{gi} : the distance between the joint i and the center of mass of link i;
- m_e : the mass of the end-effector and load;
- I_e : the moment of inertia of the end-effector and load.

It is assumed that the first joint driving torque τ_1 works between the arm base and link 1, and the second joint driving torque τ_2 works between link 1 and link 2. Taking the arm tip position $[x, y]^T$ as the manipulation vector r and letting $s_1=\sin(\theta_1)$, $s_{12}=\sin(\theta_1+\theta_2)$, $c_1=\cos(\theta_1)$, $c_{12}=\cos(\theta_1+\theta_2)$, we have

$$J = \begin{bmatrix} \ell_1 c_1+\ell_2 c_{12} & \ell_2 c_{12} \\ -\ell_1 s_1-\ell_2 s_{12} & -\ell_2 s_{12} \end{bmatrix} \tag{35}$$

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{36}$$

where

$$M_{11} = I_1+\hat{I}_2+m_1\ell_{g1}^2 + \hat{m}_2(\ell_1^2+\hat{\ell}_{g2}^2+2\ell_1\hat{\ell}_{g2}c_2) \tag{37a}$$

$$M_{12} = M_{21} = \hat{I}_2 + \hat{m}_2(\hat{\ell}_{g2}^2+\ell_1\hat{\ell}_{g2}c_2) \tag{37b}$$

$$M_{22} = \hat{I}_2+\hat{m}_2\hat{\ell}_{g2}^2 \tag{37c}$$

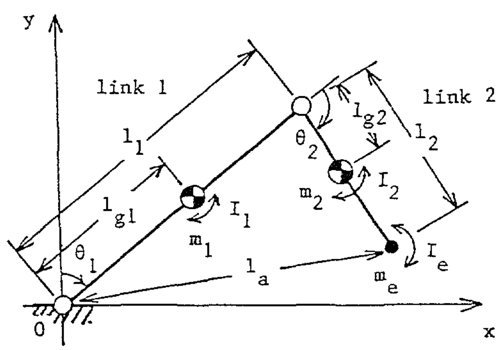

Fig.1 Two-joint link mechanism.

$$\hat{m}_2 = m_2+m_e \tag{38a}$$

$$\hat{\ell}_{g2} = (m_2\ell_{g2}+m_e\ell_2)/(m_2+m_e) \tag{38b}$$

$$\hat{I}_2 = I_2+m_2(\hat{\ell}_{g2}-\ell_{g2})^2 + I_e+m_e(\ell_2-\hat{\ell}_{g2})^2 \tag{38c}$$

Since m=n=2, from Eq.(18) we obtain

$$w_d = \ell_1\ell_2|s_2|/|\det M| \tag{39}$$

$$|\det M| = \{[I_1+m_1\ell_{g1}^2][\hat{I}_2+\hat{m}_2\hat{\ell}_{g2}^2] + \hat{I}_2\hat{m}_2\ell_1^2+\hat{m}_2^2\hat{\ell}_{g2}^2\ell_1^2 s_2^2\} \tag{40}$$

From now on we assume that the link mechanism is at a stand-still. The case where there is no gravity effect will be considered first. In this case, since the coefficient of w_d in the right hand side of Eq.(31) is a constant, it is enough to discuss w_d given by Eq.(39). If we let

$$\alpha = \frac{\ell_1\ell_2}{[I_1+m_1\ell_{g1}^2+\hat{m}_2\ell_1^2][\hat{I}_2+\hat{m}_2\hat{\ell}_{g2}^2]} \tag{41}$$

$$\beta = \frac{\hat{m}_2^2\hat{\ell}_{g2}^2\ell_1^2}{[I_1+m_1\ell_{g1}^2][\hat{I}_2+\hat{m}_2\hat{\ell}_{g2}^2]+\hat{I}_2\hat{m}_2\ell_1^2} \tag{42}$$

then Eq.(39) becomes

$$w_d = \frac{\alpha(1+\beta)|s_2|}{1+\beta s_2^2} \tag{43}$$

Therefore, the relative shape of w_d as a function of θ is uniquely determined by the parameter β, and the parameter α determines its scale.

Since the case $\beta=0$ is a special case of some interest, this case will be discussed first. The condition $\beta=0$ can be reduced to $\hat{\ell}_{g2}=0$ under the natural assumption of $\hat{m}_2\neq 0$ and $\ell_1\neq 0$. The implication of $\hat{\ell}_{g2}=0$ is that the mass center of the set of link 2, the end-effector and the load, is located precisely at joint 2 (by use of, say, a counterbalance). When $\hat{\ell}_{g2}=0$, the value $|\det M|$ is independent of θ and, as stated in the previous section, the dynamic manipulability measure w_d is equal to the manipulability measure w times the scalar $1/|\det M|$. Hence the best arm posture from the viewpoint of w (i.e., $\theta_2=\pm 90°$) is also best from the viewpoint of w_d.

Next we will consider the case where $\beta\neq 0$. Figure 2 shows the value w_d as a function of θ_2 taking β as a parameter. It can be seen from the figure that as β becomes larger starting from zero, the shape of w_d goes close to a trapezoid-like one, but that w_d still attains its maximum α at $\theta_2 = \pm 90°$ for β satisfying $0 \leq \beta \leq 1$. For $\beta > 1$, w_d attains its maximum at two values of θ_2 ; one is larger than and the other is smaller than 90° .

As we have seen, the parameter β determines the relative shape of the dynamic manipulability measure and the parameter α determines its magnitude. Hence these parameters will be useful for design and evaluation of robot arms.

A numerical example will be given now. Let $\ell_1 = \ell_2 = 1$, $m_1 = 20$, $m_2 = 10$, $m_e = 5$, $\ell_{g1} = 0.5$, $\ell_{g2} = 0.3$, $I_1 = 20/12$, $I_2 = 10/12$, $I_e = 0$, $\tau_{10} = 600$, $\tau_{20} = 200$, (in units of m, kg, and sec), and $\hat{v}_{10} = \hat{v}_{20} = 1$ (the relative importances of accelerations in x and y directions are the same). Then the dynamic manipulability ellipsoid and measure as functions of ℓ_a, the distance between the origin and the arm tip, are given by those in Figure 3. From Eq.(42) we have $\beta = 0.78$. Hence $\hat{w}_d$ be comes maximum when $\theta_2 = 90°$. As can be seen from the figure, however, the difference between $\hat{w}_d$ and the maximum value is not very large for a wide range of ℓ_a. This distribution will be desirable from the viewpoint of uniformity of the dynamic manipulability measure. So the two-link mechanism with the figures given above may be judged to be a rather good design.

Next we will consider the case where the force of gravity acts on the above mechanism in the minus y direction. Let g be the gravitational acceleration. Then the gravity term $g(\theta)$ for the arm in Figure 1 is given by

$$g(\theta) = \begin{bmatrix} m_1 \ell_{g1} s_1 + \hat{m}_2 (\ell_1 s_2 + \hat{\ell}_{g2} s_{12}) \\ \hat{m}_2 \hat{\ell}_{g2} s_{12} \end{bmatrix} g \tag{44}$$

Figure 4 shows the dynamic manipulability ellipsoid and measure calculated from J and $\hat{M}$ which is derived from Eqs.(44), (24), and (29). Due to the effect of gravity, the dynamic manipulability measure is rather small for stretched arm postures. Since this is usually not desirable, we consider a modification. If, for example, we can attain $\ell_{g1} = 0.4$, $\ell_{g2} = 0$ by a change of mass distribution of each link, then we obtain Figure 5. Comparing with the original design, the modified one has an improved dynamic manipulability measure in the region of $0.7 \leq \ell_a \leq 1.8$.

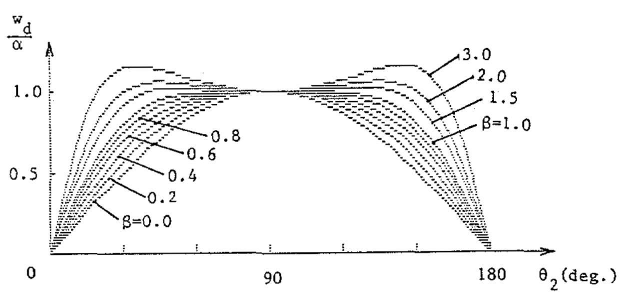

Fig.2 Dynamic manipulability measure as a function of θ_2.

CONCLUSION

The concept of dynamic manipulability measure of robot arms has been proposed as a quantitative measure of their manipulating ability in positioning and orienting the end-effectors, which takes the arm dynamics into consideration. It has been shown that this measure corresponds to the volume of the ellipsoid (dynamic manipulability ellipsoid), which is formed by the set of all realizable arm tip acceleration under certain constraint on the magnitude of joint driving force. A two-joint link mechanism has been

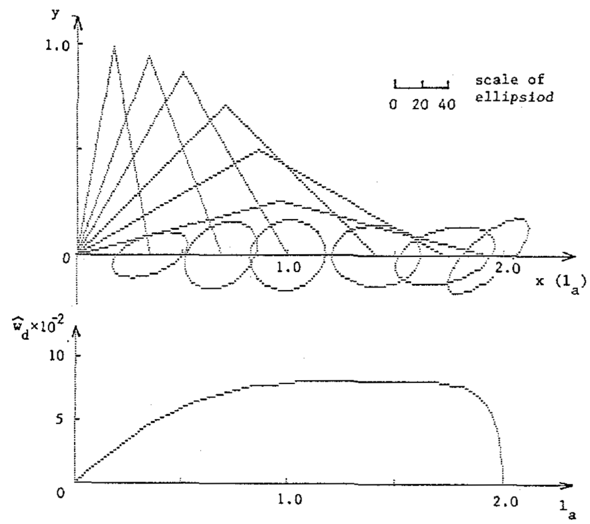

Fig.3 Dynamic manipulability ellipsoid and measure (case with no gravity).

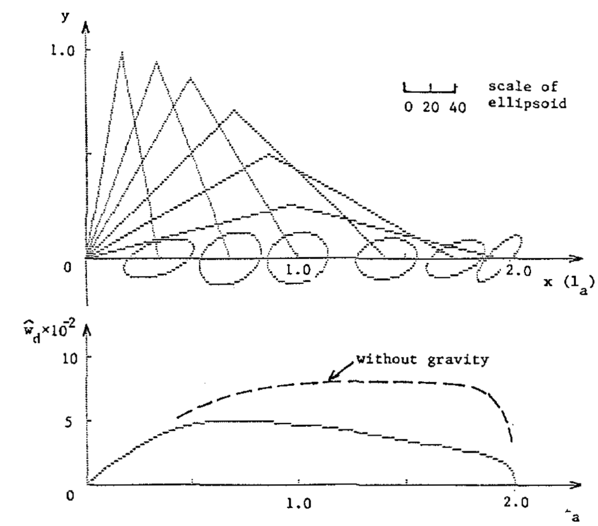

Fig.4 Dynamic manipulability ellipsoid and measure (case with gravity; $\ell_{g1} = 0.5$, $\ell_{g2} = 0.3$).

analyzed and it has been shown that if there is no gravity effect and a certain weak condition is satisfied, the mechanism attains its best posture from the viewpoint of the dynamic manipulability measure when the second joint angle is $\pm 90°$. A numerical example for the case with gravity effect has also been given to illustrate the utilization of this measure for the design of robot manipulators.

To analyze various types of manipulators from the viewpoint of this measure and to investigate in more detail its applicability to the design and control of robot arms are among the future topics.

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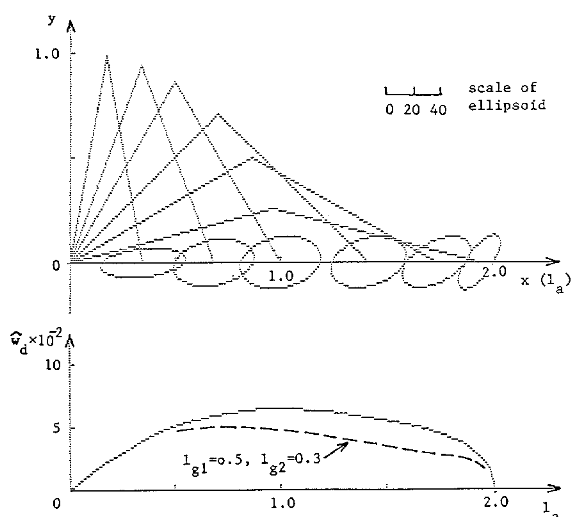

Fig.5 Dynamic manipulability ellipsoid and measure (case with gravity; ℓ_{g1}=0.4, ℓ_{g2}=0.0).

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