

Dynamic Movement Primitives—A Framework for Motor Control in Humans and Humanoid Robotics

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Abstract

Given the continuous stream of movements that biological systems exhibit in their daily activities, an account for such versatility and creativity has to assume that movement sequences consist of segments, executed either in sequence or with partial or complete overlap. Therefore, a fundamental question that has pervaded research in motor control both in artificial and biological systems revolves around identifying movement primitives (a.k.a. units of actions, basis behaviors, motor schemas, etc.). What are the fundamental building blocks that are strung together, adapted to, and created for ever new behaviors? This paper summarizes results that led to the hypothesis of Dynamic Movement Primitives (DMP). DMPs are units of action that are formalized as stable nonlinear attractor systems. They are useful for autonomous robotics as they are highly flexible in creating complex rhythmic (e.g., locomotion) and discrete (e.g., a tennis swing) behaviors that can quickly be adapted to the inevitable perturbations of a dynamically changing, stochastic environment. Moreover, DMPs provide a formal framework that also lends itself to investigations in computational neuroscience. A recent finding that allows creating DMPs with the help of well-understood statistical learning methods has elevated DMPs from a more heuristic to a principled modeling approach. Theoretical insights, evaluations on a humanoid robot, and behavioral and brain imaging data will serve to outline the framework of DMPs for a general approach to motor control in robotics and biology.

1 Introduction

When searching for a general framework of how to formalize the learning of coordinated movement, some of the ideas developed in the middle of the 20th century still remain useful. At this time, theories from optimization theory, in particular in the context of dynamic programming [1, 2], described the goal of learning control in learning a policy. A policy is formalized as a function that maps the continuous state vector \mathbf{x} of a control system and its environment, pos-

sibly in a time dependent way, to a continuous control vector \mathbf{u} :

$$\mathbf{u} = \pi(\mathbf{x}, \alpha, t) \quad (1)$$

The parameter vector α denotes the problem specific adjustable parameters in the policy π —not unlike the parameters in neural network learning. At the first glance, one might suspect that not much was gained by this overly general formulation. However, given some cost criterion that can evaluate the quality of an action \mathbf{u} in a particular state \mathbf{x} , dynamic programming, and especially its modern relative, reinforcement learning, provide a well founded set of algorithms of how to compute the policy π for complex nonlinear control problems. Unfortunately, as already noted in Bellman's original work, learning of π becomes computationally intractable for even moderately high dimensional state-action spaces. Although recent developments in reinforcement learning increased the range of complexity that can be dealt with [e.g. 3, 4, 5], it still seems that there is a long way to go to apply general policy learning to complex control problems.

In most robotics applications, the full complexity of learning a control policy is strongly reduced by providing prior information about the policy. The most common priors are in terms of a desired trajectory, $[\mathbf{x}_d(t), \dot{\mathbf{x}}_d(t)]$, usually hand-crafted by the insights of a human expert. For instance, by using a PD controller, a (explicitly time dependent) control policy can be written as:

$$\begin{aligned} \mathbf{u} &= \pi(\mathbf{x}, \alpha(t), t) = \pi(\mathbf{x}, [\mathbf{x}_d(t), \dot{\mathbf{x}}_d(t)], t) \\ &= \mathbf{K}_x(\mathbf{x}_d(t) - \mathbf{x}) + \mathbf{K}_{\dot{x}}(\dot{\mathbf{x}}_d(t) - \dot{\mathbf{x}}) \end{aligned} \quad (2)$$

For problems in which the desired trajectory is easily generated and in which the environment is static or fully predictable, as in many industrial applications, such a shortcut through the problem of policy generation is highly successful. However, since policies like in (2) are usually valid only in a local vicinity of the time course of the desired trajectory, they are not very flexible. When dealing with a dynamically changing environment in which substantial and reactive modifications of control commands are required,

one needs to modify trajectories appropriately, or even generate entirely new trajectories by generalizing from previously learned knowledge. In certain cases, it is possible to apply scaling laws in time and space to desired trajectories [6, 7], but those can provide only limited flexibility, as similarly recognized in related theories in psychology [8]. Thus, for general-purpose reactive movement, the “desired trajectory” approach seems to be too restricted.

From the viewpoint of statistical learning, Equation (1) constitutes a nonlinear function approximation problem. A typical approach to learning complex nonlinear functions is to compose them out of basis functions of reduced complexity. The same line of thinking generalizes to learning policies: a complicated policy could be learned from the combination of simpler (ideally globally valid) policies, i.e., policy primitives or movement primitives, as for instance:

$$\mathbf{u} = \pi(\mathbf{x}, \alpha, t) = \sum_{k=1}^K \pi_k(\mathbf{x}, \alpha_k, t) \quad (3)$$

Indeed, related ideas have been suggested in various fields of research, for instance in computational neuroscience as Schema Theory [9] and in mobile robotics as behavior-based or reactive robotics [10]. In particular, the latter approach also emphasized to remove the explicit time dependency of π , such that complicated “clocking” and “reset clock” mechanisms could be avoided, and the combination of policy primitives becomes simplified. Despite the successful application of policy primitives in the mobile robotics domain, so far, it remains a topic of ongoing research [11, 12] how to generate and combine primitives in a principled and autonomous way, and how such an approach generalizes to complex movement systems, like human arms and legs.

Thus, a key research topic, both in biological and artificial motor control, revolves around the question of movement primitives: what is a good set of primitives, how can they be formalized, how can they interact with perceptual input, how can they be adjusted autonomously, how can they be combined task specifically, and what is the origin of primitives? In order to address the first four of these questions, we suggest to resort to some of the most basic ideas of dynamic systems theory. The two most elementary behaviors of a nonlinear dynamic system are point attractive and limit cycle behaviors, paralleled by discrete and rhythmic movement in motor control. Would it be possible to generate complex movement just out of these two basic elements? The idea of using dynamic systems for movement generation is not new: motor pattern generators in neurobiology [13, 14], pattern generators for locomotion [15, 16], potential field approaches for planning [e.g., 17], and

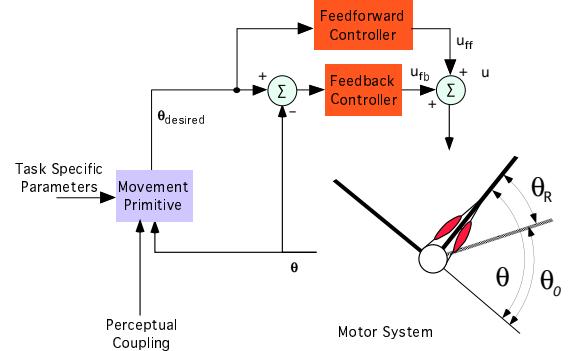


Figure 1: Sketch of control diagram with dynamic movement primitives. Each degree-of-freedom of a limb has a rest state θ_o and an oscillatory state θ_r .

more recently basis field approaches for limb movement [18] have been published. Additionally, work in the dynamic systems approach in psychology [19-23] has emphasized the usefulness of autonomous nonlinear differential equations to describe movement behavior. However, rarely have these ideas addressed both rhythmic and discrete movement in one framework, task specific planning that can exploit both intrinsic (e.g., joint) coordinates and extrinsic (e.g., Cartesian) coordinate frames, and more general purpose behavior, in particular for multi-joint arm movements. It is in these domains, that the present study offers a novel framework of how movement primitives can be formalized and used, both in the context of biological research and humanoid robotics.

2 Dynamic Movement Primitives

Using nonlinear dynamic systems as policy primitives is the most closely related to the original idea of motor pattern generators (MPG) in neurobiology. MPGs are largely thought to be hardwired with only moderately modifiable properties. In order to allow for the large flexibility of human limb control, the MPG concept needs to be augmented by a component that can be adjusted task specifically, thus leading to what we call a *Dynamic Movement Primitive* (DMP). We assume that the attractor landscape of a DMP represents the desired kinematic state of a limb, e.g., positions, velocities, and accelerations. This approach deviates from MPGs which are usually assumed to code motor commands, and is strongly related to the idea developed in the context of “mirror laws” by Bühler, Rizzi, and Koditschek [24, 25]. As shown in Figure 1, kinematic variables are converted to motor commands through an inverse dynamics model and stabilized by low gain feedback control. The motivation for this approach is largely inspired by data from neurobiology that demonstrated strong evidence for

the representation of kinematic trajectory plans in parietal cortex [26] and inverse dynamics models in the cerebellum [27, 28]. Kinematic trajectory plans are equally backed up by the discovery of the principle of motor equivalence in psychology [e.g., 29], demonstrating that different limbs (e.g., fingers, arms, legs) can produce cinematically similar patterns despite having very different dynamical properties; these findings are hard to reconcile with planning directly in motor commands. Kinematic trajectory plans, of course, are also well known in robotics from computed torque and inverse dynamics control schemes [30]. From the view point of movement primitives, kinematic representations are more advantageous than direct motor command coding since this allows for workspace independent planning, and, importantly, for the possibility to superimpose DMP. However, it should be noted that a kinematic representation of movement primitives is not necessarily independent of dynamic properties of the limb. Proprioceptive feedback can be used to modify the attractor landscape of a DMP in the same way as perceptual information [25, 31, 32].

2.1 Formalization of DMPs

In order to accommodate discrete and rhythmic movements, two kinds of DMPs are needed, a point attractive system and a limit system. Although it is possible to construct nonlinear differential equations that could realize both these behaviors in one set of equations [e.g., 33], for reasons of robustness, simplicity, functionality, and biological realism (see below), we chose an approach that separates these two regimes. Every degree-of-freedom (DOF) of a limb is described by two variables, a rest position θ_o and a superimposed oscillatory position, θ_r , as shown in Figure 1. By moving the rest position, discrete motion is generated. The change of rest position can be anchored in joint space or, by means of inverse kinematics transformations, in external space. In contrast, the rhythmic movement is produced in joint space, relative to the rest position. This dual strategy permits to exploit two different coordinate systems: joint space, which is the most efficient for rhythmic movement, and external (e.g., Cartesian) space, which is needed to reference a task to the external world. For example, it is now possible to bounce a ball on a racket by producing an oscillatory up-and-down movement in joint space, but using the discrete system to make sure the oscillatory movement remains under the ball such that the task can be accomplished—this task actually motivated our current research [34].

The key question of DMPs is how to formalize nonlinear dynamic equations such that they can be flexibly adjusted to represent arbitrarily complex motor behaviors without the need for manual parameter tuning and the danger of instability of the equations. We will develop our approach in the example of a discrete dynamic system for reaching movements. Assume we have a basic point attractive system, for instance, instantiated by the second order dynamics

$$\tau \ddot{z} = \alpha_z (\beta_z (g - z) - z), \quad \tau \dot{y} = z + f \quad (4)$$

where g is a known goal state, α_z and β_z are time constants, τ is a temporal scaling factor (see below) and y, \dot{y} correspond to the desired position and velocity generated by the equations, interpreted as a movement plan. For appropriate parameter settings and $f=0$, these equations form a globally stable linear dynamical system with g as a unique point attractor. Could we find a nonlinear function f in Equation (4) to change the rather trivial exponential convergence of y to allow more complex trajectories on the way to the goal? As such a change of Equation (4) enters the domain of nonlinear dynamics, an arbitrary complexity of the resulting equations can be expected. To the best of our knowledge, this has prevented research from employing generic learning in nonlinear dynamical systems so far. However, the introduction of an additional canonical dynamical system (x, v)

$$\tau \dot{v} = \alpha_v (\beta_v (g - x) - v), \quad \tau \dot{x} = v \quad (5)$$

and the nonlinear function f

$$f(x, v, g) = \frac{\sum_{i=1}^N \psi_i w_i v}{\sum_{i=1}^N \psi_i}, \quad (6)$$

where $\psi_i = \exp\left(-h_i\left(\frac{x}{g} - c_i\right)^2\right)$

can alleviate this problem. Equation (5) is a second order dynamical system similar to Equation (4), however, it is linear and not modulated by a nonlinear function, and, thus, its monotonic global convergence to g can be guaranteed with a proper choice of α_v and β_v , e.g., such that Equation (5) is critically damped. Assuming that all initial conditions of the state variables x, v, y, z are initially zero, the quotient $x/g \in [0, 1]$ can serve as a phase variable to anchor the Gaussian basis functions ψ_i (characterized by a center c_i and bandwidth h_i), and v can act as a “gating term” in the nonlinear function (6) such that the influence of this function vanishes at the end of the movement. Assuming boundedness of the weights w_i in Equation (6), it can be shown that the combined

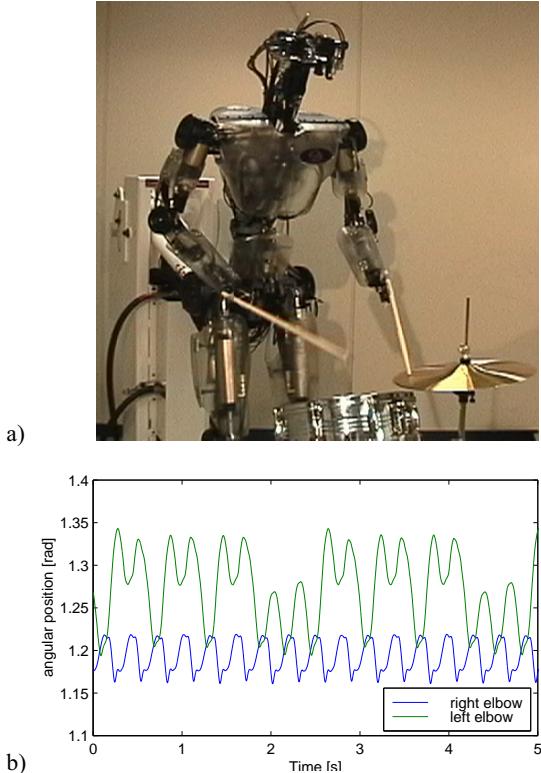


Figure 2: a) Humanoid robot in drumming task, b) coordination of left and right elbow, demonstrating the superposition of discrete and rhythmic DMPs.

system in Equations (4),(5),(6) asymptotically converges to the unique point attractor g .

Given that f is a normalized basis function representation with linear parameterization, it is obvious that this choice of a nonlinearity allows applying a variety of learning algorithms to find the w_i . For instance, if a sample trajectory is given in terms as $y_{demo}(t), \dot{y}_{demo}(t)$ and a duration T , e.g., as typical in imitation learning [35], a supervised learning problem can be formulated with the target trajectory $f_{target} = \tau \dot{y}_{demo} - z_{demo}$ for the right part of Equation (4), where z_{demo} is obtained by integrating the left part of Equation (4) with y_{demo} instead of y . The corresponding goal is $g = y_{demo}(t=T) - y_{demo}(t=0)$, i.e., the sample trajectory was translated to start at $y=0$. In order to make the nominal (i.e., assuming $f=0$) dynamics of Equations (4) and (5) span the duration T of the sample trajectory, the temporal scaling factor τ is adjusted such that the nominal dynamics achieves 95% convergence at $t=T$. For solving the function approximation problem, we chose a non-parametric regression technique from locally weighted learning (RFWR) [36] as it allows us to determine the necessary number of basis functions N ,

their centers c_i , and bandwidth h_i automatically—in essence, for every basis function ψ_i , RFWR performs a locally weighted regression of the training data to obtain an approximation of the tangent of the function to be approximated within the scope of the kernel, and a prediction for a query point is achieved by a ψ_i -weighted average of the predictions of all local models. Moreover, the parameters w_i learned by RFWR are also independent of the number of basis functions, such that they can be used robustly for categorization of different learned DMPs.

In summary, by anchoring a linear learning system with nonlinear basis functions in the *phase space* of a *canonical dynamical system with guaranteed attractor properties*, we are able to learn complex attractor landscapes of nonlinear differential equations without losing the asymptotic convergence to the goal state. Ijspeert et al [37] demonstrate how the same strategy as described for a point attractive system above can also be applied to limit cycle oscillators, thus creating oscillator systems with almost arbitrarily complex limit cycles. It is also straightforward to augment the suggested approach of DMPs to multiple DOFs: there is only one canonical system (cf. Equation (5)), but for each DOF a separate function f is learned. Even highly complex phase relationships between different DOFs, as for instance needed for locomotion, are easily and stably realizable in this approach.

2.2 Application to Humanoid Robotics

We implemented our DMP system on a 30 DOF Sarcos Humanoid robot. Desired position, velocity, and acceleration information was derived from the states of the DMPs to realize a compute-torque controller. All necessary computations run in real-time at 420Hz on a multiple processor VME bus operated by VxWorks. We realized arbitrary rhythmic “3-D drawing” patterns, sequencing of point-to-point movements and rhythmic patterns like ball bouncing with a racket. Figure 2a shows our humanoid robot in a drumming task. The robot used both arms to generate a regular rhythm on a drum and a cymbal. The arms moved in 180-degree phase difference, primarily using the elbow and wrist joints, although even the entire body was driven with oscillators for reasons of natural appearance. The left arm hit the cymbal on beat 3, 5, and 7 based on an 8-beat pattern. The velocity zero crossings of the left drum stick at the moment of impact triggered the discrete movement to the cymbal. Figure 2b shows a trajectory piece of the left and the right elbow joint angles to illustrate the drumming pattern. Given the independence of a discrete and rhythmic movement primitives, it is very



Figure 3: Left Column: Teacher demonstration of a tennis swing, Right Column: Imitated movement by the humanoid robot.

easy to create the demonstrated bimanual coordination without any problems to maintain a steady drumming rhythm.

Another example of applying the DMP is in the area of imitation learning, as outlined in the previous section. Figure 3 illustrates the teaching of a tennis forehand to our humanoid, using an exoskeleton to obtain joint angle data from the human demonstration. The learned multi-joint DMP can be re-used for different targets and at different speeds due to the flexible appearance of the goal parameter g and time scaling τ —in the example in Figure 3, the Cartesian ball position is first converted to a joint angle target by inverse kinematics algorithms, and subsequently each DOF of the robot receives a separate joint space goal state for its DMP component.

3 Parallels in Biological Research

Our ideas on dynamic movement primitives for motor control are based on biological inspiration and complex system theory, but do they carry over to biology? Over the last years, we explored various experimental setups that could actually demonstrate that dynamic movement primitives as outlined above are

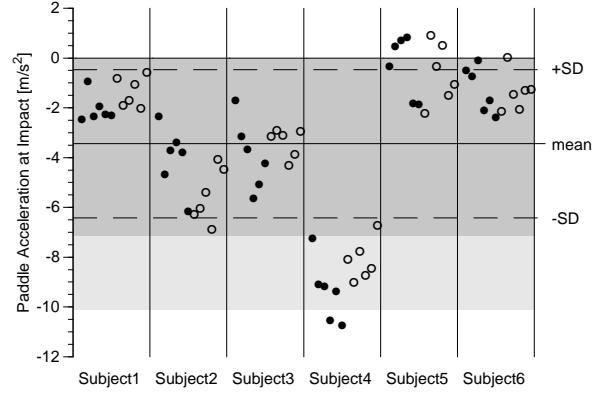


Figure 4: Trial means of acceleration values at impact, \ddot{x}_{P_n} , for all six experimental conditions grouped by subject. The symbols differentiate the data for the two gravity conditions G . The dark shading covers the range of maximal local stability for G_{reduced} the light shading the range of maximal stability for G_{normal} . The overall mean and its standard deviation refers to the mean across all subjects and all conditions.

indeed an interesting modeling approach to account for various phenomena in behavioral and even brain imaging experiments. The remainder of this paper will outline some of the results that we obtained.

3.1 Dynamic Manipulation Tasks

From the viewpoint of motor psychophysics, the task of bouncing a ball on a racket constitutes an interesting testbed to study trajectory planning and visuo-motor coordination in humans. The bouncing ball has a strong stochastic component in its behavior and requires a continuous change of motor planning in response to the partially unpredictable behavior of the ball.

In previous work [34], we examined which principles were employed by human subjects to accomplish stable ball bouncing. Three alternative movement strategies were postulated. First, the point of impact could be planned with the goal of intersecting the ball with a well-chosen movement velocity such as to restore the correct amount of energy to accomplish a steady bouncing height [38]; such a strategy is characterized by a constant velocity of the racket movement in the vicinity of the point of racket-ball impact. An alternative strategy was suggested by work in robotics: the racket movement was assumed to mirror the movement of the ball, thus impacting the ball with an increasing velocity profile, i.e., positive acceleration [25]. The dynamical movement primitives introduced above allow yet another way of accomplishing the ball bouncing task: an oscillatory racket movement creates a dynamically stable basin

of attraction for ball bouncing, thus allowing even open-loop stable ball bouncing. This movement strategy is characterized by a negative acceleration of the racket during impacting the ball [39]—a quite non-intuitive solution: why would one break the movement before hitting the ball?

Examining the behavior of six subjects revealed the surprising result that dynamic movement primitives captured the human behavior the best: all subjects reliably hit the ball with a negative acceleration at impact, as illustrated in Figure 4. Manipulations of bouncing amplitude also showed that the way the subjects accomplished such changes could easily be captured by a simple re-parameterization of the oscillatory component of the movement, similarly as suggested for our DMPs above.

3.2 Apparent Movement Segmentation

Invariants of human movement have been an important area of research for more than two decades. Here we will focus on two such invariants, the 2/3 power law and piecewise planar movement segmentation, and how a parsimonious explanation of those effects can be obtained. Studying handwriting and 2D drawing movements, Viviani and Terzuolo [40] first identified a systematic relationship between angular velocity and curvature of the endeffector traces of human movement, an observation that was subsequently formalized in the “2/3 power law” [41]:

$$a(t) = k c(t)^{2/3} \quad (7)$$

$a(t)$ denotes the angular velocity of the endpoint trajectory, and $c(t)$ the corresponding curvature; this relation can be equivalently expressed by a 1/3 power-law relating tangential velocity $v(t)$ with radius of curvature $r(t)$:

$$v(t) = k r(t)^{1/3} \quad (8)$$

Since there is no physical necessity for movement systems to satisfy this relation between kinematic and geometric properties, and since the relation has been reproduced in numerous experiments (for an overview see [42]), the 2/3-power law has been interpreted as an expression of a fundamental constraint of the CNS, although biomechanical properties may significantly contribute [43]. Additionally, Viviani and Cenzato [44] and Viviani [45] investigated the role of the proportionality constant k as a means to reveal movement segmentation: as k is approximately constant during extended parts of the movement and only shifts abruptly at certain points of the trajectory, it was interpreted as an indicator for segmented control. Since the magnitude of k also appears to correlate with the average movement velocity in a movement segment, k was termed the “velocity gain factor.”

Viviani and Cenzato [44] found that planar elliptical drawing patterns are characterized by a single k and, therefore, consist of one unit of action. However, in a fine-grained analysis of elliptic patterns of different eccentricities, Wann , Nimmo-Smith, and Wing [46] demonstrated consistent deviations from this result. Such departures were detected from an increasing variability in the log-v–log-r-regressions for estimating k and the exponent β of Equation (2), and ascribed to several movement segment each of which having a different velocity gain factor k .

The second movement segmentation hypothesis we want to address partially arose from research on the power law. Soechting and Terzuolo [47, 48] provided qualitative demonstrations that 3D rhythmic endpoint trajectories are piecewise planar. Using a curvature criterion as basis for segmentation, they confirmed and extended Morasso’s [49] results that rhythmic movements are segmented into piecewise planar strokes. After Pellizzer, Massay, Lurito, and Georgopoulos [50] demonstrated piecewise planarity even in an isometric task, movement segmentation into piecewise planar strokes has largely been accepted as one of the features of human and primate arm control.

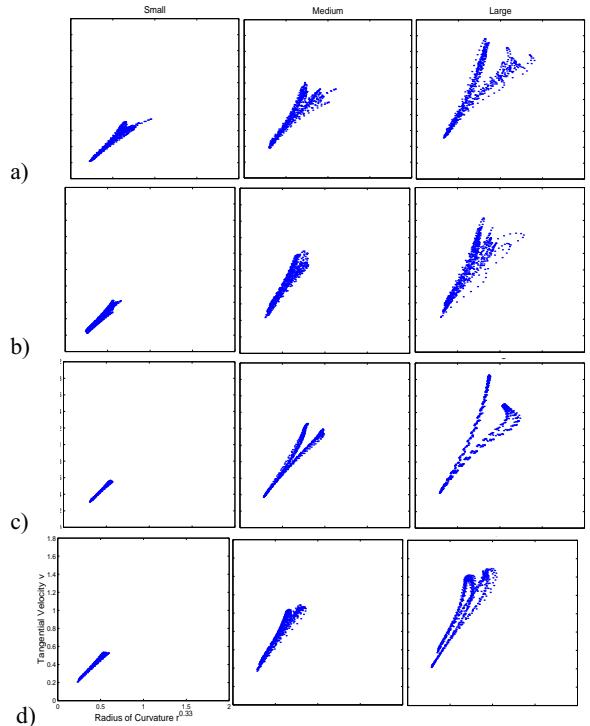


Figure 5: Tangential velocity versus radius of curvature to the power 1/3 for ellipses of small, medium, and large size for elliptical pattern orientations in the frontal and oblique workspace plane: a) human frontal; b) human oblique; c) robot frontal; d) robot oblique.

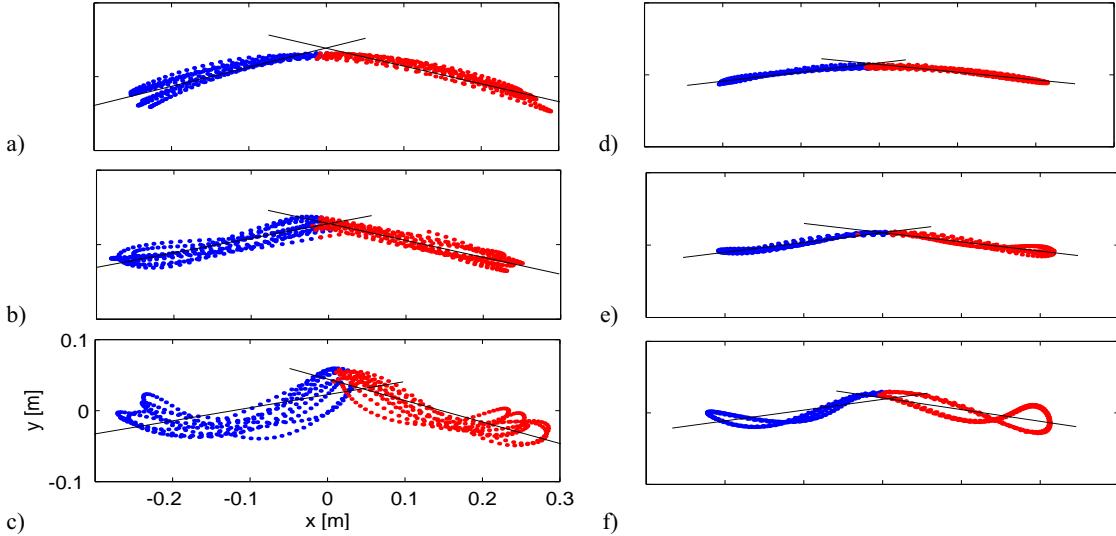


Figure 6: Planar projection of one subject’s figure-8 patterns of small, medium, and large width/height ratio: a-c) human data; d-f) corresponding robot data.

We repeated some of the experiments that led to the derivation of the power law, movement segmentation based on the power law, and movement segmentation based on piecewise planarity. We tested six human subjects when drawing elliptical patterns and figure-8 patterns in 3D space freely in front of their bodies. Additionally, we used an anthropomorphic robot arm, a Sarcos Dexterous Arm, to create similar patterns as those performed by the subjects. The robot generated the elliptical and figure-8 patterns solely out of joint-space oscillations, as described for the DMPs above. For both humans and the robot, we recorded the 3D position of the fingertip and the seven joint angles of the performing arm.

Figure 5 illustrates data traces of one human subject and the robot subject for elliptical drawing patterns of different sizes and different orientations. For every trajectory in this graph, we computed the tangential velocity of the fingertip of the arm and plotted it versus the radius of curvature raised to the power 1/3. If the power law were obeyed, all data points should lie on a straight line through the origin. Figure 5a,b clearly demonstrates that for large size patterns, this is not the case, indicating that the power seems to be violated for large size patterns. However, the development of two branches for large elliptical patterns in Figure 5a,b could be interpreted that large elliptical movement patterns are actually composed of two segments, each of which obeys the power law. The rejection of the latter point comes from the robot data in Figure 5c,d. The robot produced strikingly similar features in the trajectory realizations as the human subjects. However, the robot simply used oscillatory joint space movement to create these pat-

terns, i.e., there was no segmented movement generation strategy. Some mathematical analysis of the power law and the kinematic structure of human arms could finally establish that the power law can be interpreted as an epiphenomenon of oscillatory movement generation: as long as movement patterns are small enough, the power law holds, while for large size patterns the law breaks down [51, 52].

Using figure-8 patterns instead of elliptical patterns, we were also able to illuminate the reason for apparent piecewise-planar movement segmentation in rhythmic drawing patterns. Figure 6 shows figure-8 patterns performed by human and robot subjects. If realized with an appropriate width-to-height ratio, figure-8 patterns look indeed like piecewise planar trajectories and invite the hypothesis of movement segmentation at the node of the figure-8. However, as in the previous experiment, the robot subject produced the same features of movement segmentation despite it used solely joint space oscillations to create the patterns, i.e., no movement segmentation. Again, it was possible to explain the apparent piecewise planarity from a mathematical analysis of the kinematics of the human arm, rendering piecewise planarity to be an epiphenomenon of oscillatory joint space trajectories and the nonlinear kinematics of the human arm. [51].

3.3 Superposition of Discrete and Rhythmic Movement

In another experiment, we addressed the hypothesis of DMP that two separate movement primitives generate discrete and rhythmic movement. Subjects per-

Experiment 1

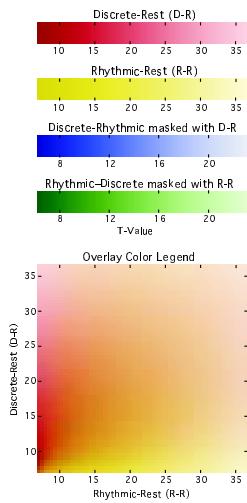


Figure 7: Difference in brain activation between discrete and rhythmic movement obtained by contrasting discrete and rhythmic wrist movement. See legend on the left of the figure for explanations of which contrasts are displayed (note that this plot may not be clear in a black-and-white printout—download a PDF version at <http://www-clmc.usc.edu/publications>). RHYTHMIC-REST and DISCRETE-REST in the middle plot of all subfigures demonstrate the main effects of brain activity during RHYTHMIC and DISCRETE movement conditions—when there is overlap between the two contrasts, the Overlay Color Legend on the left of the subfigures is used to highlight the degree of overlap. RHYTHMIC-DISCRETE shows brain areas where rhythmic movement has stronger activity than discrete movement. Analogously, DISCRETE-RHYTHMIC displays areas that showed significantly more activation than rhythmic movement. The right plot of all three subfigures shows the RHYTHMIC-DISCRETE and DISCRETE-RHYTHMIC contrasts in isolation for the sake of clarity—no overlap is possible. The left plot in all subfigures superimposes the activities from the other plots in the subfigure to allow an easy comparison of activation locations. All results shown are statistically significant at a level of $p < 0.00001$, corrected for multiple comparisons within the entire brain volume. Abbreviations are[56]: AC: anterior commissure; PC: posterior commissure; VAC: vertical line perpendicular to the AC-PC, passing through the AC; PAC: vertical line perpendicular to the AC-PC, passing through the PC; CCZ: caudal cingulate zone; RCZ: rostral cingulate zone, divided in an anterior (RCZa) and posterior (RCZp) part; SMA: caudal portion of the supplementary motor area, corresponding to SMA proper; pre-SMA: rostral portion of the supplementary motor area; M1: primary motor cortex; S1: primary sensory cortex; PMdr: rostral part of the dorsal premotor cortex; PMdc: caudal part of the dorsal premotor cortex; BA7: Brodmann area 7 in parietal cortex; BA40: Brodmann area 40 in parietal cortex.

formed oscillatory elbow movements around a given point in space and shifted the mean position of the elbow at an auditory signal to another point. In previous work [53], it was argued that such a discrete shift terminates the oscillatory elbow movement and restarts it after the shift. Using the model of dynamic movement primitives, we were able to demonstrate that a simple coupling structure between the discrete and rhythmic movement system can actually explain all the phenomena observed in this experiment, including phase resetting, a restricted set of onset phases for the discrete movement within the rhythmic

movement, and kinematic features of the trajectory after the discrete shift [54, 55].

3.4 Brain Activation in Discrete and Rhythmic Movement

A last set of experiments addressed the question whether discrete and rhythmic movements make use of different brain centers. In a 4Tesla scanner, subjects performed either continuous oscillations with the wrist at two different frequencies, or discrete flexion and extension movements with pseudo-random movement start times. Both conditions were executed either with or without metronome pacing, and even with the foot instead of the wrist in three subjects. SPM99 based data analysis, including averaging across 11 subjects, provided highly statistically significant results (Figure 7). While rhythmic movement was confined to activation in primary contralateral motor cortices, supplementary motor cortex, and ipsilateral cerebellum, discrete movement elicited additional activation in contralateral premotor and parietal areas, and also in various ipsilateral cortical regions. These results indicate that discrete movements, even as simple as wrist flexion-extension movements, recruit significantly more cortical areas than rhythmic movement, and that discrete and rhythmic movement may have different movement generating principles in the brain. Thus, the model of rhythmic and discrete movement primitives may even have physiological significance.

4 Conclusion

The present study describes research towards generating flexible movement primitives out of nonlinear dynamic attractor systems. We focused on motivating appropriate dynamic systems such that discrete and

rhythmic movements could be generated with high-dimensional movement systems. We also described some implementations of our system of Dynamic Movement Primitives on a complex anthropomorphic robot. In the last sections of the paper, we outlined various behavioral and imaging studies that resulted from our more theoretically motivated model. We believe that the combination of robotic, theoretical, and biological work that we pursued for the presented studies exemplifies a new path towards research in biomimetic robotics and computational neuroscience. Both disciplines can offer different and new ideas and techniques that will ultimately lead to reciprocal benefits in both disciplines.

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