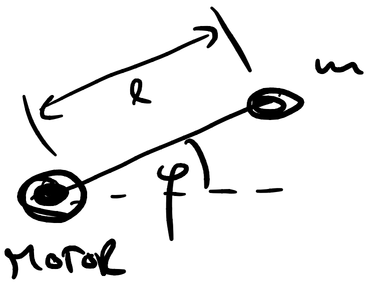
Problem 1: IDEAL MOTOR, RIGID STRUCTURE

$J = m\ell^2$ rotary inertia

τ torque



INITIAL POS

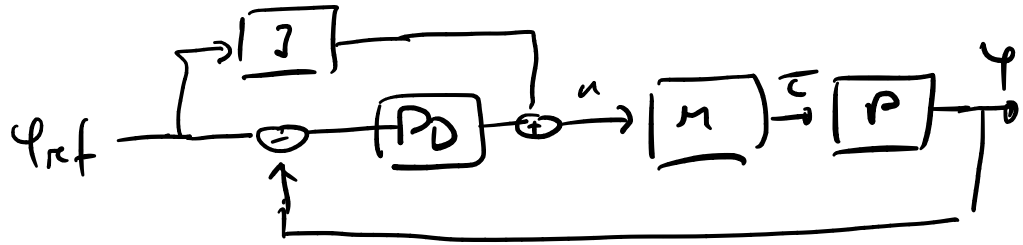
$\varphi_0 = \varphi$

TARGET POS

$\varphi_1 = \frac{\pi}{2}$

QoI: T time to move from initial to target position

DV: k_P, ~~k_D~~ fixed

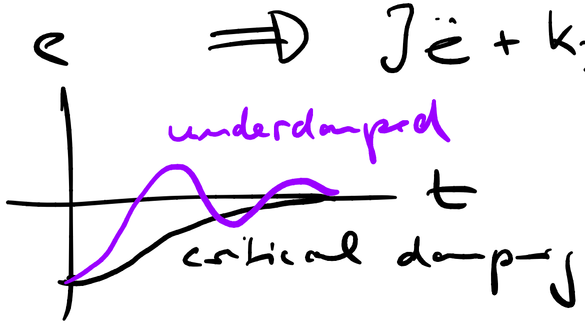


$\varphi_{ref}(t) = \varphi_1 = \text{const.}$

MODEL: $J\ddot{\varphi} = \tau$

CONTROL LAW $\tau = J\ddot{\varphi}_{ref} + k_D(\dot{\varphi}_{ref} - \dot{\varphi}) + k_P(\varphi_{ref} - \varphi)$

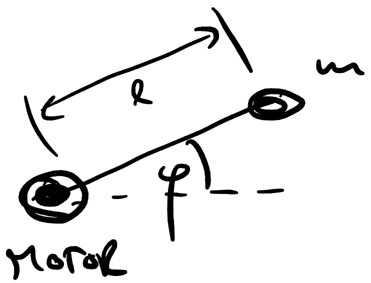
$\Rightarrow J\ddot{e} + k_D\dot{e} + k_P e = 0 \qquad e = \varphi - \varphi_{ref}$



$$\boxed{T \text{ can approach } \phi \text{ for } k_D \,\&\, k_P \to \infty}$$

BORING

PROBLEM 2: LIMITED MOTOR / RIGID STRUCTURE



INITIAL POS

$\varphi_0 = \varphi$

TARGET POS

$\varphi_1 = \frac{\pi}{2}$

QoI: T time to move from initial to target position

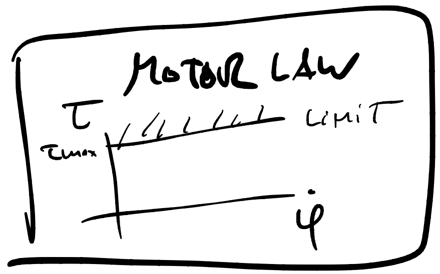
DV: $K_p, K_D, \cancel{\varphi}, \cancel{\tau_{max}}$ fixed

$\varphi_{ref}(t) = \varphi_1 = \text{const.}$

ideal torque that may not be realized

MODEL: $J\ddot{\varphi} = \tau$ $\quad \tau_{ID} = J\ddot{\varphi}_{ref} + K_D(\dot{\varphi}_{ref} - \dot{\varphi}) + K_p(\varphi_{ref} - \varphi)$

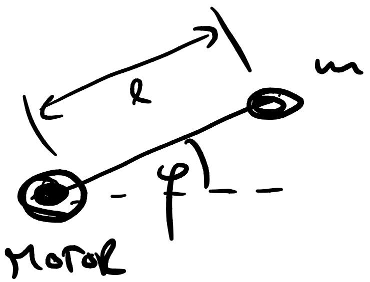
CONTROL LAW $\quad \tau = \begin{cases} \tau_{ID} & \text{if } \tau_{ID} < \tau_{max} \\ \tau_{max} & \text{if } \tau_{ID} \geq \tau_{max} \end{cases}$



?

1. What is $e(t)$
2. What a choice of K_p, K_D that minimizes T?

PROBLEM 3: LIMITED MOTOR 2, RIGID STRUCTURE



INITIAL POS

$\varphi_0 = \varphi$

TARGET POS

$\varphi_1 = \frac{\pi}{2}$

QoI: T time to move from initial to target position

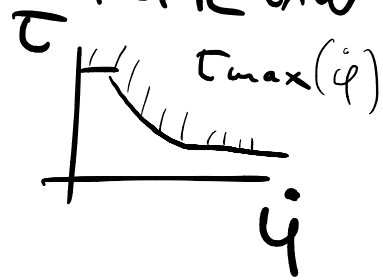
DV: $k_p, k_D, \cancel{\varphi}$ fixed

$$\varphi_{ref}(t) = \varphi_1 = \text{const.}$$

MODEL: $J\ddot{\varphi} = \tau$ $\qquad \tau_{iD} = J\ddot{\varphi}_{ref} + k_D(\dot{\varphi}_{ref} - \dot{\varphi}) + k_p(\varphi_{ref} - \varphi)$

CONTROL LAW $\quad \tau = \begin{cases} \tau_{iD} & \text{if } \tau_{iD} < \tau_{max}(\dot{\varphi}) \\ \tau_{max} & \text{if } \tau_{iD} \geq \tau_{max}(\dot{\varphi}) \end{cases}$

MOTOR LAW

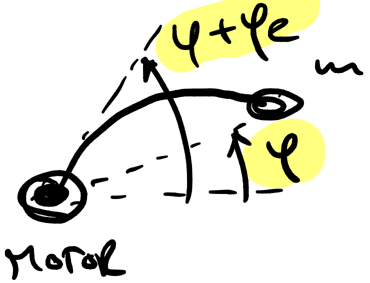


?

(1) what is $e(t)$

(2) what a choice of k_p, k_D that minimizes T?

PROBLEM 4: LIMITED MOTOR 2, FLEXIBLE STRUCTURE



INITIAL POS

$\varphi_0 = \varphi$

TARGET POS

$\varphi_1 = \frac{\pi}{2}$

φ_e denotes elastic deformation

QoI: T time to move from initial to target position

DV: k_p, k_D, $\cancel{\varphi}$ fixed, $\cancel{k}$ fixed

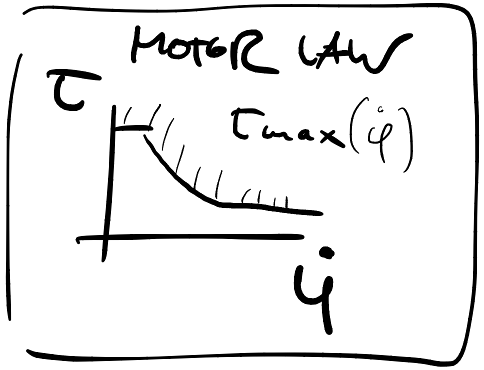
$\varphi_{ref}(t) = \varphi_1 = \text{const.}$

?

$\tau_{ID} = J\ddot{\varphi}_{ref} + \ldots + k_D(\dot{\varphi}_{ref} - \dot{\varphi}) + k_p(\varphi_{ref} - \varphi)$

MODEL: $J\ddot{\varphi} = \tau$ $\quad k\varphi_e = \tau$

CONTROL LAW $\tau = \begin{cases} \tau_{ID} & \text{if } \tau_{ID} < \tau_{max}(\dot{\varphi}+\dot{\varphi}_e) \\ \tau_{max} & \text{if } \tau_{ID} \geq \tau_{max}(\dot{\varphi}+\dot{\varphi}_e) \end{cases}$



?

1. What is $e(t)$
2. What a choice of k_p, k_D that minimizes T?