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Distribution of Singularity and Optimal Control of Redundant Parallel Manipulators

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Abstract

Singularity is a fundamental problem in the analysis of parallel mechanisms. The distribution of singularity in workspace will determine to a great extent the properties of parallel mechanisms. In this paper, we will study the distribution of actuator singularity, which can also be applied to analyze end-effector singularity. A very important observation has been made that these two kinds of singularities are caused by the parameterization of a configuration manifold by actuator coordinates or end-effector coordinates. Despite the various styles of singularities of parallel mechanisms, there are some rules which govern the behavior of stable singularities. These rules provide some useful ideas in the design of redundant parallel mechanisms so as to achieve better performance in high speed motion and improve their stiffness. Optimal kinematic and dynamic control algorithms are designed and implemented which make use of the redundancy of the parallel mechanism. Experimental results agree with our expectation.

1 Introduction

Searching and evaluation of singularities play an essential role in the design, planning and control of parallel mechanisms with singularities. The research about singularities of serial and parallel robots has a long time. However, most of these research is restricted to search singularities. More over, the ideas used by them are confined to find some Jacobian matrices and check their rank conditions [1, 2, 3, 4, 5, 6]. Few has asked the following three questions. What's the typical behavior of the singularity of parallel mechanisms? How do the number and properties of the singularity of parallel mechanisms depend on the dimension of configuration manifold and that of actuator coordinates or end-effector coordinates? How can these rules be used to help us in the design, planning and control of parallel mechanisms? In this paper we explore the behavior of singularities by studying that of parameterization of a configuration manifold by actuator coordinates or end-effector coordinates. We find that the styles of stable singularities are similar given the dimension of the configuration manifold and that of actuator coordinates or end-effector coordinates. More over, the number of stable singularities of a given parallel mechanism will decrease with the augmentation of the dimension of actuator coordinates or end-effector coordinates until no singularity exists, which coincides with Whitney embedding theorem. Basing on this fact, we propose over actuation and over constraints method in order to eliminate stable singularities and satisfy the requirements in stiffness and high speed tracking. Then by exploiting the redundancy of the parallel manipulator, optimal kinematic and dynamic control algorithms are designed. Their geometric and physical meaning are well explained. Several experiments have been implemented, which show the good performance in tracking. This paper is organized as the following: In section 2 we present some basic concepts and study the stability of parameterization singularities. In this section several examples are discussed. In section 3, we propose over actuation and over constraints in the design of parallel mechanisms. We present optimal control algorithms for the HKUST redundant parallel manipulator from kinematics and dynamics respectively in section 4. Experimental results are given in section 5. Eventually we arrive at the conclusion.

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2 Distribution of stable singularities

This section presents some basic concepts and studies the theory associated with the distribution of stable singularities.

2.1 Stable singularities

Consider a differentiable map $f : M^m \to N^n$. We know the differential df is a linear map from the tangent space of the source manifold to the tangent space of the target manifold: $df_x : T_xM^m \to T_{f(x)}N^n$. A point is called a singularity if the rank of the derivative df_x is less than the maximum possible value which is equal to the smaller of the dimensions of M and N. Next, for convenience, a coarse concept of stability of a singularity is introduced.

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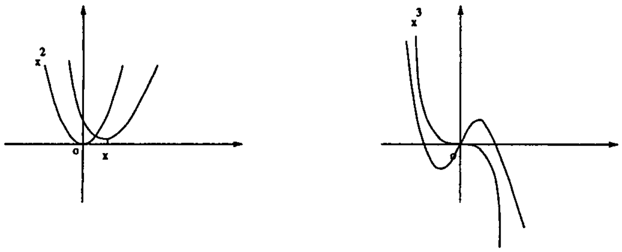


Figure 1: The singularity at zero of $y = x^2$ is stable, at zero of $y = x^3$ is unstable

Definition 1 A singularity x of a map $f : M \to N$ is said to be stable if for a sufficiently small neighborhood E of the map f , such that for any map $\tilde{f}$ in E there is a similar singularity $\tilde{x}$. The meaning of a neighborhood of a given map is the set of all maps differing from the given one when derivatives up to a fixed order are taken into account.

Example 1 The singularity at zero of the function $y = x^2$ is stable, while the singularity at zero of the function $y = x^3$ is unstable. The reason is that two stable singularities will emerge when we disturb function $y = x^3$ a little, as shown in figure (1).

2.2 Stable Parameterization Singularities

In this section, we collect some results regarding the number and the properties of actuator singularity and end-effector singularity. These results manifest in different levels the distribution of stable singularity. Several simple and practical examples have been studied which demonstrate the distribution theory. As been pointed out by Yiu[7] that actuator singularity and end-effector singularity are caused by parameterization of configuration manifold by actuator coordinates or end-effector coordinates, it is reasonable to study the properties of parameterization.

Theorem 1 There are two possible styles of stable singularities for the maps from a two-dimensional manifold to a two-dimensional manifold. (1)folding point: the null space (or kernel space) of *df* is not tangent to the singular manifold. (2)Whitney pleat: the kernel space of *df* is precisely tangent to the singular manifold. [8].

Example 2 Figure (2) is a typical 2-DOF normally actuated planar parallel mechanism if only two joints (θ_1, θ_2) are actuated. The configuration manifold is given by the following constraint equations,

$$\begin{array}{rcl} x_{A_1} + l_2 cos\theta_1 + l_1 cos\phi_1 & = & x_{A_2} + l_2 cos\theta_2 + l_1 cos\phi_2 \\ y_{A_1} + l_2 sin\theta_1 + l_1 sin\phi_1 & = & y_{A_2} + l_2 sin\theta_2 + l_1 sin\phi_2 \\ x_{A_1} + l_2 cos\theta_1 + l_1 cos\phi_1 & = & x_{A_3} + l_2 cos\theta_3 + l_1 cos\phi_3 \\ y_{A_1} + l_2 sin\theta_1 + l_1 sin\phi_1 & = & y_{A_3} + l_2 sin\theta_3 + l_1 sin\phi_3 \end{array} .$$

Actuator singularity emerges when the following Jacobian

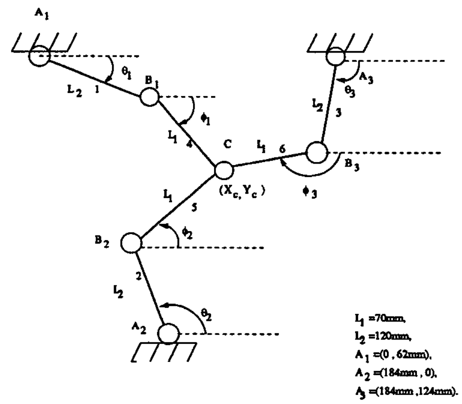
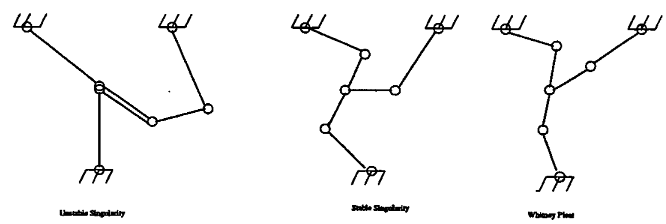


Figure 2: Planar 2-DOF parallel mechanism



(a) (b)

Figure 3: Unstable Singularity, folding point, Whitney pleat

matrix loses rank,

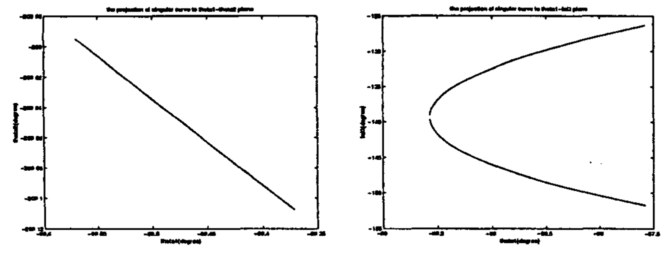
$$\begin{bmatrix} 0 & -l_1 sin\phi_1 & l_1 sin\phi_2 & 0 \\ 0 & l_1 cos\phi_1 & -l_1 cos\phi_2 & 0 \\ l_2 sin\theta_3 & -l_1 sin\phi_1 & 0 & l_1 sin\phi_3 \\ -l_2 cos\theta_3 & l_1 cos\phi_1 & 0 & -l_1 cos\phi_3 \end{bmatrix} .$$

The condition will be,

$$sin(\phi_1 - \phi_2) sin(\theta_3 - \phi_3) = 0.$$

Let's firstly consider $sin(\phi_1 - \phi_2) = 0$, there are two solutions $\phi_1 = \phi_2$ and $\phi_1 = \phi_2 + \pi$, as shown in the left two sub-figures of figure (3). Actually the first solution corresponds to unstable singularity. So we only consider the second solution. The whole stable singular points constitute a one-dimension curve which is determined by the constraint equations and $\phi_1 = \phi_2 + \pi$. Figure (4-a),(4-b),(5) give the diagrams of this curve projected into (θ_1, θ_2) plane, (θ_1, θ_3) plane and (θ_1, ϕ_3) plane respectively. We can imagine most of the points of this curve are folding points. Which point is a pleat? Pleat should be on the singular curve mentioned above where the kernel vector is tangent to this curve. In this case, the kernel vector field is given by,

$$\begin{array}{rcl} X & = & csc(\phi_1 - \theta_3) sin(\phi_3 - \theta_3) \frac{\partial}{\partial \phi_1} \\ & - & csc(\phi_1 - \theta_3) sin(\phi_3 - \theta_3) \frac{\partial}{\partial \phi_2} \\ & - & \frac{l_1 csc(\phi_1 - \theta_3) sin(\phi_1 - \phi_3)}{l_2} \frac{\partial}{\partial \theta_3} + \frac{\partial}{\partial \phi_3} \end{array} .$$



(a) (b)

Figure 4: (a) Projection to θ_1-θ_2 plane, (b) Projection to θ_1-ϕ_3 plane

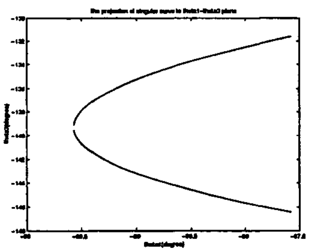


Figure 5: Projection to θ_1-θ_3 plane

It can be shown that X is always transversal to the singular curve except for the following point (Whitney pleat) which is determined by the constrained equations plus the following two equations:

$$\phi_1 = \phi_2 + \pi, \theta_3 = \phi_3.$$

The state of the parallel mechanism at this point is drawn in the right subfigure of figure (3). Where the six joints are respectively equal to : $\theta_1 = -89.57°$, $\theta_2 = 151.01°$, $\theta_3 = -139.29°$, $\phi_1 = 56.07°$, $\phi_2 = 236.07°$ and $\phi_3 = -139.29°$. From this figure we see that at this point link 4 and link 5 are aligned in the same line, so do link 3 and link 6. Usually there are four solutions in the forward kinematics given θ_1, θ_2. This number is reduced to two at a folding point, and one at Whitney pleat. Note that in this case two branches of the singular curve projected into (θ_1,θ_2) plane become one branch. This can only happen when this curve and the configuration space are embedded into high dimensional space.

The study of the singular curve $\theta_3 = \phi_3$ is symmetric to the above analysis, which gives precisely the same Whitney pleat: $\phi_1 = \phi_2 + \pi$, the intersection of these two singular curves. It is expected that the number of parameterization singularities will decrease when we use more actuators. Whitney pointed out that there is only one type of stable parameterization singularity for those maps from two dimensional manifold to three dimensional manifold: Whitney Umbrella.

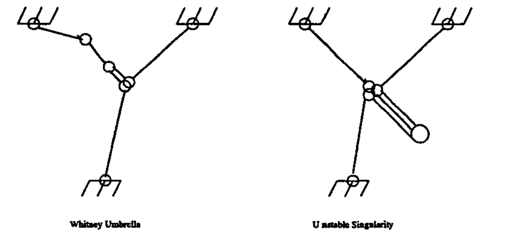


Figure 6: Whitney umbrella and unstable singularity

Example 3 We still use 2-DOF planar parallel mechanism as an example. In this case we actuate three joints $\theta_1, \theta_2, \theta_3$. There are only three possible stable parameterization singularities. They are symmetric . The two subfigures of figure (6) depict one unstable actuator singularity, one stable actuator singularity . The solution for stable singularity is : $\theta_1 = 85.88°$, $\theta_2 = 148.90°$, $\theta_3 = 211.11°$, $\phi_1 = -58.76°$, $\phi_2 = -238.76°$, $\phi_3 = -238.76°$. It satisfies $\phi_2 + \pi = \phi_3 + \pi = \phi_1$. It can be shown that the property of this singularity coincides with Whitney Umbrella [8].

The behaviors for unstable parameterization singularities could be very complicated. A common example is that the mechanism can make finite self motion even if all actuated joints are fixed or actuated joints can perform finite motion even if all passive joints (or passive coordinates) are fixed. Unstable singularities usually depend on the link length of some links or other parameters. The above discussions can be summarized as,

Theorem 2 The stable singularities of a map of an n dimensional manifold to an m dimensional manifold is decreased with the increasing of m.

When extreme case is considered that the number of actuators surmount a given number, no actuator singularity will occur. This is precisely the famous Whitney embedding theorem.

Theorem 3 Any n dimensional differentiable manifold can be differentially embedded into $2n + 1$ dimensional Euclidean space.

$2n + 1$ actuators are enough to eliminate singularities. In practical applications, it is expensive to use too many motors, and it is not practical and convenient to mount motors on the middle of each serial chain. In the next section, we will present over actuation and over constraints method which fully exploits the advantages of high-dimension parameterization. Through this method one can realize singularityless mechanism with actuators less than $2n+1$ and more than n due to the fact that singularities should also satisfy the constrained equations. The planar 2-DOF redundantly actuated parallel mechanism in HKUST intelligent control lab is such a singularityless parallel mechanism by actuating three base joints, as shown in figure (10).

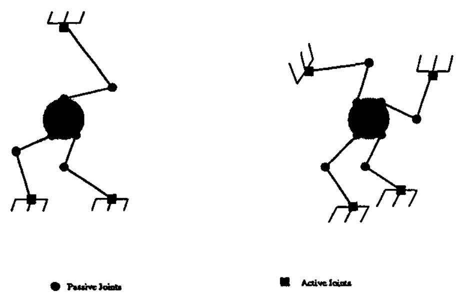


Figure 7: Over constraints and over actuation

3 Over actuation and over constraints

It has been found in the real applications that singularity will cause undesired effects to motion accuracy, Cartesian stiffness and dynamic property. Eventually it will neutralize the anticipated advantages of parallel mechanism. Although we can bypass singularities by replanning motion trajectories, such as DLS method[9], the more efficient and thorough method is to implement redundancy. In this approach, we increase the number of serial branches (over constraints) and the corresponding number of actuators (over actuation), as shown in figure (7). It is easy to see, the property of this kind of method is that the number of actuators and serial branches is larger than the DOF of the end-effector. In [10], we find the condition number becomes smaller, and the end-effector more stiff after over actuation and over constraints are used. Over constraints and over actuation are usually combined together to improve the gross performance of parallel mechanisms.

4 Optimal kinematic and dynamic control algorithms for redundant parallel manipulators

This section presents optimal control algorithms for parallel mechanisms with over actuation and over constraints from kinematics and dynamics respectively. In the former method, the motion process of the parallel mechanism is assumed to be quasi-static, and for the later one, the dynamics is rigorously considered.

4.1 Optimal kinematic control algorithm

For a parallel mechanism with over constrains and over actuation, the number of actuators is greater than the DOF of its end-effector. They can't be controlled independently. Coordinating algorithm must be introduced. More over, we assume the mentioned mechanism has a given joint stiffness, which is given by a diagonal matrix $kI_{\nu\times\nu}$, where k is a scalar and can be identified by least square method based on experiments. Then the corresponding Cartesian stiffness matrix will be $K_c = GkI_{\nu\times\nu}G^T$, where G^T is a ν by μ ($\nu > \mu$) matrix, satisfying

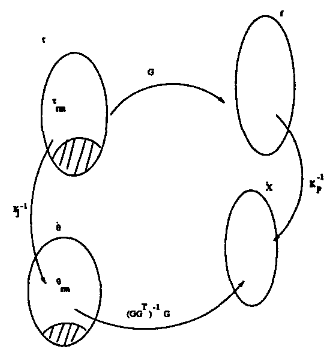


Figure 8: Four maps and their relations

$$\delta\theta_r = G^T\delta X \quad (1)$$
$$f = G\tau_r \quad (2)$$

where we denote θ_r the vector of angles of ν actuators, X the Cartesian position of the end-effector, and f the force exerted to the environment when the end-effector departs its equilibrium. τ_r is the joint torque vector. Because of its redundancy, we can divide the joint displacement into minimal-norm (optimal) displacement and null space displacement. Correspondingly, joint torque can also be split into minimal-norm (optimal) joint torque and the null space component. From equation (2), we can derive the minimal-norm torque τ_{rm} and the null space torque τ_{rn},

$$\tau_{rm} = G^T(GG^T)^{-1}G\tau_r \quad (3)$$
$$\tau_{rn} = (I_{\nu\times\nu} - G^T(GG^T)^{-1}G)\tau_r. \quad (4)$$

Applying the known joint stiffness $kI_{\nu\times\nu}$, the minimal-norm joint displacement $\delta\theta_{rm}$ and the null space one $\delta\theta_{rn}$ can be found,

$$\delta\theta_{rm} = \frac{1}{k}\tau_{rm} = \frac{1}{k}G^T(GG^T)^{-1}G\tau_r = G^T\delta X \quad (5)$$
$$\delta\theta_{rn} = \frac{1}{k}\tau_{rn} = \frac{1}{k}(I_{\nu\times\nu} - G^T(GG^T)^{-1}G)\tau_r. \quad (6)$$

For a redundant parallel mechanism, its inverse kinematics is easy to solve, which means G^T in equation (1) is easy to find. Then, its forward kinematics can be derived as $\delta X = \tilde{G}\delta\theta_r = (GG^T)^{-1}G\delta\theta_r$. We can also check $\tilde{G}\delta\theta_{rn} = 0$ and $\tilde{G}\delta\theta_{rm} = \delta X = K_c{}^{-1}f$. This fact coincides our physical intuition that we have two equivalent ways to cause the displacement of the end-effector. One is : $\tau_{rm} \rightarrow \theta_{rm} \rightarrow \delta X$, the other would be : $\tau_{rm} \rightarrow f \rightarrow \delta X$. The maps used in these two ways are all one-to-one by elimination of null space components. Figure(8) presents the diagram for these two equivalent ways, in which the null space of each map is drawn as the shaded parts. Figure (9) is a typical kinematic control diagram for a parallel mechanism, where $J^{-1} = G^T$. It is obvious that we have implemented minimal displacement algorithm here.

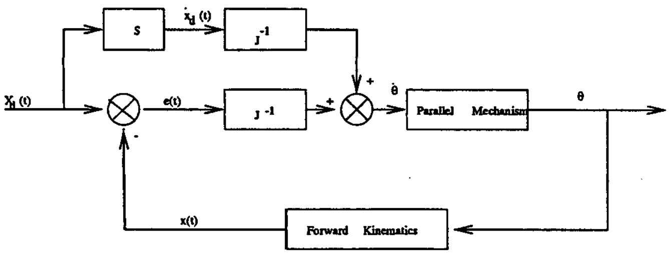


Figure 9: Ordinary kinematic control algorithm

4.2 Optimal dynamic control algorithm

Applying dynamic control, we can realize more smooth motion with high speed. Numerical research has been done in the derivation of dynamic equation of parallel manipulators, [11, 12, 7]. In this subsection, we will focus on extended Lagrangian dynamic model, from which we will derive a reduced dynamic model by direct elimination method. This dynamic model has two parts: one characterizes the motion of the parallel manipulator on the constraint submanifold, and the other involves the constraint force which can be viewed as the internal force dynamics. One modified compute torque control algorithms is proposed whose geometric structures and asymptotic stabilities are well explored.

a. Formulation of dynamics of parallel mechanisms
A Lagrangian formulation of dynamics of parallel mechanisms is utilized here. A parallel mechanism can always be cut into several serial chains. Their dynamic equations combined with the constraints constitute the dynamic equation of the parallel manipulator,

$$M\ddot{\Theta} + C(\Theta, \dot{\Theta}) + N = \tau + A^T\lambda \qquad (7)$$

$$\frac{\partial f}{\partial \Theta_a}\dot{\Theta}_a + \frac{\partial f}{\partial \Theta_p}\dot{\Theta}_p = 0 \qquad (8)$$

$$\left[\frac{\partial f}{\partial \Theta_a}, \frac{\partial f}{\partial \Theta_p}\right] = A \qquad (9)$$

where Θ_a denote the angles of actuators, and Θ_p the passive joint angles. M is the combined inertial matrix of those serial chains, and can be proved to be symmetric and positive definite. Similarly, we can compute C and N. $f(\Theta_a, \Theta_p) = 0$ is the constraint equation and defines the configuration manifold in the ambient space Θ. Without losing of generality, we assume the dimension of the ambient space parameterized by Θ is n and there are m linearly independent constraints so that A is a m by n matrix. Let's also make an assumption that $n - m + l$ joints are actuated, or the degree of redundancy is l. $\tau = (\tau_1, \cdots, \tau_{n-m+l}, 0, \cdots, 0)$. In the following, we implement purely algebraic method, namely, direct elimination method, to seek a reduced dynamic equation. The underlying geometric and physical meaning are clarified despite the complexity of its expression. The essential idea of direct elimination method is to firstly search the expression for λ and substitute it into the original equation to get a full equation without λ,

$$\lambda = (AM^{-1}A^T)^{-1}[-\dot{A}\dot{\theta} + AM^{-1}(C + N - \tau)]. \qquad (10)$$

Then the closed form dynamic equation is given by,

$$M\ddot{\Theta} + A^T(AM^{-1}A^T)^{-1}\dot{A}\dot{\Theta} + \tilde{C} + \tilde{N} = \tilde{\tau} \qquad (11)$$

$$A\dot{\Theta} = 0 \qquad (12)$$

where $\tilde{C} = P_wC$, $\tilde{N} = P_wN$, $\tilde{\tau} = P_w\tau$, and $P_w = (I - A^T(AM^{-1}A^T)^{-1}AM^{-1})$ is a projection operator. It is easy to prove the following Lemma,

Lemma 1 For the above M and A, we have,

$$M\ddot{\Theta} + A^T(AM^{-1}A^T)^{-1}\dot{A}\dot{\Theta} = P_wM\ddot{\Theta} = \tilde{M}\ddot{\Theta}$$

and the original dynamic equation is reduced (projected by P_w),

$$\tilde{M}\ddot{\Theta} + \tilde{C} + \tilde{N} = \tilde{\tau}$$

Following P_w, another map P_T can be defined,

$$P_T = I - M^{-1}A^T(AM^{-1}A^T)^{-1}A. \qquad (13)$$

Lemma 2 P_T and P_w satisfy the following properties,

$$\begin{aligned} P_wM &= MP_T \\ AP_T &= P_wA^T = 0 \\ P_T &= P_w^T. \end{aligned}$$

More over, we want to find the results of multiple action under the maps P_T and P_w.

Lemma 3 P_T and P_w satisfy,

$$\begin{aligned} {P_T}^2 &= P_TP_T = P_T \\ {P_w}^2 &= P_wP_w = P_w \\ (I - P_T)^2 &= (I - P_T)(I - P_T) = I - P_T \\ (I - P_w)^2 &= (I - P_w)(I - P_w) = I - P_w. \end{aligned}$$

From the above two Lemmas, we conclude that P_T and $I - P_w$ are normalized projection maps from ambient velocity space to free velocity space and from ambient force space to constraint force space. Similarly, the geometric meaning of $I - P_T$ and P_w can be interpreted. Now the geometric and physical meaning of equation (11) combined with Lemma 1 can be explained as the reduced dynamics by projecting the dynamic equation . Similarly, the constraint force dynamics can be extracted out by project map $I - P_w$,

$$(I - P_w)(M\ddot{\Theta} + C + N) = (I - P_w)\tau + A^T\lambda. \qquad (14)$$

b. Optimal control algorithm
Equation (11) is used to control the motion of parallel mechanism, while equation (14) is used for constraint force control.

$$\tilde{\tau} = \tilde{M}J(\ddot{\phi}_d + K_pe + K_d\dot{e}) + \hat{C} + \tilde{N} \qquad (15)$$

where ϕ is the independent coordinates, and $\dot{\Theta} = J\dot{\phi}$. The desired trajectory is represented by $\ddot{\phi}_d$, $\dot{\phi}_d$, ϕ_d. $e = \phi_d -$

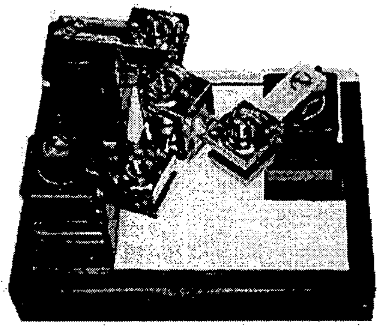


Figure 10: System structure of two DOF redundantly actuated parallel mechanism

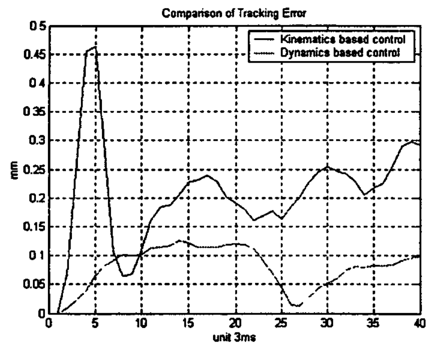


Figure 11: Comparison of tracking errors by two control algorithms

ϕ, K_d, K_p are respectively the derivative and proportional coefficient matrices. $\hat{C} = \tilde{C} + \tilde{M}\dot{J}\phi$. The error dynamics would be,

$$\tilde{M}J(\ddot{e} + K_d\dot{e} + K_p e) = 0. \tag{16}$$

It is not difficult to prove that the whole system is asymptotic stable. Let $\tau = [{\tau_a}^T, {\tau_p}^T]^T$, and $\tau_p = 0$. Split n by n matrix P_w into $[P_{w1}, P_{w2}]$, where P_{w1} is a n by $n - m + l$ matrix. So, we have, $P_w\tau = P_{w1}\tau_a = \tilde{\tau}$. Because P_w has at most rank $n-m$, the rank of P_{w1} is at most $n-m$, which means the columns of P_{w1} is linearly dependent. We can find the minimal norm solution by SVD method or other methods.

5 Experimental setup and results

Figure (10) shows a 2-DOF planar parallel mechanism with over actuation newly developed in HKUST intelligent control lab. Two experiments (kinematic trajectory control and dynamic trajectory control) are done in this research, trajectory tracking errors are compared, as shown in Fig. (11).

6 Conclusion

We study the distribution of stable singularity. Inspired by this theory, we design a kind of parallel mechanism with over actuation and over constraints in order to eliminate the undesired effects introduced by singularities. Through a direct elimination method, the dynamic equation of the parallel manipulator is decoupled. We design a modified compute torque control algorithm according to one of the two decoupled equations which can be used in stable trajectory tracking.

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