T.E (EXTC) sem V CBGS R.S.A. 13/05/15

Q.P. Code : **3374**

(3 Hours) [Total Marks : 80

N.B.:

1) **Question Number 1 is Compulsory**
2) Attempt any Three questions from the remaining Five questions
3) Assumptions made should be clearly stated.
4) Use of normal table is permitted

1 Answer the following **20**

a) **State and prove Bayes' s theorem.**

b) **A certain test for a particular cancer is known to be 95% accurate. A person submits to the test and the results are positive. Suppose that the person comes from a population of 100,000 where 2000 people suffer from that disease. What can we conclude about the probability that the person under test has that particular cancer?**

c) **Let X and Y be independent, uniform r.v.'s in (-1, 1). Compute the pdf of $V = (X+Y)^2$.**

d) **If the spectral density of a WSS process is given by**

$$S(w) = b(a-|w|)/a, \quad |w| \le a$$
$$= 0 \quad , \quad |w| > a$$

Find the autocorrelation function of the process.

2a) **State and prove Chapman-Kolmogorov equation.** **10**

b) **The joint density function of two continuous r.v.'s X and Y is** **10**

$$f(x,y) = cxy \quad 0 < x < 4,\ 1 < y < 5$$
$$= 0 \quad \text{otherwise.}$$

i) **Find the value of constant c.**
ii) **Find $P(X \ge 3, Y \le 2)$**
iii) **Find marginal distribution function of X.**

3a) **Explain strong law of large numbers and weak law of large numbers.** **05**

b) **Explain the central limit theorem.** **05**

c) **A distribution with unknown mean μ has variance equal to 1.5. Use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean.** **10**

[TURN OVER

4a) Given a r.v. Y with characteristic function **10**

$$\Phi(w) = E\{e^{jwY}\}$$

and a random process defined by $X(t) = \cos(\lambda t + Y)$, show that $X(t)$ is stationary in wide sense if

$$\Phi(1) = \Phi(2) = 0.$$

b) Define an ergodic process. Determine whether the stochastic process **10**

$X(t) = A\sin(t) + B\cos(t)$ is ergodic. Here A & B are normally distributed independent r.v.'s with zero mean and equal standard deviation.

5a) The joint probability function of two discrete r.v.'s X and Y is given by $f(x, y) = c(2x + y)$, where x and y can assume all integers such that $0 \le x \le 2$, $0 \le y \le 3$ and $f(x, y) = 0$ otherwise. Find $E(X)$, $E(Y)$, $E(XY)$, $E(X^2)$, $E(Y^2)$, $var(X)$, $var(Y)$, $cov(X, Y)$, and ρ. **10**

b) State and explain various properties of autocorrelation function and power spectral density function. **10**

6a) The transition probability matrix of Markov Chain is **10**

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \end{array}$$

Find the limiting probabilities.

b) Write notes on any two of the following: **10**

i) Markov chains
ii) Little's formula
iv)LTI systems with stochastic input
v) M/G/1 queuing system
