Matching with Costly Interviews: The Benefits of Asynchronous Offers

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Abstract

In many matching markets, matches are formed after costly interviews. We analyze the welfare implications of costly interviewing in a model of worker-firm matching. We examine the trade-offs between a centralized matching system and a decentralized one, where matches can occur at any time. Centralized matching with a common offer date leads to coordination issues in the interview stage. Each firm must incorporate the externality imposed by the interview decisions of the firms ranked above it when deciding on its interview list. As a result, low-ranked firms often fail to interview some candidates that ex-ante have high match quality. In a decentralized setting with exploding offers, the set of candidates who receive interviews differs, but the welfare generated is weakly greater than in the centralized setting. Total welfare is highest with a system that ensures firms interview and match in sequence, clearing the market for the next firm. Such asynchronicity reduces interview congestion. This system can be implemented by encouraging top firms to interview and match early and allowing candidates to renege on offers.

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1 Introduction

The National Resident Matching Program (NRMP) was established in 1952 to match medical graduates with residency programs across the United States. Annually, the NRMP facilitates the matching of tens of thousands of aspiring doctors with suitable training positions via the Gale-Shapley Deferred Acceptance algorithm. The success of the NRMP match has led to an expanding interest in the centralization of other matching markets, ranging from the academic job market to college admissions to the market for college athletes.

Much of the theoretical analysis of centralized, two-sided matching processes has focused on settings where agents have complete information about their preferences. However, a fundamental feature of labor markets is that agents are initially uncertain about their preferences and so invest in costly information acquisition — generally in the form of interviews — *prior to* submitting their rank-order lists. This process imposes significant costs on agents. For instance, in the NRMP match, medical school graduates often have to pay interview expenses out of their pocket, and the residency program doctors conducting the interviews cannot do surgeries on interview days. Moreover, due to competition for candidates, hospital interviewing decisions become calculated decisions, and much effort is devoted to thinking about which candidates are "gettable" and worth interviewing (Wapnir et al. (2021)).

We focus on how the design of matching markets affects how firms acquire information through interviews. These choices are not just about *how much* information to acquire, but *how* to acquire it: firms must not only choose how many interview invitations to issue but also which candidates will receive them. Moreover, these decisions are strategic: while a given firm cares only about the quality of the workers it is matched to, the final assignment is determined by the information that it acquires *and* the information acquired by its competitors. Consequently, the welfare produced by a matching mechanism is inextricably linked with its impact on firms' interviewing incentives.

To analyze this link, we develop a novel model of two-sided matching between firms and workers, where firms conduct costly interviews before offers are made. Interview decisions are decentralized: there is no restriction on how interviews are assigned as they are a strategic choice by firms. We show that centralized matching mechanisms where workers are matched to firms via deferred acceptance have a drawback: firms must decide which candidates to interview without knowing whether those candidates' interviews with more preferred firms will convert to offers. As a result, firms fail to interview some candidates that ex-ante have high match quality, leading to high-quality workers "falling through the cracks".

¹The assumption of complete information of preferences is substantive. Fernandez et al. (2022) highlight the fragility of classical theorems to this assumption.

To evaluate the magnitude of this inefficiency, we contrast such a centralized system with two types of decentralized systems:

- 1. A decentralized system where firms can interview at any time and make binding offers.
- 2. A decentralized system where firms can interview at any time and make public, non-binding offers.

The first is representative of the status quo in many markets, including the market for investment banking analysts, corporate law associates, and university professors. The second is a hybrid regime of sorts. Firms are free to interview at any time and make public offers, but the offers are non-binding. Effectively, workers can hold onto offers and need not make a decision until the very end of the hiring cycle. Such a system does exist, for example, in the college athletic scholarship market: high school athletes are permitted to publicly receive scholarship offers at any point after the start of their junior year. However, they cannot officially accept them until the December of their senior year.

In the decentralized regime where firms can interview at any time and make binding offers (e.g. exploding offers), we show that all firms are *ex-ante* no worse off in equilibrium than in the centralized setting. However, ex-post, firms can be worse off. In the decentralized regime where offers are public and non-binding, worker welfare is maximized in the equilibrium where firms interview and match sequentially according to their ranking. Total firm surplus is also higher in such a setting than in the equilibrium of the centralized setting. Sequential matching removes the uncertainty about which candidates will receive offers from higher-ranked competitors *before* lower-ranked firms have to make their interview decisions. The asynchronicity of offers removes firms' incentives to skip over higher-quality applicants and leads to more efficient interview decisions and final matches.

1.1 Related Literature

There is an extensive literature on two-sided matching in labor markets, particularly on the NRMP (Roth (1984); Roth (2008); Roth and Sotomayor (1992)). In much of the theoretical analysis of such models, there is complete information about preferences and no uncertainty regarding match quality (See Kojima et al. (2020) for a comprehensive survey of this literature).

Illustrative examples of the effect of incomplete information on traditional notions of stability and other classical matching properties are provided by Roth (1989) and Fernandez et al. (2022). Liu et al. (2014) and Liu (2020) also focus on a two-sided matching market with incomplete information, developing a notion of stability to accommodate such an environment. In their setting, agents draw inferences about the value of matching with others by observing their cooperative deviations. Because of workers' common preferences, this channel does not play a role in our setting. Instead, we focus on firms' incentives to gather information through interviews *before* matching.

There is an emerging body of literature on matching with search. Chade and Smith (2006) and Ali and Shorrer (2021) analyze simultaneous search problems where a student applies to colleges but is unsure whether a particular college will admit them. Our model can be viewed as a generalization of the former in two ways: firms in our model can hire multiple workers, firms must compete with other firms for the workers, and the probability of a worker being hired is *endogenous*. These features also distinguish our paper from Ali and Shorrer (2021).

Within the search literature, our model is closest to Chade et al. (2014), who (like us) consider a model where firms compete for multiple workers of unknown value, observe a costly signal about the value of some (but not all) workers, and hire the workers with the highest signals. The key differences between our papers are the presence of private information on one side of the market, the commonality of workers' values to different firms, the nature of firms' signals, and the side of the market that decides which firms will be informed about which workers. Specifically, in their model, each worker's value is *private information*, their values to different firms are *identical*, and *workers* decide which firms to reveal *noisy signals* of that value to. In our model, there is a *public noisy signal* (application rank) of each worker's value, their values to different firms are *conditionally independent* of one another, and *firms* decide which workers to acquire *perfect information* about.

Immorlica et al. (2020) also examines a college admissions setting, but students can first acquire information about a college before applying. They develop a notion of regret-free stability, which incorporates student information acquisition decisions. In their model, agents acquiring information are part of a continuum, and so each agent's decision to gather information does not influence another agent's incentives to acquire information. This is not the case in our model: a given firm's interview decision imposes a direct externality on other firms.

A nascent literature on two-sided matching with interviews has received growing interest over the last few years. Lee and Schwarz (2017) investigates the welfare consequences of interviewing in centralized mechanisms but focuses on how interview decisions affect total matches. They find that in balanced markets, maximizing total matches is achieved when firm interview sets have perfect overlap. In our model, the market is unbalanced, and no firm must worry about not filling its capacity. Instead, each firm only cares about the match value generated less the cost of interviews. Consequently, perfect overlap in our setting is suboptimal. Echenique et al. (2022) and Manjunath and Morrill (2023) also examine a two-sided matching setting where firms want to maximize match value less interview costs. However, they assume interview assignments are determined via a many-to-many deferred acceptance algorithm, which takes as inputs the ex-ante rank order list. The interviews do not provide any added information. Rather, whom a firm selects to interview restricts which agents it can list in the rank order list it submits to the clearinghouse. We do not use an exogenous interview assignment protocol. Interview assignments are determined in equilibrium. It is not the case that the outcome of a manyto-many deferred-acceptance algorithm on interview preferences aligns with the outcome in a game where firms select whom to interview.

Along these lines, there are two other works, Kadam (2015) and Erlanson and Gottardi (2023), which look at centralized matching environments where firms are free to select whom to interview.² These two papers investigate different questions. The former assumes a specific functional form on the interview technology and analyzes the effect of interview capacity constraints. He shows that relaxing interview capacities can reduce welfare due to over-interviewing. The latter looks at a two-firm environment where interviews are informative for workers and firms, and the interviewing technology for firms is equivalent to our "interviewing for bad news" in Example 1. Our paper differs because we use a general interview technology and focus on equilibrium outcomes *across* different matching protocols. In addition, in our centralized setting, equilibria are inefficient even when worker preferences are common and fixed due to the externality higher-ranked firms place on lower-ranked ones.

Lastly, Ferdowsian et al. (2022) presents a model similar to our decentralized setting except there are no exploding offers, workers can hold on to offers for as long as they like, and if a worker accepts an offer, they cannot renege. Notably, while private information and uncertainty exist in their model, there is no information acquisition stage.

2 Model

There is a finite set of F firms, indexed by $\{1, \ldots, F\}$, and a unit measure of workers. Each worker can work at one firm; each firm can hire at most $\Delta \in (0,1)$ workers. Workers have common preferences over firms: when matched with firm f, she receives a payoff of z_f , where $z_1 > z_2 > \cdots > z_F > 0$. Each worker prefers working for firm F (and thus any other firm) to remaining unmatched: If a worker does not match with any firm, she receives a payoff of $u \le 0$.

Workers are identified by their application score $a \in [0,1]$, which is common knowledge and distributed according to a continuous distribution $W(\cdot)$. Without loss of generality, we assume $W(\cdot)$ is uniform, and so the application score is simply the application rank.³

Hiring a given worker yields different payoffs for different firms, and these *match values* are not known *ex-ante* by either the workers or firms. Instead, conditional on the worker's application rank, they are i.i.d. with distribution $G(\cdot|a)$, which has support $[0,\infty)$ and a density $g(\cdot|a)$ that is continuous almost everywhere in a. We assume workers with higher application ranks are more likely to yield higher match values in the monotone likelihood ratio sense.

Assumption 2.1. The family of distributions $\{G(\cdot|a)\}_{a\in[0,1]}$ satisfies the monotone likelihood ratio property (MLRP):

$$a' > a \text{ and } s' > s \implies \frac{g(s'|a')}{g(s|a')} \ge \frac{g(s'|a)}{g(s|a)}.$$

²Our understanding of Erlanson and Gottardi (2023) is based on personal communication and viewing of a presentation in SAET 2023, where our paper was presented concurrently.

³Since the application score distribution is continuous, we can always index applications by their quantile, or percentile rank, which must be uniformly distributed on the unit interval.

Firms must interview workers before making offers to them. These interviews have constant marginal cost: interviewing a measure μ of workers costs $c \cdot \mu$. When a firm interviews a worker, it learns the value of matching with her.

Each firm's payoff is separable in the values of the workers that it matches with. When firm f interviews a mass μ of workers whose match values are described by the measure Φ , and hires all of those workers with match values above s_f , its payoff is $\int_{s_f}^{\infty} v d\Phi(v) - c \cdot \mu$.

Assumption 2.2.
$$F \cdot \Delta < 1$$
 and $\int_0^\infty s dG(s|0) > c$.

In other words, there are more workers than slots, and the lowest-ranked worker is worth interviewing.

2.1 Matching Regimes

We consider three types of matching mechanisms: *centralized*, *decentralized with binding offers*, and *decentralized with nonbinding offers*.

CENTRALIZED MATCHING. First, firms simultaneously choose which workers $I_f \subseteq [0,1]$ to interview.⁴ Then, both sides submit preferences to a centralized clearinghouse, which runs a worker-proposing Gale-Shapley algorithm. Since workers have common preferences, this algorithm is strategy-proof for both sides, and the resulting matching is the unique, stable outcome. This mechanism is equivalent to the one used in the NRMP. Furthermore, because of our independence assumption, it is also equivalent to a mechanism in which firms interview simultaneously, offers are private, and offers can be held until a common, fixed deadline.

We assume a firm can only list interviewed workers in its submitted preferences. This is what occurs in practice. We can motivate it by incorporating a small probability q that a candidate is not a good fit, and conditional on a candidate not being a good fit, a firm would never want to hire them.

DECENTRALIZED MATCHING. In a decentralized matching mechanism, there are $T \ge F$ time periods, indexed by $t \in \{1, ..., T\}$. In each period t, each firm f that has not already interviewed decides whether to interview in that period and, if so, chooses a set of candidates $I_f^t \subseteq [0, 1]$ to interview and a hiring threshold s_f^t . Then, it publicly makes employment offers to each worker it has interviewed whose match value is above s_f^t .

⁴Since $W(\cdot)$ and $G(\cdot|a)$ are continuous, it is without loss to require the firms to either interview all available applicants of rank a or none of them — and hence to represent interview decisions as a set rather than a measure.

⁵No extra information is available about candidates at later periods. The reason for this assumption is to identify the inefficiencies that arise solely due to costly interviews rather than unraveling.

We consider two varieties of decentralized matching mechanisms:

- i. With Binding Offers Firms make offers that expire at the end of the period. At the end of each period t, each worker strategically decides whether to match with one of the firms that made her an employment offer during period t or reject each of those offers. Hence, when firms choose an interview set I_f^t , they only interview those candidates with application ranks $a \in I_f^t$ who have not accepted an offer in a previous period t' < t.
- ii. With Nonbinding Offers Firms can only make non-binding offers. Equivalently, all offers expire only after the last period T, and so at the end of period T, each worker matches with the highest-ranked firm that has made her an offer. Since offers are public, when firms choose a time t and an interview set I_f^t , they can base their interview decisions on a worker's application rank and current offer set.

3 ANALYSIS

3.1 Centralized Regime

In the centralized setting, firms interview candidates and then submit preferences to a clearinghouse, which runs the standard deferred-acceptance algorithm to determine the matching.

Proposition 1. Fixing the interview strategies I_f for each firm f, deferred acceptance is strategy-proof and returns the unique stable matching.

The only strategic decision facing firms is whom to interview. Furthermore, because the workers have common preferences over firms, the worker-proposing deferred acceptance algorithm produces the same outcome as firm serial dictatorship. Hence, conditional on interviews $\{I_f\}_{f=1}^F$, the payoff to firm f is given by the following recursive rule:

$$\pi_f(\{I_h\}_{h=1}^f) \equiv \int_{I_f} \underbrace{\int_{s_f}^{\infty} sg(s|a)ds}_{Match\ Value\ from\ rank\ a\ hired\ above\ threshold\ s_f} \cdot \underbrace{\left(\prod_{h < f, a \in I_h} G(s_h|a)\right) - c\ da}_{h < f, a \in I_h} - c\ da,$$
 where $s_f = \min \left\{ s | \int_{I_f} (1 - G(s|a)) \left(\prod_{h < f, a \in I_h} G(s_h|a)\right) da \leq \Delta \right\}.$

 $^{^6}$ We avoid allowing workers to accept offers before period T because doing so would not change our analysis; indeed, it would make waiting until period T to accept offers a weakly dominant strategy for the workers.

⁷Clearly, s_f depends on the profile of interview sets $\{I_h\}_{h=1}^f$ for higher-ranked firms; we suppress the argument for notational convenience.

Definition 3.1. A (pure strategy) Nash equilibrium of a centralized matching mechanism is a strategy profile $\{I_f^*\}_{f=1}^F$ such that:⁸

$$I_f^* \in \arg\max_{I_f \subseteq [0,1]} \pi_f(\{I_h\}_{h=1}^f)$$
 for all f

The recursively defined thresholds s_f^* are the firms' optimal thresholds for hiring, given their interview set and the interview sets of higher-ranked firms: intuitively, once they have learned the match values of the workers they interviewed, they should hire those with the highest values until they are out of open positions.⁹

Given any equilibrium, one can construct another by adding a measure 0 set of workers. Such differences in equilibria are not substantive. We will consider two equilibria equivalent if the interview sets only differ on a measure 0 set of workers.

Proposition 2. There is a unique pure strategy equilibrium in the centralized matching setting.

Observe that Firm 1's decision is independent of all other firms' decisions since it is the most preferred firm. Conditional on interviewing a set of workers of size μ , it is optimal for Firm 1 to interview $I_1 = [k, 1]$, where $1 - k = \mu$. In other words, Firm 1 interviews greedily (see Lemma 3 in the appendix for a formal proof). Of the set [k, 1], Firm 1 will match with the top Δ workers based on post-interview match quality. Thus, Firm 1's optimization problem is:

$$a_1 \in \arg\max_{k} \int_{k}^{1} \left[\int_{\bar{s}_1}^{\infty} v dG(v|a) \right] da - c(1-k)$$

Where
$$\bar{s}_1(k) = \min \left\{ s | \int_k^1 (1 - G(s|a)) da \le \Delta \right\}$$

Conditional on Firm 1 interviewing $I_1^* = [a_1, 1]$, Firm 2 must consider that some of the workers in I_1^* will be taken by Firm 1 in the matching stage. Thus, the effective match value distribution for workers $a \in I_1^*$ is $G(\bar{s}_1|a)G(\cdot|a)$. Firm 2's optimization problem is:

$$I_2^* \in \arg\max_{I_2 \subset [0,1]} \int_A \psi(a) \left(\int_{\bar{s}_2(I_2)}^{\infty} G(a,s) ds \right) da - c \left(\int_{I_2} da \right)$$

$$\text{Where } \psi(a) = \begin{cases} 1 & \text{if } a \notin I_1^* \\ G(a,\bar{s}_1) & \text{if } a \in I_1^* \end{cases}, \ \ \text{and } \bar{s}_2(I_2) = \min \left\{ s | \int_{I_2} \left(1 - G(a,s) \right) \psi(a) da \leq \Delta \right\}$$

Observe that $\psi()$ is not included in the cost of interviewing because Firm 2 does not know

⁸It is without loss to restrict firms to pure strategies. Any mixed strategy $\sigma \in \Delta 2^{[0,1]}$ is payoff equivalent to the pure strategy $\mathbb{E}[\sigma]$.

⁹The cutoff score s_f would also arise in a setting where firms simultaneously selected a set of workers to interview *and* a cutoff score above which interviewed workers would receive an offer.

¹⁰Here, "distribution" should not to be interpreted in the probability sense. After all, it does not integrate to 1.

which workers in I_1^* will be matched to Firm 1. Therefore, it cannot discriminate between workers of a given rank $a \in I_1^*$.

In the proof or Proposition 2, we show that the above optimization problem has a unique solution. Intuitively, once the strategies of firms $\{1, \ldots, j\}$ are set, the optimal strategy for firm j+1 can be uniquely pinned down.

We next characterize the firms' equilibrium interview sets. Unlike the top-ranked firm, lower-ranked firms do not necessarily interview greedily: Even though the applicants interviewed by one or more higher-ranked firms have more favorable match value distributions than those that are not, they are less likely to be available. However, when the match value distribution satisfies a regularity condition, we can say that interviewing greedily is optimal *holding* constant the set of higher-ranked firms that also interview a candidate.

Recall that first-order stochastic dominance (and hence the MLRP) ensures that, fixing a hiring cutoff x, the value $\int_x^\infty sg(s|a)ds$ created by interviewing a rank-a applicant is increasing in a.¹¹ We say that G has increasing k-adjusted yields if this property still holds when we adjust the value created by an interview for the probability that the applicant will receive an offer from one of k higher-ranked firms: that is, $\int_x^\infty sg(s|a)ds\prod_{f=1}^k G(s_f|a)$ is increasing in a for each hiring threshold x and each tuple of hiring thresholds $\{s_f\}_{f=1}^k \in \mathbb{R}_{++}^k$. If G has increasing k-adjusted yields for all k, we say it has increasing adjusted yields.

We can interpret k as the *largest* number of firms whose decisions to interview a pair of applicants cannot reverse their relative attractiveness to lower-ranked firms; that is, if the property holds for k, it holds for each k' < k. Formally:

Lemma 1. If G has increasing k-adjusted yields, then it has increasing k'-adjusted yields for each k' < k.

Every distribution that satisfies our MLRP assumption has increasing 0-adjusted yields, but for many distributions, the property holds for higher k. Example 1 illustrates.

Example 1 (Exponential and "Bad News" Distributions). Several canonical distributions have increasing adjusted yields. Here, we introduce two examples of such distributions.

Exponentially-distributed match values. Suppose that given an applicant's rank, match values are exponentially distributed and the distribution has longer tails for higher-ranked applicants. Then we can write the match value distribution as $G_{\lambda}(s|a) = 1 - e^{s\lambda(a)}$, where $\lambda: [0,1] \to \mathbb{R}_+$ is a decreasing function.

Interviewing for "bad news". Suppose interviewing an applicant either reveals that they are unacceptable (s = 0) with probability β or that their pre-interview rank was accurate

 $^{^{11}}$ Note that this is not the marginal value of adding a to an interview set: Expanding the firm's interview set requires it to adjust its hiring threshold to hire the same number of workers, whereas this expression holds the hiring threshold fixed.

(s = a). We can write the match value distribution as $G_{\beta}(s|a) = \begin{cases} \beta, & s < a \\ 1, & s \ge a. \end{cases}$ Such a technology appears in Chade and Smith (2006) and Erlanson and Gottardi (2023).

Lemma 2. For any $\beta \in (0,1)$ G_{β} has increasing-adjusted yields. For any decreasing λ : $[0,1] \to \mathbb{R}_+$, G_{λ} has k-adjusted yields for some $k \ge 1$.

Proposition 3 shows that when the match value distribution exhibits increasing k-adjusted yields, the highest-ranked k+1 firms will *overlap* greedily with the interview decisions of higher-ranked firms.

Proposition 3. Suppose that G has increasing k-adjusted yields. Then every Nash equilibrium is equivalent to one $\{I_f^*\}_{f=1}^F$ in which

- i. The highest-ranked firm interviews greedily: $I_1^* = [a_1, 1]$ for some $a_1 \in [0, 1]$.
- ii. Each firm $f \in \{2, ..., k\}$
 - (a) Overlaps greedily with each set of firms above them: For $S \subseteq \{1, ..., f-1\}$, $I_f^* \cap \bigcap_{j \in S} I_i^* = [a_f^S, 1] \cap \bigcap_{j \in S} I_j$ for some $a_f^S \in [0, 1]$.
 - (b) Interviews greedily below the firm above them: There exists a decreasing sequence $\{a_f\}_{f=1}^F$ such that $I_f^* \cap [0, a_{f-1}) = [a_f, a_{f-1})$.
 - (c) Lets lowest-ranked applicants interviewed by firm above them fall through the cracks: For each $S \subseteq \{1, \ldots, f-2\}, a_{f-1}^S < a_f^{S \cup \{f-1\}}$.

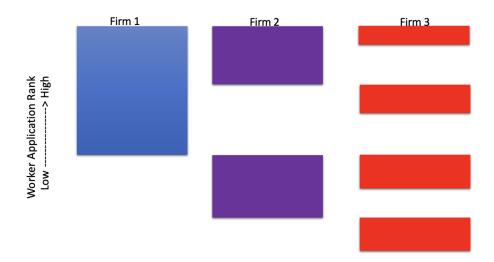
Our increasing k-adjusted yields property ensures that optimization over each overlap set is solved by a simple greedy interviewing scheme. Conceptually, when G has increasing k-adjusted yields, conditional on a given set of workers being interviewed by the same set of firms, a lower-ranked firm will view higher-ranked workers in that set as more valuable to interview.

Example 2. Consider three firms. Equilibrium interview strategies have the following form:

- Firm 1: $[a_1, 1]$
- Firm 2: $[a_2^{(1)}, a_1] \cup [a_2^{(2)}, 1]$ for $a_2^{(1)} \le a_1 \le a_2^{(2)} \le 1$.
- Firm 3: $[a_3^{(1)}, a_2^{(1)}] \cup [a_3^{(2)}, a_1] \cup [a_3^{(3)}, a_2^{(2)}] \cup [a_3^{(4)}, 1]$ for $a_3^{(1)} \le a_2^{(1)} \le a_3^{(2)} \le a_1 \le a_3^{(3)} \le a_2^{(2)} \le a_3^{(4)} \le 1$.

¹²At a technical level, Proposition 3 generalizes and extends Chade and Smith (2006) to the case of competition between decision-makers.

Pictorially, the interviewing schemes look like this:



From the graphic, one can see the coordination issues that arise in the interview stage. While Firm 2 knows whom Firm 1 will interview, it does not know which workers will be matched with Firm 1 during the deferred acceptance stage. Hence, interviewing such workers carries a risk! This prompts Firm 2 to skip over a portion of ex-ante highly ranked workers. Notice, though, that applicants of low rank do have interviewing opportunities.

Each firm's interview decision imposes an externality on the firms ranked below it. This externality is the root cause of the inefficiency of the equilibrium interviewing scheme. To see why, it suffices to consider a two-firm example. Given the equilibrium interviewing scheme for Firm 1 and Firm 2, for ε sufficiently small, the bottom ε of the candidates interviewed by Firm 1 would provide a higher marginal return for Firm 2 than Firm 1. So, to increase aggregate match values, one should prohibit Firm 1 from interviewing those ε -measure of candidates. Formally, suppose a planner could select whom each firm interviews with the objective of maximizing total match value less the cost of interviewing:

$$\left\{I_f^{sp}\right\}_{f=1}^F = \arg\max_{\left\{I_f\right\}_{f=1}^F \subseteq [0,1]} \sum_{f=1}^F \int_{I_f} \int_{s_f}^\infty sg(s|a) ds \underbrace{\left(\prod_{h < f, a \in I_h} G(s_h|a)\right)}_{\text{href}} - c \ da$$

$$\text{where } s_f = \min\left\{s|\int_{I_f} (1 - G(s|a)) \left(\prod_{h < f, a \in I_h} G(s_h|a)\right) da \le \Delta\right\}$$

Proposition 4. The planner-optimal choice does not necessarily coincide with the equilibrium interviewing schemes: $\left\{I_f^{sp}\right\}_{f=1}^F \neq \left\{I_f^*\right\}_{f=1}^F$.

3.2 Decentralized Matching with Binding Offers

The second regime is the "status quo" environment in many markets, including the academic job market. For clarity, we reiterate the description of the game:

- i. At the beginning of each period t, firms that have not been matched simultaneously decide whether to interview. Firms observe which firms went on the market in previous periods, the pool of workers yet to accept an offer, and the offers that those workers hold. Once a firm decides to interview in period t, it observes which other firms are on the market and then decides whom to interview and make offers to. Offers are exploding and expire at the end of the period. 13
- ii. At each t, workers who receive offers must decide whether to accept, hold, or reject any offers. At time t, a worker has the following information:
 - The current offers she holds.
 - The set of firms on the market in t, and the set of firms yet to be on the market.
 - The pool of unmatched workers at the beginning of time t.

Our analysis considers the extreme case where workers are sufficiently risk averse in the sense that the payoff from being unmatched is sufficiently low. Risk aversion is necessary to make exploding offers viable in equilibrium: there is no point in front-running (i.e. interviewing before another firm) to make an exploding offer if said offer will never be accepted.

Assumption 3.2.
$$u<-\frac{1-G(\underline{s}|1)}{G(\underline{s}|1)}\cdot z_1$$
 , where $\underline{s}=\min\left\{\int_0^{1-\Delta\cdot(F-1)}1-G(\underline{s}|a)da\leq\Delta\right\}$

Under this assumption, workers will accept the first offer they receive.¹⁴ In such a setting, firms will only make exploding offers: binding offers that expire immediately after the period they are made ends.

Proposition 5. A subgame perfect equilibrium exists. In equilibrium, each firm's ex-ante expected payoff is weakly greater than its expected payoff in the centralized regime. However, firms can be strictly worse-off ex-post.

In equilibrium, firm f randomizes over the times $t \in \{1, ..., f\}$ at which to interview. The probability of entering at time t is based on the observed history up until that time. While the ex-ante expected payoff for each firm is weakly higher than in the centralized setting, there may be an outcome realization such that a firm is left worse off. Furthermore, the workers the firms interview (outside of the top-ranked firm), as identified by their application rank, are *different* than in the centralized setting.

The externality imposed by higher-ranked firms on lower-ranked firms when multiple firms interview in the same period also potentially reduces worker welfare. Firms are not guaranteed

¹³Given Assumption 3.2, it is without loss to consider such offers.

¹⁴We discuss this assumption in detail in Section 4.1.

to fill their slots in the equilibrium of the centralized setting nor in the equilibrium of the decentralized setting with binding offers.

Example 3. Suppose there are three firms. Let I_1^*, I_2^* , and I_3^* be each firm's equilibrium interviewing strategy in the centralized setting. In the equilibrium of the decentralized setting with binding offers, Firm 1 interviews $I_1^* = [a_1, 1]$ in period 1. Firm 2 randomizes between interviewing I_2^* in period 1 and interviewing workers I_2^* of the available workers in period 2, where I_2^* is determined based on that pool. Firm 3 randomizes between interviewing I_3^* (not necessarily equal to I_3^*) in period 1 and interviewing workers I_3^* of the available workers in period 3, where I_3^* is determined based on that pool.

In equilibrium, the ex-ante expected payoff of each firm is weakly greater than in the centralized setting. However, one can see that there is a realization in which Firm 2 is worse off ex-post than in the centralized setting: when Firm 1 and Firm 3 interview and match in the first period, and Firm 2 interviews and matches in the second period.

3.3 Decentralized Matching with Nonbinding Offers

In this setting, offers are non-binding, public, and cannot be pulled. Workers can hold onto a set of offers until period T before deciding.

Observation 3.3. Since offers are public, when firms choose an interview set I_f^t , they only interview those candidates with application ranks $a \in I_f^t$ who have not received an offer in a previous period t' < t from a higher-ranked firm f' < f.

Observation 3.4. In any equilibrium, Firm i interviews after Firm i' if and only if $i \ge i'$. Moreover, firm i strictly prefers interviewing after firm i' if and only if i' < i.

Since the highest-ranked firm is indifferent about the timing of its interviews, there are multiple equilibria. We focus on the sequential-hiring equilibrium, where each firm f interviews and makes offers in period f.

Proposition 6. *In the sequential-hiring equilibrium:*

- i. Firms interview greedily among the set of workers who have not received an offer: For each firm f, $I_f = [a_f, 1]$ for some $a_f \in [0, 1]$.
- ii. Firms interview monotonically: $a_f \ge a_{f'}$ whenever f' > f.
- iii. There is maximal employment: all firms fill their slots.

¹⁵Conditional on all higher-ranked firms already having made offers, a given firm is indifferent about when to interview.

Firm 1 always succeeds in hiring every worker to whom it offers a job, so its payoff is independent of the other firms' strategies. Likewise, each firm f > 1 always succeeds in hiring every worker that does not receive an offer from a higher-ranked firm, so its payoff is independent of the strategies of lower-ranked firms f' > f. When firms interview in rank order, each chooses its optimal interview set over the set of workers without a dominating offer. By Lemma 3, each firm's optimal interviewing strategy is greedy. No high-ranked applicants "fall through the cracks". Moreover, the sequential-hiring equilibrium guarantees that a total of $\Delta \cdot F$ workers are hired. Thus, total equilibrium worker payoff is weakly greater in the sequential-hiring equilibrium of the decentralized regime with non-binding offers than in the centralized regime and the decentralized regime with binding offers. 16

In the sequential-hiring equilibrium, where firms interview in rank order, firms avoid the cost of interviewing workers who receive a dominating offer. The asynchronicity of the offers reduces the interview congestion and coordination frictions in the centralized regime.

Proposition 7. The sequential hiring equilibrium of the decentralized regime with nonbinding offers is more efficient than the equilibrium in the centralized regime.

4 DISCUSSION

4.1 Assumptions

Common Ranking of Firms Workers have a common ranking of firms unaffected by the interviews. This is partially for tractability but also to isolate the inefficiencies created by strategic interviewing *even when* firms know where they stand. To some degree, this feature resembles many of the motivating labor markets, such as medical residency and academia. One possible way to generalize our model while maintaining some tractability is to include a tiered ranking of firms. Formally, consider tiers T_1, \ldots, T_k , where $T_i \subset \{1, \ldots, n\}$ for all i and $T_i \cap T_j = \emptyset$ for all $i \neq j$. While each worker strictly prefers firms in T_i to firms in T_j for i < j, worker preferences over firms *within* a given tier are dependent on the interviews. As the results in Erlanson and Gottardi (2023) suggest, any centralization of interviews within a given tier would lead to potential multiplicity of equilibria, as well as inefficiency. In such a world, our results indicate that total surplus in a setting where each tier of firms interviews in sequence would be higher than in a centralized setting. However, firms within the same tier would have an incentive to front-run each other.

Independent Match Values In our model, the match value generated by a particular worker for a firm is independent across firms. With correlated match values, an adverse selection issue arises. If a candidate is available to a low-ranked firm, either that candidate was not interviewed

¹⁶How one would determine which setting is "better" for the workers depends on how a planner weighs each worker.

by better firms or was unsuccessful in their interviews, meaning they have low match quality. Thus, in the centralized regime, lower-ranked firms become more averse to interviewing candidates whom top firms are interviewing. This adverse selection issue has important implications in the decentralized regime as well. Consider the sequential interviewing procedure from Section 3.3 and suppose match values are perfectly correlated. The sequential mechanism is no longer an equilibrium. Why? The top-ranked firm has no incentive to interview and screen first. Instead, it would prefer to wait until the end to see whom the other firms extended offers to. This free-riding issue from public offers leads to the unique equilibrium being all firms interviewing and matching in the final period, mirroring the result in Ely and Siegel (2013).

Risk Aversion In the decentralized regime with binding offers, we assumed workers were sufficiently risk averse. Such an environment generates the most concern amongst labor market participants and organizers, as it makes exploding offers an effective tactic. In our model, risk aversion is captured by the relative size of the workers' payoffs from being matched to different firms and the payoff associated with being unmatched. These payoffs determine workers' incentives to accept or reject early offers. To understand the crucial role of risk aversion, consider the opposite extreme, where any worker will reject an offer from a firm if they know they will be interviewed by a better firm later. Then, it can be shown that in any equilibrium of the decentralized regime with binding offers, no firm will front-run a higher-ranked one. Hence, such a regime is equivalent to the decentralized setting with non-binding offers.

4.2 Interpretation of Results

To understand the welfare consequences of any matching mechanism and the trade-offs between matching mechanisms, one must consider their effect on interviewing decisions. A common criticism of a decentralized environment with binding offers is that exploding offers prohibit workers from interviewing at more preferable firms later in the hiring cycle. Colloquially, the usual story is "Person A received an exploding offer from Firm X, and could not interview at Firm Y. We must centralize the market so that Person A can interview at X and Y, and compare offers." But it is not necessarily the case that in the centralized system, Person A would have received an interview from any of those firms! Interviewing decisions are an equilibrium object sensitive to the matching mechanism.

In particular, the nature of a centralized matching process to make the timing of decisions simultaneous generates coordination issues at the interview stage. Each firm must incorporate the externality imposed by the interview decisions of its competitors when deciding on its interview list. This is not to say that centralized mechanisms should be done away with. For instance, recall our assumption of a continuum of workers in our analysis. Such an assumption is substantive because it implies no uncertainty about yield. By the exact law of large numbers, firms can use a cutoff score and ensure they match with exactly Δ applicants. In a discrete setting, firms must be concerned about whether more than Δ candidates accept or less than Δ

accept. An important property of centralized matching mechanisms is they guarantee that a firm will never be matched with more than its capacity.

Nevertheless, one benefit of a decentralized system is that the interview coordination issues can be mitigated due to matches occurring over time. Firm and worker exit can lead to more effective interviewing for the remaining firms. As an illustrative example, early-action programs in US undergraduate college admissions function to reduce application congestion in the regular admissions cycle. Now, in a completely decentralized environment where firms can make binding offers (e.g. exploding offers), the incentive of firms to front-run can reduce some of these benefits. Hence, the optimal system is a hybrid of these two systems. On the one hand, we want to allow firms to interview and make public offers at different times, but we also want to prohibit front-running. If this is achievable, we then need firms to interview in rank order by nudging top firms to interview first.

How would such a system be implemented in practice? Theoretically, one could institute a rule that offers are non-binding until a given date (e.g. ban exploding offers). This exists in the college athletic scholarship market. Unlike this market, though, most labor markets lack a governing body that can enforce such a rule. Thus, an alternative is to encourage a culture of reneging: explain to candidates that before a common date, they are free to change their mind about an accepted offer if a more preferred offer comes along.

A PROOFS

Proof of Proposition 1: Deferred Acceptance is guaranteed to be strategy-proof for the workers. Since workers have common preference over firms, at round f of the algorithm, all workers point to firm f. Thus, firm f holds matches with the top Δ proposals of the remaining group. In other words, at round f, firm f is a dictator and chooses the best Δ candidates amongst those available. The proposition follows.

Proof of Lemma 1: Suppose G has increasing k-adjusted yields. Then for each a' > a and each x and $\{s_f\}_{f=1}^k \in \mathbb{R}_{++}^k$, we have

$$\int_{x}^{\infty} sdG(s|a') \prod_{f=1}^{k} G(s_f|a') \ge \int_{x}^{\infty} sdG(s|a) \prod_{f=1}^{k} G(s_f|a). \tag{1}$$

Since MLRP implies first-order stochastic dominance, $G(s|a') \leq G(s|a)$ for all $s \geq 0$. Since $0 \in \text{supp}\,G(\cdot|a)$, and $s_f > 0$ for each $f \in \{1,\ldots,k\}$, we have $0 < \prod_{f=k'+1}^k G(s_f|a') \leq \prod_{f=k'+1}^k G(s_f|a)$. The statement follows by dividing the left-hand side of (1) by $\prod_{h \in \{1,\ldots,f-1\} \setminus S} G(s_h|a')$ and the right-hand side by $\prod_{h \in \{1,\ldots,f-1\} \setminus S} G(s_h|a)$.

Proof of Lemma 2: For an exponential distribution $G(x|\lambda) = 1 - e^{-\lambda x}$, consider:

$$G(s|\lambda) \cdot (1 - G(x|\lambda)) \int_{x}^{\infty} t dG(t|\lambda, t \ge x) = (1 - e^{-s\lambda}) \cdot e^{-\lambda x} \cdot (x + \frac{1}{\lambda})$$

Differentiating with respect to λ yields:

$$\frac{e^{-\lambda(s+x)}}{\lambda^2} \cdot \left[\lambda(\lambda x + 1)(s+x) - e^{\lambda s}(\lambda^2 x^2 + \lambda x + 1) + 1 \right]$$

This quantity is less than or equal to 0 if and only if:

$$\lambda(\lambda x + 1)(s + x) - e^{\lambda s}(\lambda^2 x^2 + \lambda x + 1) + 1 \le 0$$

$$\iff (\lambda^2 x^2 + \lambda x + 1)(1 - e^{-\lambda s}) + \lambda s(\lambda x + 1) \le 0 \iff 1 - e^{-\lambda s} > 1 + \lambda s$$

Thus, $G(s|\lambda) \cdot (1 - G(x|\lambda)) \int_x^\infty t dG(t|\lambda, t \ge x)$ is decreasing in λ for all x and s.

For the "Interviewing for bad news" environment, $G(x|\lambda) = \begin{cases} 1 & \text{if } x \geq \lambda \\ \beta & \text{if } x < \lambda \end{cases}$. $\Longrightarrow \Big[\Pi_{i=1}^k G(s_i|\lambda) \Big] (1 - G(x|\lambda)) \int_x^\infty t dG(t|\lambda, t \geq x) = \begin{cases} \beta \cdot (1 - \beta)\lambda & \text{if } x \leq \lambda, s_i \leq \lambda \\ 0 & \text{otherwise} \end{cases}$

This is clearly increasing in λ .

Lemma 3 (Greedy Interviewing is Optimal). Let $\underline{a}, \overline{a} \in [0,1]$ and suppose that $\phi : [0,1] \to [0,1]$ and $\psi : [0,1] \to [0,1]$ are such that either (i) $\psi(a) = 1$ on $[\underline{a}, \overline{a}]$, or (ii) $\phi(a) = 1$ and for each x, $\int_x^{\infty} sg(s|a)\psi(a)ds$ is nondecreasing in a on $[\underline{a}, \overline{a}]$. Then for any $I_f \subseteq [0,1]$ and $s_f \ge 0$, there exists $a_f \in [\underline{a}, \overline{a}]$ and $s_f' \ge 0$ such that

$$\int_{(I_{f}\setminus[\underline{a},\overline{a}])\cup[a_{f},\overline{a}]} \left(\int_{s_{f}^{\prime}}^{\infty} sg(s|a)\psi(a)ds - c \right) \phi(a)da \ge \int_{I_{f}} \left(\int_{s_{f}}^{\infty} sg(s|a)\psi(a)ds - c \right) \phi(a)da,$$

$$and \int_{(I_{f}\setminus[\underline{a},\overline{a}])\cup[a_{f},\overline{a}]} (1 - G(s_{f}^{\prime}|a))\psi(a)\phi(a)da = \int_{I_{f}} (1 - G(s_{f}|a))\psi(a)\phi(a)da. \tag{2}$$

Proof. Choose a_f so that $\int_{a_f}^{\overline{a}} \phi(a) da = \int_{I_f \cap [\underline{a}, \overline{a}]} \phi(a) da$. Let \overline{S} be a random variable with density $\overline{g}(s)$, and \underline{S} be a random variable with density g(s), where

$$\overline{g}(s) \equiv \frac{\int_{[a_f,\overline{a}]\backslash I_f} g(s|a)\psi(a)\phi(a)da}{\int_{[a_f,\overline{a}]\backslash I_f} \psi(a)\phi(a)da}, \qquad \underline{g}(s) \equiv \frac{\int_{[\underline{a},a_f]\cap I_f} g(s|a)\psi(a)\phi(a)da}{\int_{[\underline{a},a_f]\cap I_f} \psi(a)\phi(a)da}.$$

Step 1: \overline{S} dominates \underline{S} in the likelihood ratio order, and hence in the sense of first-order stochastic dominance. ¹⁷ By MLRP, for all s' > s,

$$g(s'|a')\psi(a')\phi(a')g(s|a)\psi(a)\phi(a)\geq g(s'|a)\psi(a)\phi(a)g(s|a')\psi(a')\phi(a') \text{ for all } a'>a$$

$$\int_{[a_f,\overline{a}]\backslash I_f}g(s'|a)\psi(a)\phi(a)da\int_{[\underline{a},a_f]\cap I_f}g(s|a)\psi(a)\phi(a)da\geq \int_{[\underline{a},a_f]\cap I_f}g(s'|a)\psi(a)\phi(a)da\int_{[a_f,\overline{a}]\backslash I_f}g(s|a)\psi(a)\phi(a)da$$

$$\frac{\int_{[a_f,\overline{a}]\backslash I_f}g(s'|a)\psi(a)\phi(a)da}{\int_{[a_f,\overline{a}]\backslash I_f}\psi(a)\phi(a)da}\frac{\int_{[\underline{a},a_f]\cap I_f}g(s|a)\psi(a)\phi(a)da}{\int_{[\underline{a},a_f]\cap I_f}\psi(a)\phi(a)da}\geq \frac{\int_{[\underline{a},a_f]\cap I_f}g(s'|a)\psi(a)\phi(a)da}{\int_{[\underline{a},a_f]\cap I_f}\psi(a)\phi(a)da}\frac{\int_{[a_f,\overline{a}]\backslash I_f}g(s|a)\psi(a)\phi(a)da}{\int_{[a_f,\overline{a}]\backslash I_f}\psi(a)\phi(a)da}$$

$$\overline{g}(s')g(s)\geq g(s')\overline{g}(s),$$

Step 2: In case (i), for any s, $\int_{[a_f,\overline{a}]\setminus I_f} (1-G(s|a))\psi(a)\phi(a)da \ge \int_{[\underline{a},a_f]\cap I_f} (1-G(s|a))\psi(a)\phi(a)da$. By Step 1, $\overline{S} \ge_{FOSD} \underline{S}$. It follows that

$$\frac{\int_{[a_f,\overline{a}]\backslash I_f}(1-G(s|a))\phi(a)da}{\int_{[a_f,\overline{a}]\backslash I_f}\phi(a)da} \geq \frac{\int_{[\underline{a},a_f]\cap I_f}(1-G(s|a))\phi(a)da}{\int_{[\underline{a},a_f]\cap I_f}\phi(a)da}.$$

Since $\int_{a_f}^{\overline{a}} \phi(a) da = \int_{I_f \cap [\underline{a}, \overline{a}]} \phi(a) da$ by construction, we must have $\int_{[a_f, \overline{a}] \setminus I_f} \phi(a) da = \int_{[\underline{a}, a_f] \cap I_f} \phi(a) da$. Then since in case (i), $\psi(a) = 1$ for all $a \in [\underline{a}, \overline{a}]$, the claim follows.

Step 3: There exists $\tilde{s_f} \geq 0$ such that

$$\int_{[a_f,\overline{a}]\setminus I_f} \int_{\tilde{s_f}}^{\infty} sg(s|a)\psi(a)\phi(a)dsda \ge \int_{[\underline{a},a_f]\cap I_f} \int_{s_f}^{\infty} sg(s|a)\psi(a)\phi(a)dsda,$$
and
$$\int_{[a_f,\overline{a}]\setminus I_f} (1-G(\tilde{s_f}|a))\psi(a)\phi(a)da = \int_{[\underline{a},a_f]\cap I_f} (1-G(s_f|a))\psi(a)\phi(a)da.$$

¹⁷See, e.g., Shaked and Shanthikumar (2007) Theorem 1.C.1.

Choose $\tilde{s_f}$ so that $\int_{[a_f,\overline{a}]\setminus I_f} (1-G(\tilde{s_f}|a))\psi(a)\phi(a)da = \int_{[\underline{a},a_f]\cap I_f} (1-G(s_f|a))\psi(a)\phi(a)da$; by Step 2 and since G is nondecreasing is s, either $\tilde{s_f} \geq s_f$ or (ii) holds.

First consider the case where $\tilde{s_f} \geq s_f$. From Step 1 and Shaked and Shanthikumar (2007) Theorems 1.C.5 and 1.A.15, $[\overline{S}|\overline{S} \geq \tilde{s_f}] \geq_{FOSD} [\underline{S}|\underline{S} \geq \tilde{s_f}] \geq_{FOSD} [\underline{S}|\underline{S} \geq s_f]$. It follows that

$$\frac{\int_{\widetilde{s_f}}^{\infty}\int_{[a_f,\overline{a}]\backslash I_f}sg(s|a)\psi(a)\phi(a)da}{\int_{[a_f,\overline{a}]\backslash I_f}(1-G(\widetilde{s_f}|a))\psi(a)\phi(a)da}=E\left[\overline{S}|\overline{S}\geq\widetilde{s_f}\right]\geq E\left[\underline{S}|\underline{S}\geq s_f\right]=\frac{\int_{s_f}^{\infty}\int_{[\underline{a},a_f]\cap I_f}sg(s|a)\psi(a)\phi(a)da}{\int_{[\underline{a},a_f]\cap I_f}(1-G(s_f|a))\psi(a)\phi(a)da}.$$

The claim then follows by construction of $\tilde{s_f}$.

Alternatively, consider the case where $\tilde{s_f} < s_f$. Then (ii) holds, and we have

$$\int_{s_f}^{\infty} \int_{[a_f,\overline{a}]\setminus I_f} sg(s|a)\psi(a)\phi(a)da \geq \int_{s_f}^{\infty} \int_{[a_f,\overline{a}]\setminus I_f} sg(s|a)\psi(a)\phi(a)da \geq \int_{s_f}^{\infty} \int_{[\underline{a},a_f]\cap I_f} sg(s|a)\psi(a)\phi(a)da,$$

where the first inequality holds since $sg(s|a)\psi(a) \ge 0$ for each s,a.

Step 4: $\int_{(I_f \setminus [a,\overline{a}]) \cup [a_f,\overline{a}]} c\phi(a) da = \int_{I_f} c\phi(a) da$. Follows from construction of a_f .

Step 5: *Statement of Lemma 3.* By Step 3, there exists $\tilde{s_f} \geq 0$ such that

$$\int_{I_{f}\setminus[\underline{a},a_{f}]} \int_{s_{f}}^{\infty} sg(s|a)\psi(a)\phi(a)dsda
+ \int_{[a_{f},\overline{a}]\setminus I_{f}} \int_{s_{f}}^{\infty} sg(s|a)\psi(a)\phi(a)dsda \ge \int_{I_{f}} \int_{s_{f}}^{\infty} sg(s|a)\psi(a)\phi(a)dsda,$$

$$\int_{I_{f}\setminus[\underline{a},a_{f}]} (1-G(s_{f}|a))\psi(a)\phi(a)da = \int_{I_{f}\setminus[\underline{a},a_{f}]} (1-G(s_{f}|a))\psi(a)\phi(a)da \qquad (3)$$

$$\text{and} \begin{array}{l} \int_{I_f \backslash [\underline{a},a_f]} (1-G(s_f|a)) \psi(a) \phi(a) da \\ + \int_{[a_f,\overline{a}] \backslash I_f} (1-G(\tilde{s_f}|a)) \psi(a) \phi(a) da \end{array} = \int_{I_f} (1-G(s_f|a)) \psi(a) \phi(a) da.$$

Then the left-hand side of (3) must be no greater than the value of

$$\max_{x,y} \int_{I_f \setminus [\underline{a},a_f]} \int_x^\infty sg(s|a) \psi(a) \phi(a) ds da + \int_{[a_f,\overline{a}] \setminus I_f} \int_y^\infty sg(s|a) \psi(a) \phi(a) ds da \qquad (4)$$

s.t.
$$\int_{I_f \setminus [\underline{a}, a_f]} (1 - G(x|a)) \psi(a) \phi(a) da \\ + \int_{[a_f, \overline{a}] \setminus I_f} (1 - G(y|a)) \psi(a) \phi(a) da = \int_{I_f} (1 - G(s_f|a)) \psi(a) \phi(a) da.$$
 (5)

It cannot be optimal to set $x \neq y$ in (4): Suppose x < y. Then for small enough $\varepsilon > 0$, we can decrease y to $y - \varepsilon$ and increase x to $x + \delta$ for some small $\delta \in (0, y - \varepsilon - x)$ so that (5) still holds. Then we must have

$$\int_{I_f\setminus [\underline{a},a_f]} \int_x^{x+\delta} g(s|a)\psi(a)\phi(a)dsda = \int_{[a_f,\overline{a}]\setminus I_f} \int_{y-\varepsilon}^y g(s|a)\psi(a)\phi(a)dsda,$$

and the value of (4) changes by

$$-\int_{I_f\setminus [\underline{a},a_f]}\int_x^{x+\boldsymbol{\delta}}sg(s|a)\psi(a)\phi(a)dsda+\int_{[a_f,\overline{a}]\setminus I_f}\int_{y-\varepsilon}^ysg(s|a)\psi(a)\phi(a)dsda,$$

which has the same sign as

$$-\frac{\int_{I_{f}\setminus[\underline{a},a_{f}]}\int_{x}^{x+\delta}sg(s|a)\psi(a)\phi(a)dsda}{\int_{I_{f}\setminus[\underline{a},a_{f}]}\int_{x}^{x+\delta}g(s|a)\psi(a)\phi(a)dsda} + \frac{\int_{[a_{f},\overline{a}]\setminus I_{f}}\int_{y-\varepsilon}^{y}sg(s|a)\psi(a)\phi(a)dsda}{\int_{[a_{f},\overline{a}]\setminus I_{f}}\int_{y-\varepsilon}^{y}g(s|a)\psi(a)\phi(a)dsda} \\ \geq -(x+\delta) + y - \varepsilon > 0.$$

An analogous argument applies when x > y. Then let x^* be such that (x^*, x^*) solves (4), and choose $s'_f = x^*$; the statement of Lemma 3 follows from Step 4.

The intuition for Lemma 3 is simplest when $\phi(a) = \psi(a) = 1$ for all a — the relevant case for firm 1. Given an interview set I, we can construct a greedy interview set $[1 - \mu(I), 1]$ with the same mass, and keep the hiring threshold the same for those workers interviewed in both sets. Then, for the workers that are interviewed in the greedy set but not in I, we choose a new hiring threshold, so that the firm hires the same mass of them as it would have hired from $I \setminus [1 - \mu(I), 1]$ (the workers interviewed in I but not in the greedy set). MLRP then ensures that conditional on being hired, the average match value of workers only interviewed in the greedy set is higher than the average match value of workers only interviewed in the original set I.

Proof of Proposition 2: Each (possibly mixed) interview strategy I_f can be represented as a function $\mu:[0,1]\to[0,1]$. Let $\mathscr Y$ be the set of all such functions. $\mathscr Y$ is compact because it is a complete metric space and totally bounded under the sup-norm. Therefore, a solution exists to the following recursive system:

$$I_f^* \in \arg\max_{I_f \in \mathscr{Y}} \int_0^1 I_f(a) \left[\int_{s_f^*(I_f)}^{\infty} sg(s|a) ds \left(\prod_{h < f} \left(1 - I_h^*(a) (1 - G(s_h^*|a) \right) \right) - c \right] da$$

$$\text{ where } s_f^*(I_f) = \min \left\{ s | \int_0^1 I_f(a) \left[(1 - G(s|a)) \left(\prod_{h < f} \left(1 - I_h^*(a) (1 - G(s_h^*|a)) \right) \right] da \le \Delta \right\}$$

It is clear that any function $I_f^* \in \mathscr{Y}$ such that $I_f(W) \notin \{0,1\}$ for some measurable set W can not be optimal. Therefore, the recursive system above is equivalent to:

$$I_f^* \in \arg\max_{I_f \subseteq [0,1]} \int_{I_f} \int_{s_f^*}^{\infty} sg(s|a) ds \left(\prod_{h < f, a \in I_h^*} G(s_h^*|a) \right) - c \ da$$
 where $s_f^* = \min \left\{ s | \int_{I_f} (1 - G(s|a)) \left(\prod_{h < f, a \in I_h^*} G(s_h^*|a) \right) da \le \Delta \right\}$

Proof of Proposition 3:

(i): By Lemma 3 (letting $\underline{a} = 0$, $\overline{a} = 1$, and $\psi(a) = \phi(a) = 1$ for each a), for any $I_1 \subseteq [0,1]$ and $s_1 \ge 0$, there exists $a_1 \in [0,1]$ and $s_1' \ge 0$ such that

$$\int_{a_1}^{1} \int_{s_1'}^{\infty} sg(s|a)ds - cda \ge \int_{I_1} \int_{s_1}^{\infty} sg(s|a)ds - cda; \quad \int_{a_1}^{1} 1 - G(s_1'|a)da = \int_{I_1} 1 - G(s_1|a)da.$$

$$(6)$$

Since $sg(s|a) \ge 0$ for all s,a, and G(s|a) is increasing in s for each a, if $\int_{I_1} 1 - G(s_1|a) da = \Delta$, (6) holds for $s_1' = \min \left\{ s \mid \int_{a_1}^1 1 - G(s|a) da \le \Delta \right\}$. For the same reason, if $\int_{I_1} 1 - G(s_1|a) da < \Delta$, the inequality in (6) holds for $s_1' = \min \left\{ s \mid \int_{a_1}^1 1 - G(s|a) da \le \Delta \right\}$. It follows that for some $a_1^* \in [0,1], [a_1^*,1] \in \arg\max_{I_1} \pi_1(I_1)$.

(iia) and (iib): We proceed by induction, beginning with f=2. Since G has increasing k-adjusted yields, by Lemma 1, it has increasing 1-adjusted yields. Then by Lemma 3 (letting $\underline{a}=a_1^*, \overline{a}=1, \ \phi(a)=1$ for each a, and $\psi(a)=\begin{cases} G(s_1^*|a), & a\geq a_1^*;\\ 1, & a< a_1^* \end{cases}$), for any $I_2\subseteq [0,1]$ and $s_2\geq 0$, there exists $a_2^{\{1\}}\in [a_1^*,1]$ and $s_2'\geq 0$ such that

$$\begin{split} &\int_{(I_2\setminus I_1^*)\cup [a_2^{\{1\}},\overline{a}]} \int_{s_2'}^\infty sg(s|a)G(s_1^*|a)ds - cda \geq \int_{I_2} \int_{s_2}^\infty sg(s|a)G(s_1^*|a)ds - cda, \\ &\text{and } \int_{(I_2\setminus I_1^*)\cup [a_2^{\{1\}},\overline{a}]} (1-G(s_2'|a))G(s_1^*|a)da = \int_{I_2} (1-G(s_2|a))G(s_1^*|a)da. \end{split}$$

Another application of Lemma 3, keeping ψ and ϕ the same but letting $\underline{a} = 0$, $\overline{a} = a_1^*$, shows that there exists $a_2 \in [0, a_1^*]$ and $s_2'' \ge 0$ such that

$$\begin{split} &\int_{[a_{2},a_{1}^{*}]\cup[a_{2}^{\{1\}},\overline{a}]}\int_{s_{2}''}^{\infty}sg(s|a)G(s_{1}^{*}|a)ds-cda \geq \int_{I_{2}}\int_{s_{2}}^{\infty}sg(s|a)G(s_{1}^{*}|a)ds-cda, \\ &\text{and } \int_{[a_{2},a_{1}^{*}]\cup[a_{2}^{\{1\}},\overline{a}]}(1-G(s_{2}''|a))G(s_{1}^{*}|a)da = \int_{I_{2}}(1-G(s_{2}|a))G(s_{1}^{*}|a)da. \end{split}$$

If $\int_{I_2} 1 - G(s_2|a) da = \Delta$, then without loss, choose $s_2'' = \min \left\{ s \mid \int_{[a_2,a_1^*] \cup [a_2^{\{1\}},\overline{a}]} 1 - G(s|a) da \leq \Delta \right\}$; if $\int_{I_2} 1 - G(s_2|a) da < \Delta$, observe that (7) holds for $s_2'' = \min \left\{ s \mid \int_{[a_2,a_1^*] \cup [a_2^{\{1\}},\overline{a}]} 1 - G(s|a) da \leq \Delta \right\}$. It follows that for some $a_2^{\{1\}^*} \in [a_1^*,1]$ and $a_2^* \in [0,a_1^*]$, $[a_2^*,a_1^*) \cup [a_2^{\{1\}^*},1] \in \arg\max_{I_2} \pi_2(I_1^*,I_2)$. Hence, (iia) and (iib) hold for f=2.

Now consider $2 < f \le k$, and suppose that (iia) and (iib) hold for each $2 \le f' < f$. Then for each $S \subseteq \{1, \ldots, f-1\}$, $\bigcap_{j \in S} I_j^*$ is an interval. Since G has increasing k-adjusted yields, by Lemma 1, it has increasing k'-adjusted yields for each k' < k. Then applying Lemma 3 once for each $S \subseteq \{1, \ldots, f-1\}$ (letting $\underline{a} = \min \bigcap_{j \in S} I_j^*$, $\overline{a} = \max \bigcap_{j \in S} I_j^*$, $\phi(a) = 1$ for each a, and

 $\psi(a) = \prod_{h < f, a \in I_h^*} G(s_h^*|a), \text{ for any } I_f \subseteq [0,1] \text{ and } s_f \ge 0, \text{ there exist } \{a_f^S\}_{S \subseteq \{1,...,f-1\}} \text{ and } s_f' \ge 0$ such that for $I_f' = \bigcup_{S \subseteq \{1,...,f-1\}} \left([a_f^S,1] \cap \bigcap_{j \in S} I_j^* \right),$

$$\begin{split} & \int_{I_f'} \int_{s_f'}^{\infty} sg(s|a) \left(\prod_{h < f \atop a \in I_h^*} G(s_h^*|a) \right) ds - cda \geq \int_{I_2} \int_{s_2}^{\infty} sg(s|a) \left(\prod_{h < f \atop a \in I_h^*} G(s_h^*|a) \right) ds - cda, \quad (8) \\ & \text{and } \int_{I_f'} (1 - G(s_f'|a)) \left(\prod_{h < f \atop a \in I_h^*} G(s_h^*|a) \right) da = \int_{I_2} (1 - G(s_2|a)) \left(\prod_{h < f \atop a \in I_h^*} G(s_h^*|a) \right) da. \end{split}$$

Then if $\int_{I_2} (1 - G(s_2|a)) \left(\prod_{h < f, a \in I_h^*} G(s_h^*|a) \right) da = \Delta$, it follows that (8) holds for

$$s_f' = \min \left\{ s \mid \int_{I_f'} (1 - G(s|a)) \left(\prod_{h < f, a \in I_h^*} G(s_h^*|a) \right) da \leq \Delta \right\}.$$

Hence, for some $\{a_f^S\}_{S\subseteq\{1,\dots,f-1\}}$, $\bigcup_{S\subseteq\{1,\dots,f-1\}}\left([a_f^S,1]\cap\bigcap_{j\in S}I_j^*\right)\in\arg\max_{I_f}\pi_f(I_f,\{I_h^*\}_{h< f})$. The claim follows by induction.

To prove Propositions 4-6, we will need to make use of the submodularity of a firm's payoff function. In this context, submodularity means the marginal gain from interviewing an additional applicant is higher for firms interviewing weakly fewer and worse applicants.

Definition A.1. A function f is submodular if for any $X,Y,Z \subset [0,1]$ such that $X \subset Y$ and $Z \cap Y = \emptyset$ then:

$$f(X \cup Z) - f(X) > f(Y \cup Z) - f(Y)$$

Lemma 4. For any $\psi: [0,1] \to [0,1]$, consider the following function f:

$$f(X, \psi) = \int_X \psi(a) \left[\int_{s^*}^{\infty} sg(s|a) ds - c \right] da$$

where
$$s^*(X, \psi) = \min \left\{ s | \int_X (1 - G(s|a)) \psi(a) da \le \Delta \right\}$$

 $i \ f(\cdot, \psi)$ and $f(X, \psi) + \int_X \psi(a) c da$ are submodular.

ii For any $X \subset [0,1]$, $\psi : [0,1] \to [0,1]$, $\hat{\psi} : [0,1] \to [0,1]$ such that $\hat{\psi}(a) \leq \psi(a)$ for all $a \in X$ and $\hat{\psi}(a) = \psi(a)$ for all $a \notin X$, then:

$$f(X \cup Z, \hat{\psi}) - f(X, \hat{\psi}) > f(X \cup Z, \psi) - f(X, \psi)$$

$$for all Z \subset [0, 1], Z \cap X = \emptyset$$

Proof of Proposition 4: We consider a two-firm setting with G exhibiting k-adjusted yields such that Firm 1 interviews $I_1^* = [a_1, 1]$ and Firm 2 interviews the set $I_2^* = [a_2^{(1)}, 1] \cup [a_2^{(2)}, a_1]$ in equilibrium, with $a_2^{(1)} > a_1$.

If Firm 1 interviews a set X, define $\psi_2(a,X) = G(s_1^*|a)$, where we suppress dependence of s_1^* on X for notational convenience.

Given an interview set X_1 and X_2 Denote the payoffs to firm 1 by $\pi_1(X_1)$ and the payoffs to firm 2 by $\pi_2(X_2, \psi_2(\cdot, X_1))$. Notice that firm 2's payoff depends on firm 1's interview set through ψ_2 . Take $Y = [1 - \varepsilon, 1]$. If $1 - a_1 < 1 - a_2^1 + a_1 - a_2^2$ then for sufficiently small $\varepsilon > 0$, Lemma 4 implies:

$$\pi_1(I_1^*) - \pi_1(I_1^* \setminus Y) < \pi_2\Big(I_2^* \cup Y, \psi_2(\cdot, I_1^* \setminus Y)\Big) - \pi_2\Big(I_2^*, \psi_2(\cdot, I_1^*)\Big)$$

The proposition follows.

Proof of Proposition 5: Define the following objects:

- 1. For any $S \subset \{1, ..., n\}$ define r(S, i) to be the rank of firm i in S.
- 2. For any $\psi:[0,1] \to [0,1]$ such that $\int_0^1 \psi(a) da \le 1$, define the candidate pool associated with ψ to be the pair $(W_{\psi}, \int_0^1 \psi(a) da)$, where $W_{\psi}(a) = \frac{a}{\int_0^a \psi(s) ds}$. In other words, there is a measure $\int_0^1 \psi(a) da$ of applicants on [0,1], and the conditional distribution is given by W_{ψ} . Due to the one-to-one correspondence, we will refer to ψ as the applicant pool. Thus, $\psi(a) = 1$ is the original applicant pool. In an abuse of notation, we will also use ψ to refer to the actual set of applicants.
- 3. For any $i \in \{1, ..., n\}$ and candidate pool ψ , let $\sigma(i, \psi)$ be the optimal interview strategy for an i-th ranked firm in a centralized setting with candidate pool ψ .
- 4. Given a subset of firms S and applicant pool ψ , for each $i \in S$, let $C_i(S, \psi)$ denote the equilibrium payoff in a centralized matching setting where firms S face candidate pool ψ . $C_i(S, \psi)$ is pinned down due to firm $j \in S$ using equilibrium strategy $\sigma(r(S, j), \psi)$.
- 5. Consider a centralized matching setting with firms $S \subset \{1, ..., F\}$ and an applicant pool ψ , Let $\mu(S, \psi)$ be the set of applicants firms in S match with in equilibrium. Define $\Xi(S, \psi) = \psi \setminus \mu(S, \psi)$.

We proceed by induction on the number of time periods.

Base Case: T = 2

Firms only have two choices: interview in time period 1 or 2. Let $A \in \{0,1\}^n$ denote the choice profile for each firm, where $A_i = 0$ means firm i is choosing to interview in period 1. Notice then, that for any A, firm i's payoff is uniquely pinned down:

1. If
$$A_i = 0$$
, then firm *i*'s payoff is $\pi_i(A) = C_i(\{j|A_j = 0\}, \psi)$

2. If
$$A_i = 1$$
, then firm i's payoff is $\pi_i(A) = C_i(\{j|A_j = 1\}, \Xi(\{j|A_j = 0\}, \psi))$

A mixed strategy equilibrium exists since there are a finite number of players and actions!

Inductive Step: Assume that a subgame perfect equilibrium exists for all $T \le k$

For any subset of firms S and applicant pool ψ , let $E_i(T, S, \psi)$ denote the equilibrium payoff to firm i in a game with T time periods.

Case T = k + 1:

Now, consider the following game: firms can choose to interview in time period 1 or defer to a later period. Let $A \in \{0,1\}^F$ denote the choice profile for each firm, where $A_i = 0$ means firm i is choosing to interview in period 1. Notice, then, that for any A, firm i's payoff is uniquely pinned down by the inductive hypothesis:

1. If
$$A_i = 0$$
, then firm *i*'s payoff is $\pi_i(A) = C_i(\{j|A_j = 0\}, \psi)$

2. If
$$A_i = 1$$
, then firm *i*'s payoff is $\pi_i(A) = E_i\left(k, \{j|A_j = 1\}, \Xi(\{j|A_j = 0\}, \psi)\right)$

A mixed strategy equilibrium exists in this game as there are two actions and F players. Equilibrium payoff must be the subgame perfect equilibrium payoffs.

Finally, in any mixed strategy equilibrium, each firm must weakly prefer playing the strategy to interviewing in period 1. Notice, though, that by interviewing in period 1, each firm i is guaranteed a payoff of at least $C_i(\{1,\ldots,n\},\psi)$.

Proof of Proposition 6: Part (i): Given the firms' interview timing decisions, equilibrium strategies are unique. To show this — and that they have the form in (i) — we proceed by induction. For our initial step, observe that firm 1's problem is identical to that in the centralized case; then by Proposition 3 (i), there exists $a_1^* \in [0,1]$ such that $[a_1^*,1] \in \arg\max_{I_1} u_1(I_1)$.

Now consider f > 1, and suppose that for each $1 \le h < f$, there exists $a_h^* \in [0,1]$ such that $I_h = [a_h^*, 1]$ is a best response for firm h to $\{[a_\ell^*, 1]\}_{\ell < h}$. Then by Lemma 3, letting $\underline{a} = 0$, $\overline{a} = 1$,

 $\psi(a) = 1$ for each a, and $\phi(a) = \left(\prod_{h < f} G(s_h^*|a) \mathbf{1}_{a \ge a_h^*}\right)$, for any $I_f \subseteq [0, 1]$ and $s_f \ge 0$, there exists $a_f \in [0, 1]$ and $s_f' \ge 0$ such that

$$\int_{a_f}^{1} \left(\int_{s_f'}^{\infty} sg(s|a)ds - c \right) \left(\prod_{\substack{h < f \\ a \ge a_h^*}} G(s_h^*|a) \right) da \ge \int_{I_f} \left(\int_{s_f}^{\infty} sg(s|a)ds - c \right) \left(\prod_{\substack{h < f \\ a \ge a_h^*}} G(s_h^*|a) \right) da;$$

$$\int_{a_f}^{1} (1 - G(s_f'|a)) \left(\prod_{\substack{h < f \\ a \ge a^*}} G(s_h^*|a) \right) da = \int_{I_f} (1 - G(s_f|a)) \left(\prod_{\substack{h < f \\ a \ge a^*}} G(s_h^*|a) \right) da.$$
(9)

If $\int_{I_f} (1-G(s|a)) \left(\prod_{h < f, a \geq a_h^*} G(s_h^*|a)\right) da = \Delta$, then since $sg(s|a) \geq 0$ for all s, a, and G(s|a) is increasing in s for each a, (9) holds for $s_f' = \min\left\{s \mid \int_{a_f}^1 1 - G(s|a) da \leq \Delta\right\}$. For the same reason, if $\int_{I_f} (1-G(s|a)) \left(\prod_{h < f, a \geq a_h^*} G(s_h^*|a)\right) da < \Delta$, the inequality in (9) holds for $s_1' = \min\left\{s \mid \int_{a_f}^1 (1-G(s|a)) \left(\prod_{h < f, a \geq a_h^*} G(s_h^*|a)\right) \leq \Delta\right\}$. The claim then follows by induction.

To prove (ii), notice that firm i faces a much smaller applicant pool than the firms ranked above it. If we let $\psi^{(i)}(a) = \prod_{h < i, a \in I_h^*} G(s_h^*|a)$ be the probability applicant a is available by the time firm i interviews, the optimization problem for firm i is of the form:

$$a_i \in \arg\max_k \int_k^1 \psi^{(i)}(a) \left[\int_{\bar{s}_i}^\infty v dG(v|a) \right] da - c \int_k^1 d\psi^{(i)}(a)$$

$$\bar{s}_i(k) = \min\left\{ s | \int_k^1 (1 - G(s|a)) d\psi^{(i)}(a) \le \Delta \right\} \text{ and } \psi^{(i)} \in \Delta[0, 1]$$

Recognize the following two properties of $\psi^{(i)}(\cdot)$:

- 1. $\psi^{(i)}(a) \le \psi^{(i')}(a)$ for $i \ge i'$ since firm i interviews after firm i' for all $i' \le i$.
- 2. $\psi^{(i)}(a) < \psi^{(i)}(a')$ for a > a'

Letting $\pi_i(k) = \int_k^1 \psi^{(i)}(a) [\int_{\bar{s}_1}^{\infty} v dG(v|a)] da - c \int_k^1 d\psi^{(i)}(a)$, Lemma 4 implies that for sufficiently small $\varepsilon > 0$:

$$\pi_{i+1}(a_i) - \pi_{i+1}(a_i + \varepsilon) > \pi_i(a_i) - \pi_i(a_i + \varepsilon)$$

Since interviewing $[0, a_i]$ is optimal for firm i, $\pi_i(a_i) - \pi_i(a_i + \varepsilon) \ge c\varepsilon \Longrightarrow a_{i+1} \ge a_i$.

Finally, (iii) follows immediately from (i) and Assumption 2.2.

Proof of Proposition 7: Let a firm's interview decision be given by $\mu_f : [0,1] \to [0,1]$, where $\mu_f(a)$ is the fraction of *available* applicant *a*'s that firm *f* interviews. Let μ denote the profile of firm interview decisions and let $\psi_i(\mu, a) = \psi_1(\mu, a) \cdot \left[\prod_{h < i} (1 - \mu_h(a)(1 - G(s_h|a)) \right] da$. Total surplus in a setting where firms interview sequentially according to μ is given by:

$$S(\mu, \psi_1) = \sum_{i=1}^F \int_0^1 \mu_i(a) \psi_i(\mu, a) \left[\int_{s_i}^\infty sg(s|a) ds \right] da - \sum_{i=1}^F c \cdot \int_0^1 \mu_i(a) \psi_i(\mu, a) da$$
where $s_i = \min \left\{ s | \int_0^1 \mu_i(a) \psi_i(\mu, a) \le \Delta \right\}$

Let μ_f^* be firm f's interview decision in the sequential-hiring equilibrium.

Consider the equilibrium of the centralized setting when the pool of workers is given by ψ_1 . Denote the equilibrium interview strategy of a firm ranked f amongst those interviewing by $I_f^*(\psi_1)$. Total surplus generated by F firms is $Cent(\left\{I_f^*\right\}_{f=1}^F, \psi_1)$.

Initialize $\psi_1(a) = 1$ (i.e. the pool of workers is the initial measure 1 set distributed uniformly on [0,1]). Now, it is easy to see that since $\mu_1^* = I_1^*$, we have:

$$Cent(I_f^*, \psi_1) \leq \int_0^1 \mu_1^*(a) \psi_1 \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] da - c \cdot \int_0^1 \mu_1^*(a) \psi_1 da + Cent(\{I_{f-1}^*\}_{f=2}^F, \psi_2) \Big] ds + Cent(I_f^*, \psi_1) \leq \int_0^1 \mu_1^*(a) \psi_1 \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] da - c \cdot \int_0^1 \mu_1^*(a) \psi_1 da + Cent(\{I_{f-1}^*\}_{f=2}^F, \psi_2) \Big] ds + Cent(I_f^*, \psi_1) \leq \int_0^1 \mu_1^*(a) \psi_1 \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] da - c \cdot \int_0^1 \mu_1^*(a) \psi_1 da + Cent(\{I_{f-1}^*\}_{f=2}^F, \psi_2) \Big] ds + Cent(I_f^*, \psi_1) \leq \int_0^1 \mu_1^*(a) \psi_1 \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \leq \int_0^1 \mu_1^*(a) \psi_1 \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \leq \int_0^1 \mu_1^*(a) \psi_1 \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int_{s_1^*}^{\infty} sg(s|a) ds \Big] ds + Cent(I_f^*, \psi_1) \Big[\int$$

Observe that by recursion, the right-hand side is weakly less than $S(\mu^*)$.

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