

Losing Money to Make Money: The Benefits of Redistribution in Collective Bargaining in Sports *

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Abstract

I develop a model of majority-rule collective bargaining between a sports league and its players when delay costs incurred by players are wealth-dependent. I propose a refinement of subgame perfect equilibrium that requires equilibrium strategies to be immune to deviations by any majority subgroup. I show this is equivalent to giving the player with median bargaining power the unilateral ability to negotiate with the league. Using this model, I demonstrate that policies reallocating surplus from high-talent to moderate-talent players, such as maximum contracts in professional sports, can improve the welfare of *all* players. Redistribution of surplus harmonizes players' interests, giving a majority of them a greater stake in the bargaining outcome. The model highlights the gains to be had if a heterogeneous group agrees to concessions that increase the alignment of their individual interests.

Keywords: Majority-Rule Bargaining, Collective Bargaining, Negotiation, Unions

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You had some guys who were making a lot of money that wanted to hold the line. Then you had other guys saying, 'I got to get back to work. I got a wife. I got kids. I got family members that I have to help.'

- Aaron McKie, NBA player 1994-2007

1 INTRODUCTION

On January 20th, 1999, the longest lockout in the National Basketball Association's history ended after 205 days. The league lost \$1 billion in revenue, while players forfeited \$500 million in salary. The crux of the dispute was how revenue should be divided between owners and players. The stumbling block was the maximum contract, a limit on an individual's salary as a percentage of total revenue accorded to the players; this pitted players against each other. As time wore on, the cohesion of the union fractured, and the NBA became the first major league to adopt maximum contracts. While such contracts depress the salaries of stars and increase those of non-stars in the immediate term, *all* players may benefit in future collective bargaining negotiations over the split of revenue. For players to have bargaining power, they must be sufficiently patient. However, delays in bargaining affect wealthier players differently than poorer ones. Bargaining in the NBA is not simply a negotiation "between short billionaires and tall millionaires," as Washington Post writer Tony Kornheiser remarked. Not all the tall ones are millionaires. Redistribution of the salary through maximum contracts reduces inequality in patience in bargaining, which may benefit the players in future collective bargaining.

I demonstrate this in a model of collective bargaining between a coalition of heterogeneous players and the league. Players are either high or moderate talent. They receive rewards as a function of the current share of surplus dedicated to all the players. The term "reward" should be interpreted to include wages, bonuses, and other amenities. High-talent players earn larger rewards than moderate-talent players.

Crucially, players earn rewards *before* the bargaining date. This allows me to identify how the initial distribution of rewards impacts subsequent bargaining outcomes. At a known future date, the players enter into a negotiation with the league, bargaining over the size of the future surplus. The league and the players' coalition alternate between proposing offers, with negotiations ending when one side accepts the other's proposal. I show that redistributive policies have an overlooked benefit: they improve the bargaining power of the players.

A difficulty in many models of coalitional bargaining is the need to specify protocols for how agents in a coalition generate and agree on proposals. Generally, bargaining outcomes

are sensitive to the chosen protocol ([Eraslan and Evdokimov \[2019\]](#)). I sidestep this difficulty by abstracting from the method by which the players' coalition generates proposals. Instead, I require as an equilibrium condition that there be no profitable deviation for any majority subgroup. Importantly, I show that this is equivalent to giving the player with median bargaining power the right to make, accept, and reject proposals. If delay costs in bargaining are wealth-dependent and high-talent players are longer-lived than those of lesser talent, redistributive policies strengthen the coalition's bargaining position by increasing median bargaining power.

Contribution My model has direct applications to professional sports markets in the United States. Much of the literature in sports economics focuses on the effect of contract structures on salary and talent distribution (e.g. [Fort and Quirk \[1995\]](#); [Dietl, Lang, and Rathke \[2009\]](#)).¹ Such papers do not include a bargaining stage. My paper demonstrates that when one includes a bargaining stage, maximum contracts that redistribute wealth from stars to less talented players can benefit all players. This has immediate policy implications for US sports leagues. For example, the NBA players' union is seen as more effective than its NFL counterpart. A critical distinction between the leagues is that the NFL operates with a salary cap and no limit on the size of individual contracts. My results support an argument that NFL players would benefit from maximum contracts. Such contracts will increase their leverage in future negotiations, leading to players receiving a higher percentage of league revenue than they would otherwise.

My paper contributes to the literature on majority-rule bargaining where one side is comprised of heterogeneous agents (see [Card \[1990\]](#), [Cramton and Tracy \[2003\]](#), and [Serrano \[2007\]](#) for surveys in bargaining). I extend [Rubinstein \(1982\)](#) to incorporate a majority-rule approval mechanism on one side where members have heterogeneous discounting. It is related to the majority-rule legislative bargaining literature (see [Eraslan and Evdokimov \[2019\]](#) for a survey). In those papers, individuals are randomly selected as proposers (e.g. [Baron and Ferejohn \[1989\]](#); [Haller and Holden \[1997\]](#)). In mine, I abstract from how the coalition crafts its proposals but require that there be no profitable deviation by any majority subgroup as an equilibrium condition. I demonstrate that this is equivalent to the player of "median bargaining power" being given the right to make and reject proposals. In the same vein, [Compte and Jehiel \(2010\)](#) look at a majority-rule bargaining game, showing that there is a "key-player" who determines the outcome. In their paper, the coalition searches for proposals, and the majority dictates when to halt search. My model requires a majority of players to accept any proposal

¹[Madden \(2019\)](#) looks at collective bargaining but using the framework from [McDonald and Solow \(1981\)](#).

from the league. However, it does not specify how the players craft their counter-proposals. My model can be viewed as one where the players can not commit to any protocol by which to do this, as various subsets can “defect” at any time. Thus, I look at proposal strategies that satisfy a refinement of subgame perfect equilibrium to pin down decisions that are immune to such deviations by a majority subgroup.

Early models of collective bargaining treat agents as homogenous (see [Kaufman \(2002\)](#) for a survey). Heterogeneity is a critical component of my model because it affects the bargaining outcome, as some players are more patient than others. Furthermore, heterogeneity is crucial to identifying the consequences of redistributive policies across players. Related models of collective bargaining that include heterogeneity also have the outcome determined by a median voter (e.g. [Booth \[1995\]](#); [Lee and Mas \[2012\]](#)). However, in mine, the “median voter” is not *exogenously selected* as an arbitrator. My model can be viewed as a microfoundation for this choice. While there is no explicit selection of an arbitrator, the equilibrium outcome is equivalent to the setting where the player with median bargaining power is delegated to bargain on behalf of the coalition. [Manzini and Mariotti \(2005\)](#), [Romer and Rosenthal \(1978\)](#), and [Callandar and Martin \(2017\)](#) also posit the median voter is decisive. My paper departs from [Manzini and Mariotti \(2005\)](#) by developing an equilibrium refinement equivalent to a generalized one-shot deviation principle. [Romer and Rosenthal \(1978\)](#) and [Callandar and Martin \(2017\)](#) consider a single agenda-setter negotiating with voters, but only the former has proposal power. I show that the median voter remains decisive even when voters have proposal power.

A key distinction between my paper and previous bargaining models is that I examine how bargaining outcomes depend on redistribution policies *before* bargaining begins. In particular, redistributing surplus from players with a high share to those with a low share can improve bargaining outcomes by increasing the bargaining power of the median player. While framed in a labor market context, the ideas translate to environments where a group of heterogeneous agents negotiates over surplus with an institution. The model highlights the benefits to the group from agreeing to concessions that improve the alignment of members’ interests.² Interpreting my results through the lens of [Grossman and Hart \(1986\)](#), [Penceval \(1991\)](#), and [Muthoo \(2004\)](#), I demonstrate that redistributing property rights incentivizes more of the coalition members to work harder for the collective good (in this case, not agree to low proposals).³

²In this sense, one can view redistribution as how unions “coordinate bargaining” ([Ahlquist \[2017\]](#)).

³[Sandroni and Urgan \(2018\)](#) look at the effect of patience on committing destructive acts. Bargaining delay costs represent patience, while wealth transfers caused by redistribution can be viewed as “destructive acts”.

The next section describes the model. [Section 3](#) analyzes the collective bargaining game and welfare effects. [Section 4](#) discusses the assumptions of the model as well as its applications.

2 MODEL OVERVIEW

There is a league employing a unit mass of players. Players are either high or moderate-talent, denoted by $\theta = h$ and $\theta = l$, respectively; there are $l_h < \frac{1}{2}$ high-talent players. Time is continuous, starting at $t = 0$. The league generates a unit surplus at each t . While working, some players exit the league, and this is modeled as a Poisson process with intensity λ_θ . Exiting players are replaced by those of the same type. Let π denote the share of surplus accorded to the players at each t , and let μ_0 be the exogenously specified fraction of π allotted to the high-talent players. The quantity μ_0 is known as the maximum contract.

At each t , a moderate-talent player receives reward $s_l = \max \left\{ \frac{(1-\mu_0)\pi}{1-l_h}, s_{min} \right\}$, and a high-talent player receives $s_h = \min \left\{ \frac{\mu_0\pi}{l_h}, \frac{\pi - s_{min}(1-l_h)}{l_h} \right\}$, where $s_{min} \geq 0$ is the *reservation reward*. I assume μ_0 is such that $s_h \geq s_l$.⁴ Thus, s_h and s_l specify how much of the share of surplus *each* player gets. The moderate-talent player is guaranteed to receive s_{min} . I provide a formal microfoundation of these reward functions in [Section 2.1](#) with a model of a sports league.

One can interpret μ_0 as the outcome of prior negotiations or internal union discussions. More generally, the functional forms of s_h and s_l can be interpreted as reduced form representations of what might emerge in an equilibrium model of wage-setting in labor markets (e.g. [Fernandez and Glazer \[1990\]](#); [Mortensen and Pissarides \[1994\]](#); [Houba and van Lomwel \[2001\]](#)). They also arise in settings where the surplus at stake is the total revenue generated, and the involved parties are participants in a revenue-sharing agreement (e.g. [Feiveson \[2015\]](#)).

The initial share of surplus accorded to players is π_0 . The future share of surplus awarded to them is determined via collective bargaining, which I model as follows. At a known time \bar{t} in the future, a bargaining game is initiated. Players collectively negotiate with the league over the future division of surplus. The league and players compete in a Rubinstein bargaining game, making alternating take-it-or-leave-it offers specifying the fraction of surplus the players will receive. Bargaining ends when the league accepts an offer supported by a majority of the players or when a majority of players accept the offer made by the league. The new share of surplus is denoted π_A . Players then earn a reward stream according to π_A .⁵ No rewards are

⁴I suppress dependence of s_θ on the parameters for ease of exposition.

⁵It is assumed that π_0 and π_A must always be large enough to guarantee the minimum reward to each player.

received until an offer is accepted. Offers are made at discrete intervals of length $\Delta > 0$ to account for the fact that proposals require time to craft and analyze. A novel feature of the model is that costs associated with delays in bargaining depend on player wealth, which is the cumulative rewards he has received up until the start of bargaining. A player's delay cost in bargaining is represented by a decreasing function $\delta(\cdot)$ of his wealth. This is for economy of exposition. All theorems and proofs incorporate type-dependent bargaining delay costs of the form $\delta(\theta, w)$, where $\delta(\theta, \cdot)$ is decreasing in wealth.⁶ The league discounts at rate $\rho > 0$.

Using this model, I answer the following questions:

1. What is the bargaining outcome, and how does wealth-dependent discounting affect it?
2. While the initial μ_0 is exogenous, if the players could collectively reduce μ_0 and increase redistribution, how would that affect bargaining?

2.1 Example of a Sports League

There are N symmetric teams, a limited supply l_h of high-talent players, and an unlimited supply of moderate-talent players. Each team must sign a unit measure of players. If each team j signs x_j high-talent and y_j moderate-talent players, team i 's revenue is $R(x_i, y_i; (x_{-i}, y_{-i}))$. Assume team revenue is increasing and concave in its talent, and the marginal revenue of signing a high-talent player is greater than that of a moderate-talent one. Finally, assume team i 's revenue depends on its talent and the distribution of talent across other teams, independent of team name. Team costs are player salaries. As in most US leagues, there is a salary cap specifying the amount each team must pay its players. Given salary cap C , players are thus entitled to a share $\pi = \frac{NC}{\sum_{i=1}^N R(x_{it}, y_{it}; (x_{-it}, y_{-it}))}$ of total revenue.

Késenne (2000) analyzes such a league under a Walrasian framework.⁷ Equilibrium salaries are a function of talent supply, salary cap, and minimum salary:

$$s_h = \min \left\{ \frac{NC}{l_h}, \frac{NC - s_{\min}(N - l_h)}{l_h} \right\} \quad (1)$$

$$s_l = s_{\min} \quad (2)$$

⁶There is much empirical evidence that poorer individuals discount the future more than wealthier ones (Fredrick et al. [2002]; Card, Chetty, and Weber [2007]). In a bargaining context, Hardy et al. (2020) observe that discount rates in bargaining decline with income level. In Appendix C, I provide a microfoundation for the existence of wealth-dependent delay costs in bargaining.

⁷Burguet and Sákovic (2019) demonstrate that even when teams can offer salary schedules discriminating between players of the same type, and players can choose whom to play for, the equilibrium salary schedule is equal to the Walrasian one.

If a redistributive policy in the form of a maximum contract μ_0 is implemented (limit on the fraction of the salary cap that high-talent players can receive), salaries would be:

$$s_h = \min \left\{ \mu_0 \frac{NC}{l_h}, \frac{NC - s_{min}(N - l_h)}{l_h} \right\} \quad (3)$$

$$s_l = \max \left\{ \frac{(1 - \mu_0)NC}{N - l_h}, s_{min} \right\} \quad (4)$$

Players bargain with the league over the value of C (the share of surplus).

2.2 Payoffs

Consider the following sequence of events for a player of type θ :

1. Receives reward $s_\theta(\pi_0, \mu_0)$ at times $t \leq \bar{t}$.
2. Collective bargaining begins at \bar{t} and an agreement is reached at time $\bar{t} + k\Delta$ resulting in a share π_A of surplus to the players. No rewards are earned during the delays in bargaining.
3. Earns new reward $s_\theta(\pi_A, \mu_0)$ at times $t \geq \bar{t} + k\Delta$.

The expected payoff to the player that enters at time $\hat{t} \leq \bar{t}$:⁸

$$U(\theta, k, \pi_0, \pi_A, \mu_0) = \underbrace{\int_{\hat{t}}^{\bar{t}} \lambda_\theta e^{-\lambda_\theta(t-\hat{t})} s_\theta(\pi_0, \mu_0) dt}_{\text{Discounting up to } \bar{t}} + \underbrace{e^{-k\Delta\delta(w)}}_{\text{Delay in bargaining}} \cdot \underbrace{\int_{\bar{t}+k\Delta}^{\infty} \lambda_\theta e^{-\lambda_\theta(t-\hat{t}-k\Delta)} s_\theta(\pi_A, \mu_0) dt}_{\text{Discounting from } \bar{t} + k\Delta}$$

where $w = \underbrace{s_\theta(\pi_0, \mu_0)(\bar{t} - \hat{t})}_{\text{Wealth at } \bar{t}}$

3 COLLECTIVE BARGAINING

3.1 Solution Concept

At the time of bargaining, the players as a collective negotiate with the league over the division of surplus. I model bargaining as a modified Rubinstein bargaining process in which the league and players alternate between making offers to one another until an offer is accepted.

⁸One may wonder what happens if players value rewards non-linearly. I discuss the robustness of my results to such a specification in [Appendix B](#). Most results remain true, though some comparative statics regarding the bargaining solution depend on properties of the value function.

I assume that the league is the initial proposer in the first period. An offer by the league is accepted if a majority of the players choose to accept.⁹ Similarly, an offer by the players is accepted if the league agrees to it.

The history-contingent strategies of the league and players' coalition are denoted σ_F and σ_W , respectively. As the league makes the first offer, σ_F specifies offers in each *odd* period; in *even* periods, it specifies an acceptance/rejection decision in response to the counteroffer. The players' coalition strategy, σ_W , is defined analogously. Formal definitions are in [Appendix A](#).

The players' coalition is the set of all players at the time of bargaining. Any player at that time can be identified by their talent (θ) and accumulated wealth (w). Thus, **the players' coalition** is described by a distribution G over $\{(\theta, w) | \theta \in \{h, l\} \text{ and } w \leq s_\theta \bar{t}\}$, which accounts for there being players of varying career lengths in the coalition. Each redistribution level μ_0 induces a different G since μ_0 changes the initial reward stream of the players and, therefore, the distribution of accumulated wealth. The distribution G is crucial, as one needs to know the individual players' preferences to understand the majority's actions.

I now define the notion of equilibrium, which will require that any decision by the players' coalition be supported by a simple majority.

Definition 3.1 *A subgame perfect majority-rule equilibrium (SPMRE) is a pair of strategies σ_W and σ_F such that:*

1. *There is no closed $M \subset \{(\theta, w) | \theta \in \{h, l\} \text{ and } w \leq s_\theta \bar{t}\}$, with $\mathbb{P}_G(M) \geq \frac{1}{2}$, such that after some history, there is a deviation that leaves all members of M strictly better off.*
2. *At any history, there is no deviation by the league that leaves it better off.*

While the coalition is a singular entity, I must rule out deviations by majority subsets. This is because the league could target its proposals toward certain players to gain a more favorable deal. [Definition 3.1](#) incorporates this feature. Moreover, I can abstract from the method by which the players' coalition generates proposals by using the equilibrium criterion above.¹⁰

⁹In the body of the paper, I assume that the threshold for agreement is a simple majority. However, I demonstrate in the [appendix](#) that [Theorems 3.3](#) and [Theorem 3.4](#) generalize easily for an arbitrary threshold $q \geq \frac{1}{2}$. I defer discussion of varying the threshold to [Section 4](#).

¹⁰This refinement leads to deterministic subgames of the bargaining game, which is not the case with probabilistic proposal mechanisms. The importance of this is discussed in the appendix following [Lemma A.7](#).

3.2 Necessity of Redistribution

To highlight incentives, consider a setting with no redistribution ($\mu_0 = 1$). Moderate-talent players receive the reservation salary, and high-talent players receive the maximum possible reward. Collective bargaining leads to players receiving the reservation share of surplus $\pi_A = \pi_{min} = s_{min}$. Post-bargaining rewards are $s_\theta(\pi_{min}) = s_{min}$ for all θ .

Proposition 3.2 *Suppose $\mu_0 = 1$. Then the post-bargaining share of surplus is $\pi_A = \pi_{min}$.*

Proof: Moderate-talent players earn the minimum reward independent of the share of surplus. Thus, without redistribution, moderate-talent players receive s_{min} at each t regardless of the bargaining outcome. The league can credibly offer the reservation share of surplus as a result.¹¹ Hence, the unique subgame-perfect majority-rule equilibrium has the league offering the reservation share of surplus and all the moderate-talent players accepting immediately. ■

To counter this, the players' coalition can institute a minimal level of redistribution, which may increase rewards from bargaining for moderate-talent players without high-talent players sacrificing initial rewards. The minimal level of redistribution is given by an *upper bound* $\bar{\mu}$ on the share of surplus that high-talent players can receive. The quantity $\bar{\mu}$ is defined by:

$$\bar{\mu} \frac{\pi_0}{l_h} = \frac{\pi_0 - s_{min}(1 - l_h)}{l_h}$$

The minimal level of redistribution $\bar{\mu}$ has the feature that when $\mu_0 = \bar{\mu}$, rewards at times $t \leq \bar{t}$ are the same as those in a regime with no redistribution. It does not affect the initial stream of payments before \bar{t} but may increase rewards for moderate-talent players *after* bargaining. If delay costs are constant, there is no benefit in the bargaining stage from redistribution beyond the minimal level. Incorporating wealth-dependent discounting enriches the model and yields insights into the benefits of *higher levels* of redistribution beyond the minimal level. In particular, given the previous discussion, one can restrict attention to $\mu_0 \leq \bar{\mu}$.

3.3 Bargaining Outcome

Consider a hypothetical situation where a type θ player with wealth w negotiates on behalf of the coalition. From [Rubinstein \(1982\)](#), there is a unique subgame perfect equilibrium where the players receive a share of surplus π^* such that:

¹¹ Suppose high-talent players had a different reservation reward. The reservation share of surplus is simply the minimum share that allows high-talent and moderate-talent players to receive their respective reservation rewards.

$$\pi^*(\theta, w) = \frac{e^{-\Delta\delta(w)}(1 - e^{-\Delta\rho})}{1 - e^{-\Delta(\delta(w)+\rho)}}$$

π^* quantifies the bargaining power of a type θ player with wealth w . Notice π^* depends on (θ, w) only through the delay cost in bargaining $\delta(w)$. Thus, bargaining power is entirely characterized by $\delta(w)$. A player with a *high value* of $\delta(w)$ (high delay cost) has *low bargaining power*: $\pi^*(\theta, w) > \pi^*(\theta', w')$ if and only if $\delta(w) < \delta(w')$. A player with **median bargaining power** has delay cost δ^* such that:

$$\delta^* = \sup \left\{ \hat{\delta} : \mathbb{P}_G(\{(\theta, w) | \delta(w) > \hat{\delta}\}) \geq \frac{1}{2} \right\}$$

Given the reasoning above, the player with median bargaining power has median wealth w^* :

$$w^* = \delta^{-1}(\delta^*) = \inf \left\{ \hat{w} : \mathbb{P}_G(\{(\theta, w) | w < \hat{w}\}) \geq \frac{1}{2} \right\}$$

Remark: The supremum and infimum are used since the distribution G has two atoms: some players of each type who started at time 0 will not have exited by the bargaining time \bar{t} .

While the coalition may include a high-talent player with median bargaining power, there will always be a moderate-talent player with median bargaining power since the measure of high-talent players is $l_h < \frac{1}{2}$. There will always be a moderate-talent player with wealth w^* .

I rank players based on how each fares in an individual bargaining game against the league. Intuitively, since the moderate-talent player with wealth w^* is the median of this ranking, his individual bargaining outcome will be accepted by a majority. However, because there is no actual representative in the collective bargaining game and no commitment device to select such a representative, it is not obvious that this player's preference will determine outcomes. In other models of majority-rule bargaining, the “key” individual that determines the outcome is sensitive to the proposal-construction process (e.g. [Baron and Ferejohn \[1989\]](#); [Compte and Jehiel \[2010\]](#)). I show that the equilibrium refinement requires the outcome to be determined by a player with median bargaining power. Importantly, I demonstrate that the equilibrium outcome is equivalent to the solution to a Rubinstein bargaining game where the player with median bargaining power negotiates on behalf of the coalition.

Theorem 3.3 *In any SPMRE, collective bargaining results in the players receiving a fraction π_A of future surplus:*

$$\pi_A = \begin{cases} \max \left\{ \frac{e^{-\Delta\delta^*}(1-e^{-\Delta\rho})}{1-e^{-\Delta(\delta^*+\rho)}}, \pi_{min} \right\} & \text{if } \frac{1-e^{-\Delta\rho}}{1-e^{-\Delta(\delta^*+\rho)}} > \frac{s_{min}(1-l_h)}{1-\mu_0} \\ \pi_{min} & \text{if } \frac{1-e^{-\Delta\rho}}{1-e^{-\Delta(\delta^*+\rho)}} \leq \frac{s_{min}(1-l_h)}{1-\mu_0} \end{cases}$$

In particular, if $s_{min} = 0$, then $\pi_A = \frac{e^{-\Delta\delta^}(1-e^{-\Delta\rho})}{1-e^{-\Delta(\delta^*+\rho)}}$.*

Proof: [See Appendix.](#) ■

3.4 Player Share of Future Surplus

The players' coalition aims to prevent the league from extracting maximal surplus in collective bargaining and depressing the players' share of surplus to the reservation level. Without redistribution, post-bargaining share of surplus is $\pi_A = \pi_{min}$. By [Theorem 3.3](#), post-bargaining player share of surplus is decreasing in δ^* and increasing in w^* . It follows that increasing redistribution increases post-bargaining share of surplus only if $\frac{\partial w^*}{\partial(1-\mu_0)} > 0$. It may appear obvious that increasing redistribution increases median wealth as there are more moderate-talent players than high-talent players. However, this is not correct. Since players exit and enter at different rates, there will be a distribution of players of various ages and hence wealth levels at the time of bargaining. Furthermore, when high-talent players give up a unit of their reward, each moderate-talent player only receives a fraction of this unit since there are more moderate-talent players than high-talent ones. Hence, for the median wealth w^* to increase, the upward shift in the distribution of *accumulated wealth* of the moderate-talent players must outweigh the downward shift in *accumulated wealth* of the high-talent players. Quantifying these shifts is not trivial because players exit according to a Poisson process, and so the wealth distribution is determined by the distribution of players' ages at the time of bargaining.

Intuitively, reducing μ_0 increases median wealth when there is “sufficient wealth disparity” between the distributions of wealth at the time of bargaining between high and moderate-talent players. The parameters affecting wealth disparity at the time of bargaining are player exit rates (λ_l, λ_h) , the number of high-talent players (l_h), and the time of bargaining (\bar{t}). The exit rates affect how easy it is to accumulate wealth. The time of bargaining caps how much wealth can be achieved. The number of high-talent players affects the reward stream levels. The wealth dis-

parity needed occurs when high-talent players are sufficiently long-lived relative to moderate-talent ones. How much longer-lived depends on these other parameters. For instance, when high-talent players are scarce, they receive large rewards, leading to large disparity. Hence, high-talent players need not be much longer-lived than moderate-talent ones. [Theorem 3.4](#) formalizes this idea to yield an intuitive condition for when decreasing μ_0 increases w^* .

Theorem 3.4 *If $\mu_0 \leq \bar{\mu}$, there exists $k(\lambda_l, l_h, \bar{t})$ such that if $\lambda_h \leq \min\{\lambda_l, k\}$, then $\frac{\partial w^*}{\partial(1-\mu_0)} > 0$. In particular, if $l_h < \frac{e-2}{2e-2} \approx 0.21$, then $\lambda_h \leq \lambda_l \implies \frac{\partial w^*}{\partial(1-\mu_0)} > 0$.¹²*

Proof: [See Appendix](#). ■

Proof Sketch: I describe my approach and the intuition behind the result. Varying μ_0 changes the initial rewards each player receives, thereby changing the distribution of accumulated wealth at the time of bargaining and, in particular, w^* . Since players exit according to a Poisson process, and the fraction of type θ players is constant, I can compute the distribution of wealth at the time of bargaining for each player type. I then characterize w^* , the median wealth level corresponding to median bargaining power.

As redistribution increases, initial rewards are shifted towards moderate-talent players. The shift is not “one-to-one” since there are more moderate-talent players than high-talent ones. As high-talent rewards decline, high-talent players become less patient. For median bargaining power to increase, the gain in the measure of moderate-talent players that have accumulated more wealth must outweigh the increase in high-talent players willing to “settle”. This is guaranteed when high-talent players are sufficiently longer-lived than moderate-talent ones. When talent is scarce, high-talent players need only be at least as long-lived. This is the case in many industries. In professional sports, for example, it is well-documented that high-talent players have longer professional careers than those with lesser talent.¹³

It is critical to note that this does not mean that the post-bargaining share of surplus will be higher than the initial share. Rather, the post-bargaining share of revenue will be higher than *what it would have been* if there were less redistribution.

¹²Without exit, the player with median bargaining power is moderately talented with wealth $s_l \bar{t}$. In this case, reducing maximum contracts always increases median bargaining power. With exit, this is not guaranteed, even when high-talent players are longer lived than moderately talented ones.

¹³<https://www.businessinsider.com/nfls-spin-average-career-length-2011-4>

3.5 Welfare

Theorem 3.4 shows that under plausible conditions, reducing μ_0 and increasing redistribution will increase median wealth w^* . Hence, moderate-talent players will always prefer to reduce μ_0 as they receive both higher initial rewards and an improved bargaining position in the future. Since they are in the majority, why can't they just impose such redistribution?

A slight perturbation of the model offers a rationale. Implicitly assumed is that player type is immediately known. This may not always be the case. For example, in sports, talent may be ascertained only after some games are played. Such a feature does not affect any of the earlier results, but it does explain why at $t = 0$, the majority of players may not, ex-ante, agree to maximum contracts. Suppose there was a lag before player type is realized. Then, player decisions on increasing the intensity of redistribution depend on the distribution of beliefs about their ability. Since rewards are type-contingent, they are determined by the *expected* size of the high-talent pool. However, player stance on redistribution is determined by *beliefs*. Ex-ante, a majority of players may assign a sufficiently high probability of being high-talent.

Example 1 Recall the sports example from [Section 2.1](#). Suppose players at $t = 0$ are unsure about their talent. Let $F(\cdot)$ be the distribution of their beliefs about their ability. The players must decide on whether to increase redistribution (decrease μ_0) beyond $\bar{\mu}$, where μ_0 represents the maximum contract: the maximum salary high-talent players can receive as a fraction of the salary cap. After μ_0 is set, ability is realized after an infinitesimal lag.

Suppose salaries s_h and s_l are dependent on the expected number of high and moderate-talent players. This is true in a Walrasian framework. The fraction of high-talent players will be $l_h = \mathbb{E}_F[b]$ almost surely ([Duffie and Sun \[2004\]](#)). However, a player with belief b has expected payoff $bs_h(l_h) + (1 - b)s_l$. If there is no future bargaining, such a player is against further salary limits if $b > l_h$. Hence, if $F^{-1}(\frac{1}{2}) > \mathbb{E}_F[b]$, a majority will not want to decrease μ_0 . With future bargaining, a player of belief b may still vote no if b is sufficiently high, and if conditional on being high-talent, gains in bargaining do not offset the loss of initial salary.

The example above demonstrates that when types are initially unknown, a majority at $t = 0$ will not approve of redistribution if:

1. A majority believe they are likely to be high-talent with sufficient probability.
2. Conditional on being high-talent, the gains in bargaining are not sufficient to offset the loss in the initial reward stream.

Thus, while redistribution always makes moderate-talent players better off, it is important to also think about when it will make high-talent players better off. Otherwise, a majority of players may not support redistribution. If high-talent players are also better off, then *all* players would prefer more redistribution *even if* there is uncertainty about player-type.

Proposition 3.5 *Suppose $\mu_0 \leq \bar{\mu}$ and $\pi_A > \pi_{min}$ at μ_0 . All players agree to increase redistribution (lowering μ_0) if:*¹⁴

$$\left(\mu_0 \cdot \frac{-\Delta \delta'(w^*)}{1 - e^{-\Delta(\delta^* + \rho)}} \cdot \frac{\partial w^*}{\partial(1 - \mu_0)} - 1 \right) \pi_A > (e^{\lambda_h \bar{t}} - 1) \pi_0 \quad (5)$$

Proof: Unanimous agreement can be achieved if redistribution also improves the payoff to players conditional on them being high-talent. Consider the payoff to a high-talent player:

$$U(h) = \int_0^{\bar{t}} \lambda_h e^{-\lambda_h t} s_h(\mu_0, \pi_0) dt + \int_{\bar{t}}^{\infty} \lambda_h e^{-\lambda_h t} s_h(\mu_0, \pi_A) dt = \mu_0 \frac{\pi_0}{l_h} \left(1 - e^{-\lambda_h \bar{t}} \right) + \mu_0 \frac{\pi_A}{l_h} e^{-\lambda_h \bar{t}}$$

The payoff increases with redistribution if the derivative with respect to $1 - \mu_0$ is positive:

$$e^{-\lambda_h \bar{t}} \mu_0 \cdot \frac{\partial \pi_A}{\partial w^*} \cdot \frac{\partial w^*}{\partial(1 - \mu_0)} > \pi_0 + e^{-\lambda_h \bar{t}} (\pi_A - \pi_0)$$

Using the expression for π_A in [Theorem 3.3](#), the above inequality is equivalent to:

$$\left(\mu_0 \cdot \frac{-\Delta \delta'(w^*)}{1 - e^{-\Delta(\delta^* + \rho)}} \cdot \frac{\partial w^*}{\partial(1 - \mu_0)} - 1 \right) \pi_A > (e^{\lambda_h \bar{t}} - 1) \pi_0 \quad \blacksquare$$

Even if players are uncertain about their ability, they will unanimously vote to increase redistribution if the condition in [Proposition 3.5](#) holds. The condition reflects the central trade-off for high-talent players: sacrifice initial rewards and a larger stake in the future share surplus for better bargaining power and a larger future share of surplus. Notice that $\frac{\partial w^*}{\partial(1 - \mu_0)} > 0$ is necessary for the inequality to hold. Inequality (5) highlights a key quantity of interest:

$$\underbrace{\frac{\frac{\partial \pi_A}{\partial(1 - \mu_0)}}{\pi_A}}_{\text{Relative change in surplus share}} = \underbrace{\frac{-\Delta \delta'(w^*)}{1 - e^{-\Delta(\delta^* + \rho)}}}_{\text{Change in Patience}} \cdot \underbrace{\frac{\partial w^*}{\partial(1 - \mu_0)}}_{\text{Gain in wealth}}$$

The left-hand side measures the increase in future share of surplus as a result of redistribution. Since $1 - e^{-\Delta(\delta^* + \rho)} \in (1 - e^{-\Delta(\rho)}, 1)$ for all parameter values, the crucial term is

¹⁴This proposition holds with general delay costs $\delta(\theta, w)$. In the general case, δ^* and w^* are defined as in the [Appendix](#). Then $\delta'(l, w^*)$ would replace $\delta'(w^*)$ in the proposition.

$-\delta'(w^*) \frac{\partial w^*}{\partial(1-\mu_0)}$: the product of the magnitude of the reduction in bargaining delay cost and the increase in w^* .

Thus, [Proposition 3.5](#) shows that increasing redistribution is Pareto improving if high-talent players are sufficiently long-lived relative to the time of bargaining (right-hand side of inequality (5) is low), and $-\delta'(w^*) \frac{\partial w^*}{\partial(1-\mu_0)}$ is sufficiently high. The magnitude of $\delta'(w^*) \frac{\partial w^*}{\partial(1-\mu_0)}$ depends critically on w^* . The proof of [Theorem 3.4](#) characterizes w^* and shows its dependence on the model's primitives. When λ_l is large relative to the time of bargaining \bar{t} , w^* is small. However, since moderate-talent players have short careers, it is difficult for them to accumulate wealth $\implies \frac{\partial w^*}{\partial(1-\mu_0)}$ is also small. Therefore, $-\delta'(w^*)$ must be large at low wealth levels for redistribution to benefit high-talent players. On the other hand, if λ_l is small relative to the time of bargaining (moderate-talent players have long careers), then w^* and $\frac{\partial w^*}{\partial(1-\mu_0)}$ are large: moderate-talent players can accumulate wealth. However, $-\delta'(w^*)$ may be small because w^* is already high.¹⁵

4 DISCUSSION

4.1 Model Assumptions

Parameterization In my model, players bargain with the league over “surplus”. Surplus is an abstraction given that collective bargaining in labor markets often includes bargaining over many things: wages, benefits, insurance coverage, and other amenities ([Cramton, Mehran, and Tracy \[2015\]](#)). While I assume the size of the available surplus is constant, if it grows over time, the rewards from a better bargaining position are greater. Finally, there is only a single instance of bargaining. I do this for tractability. Generally, collective bargaining takes place every fixed number of years. Allowing for multiple bargaining periods can make reducing maximum contracts more lucrative to *high-talent players* as the accumulation of wealth in each period between bargaining has a ratchet effect that helps in *subsequent* bargaining periods.

A substantive assumption is that players only bargain with the league over the share of surplus. What if players could negotiate over a new *maximum contract* (i.e. a “ μ_A ”)? Since there is only a single bargaining period, players do not need to think about how μ_A affects future bargaining. If there is no uncertainty about player type, all moderate-talent players will vote for more restrictive maximum contracts, while high-talent players will vote against. Furthermore,

¹⁵This would be the case if $\delta(\cdot)$ is convex. Since $\delta(\cdot)$ is decreasing, convexity implies $-\delta'(\cdot) > 0$, $-\delta''(\cdot) < 0$.

in a multi-dimensional bargaining setting, the model suggests that the league may also be in favor of more restrictive maximum contracts. The league only cares about the share of surplus π_A accorded to the players, while the pivotal player cares about μ_A and π_A . Thus, the league could take advantage of this by offering a lower π_A and a lower μ_A . On the other hand, as [Example 1](#) indicates, if there is uncertainty about ability, a majority of players may prefer to make the maximum contract less restrictive (reducing the maximum contract served its initial purpose of increasing players' bargaining power and can now be relaxed).

The assumption of two player types is for simplicity only. With multiple types, one must specify which types are giving up rewards and which types are the beneficiaries. With multiple types, maximum contracts would shift dollars downwards to the *next* highest player type. The lowest types would not enjoy any of these gains and continue to earn the minimum salary.

Player-Exit In the paper, I assume there is no exit in the bargaining stage when delays occur. However, I will argue that the main economic forces remain even when this is permitted. Suppose players exit according to a Poisson process during the bargaining stage and are replaced by players of the same talent. These new players have no accumulated wealth. As a result, median bargaining power of the coalition declines in each successive period. Let $w^*(k)$ be the median wealth level of the players' coalition after k periods of delay in bargaining. Set $k^* = \min \{k : w^*(k) = 0, k \text{ odd}\}$. Because $w^*(k)$ is decreasing, $w^*(k) = w^*(k^*)$ for all $k > k^*$. By [Theorem 3.3](#), at time k^* , the league offers $\max \left\{ \frac{e^{-\Delta\delta(0)}(1-e^{-\Delta\rho})}{1-e^{-\Delta(\delta(0)+\rho)}}, \pi_{min} \right\}$ and the coalition accepts. Since one knows the median wealth level at time $k^* - 1$, one can compute the players' proposal in period $k^* - 1$ (the median remains decisive). Proceeding via backward induction yields the equilibrium outcome in the first period. Now, suppose the maximum contract μ_0 declines, and let \hat{k}^* and $\hat{w}^*(\cdot)$ denote the corresponding objects in the new environment. The conditions outlined in [Theorem 3.4](#) imply that $\hat{k}^* \geq k^*$ and $\hat{w}^*(k) \geq w^*(k)$ for all $k \leq k^*$. This leads to an increase in player-share of surplus. Therefore, the result still holds.

Player-exit in non-bargaining periods follows a Poisson process. One might criticize this on the grounds that exit time should be age-dependent. Inclusion of this feature changes the distribution of wealth at the time of bargaining. The incentive to sacrifice current salary to improve bargaining position remains. However, the date of bargaining and functional form of time-varying discounting will matter.¹⁶ The Poisson assumption highlights the effects of wealth-dependent discounting. If exit *during* bargaining depends on age as well, the model

¹⁶[Schweighofer-Kodritsch \(2018\)](#) studies Rubinstein bargaining under time-dependent discounting.

becomes more complex. Bargaining power of the coalition is not necessarily decreasing with delay since the characteristics of the coalition change across two dimensions: wealth and age.

Bargaining Process In the model, decisions are made via simple majority-rule, which is the criteria used in the NBA’s collective bargaining agreement. However, one may argue that the pivotal voter is not simply the one with median bargaining power. After all, some players follow their teammates or the guidance of agents who represent groups of players. Hence, it makes sense to consider decision rules for any threshold $q > \frac{1}{2}$.¹⁷ As pointed out in the appendix, [Theorems 3.3](#) and [3.4](#) generalize easily (the pivotal voter is simply the player with the q^{th} -lowest bargaining power). The higher the threshold q , the better the players’ bargaining ability. Fixing a maximum contract level μ_0 , players will always prefer a decision rule with threshold q to one with threshold \hat{q} for $q > \hat{q}$.¹⁸

This raises the following question: why doesn’t the player coalition ex-ante adopt a unanimous threshold decision rule? The primary obstacle is the lack of enforceability. In reality, the decision rule utilized by the players’ union can always be changed. It is doubtful that it could commit to a rule that delegates authority to the wealthiest, high-talent player. Hence, the maximum threshold that could be enforceable may be the value q such that the league can operate if a measure q players want to play. If q is sufficiently large, the only player with the threshold level of bargaining power is of high-talent. Then, maximum contracts would *reduce* the coalition’s bargaining power. In this setting, high-talent players would remain opposed to redistribution, but moderate-talent players now face a trade-off: sacrifice future bargaining power to increase current rewards.

4.2 Implications and Conclusion

The story of negotiations between players and the league illustrates two fundamental aspects of collective bargaining:

1. It is often between a coalition of heterogeneous agents and a single counterparty.

¹⁷To be clear, this “herding” argument implies that a small coalition of players must be placated to reach an agreement. Suppose this small coalition comprises the wealthiest and most powerful players. Having to placate them is outcome-equivalent to *increasing* the threshold q . More generally, if the league must appease a fraction of an arbitrary subset Q of players, then one need only identify the pivotal voter in that subset.

¹⁸Increasing the threshold q is outcome-equivalent to permitting a bounded amount of side payments where wealthy players give moderately-talent players wealth now to increase their patience. Notice that allowing side payments is not equivalent to decreasing the maximum contract. When the maximum contract is lowered, the salary of all moderate-talent players is increased. With side payments, the most patient players need only subsidize the subset of players with bargaining power greater than or equal to the pivotal value.

2. While the union negotiates over the share of surplus accorded to the players as a whole, equally important is how that share is distributed amongst the players.

Thus, while the model is framed in the context of professional sports collective bargaining, the insights apply more broadly to settings where a coalition of heterogeneous agents negotiates with a single institution over a share of tomorrow's surplus. Conditional on the share of today's surplus allocated to agents, how that share is distributed amongst the agents is critical. Importantly, redistributive policies before bargaining are akin to reallocating property rights over the share of surplus won. Such redistribution can harmonize agents' interests by decreasing delay costs and incentivizing those who would have been more inclined to settle to be more aggressive. For instance, consider legislative environments where a single agenda setter negotiates with a group of voters. My results suggest that voters can all benefit from ex-ante redistribution amongst themselves.

My paper has immediate policy consequences for professional sports leagues. Within the United States, the NBA has instituted maximum contracts, limiting salaries of high-talent players to a percentage of the salary cap (i.e. the share of surplus accorded to the players as a collective). The impact of maximum contracts in the NBA is reflected in the outcomes of recent negotiations and public opinion of the strength of the players' coalition. Since the 1998-99 lockout, player share of revenue declined from 55% to 50%, where it has remained stable for over 14 years. This is not at odds with my model. Rather, my model predicts that without such contracts, owners would have been able to extract even more rents. Furthermore, while revenue in my model (size of surplus) is the same at each time t , it has actually increased over the years due to growing viewership, making deterring owner rent extraction crucial.¹⁹

Comparing across sports, the NBA players' union is seen as more effective than the NFL players' union. A key distinction between the two is that the NFL operates with a salary cap and no limit on the size of individual contracts. Oddly, it is the NFL owners who have often expressed a desire for maximum contracts. Players seem to be wary.²⁰ The most recent NFL collective bargaining agreement extended the season by a game and shifted little of the additional revenue to the players. Before the deal was signed, stars protested the proposal but

¹⁹One may wonder if the model's findings truly apply to the NBA. Since the minimum salary of NBA players is extremely high (in the millions), the discount factors will not vary much across players. However, current discount levels in the NBA are endogenous. They are a positive byproduct of the outcomes of previous collective bargaining negotiations. My model suggests that maximum contracts improved player bargaining power over time, naturally leading to little variance in discount factors.

²⁰<https://bleacherreport.com/articles/2884885-nfl-owners-reportedly-wanted-nba-like-max-contracts-in-new-cba>

could not persuade others to join them. Most NFL players have short careers relative to the stars. Many live paycheck to paycheck and cannot afford a lockout.²¹ Moreover, the fraction of stars (i.e. Quarterbacks) relative to the total number of players in the league is small. The NFL is similar to the “pre-max-contract” NBA in terms of the wealth disparity between high and moderate-talent players. My model suggests that NFL players may benefit from maximum contracts as such contracts will increase leverage in future negotiations, leading to players receiving a higher percentage of league revenue.

The implications of my model extend to other labor markets as well. In traditional labor markets, unions can implement direct and indirect redistributive policies without requiring management’s approval.²² My model also points to the benefits of forming bargaining coalitions in industries without unions. Especially relevant are industries where employees share in league profits: traders, law associates, and investment bankers. The dominant fraction of take-home compensation comes from bonus pay. The mechanism by which they receive this compensation is similar to that of a sports team. Each group within a league is allocated a pool of money in proportion to revenue generated (e.g. a salary cap). From there, the director of the group pays the employees. My model suggests that creating a coalition in these industries and instituting a cap on bonus compensation will improve the welfare of all players.

²¹http://www.nbcnews.com/id/41855264/ns/business-personal_finance/t/nfl-owners-wont-run-hurry-up-offense-vs-players

²²Examples include profit-sharing, implicit or explicit caps on individual salary, discriminatory fees, and additional funds that are paid to players in the form of benefits (Pencavel [1991]).

A COLLECTIVE BARGAINING APPENDIX

All notation refers to the same objects as before, unless otherwise stated. I refer to a player by their talent level and wealth at the time of bargaining. A player is an ordered-pair $(\theta, w) \in \text{Supp}(G)$. A group of players is a subset of $\{(\theta, w) | \theta \in \{l, h\} \text{ and } w \leq s_{\theta} \bar{t}\}$.

A.1 Subgame Perfect Majority Rule

Definition A.1 A history h^n is a sequence of n offers and acceptance/rejection decisions at each time $t \leq n$.

Definition A.2 Strategies for the players' coalition (σ_W) and the league (σ_F) are mappings from each h^n to an offer OR acceptance/rejection choice.

1. $\sigma_W(h^{2n}) \in [0, 1]$ and $\sigma_W(h^{2n-1}) \in \{A, R\}$, $\forall n \geq 1$.
2. $\sigma_F(h^{2n-1}) \in [0, 1]$ and $\sigma_F(h^{2n}) \in \{A, R\}$, $\forall n \geq 1$.

Definition A.3 Given σ_W and σ_F , a path of play z is the realized sequence of offers and acceptance/rejection decisions. Since the game terminates when there is an acceptance, t_z is the time at which an agreement is reached. The outcome is characterized by the accepted offer and time of acceptance.

Definition A.4 Fixing a players' coalition G , define $X(m, z)$ as the payoff to player $m \in \text{Supp}(G)$ under path of play z .

Definition A.5 Fix a strategy σ_F for the league. A strategy σ_W for players is said to violate the majority if there is a deviation after some history such that there exists a closed group of players M , $G(M) \geq \frac{1}{2}$, with members of M strictly better off.²³

Definition A.6 An SPMRE is a pair of strategies σ_W and σ_F such that σ_W is not in violation of the majority, and at any history, there is no deviation by the league that leaves it better off.

Lemma A.7 Fix σ_F and σ_W . If player (θ, w) prefers $\hat{\sigma}_W$ to σ_W , then at least one of the following is true: all players (θ, \hat{w}) , $\hat{w} \leq w$, OR all players (θ, \hat{w}) , $\hat{w} \geq w$, prefer $\hat{\sigma}_W$ to σ_W .

²³If G were continuous, the condition can be relaxed to require that there be no M that is weakly better off with a positive measure subset strictly better off. Lemma A.8 and its proof remain intact. In this case, the condition for an SPMRE on the players' side is analogous to requiring that every set M with $G(M) \geq \frac{1}{2}$ use a weakly undominated voting strategy. The condition can not be relaxed in general because issues arise when G contains atoms. The proof of Lemma A.8 breaks down, and the one-shot deviation principle does not hold.

Proof: Under σ_W , the realized path of play is z , and the outcome is π_z at time t_z . Consider an alternative strategy $\hat{\sigma}_W$ that induces a path of play \hat{z} and outcome $\pi_{\hat{z}}$ at time $t_{\hat{z}}$. Suppose player (θ, w) prefers $\hat{\sigma}_W$ to σ_W . If $t_{\hat{z}} \geq t_z$, all players with $\delta(\theta, \hat{w}) \leq \delta(\theta, w)$ will prefer $\hat{\sigma}_W$. If $t_{\hat{z}} \leq t_z$, all players with wealth *lower than* w will prefer $\hat{\sigma}_W$. ■

Lemma A.7 implies we can restrict to M such that $\{(l, w) | (l, w) \in M\}$ and $\{(h, w) | (h, w) \in M\}$ are path-connected in wealth.²⁴ Since the atoms of G occur at the maximum wealth levels of each type, Lemma A.7 also implies that when checking SPMRE, it is sufficient to look at deviating coalitions of the form:

$$\left\{ M \subset \text{supp}(G) | M \text{ is closed, } G(M) \geq \frac{1}{2} \text{ and } G(\hat{M}) < \frac{1}{2}, \forall \hat{M} \subset M \right\}$$

Given a path of play \bar{z} , consider a modified bilateral Rubinstein bargaining game, $\mathcal{R}(M, \bar{z})$ for any such $M \in S$. In $\mathcal{R}(M, \bar{z})$, strategies for Player 1 and Player 2 are labeled Σ_1 and Σ_2 , respectively. Player 1's payoff is equivalent to that of the league. Player 2's payoff, P_2 , is defined over each possible path of play z :

$$P_2(z) = \inf_{m \in M} X(m, z) - X(m, \bar{z})$$

Since M is closed, there exists $m_z^* \in M$ such that $P_2(z) = X(m_z^*, z) - X(m_z^*, \bar{z})$.²⁵ Payoff function P_2 satisfies consistency and continuity as defined in Fudenberg and Tirole (1991) and Ray (2003). It follows that if a subgame perfect equilibrium exists in $\mathcal{R}(M, \bar{z})$, then the one-shot deviation principle holds (and vice-versa).

Lemma A.8 *The profile (σ_F, σ_W) is an SPMRE if and only if $(\Sigma_1, \Sigma_2) = (\sigma_F, \sigma_W)$ is a subgame perfect equilibrium in $\mathcal{R}(M, \bar{z}) \forall M \in S$ and paths of play \bar{z} in the collective bargaining game under (σ_F, σ_W) .*

Proof: Suppose (σ_F, σ_W) is an SPMRE. Fix a minimal deviating coalition $M \in S$. I need only show that (σ_F, σ_W) is subgame optimal in $\mathcal{R}(M, \bar{z})$. Consider the strategy profile $\Sigma_1 = \sigma_F$ and $\Sigma_2 = \sigma_W$. From Player 1's perspective, Σ_1 is optimal. Suppose $\Sigma_2 = \sigma_W$ is *not* subgame optimal

²⁴Suppose one requires that a player be selected at random to make an offer on behalf of the coalition. Each player's reservation value then involves an expectation over the identity of the future proposer, leading to a non-degenerate lottery over future payoffs. If one player prefers one lottery to another, it is not guaranteed that the set of players who also prefer that lottery is path-connected (Banks and Duggan [2006]; Duggan [2014]). One would require additional assumptions on $\delta(\cdot)$ for it to remain true.

²⁵The subscript reflects dependence on z .

in $\mathcal{R}(M, \bar{z})$. Then there is a one-shot deviation strategy $\hat{\Sigma}_2$ with $P_2(\Sigma_1, \hat{\Sigma}_2) > P_2(\Sigma_1, \Sigma_2) = P_2(\sigma_F, \sigma_W) = 0$. This means (σ_F, σ_W) is not an SPMRE in the collective bargaining game. The reverse direction follows trivially. ■

Since a one-shot deviation in the collective bargaining game for any M is equivalent to a one-shot deviation in $\mathcal{R}(M)$, it follows that: (σ_F, σ_W) is an SPMRE if and only if:

1. $\forall M \in S$, σ_W is unimprovable with respect to players in M via a one-shot deviation.²⁶
2. σ_F is unimprovable via a one-shot deviation for the league.

Remark: The above proofs do not rely on the threshold being $\frac{1}{2}$. If the threshold is $q > \frac{1}{2}$, one simply needs to adjust the definition of the minimal deviating coalition to be of size q .

A.2 Collective Bargaining Outcome

I prove the collective bargaining results for general bargaining delay costs $\delta(\theta, w)$ where δ is decreasing in wealth for all θ . I assume that for any w , there exists $w' \geq w$ such that $\delta(l, w') = \delta(h, w)$.²⁷ This is not needed and is purely for exposition. I will also use the notation $\delta_\theta(w) = \delta(\theta, w)$ when necessary. For completeness, I will repeat some of the arguments in the body of the paper, but this time with the general delay cost δ .

Consider a hypothetical situation where $s_{min} = 0$ and a type θ player with wealth w negotiates on behalf of the coalition. From [Rubinstein \(1982\)](#), there is a unique subgame perfect equilibrium where the players receive:

$$\pi^*(\theta, w) = \frac{e^{-\Delta\delta(\theta, w)}(1 - e^{-\Delta\rho})}{1 - e^{-\Delta(\delta(\theta, w) + \rho)}}$$

π^* quantifies the bargaining power of a type θ player with wealth w . Notice π^* depends on (θ, w) only through the delay cost in bargaining $\delta(\theta, w)$. A player with a *high value* of $\delta(\theta, w)$ (high delay cost) has *low bargaining power*: $\pi^*(\theta, w) > \pi^*(\theta, w')$ if and only if $\delta(\theta, w) < \delta(\theta, w')$. A player with **median bargaining power** has delay cost δ^* such that:

$$\delta^* = \sup \left\{ \hat{\delta} : \mathbb{P}_G(\{(\theta, w) | \delta(\theta, w) > \hat{\delta}\}) \geq \frac{1}{2} \right\}$$

²⁶This is an extension of the one-shot deviation property in [Blackwell \(1965\)](#), except such deviations must be checked for each minimal deviating coalition.

²⁷In other words, for any wealth level w of a high-talent player, there is a larger wealth level w' such that a moderate-talent player with wealth w' has the same delay cost.

One can express δ^* in terms of player type and wealth level. First, recognize that a high-talent player can have the same bargaining power as a moderate-talent player since both could have the same delay cost due to different wealth levels. This is not significant because what matters is the bargaining outcome itself which depends only on the delay cost. While there may be a high-talent player with median bargaining power, there will always be a moderate-talent player with median bargaining power since the measure of high-talent players is $l_h < \frac{1}{2}$. Thus, there exists w^* such that $\delta(l, w^*) = \delta^*$.

Lemma A.9 *Consider a moderate-talent player with wealth w^* . All moderate-talent players with wealth lower than w^* have less bargaining power. All high-talent players with wealth less than $\delta_h^{-1}(\delta_l(w^*))$ have less bargaining power.*

Proof: Such a player has delay cost $\delta(l, w^*)$. Since the function is decreasing in wealth, moderate-talent players with wealth less than w^* have higher delay costs and, therefore, lower bargaining power. Now, notice that $\delta(h, w) \leq \delta(l, w^*) \iff w \leq \delta_h^{-1}(\delta_l(w^*))$. ■

When $s_{min} = 0$, each player's payoff increases linearly with the share of player surplus. In this case, the outcome of a bargaining game between the league and any individual delegate depends only on $\delta(\theta, w)$. If $s_{min} > 0$, we need to incorporate the fact that players are guaranteed at least s_{min} . Since there is a range of player surplus shares leading to the moderate-talent player receiving s_{min} , their value for a share of surplus π is piece-wise linear: constant for $\pi \in [\pi_{min}, \frac{s_{min}(1-l_h)}{1-\mu_0}]$ and linearly increasing in π when $\pi > \frac{s_{min}(1-l_h)}{1-\mu_0}$.²⁸ Thus, if a moderate-talent player with wealth w negotiates on behalf of the coalition, the resulting outcome is:²⁹

$$\pi^*(l, w) = \begin{cases} \max \left\{ \frac{e^{-\Delta\delta(l, w)}(1-e^{-\Delta\rho})}{1-e^{-\Delta(\delta(l, w)+\rho)}}, s_{min} \right\} & \text{if } \frac{1-e^{-\Delta\rho}}{1-e^{-\Delta(\delta(l, w)+\rho)}} > \frac{s_{min}(1-l_h)}{1-\mu_0} \\ \pi_{min} & \text{if } \frac{1-e^{-\Delta\rho}}{1-e^{-\Delta(\delta(l, w)+\rho)}} \leq \frac{s_{min}(1-l_h)}{1-\mu_0} \end{cases}$$

Proof of Theorem 3.3: Let (σ_W, σ_F) denote the SPE strategy profile in a traditional Rubinstein bargaining game where the moderate-talent player of wealth w^* negotiates with the league on behalf of the players (the outcome is given by $\pi^*(l, w)$, which is defined above). Given Lemma A.7 and because this is an SPE in the traditional Rubinstein Bargaining game, any deviating

²⁸The term $\frac{s_{min}(1-l_h)}{1-\mu_0}$ is the maximum fraction of surplus that still guarantees the moderate-talent players receive the minimum salary.

²⁹This is an immediate consequence of Rubinstein (1982). Intuitively, $\frac{1-e^{-\Delta\rho}}{1-e^{-\Delta(\delta(l, w)+\rho)}} \leq \frac{s_{min}(1-l_h)}{1-\mu_0}$ means the moderate-players have little bargaining power, and the league will offer π_{min} .

coalition that could do better either excludes $\{(\theta, w) : \theta = l, w \leq w^*, \text{ and } \theta = h, w \leq \delta_h^{-1}(\delta_l(w^*))\}$ or the set of all moderate-talent players. These sets have a minimum size of $\frac{1}{2}$ by definition of w^* . Thus, (σ_W, σ_F) must be an SPMRE.

Next, I demonstrate uniqueness. Denote the moderate-talent player of median bargaining power as m_b . Lemmas A.7 implies that in any SPMRE, m_b must approve of the outcome. Let v_{hi} and v_{lo} be the maximum and minimum value to the league in any SPMRE *starting in a period when the league makes an offer*. Since the league's payoff is just the share of surplus accorded to them, $1 - v_{lo}$ and $1 - v_{hi}$ are the players' share of surplus. Moreover, $v_{hi} \leq 1 - \pi_{min}$ as the players are guaranteed at least π_{min} . As player m_b must approve of the outcome define the function $f(\pi)$ to be the value to m_b from the players receiving a share of surplus π .³⁰

Consider a period when the players' coalition makes an offer. The league will accept any offer of at least $e^{-\Delta\rho} v_{hi}$ and reject any offer below $e^{-\Delta\rho} v_{lo}$. This means player m_b can secure a payoff of at least $f(1 - e^{-\Delta\rho} v_{hi})$ and at most $f(1 - e^{-\Delta\rho} v_{lo})$. Now, consider a period when the league makes an offer. To get m_b to accept, the league must offer at least $\max\left\{e^{-\Delta\delta(l, w^*)} \cdot \min\left\{\pi : f(\pi) = f(1 - e^{-\Delta\rho} v_{hi})\right\}, \pi_{min}\right\}$.³¹ Moreover, m_b will accept if the league offers $\max\left\{e^{-\Delta\delta(l, w^*)} \cdot \min\left\{\pi : f(\pi) = f(1 - e^{-\Delta\rho} v_{lo})\right\}, \pi_{min}\right\}$. Putting this together:

$$v_{hi} \leq 1 - \max\left\{e^{-\Delta\delta(l, w^*)} \cdot \min\left\{\pi : f(\pi) = f(1 - e^{-\Delta\rho} v_{hi})\right\}, \pi_{min}\right\}$$

$$v_{lo} \geq 1 - \max\left\{e^{-\Delta\delta(l, w^*)} \cdot \min\left\{\pi : f(\pi) = f(1 - e^{-\Delta\rho} v_{lo})\right\}, \pi_{min}\right\}$$

If $e^{-\Delta\delta(l, w^*)} \cdot \min\left\{\pi : f(\pi) = f(1 - e^{-\Delta\rho} v_{lo})\right\} > \pi_{min}$, combining the two inequalities yields:

$$v_{lo} \geq \frac{1 - e^{-\Delta\delta(l, w^*)}}{1 - e^{-\Delta(\delta(l, w^*) + \rho)}} \geq v_{hi} \implies v_{lo} = v_{hi} = \frac{1 - e^{-\Delta\delta(l, w^*)}}{1 - e^{-\Delta(\delta(l, w^*) + \rho)}}$$

If $e^{-\Delta\delta(l, w^*)} \cdot \min\left\{\pi : f(\pi) = f(1 - e^{-\Delta\rho} v_{lo})\right\} < \pi_{min} \implies v_{lo} \geq 1 - \pi_{min} \geq v_{hi}$. Therefore, $v_{hi} = v_{lo} = 1 - \pi_{min}$. A symmetric argument for player m_b completes the proof. ■

Remark: The results here extend easily to thresholds $q \geq \frac{1}{2}$ (as long as δ^* is defined with respect to q). The only new case is when q is sufficiently high. Then there will only be a high-talent player with the threshold level of bargaining power. This high-talent player determines the outcome of bargaining.

³⁰Recognize that $f(\pi) = s_{min}$ whenever $\pi \in [\pi_{min}, \frac{s_{min}(1-l_h)}{1-\mu_0}]$.

³¹The reason for the use of the set $\{\pi : f(\pi) = f(1 - e^{-\Delta\rho} v_{hi})\}$ is because f is one-to-one $\iff \pi > \frac{s_{min}(1-l_h)}{1-\mu_0}$.

Proof of Theorem 3.4: I prove the theorem for when delay costs are $\delta(\theta, w)$ and the threshold is some $q \geq \frac{1}{2}$.

Given time is continuous, and there is a continuum of players of fixed measure, the exact law of large numbers holds (Duffie and Sun [2004]). The wealth distribution of type θ players at time \bar{t} is:

$$\begin{aligned}\mathbb{P}(w_\theta(\bar{t}) \geq u) &= e^{-\lambda_\theta \frac{q}{s_\theta}} \text{ for } u < s_\theta \bar{t} \\ \mathbb{P}(w_\theta(\bar{t}) = s_\theta \bar{t}) &= e^{-\lambda_\theta \bar{t}}\end{aligned}$$

Define $\bar{w}(w) = \max \{0, \delta_h^{-1}(\delta_l(w))\}$.³² Recall that $\delta_h^{-1}(\delta_l(w))$ is the wealth level of a high-talent player with the same bargaining power as a moderate-talent player of wealth w . The threshold-level of bargaining power is $\delta^* = \delta(l, w^*)$, where:

$$w^* = \inf \left\{ w : (1 - l_h)(1 - e^{-\lambda_l \frac{w}{s_l}}) \chi_{w \leq s_l \bar{t}} + (1 - l_h)e^{-\lambda_l \bar{t}} \chi_{w = s_l \bar{t}} + l_h(1 - e^{-\lambda_h \frac{\bar{w}(w)}{s_h}}) \geq q \right\} \quad (6)$$

The expression is complex due to an atom at the wealth level $s_l \bar{t}$. There are three cases to consider: $w^* = s_l \bar{t}$, $w^* > s_l \bar{t}$, and $w^* < s_l \bar{t}$

Case #1:

If $w^* = s_l \bar{t}$ then $\mathbb{P}_G(\{(\theta, w) | \delta(\theta, w) > \delta^*\}) > q$. Any small change in redistribution keeps the wealth level of the moderate-talent player with the threshold-level of bargaining power at the atom. Therefore, $\frac{\partial w^*}{(\partial 1 - \mu_0)} = \frac{\partial s_l}{(\partial 1 - \mu_0)} \bar{t} > 0$. The wealth level w^* occurs at the atom when sufficiently many moderate-talent players that started at $t = 0$ live until the time of bargaining. Thus, there exists ε such that $\lambda_h \leq \lambda_l \leq \varepsilon \implies w^* = s_l \bar{t}$.

Case #2:

If $w^* > s_l \bar{t}$, then there is no moderate-talent player with the threshold-level of bargaining power. Lowering the maximum contract reduces bargaining power as it decreases the wealth of high-talent players. This case does not arise when $q = \frac{1}{2}$.

Case #3:

For the third case, $w^* < s_l \bar{t} \implies$:

$$(1 - l_h)(1 - e^{-\lambda_l \frac{w^*}{s_l}}) + l_h(1 - e^{-\lambda_h \frac{\bar{w}(w^*)}{s_h}}) = q \quad (7)$$

³²If $\delta_h^{-1}(\delta_l(w))$ does not exist, take $\bar{w}(w) = 0$.

Holding w^* fixed and differentiating (7) with respect to $1 - \mu_0$ yields the following necessary and sufficient condition for $\frac{\partial w^*}{\partial(1-\mu_0)} > 0$:

$$-e^{-\lambda_l \frac{w^*}{s_l}} \lambda_l w^* \frac{1}{s_l^2} + e^{-\lambda_h \frac{\bar{w}}{s_h}} \lambda_h \bar{w} \frac{1}{s_h^2} < 0 \quad (8)$$

Suppose $\lambda_h \leq \lambda_l$. Since $\delta(h, w) \leq \delta(l, w)$, it follows that $\bar{w}(w^*) \leq w^*$. Using Equation (6), one can compute the following bounds on $\lambda_l \frac{w^*}{s_l}$:

$$\lambda_l \frac{w^*}{s_l} \in \left[-\log(q), \min \left\{ \lambda_l \bar{t}, \log \left(\frac{1-l_h}{q-l_h} \right) \right\} \right]$$

Since this set is compact, the function $e^{-\lambda_l \frac{w^*}{s_l}} \lambda_l w^* \frac{1}{s_l}$ achieves a minimum at some $\lambda_l \frac{w^*}{s_l} = y$. In fact, y will be one of the endpoints of the interval because the function $e^{-x}x$ attains its maximum at $x = 1$. Inequality (8) is guaranteed to hold when:

$$-e^y y \frac{1}{s_l} + e^{-\lambda_h \frac{\bar{w}}{s_h}} \lambda_h \bar{w} \frac{1}{s_h^2} < 0 \quad (9)$$

As λ_h declines, $e^{-\lambda_h \frac{\bar{w}}{s_h}} \lambda_h \bar{w} \frac{1}{s_h^2}$ approaches 0 \implies there exists k such that Inequality (9) holds for $\lambda_h \leq k$.³³ Hence, $\lambda_h \leq \min \{l_h, k\} \implies$ a reduction in μ_0 leads to an increase in w^* .

In particular, when $l_h < \frac{qe-1}{e-1}$, then $\lambda_h \leq \lambda_l$ is all that is required. To see this, notice that $l_h < \frac{e-2}{2e-2} \implies \lambda_l \frac{w^*}{s_l} \leq 1 \implies e^{-\lambda_l \frac{w^*}{s_l}} \lambda_l w^* \frac{1}{s_l} > e^{-\lambda_h \frac{\bar{w}(w^*)}{s_h}} \lambda_h \bar{w}(w^*) \frac{1}{s_h^2}$. The last implication stems from the fact that $e^{-x}x$ is increasing for $x \in [0, 1]$. Since $s_l < s_h$, it follows that:

$$e^{-\lambda_l \frac{w^*}{s_l}} \lambda_l w^* \frac{1}{s_l^2} > e^{-\lambda_h \frac{\bar{w}(w^*)}{s_h}} \lambda_h \bar{w}(w^*) \frac{1}{s_h^2}$$

\implies Inequality (8) holds.

Thus, if $l_h < \frac{qe-1}{e-1}$, then $\lambda_h \leq \lambda_l$ implies that increasing redistribution increases w^* . ■

³³Notice that k may depend on l_h , λ_l , and \bar{t} .

B ROBUSTNESS

Given a share of surplus π , the reward stream for a type θ player is $\alpha_\theta \pi$, where $\alpha_h = \mu_0$ and $\alpha_l = 1 - \mu_0$. Suppose players value reward s as $V(s)$. The league values a share of surplus $1 - \pi$ at $F(1 - \pi)$. Assume F and V are increasing, continuous, and $F(0) = V(0) = 0$.

Proposition B.1 *Consider a Rubinstein bargaining game between the league and a type- θ player with discount rate δ_W . There is a subgame perfect equilibrium where the player receives π_A :*

$$1 - \frac{1}{\alpha_\theta} V^{-1} \left(\frac{1}{\delta_W} V(\alpha_\theta \pi_A) \right) = \rho F(1 - \pi_A)$$

Proof: Let f denote the equilibrium value to the league when proposing first and v the equilibrium value to the player when proposing first. [Rubinstein \(1982\)](#) implies that equilibrium values are characterized by the solution to the following system:

$$V \left(\alpha_\theta (1 - F^{-1}(f)) \right) = \delta_W v \tag{10}$$

$$1 - \frac{1}{\alpha_\theta} (V^{-1}(v)) = \rho f \tag{11}$$

Solving for v in (10) and substituting it into (11) yields:

$$1 - \frac{1}{\alpha_\theta} V^{-1} \left(\frac{1}{\delta_W} V(\alpha_\theta (1 - F^{-1}(f))) \right) = \rho f$$

Since $F^{-1}(f) = 1 - \pi_A$ and the league makes offers first, the proposition follows. ■

When $V(\alpha_\theta \pi) = \alpha_\theta \pi$ and $F(1 - \pi) = 1 - \pi$, it reduces to the model discussed in the paper. In general, as long as V is increasing, [Theorem 3.3](#) and [Theorem 3.4](#) hold. What is not guaranteed is that bargaining power is determined entirely by the delay cost in bargaining. Before providing insight into why, I highlight when this would remain the case.

Proposition B.2 *If $V(xy) = V(x)V(y)$, then independent of the functional form of F , all the results from the paper still hold.*

Proof: $V(xy) = V(x)V(y) \implies \frac{1}{\alpha_\theta} V^{-1} \left(\frac{1}{\delta_W} V(\alpha_\theta \pi_A) \right) = V^{-1} \left(\frac{1}{\delta_W} \right) \pi_A$

$$\implies \pi_A = 1 - \frac{1}{\rho} F^{-1} \left(1 - V^{-1} \left(\frac{1}{\delta_W} \right) \pi_A \right)$$

$\implies \pi_A$ depends on the level of redistribution through the wealth-dependent discount rate. ■

The above proposition demonstrates that for common functions like $V(x) = x^k$ for $k > 0$, all the results from the paper still hold. However, it does reveal where complications may arise. To provide intuition, fix the payoff function for the league to be $F(1 - \pi) = \pi$. Then the equilibrium share of surplus accorded to the players is:

$$1 - \rho = \frac{1}{\alpha_\theta} V^{-1} \left(\frac{1}{\delta_W} V(\alpha_\theta \pi_A) \right) - \rho \pi_A$$

How $\frac{1}{\alpha_\theta} V^{-1} \left(\frac{1}{\delta_W} V(\alpha_\theta \pi_A) \right)$ changes when α_θ increases will depend on the properties of the function V . α_θ affects this quantity directly and through δ_W ! Before, α_θ only affected bargaining through δ_W by changing the initial reward stream and thus changing the accumulated wealth by the time of bargaining.

Example 2 Let $V(\alpha_\theta \pi) = \log(1 + \alpha_\theta \pi)$ and $F(1 - \pi) = 1 - \pi$. Suppose a type θ player represents the coalition and engages in bargaining with the league. The equilibrium equation is:

$$1 - \rho = \frac{1}{\alpha_\theta} (1 + \alpha_\theta \pi_A)^{\frac{1}{\delta_W}} - \rho \pi_A$$

The term $\frac{1}{\alpha_\theta} (1 + \alpha_\theta \pi_A)^{\frac{1}{\delta_W}}$ represents the share of surplus such that the player would be indifferent between accepting that share in the next period and accepting π_A now. If α_θ increases so that the representative earns a higher share of the surplus accorded to the players, π_A may go down!³⁴ As α_θ increases, the player has a strictly larger payoff at every share of surplus, but $\frac{1}{\alpha_\theta} V^{-1} \left(\frac{1}{\delta_W} V(\alpha_\theta \pi_A) \right)$ is sufficiently concave in π_A at the higher levels of α_θ . The league can credibly reduce the share it accords the player.

The example illustrates how redistribution can have bidirectional effects for some $V(\cdot)$. Increasing redistribution helps in bargaining by increasing wealth levels and reducing median delay costs. However, it also increases α_l , which may make moderate-talent players more passive and incentivize the league to *reduce* the share of surplus accorded to the players.

³⁴Take $\delta_W = 0.4$, $\rho = 0.9$, and vary α_θ from $\frac{1}{2}$ to $\frac{2}{3}$. The player share of surplus declines.

C MICROFOUNDING WEALTH-DEPENDENT DISCOUNTING

In the paper, I highlight empirical evidence that wealthier individuals discount the future less than poorer individuals. Here, I argue that the assumption of a constant wealth-dependent discount factor can be motivated by a consumption-saving type model. Recognize that in any such model, players need not worry how their individual decisions affect bargaining.

Many major expenditures involve periodic payments, including mortgages, car leases, yacht payments, wealth management fees, and familial loans. Therefore, wealth declines each period when a player is not earning a salary. To cover these costs, a player may borrow against their assets, and the interest rate a player would be charged is a function of their current wealth in that period. A player at the start of bargaining has initial wealth w_0 and must pay costs c_t each period t . I assume c_t is dependent on initial wealth w_0 . The case where c_t is constant reflects an environment where the player has constant yearly bills and consumption. Each period, a player's discount factor is given by a decreasing function $f(\cdot)$ (interest rates are higher in periods where the player has less wealth) bounded below by some f_{min} . For simplicity, assume that there is an m such that $f(w) = f_{min}$ when $w < m$.

If an agreement isn't reached until period k , the future payoff of a player is reduced by $f(w - c_1) \cdot f(w - c_1 - c_2) \cdots f(w - \sum_{t=1}^k c_t)$, where $f(w - \sum_{t=1}^i c_t) = f_{min}$ when $w - \sum_{t=1}^i c_t < m$. The per-period discount factor declines in each successive period. If wealthier players have more wealth, even after costs are accounted for, the median discount factor declines each period. Formally, the sufficient condition for this is $w - \sum_{t=1}^i c_t > \hat{w} - \sum_{t=1}^i \hat{c}_t$ for each i when $w > w'$.

Define f_t^{med} to be the median value of $f(w_0 - \sum_{t=1}^k c_t)$ amongst the player's coalition in period t , and let $k^* = \min \{t : f_t^{med} = f_{min}, k \text{ odd}\}$. By [Theorem 3.3](#), at time k^* , the league offers $\pi_A(f_{min})$, and the players accept. Since one knows the median discount factor at all times before k^* , backward induction yields the equilibrium outcome in the first period. Now, suppose maximum contracts become more restrictive. Let \hat{k}^* and $\hat{w}^*(\cdot)$ denote the corresponding objects in the new environment. Under the conditions of [Theorem 3.4](#), $\hat{k}^* \geq k^*$ and $\hat{w}^*(k) \geq w^*(k)$ for $k \leq k^*$. This leads to player-share of surplus *increasing*.

To obtain closed-form solutions, I take discounting to be constant and dependent only on initial wealth. In other words, $f(\cdot)$ and the cost stream $\{c_t\}$ are well-behaved so that there exists a function $\delta(w)$ such that $\delta(w)^k$ best-approximates $f(w - c_1) \cdot f(w - c_1 - c_2) \cdots f(w - \sum_{t=1}^k c_t)$ for $k \leq k^*$.

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