Majority-Rule Collective Bargaining and the Benefits of Redistribution*

Akhil Vohra†

September 16, 2021

Abstract

I develop a model of majority-rule collective bargaining between a firm and its workers when delay costs incurred by workers are wealth-dependent. Such costs arise from liquidity constraints reducing workers' ability to hold out at the bargaining table. I propose a refinement of subgame perfect equilibrium that requires equilibrium strategies to be immune to deviations by any majority subgroup. I show this is equivalent to giving the worker with median bargaining power the unilateral ability to negotiate with the firm. Using this model, I demonstrate that policies reallocating surplus from high-talent to moderate-talent workers, such as maximum contracts in professional sports, can improve the welfare of *all* workers. Redistribution of surplus harmonizes workers' interests, giving a majority of them a greater stake in the bargaining outcome. The model highlights the gains to be had if a heterogeneous group agrees to concessions that increase the alignment of their individual interests.

Keywords: Majority-Rule Bargaining, Collective Bargaining, Negotiation, Unions

JEL Classification: C7, D7, J5, Z22.

*Previously circulated under the title "Losing Money to Make Money: The Benefits of Salary Compression"

†I thank Roger Noll for numerous discussions throughout the process of writing this paper. I am grateful to Matthew Jackson, Alvin Roth, and Jonathan Weinstein for helpful comments and suggestions. This work was supported in part by ERC Grant #757229 under the European Union's Horizon 2020 research and innovation program.

Dept. of Economics, University of Cambridge email: av615@cam.ac.uk

You had some guys, who were making a lot of money, that wanted to hold the line... Then you had other guys saying, 'I got to get back to work. I got a wife. I got kids. I got family members that I have to help.'

- Aaron Mckie, NBA player 1994-2007

1 Introduction

On January 20th, 1999, the longest lockout in the National Basketball Association's history ended after 205 days. The league lost $1 billion in revenue, while players forfeited $500 million in salary. The crux of the dispute was how revenue should be divided between owners and players. The stumbling block was the maximum contract, a limit on an individual's salary as a percentage of total revenue accorded to the players; this pitted players against each other. As time wore on, the cohesion of the union fractured, and the NBA became the first major league to adopt maximum contracts. While such contracts depress the salaries of stars and increase those of non-stars in the immediate term, *all* players may benefit in future collective bargaining negotiations over the split of revenue. For players to have bargaining power, they must be sufficiently patient. However, delays in bargaining affect wealthier players differently than poorer ones. Bargaining in the NBA is not simply a negotiation "between short billionaires and tall millionaires", as Washington Post writer Tony Kornheiser remarked. Not all the tall ones are millionaires. Redistribution of the salary through maximum contracts reduces inequality in patience in bargaining, which may benefit the players in future collective bargaining.

The story illustrates two fundamental aspects of collective bargaining:

1. Negotiations are between a coalition of heterogeneous workers against their employer.
2. While the union negotiates over the share of surplus accorded to workers as a whole, equally important is how that share is distributed amongst the workers.[1]

Two questions arise. First, how does a collection of workers bargain with a firm over the share of surplus accorded to them? Second, how will the division of that share of surplus amongst the workers affect bargaining?

I answer these questions in a model of collective bargaining between a coalition of heterogeneous workers and a firm. Workers are either high or moderate talent. They receive rewards as a function of the current share of surplus dedicated to the entire workforce. The term "reward" should be interpreted to include wages, bonuses, and other amenities. I take as given that

[1]While redistributive policies are sometimes done within collective bargaining, unions can often implement them independently. Examples include profit-sharing, implicit or explicit caps on individual salary, discriminatory fees, and additional funds that are paid to workers in the form of benefits (Pencavel [1991]).

high-talent workers earn larger rewards than moderate-talent workers but provide a microfoundation in Section 2.1. One can interpret this as a reduced-form representation of settings where high-talent workers are scarce and are more productive than moderate-talent workers. The firm must give them greater rewards due to their higher outside options.

Crucially, workers earn rewards *before* the bargaining date. This allows me to identify how the initial distribution of rewards impacts subsequent bargaining outcomes. At a known future date, the workers enter into a negotiation with the firm, bargaining over the size of the future surplus. The firm and the workers' coalition alternate between proposing offers, with negotiations ending when one side accepts the other's proposal. I show that redistributive policies have an overlooked benefit: they improve the bargaining power of the workers.

A difficulty in many models of coalitional bargaining is the need to specify protocols for how agents in a coalition generate and agree on proposals. Generally, bargaining outcomes are sensitive to the chosen protocol (Eraslan and Evdokimov [2019]). I sidestep this difficulty by abstracting from the method by which the workers' coalition generates proposals. Instead, I require as an equilibrium condition that there be no profitable deviation for any majority subgroup. Importantly, I show that this is equivalent to giving the worker with median bargaining power the right to make, accept, and reject proposals. If delay costs in bargaining are wealth-dependent and high-talent workers are longer-lived than those of lesser talent, redistributive policies strengthen the coalition's bargaining position by increasing median bargaining power.

Contribution My paper contributes to the literature on majority-rule bargaining where one side is comprised of heterogeneous agents (see Card [1990], Cramton and Tracy [2003], and Serrano [2007] for surveys in collective bargaining and bargaining theory). I extend Rubinstein (1982) to incorporate a majority-rule approval mechanism on one side where members have heterogeneous discounting. It is related to the majority-rule legislative bargaining literature (see Eraslan and Evdokimov [2019] for a survey). In those papers, individuals are randomly selected as proposers (e.g. Baron and Ferejohn [1989]; Haller and Holden [1997]). In mine, I abstract from how the coalition crafts its proposals but require that there be no profitable deviation by any majority subgroup as an equilibrium condition. I demonstrate that this is equivalent to the worker of "median bargaining power" being given the right to make and reject proposals. In the same vein, Compte and Jehiel (2010) look at a majority-rule bargaining game, showing that there is a "key-worker" who determines the outcome. In their paper, the coalition searches for proposals, and the majority dictates when to halt search. My model requires a majority of workers to accept any proposal offer from the firm. However, it does not specify how the workers craft their counter-proposals. My model can be viewed as one where the workers can not commit to any protocol by which to do this, as various subsets can "defect" at any time. Thus, I look at proposal strategies that satisfy a refinement of subgame perfect equilibrium to pindown decisions that are immune to such deviations by a majority subgroup.

Early models of collective bargaining treat workers as homogenous.[2] Heterogeneity is a critical component of my model because it affects the bargaining outcome as some workers are more patient than others. Furthermore, heterogeneity is crucial to identifying the consequences of redistributive policies across workers. Related models of collective bargaining that include heterogeneity also have the outcome determined by a median voter (e.g. Booth [1995]; Kaufman [2002]; Manzini and Mariotti [2005]; Lee and Mas [2012]). However, in mine, the "median voter" is not *exogenously selected* as an arbitrator. My model can be viewed as a microfoundation for this choice. While there is no explicit selection of an arbitrator, the equilibrium outcome is equivalent to the setting where the worker with median bargaining power is delegated to bargain on behalf of the coalition.

A key distinction between my paper and previous bargaining models is that I examine how bargaining outcomes depend on redistribution policies *before* bargaining begins. In particular, redistributing surplus from workers with a high share to those with a low share can improve bargaining outcomes by increasing the bargaining power of the median worker. While framed in a labor market context, the ideas translate to environments where a group of heterogeneous agents negotiates over surplus with an institution. The model highlights the benefits to the group from agreeing to concessions that improve the alignment of members' interests.[3] Interpreting my results through the lens of Grossman and Hart (1986), Penceval (1991), and Muthoo (2004), I demonstrate that redistributing property rights incentivizes more of the coalition members to work harder for the collective good (in this case, not agree to low proposals).[4]

My model has applications to professional sports markets in the United States. Much of the literature in sports economics focuses on the effect of contract structures on salary and talent distribution (e.g. Fort and Quirk [1995]; Dietl, Lang, and Rathke [2009]).[5] Such papers do not include a bargaining stage. As my paper demonstrates, when one includes a bargaining

---

[2] See Kaufman (2002) for examples.

[3] In this sense, one can view redistribution as how unions "coordinate bargaining" (Ahlquist [2017]).

[4] Sandroni and Urgun (2018) look at the effect of patience on committing destructive acts. Bargaining delay costs represent patience, while wealth transfers caused by redistribution can be viewed as "destructive acts".

[5] Madden (2019) looks at collective bargaining but using the framework from McDonald and Solow (1981).

stage, maximum contracts that redistribute wealth from stars to less talented players can benefit all players. This has immediate policy implications for US sports leagues. For example, the NBA players' union is seen as more effective than its NFL counterpart. A critical distinction between the leagues is that the NFL operates with a salary cap and no limit on the size of individual contracts. My results support an argument that NFL players would benefit from maximum contracts. Such contracts will increase their leverage in future negotiations, leading to players receiving a higher percentage of league revenue than they would otherwise.

The next section outlines the model. Section 3 analyzes the collective bargaining game and welfare effects. Section 4 discusses implications and applications of the results.

2 Model Overview

I present the model and defer discussion of its assumptions to Section 2.3. There is a firm employing a unit mass of workers. Workers are either high or moderate-talent, denoted by $\theta = h$ and $\theta = l$, respectively; there are $l_h < \frac{1}{2}$ high-talent workers. Time is continuous, starting at $t = 0$. The firm generates a unit surplus at each t. While working, some workers exit the firm, and this is modeled as a Poisson process with intensity λ_θ. Exiting workers are replaced by those of the same type. Let π denote the share of surplus accorded to the workers at each t, and let μ_0 be the exogenously specified fraction of π allotted to the high-talent workers.

At each t, a moderate-talent worker receives reward $s_l = \min\left\{\frac{(1-\mu_0)\pi}{1-l_h}, s_{min}\right\}$, and a high-talent worker receives $s_h = \min\left\{\frac{\mu_0\pi}{l_h}, \frac{\pi - s_{min}(1-l_h)}{l_h}\right\}$, where $s_{min} \geq 0$ is the *reservation reward.* I assume μ_0 is such that $s_h > s_l$. Thus, s_h and s_l specify how much of the share of surplus each worker gets. The moderate-talent worker is guaranteed to receive s_{min}.

One can interpret μ_0 as the outcome of prior collective bargaining negotiations or internal union discussions. Another interpretation is that the functional form of s_h and s_l are reduced form representations of what might emerge in an equilibrium model of wage-setting in labor markets (e.g. Fernandez and Glazer [1990]; Mortensen and Pissarides [1994]; Houba and van Lomwel [2001]). Suppose a firm must hire a unit measure of workers, and high-talent workers contribute more than moderate-talent workers. However, high-talent workers are in scarce supply. In equilibrium, l_h high-talent workers and $1 - l_h$ moderate-talent workers are hired, receiving rewards s_h and s_l, respectively. Due to competition with other firms for high-talent workers, such workers receive higher rewards. If π represents the share of surplus accorded to the workers, then $\pi = s_h l_h + s_l(1-l_h)$ and $\mu_0 = \frac{s_h l_h}{\pi}$ is the fraction of π given to the high-talent workers. Hence, μ_0 can be the equilibrium outcome of wage competition, and the rewards can then be written as above. The key property needed is that as the share of surplus π increases, high-talent workers receive a fraction μ_0 of the gains.

In Section 2.1, I provide a microfoundation for this representation using a model of a sports league where surplus is total revenue, and total player salaries must constitute a fraction of total revenue. In equilibrium, salaries take this functional form. In particular, equilibrium behavior results in $\mu_0 = 1$. The example corresponds to settings where moderate-talent workers are in abundant supply, and get paid the reservation salary. High-talent workers are scarce and receive the maximum possible reward as a result of competition for their services. Such functional forms for s_h and s_l also arise in settings where the surplus at stake is the total revenue generated, and the involved parties are participants in a revenue-sharing agreement (e.g. Feiveson [2015]).

The initial share of surplus accorded to workers is π_0. The future share of surplus awarded to them is determined via collective bargaining, which I model as follows. At a known time $\bar{t}$ in the future, a bargaining game is initiated. Workers as a collective negotiate with the firm over the future division of surplus. The firm and workers compete in a Rubinstein bargaining game, making alternating take-it-or-leave-it offers $(1-\pi,\pi)$ specifying the fraction π of surplus the workers will receive. Bargaining ends when the firm accepts an offer supported by a majority of the workers or when a majority of workers accept the offer made by the firm. Workers then earn a reward stream according to the new share of surplus. No rewards are received until an offer is accepted. Offers are made at discrete intervals of length $\Delta > 0$ to account for the fact that proposals require time to craft and analyze. A novel feature of the model is that costs associated with delays in bargaining depend on worker wealth, which is the cumulative rewards he has received up until the start of bargaining. A worker's delay cost in bargaining is represented by a decreasing function $\delta(\cdot)$ of his wealth. All theorems extend to type-dependent bargaining delay costs of the form $\delta(\theta,w)$, where $\delta(\theta,\cdot)$ is decreasing in wealth. For economy of exposition, I do not incorporate that in the body of the paper. However, all proofs in the appendix incorporate type-dependent discounting. The firm discounts at rate $\rho > 0$.

Using this model, I answer the following questions:

1. What is the bargaining outcome, and how does wealth-dependent discounting affect it?
2. While the initial μ_0 is exogenous, if the union or workers could collectively reduce μ_0 and increase redistribution, how will that affect bargaining?

2.1 Illustrative Example: Professional Sports in the United States

To provide an application as well as a microfoundation of the reward stream, consider the model of a sports league in Késenne (2000). There are N symmetric teams, a limited supply l_h of high-talent players, and an unlimited supply of moderate-talent players. Each team must sign a unit measure of players. If each team j signs x_j high-talent and y_j moderate-talent players, team i's revenue is $R(x_i, y_i; (x_{-i}, y_{-i}))$. Assume team revenue is increasing and concave in its talent, and the marginal revenue of signing a high-talent player is greater than that of a moderate-talent one. Finally, assume team i's revenue depends on its talent and the distribution of talent across other teams, independent of team name. Team costs are the player salaries. As in most US leagues, there is a salary cap specifying the amount each team must pay its players. Given salary cap C, players are thus entitled to a share $\pi = \frac{NC}{\sum_{i=1}^{N} R(x_{it}, y_{it}; (x_{-it}, y_{-it}))}$ of total revenue.

Késenne (2000) analyzes the league under a Walrasian framework.[6] Equilibrium salaries are a function of talent supply, salary cap, and minimum salary:

$$s_h = \min\left\{\frac{NC}{l_h}, \frac{NC - s_{min}(N - l_h)}{l_h}\right\} \tag{1}$$

$$s_l = s_{min} \tag{2}$$

Normalizing revenue to 1 as it is a constant in equilibrium (salary cap levels do not affect the distribution of talent across teams in equilibrium), yields the exact functional form described in the beginning for the case where $\mu_0 = 1$. If the players' coalition instituted their own redistributive policy in the form of a maximum contract μ_0 (limit on the fraction of the salary cap that can be rewarded to high-talent players), then salaries would be:

$$s_h = \min\left\{\mu_0 \frac{NC}{l_h}, \frac{NC - s_{min}(N - l_h)}{l_h}\right\} \tag{3}$$

$$s_l = \max\left\{\frac{(1-\mu_0)NC}{N - l_h}, s_{min}\right\} \tag{4}$$

Within professional sports, the players bargain with the league over the size of the salary cap (i.e. the share of the surplus π accorded to them).

[6] Burguet and Sákovics (2019) demonstrate that even when teams can offer salary schedules discriminating between players of the same type, and players can choose whom to play for, the equilibrium salary schedule is equal to the Walrasian one.

2.2 Payoffs

Consider the following sequence of events for a worker of type θ:

1. Receives reward $s_\theta(\pi_0, \mu_0)$ at times $t \le \bar{t}$.

2. Collective bargaining begins at $\bar{t}$ and an agreement is reached at time $\bar{t} + k\Delta$ resulting in a share π_A of surplus accorded to the workers. No rewards are earned during the delays in bargaining.

3. Earns new reward $s_\theta(\pi_A, \mu_0)$ at times $t \ge \bar{t} + k\Delta$.

The payoff to the worker[7] :

$$U(\theta, k, \pi_0, \pi_A, \mu_0) = \underbrace{\int_0^{\bar{t}} \lambda_\theta e^{-\lambda_\theta t} s_\theta(\pi_0, \mu_0) dt +}_{\text{Standard discounting up to } \bar{t}} \underbrace{e^{-k\Delta\delta(w)}}_{\text{Delay in bargaining}} \cdot \underbrace{\int_{\bar{t}+k}^{\infty} \lambda_\theta e^{-\lambda_\theta (t-k)} s_\theta(\pi_A, \mu_0) dt}_{\text{Standard discounting from } \bar{t}+k}$$

$$\text{where } \underbrace{w = s_\theta(\pi_0, \mu_0)\bar{t}}_{\text{Wealth at } \bar{t}}$$

During bargaining, discounting depends on cumulative rewards up until the start of the negotiation. I assume $\delta(w)$ is decreasing and differentiable. There is much evidence demonstrating the dependence of discount rates on income, with poorer individuals discounting the future more than wealthier ones (Frederick et al. [2002]). For example, Card, Chetty, and Weber (2007) observe liquidity constraints as a function of cash-on-hand. Within a bargaining setting, Hardy et al. (2020) observe that discount rates in bargaining decline with income level.

Again, all conclusions of the model hold with type-dependent delay costs of the form $\delta(\theta, w)$. The proofs of the theorems, which are in the appendix, are for general $\delta(\theta, w)$.[8]

2.3 Discussion of Assumptions

In my model, workers bargain with the firm over "surplus". Surplus is an abstraction given that collective bargaining in labor markets often includes bargaining over many things: wages, benefits, insurance coverage, and other amenities (Cramton, Mehran, and Tracy [2015]). While I assume the size of the available surplus is constant, the results are stronger if the surplus grows

[7] One may wonder what happens if workers value rewards non-linearly according to some increasing function V. I discuss the robustness of my results to such a specification in Appendix B. Most results remain true, though some comparative statics regarding the bargaining solution depend on properties of V.

[8] In particular, everything holds when incorporating exit-rate in the delay cost (i.e. $\delta(\theta, w) = \lambda_\theta + \hat{\delta}(w)$).

over time, as then the rewards from a better bargaining position are greater. Finally, there is only a single instance of bargaining. I do this for tractability. Generally, collective bargaining takes place every fixed number of years. Allowing for multiple bargaining periods strengthens my results. Accumulation of wealth in each period between bargaining has a ratchet effect that helps in subsequent bargaining periods.

The assumption of two worker types is for simplicity only. When incorporating multiple types, one must specify which types are giving up rewards and which types are the beneficiaries. Returning to the sports example, with multiple types, maximum contracts would shift dollars downwards to the *next* highest player type. The lowest types would not enjoy any of these gains and continue to earn the minimum salary.

Worker-exit during non-bargaining periods follows a Poisson process. One might criticize the Poisson exit assumption on the grounds that exit time should be age-dependent. However, the model can easily accommodate this and other elements like promotion (i.e. moderate-talent workers become high-talent). Inclusion of these features does not change the qualitative conclusions of the model. What they do change is the distribution of wealth at the time of bargaining. The incentive to sacrifice current salary to improve bargaining position remains. The difference is that the date of bargaining matters and the functional form of time-varying discounting matter.[9] The Poisson assumption highlights the effects of wealth-dependent discounting.

I allow the union to exogenously vary this parameter to capture the fact that unions can implement their own redistributive policies (see Penceval [1991] for evidence of this).

3 Collective Bargaining

3.1 Solution Concept

At the time of bargaining, the workers as a collective negotiate with the firm over the division of surplus. Recall that π_0 is the *initial*, pre-bargaining share of surplus, and let π_A denote the post-bargaining share of surplus. I model bargaining as a modified Rubinstein bargaining process in which the firm and workers alternate between making offers to one another until an offer is accepted. I assume that the firm is the initial proposer in the first period. An offer by the firm is accepted if a majority of the workers choose to accept. Similarly, an offer by the workers is accepted if the firm agrees to it.

[9] Schweighofer-Kodritsch (2018) studies Rubinstein bargaining under time-dependent discounting.

The history-dependent strategies of the firm and workers' coalition are denoted by σ_F and σ_W, respectively. As the firm makes the first offer, σ_F specifies an offer in each *odd* period; in *even* periods, it specifies an acceptance/rejection decision in response to the opposition's offer. Decisions are contingent on the observed history of proposals and counter-proposals. Since I abstract from the process by which the workers generate proposals, σ_W is defined analogously. Formal definitions of the strategies can be found in Appendix A.

The workers' coalition is the set of all workers at the time of bargaining. Any worker at that time can be identified by his talent (θ) and accumulated wealth (w). Thus, **the workers' coalition** is described by a distribution G over $\{(\theta, w) | \theta \in \{h, l\}$ and $w \leq s_h \bar{t}\}$, which accounts for there being workers of varying career lengths in the coalition. Each redistribution level μ_0 induces a different G since μ_0 changes the initial reward stream of the workers and, therefore, the distribution of accumulated wealth. The distribution G is crucial, as one needs to know the individual workers' preferences to understand the majority's actions.

I now define the notion of equilibrium, which will require that any decision by the workers' coalition be supported by a simple majority.

Definition 3.1 *A* **subgame perfect majority-rule equilibrium** *(SPMRE) is a pair of strategies σ_W and σ_F such that:*

1. *There is no closed $M \subset \{(\theta, w) | \theta \in \{h, l\}$ and $w \leq s_h \bar{t}\}$, with $\mathbb{P}_G(M) \geq \frac{1}{2}$, such that after some history, there is a deviation that leaves all members of M strictly better off.*[10]

2. *At any history, there is no deviation by the firm that leaves it better off.*

While the coalition is a singular entity, I must rule out deviations by majority subsets. This is because the firm could target its proposals towards certain workers to gain a more favorable deal. Definition 3.1 incorporates this feature. In other models of majority-rule bargaining, a representative is selected with some probability to propose a policy (e.g. Baron and Ferejohn [1989]). I can abstract from the method by which the workers' coalition generates proposals by using the equilibrium criterion above.

[10]The necessity of a deviating coalition to be strictly better off is solely because the distribution G is not continuous. If G were continuous, the definition would only require that a positive measure of such an M be strictly better off. This is discussed in the appendix. The reason for utilizing closed sets is to avoid tie-breaking issues that would arise given that I am working in the continuum.

3.2 Necessity of Redistribution

To highlight incentives, consider a setting with no redistribution (i.e. $\mu_0 = 1$). This is the case in the sports example, where, in equilibrium, moderate-talent players receive the reservation salary, and high-talent players receive maximal reward possible given the share of surplus accorded to the players. Collective bargaining leads to workers receiving the reservation share of surplus $\pi_A = \pi_{min} = s_{min}$. Post-bargaining rewards are $s_\theta(\pi_{min}) = s_{min}$ for all θ.

Proposition 3.2 *Suppose $\mu_0 = 1$. Then the post-bargaining share of surplus is $\pi_A = \pi_{min}$.*

Proof: Moderate-talent workers earn the minimum reward independent of the share of surplus. Thus, without any redistribution, moderate-talent workers receive s_{min} at each t regardless of the bargaining outcome. The firm can credibly offer the reservation share of surplus as a result.[11] Hence, the unique subgame-perfect majority-rule equilibrium has the firm offering the reservation share of surplus and all the moderate-talent workers accepting immediately. ■

To counter this, the workers' coalition can institute a minimal level of redistribution, which may increase rewards from bargaining for moderate-talent workers without high-talent workers sacrificing initial rewards. The minimal level of redistribution is given by an *upper bound* $\overline{\mu}$ on the share of surplus that high-talent workers can receive. The quantity $\overline{\mu}$ is defined by:

$$\overline{\mu}\frac{\pi_0}{l_h} = \frac{\pi_0 - s_{min}(1-l_h)}{l_h}$$

The minimal level of redistribution $\overline{\mu}$ has the feature that when $\mu_0 = \overline{\mu}$, rewards at times $t \leq \bar{t}$ are the same as those in a regime with no redistribution (i.e. $\mu_0 = 1$). It does not affect the initial stream of payments before $\bar{t}$, but may increase rewards for moderate-talent workers *after* bargaining. If delay costs are constant, there is no benefit from redistribution beyond the minimal level. Incorporating wealth-dependent discounting not only enriches the model but yields insights into the benefits of *higher levels* of redistribution beyond the minimal level. In particular, given the previous discussion, one can restrict attention to $\mu_0 \leq \overline{\mu}$.

3.3 Bargaining Outcome

To understand how the bargaining outcome changes as μ_0 varies, I first provide a general characterization of the equilibrium of the bargaining game. I will show that the collective

[11] Suppose high-talent workers had a different reservation reward. The reservation share of surplus is simply the minimum share that allows high-talent and moderate-talent workers to receive their respective reservation rewards.

bargaining outcome is determined by a “swing-voter”. Consider a hypothetical situation where a type θ worker with wealth w negotiates on behalf of the coalition. From Rubinstein (1982), there is a unique subgame perfect equilibrium where the workers receive a share of surplus π^* such that:

$$\pi^*(\theta,w) = \frac{e^{-\Delta\delta(w)}(1-e^{-\Delta\rho})}{1-e^{-\Delta(\delta(w)+\rho)}}$$

π^* quantifies the bargaining power of a type θ worker with wealth w. Notice π^* depends on (θ,w) only through the delay cost in bargaining $\delta(w)$. Thus, bargaining power is entirely characterized by $\delta(w)$. A worker with *a high value* of $\delta(w)$ (high delay cost) has *low bargaining power*: $\pi^*(\theta,w) > \pi^*(\theta',w')$ if and only if $\delta(w) < \delta(w')$. A worker with **median bargaining power** has delay cost δ^* such that:

$$\delta^* = \sup\left\{\hat{\delta} : \mathbb{P}_G(\left\{(\theta,w)|\delta(w) > \hat{\delta}\right\}) \geq \frac{1}{2}\right\}$$

Given the reasoning above, the worker with median bargaining power has median wealth w^*:

$$w^* = \delta^{-1}(\delta^*) = \inf\left\{\hat{w} : \mathbb{P}_G(\{(\theta,w)|w < \hat{w}\}) \geq \frac{1}{2}\right\}$$

Remark: The supremum and infimum are used since the distribution G has two atoms: some workers of each type who started at time 0 will not have exited by the bargaining time $\bar{t}$.

While the coalition may include a high-talent player with median bargaining power, there will always be a moderate-talent player with median bargaining power since the measure of high-talent workers is $l_h < \frac{1}{2}$. There will always be a moderate-talent worker with wealth w^*.

I rank workers based on how each fares in an individual bargaining game against the firm. Intuitively, since the worker with wealth w^* is the median of this ranking, his individual bargaining outcome will be accepted by a majority. However, because there is no actual representative in the collective bargaining game and no commitment device to select such a representative, it is not obvious that this worker’s preference will determine outcomes. In other models of majority-rule bargaining, the “key” individual that determines the outcome is sensitive to the proposal-construction process (e.g. Baron and Ferejohn [1989]; Compte and Jehiel [2010]). I abstract from how proposals are crafted, showing that the equilibrium refinement requires the outcome to be determined by a worker with median bargaining power.

Theorem 3.3 *Suppose $\mu_0 < 1$. Then there is a unique SPMRE where collective bargaining results in workers receiving a fraction π_A of future surplus:*

$$\pi_A = \max\left\{\frac{e^{-\Delta\delta^*}(1-e^{-\Delta\rho})}{1-e^{-\Delta(\delta^*+\rho)}}, \pi_{min}\right\}$$

Proof: *See Appendix.* ■

I demonstrate that the SPMRE is equivalent to the solution to a Rubinstein bargaining game where the worker with median bargaining power negotiates on behalf of the coalition.

3.4 Worker Share of Future Surplus

The workers' coalition aims to prevent the firm from extracting maximal surplus in collective bargaining and depressing the workers' share of surplus to the reservation level. Without redistribution, post-bargaining share of surplus is $\pi_A = \pi_{min}$. By Theorem 3.3, post-bargaining worker share of surplus is decreasing in δ^* and increasing in w^*. It follows that increasing redistribution increases post-bargaining share of surplus if and only if $\frac{\partial w^*}{\partial(1-\mu_0)} > 0$. It may appear obvious that increasing redistribution increases median wealth as there are more moderate-talent workers than high-talent workers. However, this is not correct. Since workers exit and enter at different rates, there will be a distribution of workers of various ages and hence wealth levels at the time of bargaining. Furthermore, when high-talent workers give up a unit of their reward, each moderate-talent worker only receives a fraction of this unit since there are more moderate-talent workers than high-talent ones. Hence, for the median wealth w^* to increase, the upward shift in the distribution of *accumulated wealth* of the moderate-talent workers must outweigh the downward shift in *accumulated wealth* of the high-talent workers. Quantifying these shifts is not trivial because workers exit according to a Poisson process, and so the wealth distribution is determined by the distribution of workers' ages at the time of bargaining.

Intuitively, reducing μ_0 increases median wealth when there is "sufficient wealth disparity" between the distributions of wealth at the time of bargaining between high and moderate-talent workers. The parameters affecting wealth disparity at the time of bargaining are worker exit-rates (λ_l, λ_h), the number of high-talent workers (l_h), and the time of bargaining $(\bar{t})$. The exit-rates affect how easy it is to accumulate wealth. The time of bargaining caps how much wealth can be achieved. The number of high-talent workers affects the reward stream levels. The wealth disparity needed occurs when high-talent workers are sufficiently long-lived relative to moderate-talent ones. How much longer-lived depends on these other parameters. For instance, when high-talent workers are scarce, they receive large rewards, leading to large disparity. Hence, high-talent workers need not be much longer-lived than moderate-talent ones. Theorem 3.4 formalizes this idea to yield an intuitive condition for when decreasing μ_0 increases w^*.

Theorem 3.4 *There exists $k(\lambda_l, l_h, \bar{t})$ such that if $\lambda_h \leq \min\{\lambda_l, k\}$, then $\frac{\partial w^*}{\partial(1-\mu_0)} > 0$. In particular, if $l_h < \frac{e-2}{2e-2} \approx 0.21$, then $\lambda_h \leq \lambda_l \implies \frac{\partial w^*}{\partial(1-\mu_0)} > 0$.*

Proof: *See Appendix.* ■

Proof Sketch: I describe my approach and the intuition behind the result. Varying μ_0 changes the initial rewards each worker receives, thereby changing the distribution of accumulated wealth at the time of bargaining, and, in particular, w^*. Since workers exit according to a Poisson process, and the fraction of type θ workers is constant, I can compute the distribution of wealth at the time of bargaining for each worker type. I then provide a characterization of w^*, the median wealth level corresponding to median bargaining power.

As redistribution increases, initial rewards are shifted towards moderate-talent workers. The shift is not "one-to-one" since there are more moderate-talent workers than high-talent ones. As high-talent rewards decline, high-talent workers become less patient. For median bargaining power to increase, the gain in the measure of moderate-talent workers that have accumulated more wealth must outweigh the increase in high-talent workers willing to "settle". This is guaranteed when high-talent workers are sufficiently longer-lived than moderate-talent ones. When talent is scarce, high-talent workers need only be at least as long-lived. This is the case in many industries. In professional sports, for example, it is well-documented that high talent players have longer professional careers than those with lesser talent.[12]

It is critical to note that this does not mean that the post-bargaining share of surplus will be higher than the initial share. Rather, the post-bargaining share of revenue will be higher than *what it would have been* if there were less redistribution.

3.5 Welfare

Theorem 3.4 shows that under plausible conditions, reducing μ_0 and increasing redistribution will increase median wealth w^*. Hence, moderate-talent workers will always prefer to

[12] https://www.businessinsider.com/nfls-spin-average-career-length-2011-4

reduce μ_0 as they receive both higher initial rewards and an improved bargaining position in the future. Since they are in the majority, why can't they just impose such redistribution?

A slight perturbation of the model offers a rationale. Implicitly assumed is that worker type is immediately known. This may not always be the case. For example, in sports, talent may be ascertained only after some games are played. Such a feature does not affect any of the earlier results, but it does explain why at $t = 0$, the majority of workers may not, ex-ante, agree to maximum contracts. Suppose there was a lag before worker type was realized. Then, worker decisions on increasing the intensity of redistribution depend on the distribution of beliefs about their ability. Since rewards are type-contingent, they are determined by the *expected* size of the high-talent pool. However, worker stance on redistribution is determined by *beliefs*. Ex-ante, a majority of workers may assign a sufficiently high probability to being high-talent.

Example 1 *Recall the sports example from Section 2.1. Suppose players at $t = 0$ are unsure about their talent. Let $F(\cdot)$ be the distribution of their beliefs about their ability. The players must decide on whether to increase redistribution (decrease μ_0) beyond $\bar{\mu}$, where μ_0 represents the maximum contract: the maximum salary high-talent players can receive as a fraction of the salary cap. After μ_0 is set, ability is realized after an infinitesimal lag.*

Suppose salaries s_h and s_l are dependent on the expected number of high and moderate-talent players. This is true in a Walrasian framework. The fraction of high-talent players will be $l_h = \mathbb{E}_F[b]$ almost surely (Duffie and Sun [2004]). However, a player with belief b has expected payoff $bs_h(l_h) + (1-b)s_l$. If there is no future bargaining, such a player is against further salary limits if $b > l_h$. Hence, if $F^{-1}(\frac{1}{2}) > \mathbb{E}_F[b]$, a majority will not want to decrease μ_0. With future bargaining, a player of belief b may still vote no if b is sufficiently high, and if conditional on being high-talent, gains in bargaining do not offset the loss of initial salary.

Thus, if types are initially unknown, a majority at $t = 0$ will not approve of redistribution if:

1. A majority believe they are likely to be high-talent with sufficient probability.

2. Conditional on being high-talent, the gains in bargaining are not sufficient to offset the loss in the initial reward stream.

Proposition 3.5 *Workers will all agree to increase redistribution (lowering μ_0) if:*[13]

$$\left(\mu_0 \cdot \frac{-\Delta\delta'(w^*)}{1-e^{-\Delta(\delta^*+\rho)}} \cdot \frac{\partial w^*}{\partial(1-\mu_0)} - 1\right)\pi_A > (e^{\lambda_h \bar{t}} - 1)\pi_0 \tag{5}$$

Proof: Unanimous agreement can be achieved if redistribution also improves the payoff to workers conditional on them being high-talent. Consider the payoff to a high-talent worker:

$$U(h) = \int_0^{\bar{t}} \lambda_h e^{-\lambda_h t} s_h(\mu_0, \pi_0) dt + \int_{\bar{t}}^{\infty} \lambda_h e^{-\lambda_h t} s_h(\mu_0, \pi_A) dt = \mu_0 \frac{\pi_0}{l_h}\left(1 - e^{-\lambda_h \bar{t}}\right) + \mu_0 \frac{\pi_A}{l_h} e^{-\lambda_h \bar{t}}$$

The payoff increases with redistribution if the derivative with respect to $1-\mu_0$ is positive:

$$e^{-\lambda_h \bar{t}} \mu_0 \cdot \frac{\partial \pi_A}{\partial w^*} \cdot \frac{\partial w^*}{\partial(1-\mu_0)} > \pi_0 + e^{-\lambda_h \bar{t}}(\pi_A - \pi_0)$$

Using the expression for π_A in Theorem 3.3, the above inequality is equivalent to:

$$\left(\mu_0 \cdot \frac{-\Delta\delta'(w^*)}{1-e^{-\Delta(\delta^*+\rho)}} \cdot \frac{\partial w^*}{\partial(1-\mu_0)} - 1\right)\pi_A > (e^{\lambda_h \bar{t}} - 1)\pi_0 \qquad \blacksquare$$

Even if workers are uncertain about their ability, they will unanimously vote to increase redistribution if the condition in Proposition 3.5 holds. The condition reflects the central trade-off for high-talent workers: sacrifice initial rewards and a larger stake in the future share surplus for better bargaining power and a larger future share of surplus. Notice that $\frac{\partial w^*}{\partial(1-\mu_0)} > 0$ is necessary for the inequality to hold. Inequality (5) highlights a key quantity of interest:

$$\underbrace{\frac{\frac{\partial \pi_A}{\partial(1-\mu_0)}}{\pi_A}}_{\text{Relative change in surplus share}} = \underbrace{\frac{-\Delta\delta'(w^*)}{1-e^{-\Delta(\delta^*+\rho)}}}_{\text{Change in Patience}} \cdot \underbrace{\frac{\partial w^*}{\partial(1-\mu_0)}}_{\text{Gain in wealth}}$$

The left-hand side measures the increase in future share of surplus as a result of redistribution. Since $1-e^{-\Delta(\delta^*+\rho)} \in (1-e^{-\Delta(\rho)}, 1)$ for all parameter values, the crucial term is $-\delta'(w^*)\frac{\partial w^*}{\partial(1-\mu_0)}$: the product of the magnitude of the reduction in bargaining delay cost and the increase in w^*.

Thus, Proposition 3.5 shows that increasing redistribution is Pareto improving if high-talent players are sufficiently long-lived relative to the time of bargaining (right-hand side of inequality (5) is low), and -$\delta'(w^*)\frac{\partial w^*}{\partial(1-\mu_0)}$ is sufficiently high. The magnitude of $\delta'(w^*)\frac{\partial w^*}{\partial(1-\mu_0)}$ de-

[13] This proposition holds with general delay costs $\delta(\theta, w)$. In the general case, δ^* and w^* are defined as in the Appendix. Then $\delta'(l, w^*)$ would replace $\delta'(w^*)$ in the proposition.

pends critically on w^*. The proof of Theorem 3.4 characterizes w^* and shows its dependence on the model's primitives. When λ_l is large relative to the time of bargaining $\bar{t}$, w^* is small. However, since moderate-talent workers have short careers, it is difficult for them to accumulate wealth $\Longrightarrow \frac{\partial w^*}{\partial(1-\mu_0)}$ is also small. Therefore, $-\delta'(w^*)$ must be large at low wealth levels for redistribution to benefit high-talent workers. On the other hand, if λ_l is small relative to the time of bargaining (moderate-talent workers have long-careers), then w^* and $\frac{\partial w^*}{\partial(1-\mu_0)}$ are large: moderate-talent workers are able to accumulate wealth. However, $-\delta'(w^*)$ may be small because w^* is already high.[14]

.

4 IMPLICATIONS AND CONCLUSION

This paper introduced a novel refinement of a majority-rule Rubinstein bargaining game with heterogeneous agents that allows one to abstract from the precise proposal mechanism. In this setting, a generalized notion of subgame perfect equilibrium is equivalent to delegating bargaining to the worker with median bargaining power. Furthermore, the model allows one to understand how redistributive policies before bargaining change the level of median bargaining power, thereby affecting bargaining outcomes.

Though framed in the context of labor market negotiations, these insights apply to any setting where a coalition of heterogeneous agents negotiates with a single institution over a share of surplus. Conditional on the share of surplus allocated to agents, how that share is distributed amongst the agents is critical. Importantly, redistributive policies before bargaining are akin to reallocating property rights over the share of surplus won. Such redistribution can harmonize agents' interests by decreasing delay costs and incentivizing those who would have been more inclined to settle to be more aggressive.

These results have immediate policy consequences for many labor markets, and in particular, professional sports. Within the United States, the NBA has instituted maximum contracts, limiting salaries of high-talent players to a percentage of the salary cap (i.e. the share of surplus accorded to the players as a collective). The impact of maximum contracts in the NBA is reflected in the outcomes of recent negotiations and public opinion of the strength of the players' coalition. Since the 1998-99 lockout, player share of revenue declined from 55% to 50%,

[14] This would be the case if $\delta(\cdot)$ is convex. Since $\delta(\cdot)$ is decreasing, convexity implies $-\delta'(\cdot) > 0$, $-\delta''(\cdot) < 0$.

where it has remained stable for over 14 years. This is not at odds with my model. Rather, my model predicts that without such contracts, owners would have been able to extract even more rents. Furthermore, while revenue in my model (size of surplus) is the same at each time t, it has actually increased over the years due to growing viewership. This makes deterring owner rent extraction crucial.

Comparing across sports, the NBA players' union is seen as more effective than the NFL players' union. A key distinction between the two is that the NFL operates with a salary cap and no limit on the size of individual contracts. Oddly, it is the NFL owners who have often expressed a desire for maximum contracts. Players seem to be wary.[15] The most recent NFL collective bargaining agreement extended the season by a game and shifted little of the additional revenue to the players. Before the deal was signed, stars protested the proposal but could not persuade others to join them. Most NFL players have short careers relative to the stars. Many live paycheck to paycheck and cannot afford a lockout.[16] Moreover, the fraction of stars (i.e. Quarterbacks) relative to the total number of players in the league is small. The NFL is similar to the "pre-max-contract" NBA in terms of the wealth disparity between high and moderate-talent players. My model suggests that NFL players may benefit from maximum contracts as such contracts will increase leverage in future negotiations, leading to players receiving a higher percentage of league revenue.

The idea of "redistributing patience" to improve bargaining outcomes applies to other labor markets. Moreover, unlike in sports, such unions can implement direct and indirect redistributive policies without necessarily requiring the approval of management. My model also points to the benefits of forming bargaining coalitions in industries where there are no unions. Especially relevant are industries where employees share in firm profits: traders, law associates, and investment bankers. The dominant fraction of take-home compensation comes from bonus pay. The mechanism by which they receive this compensation is similar to that of a sports team. Each group within a firm is allocated a pool of money in proportion to revenue generated (e.g. salary cap). From there, the director of the group pays the employees. My model suggests that creating a coalition in these industries and instituting a cap on bonus compensation will improve welfare for all workers.

[15] https://bleacherreport.com/articles/2884885-nfl-owners-reportedly-wanted-nba-like-max-contracts-in-new-cba

[16] http://www.nbcnews.com/id/41855264/ns/business-personal_finance/t/nfl-owners-wont-run-hurry-up-offense-vs-workers

A Collective Bargaining Appendix

All notation refers to the same objects as before, unless otherwise stated. I refer to a worker by his talent level and wealth at the time of bargaining. A worker is simply an ordered-pair $(\theta, w) \in Supp(G)$. A group of workers is a subset of $\{(\theta, w) | \theta \in \{l, h\} \text{ and } w \leq s_h \bar{t}\}$.

A.1 Subgame Perfect Majority Rule

Definition A.1 *A history $\mathfrak{h}^n$ is a sequence of n offers and acceptance/rejection decisions at each time $t \leq n$.*

Definition A.2 *Strategies for the workers' coalition (σ_W) and the firm (σ_F) are mappings from each $\mathfrak{h}^n$ to an offer* OR *acceptance/rejection choice.*

1. $\sigma_W(\mathfrak{h}^{2n}) \in [0,1]$ *and* $\sigma_W(\mathfrak{h}^{2n-1}) \in \{Y, R\}$, $\forall\, n \geq 1$.
2. $\sigma_F(\mathfrak{h}^{2n-1}) \in [0,1]$ *and* $\sigma_F(\mathfrak{h}^{2n}) \in \{Y, R\}$, $\forall\, n \geq 1$.

Definition A.3 *Given σ_W and σ_F, a path of play z is the realized sequence of offers and acceptance/rejection decisions. Since the game terminates when there is an acceptance, $|z|$ is the time at which an agreement is reached. The outcome is characterized by the accepted offer and time of acceptance.*

Definition A.4 *Define $U(m, z)$ as the payoff to worker $m \in Supp(G)$ under path of play z.*

Definition A.5 *Fix a strategy σ_F for the firm. A strategy σ_W for workers is said to violate the majority if there is a deviation after some history such that there exists a closed group of workers M, $G(M) \geq \frac{1}{2}$, with members of M strictly better off.*[17]

Definition A.6 *A SPMRE is a pair of strategies σ_W and σ_F such that σ_W is not in violation of the majority, and at any history, there is no deviation by the firm that leaves it better off.*

Lemma A.7 *Fix σ_F and σ_W. Suppose worker (θ, w) prefers $\hat{\sigma}_W$. At least one of the following is true: all workers $(\theta, \hat{w})$, $\hat{w} \leq w$, prefer $\hat{\sigma}_W$, OR all workers $(\theta, \hat{w})$, $\hat{w} \geq w$, prefer $\hat{\sigma}_W$.*

[17]The necessity of a deviating coalition to be strictly better off is because the distribution G is not continuous. If G were continuous, the definition would only require that a positive measure of such an M be strictly better off.

Proof: Under σ_W, the realized path of play is z, and the outcome is π_z at time $|z|$. Consider an alternative strategy $\hat{\sigma}_W$ that induces a path of play $\hat{z}$ and outcome $\pi_{\hat{z}}$ at time $|\hat{z}|$. Suppose worker (θ, w) prefers $\hat{\sigma}_W$ to σ_W. If $|\hat{z}| \geq |z|$, all workers with $\delta(\theta, \hat{w}) \leq \delta(\theta, w)$ will prefer $\hat{\sigma}_W$. If $|\hat{z}| \leq |z|$, all workers with wealth *lower than* w will prefer $\hat{\sigma}_W$. ■

Lemma A.7 implies we can restrict to M such that $\{(l,w)|(l,w) \in M\}$ and $\{(h,w)|(h,w) \in M\}$ are path-connected in wealth. Since the atoms of G occur at the maximum wealth levels of each type, Lemma *A.7* also implies that when checking SPMRE, it is sufficient to look at deviating coalitions of the form:

$$\left\{M \subset supp(G) | M \text{ is closed}, G(M) \geq \frac{1}{2} \text{ and } G(\hat{M}) < \frac{1}{2}, \forall \hat{M} \subset M\right\}$$

Given a path of play $\bar{z}$, consider a modified bilateral Rubinstein bargaining game, $\mathscr{R}(M, \bar{z})$ for any such $M \in S$. In $\mathscr{R}(M, \bar{z})$, strategies for Player 1 and Player 2 are labeled Σ_1 and Σ_2, respectively. Player 1's payoff is equivalent to that of the firm. Player 2's payoff, P_2, is defined over each possible path of play z:

$$P_2(z) = \inf_{m \in M} U(m, z) - U(m, \bar{z})$$

Since M is closed, there exists $m_z^* \in M$ such that $P_2(z) = U(m_z^*, z) - U(m, \bar{z})$.[18] Payoff function P_2 satisfies consistency and continuity as defined in Ray (2003). It follows that a subgame perfect equilibrium exists in $\mathscr{R}(M, \bar{z})$, and the one-shot deviation principle holds.

Lemma A.8 *The profile (σ_F, σ_W) is an SPMRE if and only if $(\Sigma_1, \Sigma_2) = (\sigma_F, \sigma_W)$ is a subgame perfect equilibrium in $\mathscr{R}(M, \bar{z})$ $\forall$ $M \in S$, where $\bar{z}$ is the path of play in the collective bargaining game under (σ_F, σ_W).*

Proof: Suppose (σ_F, σ_W) is an SPMRE. Fix a minimal deviating coalition $M \in S$. I need only show that (σ_F, σ_W) is subgame optimal in $\mathscr{R}(M, \bar{z})$. Consider the strategy profile $\Sigma_1 = \sigma_F$ and $\Sigma_2 = \sigma_W$. From Player 1's perspective, Σ_1 is optimal. Suppose $\Sigma_2 = \sigma_W$ is *not* subgame optimal in $\mathscr{R}(M, \bar{z})$. Then there is a one-shot deviation strategy $\hat{\Sigma}_2$ with $P_2(\Sigma_1, \hat{\Sigma}_2) > P_2(\Sigma_1, \Sigma_2) = P_2(\sigma_F, \sigma_W) = 0$. This means (σ_F, σ_W) is not an SPMRE in the collective bargaining game. The reverse direction follows trivially. ■

[18]The subscript reflects dependence on z.

Since a one-shot deviation in the collective bargaining game for any M is equivalent to a one-shot deviation in $\mathscr{R}(M)$, it follows that: (σ_F, σ_W) is an SPMRE if and only if:

1. $\forall M \in S$, σ_W is unimprovable with respect to workers in M via a one-shot deviation.[19]

2. σ_F is unimprovable via a one-shot deviation for the firm.

A.2 Collective Bargaining Outcome

I prove the collective bargaining results for general bargaining delay costs $\delta(\theta, w)$ where δ is decreasing in wealth for all θ. In the body of the paper, $\delta(\theta, w) = \delta(w)$. For notational convenience, I will use $\delta_\theta(w) = \delta(\theta, w)$ when necessary. For completeness, I will repeat some of the arguments in the body of the paper, but this time with the general delay cost δ.

Consider a hypothetical situation where a type θ worker with wealth w negotiates on behalf of the coalition. From Rubinstein (1982), there is a unique subgame perfect equilibrium where the workers receive:

$$\pi^*(\theta, w) = \frac{e^{-\Delta\delta(\theta,w)}(1 - e^{-\Delta\rho})}{1 - e^{-\Delta(\delta(\theta,w)+\rho)}}$$

π^* quantifies the bargaining power of a type θ worker with wealth w. Notice π^* depends on (θ, w) only through the delay cost in bargaining $\delta(\theta, w)$. A worker with *a high value* of $\delta(\theta, w)$ (high delay cost) has *low bargaining power*: $\pi^*(\theta, w) > \pi^*(\theta', w')$ if and only if $\delta(\theta, w) < \delta(\theta', w')$. A worker with **median bargaining power** has delay cost δ^* such that:

$$\delta^* = \sup\left\{\hat{\delta} : \mathbb{P}_G(\left\{(\theta, w) | \delta(\theta, w) > \hat{\delta}\right\}) \geq \frac{1}{2}\right\}$$

One can express δ^* in terms of worker type and wealth level. First, recognize that a high-talent worker can have the same bargaining power as a moderate-talent worker since both could have the same delay cost due to different wealth levels. This is not significant because what matters is the bargaining outcome itself which depends only on the delay cost. While there may be a high-talent player with median bargaining power, there will always be a moderate-talent player with median bargaining power since the measure of high-talent workers is $l_h < \frac{1}{2}$. Thus, there exists w^* such that $\delta(l, w^*) = \delta^*$.

[19]This is an extension of the one-shot deviation property in Blackwell (1965), except such deviations must be checked for each minimal deviating coalition.

Lemma A.9 *Consider a moderate-talent worker with wealth w^*. All moderate-talent workers with wealth lower than w^* have less bargaining power. All high-talent workers with wealth less than $\delta_h^{-1}(\delta_l(w^*))$ have less bargaining power.*

Proof: Such a worker has delay cost $\delta(l,w^*)$. Since the function is decreasing in wealth, moderate-talent workers with wealth less than w^* have higher delay costs, and therefore lower bargaining power. Now, notice that $\delta(h,w) \leq \delta(l,w^*) \iff w \leq \delta_h^{-1}(\delta_l(w^*))$. ■

Proof of Theorem 3.3: Let (σ_W, σ_F) denote the SPE strategy profile in a traditional Rubinstein bargaining game where the moderate-talent worker of wealth w^* negotiates with the firm on behalf of the workers. Given Lemma A.7 and because this is a SPE in the traditional Rubinstein Bargaining game, any deviating coalition that could do better excludes $\{(\theta,w) : \theta = l, w \leq w^*, \text{ and } \theta = h, w \leq \delta_h^{-1}(\delta_l(w^*))\}$. This set has a minimum size of $\frac{1}{2}$ by definition of w^*. Thus, (σ_W, σ_F) must be an SPMRE.

Next, I demonstrate uniqueness. Denote the worker of median bargaining power as m_b. Lemmas *A.7* implies that in any SPMRE, m_b must approve of the outcome. Let v_{lo} and v_{hi} be the minimum and maximum value to m_b in any SPMRE *starting in a period where the workers' coalition makes an offer*. Consider a period where the firm makes an offer. All workers with bargaining power less than m_b will accept an offer greater than $e^{-\Delta\delta(l,w^*)}v_{hi}$. Offers less than $e^{-\Delta\delta(l,w^*)}v_{lo}$ won't have majority approval.

Starting from this period, the firm can secure at least $1 - e^{-\Delta\delta(l,w^*)}v_{hi}$ and at most $1 - e^{-\Delta\delta(l,w^*)}v_{lo}$. Now, consider a period when the workers' coalition makes an offer. For the firm to accept, it must offer at least $e^{-\Delta\rho}(1 - e^{-\Delta\delta(l,w^*)}v_{hi}) \implies v_{hi} \leq 1 - e^{-\Delta\rho}(1 - e^{-\Delta\delta(l,w^*)}v_{hi})$. The firm will accept if offered more than $e^{-\Delta\rho}(1 - e^{-\Delta\delta(l,w^*)}v_{lo})$:

$$\implies v_{lo} \geq 1 - e^{-\Delta\rho}(1 - e^{-\Delta\delta(l,w^*)}v_{lo})$$

Combining the two inequalities yields:

$$v_{lo} \geq \frac{1 - e^{-\Delta\rho}}{1 - e^{-\Delta\rho}e^{-\Delta\delta(l,w^*)}} \geq v_{hi} \geq v_{lo} \implies v_{hi} = v_{lo}$$

A symmetric argument for the firm completes the proof. ■

Proof of Theorem 3.4: I prove a general version of the theorem when delay costs are $\delta(\theta,w)$. The conditions in the theorem are sufficient whenever $\delta(h,w) \leq \delta(l,w)$: if a high-talent and moderate-talent player have the same wealth, the high-talent player has weakly lower delay cost. This is trivially satisfied when $\delta(h,w) = \delta(l,w)$, as in the paper. For the remainder of this proof, assume $\delta(h,w) \leq \delta(l,w)$.

Given time is continuous and there is a continuum of workers of fixed measure, the exact law of large numbers holds (Duffie and Sun [2004]). The wealth distribution of type θ workers at time $\bar{t}$ is:

$$\mathbb{P}(w_\theta(\bar{t}) \geq q) = e^{-\lambda_\theta \frac{q}{s_\theta}} \text{ for } q < s_\theta \bar{t}$$
$$\mathbb{P}(w_\theta(\bar{t}) = s_\theta \bar{t}) = e^{-\lambda_\theta \bar{t}}$$

Define $\overline{w}(w) = \max\left\{0, \delta_h^{-1}(\delta_l(w))\right\}$. Recall that $\delta_h^{-1}(\delta_l(w))$ is the wealth level of a high-talent player with the same bargaining power as a moderate-talent player of wealth w. The median bargaining power is $\delta^* = \delta(l,w^*)$, where:

$$w^* = \inf\left\{w : (1-l_h)(1-e^{-\lambda_l \frac{w}{s_l}})\chi_{w \leq s_l \bar{t}} + (1-l_h)e^{-\lambda_l \bar{t}}\chi_{w=s_l\bar{t}} + l_h(1-e^{-\lambda_h \frac{\overline{w}(w)}{s_h}}) \geq \frac{1}{2}\right\} \tag{6}$$

The expression is complex due to the presence of an atom at the wealth level $s_l\bar{t}$. There are two cases to consider: $w^* < s_l\bar{t}$ and $w^* = s_l\bar{t}$.

Case #1:

First consider $w^* = s_l\bar{t}$. This implies that $\mathbb{P}_G(\{(\theta,w)|\delta(\theta,w) > \delta^*\}) > \frac{1}{2}$. Any small change in redistribution keeps the wealth level of the moderate-talent worker with median bargaining power at the atom. Therefore, $\frac{\partial w^*}{(\partial 1-\mu_0)} = \frac{\partial s_l}{(\partial 1-\mu_0)}\bar{t} > 0$. The wealth level w^* occurs at the atom when sufficiently many moderate-talent players that started at $t=0$, live until the time of bargaining. Thus, there exists ε such that $\lambda_h \leq \lambda_l \leq \varepsilon \implies w^* = s_l\bar{t}$.

Case #2:

For the second case, $w^* < s_l\bar{t} \implies$:

$$(1-l_h)(1-e^{-\lambda_l \frac{w^*}{s_l}}) + l_h(1-e^{-\lambda_h \frac{\overline{w}(w)}{s_h}}) = \frac{1}{2} \tag{7}$$

Holding w^* fixed and differentiating (7) with respect to $1-\mu_0$ yields the following necessary and sufficient condition for $\frac{\partial w^*}{\partial(1-\mu_0)} > 0$:

$$-e^{-\lambda_l \frac{w^*}{s_l}} \lambda_l w^* \frac{1}{{s_l}^2} + e^{-\lambda_h \frac{\overline{w}}{s_h}} \lambda_h \overline{w} \frac{1}{{s_h}^2} < 0 \tag{8}$$

Suppose $\lambda_h \leq \lambda_l$. Since $\delta(h,w) \leq \delta(l,w)$, it follows that $\overline{w}(w^*) \leq w^*$. Using Equation (6), one can compute the following bounds on $\lambda_l \frac{w^*}{s_l}$:

$$\lambda_l \frac{w^*}{s_l} \in \left[log(2),\ \min\left\{ \lambda_l \bar{t}, log\left(\frac{2-2l_h}{1-2l_h} \right) \right\} \right]$$

Since this set is compact, the function $e^{-\lambda_l \frac{w^*}{s_l}} \lambda_l w^* \frac{1}{s_l}$ achieves a minimum at some $\lambda_l \frac{w^*}{s_l} = y$. In fact, y will be one of the endpoints of the interval because the function $e^{-x}x$ attains its maximum at $x = 1$. Inequality (8) is guaranteed to hold when:

$$-e^y y \frac{1}{s_l} + e^{-\lambda_h \frac{\overline{w}}{s_h}} \lambda_h \overline{w} \frac{1}{{s_h}^2} < 0 \tag{9}$$

As λ_h declines, $e^{-\lambda_h \frac{\overline{w}}{s_h}} \lambda_h \overline{w} \frac{1}{{s_h}^2}$ approaches $0 \implies$ there exists k such that Inequality (9) holds for $\lambda_h \leq k$.[20] Hence, $\lambda_h \leq \min\{l_h, k\} \implies$ a reduction in μ_0 leads to an increase in w^*.

In particular, when $l_h < \frac{e-2}{2e-2}$, then $\lambda_h \leq \lambda_l$ is all that is required. To see this, notice that $l_h < \frac{e-2}{2e-2} \implies \lambda_l \frac{w^*}{s_l} \leq 1 \implies e^{-\lambda_l \frac{w^*}{s_l}} \lambda_l w^* \frac{1}{s_l} > e^{-\lambda_l \frac{\overline{w}(w^*)}{s_h}} \lambda_h \overline{w}(w^*) \frac{1}{s_h}$. The last implication stems from the fact that $e^{-x}x$ is increasing for $x \in [0,1]$. Since $s_l < s_h$, it follows that:

$$e^{-\lambda_l \frac{w^*}{s_l}} \lambda_l w^* \frac{1}{{s_l}^2} > e^{-\lambda_l \frac{\overline{w}(w^*)}{s_h}} \lambda_h \overline{w}(w^*) \frac{1}{{s_h}^2}$$

$\implies$ Inequality (8) holds.

Thus, if $l_h < \frac{e-2}{2e-2}$, then $\lambda_h \leq \lambda_l$ implies that increasing redistribution increases w^*. ■

[20] Notice that k may depend on l_h, λ_l, and $\bar{t}$.

B Robustness

Given a share of surplus π, the reward stream for a type θ worker is $\alpha_\theta \pi$, where $\alpha_h = \mu_0$ and $\alpha_l = 1 - \mu_0$. Suppose workers value reward s as $V(s)$. The firm values a share of surplus $1-\pi$ at $F(1-\pi)$. Assume F and V are increasing, continuous, and $F(0) = V(0) = 0$.

Proposition B.1 *Consider a Rubinstein bargaining game between the firm and a type-θ worker with discount rate δ_W. There is a subgame perfect equilibrium where the worker receives π_A:*

$$1 - \frac{1}{\alpha_\theta} V^{-1}\Big(\frac{1}{\delta_W} V(\alpha_\theta \pi_A)\Big) = \rho F(1-\pi_A)$$

Proof: Let f denote the equilibrium value to the firm when proposing first and v the equilibrium value to the worker when proposing first. Rubinstein (1982) implies that equilibrium values are characterized by the solution to the following system:

$$V\Big(\alpha_\theta(1 - F^{-1}(f))\Big) = \delta_W v \tag{10}$$

$$1 - \frac{1}{\alpha_\theta}(V^{-1}(v)) = \rho f \tag{11}$$

Solving for v in (10) and substituting it into (11) yields:

$$1 - \frac{1}{\alpha_\theta} V^{-1}\Big(\frac{1}{\delta_W} V\big(\alpha_\theta(1 - F^{-1}(f))\big)\Big) = \rho f$$

Since $F^{-1}(f) = 1 - \pi_A$ and the firm makes offers first in the game, the proposition follows. ■

When $V(\alpha_\theta \pi) = \alpha_\theta \pi$ and $F(1-\pi) = 1-\pi$, it reduces to the model discussed in the paper. In general, as long as V is increasing, Theorem 3.3 and Theorem 3.4 hold. What is not guaranteed is that bargaining power is determined entirely by the delay cost in bargaining. Before providing insight into why, I highlight when this would remain the case.

Proposition B.2 *If $V(xy) = V(x)V(y)$, then independent of the functional form of F, all the results from the paper still hold.*

Proof: $V(xy) = V(x)V(y) \implies \frac{1}{\alpha_\theta} V^{-1}\Big(\frac{1}{\delta_W} V(\alpha_\theta \pi_A)\Big) = V^{-1}(\frac{1}{\delta_W})\pi_A$

$$\implies \pi_A = 1 - \frac{1}{\rho} F^{-1}\Big(1 - V^{-1}(\frac{1}{\delta_W})\pi_A\Big)$$

$\implies \pi_A$ depends on the level of redistribution through the wealth-dependent discount rate. ■

The above proposition demonstrates that for common functions like $V(x) = x^k$ for $k > 0$, all the results from the paper still hold. However, it does reveal where complications may arise. To provide intuition, fix the payoff function for the firm to be $F(1-\pi) = \pi$. Then the equilibrium share of surplus accorded to the workers is:

$$1-\rho = \frac{1}{\alpha_\theta} V^{-1}\Big(\frac{1}{\delta_W} V(\alpha_\theta \pi_A)\Big) - \rho\pi_A$$

How $\frac{1}{\alpha_\theta} V^{-1}\Big(\frac{1}{\delta_W} V(\alpha_\theta \pi_A)\Big)$ changes when α_θ increases will depend on the properties of the function V. α_θ affects this quantity directly as well as through δ_W! Before, α_θ only affected bargaining through δ_W by changing the the initial reward stream and thus changing the accumulated wealth by the time of bargaining.

Example 2 *Let $V(\alpha_\theta \pi) = log(1+\alpha_\theta \pi)$ and $F(1-\pi) = 1-\pi$. Suppose a type θ worker represents the coalition and engages in bargaining with the firm. The equilibrium equation is:*

$$1-\rho = \frac{1}{\alpha_\theta}(1+\alpha_\theta \pi_A)^{\frac{1}{\delta_W}} - \rho\pi_A$$

The term $\frac{1}{\alpha_\theta}(1+\alpha_\theta \pi_A)^{\frac{1}{\delta_W}}$ represents the share of surplus such that the worker would be indifferent between accepting that share in the next period and accepting π_A now. If a_θ increases, so that the representative earns a higher share of the surplus accorded to the workers, π_A may actually go down![21] *As α_θ increases, the worker has a strictly larger payoff at every share of surplus, but $\frac{1}{\alpha_\theta} V^{-1}\Big(\frac{1}{\delta_W} V(\alpha_\theta \pi_A)\Big)$ is sufficiently concave in π_A at the higher levels of α_θ. The firm can credibly reduce the share it accords the worker.*

The example illustrates how redistribution can have bidirectional effects for some $V(\cdot)$. Increasing redistribution helps in bargaining by increasing wealth levels and reducing median delay costs. However, it also increases α_l, which may make moderate-talent workers more passive and giving the firm an incentive to *reduce* the share of surplus accorded to the workers.

[21] Take $\delta_W = 0.4$, $\rho = 0.9$, and vary α_θ from $\frac{1}{2}$ to $\frac{2}{3}$. The worker share of surplus declines.

REFERENCES

1. Ahlquist, J. (2017). “Labor Unions, Political Representation, and Economic Inequality,” Annual Review of Political Science 20, 409-432.

2. Baron, D.P. and J. Ferejohn (1989). “Bargaining in legislatures,” American Political Science Review 83(4), 1181-1206.

3. Blackwell, D. (1965). “Discounted Dynamic Programming,” Annals of Mathematics and Statistics 36(1), 226-235.

4. Booth, A. (1995). “The economics of the trade union,” Cambridge University Press.

5. Burguet, R. and J. Sákovics (2019). “Bidding for talent in sport,” Economic Inquiry 57(1), 85-102.

6. Card, D. (1990). “Strikes and Bargaining: A Survey of the Recent Empirical Literature,” American Economic Review Papers & Proceedings 80(2), 410-415.

7. Card, D., R. Chetty, and A. Weber (2007). “Cash-on-Hand and Competing Models of Intertemporal Behavior: New Evidence from the Labor Market,” Quarterly Journal of Economics 122(4), 1511-1560.

8. Compte, O. and P. Jehiel (2010). “Bargaining and Majority Rules: A Collective Search Perspective,” Journal of Political Economy, 118(2), 189-221.

9. Cramton, P. and J. Tracy (2003). “Unions, Bargaining, and Strikes,” International Handbook of Trade Unions.

10. Cramton, P., H. Mehran, and J. Tracy (2015). “Bargaining with a Shared Interest: The Impact of Employee Stock Ownership Plans on Labor Disputes,” Working Paper.

11. Dietl, H., M. Lang, and A. Rathke (2009). “The effect of salary caps in professional team sports on social welfare,” B.E. Journal of Economic Analysis and Policy 9(1), 129-151.

12. Duffie, D. and Y. Sun (2004). “The Exact Law of Large Numbers for Independent Random Matching,” Working Paper, Stanford University Graduate School of Business.

13. Eraslan, H. and K.S. Evdokimov (2019). “Legislative and Multilateral Bargaining,” Annual Review of Economics 11, 443-472.

14. Feiveson, L. (2015). "General Revenue Sharing and Public Sector Unions," Journal of Public Economics 125(), 28-45.

15. Fernandez, R. and J. Glazer (1991). "Striking for a bargain between two completely informed agents," American Economic Review 81(1), 240-252.

16. Frederick, Shane, George Loewenstein, and Ted O'Donoghue, "Time Discounting and Time Preference: A Critical Review," Journal of Economic Literature 40(2002), 351-401.

17. Fort, R. and J. Quirk (1995). "Cross-subsidization, incentives, and outcomes in professional team sports leagues," Journal of Economic Literature 33(3), 1265-1299.

18. Grossman, S.J. and O.D. Hart (1986). "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," Journal of Political Economy 94(4), 691-719.

19. Haller, H. and S. Holden (1997). "Ratification requirement and bargaining power," International Economic Review 38(4), 825-851.

20. Hardy, M., G. Kady, and L. Song (2021). "Gotta' Have Money to Make Money? Bargaining Behavior and Financial Need of Microentrepreneurs," American Economic Review: Insights, *forthcoming*.

21. Houba, H. and G. van Lomwel (2001). "Counter Intuitive Results in a Simple Model of Wage Negotiation," Economic Theory 17(1), 81-99.

22. Kaufman, B. (2002). "Models of Union Wage Determination: What Have We Learned Since Dunlop and Ross?," Industrial Relations 41(1), 110-158.

23. Késenne, S. (2000). "The Impact of Salary Caps in Professional Team Sports," Scottish Journal of Political Economy 47(4), 422-430.

24. Lee, D. and A. Mas (2012). "Long-run impacts of unions on firms: New evidence from financial markets, 1961-1999," Quarterly Journal of Economics 127(1), 333-378.

25. Madden, P. (2019). " Collective Bargaining in a Basic North American Sports League Model With Broadcasting Revenue," Journal of Sports Economics 20(8), 1088-1118.

26. Manzini, P. and M. Mariotti (2005). “Alliances and Negotiations,” Journal of Economic Theory 121, 128-141.

27. McDonald, I. and R. Solow (1981). “Wage Bargaining and Employment,” American Economic Review 71(5), 896-908.

28. Mortensen, D. and C. Pissarides (1994). “Job Creation and Job Destruction in the Theory of Unemployment,” Review of Economics Studies 61(3), 397-415.

29. Muthoo, A. (2004). “A model of the origins of basic property rights,” Games and Economic Behavior 49(2), 288-312.

30. Penceval, J. (1991). “Labor Markets Under Trade Unionism,” Oxford: Basil Blackwell.

31. Ray, D. (2003). “Notes on Game Theory,” *Dept. of Economics, New York University*.

32. Rubinstein, A. (1982). “Perfect Equilibrium in a Bargaining Model,” Econometrica 50(1), 97-109.

33. Sandroni, A. and C. Urgun (2018). “When to Confront: The Role of Patience,” American Economic Journal: Microeconomics 10(3), 219-252.

34. Schweighofer-Kodritsch, S. (2018). “Time Preferences and Bargaining,” Econometrica 86(1), 173-217.

35. Serrano, R. (2007). “Bargaining,” Instituto Madrileño de Estudios Avanzados (IMDEA) Ciencias Sociales.