Unraveling and Inefficient Matching

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Abstract

Labor markets are said to unravel if the matches between workers and firms occur inefficiently early, based on limited information. I argue that a significant determinant of unraveling is the transparency of the secondary market, where firms can poach workers employed by other firms. I propose a model of interviewing and hiring that allows firms to hire on the secondary market *as well as* at the entry-level. Unraveling arises as a strategic decision by low-tier firms to prevent poaching. While early matching reduces the probability of hiring a high type worker, it prevents rivals from learning about the worker, making poaching difficult. As a result, unraveling can occur even in labor markets without a shortage of talent. When secondary markets are very transparent, unraveling disappears. However, the resulting matching is still inefficient due to the incentives of low-tier firms to communicate that they have *not* hired top-quality workers. Coordinating the timing of hiring does not mitigate the inefficiencies because firms continue to act strategically to prevent poaching.

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1 Introduction

Labor markets in which matches between firms and workers occur inefficiently early due to limited information are said to have unraveled. A classic example is the market for appellate court judicial clerks in the United States. Judges rush to make offers to law students as early as two years before the start date, at which point information about a student's legal writing is non-existent (Avery et al. [2001]). Unraveling is often ascribed to the combination of intense competition for scarce, high-quality workers and applicant uncertainty, which drives them to accept early offers (Roth and Xing [1994]).

However, there are markets that experience unraveling though employers *do not* face a scarcity of talent. In corporate law associate hiring, students also receive offers in their first year of law school. New investment banking analysts obtain offers as college sophomores. At the same time, not all markets unravel. For instance, until recently, there was general agreement that the market for assistant professors in economics does not experience unraveling.[1] Table 1 lists some labor markets and whether they unravel.

Markets	**Unraveling**[2]
Corporate Law Associates	✓
Private Equity Associates	✓
Investment Banking Analysts	✓
High-end Chefs and Line Cooks	✓
Hedge Fund Traders	X
Assistant Professors in Economics	X
Management Consulting	X
Programmers and Software Engineers	X

Table 1

[1] Post-pandemic debate about whether the assistant professor market has unraveled has grown. I will argue that even with the switch to virtual interviews, we should not expect significant unraveling to arise. Moreover, I highlight that the central concern is actually one of coordination due to costly interviewing. This affects markets with a central clearinghouse (e.g. medical matching market).

[2] In corporate law and banking, firms cap the number of hires from each university (i.e. in 2014, the BAML S&T division in New York capped Stanford hires at two). It is difficult to believe that this policy would exist if firms believed talent was scarce. For unraveling in law markets, see Ginsburg and Wolf (2004). Herald Chen, former managing director at KKR, provided testimony on private equity unraveling. Hiring timelines were also provided by the Stanford career center.

Why do some markets with an abundance of talent unravel? And why do some markets unravel while others do not? My paper answers this question by identifying a new channel by which unraveling can propagate. I argue that unraveling can be caused by the presence and characteristics of a *secondary market*, whereby firms may poach workers currently employed by other firms. Poaching is a prevalent form of rematching in many industries: investment banks can recruit analysts from other banks, law firms can attract associates and partners from competitors, universities can hire professors laterally, and large venture capital firms can poach startups from smaller venture capital firms during series funding rounds. In models with a single stage of hiring and matching, the main driver of firm behavior is the desire to acquire top talent. With a secondary market, firms must also be concerned with their ability to *retain* the talent they hire. A secondary market might mitigate unraveling because it allows for rematching, but this depends on its transparency. In fact, moderately transparent secondary markets can *promote* unraveling as early hiring prevents rivals from learning about the worker. When secondary markets are highly transparent, unraveling disappears, but inefficiencies remain because of the threat of poaching.

These conclusions are based on a model of firm interviewing and hiring when there is a secondary market that allows firms to hire laterally instead of at the entry-level. I consider a situation with a high-tier and low-tier firm, each in need of a single worker from a large pool of applicants. Workers are of high or low type.[3] Time is divided into two stages. The first is the *primary market stage*, and the second is the *secondary market stage*. The former is analogous to the time students are in law school, while the latter is the time they work as associates post-graduation. In the primary market, firms can select a time to interview candidates and make a hiring decision; the worker begins working at the end of the primary market. The later a firm interviews, the better it can distinguish between them. A key feature of the model is that firms can choose whether to hire in the primary market or the secondary market, where they can monitor the worker hired by its competitor and "poach" her. This reflects the fact that firms receive signals about the quality of workers employed *at other firms*. The clarity of

[3]The model applies to other two-sided-matching markets. For example, "worker" could be replaced by "startup" and "firm" by "venture capitalist".

these signals varies across industries.[4]

The existence of a monitoring technology and an additional stage of mobility introduce a strategic element overlooked in prior work. Firms now must be mindful of losing a worker in the secondary market. To forestall poaching, a firm has two levers at its disposal: increase the hired worker's wage or increase uncertainty about the quality of the hired worker. Raising the wage makes the worker more costly to poach. Similarly, obscuring the quality of the worker it hires discourages poaching due to the increased uncertainty about the worker's ability. I demonstrate that when the secondary market is not too transparent, the best way to prevent poaching is via the latter action.

But how does a firm *credibly* increase uncertainty about the quality of the worker it hires? By hiring early, when all parties involved have less information.

Consequently, unraveling arises because early hiring acts as a signal-jamming mechanism. Low-tier firms interview early to make monitoring and poaching in the secondary market more challenging for high-tier firms. As the secondary market becomes more transparent (i.e., the monitoring technology improves), unraveling disappears in equilibrium due to the low-tier firm's incentive to communicate that they have *not* hired top-quality applicants. Low-tier firms interview candidates at the end of the primary market to ensure the hiring of applicants that are unlikely to be of high quality. This has stark welfare implications. A highly transparent secondary market decreases total match quality as it creates an adverse signaling incentive for the low-tier firm.

Could one improve match quality via coordination of the hiring times in the primary market? Not necessarily. Unraveling is a strategic response to the threat of poaching, and coordination on hiring time does not fully mitigate the threat. Moreover, such coordination may *reduce* match quality in comparison to the decentralized setting. This indicates that to increase ex-ante match quality, the focus should be on the secondary market rather than controlling timing in the primary market. Finally, my analysis has consequences in other markets where assets of uncertain quality are mobile, and counterparties must make costly investments to ascertain their quality. "Unraveling" occurs in

[4] In the market for economics professors, it is easy for universities to monitor professors at competing institutions: papers are published, and research is presented. On the other hand, it is more difficult for corporate law firms to ascertain the ability of associates at rival firms.

these environments in the form of under-investment in screening.

In the next section, I describe my model and its relation to the relevant literature. Sections 3 and 4 focus on the equilibrium analysis, and Section 5 discusses applications of my results. Finally, Section 6 discusses the assumptions of the model and the robustness of the results. Proofs are relegated to the appendix.

2 Model

Two firms, F_H (high-tier) and F_L (low-tier), each need a single worker from a finite pool of size N. Workers are either of high or low type, represented by $\theta \in \{H, L\}$. Each worker prefers to work for the high-tier firm, all else equal. The probability a worker is of high type is β, independent of the others. I assume N is sufficiently large so that if types were realized, there would be more high type workers than available slots with probability sufficiently close to 1. For expositional purposes I assume that the probability is exactly 1 (corresponding to the case where $N \to \infty$). [5]

Time is continuous from $[-T, \infty)$ and divided into two stages: $[-T, 0]$, which I call the primary market stage, and $(0, \infty)$, which I call the secondary market stage. Hiring can take place in each of the stages. If a firm approaches a worker at time $t \in [-T, 0]$ with an offer at wage w, and the worker accepts, the worker exits the market and *begins working at time 0*. A firm that fails to hire in the primary market can choose to 'poach' the employed worker at any time $t \in [0, \infty)$ in the secondary market.

2.1 Information in Primary and Secondary Market Stages

Primary Market

A firm choosing to hire in the primary market selects a time $t \in (-T, 0]$ at which to conduct interviews. Interviews are more informative the later they occur. One can think of interviews as a sequence of progressively more informative binary tests that return a high or low-signal depending on the worker's true type. This can be represented by

[5] The probability that there are less than two high type workers in the population converges to 0 as N gets large. All results in this paper hold for a sufficiently large finite N. See Appendix C..

a function $M : [-T,0] \longrightarrow [0,1] \times [0,1]$ that maps interview time to the probability the worker is of high type conditional on a high-signal, and the probability the worker is of high type conditional on a low-signal, respectively.

More generally, consider any mapping $M : [-T,0] \longrightarrow [0,1] \times [0,1]$ satisfying:

1. $M(t) = (M_{high}(t), M_{low}(t))$ is continuous.
2. $M_{high}(t)$ is increasing in t and $M_{low}(t)$ is decreasing in t.
3. For any $t > t'$, $M_{high}(t) - M_{high}(t') > 0$ if and only if $M_{low}(t) - M_{low}(t') < 0$.
4. $M_{high}(-T) = M_{low}(-T) = \beta$.

Such a mapping M is a reduced form representation of how well firms can sort workers at time t. The maximum probability that a worker is of high type given the results of any screening mechanism at time t is $M_{high}(t)$. The minimum probability that a worker is of high type given the results of any screening mechanism at time t is $M_{low}(t)$. The fourth condition indicates that there is no ability to sort at the start. In Appendix C, I show that any such M is equivalent to a sequence of progressively more informative binary tests. Thus, I define a **high-signal** worker at time t to be a worker that is high type with probability $M_{high}(t)$. Similarly, a **low-signal** worker at time t is one that is high type with probability $M_{low}(t)$. At a given time t, firms can interview all workers in the primary market costlessly, which means they can hire a high-signal or low-signal worker almost surely.[6] Given sorting is best at the end of the primary market, $M_{high}(0)$ and $M_{low}(0)$ are the maximum and minimum probability with which a firm can be sure that it has hired a high type worker, respectively.

Fix an M. If a firm hires a worker at time $t < 0$ and the worker is high type with probability $p < M_{high}(0)$, I say that the market has **unraveled**.

Secondary Market

Consider a firm that does not hire in the primary market, instead choosing to operate in the secondary market where it can monitor the worker hired by the other firm. The

[6]Under the binary test interpretation, randomization allows firms to hire a worker that is high type with probability $p \in [M_{low}(t), M_{high}(t)]$.

monitoring firm observes a signaling process yielding information about the employed worker's type. To formalize this, consider a worker whose probability of being of type $\theta = H$ is p_0. The monitoring technology is represented by an observable process $\{\pi_t\}$:[7]

$$d\pi_t = \mu_\theta dt + \sigma dB_t$$
$$\pi_0 = 0$$

One can interpret π_t as a noisy signal of visible worker output. The type-dependent drift satisfies $\mu_H \geq \mu_L$, reflecting expected differences in output between worker types. The quantity $\alpha = \dfrac{\mu_H - \mu_L}{2\sigma^2} = \dfrac{\bar{\mu}}{\sigma^2}$ represents the **transparency of the secondary market**.

2.2 Payoffs

Consider a type θ worker hired by F_i in the primary market at wage w. She will start working at time 0. Suppose at time t, firm F_{-i} approaches her with an offer of wage w'. If she accepts the offer, payoffs *from a time* 0 *perspective* are:

$$\left.\begin{aligned} \text{Worker}: \ & r\int_0^t e^{-r\hat{t}}(w + \delta_{i=H})d\hat{t} \ + \\ & re^{-rt}\int_0^\infty (w' + \delta_{-i=H})d\hat{t} \end{aligned}\right\} \delta \text{ is added payoff from working at } F_H.$$

$$\left.\begin{aligned} F_i: \ & r\int_0^t e^{-r\hat{t}}(Z_\theta^i - w)d\hat{t} \\ F_{-i}: \ & re^{-rt}\int_0^\infty e^{-r\hat{t}}(Z_\theta^{-i} - w')d\hat{t} \end{aligned}\right\} Z_\theta^i \text{ represents } \textit{match quality} \text{ to the firm.}$$

Match quality encapsulates productivity and output. I assume:

$$Z_H^H \geq Z_H^L \geq Z_L^L > 0 > Z_L^H$$

The inequalities reflect firm preferences and incorporate a notion of supermodularity in match quality. Both firms prefer high type workers. Notably, the high-tier firm never wants to employ a low type worker, while the low-tier firm finds such a worker acceptable. This is a natural assumption, as high-tier firms may have reputational concerns, so hiring a low type worker is especially undesirable.

Workers not hired in the primary market receive a payoff normalized to 0 and leave the game. Firms that are unmatched receive a flow payoff of 0 for the duration they are

[7] Construct a probability space $(\Omega, \mathscr{F}, \mathbb{P}_0)$, where $\mathbb{P}_0$ is the measure induced by p_0. Let B_t be a Brownian motion with respect to $\mathbb{P}_0$ independent of θ. The process $\{\pi_t\}$ is defined on $(\Omega, \mathscr{F}, \mathbb{P}_0)$.

unmatched. I assume that once a worker is hired and begins working for a firm, she can never be fired. This is without loss, as F_L will never choose to fire a worker, and giving F_H the power to terminate a worker is analogous to a rescaling of the match quality.

Lastly, I impose the trivial condition that $M_{high}(0)Z_H^H + (1 - M_{high}(0))Z_L^H > 0$, as otherwise the high-tier firm would never want to hire in the primary market.

2.3 Strategies

Due to continuous time, formal definitions of strategies require care. The technicalities, which I omit here, can be found in Appendix A. Each worker strategy consists of the following decisions: accept or reject offers in the primary market, and, if hired, whether to accept a lateral offer. **F_L's strategy** consists of:

1. A time t in the primary market at which to interview.
2. Conditional on t, whether to make an offer to a high or low-signal worker.

The low-tier firm will always choose to hire conditional on interviewing (as it will be unable to "poach" a worker from F_H). It is clear F_H never enters the primary market before $t = 0$. Therefore, its decision is whether to operate in the secondary market or hire in the primary market at $t = 0$, and it can make this choice based on the observed time at which F_L hires and the offered wage. Upon this observation, F_H formulates beliefs about whether F_L hired a high-signal or low-signal worker. Thus, **F_H's strategy** *maps the observed time at which F_L hires* to:

1. A probability of hiring in the primary market.
2. A poaching rule conditional on operating in the secondary market.
3. A belief about the worker hired by F_L.

Within the secondary market, the high-tier firm must decide at each time whether to hire the worker at F_L or not. Hence, such a decision is equivalent to a stopping time where F_H hires the worker when the process stops. Formally, a **poaching rule** is a stopping time τ adapted to the filtration $\mathscr{F}_t^{\pi}$.

A pair of firm strategies constitute an **equilibrium** if each firm is best-responding at each information set, and F_H's beliefs are consistent on-path.[8]

OBSERVATION 2.1 *A worker that receives a primary market offer always accepts it.*

The proof is in Appendix C. One might think that a worker receiving an offer is a signal of her type, allowing for the opportunity to strategically reject the offer. When N is large, this incentive disappears as the probability of receiving a future offer is ≈ 0.

Before discussing the dynamics of the model, I highlight the benchmark, which serves as a comparison to the equilibrium findings. Suppose no secondary market exists.

OBSERVATION 2.2 *If a secondary market does not exist, there is no unraveling.*

Since N is large, there will be βN high types with probability close to 1. If the firms interview at $t = 0$, each will be able to match with a high-signal worker almost surely. Total match value generated is $M_{high}(0) \cdot (Z_H^L + Z_H^H) + (1 - M_{high}(0)) \cdot (Z_L^L + Z_H^L)$.

When there is no secondary market and an abundance of talent, there is no unraveling. This aligns with Roth and Xing (1994) and Niederle et al. (2013). Each firm can hire a high-signal worker at the end of the primary market.

2.4 Relation to the Literature

An extensive literature on market unraveling was spawned by Roth and Xing (1994), who identified the phenomenon and several dozen markets that had experienced an unraveling of appointment dates. Along with Avery et al. (2001), they conjecture that firms "jump the gun" to acquire top talent. Niederle et al. (2013) formalize this intuition in a market with *comparable* supply and demand, where firms and workers both believe that they are on the long-side of the market. My paper demonstrates that unraveling can occur *even when talent is plentiful*, and there is a supply and demand *imbalance*.[9]

[8] The equilibrium concept used is Perfect Bayesian Equilibrium.

[9] Other papers propose different causes. Damiano et al. (2005) examine a search and matching model, where introducing participation costs decrease the fraction of low types searching in early periods. Firms are incentivized to match early or face a pool of workers bereft of talent. Halaburda (2010) and Echenique and Pereyra (2016) view unraveling similar to a bank run: unraveling by one firm incentivizes unraveling by others. Fainmesser (2013) highlights the effect of networks and social connections on unraveling.

A common theme in the unraveling literature is the presence of informational uncertainty. Li and Rosen (1998) and Li and Suen (2000) examine matching markets with one-sided and two-sided uncertainty, respectively. In these papers, unraveling acts as insurance against being unmatched. In my paper, unraveling is insurance against competitors poaching a hired worker. However, due to the threat of poaching, coordinating on the time of contracting does not necessarily increase match quality.

The relationship between strategic signaling incentives in labor markets and unraveling builds on Milgrom and Oster (1987), Ostrovsky and Schwarz (2010), and Ely and Siegel (2013). The former develop a model where firms profit by placing talented workers in less visible positions to prevent wage increases from competition. They do not focus on unraveling or screening. In my model, a wage increase is beneficial in that it makes poaching more costly. Moreover, firms can not limit employee visibility, but they can control the flow of information by affecting the initial signal of a worker's ability. Ostrovsky and Schwarz (2010) endogenize information revelation in the primary market to show that optimal information disclosure prevents informational unraveling. They do not consider the presence of a secondary market where more information could be revealed in the future. While unraveling in their context is different than in mine, a secondary market in their setting *can counteract* the benefits of the informational disclosure policy. Ely and Siegel (2013) examine a common-value labor market where firms first observe a private signal about workers and then decide whether to interview. When interview decisions are public, adverse selection arises, leading to low-tier firms never hiring. A crucial feature of my model is that it is not a common-value setting: high-tier firms are averse to hiring certain worker types. This aversion, coupled with the presence of the secondary market, generates adverse signaling incentives. High-tier firms can opt out of the primary market, monitor workers at low-tier firms, and potentially poach said workers in the future. The low-tier firm uses early interviewing to credibly reduce its own ability to sort effectively to disincentivize the high-tier firm from choosing to poach in the secondary market.

Much of the theoretical literature described assumes wages to be fixed. Du and Livne (2016) find that when transfers are flexible, early contracting is mitigated. I allow for flexible wage-setting, yet unraveling is unabated due to the secondary market. While increasing the wage deters poaching, it is not as effective as early matching.

None of the above papers allow for rematching between workers and firms. By dividing time into two stages, I highlight the strategic signaling incentives induced by the secondary market. Moreover, my model yields a characterization of the time at which unraveling occurs and precise comparative statics regarding hiring times.

My paper fits into a broader literature on poaching in labor markets. Most papers in this area study how the presence of poaching affects firm investment in the development of their workers (e.g. Moen and Rosén [2004]; Leuven [2005]). Since general skill training makes a firm's worker more attractive to outsiders, poaching reduces the firm's return from such training. Battiston, Espinosa, and Liu (2020) provide empirical evidence showing how firms deter poaching by strategically rotating workers across clients so as to ensure they are not too productive for a particular client. My paper differs from these in that I focus on how poaching impacts firms' screening incentives. The presence of poaching affects the type of worker firms hire in the first place.

Finally, at a higher level, my paper relates to the research on the strategic incentives generated by aftermarkets and resale markets.[10] One can think of the secondary market in my paper as an aftermarket where extra information becomes available. A critical difference between my model and this literature is that firms rank-order objects similarly and the objects can become unavailable.

3 Poaching and Incentives

I begin by characterizing the optimal poaching rule *conditional on F_H operating on the secondary market*. That is, suppose F_L has hired a worker in the primary market, and F_H is monitoring the worker. If the worker is earning a wage w, to successfully poach at any time, F_H must offer $\max\{w-\delta,0\}$. Thus, F_H's decision problem is:

$$\Gamma_H(p_0,w) = \max_{\tau} \mathbb{E}[e^{-r\tau}(Z_\theta^H - \max\{w-\delta,0\})|\mathscr{F}_t^\pi, p_0, w]$$

To determine the optimal stopping rule for F_H, I map π_t to the space of posterior

[10] Ausubel and Cramton (1999), Halafir and Krishna (2009), Carroll and Segal (2019).

beliefs.[11] Given initial belief p_0, let $p_t = \mathbb{P}(\theta = H|\mathscr{F}_t^{\pi})$ denote the posterior belief that the worker is of high type at time t given the observations from the process $\{\pi_t\}$.

Proposition 3.1 *The optimal poaching rule is a threshold stopping time of the form:*

$$\tau^* = \inf\{t \geq 0 : p_t \geq B^*\}$$

Where B^* depends on (α, w, Z_θ^H), is time-invariant, and independent of p_0.

Proof: *See Appendix B.*

The decision to poach depends solely on whether the belief about the worker is above a static threshold B^* that is independent of the initial belief p_0. The sharp characterization of τ^* elucidates the close relationship between poaching and the secondary market's informativeness. As the transparency of the secondary market increases (α increases), B^* increases. With a more informative signal, the high-tier firm can afford to wait for a higher posterior. Note that while w and α both affect the value of B^*, only α affects the *speed* of reaching a given threshold.

3.1 High-tier Firm Hires Laterally Only

To understand the incentives at work, consider a setting where the low-tier firm F_L is the only participant in the primary market, with the high-tier firm F_H only hires laterally. Out of the pool of available workers, F_L hires one and understands that she may eventually be poached by F_H.

The high-tier firm uses the poaching rule τ^*. Suppose the high-tier firm has initial belief $\tilde{p}_0$ about the worker F_L has hired. The payoff to the low-tier firm from employing a worker at wage w with probability p_0 of being a high type is:

$$\Sigma_L(p_0, \tilde{p}_0, w) = \underbrace{p_0(Z_H^L - w) + (1 - p_0)(Z_L^L - w)}_{\text{Expected Match Value}} - \textit{Loss due to Poaching}$$

The first term reflects the expected net match quality conditional on the low-tier firm keeping the worker forever. The second term is the expected loss due to poaching by F_H. The loss due to poaching is dependent on the actual probability (p_0) that the worker is of high type as well as the high-tier firm's belief ($\tilde{p}_0$) that the worker is of high type.

[11] Related is the experimentation literature (Wald [1947]; Moscarini and Smith [2001]).

Of particular interest is the function $\Gamma_L(p_0, w) = \Sigma_L(p_0, p_0, w)$: the expected payoff to the low-tier firm when its belief is consistent with the high-tier firm's belief about the worker. Considering Γ_L, observe that the two parameters that F_L can control are the prior on the worker it hires and the wage. Increasing the wage increases the cost of poaching for F_H, thereby raising the belief threshold B^* needed before poaching can occur. On the other hand, changing the probability that the worker is of high type delays the time until poaching. It is not obvious which lever the low-tier firm should pull.

Proposition 3.2 ***"Obscuring the quality of the worker is best."***

Fix an α. There exists p^* depending on α such that:

$$(p^*, 0) = arg \max_{p_0 \in [0,1], w \geq 0} \Gamma_L(p_0, w)$$

Proof: *See Appendix B.*

Increasing the wage increases F_L's costs but also makes poaching more costly. By increasing w, F_L can artificially increase B^* and lengthen the expected time it employs a worker. However, what Proposition 3.2 shows is that the best way to make poaching more costly is to hire a worker with a different expected match quality and pay her a wage of 0. The key is to show that the wage is a suboptimal tool to deter poaching.

The quantity p^* defined in Proposition 3.2 represents the optimal induced prior in a game where the high-tier firm is committed to hiring in the secondary market *and knows* the probability that the worker is of high type at the time it was hired by the low-tier firm. The intuition behind the proposition is that in order to increase the threshold belief, the low-tier firm must increase the wage in magnitude proportional to $Z_H^H - Z_L^H$, which is quite costly. On the other hand, by changing the prior probability the hired worker is of high type, the reduction is proportional to $Z_H^L - Z_L^L$.

The proof of Proposition 3.2 also illustrates how the optimal induced prior p^* varies naturally with α, the transparency of the secondary market. As $\alpha \to 0$ (low transparency), the low-tier firm understands that poaching is more challenging; it is more willing to hire potentially high-quality workers in the primary market. Conversely, as $\alpha \to \infty$ (high transparency), F_L seeks to hire a high type worker with low probability to ensure that it can keep the worker for a long time.

Proposition 3.3 *There exists $\bar{\alpha} > 0$ such that:*

1. *For $\alpha \in [0, \bar{\alpha}]$, the market unravels. The low-tier firm hires at $t^* < 0$, where $M_{high}(t^*) = p^*(\alpha)$. The wage is zero.*
2. *For $\alpha \in (\bar{\alpha}, \infty)$, the low-tier firm hires a worker that is high type with probability $p(\alpha)$, the belief the high-tier firm holds to make the low-tier firm indifferent between worker types.*[12] *Unraveling and non-unraveling equilibria exist.*

Proof: *See Appendix B.*

4 Equilibrium and Match Quality

The previous section examined the strategic decisions made when F_H specializes in hiring in the secondary market. I now analyze the equilibrium dynamics when F_H can choose whether to hire on the primary market *as a function of the history it observes*.

The decision to operate on the secondary market depends on the effectiveness of screening in the primary market. Suppose at the end of the primary market that the screening ability is such that the posterior belief is already above the poaching threshold. In that case, the high-tier firm will not hire in the secondary market. Since the labor market supply is large, both firms will interview at $t = 0$ and hire a high-signal worker.

OBSERVATION 4.1 *If $M_{high}(0) > B^*$, the market does not unravel.*

In Observation 4.1, if the threshold belief for poaching, B^*, is lower than the belief about a high-signal worker at the end of the primary market, the high-tier firm will always choose to hire in the primary market. In this case, the secondary market provides no value to the high-tier firm; the monitoring technology does no better than what can be achieved with screening in the primary market. This serves as the basis for a definition of opaqueness. Let α_{opaque} denote the value of transparency such that $B^* = M_{high}(0)$. Hence, for $\alpha \leq \alpha_{opaque}$, the high-tier firm never operates in the secondary market.

At transparency levels $\alpha > \alpha_{opaque}$, the insight that the secondary market incentivizes the low-tier firm to hire in a manner to prevent poaching still holds. However, recognize that the high-tier firm will not necessarily ex-ante commit to operating in the

[12] To do so, the low-tier firm mixes between hiring high and low signal workers

secondary market. Since the high-tier firm can only observe the time at which the low-tier firm hires, its belief about the worker hired can only be contingent on the hiring time t and its knowledge of the interviewing technology $M(t)$. It becomes crucial, then, to pin-down the firms' "indifference beliefs":

1. When is the high-tier firm indifferent between operating in the secondary market and hiring at the end of the primary?
2. Conditional on the high-tier firm operating in the secondary market, what belief does it need to have to make the low-tier firm indifferent between worker types?

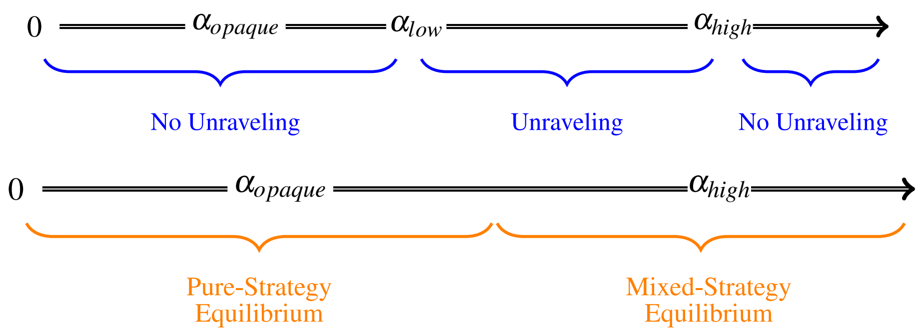
Let $\bar{p}$ be the value such that F_H is indifferent between primary market hiring and hiring on the secondary market when F_L hires a worker of type $\bar{p}$. The qualitative results are provided below in Theorem 4.2. In Appendix B, I list all the equilibria that exist as a function of these quantities. Importantly, for a large class of the parameter values, the set of equilibria will reduce substantially.

For intuition, I describe an equilibrium that will emerge: a **pure strategy unraveling equilibrium**. Consider the situation where the high-tier firm *would not* want to operate in the secondary market if F_L hired a worker that was high type with probability p (i.e. $\bar{p} > p^*$). The low-tier firm could interview later, screen more effectively, and hire a worker that is high type with probability $p' \in (p, \bar{p})$. Therefore, in any pure strategy equilibrium, it must be the case that the low-tier firm hires at $\bar{t}$ to induce belief $\bar{p}$ (the point where the high-tier firm is indifferent between operating in the secondary market and hiring in the primary market). When $\bar{p} \in (\beta, M_{high}(0))$, the low-tier firm interviews at $\bar{t} < 0$ where $M_{high}(\bar{t}) = \bar{p}$. The high-tier firm's equilibrium strategy is to hire on the primary market if the low-tier firm hires at $t \leq \bar{t}$ and to specialize in secondary market hiring if the low-tier firm does not hire by $\bar{t}$. This equilibrium outcome is salient because it mirrors the dynamics of the unraveled industries described in the introduction: the market for corporate law associates, private equity associates, and investment banking analysts.

The next theorem summarizes the qualitative features of the various equilibria. Since the "indifference beliefs" are determined by the transparency level α, I can map the values of α to the types of equilibria that arise.

Theorem 4.2 *There exist thresholds* $0 \leq \alpha_{opaque} \leq \alpha_{low} \leq \alpha_{high}$ *such that:*

1. *For* $\alpha \in [0, \alpha_{opaque})$, *there is no unraveling in equilibrium and the wage is* 0.
2. *For* $\alpha \in [\alpha_{opaque}, \alpha_{low}]$, *there is no unraveling in equilibrium but the low-tier firm pays a positive wage* $w > 0$.
3. *For* $\alpha \in (\alpha_{low}, \alpha_{high}]$, *there is unraveling in equilibrium.*
4. *For* $\alpha \in (\alpha_{high}, \infty)$, *there is no unraveling in equilibrium.*



Proof: *See Appendix B.*

When the high-tier firm exclusively hires laterally, changing the wage is a suboptimal lever to deter poaching (see Proposition 3.2). When the high-tier firm has a choice between hiring at the end of the primary market or operating in the secondary market, the low-tier firm can now use the wage to deter poaching. By increasing the wage, the low-tier firm can force the high-tier firm to hire at the end of the primary market. The low-tier firm's worker will never be poached and payoffs increase discontinuously. Therefore, wage flexibility can allow for some reduction in unraveling in the low transparency environments $[\alpha \in (\alpha_{opaque}, \alpha_{low})]$. However, it is not enough to entirely stop unraveling. Obviously, wages are bounded above by Z_H^L. Moreover, a wage w only serves as a potential deterrent when $w \geq \delta$, the intensity of worker preference for the high-tier firm. When δ is negligible, as transparency increases, even a wage of Z_H^L is not sufficient to mitigate poaching.

As the monitoring technology in the secondary market improves, the low-tier firm wants to induce a lower belief about the worker it hires to prevent poaching. Hiring a low type worker with high probability requires being able to sort very well. As a result, it chooses to hire at the end of the primary market. However, if the low-tier firm is screening to hire a low type worker with high probability, there is no incentive for the high-tier firm to operate in the secondary market. On the other hand, if the high-tier firm chooses to hire at the end of the primary market, the threat of poaching vanishes, and so the low-tier firm no longer has an incentive to hire the worker with a low-signal! Thus, to support the non-unraveling equilibrium when the secondary market is sufficiently informative, the low-tier firm hires at the end of the primary market *but mixes* between hiring a high-signal and low-signal worker. The high-tier firm mixes between operating in the secondary market and hiring at the end of the primary market.[13]

Recall that α_{opaque} and α_{high} depend on $M_{high}(0)$ and $M_{low}(0)$. In other words, the transparency thresholds are determined *relative* to firms' sorting ability in the primary market. In the extreme, if firms were able to perfectly distinguish between types in the primary market, then there would never be unraveling and $\alpha_{opaque} = \infty$. The transparency thresholds are increasing in $(M_{high}(0), 1 - M_{low}(0))$.

Furthermore, the value in sorting is not just about identifying high types but the ability to identify low types as well. If β is very small so that $\bar{p} > \beta$ for all α, then as transparency increases, unraveling will not dissipate. Why? Because while talent is not scarce from a realization perspective, it is rare. Sorting to find a low type worker with high probability is extremely easy: random selection yields a low type with probability $1 - \beta$, which is high. Consequently, there is little incentive to sort to hire a low-quality worker. For β sufficiently small, $\alpha_{high} \to \infty$.[14]

One may wonder whether the equilibrium strategies are realistic depictions of firm behavior. That is, do top firms condition their decisions on whether a lower-tier competitor hired a first-year law student in February? In some matching markets, where matching processes are very public (e.g., Venture Capital funding), such strategies are indeed realistic. However, in labor markets, one should view the equilibrium strategies and outcomes as limit points of a long-run process that involves learning. Over time,

[13]Since the optimal poaching rule is independent of initial beliefs, F_H's decision reduces to whether to operate on the secondary market or the primary market. It follows from Hendon et al. (1996) that any Perfect Bayesian Equilibrium will also be a sequential equilibrium.

[14]The threshold values are necessarily distinct when $\beta Z_H^H \geq M_{high}(0) Z_H^H + (1 - M_{high}(0)) Z_L^H$.

a firm can observe the quality of workers at its competitors and deduce how well its competitors are screening.[15]

4.1 Match Quality

Using the characterization of the equilibria in Theorem 4.2, I compare the total equilibrium match quality to the benchmark-setting where there is no secondary market.

In Theorem 4.3, I provide weak conditions that are sufficient for the total match quality in any equilibrium to be lower than the benchmark-setting. Under weak conditions on the relation between the payoffs and maximal sorting in the primary market, I can exclude the other types of equilibria described in Appendix B. These conditions rule out certain “corner cases".

Theorem 4.3 *Suppose the following two conditions hold:*

1. $M_{low}(0)Z_H^H < M_{high}(0)Z_H^H + (1 - M_{high})Z_L^H$.
2. $\dfrac{Z_H^L}{Z_L^L} < -\dfrac{Z_H^H}{Z_L^H} \cdot \dfrac{M_{high}(0)}{1 - M_{high}(0)}$.

Then for any $\alpha \geq 0$, the total ex-ante match value is lower than the ex-ante match value when no secondary market exists.

Proof: *See Appendix B.*

Suppose types are realized immediately after the primary market ends ($\alpha = \infty$). Condition #1 rules out the situation where the high-tier firm would be happy to poach from the low-tier firm even if it knew that the low-tier firm hired the applicant most likely to be of low type. It holds when the interviews at the end of the primary market are successful at identifying low types (i.e., when $M_{low}(0)$ is small). Importantly, condition #1 implies that when the secondary market is highly transparent, the low-tier firm has the opportunity to hire low-quality workers with high enough probability to deter the high-tier firm from poaching. As a result, unraveling dissipates in highly transparent

[15]This interpretation echoes Green and Porter (1984), where firms can not observe competitors' prices, but instead see noisy estimates of demand, which they use to deduce said prices.

markets *because* the low-tier firm has an incentive to screen for lower quality workers. This causes a reduction in total match quality relative to the benchmark. As the proof of the theorem demonstrates, any equilibrium where the high-tier firm hires in the primary market with positive probability generates a lower total match value than when no secondary market exists. The conditions guarantee that in every equilibrium, the high-tier firm hires in the primary market with positive probability.

When condition #1 does not hold, extremely transparent secondary markets lead to the high-tier firm choosing to monitor and poach even when the low-tier firm hires a worker that is high type with the minimal probability $M_{low}(0)$. Total match quality is $\approx M_{low}(0)Z_H^H + (1 - M_{low}(0))Z_H^L$, which may or may not be higher than the benchmark.

Condition #2 is important in the moderately transparent range. It excludes "corner cases" where for some values of α, the low-tier firm is ok with hiring high signal workers even though it knows it will be poached with probability 1. These cases arise when the incentives of the firms are aligned with each other. The high-tier firm has a strong desire to screen in the secondary market, *and* the low-tier firm has strong preferences between worker types. Thus, condition #2 is a condition on the preferences of the low-tier firm *in relation* to the high-tier firm's desire to screen. In these moderately transparent regimes, equilibria where the market unravels emerge. Low-tier firms hire high-signal workers but at earlier times, and so the interviews returning these high-signals are not as informative.

In the market design literature, reductions in match quality in unraveled markets are often seen as the product of the timing of the matches (e.g., Roth and Xing [1994]; Li and Rosen [1998]). If the timing issue is resolved, will total match quality increase?

Theorem 4.4 *Mandating an interviewing and hiring time can reduce the match quality relative to an unraveled market.*

Proof: *See Appendix B.*

Consider a pure-strategy unraveling equilibrium where both firms hire in the primary market: the low-tier firm hires at time $t < 0$ and the high-tier firm hires at $t = 0$. Suppose a third party could ensure that all interviewing in the primary market must occur at $t = 0$. By coordinating the hiring date, one gives more incentives for the high-tier firm to screen. Why? Because the low-tier firm now has access to higher-quality applicants! Since interviews are more informative at $t = 0$, if the low-tier firm hired a high signal worker now, the high-tier firm would want to operate in the secondary market. The threat of poaching is now more serious, and so the low-tier firm has an incentive to hire low type workers. A mixed strategy equilibrium must exist but with the high-tier firm poaching with non-zero probability. The low-tier firm mixes between hiring a high-signal and low-signal worker. With positive probability, only a single worker is hired by the end of the primary market. This reduces total match quality relative to the pure-strategy unraveled equilibrium that exists without the mandate.

5 Applications

5.1 Labor Markets

The qualitative characterization of the equilibrium dynamics in my model sheds light on which labor markets may be subject to unraveling. Returning to Table 1, consider the subset of labor markets that unravel:

Markets	**Unraveling**
Corporate Law Associates	✓
Private Equity Associates	✓
Investment Banking Analysts	✓
High-End Line Cooks	✓

Table 2

These industries have what one might consider moderately transparent secondary markets. Why is it reasonable to describe these industries as such? Consider the world of investment banking. Banks generally have an understanding of the activity of their competitors. For instance, during an IPO of a company, it is publicly known which investment banks are working on the offering. Importantly, banks generally know the specific groups that are working on particular deals. However, it is difficult to observe how much an individual contributed, especially at the analyst and associate levels. Did he merely bring coffee for his bosses, akin to an intern, or was he actively engaged in the deal-structuring process? Similarly, in corporate law, while a high-tier firm can monitor associates at other firms, it is not as easy to assess associate quality compared to a market like that for academic professors, where research is published for public view. Thus, my model predicts that the market for corporate law associates and investment banks will not only unravel, but there will be little poaching in the secondary market in equilibrium. This is consistent with the observation that unraveling in the market has become more extensive, while the lateral movement of associates has decreased substantially over the last few decades.[16]

Now, consider Table 3, which describes the observed characteristics of industries that do not experience unraveling:

Markets	Opaque Secondary Market	Highly Transparent Secondary Market	Unraveling
Hedge Fund Traders	X	✓	X
Assistant Professors in Economics	X	✓	X
Management Consulting	✓	X	X
Programmers and Software Engineers	✓	X	X

Table 3

The market for assistant professors in economics has received a lot of attention

[16]See https://www.nalp.org/entry-lateral.

due to the shift to virtual interviews. My model suggests that unraveling should not be considered a significant issue in this setting because the secondary market is very transparent. In fact, my model highlights that it is the transparency of the secondary market that is critical to preventing unraveling and not the existence of a centralized system (e.g. the ASSA meeting). Such a centralized system is sustainable because of the transparency of the secondary market. Now, some may point to the difference in timing of interviews and offers as evidence of unraveling. This is not the case. There is no lack of information even with virtual interviews: recommendation letters are in, applications are submitted, and papers are available. Interviews and offers occurring at slightly different times is not indicative of unraveling. That particular issue is due to a lack of coordination caused by limitations in interviewing. For example, even if one enforced a common date at which candidates need to make a decision, inefficiencies will still arise due to this coordination issue (see the concerns medical schools have though the market there does not unravel Wapnir et al. [2021]).

On the opposite side of the spectrum is managerial consulting, which has an opaque secondary market. This is because casework in consulting is entirely private. Consulting firms are barred from revealing their clients.

Critically, when there is abundant talent, there are two settings in which labor markets will not unravel. The first is when there is a complete absence of a secondary market (i.e., one that is sufficiently opaque). The other is when there is a secondary market that is sufficiently transparent. Though non-unraveling occurs in both settings, the equilibrium matches are vastly different. In the former, both firms hire at the end of the primary market, while in the latter, there is the type of mixed strategy equilibrium described in Theorem 4.2. Hence, one would expect to see differences in the frequency of junior-level lateral hiring in these industries. Industries with transparent secondary markets will have more lateral hiring than industries with opaque or inactive secondary markets. This is the case when comparing markets for managerial consultants and software engineers to markets for assistant professors in economics and hedge fund traders.

Markets	Lateral Hiring
Hedge Fund Traders	✓
Assistant Professors in Economics	✓
Management Consulting	X
Programmers and Software Engineers	X

Table 4: Lateral Hiring in Markets that do not Unravel

It is important to note that my model does not claim that the secondary market's characteristics alone determine whether unraveling occurs or not. Rather, it highlights another avenue by which unraveling can arise. Importantly, it illustrates how unraveling is a phenomenon that is present in markets where firms are *not* worried about whether there will be a shortage of high-quality workers at the end of the primary market. A case where these insights do not apply is the hiring of appellate court judicial clerks. There is no viable secondary market there, yet substantial unraveling occurs. This does not contradict my model. In my model, there is a "short-side" of the market and a "long-side". Unraveling does not occur because the firms are on the long-side. In the judicial clerk market, the size of the viable pool of applicants is not large; firms and applicants fear they are on the long-side of the market. Thus, explanations provided by Niederle et al. (2013) and Ambuehl and Groves (2020) are better suited for this setting.

While the model is described in the context of a labor market, it applies to other two-sided matching markets. For example, in venture capital, one can think of the primary market as the pool of early-stage, pre-seed startups. The secondary market consists of startups that have already received funding and are looking for future series rounds. In sports, the primary market refers to the early-scouting of pre-professional players, while the secondary market refers to professional players' movement across teams.

In venture capital, the firms that find it difficult to earn large returns are typically the smaller, lesser-known ones. It is not that they are unable to find promising startups, but that they are unable to maintain investment relations with the successful startups.[17] More prominent venture firms utilize the smaller ones as screening devices, poaching the "winners" in later series' rounds. As a result, the market has unraveled, with the lesser-known firms investing in startups earlier in their life cycle to prevent dilution.

In sports, the secondary market is transparent because player ability is on public display. My model predicts that not only would little unraveling occur, but mandated "interview dates" (i.e., draft days) would cause inefficient matchings. Low-tier teams (small-market teams) would screen to draft non-star players. However, this is not seen in practice. Is this inconsistent with the model? No. A crucial feature of the model is the worker's freedom to move between firms in the secondary market. Implicitly assumed is that the contracts available to the firms can not prevent mobility. Thus, my model corresponds to a sports league with no restrictive contracts. If players were free to move across teams, inefficient matchings as a result of the adverse informational incentives would emerge (Rottenberg [1956]; El-Hodiri and Quirk [1971]). The reason such inefficiencies are not observed is due to the existence of contracts preventing mobility. Therefore, teams no longer need to be as concerned with players being poached.[18] If such restrictive contracts did not exist, teams would underinvest in screening for talent (Feess and Muehlheusser [2003]). I discuss the relationship between contract characteristics, unraveling, and investment in screening in more detail in Section 5.3.

5.2 Beyond Labor Markets: Relation to Innovation

Theorems 4.3 and 4.4 highlight a general phenomenon regarding markets with mobile assets of unknown quality. Within the labor market context, time is the crucial dimension that affects the ability to screen the assets (i.e., workers). However, in markets involving innovation, effective screening may be contingent on costly investment.

Prospective employees are analogous to "potential ideas" that companies can screen

[17] I am grateful to Tomasz Tunguz (Partner at Redpoint Ventures) and Aaron Gershenberg (General Partner at Silicon Valley Bank Capital) for this point.

[18] Under free agency rules, such movement can not be prohibited indefinitely (i.e., the restrictive contracts only last for a fixed number of years). However, leagues such as the NBA have implemented rules that allow teams to pay their players on expiring contracts significantly more than any competitor.

and choose to develop. The low-tier firm, F_L, corresponds to an entrant in the market, while the high-tier firm, F_H, corresponds to an incumbent. Conditional on "matching" (selecting a potential innovation to develop), the innovation generates profits for the company. Competitors can observe informative signals regarding the quality of the innovation and make a "poaching decision", which corresponds to developing a substitute themselves. This would reduce the profits of the innovator.

The threat of copycat innovation is particularly detrimental to the entrant. As a result, an informative secondary market discourages investment in the screening of potential ideas. Furthermore, interpreting Theorem 4.4 through this lens demonstrates that the entrant will develop most innovations. This is consistent with the observation that incumbents are less likely to develop innovations compared to entrants (e.g., Bresnahan et al. [2012]; Awaya and Krishna [2020]).

Example 1 *Time is continuous from* $[0,\infty]$*, and there is a set of* N *ideas. Time* $t=0$ *represents the "primary market stage", and* $(0,\infty)$ *represents the "secondary market stage". Each idea has i.i.d probability* β *of being turned into a novel innovation (high type); otherwise, it becomes an average innovation (low type). At* $t=0$*, firms exert effort* $e\geq 0$ *to screen ideas. Screening is modeled by a function* M *as in Section 2.2, except that it is a function of effort rather than time. Effort is costly, represented by a convex cost function* $c(e)$*. The flow payoff for firm* F_i *with an innovation of type* θ *is* Z^i_θ*. These payoffs have the same structure as in Section 2.3.*[19]

Once the idea is selected, the innovation is realized, and the secondary market stage begins. A public signal regarding the innovation's quality is observed:

$$d\pi_t = \mu_\theta dt + \sigma dB_t$$

This description is analogous to a specific instance of my model. While there is no time dimension in the primary market stage, the existence of an effort cost indirectly caps the firms' screening ability at $t=0$*. Therefore, effort operates in the same way as time-selection does in my model. At a technical level, the solution to the innovation*

[19]The negative payoff to the high-tier firm from implementing an average innovation represents opportunity and reputational cost.

game is equivalent to the equilibrium found under a mandated hiring time (Theorem 4.4). Thus, the unique equilibrium of the innovation game has the low-tier firm choosing an effort level and a non-unit probability of selecting a high-signal idea. The high-tier firm mixes between poaching (exerting no effort in the primary market) and screening at the optimal effort level. This equilibrium is inefficient relative to the setting with no secondary market (i.e., a setting with long-lasting patents).

5.3 Potential Policy Solutions

Two crucial features of the model are the low-tier firm's ability to block off information in the primary market once it matches with a worker and the freedom of the worker to move between firms in the secondary market. Hence, two interventions may mitigate unraveling and increase efficiency:

1. Improving the Flow of Information in the Primary Market.
2. Controlling Mobility in the Secondary Market.

With regards to the former, the growth of the internet has greatly facilitated communication: websites such as LinkedIn and Github have increased transparency substantially. While these could alleviate unraveling in the primary market, they also improve monitoring ability in the secondary market. This can actually increase inefficiency due to the findings in Theorems 4.3 and 4.4.

The second intervention concerns the issue of labor mobility. While public opinion on labor mobility is positive, there are several papers highlighting bidirectional effects associated with either permitting or restricting worker movement (see Jeffers [2019]). My paper points to an inefficiency caused by strategic responses to mobility: unraveling and reduced screening. A simple way to increase match quality in my model is to allow firms to offer long-term, restrictive contracts that prevent workers from being poached. The inclusion of long-term contracts is especially significant in settings where the secondary market is highly transparent. Without them, the high-tier firm mixes between operating on the primary and secondary market, and the low-tier firm mixes between hiring high and low signal workers. If firms could offer contracts prohibiting workers from leaving for a certain period of time, the resulting equilibrium would have both firms hiring high signal workers at the end of the primary market. In practice, long-term, restrictive contracts are difficult to implement due to legality issues. However, in some industries, clauses that attempt to mimic their structure are utilized. For example, Burguet, Caminal, and Matutes (2002) find that in markets with a high degree of transparency, firms will include clauses that confer high quitting costs on employees. Another method by which firms attempt to prevent movement is via non-compete clauses.[20] Non-compete clauses have been viewed negatively, but this is partly because their effect has been analyzed from the perspective of workers *that are already employed* (e.g. Dougherty [2017]; Balasubramanian et al. [2020]). My model points out that the existence of non-competes may change the *initial* matchings: workers not hired in the old regime would be hired in an environment that permitted long-term contracts or non-competes.[21] This is especially significant for markets with a surplus of talent and an active secondary. Even though high talent workers are abundant, low-tier firms will frequently match with lesser talented workers due to the threat of poaching. In professional sports, for instance, where the secondary market is active and highly transparent, incentivizing small-market teams to screen, draft, and train high-talent players will be difficult if there is no restriction on player movement. Hence the need for long-term, restrictive contracts. In the financial and tech industry, firms use deferred compensation, discretionary bonuses, and stock option-vesting periods to reduce movement. In areas of innovation, patents play this role.

[20] Within the United States, non-competes are not necessarily enforced (in California, for instance). Furthermore, in industries such as corporate law, non-competes are generally only used at the partner level. There is no equivalent contract for associate-level positions, which is where unraveling occurs.

[21] This line of reasoning is related to that in papers that highlight how non-competes can help incentivize firm-sponsored general-skill provision (e.g. Aghion and Bolton [1987]; Marx et al. [2009]; Garmaise [2011]; Mukherjee and Vasconcelos [2012]). In my model, non-competes can incentivize better screening at the initial stage, leading to more improved matches at the outset.

6 Discussion

6.1 Assumptions

The first assumption is that primary market hiring prevents a competing firm from learning about the hired worker before the secondary market begins. This is the reality in many two-sided matching markets. For example, in hiring at the university level, once an offer is accepted, students are not permitted to interview with other employers through the university placement office. The second assumption is that firms can not fire workers. This is without loss because the low-tier firm would never fire a worker, and giving F_H the power to terminate a worker is analogous to rescaling the match quality. All that is required is some delay before a worker can be fired. The third major assumption is that N is "sufficiently large".[22] When N is large, I need not consider the case of all candidates failing or passing a given test at any stage in the primary market. The probability of such an event tends rapidly to 0 as N increases. In addition, suppose F_L interviews candidates at time t, making an offer according to a known hiring rule. If F_H interviews the remaining applicants at a later date, its beliefs about the applicant hired by F_L will not be affected. This isolates the effect of the informativeness of the secondary market on firm behavior in the primary market. When the number of workers is small, one must account for strategic rejection of offers. If a worker receives an early offer from the low-tier firm, she may infer something about her type and, therefore, her ability to receive an offer from the high-tier firm later. This causes further unraveling as the low-tier firm must move even earlier to ensure the worker accepts the offer.

6.2 Multiple Firms

The model of unraveling developed in this paper captures the rich informational incentives at play. The assumption of only two firms in the market allows for a complete closed-form characterization of the informational dynamics. Allowing for multiple firms does not change the qualitative features of the results above. However, it does elucidate the importance of the *risk of poaching* to unraveling. The cost of adding mul-

[22] The precise threshold is given in the proof of Observation 2.1.

tiple firms to the model is the loss of closed-form solutions. In the model with only a single low-tier and high-tier firm, poaching is essentially guaranteed to occur if the high-tier firm operates in the secondary market. If another low-tier firm is added, the high-tier firm's payoff from poaching increases because of optionality. However, that same optionality reduces the threat of poaching as viewed by each low-tier firm. To generalize this intuition, if there are very few firms poaching relative to the number of firms available to poach from, then poaching is not a serious threat. When the individual threat of poaching reduces to approximately 0, and there is no shortage of talent, the market will not unravel. Moreover, the lack of a threat of poaching eliminates the adverse signaling incentives, allowing the ex-ante efficient matching to be achieved.

Unraveling is a phenomenon that occurs when there is a hierarchy of firms, and poaching is a credible event. Some well-known markets that fit this description are the private equity space, academia, corporate law, venture capital, and professional sports.

6.3 Adverse Selection

One may be concerned about the presence of adverse selection when poaching (or the lack thereof in my model). Adverse selection is not present because poaching is modeled as an offer by the high-tier firm followed by the worker's decision to either stay with the low-tier firm *at the current wage* or to leave for the high-tier firm. The low-tier firm has no chance to make a counteroffer. Adverse selection occurs if the worker can hold a competition between the two firms for her services. Suppose that when F_H decides to poach, it competes with F_L in retaining the worker's services. However, F_L has private information: it knows the worker's type. If F_H offers a wage $w < Z_H^L - \delta$ and successfully poaches the worker, it knows that the worker is low type. Thus, if there is renegotiation, the high-tier firm will only poach once it is willing to pay a wage of $Z_H^L - \delta$. As a result, $B^* = -log\left(\frac{Z_H^H - Z_H^L + \delta}{Z_L^H - Z_H^L + \delta} \cdot \frac{R_1}{R_2}\right)$. Since B^* is independent of the initial wage, F_L offers $w = 0$ in equilibrium.

All the results in the previous sections still go through but with new transparency thresholds α'_{low} and α'_{high}, where $\alpha'_{low} \geq \alpha_{low}$ and $\alpha'_{high} \geq \alpha_{high}$. Adverse selection reduces the threat of poaching and therefore mitigates unraveling.

7 Conclusion

In most industries, the initial match between an employee and a firm is not permanent. After a worker is hired, it is often the case that she will receive offers from competing firms. The addition of this secondary market, whereby firms can poach workers from other firms, introduces a new channel by which unraveling can occur. Unraveling is no longer a race to acquire top talent but a strategic decision made by low-tier firms to retain the worker it does hire. However, even with an active and fully transparent secondary market, there may still be adverse effects. While transparency decreases unraveling, it does so at the expense of efficiency. With an abundance of talent, both firms should match with high-quality workers. A highly transparent secondary market incentivizes the low-tier firm to screen workers to ensure that it has *not* hired one.

A Strategies and Equilibrium

A.1 Definitions

Conceptually, the strategies available to the firms are intuitive. Given F_H will never enter the primary market before $t = 0$, its decision is whether to operate in the secondary market or the primary market at $t = 0$. It can make this choice based on the time at which F_L chooses to hire. To formalize this requires additional care. Endow the interval $\mathscr{E} = [-T, 0]$ with the Borel Σ-algebra $\mathscr{B}$. For rigor, we also define the set $E = [-T, 0]$ and endow E with the Σ-algebra $\mathscr{B}$ as well. I refer to $\mathscr{E}$ as the sample space and E the outcome space.[23]

Definition A.1 *A* **hiring policy** *adapted to a set* $\hat{\mathscr{E}} \subset \mathscr{E}$*, is a function* $h : \hat{\mathscr{E}} \longrightarrow [0, 1]$.

The function $h(t)$ represents the probability of choosing a high-signal worker when choosing to interview and hire at any time $t \in \hat{\mathscr{E}}$.

Definition A.2 *A* **primary-market strategy** *is a probability measure* μ *on* $(\mathscr{E}, \mathscr{B})$ *and a hiring policy h adapted to* $supp(\mu)$.

F_L chooses a probability measure over $([-T, 0], \mathscr{B})$. The random variable $X_\mu : \mathscr{E} \longrightarrow E$, where $X_\mu(\omega) = \omega$, has distribution F_μ. The realization of X is the time at which F_L enters the primary market. Upon observing the realization $x \in E$, F_H formulates beliefs about whether F_L hired out of the high-pool or low-pool.

Definition A.3 *A* **belief mapping** $f : E \longrightarrow [0, 1]$ *represents the probability* F_H *attaches to* F_L *hiring a worker out of the high-pool, conditional on the time* $x \in E$ *at which* F_L *entered the primary market.*

Conceptually, a strategy for F_H is a decision based on the observed history and its belief about the worker hired by F_L. The strategy will dictate whether F_H hires on the primary market, and the poaching rule it will use conditional on operating in the secondary market. If F_H has belief p_0 about the worker hired by F_L, let $ST(p_0)$ denote the set of all possible poaching rules.

[23] See Karatzas and Shreve (1998) for definitions of the mathematical terms.

Definition A.4 *A* **strategy for** F_H *is a mapping* $W : E \times [0,1] \longrightarrow [0,1] \times ST$ *from observations and beliefs to a probability of operating in the primary market and a poaching rule conditional on operating on the secondary market. In other words,* $W(x,g) = (m,\tau)$ *where* $m \in [0,1]$ *and* $\tau \in ST(g)$.

Given there is no commitment, F_H chooses whether to operate on the primary market at $t = 0$ or on the secondary market using poaching rule τ.

Definition A.5 *An* **equilibrium** *is a primary-market strategy* (μ, h) *for* F_L *and a strategy-belief pair* (W, f) *for* F_H*, such that:*

1. *Each firm is best-responding at each information set.*

2. F_H*'s beliefs are consistent on the support of* μ.

The equilibrium concept used is Perfect Bayesian Equilibrium. Since the strategy for F_H is to choose whether to operate on the secondary market or the primary market, it follows from Hendon et al. (1996) that any perfect bayesian equilibrium will also be a sequential equilibrium.

B Proofs

The table below lists parameters of the model and additional terms. Parameters are highlighted in blue. Terms highlighted in red are substantial ones used throughout. Terms in black are simply defined for notational convenience.

Terms	Meaning
Z^i_θ	Payoff to firm F_i from hiring worker of type θ
α	Transparency of the Secondary Market; Signal-to-noise ratio ($\frac{\mu_H-\mu_L}{2\sigma^2}$)
$M_{high}(t)$, $M_{low}(t)$	$M_{high}(t)$ is the highest feasible probability with which a hired worker can be of high type. $M_{low}(t)$ is the lowest feasible probability. At time t, a firm can identify a worker that has a probability of $M_{high}(t)$ or $M_{low}(t)$ of being high type.
R_1, R_2	$R_1 = \frac{1-\sqrt{1+\frac{2r}{\alpha}}}{2}$, $R_2 = \frac{1+\sqrt{1+\frac{2r}{\alpha}}}{2}$
p, Q	For computational convenience, I will sometimes work in the log-odds space of the beliefs, $Q = log(\frac{p}{1-p})$. Due to this isomorphism, I refer to both Q and p as the "belief".
$\Pi_i(p,w)$	The expected payoff to firm F_i from employing a worker forever that is high type with probability p at a wage w. $\Pi_i(p,w) = pZ^i_H + (1-p)Z^i_L - w = \frac{1}{1+e^Q}(e^Q Z^i_H + Z^i_L) - w$.
$\Sigma_i(p,p',w)$	The expected payoff to firm F_i when F_H operates on the secondary market holding initial belief p' about the worker and F_L has hired a worker at wage w that is high type with probability p.
$\Gamma_i(p,w)$	$\Gamma_i(p,w) = \Sigma_i(p,p,w)$. Γ_H and Σ_H are the high-tier firm's payoffs from poaching. Γ_L and Σ_L are the low-tier firm's payoff conditional on the high-tier firm operating on the secondary market.
$\widetilde{Z}^H_\theta$	$\widetilde{Z}^H_\theta = Z^H_\theta - w$
d^H, $\tilde{d}^{F_H}$, d^L	$d^H = -\frac{Z^H_H}{Z^H_L}$, $\tilde{d}^{F_H} = -\frac{\widetilde{Z}^H_H}{\widetilde{Z}^H_L}$, $d^L = \frac{Z^L_H}{Z^L_L}$

The proofs in this appendix assume $\delta \to 0$ (i.e. the high-tier firm must match the wage set by the low-tier firm in order to poach the worker). This is the most restrictive case. The proofs all go through for any $\delta > 0$ as $\delta > 0$ mitigates the effects of increasing the wage to deter poaching.

B.1 Optimal Poaching

Lemma B.1 *The belief process $\{p_t\}$ has the strong markov property.*

Proof: Since $\{\pi_t\}$ is Markovian, p_t depends only on π_t. Bayes' rule yields:

$$p_t = \frac{p_0 f_t(\pi_t|\theta = H)}{p_0 f_t(\pi_t|\theta = H) + (1-p_0) f_t(\pi_t|\theta = L)}$$

Using Ito's Lemma and the Innovation Theorem:

$$dp_t = \frac{2\bar{\mu}}{\sigma}(1-p_t)p_t d\hat{B}_t$$

$$\text{where } \hat{B}_t = \frac{1}{\sigma}(\pi_t - 2\bar{\mu}\int_0^t p_s ds) \text{ is the innovation process}$$

The innovation theorem implies that the innovation process $\hat{B}_t$ is a Brownian motion with respect to the filtration $\{\mathscr{F}^{\pi_t}\}$. The lemma follows. ■

Lemma B.2 *Given a Markov process $\{x_t\}$ and a continuous function g, consider the optimal stopping problem:*

$$\sup_{\tau} \mathbb{E}[e^{-r\tau} g(x_t)|x_0]$$

There exists a solution of the form $\tau = \inf\{t|x_t \notin (a,b)\}$.

Proof: Given there is exponential discounting and the process $\{x_t\}$ is Markov, the value function V takes the form:

$$V(x_0) = \sup_{\tau} \mathbb{E}[e^{-r\tau} g(x_t)|x_0]$$

By standard arguments, the continuation region is given by $C = \{x : V(x) > g(x)\}$ and the stopping region by $S = \{V(x) = g(x)\}$. Continuity of $\{x_t\}$ means I can restrict attention to a *connected* subset of C around x_0. This continuation region provides the same expected value $\Longrightarrow$ there is an optimal stopping time of the desired form. ■

Proof of Proposition 3.1: Using a standard change of variables, define the log-odds ratio $Q_t = log(\frac{p_t}{1-p_t})$. Applying Bayes' rule yields:

$$Q_t = log\Big(\frac{p_0}{1-p_0}\Big) + log\Big(\frac{f_t(\pi_t|\theta = H)}{f_t(\pi_t|\theta = L)}\Big)$$

$$\Longrightarrow Q_t = Q_0 + \frac{\mu_H - \mu_L}{\sigma^2}\pi_t + \frac{\mu_L^2 - \mu_H^2}{2\sigma^2}t$$
$$\Longrightarrow dQ_t = \frac{(\mu_H - \mu_L)([\mu_\theta - \mu_L] + [\mu_\theta - \mu_H])}{2\sigma^2}dt + \frac{\mu_H - \mu_L}{\sigma}B_t$$

From Peskir and Shiryaev (2006), it follows that Q_t has the strong Markov property. By Lemma B.2, the optimal poaching rule τ is characterized by a continuation region around Q_0. There is no cost associated with observation, which implies that there is no "rejection" threshold. Thus, the continuation region is of the form $(-\infty, B)$. It follows that the optimal poaching rule for F_H is a stopping time τ of the form $\tau = \inf\{t \geq 0 : Q_t \geq B\}$, for some $B > 0$.

Given a worker at F_L with wage w and probability Q_0 of being high type, let $\Gamma_H(Q_0, w, \tau)$ denote the payoff to F_H from following τ. I first explicitly compute $\Gamma_H(Q_0, w, \tau)$, and then maximize it over all threshold stopping times.

Consider a poaching rule where F_H hires the worker if its belief about the worker reaches a threshold B, and commits to never hiring once beliefs fall below b. Such a poaching rule can be represented by the stopping time $\tau = \inf\{t \geq 0 : Q_t \notin (b, B)\}$. With initial condition $Q_0 \in (b, B)$, it follows:

$$Pr(Q_\tau = B|\theta = H)\mathbb{E}[e^{-r\tau}|\theta = H, Q_\tau = B] = \frac{e^{-R_1(Q_0-b)} - e^{-R_2(Q_0-b)}}{e^{-R_1(B-b)} - e^{-R_2(B-b)}} = \xi(Q_0, b, B)$$

$$Pr(Q_\tau = B|\theta = L)\mathbb{E}[e^{-r\tau}|\theta = L, Q_\tau = B] = \frac{e^{-R_1(Q_0-b)} - e^{-R_2(Q_0-b)}}{e^{-R_1(B-b)} - e^{-R_2(B-b)}} = \xi(Q_0, b, B)e^{Q_0-B}$$

Since the optimal poaching rule has $b = -\infty$, taking limits shows that the payoff under a stopping time of the form $\tau = \inf\{t \geq 0 : Q_t \geq B\}$ with initial condition Q_0 is precisely:

$$\Gamma_H(Q_0, w) = \widetilde{Z}_H^H \frac{e^{Q_0}}{1 + e^{Q_0}} e^{R_1(B-Q_0)} + \frac{Z_L^H}{1 + e^{Q_0}} e^{R_2(Q_0-B)}$$

While this function is not concave, it attains its maximum in the interior[24], so taking first order conditions to calculate the optimal threshold B^*, yields $B^* = -log\left(\frac{\widetilde{Z}_H^H R_1}{\widetilde{Z}_L^H R_2}\right)$.

Thus, the optimal poaching rule is:

$$\tau^* = \inf\left\{t \geq 0 : Q_t \geq -log\left(\frac{\widetilde{Z}_H^H R_1}{\widetilde{Z}_L^H R_2}\right)\right\} = \inf\left\{t \geq 0 : p_t \geq \frac{\widetilde{Z}_L^H R_2}{\widetilde{Z}_H^H R_1 + \widetilde{Z}_L^H R_2}\right\} \qquad \blacksquare$$

[24] As $B \longrightarrow -\infty$, it approaches $-\infty$. As $B \longrightarrow \infty$, it approaches 0. Finally, due to the restrictions on k, there exists B such that the expression is positive. Thus, a global maximum is attained in the interior.

B.2 Payoff Functions

Note the following. As the high-tier firm's payoff function depends on its belief, consistency is assumed. For the low-tier firm, the probability that the worker is of high type depends on its action choice. The payoff to the low-tier firm depends on this as well as the high-tier firm's belief.

$$\Gamma_H(\tilde{p},w) = \tilde{p}(Z_H^H - w)\xi(\tilde{Q},B^*) + (1-\tilde{p})(Z_L^H - w)\xi(\tilde{Q},B^*)e^{-(B^*-\tilde{Q})}$$

$$\Sigma_L(p,\tilde{p},w) = p(Z_H^L - w)\Big(1-\xi(\tilde{Q},B^*)\Big) + (1-p)(Z_L^L - w)\Big(1-\xi(\tilde{Q},B^*)e^{-(B^*-\tilde{Q})}\Big)$$

Lemma B.3 *Suppose F_H operates in the secondary market using poaching rule τ^*. The payoff to F_L from employing a worker under initial belief Q_0 and wage w is:*

$$\Gamma_L(Q_0,w) = \underbrace{\frac{1}{1+e^{Q_0}}(e^{Q_0}(Z_H^L - w) + (Z_L^L - w))}_{\text{Expected Productivity}} - \underbrace{\frac{e^{R_2(Q_0-B^*)}}{1+e^{Q_0}}\left[(Z_H^L - w)\frac{\widetilde{Z}_L^H R_2}{\widetilde{Z}_H^H R_1} + (Z_L^L - w)\right]}_{\substack{\text{Loss due to possibility} \\ \text{of losing worker to } F_H}}$$

Proof: F_H will use the stopping rule τ^*. The payoff to F_L from employing a worker at wage w with probability Q_0 of being of a high type is:

$$\Gamma_L(Q_0,w) = p_0(Z_H^L - w)Pr(Q_{\tau^*} = B^*|\theta = H)(1-\mathbb{E}[e^{-r\tau^*}|\theta = H, Q_{\tau^*} = B^*])$$

$$+(1-p_0)(Z_L^L - w)\left[Pr(Q_{\tau^*} = B^*|\theta = L)(1-\mathbb{E}[e^{-r\tau^*}|\theta = L, Q_{\tau^*} = B^*]) + 1 - Pr(Q_{\tau^*} = B^*|\theta = L)\right]$$

$$\implies \Gamma_L(Q_0,w) = p_0(Z_H^L - w)(1-\xi(Q_0,-\infty,B^*)) + (1-p_0)(Z_L^L - w)(1-\xi(Q_0,-\infty,B^*)e^{Q_0-B^*})$$

$$= p_0(Z_H^L - w)(1-e^{R_1(B^*-Q_0)}) + (1-p_0)(Z_L^L - w)(1-e^{Q_0-B^*}e^{R_1(B^*-Q_0)})$$

$$= p_0(Z_H^L - w)(1-e^{R_1(B^*-Q_0)}) + (1-p_0)(Z_L^L - w)(1-e^{R_2(Q_0-B^*)})$$

$$= \frac{1}{1+e^{Q_0}}(e^{Q_0}(Z_H^L - w) + (Z_L^L - w)) - \frac{e^{R_2(Q_0-B^*)}}{1+e^{Q_0}}\left[(Z_H^L - w)e^{B^*} + (Z_L^L - w)\right]$$

$$\underbrace{\frac{1}{1+e^{Q_0}}(e^{Q_0}(Z_H^L - w) + (Z_L^L - w))}_{\text{Expected Productivity}} - \underbrace{\frac{e^{R_2(Q_0-B^*)}}{1+e^{Q_0}}\left[(Z_H^L - w)\frac{\widetilde{Z}_L^H R_2}{\widetilde{Z}_H^H R_1} + (Z_L^L - w)\right]}_{\substack{\text{Loss due to possibility} \\ \text{of losing worker to } F_H}} \qquad \blacksquare$$

Lemma B.4 *For any p_0 and $w \geq 0$, there exists p_0' such that $\Gamma_L(p_0', 0) \geq \Gamma_L(p_0, w)$.*

Proof: Conditional on worker type and F_H using a threshold poaching rule, the expected discounted probabilities of being poached are:

$$Pr(Q_\tau = B|\theta = H)\mathbb{E}[e^{-r\tau}|\theta = H, Q_\tau = B] = \lim_{b \to -\infty} \xi(Q_0, b, B) = e^{R_1(B-Q_0)}$$

$$Pr(Q_\tau = B|\theta = L)\mathbb{E}[e^{-r\tau}|\theta = L, Q_\tau = B] = \lim_{b \to -\infty} \xi(Q_0, b, B)e^{Q_0 - B} = e^{-R_2(B-Q_0)}$$

Recognize that each of the above quantities depends on the *difference* between the threshold belief and the initial belief that the worker is of high type.

Consider a worker that is of type Q_0. Let $B^*(w)$ denote the poaching threshold when F_L pays the worker a wage $w \geq 0$. For any $\Delta > 0$, I will show that $\Gamma_L(Q_0 - \Delta, 0) \geq \Gamma_L(Q_0, w_\Delta)$, where w_Δ is such that $B^*(w_\Delta) - B^*(0) = \Delta$. Writing out the expressions for $\Gamma_L(Q_0 - \Delta, 0)$ and $\Gamma_L(Q_0, w_\Delta)$ yields:

$$\Gamma_L(Q_0 - \Delta, 0) = \frac{e^{Q_0 - \Delta}}{1 + e^{Q_0 - \Delta}} Z_H^L (1 - e^{R_1(B^*(0) + \Delta - Q_0)}) + \frac{1}{1 + e^{Q_0 - \Delta}} Z_L^L (1 - e^{R_2(Q_0 - B^*(0) - \Delta)})$$

$$\Gamma_L(Q_0, w_\Delta) \geq \frac{e^{Q_0}}{1 + e^{Q_0}} (Z_H^L - w)(1 - e^{R_1(B^*(0) + \Delta - Q_0)}) + \frac{1}{1 + e^{Q_0}} (Z_L^L - w)(1 - e^{R_2(Q_0 - B^*(0) - \Delta)})$$

Letting $x = B^*(0) + \Delta - Q_0$, simplifying the expressions above yields the following sufficient condition for $\Gamma_L(Q_0 - \Delta, 0) \geq \Gamma_L(Q_0, w_\Delta)$:

$$e^{Q_0}(e^{-\Delta} - 1)(Z_H^L - Z_L^L) > -w(1 + e^{Q_0})\left[\frac{1 - e^{-R_2 x}}{1 - e^{R_1 x}} + 1\right]$$

Now, $w = \dfrac{Z_L^H R_2 - R_1 Z_H^H e^{B^*(0) + \Delta}}{R_2 - R_1 e^{B^*(0) + \Delta}}$. Using the fact that $Z_H^H \geq Z_H^L \geq Z_L^L \geq Z_L^H$ and $Z_L^H < 0$, it follows that:

$$w \geq -\frac{R_1}{R_2} \cdot \left(\frac{Z_H^L e^x - Z_L^L}{1 + e^x}\right) > -\frac{R_1}{R_2} \cdot \left(\frac{Z_H^L - Z_L^L}{1 + e^x}\right)$$

$$\implies -w(1 + e^{Q_0})\left[\frac{1 - e^{-R_2 x}}{1 - e^{R_1 x}} + 1\right] < \frac{R_1}{R_2} \cdot \left(\frac{Z_H^L - Z_L^L}{1 + e^x}\right) \cdot (1 + e^{Q})\left[\frac{1 - e^{-R_2 x}}{1 - e^{R_1 x}} + 1\right]$$

$$< -(1 + \frac{R_1}{R_2})(1 + e^{Q_0})(Z_H^L - Z_L^L) < -(1 + e^{Q_0})(Z_H^L - Z_L^L)$$

Since $e^{Q_0}(e^{-\Delta} - 1) < -(1 + e^{Q_0})$, the inequality follows. ■

Lemma B.5 *$\Gamma_L(p_0, 0)$ is single-peaked in p_0.*

Proof: With the wage set at 0, Γ_L can be viewed as a function of a single variable, p_0. As before, I will use the change of variables $Q = log(\frac{p}{1-p})$ for algebraic convenience. Taking the expression for Γ_L in Lemma B.3, I can derive the following closed form expression for the derivative of $\Gamma_L(Q)$:

$$\frac{\partial \Gamma_L}{\partial Q} = \frac{e^Q}{(1+e^Q)^2}(Z_H^L - Z_L^L) - e^{R_2 Q}\left(\frac{R_2 + R_2 e^Q - e^Q}{(1+e^Q)^2}\right)\left(-\tilde{d}^{F_H} \cdot \frac{R_1}{R_2}\right)^{R_2-1}\left(Z_H^L - Z_L^L \frac{R_1}{R_2}\tilde{d}^{F_H}\right)$$

Therefore, $\Gamma_L(Q)$ is decreasing in Q whenever:

$$e^Q(Z_H^L - Z_L^L) - e^{R_2 Q}\left(R_2 - R_1 e^Q\right)\left(-\tilde{d}^{F_H} \cdot \frac{R_1}{R_2}\right)^{R_2-1}\left(Z_H^L - Z_L^L \frac{R_1}{R_2}\tilde{d}^{F_H}\right) < 0$$

$$\iff -1 + d^L - e^{(R_2-1)Q}\left(R_2 - R_1 e^Q\right)\left(-\tilde{d}^{F_H} \cdot \frac{R_1}{R_2}\right)^{R_2-1}\left(d^L - \frac{R_1}{R_2}\tilde{d}^{F_H}\right) < 0$$

$$\iff (R_2-1)Q + log(R_2 - R_1 e^Q) + (R_2-1)log(\tilde{d}^{F_H}) + (R_2-1)log\left(-\frac{R_1}{R_2}\right) + log\left(d^L - \frac{R_1}{R_2}\tilde{d}^{F_H}\right)$$
$$> log(d^L - 1)$$

Notice that the left-hand side is increasing in Q, holding everything else fixed. Therefore, Γ_L is single-peaked in Q for $Q \leq B^*$. ■

Proof of Proposition 3.2: This is a consequence of Lemma B.4. Since $\Gamma_L(p_0, 0))$ is single-peaked, p_0^* is unique and satisfies the first-order condition. ■

Lemma B.6 ***Comparative Statics***

1. $\lim_{\alpha \to 0} Q^* = B^*$

2. $\lim_{\alpha \to \infty} Q^* = -\infty$

3. $\lim_{d^L \to \infty} Q^* = K(\alpha, \tilde{d}^H)$ *for some constant* $K(\alpha, \tilde{d}^H)$

Proof: Follows immediately from the first-order condition identified in Lemma B.5. ■

B.3 Indifference Beliefs

When is the low-tier firm indifferent between the type of worker it hires? Similarly, when is the high-tier firm indifferent between poaching on the secondary market and hiring at the end of the primary market?

Define $p_{ind}(w)$ and $\bar{p}(w)$ to be these beliefs, respectively:

$$\Sigma_L(p, p_{ind}(w), w) = \Sigma_L(p', p_{ind}(w), w) \text{ for all } p, p' \in (0,1)$$

$$\Gamma_H(\bar{p}(w), w) = \Pi_H(M_{high}(0), w)$$

The indifference beliefs are endogenous, depending crucially on α, the wage w, the match quality values, and sorting ability at the end of the primary market ($M_{high}(0)$). For exposition, I suppress dependence on these quantities unless necessary.

Lemma B.7 *The following is always true for $\alpha \geq \alpha_{opaque}$:*

$$p_{ind}(w) \geq p^*$$

Proof: Suppose otherwise. Then:

$$\Gamma_L(p^*) = \Sigma_L(p^*, p^*, 0) < \Sigma_L(p_{ind}(0), p^*, 0) \leq \Sigma(p_{ind}(0), p_{ind}(0), 0) = \Gamma_L(p_{ind}(0), 0)$$

Which is a contradiction since $(p^*, 0)$ maximizes $\Gamma_L(p, 0)$. Therefore, $p_{ind}(0) \geq p^*$. Since $p_{ind}(\cdot)$ is increasing in wage, the lemma follows. ■

Lemma B.8 *If* $\frac{Z_H^L}{Z_L^L} < -\frac{Z_H^H}{Z_L^H} \cdot \frac{M_{high}(0)}{1 - M_{high}(0)}$*, then $\bar{p}(0) > p_{ind}(0) > p^*$ for all $\alpha > \alpha_{opaque}$.*

Proof: It suffices to show that $\bar{p}(0) > p_{ind}(0)$. Consider $R_2(\alpha)$ and $R_1(\alpha)$, where these quantities are written as functions of α. I suppress dependence for notational convenience. From the definition of p_{ind}, it follows that $p_{ind}(0) < \bar{p}(0)$ if and only if:

$$\frac{Z_H^L}{Z_L^L} < \frac{1 - e^{-R_2(B^* - \bar{Q})}}{1 - e^{R_1(B^* - \bar{Q})}} \text{ where } \bar{Q} = \frac{e^{\bar{p}}}{1 + e^{\bar{p}}}$$

Since $\bar{p} \geq M_{high}(0)$ for $\alpha \leq \alpha_{opaque}$ and is decreasing in α for $\alpha > \alpha_{opaque}$, it follows that $\frac{1 - e^{-R_2(B^* - \bar{Q})}}{1 - e^{R_1(B^* - \bar{Q})}}$ is increasing in α for $\alpha \geq \alpha_{opaque}$. L'Hospital's Rule yields:

$$lim_{\alpha \to \alpha_{opaque}^{+}} \frac{1-e^{-R_2(B^* - \bar{Q})}}{1-e^{R_1(B^* - \bar{Q})}} = \frac{R_2(\alpha_{opaque})}{-R_1(\alpha_{opaque})}$$

Thus, $\frac{Z_H^L}{Z_L^L} < \frac{R_2(\alpha_{opaque})}{-R_1(\alpha_{opaque})} = -\frac{Z_H^H}{Z_L^H} \cdot \frac{M_{high}(0)}{1-M_{high}(0)} \implies p_{ind}(0) < \bar{p}(0)$. ■

Proof of Proposition 3.3:

In any PBE, if the low-tier firm hires at t where $p^* < M_{high}(t)$, then the high-tier firm can not believe that the low-tier firm is hiring a worker with probability p^* of being high type. Otherwise, at the time of hiring, the low-tier firm would deviate to hiring the high-signal worker. Therefore, in any perfect bayesian equilibrium, if the low-tier firm hires at time t, the high-tier firm's belief must be that at a wage of w, the low-tier firm hires a worker with probability $\max\{M_{low}(t), \min\{p_{ind}(w), M_{high}(t)\}\}$ of being high type.

Let $\bar{\alpha} = p^{*-1}(\beta)$. For $\alpha \leq \bar{\alpha}$, there exists t^* with $M_{high}(t^*) = p^*$. By Proposition 3.2, it is optimal for the low-tier firm to hire the high signal worker at t^* at a wage of 0.

Now, consider $\alpha > \bar{\alpha} \implies p^* < \beta < M_{high}(0)$. From the definition of $p_{ind}(\cdot)$, $log(\frac{p_{ind}(w)}{1-p_{ind}(w)}) - log(\frac{p_{ind}(0)}{1-p_{ind}(0)}) > B^*(w) - B^*(0)$. By the proof of Lemma B.4, there exists $\tilde{p} \in (p_{ind}(0), p_{ind}(w))$ such that $\Sigma_L(\tilde{p}, \tilde{p}, 0) > \Sigma_L(p_{ind}(w), p_{ind}(w), w)$:

$$\implies \Sigma_L(p_{ind}(w), p_{ind}(w), w) < \Sigma_L(\tilde{p}, \tilde{p}, 0) < \Sigma_L(p_{ind}(0), \tilde{p}, 0) < \Sigma_L(p_{ind}(0), p_{ind}(0), 0)$$

Hence, the low-tier firm will never choose to induce a belief $p_{ind}(w) \in (p_{ind}(0), M_{high}(t))$.

To construct the PBE strategy for the low-tier firm, first consider the constrained game, where the low-tier firm is required to hire at time t (this will pin down off-path beliefs for the high-tier firm). First, define $\hat{w}(t) = \max_w \Gamma_L(M_{high}(t), M_{high}(t), w)$. Let S_t be the following strategy:

1. If $\Gamma_L\Big(M_{high}(t), \hat{w}(t)\Big) > \Gamma_L(p_{ind}, 0)$, then the low-tier firm hires the high-signal worker at wage $\hat{w}(t)$.

2. If $\Gamma_L\Big(M_{high}(t), \hat{w}(t)\Big) \leq \Gamma_L(p_{ind}, 0)$, the low-tier firm pays wage 0 and mixes between hiring high and low-signal workers so that the expected probability with which the worker is of high type is $p_{ind}(0)$. The low-tier firm offers a wage of 0.

S_t describes a PBE in the constrained game. Returning to the unconstrained game, the low-tier firm's equilibrium strategy requires selecting the times t with the maximum constrained game payoffs. Any equilibrium consists of the low-tier firm choosing a distribution over $argmax_t \Gamma_L(S_t)$ and following S_t based on the realized time t.[25] ■

B.4 Proof of Theorem 4.2

I characterize the equilibrium for all settings. I also highlight mild conditions under which the characterization simplifies substantially. Let p^*, $\bar{p}(w)$, and $p_{ind}(w)$ indicate the usual quantities. Once α is fixed, $\bar{p}(w)$, and $p_{ind}(w)$ depend only on the wage.

When making its decision, the high-tier firm considers its belief about the hired worker and its observation of the offered wage. If $\Gamma_H(p,w) > \Pi_H(M_{high}(0))$, it operates on the secondary market. If $\Pi_H(M_{high}(0)) > \Gamma_H(p,w)$, it hires a high signal worker in the primary market at $t = 0$. Based on this reasoning, I will only specify on-path outcomes and off-path beliefs when describing the equilibria. It is also implicitly assumed the high-tier firm will follow the poaching rule τ^*.

Intuition: To provide intuition behind how the equilibrium is constructed, I describe, in words, some of the dynamics that arise. The quantities p^*, $\bar{p}(w)$, and $p_{ind}(w)$ are the possible on-path beliefs that *could* be sustained in an equilibrium for a given α. Fix an $\alpha \geq \alpha_{opqaue}$. Suppose F_L hires a worker that is high type with probability p at time t, offering wage w, where $p \in (M_{low}(t), M_{high}(t))$. For beliefs to be consistent, F_H must believe the worker is high type with probability p. Now, what should F_H do? Depending on whether $p > \bar{p}(w)$, the high-tier firm will choose to operate at the end of the primary market or on the secondary market. If $p < \bar{p}(w)$, the low-tier firm would want to deviate *within the same time period* by hiring a worker that is high type with probability $p' > p$. Such a deviation is undetectable. Likewise, if $p > \bar{p}(w)$ and the high-tier firm operates on the secondary market with initial belief p, the low-tier firm will deviate *within the same time period* if $p \neq p_{ind}(w)$.

More generally, when the low-tier firm hires at time t, it has access to workers that

[25] In the edge case where $p_{ind}(0) < M_{low}(0)$, the low-tier firm will hire a worker that is high type with probability $M_{low}(0)$. The high-tier firm's belief is obviously $M_{low}(0)$.

are high type with a probability between $M_{low}(t)$ and $M_{high}(t)$. Thus, once the low-tier firm hires a worker at time t at a wage w, the high-tier firm forms an initial belief about the hired worker. Conditional on the high-tier firm's initial belief and decision, I must ensure that the low-tier firm does not want to hire a worker with a different probability of being a high type.

The ideas described above are formalized in the following three lemmas. Pinning down the equilibrium requires understanding the ordinal properties of $\bar{p}(w)$ and $p_{ind}(w)$ and which belief the low-tier firm would prefer to induce in the high-tier firm.

Lemma B.9 *If $\bar{p}(0) \geq \beta$, then in any equilibrium where the low-tier firm hires at time t, it can not hire a worker that is high type with probability $p_{ind}(w)$ at a wage w.*

Proof: Suppose in equilibrium F_L hired a worker that is high type with probability $p_{ind}(w)$ at time t at a wage w. If $p_{ind}(w) > \bar{p}(w)$ then the F_H operates in the secondary market. However, $\Gamma_L(p_{ind}(w), w) = \Sigma_L(\bar{p}(w), p_{ind}(w), w) < \Pi_L(\bar{p}(w))$. This means there is a $t' < t$ such that $\bar{p}(w) = M_{high}(t')$. The low-tier firm would interview at that time and hire a worker that is high type with probability $M_{high}(t')$ at a wage w. The high-tier firm would be forced to operate in the primary market, allowing the low-tier firm to make $\Pi_L(\bar{p}(w))$.

The other case that must be considered is $p_{ind}(w) < \bar{p}(w)$. In this case, the high-tier firm must hire in the primary market in equilibrium. This means the low-tier firm has an incentive to deviate to hiring at any time $t' \leq t$ such that $p_{ind}(w) < M_{high}(t') < \bar{p}(w)$ (which exists by continuity of M_{high}). ■

Lemma B.10 *Suppose $\bar{p}(0) < \beta$. Suppose the low-tier firm is constrained to hiring at time t. The equilibrium in this constrained game is given by $\chi_{prim}(w,t)$ representing the probability the high-tier firm hires in the primary market as a function of t and the wage the low-tier firm offers. The low-tier firm hires a worker at wage $w^*(t)$, hiring a high-signal worker with probability $\chi_{hi}(t)$.*

Proof: Define $\hat{p}(w,t) = \min\left\{\max\left\{p_{ind}(w), \bar{p}(w)\right\}, M_{high}(t)\right\}$. This represents the belief that must be induced in equilibrium in the constrained game.

Let $\hat{\chi}_{hi}(\hat{p}, w, t)$ denote the probability the low-tier firm hires a high-signal worker at time t given a wage of w. Let $\chi_{prim}(\hat{p}, w, t)$ be the probability the high-tier firm hires at the end of the primary markets after observing a wage w at time t. Since $\hat{p}$ depends on w and t, I suppress dependence of $\hat{\chi}_{hi}(\hat{p}, w, t)$ and $\chi_{prim}(\hat{p}, w, t)$ on $\hat{p}$. The functions are defined as follows:

1. If $\hat{p}(w,t) > \bar{p}(w)$, $\chi_{prim}(\hat{p}, w, t) = 0$, and the high-tier firm believes the worker hired by the low-tier firm is high type with probability $\hat{p}(w,t)$.

2. If $\hat{p}(w,t) = \bar{p}(w)$, $\chi_{prim}(\hat{p}, w, t)$ is defined by the unique value of Y such that $\bar{p}(w) = \arg max_p Y\Pi_L(p, w) + (1-Y)\Sigma_L(p, \bar{p}(w), w)$. In other words, the low-tier firm is indifferent across workers.

3. The low-tier firm mixes so that $\hat{\chi}_{hi}(\hat{p}, w, t)M_{high}(t) + (1 - \hat{\chi}_{hi}(\hat{p}, w, t))M_{low}(t) = \hat{p}$.

The strategy χ_{prim} is clearly the equilibrium strategy for the high-tier firm. Now, the low-tier firm ultimately chooses $w^* \geq 0$ such that:

$$w^* = arg\max_{w} \chi_{prim}(w,t)\Pi_L(\hat{p}, w) + (1 - \chi_{prim}(w,t))\Gamma_L(\hat{p}, w)$$

This w^* is unique. Defining $\chi_{hi}(t) = \hat{\chi}_{hi}(w^*, t)$ completes the proof. ■

Lemma B.11 *If $\bar{p}(w) > p_{ind}(w)$ for some w and $\bar{p}(w) > M_{low}(0)$, then in equilibrium the low-tier firm will never pay a wage w' and hire a worker that is high type with probability $p_{ind}(w')$ for $w' > w$.*

Proof: Suppose the low-tier firm hires a worker at time t at a wage of $w' > w$ that is high type with probability $p_{ind}(w')$. If $p_{ind}(w') < \bar{p}(w)$ then the high-tier firm must hire in the primary market in equilibrium. There is then a profitable deviation by the same argument in Lemma B.10.

If $p_{ind}(w') > \bar{p}(w')$, then the high-tier firm must operate in the secondary market, giving a payoff of $\Sigma_L(p_{ind}(w'), p_{ind}(w'), w')$ to the low-tier firm. Since $p_{ind}(w) < \bar{p}(w)$ and $\bar{p}(w') < p_{ind}(w')$, there is a $\tilde{w}$ such that $p_{ind}(\tilde{w}) = \bar{p}(\tilde{w})$. Consider what happens if the low-tier firm offers a wage $\tilde{w}$. On this path of play, the high-tier firm believes the low-tier firm hires a worker that is high type with probability $\bar{p}(\tilde{w}) = p_{ind}(\tilde{w})$. By Lemma B.10 the high-tier firm must operate in the secondary market, giving a payoff of $\Sigma_L(p_{ind}(\tilde{w}), p_{ind}(\tilde{w}), \tilde{w})$ to the low-tier firm.

From the proof of Lemma B.4, there exists $\tilde{p} \in (p_{ind}(\tilde{w}), p_{ind}(w'))$ such that:

$$\Sigma_L(\tilde{p}, \tilde{p}, \tilde{w}) > \Sigma_L(p_{ind}(w'), p_{ind}(w'), w')$$

$$\Longrightarrow \Sigma_L(p_{ind}(w'), p_{ind}(w'), w') < \Sigma_L(\tilde{p}, \tilde{p}, \tilde{w}) < \Sigma_L(p_{ind}(\tilde{w}), \tilde{p}, \tilde{w})$$

$$< \Sigma_L(p_{ind}(\tilde{w}), p_{ind}(\tilde{w}), \tilde{w}) \qquad \blacksquare$$

With these lemmas in hand, I will proceed to the proof of Theorem 4.2. For any given α, there are three cases. The first two correspond to settings where there is an incentive for the low-tier firm to sort and hire high-signal workers. This ensures that the only equilibrium is in pure strategies. The third case corresponds to the setting where the low-tier firm is incentivized to sort workers and hire those likely to be of low type in order to deter the high-tier firm from operating on the secondary market. However, the low-tier firm can always deviate by hiring a high-signal worker *at that time*.

Case #1: $p^*, \bar{p}(0) \geq \beta$, $\bar{p}(0) \geq \beta > p^*$

Under these conditions, the low-tier firm is incentivized to sort a little and hire high-signal workers. Pure strategy equilibrium will exist. Whether the pure strategy equilibrium involves F_H monitoring in the secondary market or not depends on the relationship between p^* and $\bar{p}$. Define $w_i(t) = \Gamma_H^{-1}(M_{high}(t), \Pi_H(M_{high}(0)))$ to be the wage the low-tier firm needs to pay to make the high-tier firm indifferent between operating on the secondary market and hiring on primary market.

(a) **Suppose $\Pi_L(\bar{p}(0)) > \Gamma_L(p^*)$:**

This is trivially satisfied when $\bar{p}(0) > p^*$. Let $\bar{t}_0$ be the time such that $M_{high}(\bar{t}_0) = \bar{p}_0(0)$. Therefore, $w_i(\bar{t}_0) = 0$. Next, define $t_{eq} = argmax_{t \geq \bar{t}_0} \Pi_L(M_{high}(t), w_i(t))$. This will be the equilibrium hiring time for the low-tier firm.

The equilibrium strategies are as follows:

- F_L hires a high-signal worker at time t_{eq} and offers wage $w_i(t_{eq})$.
- If F_H observes that F_L hires at any time $t \leq \bar{t}_0$, it also hires at $t = 0$. If it observes F_L hire in $t > \bar{t}_0$ at wage w, it believes F_L hired a worker that is high type with probability $M_{high}(t)$. F_H hires at $t = 0$ if and only if $\Pi_H(M_{high}(0)) \geq \Gamma_H(M_{high}(t), w)$.

This equilibrium is clearly a PBE. On path, F_L hires a high-signal worker at time t_{eq} and F_L hires a high-signal worker at time $t = 0$. For moderately transparent environments, t_{eq} will be strictly less than 0, demonstrating that not only will there be a pure strategy unraveling equilibrium, but flexible wage-setting is not enough to stop unraveling from happening. An example where this occurs is when the intensity of worker preference for the high-tier firm (δ) is large. Then the maximum wage a low-tier firm could conceivably offer, Z_H^L, will have a marginal effect.

(b) **Suppose $\Gamma_L(p^*) > \Pi_L(\bar{p}(0))$:**

It is necessary that $p^* > \bar{p}(0)$. In this scenario, the low-tier firm may be ok with screening more, even though it means getting poached. Intuitively, this occurs when the high-tier firm's payoff from primary market hiring is close to 0.

Let t^* be the time such that $M_{high}(t^*) = p^*$. Let $\hat{t} = argmax_{t \geq \bar{t}_0} \Pi_L(M_{high}(t), w_i(t))$. The equilibrium payoff to the low-tier firm is:

$$\max\left\{\Gamma_L(p^*, 0), \Pi_H(M_{high}(\hat{t}), w_i(\hat{t}))\right\}$$

If the maximal value is $\Gamma_L(p^*, 0)$, the equilibrium involves the low-tier firm hiring at t^*. By Lemma B.5, the equilibrium wage will be 0. The high-tier firm operates on the secondary market with belief p^*. Otherwise, the low-tier firm hires at time $\hat{t}$ at wage $w_i(\hat{t})$. The high-tier firm hires at the end of the primary market.[26]

[26] I break indifference here by having the high-tier firm operate on the primary market. This is not necessary. Mixing results in the same equilibrium payoffs.

Case #2: $p^* > \beta \geq \bar{p}(0)$

First, I show that the low-tier firm can not hire a worker that is high type with probability $\bar{p}(w)$ at wage w if $\bar{p}(w) < \beta$. Assume otherwise $\implies$ the high-tier firm must hold belief $\bar{p}(0)$ about the low-tier firm's worker. Hence, the low-tier firm has an incentive to hire a worker that is high type with probability $M_{high}(t) > \bar{p}(w)$, regardless of F_H's decision.

Therefore, if the low-tier firm hires a worker at time t at wage 0, the high-tier firm will believe the low-tier firm is hiring a worker that is high type with probability $\min\left\{p_{ind}(0), M_{high}(t)\right\}$. On-path, it is not optimal for the low-tier firm to hire a worker that is high type with probability $p_{ind}(w)$ at wage w. This is because the high-tier firm must then operate on the secondary market, resulting in the low-tier firm receiving a payoff of $\Sigma_L(p_{ind}(w), p_{ind}(w), w) < \Sigma_L(p^*, p^*, 0)$. Hence, conditional on offering a wage of 0, the low-tier firm will choose to hire at time t such that $M_{high}(t) = p^*$.

Case #3: $p^*, \bar{p}(0) < \beta$

An important feature of this case is that "*within* time deviations" must be considered. While the low-tier firm is incentivized to hire low-signal workers, it can deviate within the same hiring time. If the high-tier firm holds a low initial belief about the worker and so chooses to hire at the end of the primary market, the low-tier firm may deviate to hire a high signal worker. The high-tier firm can not detect such a deviation! As a result, there is no pure strategy equilibrium where F_H hires at the end of the primary market with probability 1. This is because in any such candidate equilibrium, F_L would need to hire a worker at time t that is high type with probability less than or equal to $\bar{p}$. However, if F_H is hiring at the end of the primary market, F_L has no incentive to hire such a worker. It can achieve a higher payoff by hiring the worker that is high type with probability $M_{high}(t) > \beta > \bar{p}$.

Using the functions constructed in Lemma B.9 on the following page, it is clear that F_H must follow $\chi_{prim}(w,t)$. Its beliefs on and off-path are described by $\hat{p}(w,t)$. For the low-tier firm, define the set D of possible equilibrium hiring times:

$$D = \arg max_{t\in[-T,0]} \chi_{prim}(w^*,t)\Pi_L(\hat{p}(w^*,t),w^*) + (1-\chi_{prim}(w,t))\Sigma_L(\hat{p}(w^*,t),\hat{p}(w^*,t),w^*)$$

Therefore all equilibria can be described as follows:

1. F_H hires in the primary market with probability $\chi_{prim}(w,t)$, where w is the wage paid by the low-tier firm and t is the time at which the low-tier firm hired the worker. F_H has belief $\hat{p}(w,t)$ about the worker.

2. F_L chooses a distribution over D and, based on the realization, selects a high-signal worker with probability $\chi_{hi}(t)$ and a low-signal worker with probability $1-\chi_{hi}(t)$. The worker is paid $w^*(t)$.

The three quantities $\{\bar{p}, p_{ind}, p^*\}$ are decreasing in α. By Lemmas B.5 and B.6, there is a threshold α_{high} such that for all $\alpha > \alpha_{high}$, $p_{ind}(0) < \bar{p}(0) < \beta$. As a result, in highly transparent environments, the equilibrium is in mixed strategies. Importantly, there is a mixed-strategy equilibrium with no unraveling. As $\alpha \to \infty$, $\bar{p} \to k$ where k satisfies $kZ_H^H = M_{high}(0)Z_H^H + (1-M_{high}(0))Z_L^H$. If $k \leq M_{low}(0)$ then for α's sufficiently high, the equilibrium is in pure strategies. The high-tier firm monitors with probability 1, believing that the worker hired by F_L is of high type with probability $M_{low}(0)$. ■

COROLLARY B.12 *Fix an $\alpha \geq \alpha_{opaque}$:*

1. *If $\bar{p}(0) > \max\{p_{ind}(0), M_{low}(0)\}$, then in equilibrium, F_L hires a worker at wage $w \geq 0$ that is high type with probability $\bar{p}(w)$.*

2. *For $\alpha' > \alpha$, F_L hires a worker at wage $w' \geq 0$ that is high type with probability $\bar{p}(w', \alpha') > \bar{p}(w, \alpha)$.*

Proof: The condition $\alpha \geq \alpha_{opaque}$ implies $\bar{p}(0) \leq M_{high}(0)$. The condition $\bar{p}(0) > \max\{p_{ind}(0), M_{low}(0)\}$ implies $\bar{p}(0) \geq M_{low}(0)$ and $\bar{p}(0) > p_{ind}(0) \geq p^*$. The first part of the corollary follows immediately after the lemmas and the proof of Theorem 4.2. To prove the second statement, assume otherwise, so that for an $\alpha' > \alpha$, the low-tier firm hired a worker that is high type with probability $\bar{p}(w', \alpha') > \bar{p}(w, \alpha) \implies \bar{p}(w', \alpha) > \bar{p}(w', \alpha') > \bar{p}(w, \alpha) \implies$ the low-tier firm had a feasible profitable deviation at α, contradicting the definition of an equilibrium. ■

B.5 Proofs of Theorem 4.3 and 4.4

Proof of Theorem 4.3: When $\alpha \leq \alpha_{opaque}$, the equilibrium is identical to the benchmark-setting, with equilibrium wages as 0. Therefore, I restrict attention to $\alpha > \alpha_{opaque}$.

The first condition in Theorem 4.3 rules out the existence of sufficiently high transparency levels where the equilibrium involves the high-tier firm operating in the secondary market with probability 1 ($\bar{p}(0) > M_{low}(0)$). If the second condition in Theorem 4.3 holds, Lemma B.8 implies that $\bar{p}(0) > p_{ind}(0) \geq p^*$. Therefore, by Theorem 4.2, there are only pure strategy equilibria and equilibria in strict mixed-strategies.

The high-tier firm in all equilibria makes the same payoff as in the benchmark-setting: at every equilibrium, it is indifferent between operating on the secondary market and hiring at the end of the primary market. On the other hand, the low-tier firm makes strictly lower payoffs in any strict mixed-strategy equilibrium and pure-strategy unraveling equilibrium. In any pure strategy non-unraveling equilibrium for $\alpha > \alpha_{opaque}$, the wage is positive, and so the low-tier firm also earns a strictly lower payoff. ■

Proof of Theorem 4.4: I demonstrate that coordination can reduce the low-tier firm's payoffs when the original equilibrium is a pure-strategy unraveling equilibrium.

Consider any primary and secondary market environment with the following properties:

1. $\dfrac{Z_H^L}{Z_L^L} < -\dfrac{Z_H^H}{Z_L^H} \cdot \dfrac{M_{high}(0)}{1 - M_{high}(0)}$.

2. At some α, there exists a pure strategy unraveling equilibrium.

Since a pure strategy unraveling equilibrium exists when the transparency level is α, let $\bar{t}_0 < 0$ denote the time at which the low-tier firm hires. The hired worker is high type with probability $M_{high}(\bar{t}_0)$. In this pure-strategy unraveling equilibrium, the high-tier firm earns a payoff of $\Pi_H(M_{high}(0))$. The low-tier firm earns a payoff of $\Pi_H(M_{high}(\bar{t}_0), w_i(\bar{t}_0))$.

If firms are constrained to hire and interview at $t = 0$, the new equilibrium must be in strict mixed strategies. Call these equilibria the *centralized equilibria*. An example of one is the following:

1. The low-tier firm mixes between hiring a low-signal and high-signal worker such that the net probability that the worker is of high type is $\bar{p}_0 = M_{high}(\bar{t}_0)$. The wage is $w_i(\bar{t}_0)$, where $w_i(\cdot)$ is defined as in Theorem 4.2.

2. The high-tier firm F_H mixes between hiring on the primary market and operating on the secondary market using the strategy $\chi_{prim}(w,t)$. It mixes so that the low-tier firm is indifferent about which signal worker it hires. The high-tier firm believes that the low-tier firm's worker is high type with probability $\bar{p}(\cdot)$.

The payoff to the low-tier firm in this centralized equilibrium is:

$$(1-\chi_{prim})\Gamma_L(\bar{p}_0, w_i(\bar{t}_0)) + \chi_{prim}\Pi_L(\bar{p}_0, w_i(\bar{t}_0))$$

This is strictly lower than the equilibrium payoff in the unraveled market. This is not the only equilibrium. I will demonstrate that no matter the equilibrium one chooses, the low-tier firm will always receive a reduced payoff. The other centralized equilibria must involve the low-tier firm paying a positive wage w and hiring a worker with probability $\bar{p}(w)$ of being high type. The high-tier firm, though, must mix. Therefore, equilibrium payoffs for the low-tier firm are strictly lower than $\Pi_L(\bar{p}(w), w)$. Letting $t = w_i^{-1}(w)$, it follows that the payoffs are bounded by $\Pi_L(M_{high}(t), w_i(t))$. But this is bounded above by $\Pi_L(M_{high}(\bar{t}_0), w_i(\bar{t})$ as $\Pi_L(M_{high}(\bar{t}_0), w_i(\bar{t}))$ is the payoff from the pure strategy unraveling equilibrium (see Theorem 4.2). ■

C Primary and Secondary Market Microfoundation

C.1 Primary Market

If a firm chooses to hire in the primary market, it selects a time to conduct the interview and makes an offer based on the interview. The primary market must be such that the earlier a firm interviews, the lower its ability to sort between workers of high and low type. To capture this, I model interviewing as a probabilistic test on each worker that returns a *high* or *low* signal (denoted by lower-case h and l) depending on the true type of the worker. The associated conditional probabilities of a worker being of high type reflect the ability to sort at a given time t in the primary market.

Definition C.1 *Given* $x_h, x_l \in [0,1]$*, an* $(\mathbf{x_h}, \mathbf{x_l})$**-test** *is a signal applied to each worker that returns h (high) or l (low)*

$$P(h|\theta = H) = x_h$$

$$P(l|\theta = H) = x_l$$

Any (x_h, x_l)-test induces an ordered pair (p_h, p_l) where $p_h = P(\theta = H|h)$ and $p_l = P(\theta = H|l)$ are the conditional probabilities a worker is of high type given the results of the test. A *partial ordering* can be defined on the space of (x_h, x_l)-tests:

Definition C.2 *An* (x_h, x_l)*-test is* **more powerful** *than an* $(\hat{x}_h, \hat{x}_l)$*-test if and only if* $p_h \geq \hat{p}_h$ *and* $p_l \leq \hat{p}_l$*.*

Consider any mapping $Y : [-T, 0] \longrightarrow [0,1] \times [0,1]$ such that $Y(t)$ returns an (x_h, x_l)-test. Thus, Y associates with each time t a binary test that can be implemented for sorting between worker types. Since it is necessary to incorporate the feature that one can sort more effectively at later times, I impose the constraint that for such a Y to be admissible, it must be that $Y(t)$ is more powerful than $Y(t')$ for any t and t' such that $t \geq t'$. Call such a Y a testing-map.

It follows from Bayes' rule that any testing-map Y is equivalent to a unique mapping M. Likewise, any mapping M corresponds to a unique testing-map Y.

Proof of Observation 2.1:

Since N is discrete, there is a technicality that must be considered. There is a non-zero probability that all individuals emit the same signal in an interview. That is, after interviewing, the firm sees only low-signals or only high-signals. In this situation, I assume that the firm randomizes between whom it hires. Recognize that the probability of this event occurring tends to 0 rapidly (e.g., converges exponentially). Given the primary market's informational structure M, consider the corresponding mapping from time to probabilistic binary tests.

I will show that all workers will accept any offer they receive in the primary market when N is sufficiently large. Intuitively, for N small, receiving an offer provides the worker with more information about her type. She may strategically reject because she now believes she has a better chance of receiving an offer from the high-tier firm. For N sufficiently large, there is no gain from such "strategic rejection".

Define the function $\hat{\beta}(t,N)$ as the posterior probability that a worker is of type $\theta = H$ conditional on receiving an offer at time t when there are N workers available. Let $\Delta_{t,\beta} = \beta(1-x_H^t)+(1-\beta)(1-x_L^t)$ denote the ex-ante probability of failing the test at time t when the probability a worker is of type $\theta = H$ is β. It follows that:

1. $\hat{\beta}(t,N+1) = (N+1)\beta \cdot \left(x_H \sum_{k=0}^{N} \binom{N}{k} \dfrac{\Delta_{t,\beta}^{N-k}(1-\Delta_t)^k}{k+1} + \dfrac{(1-x_H)\Delta_{t,\beta}^N}{N+1}\right)$
2. $\hat{\beta}$ is weakly increasing in t and N
3. $\lim_{N\to\infty} \hat{\beta}(t,N) = \frac{\beta x_H}{\beta x_H + (1-\beta)x_L}$

Suppose there are $N+1$ potential workers, and all workers besides worker i accept any offer. From worker i's perspective, if she received an offer from F_L at time t and rejected it, her expected payoff is the probability of receiving an offer at time 0 from the high-tier firm multiplied by δ. Since δ is a constant and normalize it to 1.

$$\text{Payoff from Rejection} = \left(1-\Delta_{t,\hat{\beta}(t,N+1)}\right) \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{\Delta_{t,\beta}^{N-k-1}(1-\Delta_{t,\beta})^k}{k+1} + \frac{\Delta_{t,\hat{\beta}(t,N+1)}\Delta_t^{N-1}}{N}$$

$$< \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{\Delta_{t,\beta}^{N-k-1}(1-\Delta_{t,\beta})^k}{k+1} + \frac{1}{N}$$

$$< \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{\Delta_{t,\beta}^{N-k-1}(1-\Delta_{t,\beta})^k}{k+1} + \frac{1}{N}$$

$$\text{For large } N \Longrightarrow \; < N \binom{N-1}{\frac{N-1}{2}} \frac{2^{1-N}}{\frac{N-1}{2}+1} + \frac{1}{N}$$

Using Stirling's formula, the expression above approaches 0 as N grows large. ■

C.2 Secondary Market

For the definition of the secondary market to make sense, it must be the case that B_t is a Brownian motion with respect to all measures induced by $p_0 \in [0,1]$.

Lemma C.3 *There exists a probability space $(\Omega, \Sigma_\Omega, \mathbb{P}_0)$, where $\mathbb{P}_0 = p_0 \mathbb{P}_H + (1-p_0)P_L$, and a process B_t, such thatB_t is a Brownian motion with respect to all measures $\mathbb{P}_0$ induced by $p_0 \in [0,1]$.*

Proof: Consider the following two probability spaces:

1. A sufficiently rich space $(\mathbb{R}, \mathscr{B}, \lambda)$, where $\mathscr{B}$ is the Borel σ-algebra and λ is the lebesgue measure.
2. The space $(\{H, L\}, \Sigma, \hat{\mathbb{P}}_0)$, where $\hat{\mathbb{P}}(H) = p_0$, and Σ is the natural σ-algebra.

Let B_t be a brownian motion on $(\mathbb{R}, \mathscr{B}, \lambda)$. Let $\Omega = \{(\theta, a) : \theta \in \{H, L\} \text{ and } a \in \mathbb{R}\}$. Let Σ_Ω de the σ-algebra on Ω generated by Σ and $\mathscr{B}$. Finally, define the measure $\mathbb{P}_0$ to be the measure on (Ω, Σ) induced by $\hat{\mathbb{P}}_0$ and λ.

It follows that the measure $\mathbb{P}_0$ satisfies $\mathbb{P}_0(\theta = H) = p_0$ and that B_t is a brownian motion with respect to $(\Omega, \Sigma_\Omega, \mathbb{P}_0)$ for any $p_0 \in [0,1]$. ■

It is crucial to point out that the idea of a binary test is just one interpretation of the mapping M. Randomization allows for the selection of a worker with probability of being a high type $p \in [M_{low}(t), M_{high}(t)]$. Thus, what is really assumed is that there is a cap on how well a firm can identify a high type worker at time t in the primary market. The mapping M is simply a reduced-form representation of this notion.

Now, within the secondary market, firms' ability to identify a high type worker is uncapped. The difference between the primary and secondary markets is that more information can be observed in the latter. Thus, my model applies to settings where there is a difference in the work done in each stage. Many labor markets have this feature since firms hire directly at the university level.

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