Unraveling and Inefficient Matching

Akhil Vohra *†

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Abstract

Labor markets unravel when workers and firms match inefficiently early, under limited information. I argue that a significant determinant of unraveling is the presence of a secondary market, where firms can poach workers, and its transparency: how well firms can ascertain workers' value once they are employed by competitors. While early hiring reduces the probability of hiring a high-type worker, it prevents rivals from learning about the worker, making poaching difficult. When secondary markets are very transparent, unraveling disappears. However, the matching remains inefficient due to the incentives of low-tier firms to communicate that they have *not* hired top-quality workers.

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* *University of Georgia, Terry College of Business* and the *University of Exeter Business School*

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`email: akhil.vohra@uga.edu`

1 Introduction

Labor markets in which matches between firms and workers occur inefficiently early due to limited information are said to have unraveled. A classic example is the market for appellate court judicial clerks in the United States. Judges rush to make offers to law students as early as two years before the start date, at which point information about a student's legal writing is non-existent (Avery et al. [2001]). Unraveling is often ascribed to the combination of intense competition for scarce, high-quality workers and applicant uncertainty, which drives them to accept early offers (Roth and Xing [1994]).

However, there are markets that experience unraveling though employers *do not* face an initial scarcity of talent. In investment banking recruiting, banks scramble to contract with college sophomores despite the abundance of talent relative to the number of available entry-level positions.[1] In corporate law, students receive associate offers in their first year of law school.[2] At the same time, not all markets unravel. For instance, the market for academic economists and managerial consultants has not been prone to the same type of early hiring that plagues investment bank recruiting.

Why do some markets unravel while others do not, even if they appear to have abundant talent? My paper answers this question by identifying a new channel by which unraveling can propagate. I argue that unraveling can be caused by the presence and characteristics of a secondary market, whereby firms may poach workers currently employed by other firms. Poaching is a prevalent form of rematching in many industries: private equity firms recruit analysts from competitors and investment banks, law firms attract associates and partners from competitors, universities hire professors laterally, and large venture capital firms poach startups from smaller ones during series funding rounds. In models with a single stage of hiring and matching, the main driver of firm

[1] Annually, JP Morgan has ≈ 450 positions globally (for undergraduate and graduate students), while Goldman Sachs has ≈ 200. Wharton alone produces more than 650 undergraduates in finance. See Clarke (2021) and https://www.efinancialcareers.com/news/2023/02/goldman-sachs-acceptance-rate.

[2] The rationale that accelerated hiring is to secure top talent does not fully align with certain recruitment practices. Banks often cap the number of hires from each university (Rivera [2015]). It is difficult to believe such a practice would exist if firms believed talent was scarce. Additionally, the trend of advancing hiring timelines occurs irrespective of the economy or reductions in total slots (Rainey [2013]).

behavior is the desire to acquire top talent. With a secondary market, firms must also be concerned with their ability to *retain* the talent they hire.

Talent retention is a critical driver of firm behavior and influences hiring and screening incentives. Firms opt not to hire overqualified candidates due to their turnover risk, despite their high productivity on the job (Maynard et al. [2006]; Luo [2010]; Erdogan et al. [2011]). Firms even sacrifice match quality by strategically rotating workers across jobs and clients to ensure they are not too productive for a particular client (Battiston et al. [2024]). Likewise, firms may reduce investment in talent discovery if worker movement is prevalent (Terviö [2009]). Early hiring can also function as a talent-retention instrument by blocking competitors' access to desirable candidates. Many Japanese companies engage in what is called "kosoku": they contract with students early and schedule required events when other firms are interviewing, preventing contact between students and competitors (Roth and Xing [1994]). Such a strategy is compelling when talent is scarce but unproductive when plentiful. However, there is an added benefit from blocking initial information about a hired candidate: it deters competitors from using a recruiting strategy based on poaching hired workers.

To illustrate, consider a private equity firm and an investment bank, each looking to hire an analyst from a future university graduating class. Students are high or low type. As they progress through university, their academic performance allows firms to better sort between student types. As in reality, the private equity firm need not hire a graduating student. Rather, it can opt to keep tabs on the student the bank hires, gauging her quality over time through observable, yet noisy, metrics before poaching. The bank prefers to hire students in their final year when it can best distinguish between student types. This strategy, though, leaves them vulnerable to poaching, as the private equity firm can free-ride off its efforts in screening. To combat this, the bank could choose candidates less likely to be high type. Doing so leads to the bank retaining workers longer as the private equity firm must wait for more evidence before poaching. But how can the bank credibly choose candidates less likely to be high type? By hiring early, when all parties have less information and screening ability is capped.

I formalize these ideas in a model of firm interviewing and hiring when there is a secondary market allowing firms to hire laterally instead of at the entry level. Unraveling arises as a strategic decision to deter future poaching because firms are willing to trade off quality in favor of retention. What determines the extent of unraveling is the transparency of the secondary market: how well firms can ascertain the value of workers employed at other firms. Firms often receive signals about the quality of workers employed at competitors, and the clarity of such signals determines the intensity of the threat of poaching and the strategic importance of deterring it (Wu [2024]). When the secondary market is moderately transparent, unraveling makes underinvestments in talent discovery credible. As the secondary market becomes more transparent, unraveling disappears in equilibrium due to the low-tier firm's incentive to communicate that they have *not* hired top-quality applicants. Low-tier firms interview candidates at the end of the primary market to ensure the hiring of applicants unlikely to be of high quality. This has stark welfare implications. A highly transparent secondary market decreases total match quality by creating an adverse signaling incentive for the low-tier firm.

Could one improve match quality by coordinating hiring times in the primary market? Not necessarily. Unraveling is a strategic response to the threat of poaching, and coordination on hiring time does not fully mitigate the threat. Mandating interviewing and hiring to occur at a common date may reduce match quality compared to the decentralized setting in which unraveling occurs. This indicates that to increase ex-ante match quality, the focus should be on the secondary market rather than controlling timing in the primary market.

I describe my model next. Sections 3 and 4 focus on equilibrium analysis and show how the strategies mirror real-world hiring practices. Section 5 explores applications. I defer discussion of the relevant literature to Section 6. Finally, Section 7 discusses the substantive assumptions of the model. Proofs are in the appendix.

2 Model

Two firms, F_H (high-tier) and F_L (low-tier), each need a single worker. Workers are high or low type, represented by $\theta \in \{H, L\}$. Workers prefer to work for the high-tier firm. A worker is high type with probability $\beta \in (0,1)$, independent of the others. Let N be the total number of workers. For technical convenience, I consider the case as $N \to \infty$.

Time is continuous from $-T$ to ∞ and divided into two stages: $[-T,0]$, which I call the primary market, and $(0,\infty)$, which I call the secondary market. Matching can occur in each stage. If a firm approaches a worker at time $t \in [-T,0]$, and the worker accepts, the worker exits the market and *begins working at time 0*. A firm failing to match in the primary market can choose to poach the employed worker at any time $t \in [0,\infty)$ in the secondary market.

Primary Market A firm choosing to match in the primary market selects a time $t \in [-T,0]$ to conduct interviews. Interviews are more informative the later they occur. I represent this via a function $M : [-T,0] \to [0,1] \times [0,1]$, where $M(t) := (M_{high}(t), M_{low}(t))$ maps interview times to the highest and lowest probability with which firms can identify a worker of being high type. More generally, consider any $M(\cdot)$ satisfying:

1. $M_{high}(t)$ and $M_{low}(t)$ are continuous.
2. $M_{high}(t)$ is strictly increasing, and $M_{low}(t)$ is decreasing.
3. $M_{high}(-T) = M_{low}(-T) = \beta$.

Mapping M is a reduced-form representation of how well firms can sort workers at time t. The first two conditions ensure better sorting at later times. The third says there is no ability to sort at the start. In Appendix A, I show that any such M is equivalent to a sequence of progressively more informative binary tests that return a high or low signal depending on the worker's true type. Thus, I define a **high-signal** worker at time t to be a worker that is high type with probability $M_{high}(t)$. Similarly, a **low-signal** worker at time t is one that is high type with probability $M_{low}(t)$. Note that randomizing over high and low-signal workers at time t allows for hiring a worker that is high type with probability $p \in [M_{low}(t), M_{high}(t)]$. At any time t, firms can interview all workers in the primary market costlessly, and so they can hire a high-signal or low-signal worker with probability 1 as $N \to \infty$. Given sorting is best at the end of the primary market, $M_{high}(0)$ and $M_{low}(0)$ are, respectively, the maximum and minimum probability with which a firm can be sure it has hired a high-type worker.

Fix a mapping M. If a firm hires a worker at time $t < 0$ and the worker is high type with probability $p < M_{high}(0)$, I say that the market has **unraveled**.

Secondary Market Consider a firm that does not hire in the primary market, instead choosing to operate in the secondary market where it can monitor the worker hired by the other firm. The monitoring firm observes a signaling process yielding information about the employed worker's type. Formally, consider a worker that is high type with probability p_0. The monitoring technology is given by an observable process $\{\pi_t\}$:[3]

$$d\pi_t = \mu_\theta dt + \sigma dB_t,\ \pi_0 = 0,$$
$$\text{where } B_t \text{ is a standard Brownian motion}$$

One can interpret π_t as a noisy signal of visible worker output. The type-dependent drift satisfies $\mu_H \geq \mu_L$, reflecting expected differences in output between worker types. The diffusion process is a continuous analog of the case where the high-tier firm periodically receives signals of the worker's quality. The quantity $\alpha := \dfrac{\mu_H - \mu_L}{2\sigma}$ measures the informativeness of π_t and represents the **transparency of the secondary market**.[4]

2.1 Payoffs

Consider a type θ worker hired by F_i in the primary market. If, at some time $t \geq 0$, the other firm F_{-i} offers a position and she accepts, payoffs from a time 0 perspective are:

$$\begin{aligned}
\text{Worker}: &\ r\left[\int_0^t e^{-r\hat{t}}(\delta \cdot \mathbb{1}_{i=H})d\hat{t} + \int_t^\infty e^{-r\hat{t}}(\delta \cdot \mathbb{1}_{-i=H})d\hat{t}\right] \\
F_i: &\ r\int_0^t e^{-r\hat{t}} Z_\theta^i d\hat{t} \\
F_{-i}: &\ r\int_t^\infty e^{-r\hat{t}} Z_\theta^{-i} d\hat{t}
\end{aligned}$$

Here, r is the common discount factor, δ is the added payoff to the worker from working at the high-tier firm, and Z_θ^i is the match quality to firm i from hiring a worker of type θ. Match quality encapsulates productivity. I assume:

$$Z_H^H \geq Z_H^L > Z_L^L > 0 > Z_L^H$$

The inequalities reflect firm preferences and incorporate a notion of supermodularity in match quality. Both firms prefer high-type workers. Notably, the high-tier firm never

[3]The implication of the diffusion process is that the secondary market is more informative than the primary market. Therefore, my model applies to settings where there is a difference in the work done in each stage. This is a feature of entry-level markets since firms often hire directly from universities.

[4]One can see this from the log-odds process $Q_t = log\left(\frac{Pr(\theta=H|\mathscr{F}_t^\pi)}{1-Pr(\theta=H|\mathscr{F}_t^\pi)}\right)$, where $\mathscr{F}_t^\pi$ is the natural filtration with respect to $\{\pi_t\}$. The log-odds process satisfies $dQ_t = \frac{2\alpha}{\sigma}d\pi_t$.

wants to employ a low-type worker. The low-tier firm finds such a worker acceptable. This is a natural assumption, as high-tier firms may have reputational concerns, so hiring a low-type worker is especially undesirable. A different interpretation is that high-type workers are qualified for the job at the high-tier firm but over-qualified for the job at the low-tier firm, while low-type workers are only qualified for the job at the low-tier firm.[5]

Workers not hired in the primary market receive a payoff normalized to 0 and exit. Unmatched firms receive a flow payoff of 0 for the duration they are unmatched. I assume that once a worker is hired, she can not be fired. In Appendix F, I explain why giving firms the ability to fire workers does not change the qualitative features of the results. Lastly, I impose the trivial condition $M_{high}(0)Z_H^H + (1 - M_{high}(0))Z_L^H > 0$, as otherwise the high-tier firm would never want to hire in the primary market.

2.2 Strategies

Worker strategies consist of accepting or rejecting offers in the primary market and, if hired, accepting a lateral offer. Firm strategies specify primary and secondary market strategies, describing whether, when, and how to hire in each stage. Formal definitions are in Appendix A.2.

Primary Market Strategy At each $t \in [-T, 0]$, if F_i has not hired up until t, it selects a probability to interview and, conditional on interviewing, to whom to make an offer. This selection can be based on whether F_{-i} has interviewed at any point before t. Upon this observation, F_i formulates beliefs about the type of worker F_{-i} hired.

Secondary Market Strategy Conditional on operating on the secondary market, F_i's strategy is a poaching rule: a decision at each $t \in (0, \infty)$ whether to hire the worker at F_{-i}. Hence, a poaching rule is a stopping time adapted to the filtration $\mathscr{F}_t^\pi$.

A pair of firm strategies constitute an **equilibrium** if each firm is best-responding at each information set, and F_H's beliefs are consistent. See Appendix A.2 for a formal definition. The next two observations reduce the strategy sets I need to consider.

OBSERVATION 2.1 *A worker who receives a primary market offer always accepts it.*

[5] As will be made apparent later, $Z_L^H < 0$ is necessary for the analysis. If the payoff from not hiring a worker is 0, and hiring any worker gives a positive payoff, there is no incentive to operate in the secondary market and wait for information to be revealed about a hired worker.

One might think that a worker receiving an offer is a signal of her type, allowing for the opportunity to strategically reject the offer. When N is large, this incentive disappears as the probability of receiving a future offer is ≈ 0.

OBSERVATION 2.2 *The low-tier firm will always hire in the primary market.*

Consequently, I restrict attention to low-tier firm strategies specifying the time t in the primary market at which to interview and, conditional on t, to whom to make an offer. It follows that the high-tier firm will never interview in the primary market before $t = 0$. So, I can restrict attention to high-tier firm strategies that *map the observed time at which the low-tier firm hires* to a probability of hiring in the primary market and a poaching rule conditional on operating in the secondary market.

OBSERVATION 2.3 *If a secondary market does not exist, there is no unraveling.*

By interviewing at $t = 0$, each firm matches with a high-signal worker with probability 1 as $N \to \infty$. Total match value is $M_{high}(0) \cdot (Z_H^L + Z_H^H) + (1 - M_{high}(0)) \cdot (Z_L^L + Z_L^H)$. When there is no secondary market and an abundance of talent, there is no unraveling. This serves as a benchmark for comparison to the equilibrium findings.

3 Poaching and Incentives

I begin by characterizing the optimal poaching rule *conditional on the high-tier firm operating on the secondary market*. Suppose the low-tier firm has hired a worker in the primary market, and the high-tier firm's initial belief that the worker is high type is p_0. The high-tier firm's decision problem is:

$$\Gamma_H(p_0) := \max_{\tau} \mathbb{E}[e^{-r\tau} Z_\theta^H | \mathscr{F}_t^\pi, p_0]$$

To determine the optimal poaching rule, I map π_t to the space of posterior beliefs. Given initial belief p_0, let $p_t := \mathbb{P}(\theta = H | \mathscr{F}_t^\pi)$ denote the posterior belief that the worker is of high type at time t given the observations from the process $\{\pi_t\}$.

Proposition 3.1 *The optimal poaching rule is a threshold rule $\tau^* := \inf\{t \geq 0 : p_t \geq B^*\}$, where B^* is time-invariant, independent of p_0, and increasing in α.*

The decision to poach depends only on whether the belief about the worker is above a static threshold B^*, which is independent of the initial belief p_0. This is not to say that p_0 plays no role in the high-tier firm's "thought process". The value of the initial belief affects the value of the signal π_t the firm must see to update its belief to B^*. The transparency of the secondary market, α, affects the speed of learning. As α increases, B^* increases, but the expected payoff increases for the high-tier firm. With a more informative signal, the high-tier firm can afford to wait for a higher posterior.

The characterization of the optimal poaching rule allows for a clean description of firm payoffs when the high-tier firm commits to not hiring in the primary market. As the high-tier firm's payoff function depends on its initial belief (p_0), consistency is assumed. Its payoff from operating on the secondary market is:

$$\Gamma_H(p_0) = p_0 Z_H^H \cdot \mathbb{E}[e^{-r\tau^*}|H, p_0] + (1-p_0)Z_L^H \cdot \mathbb{E}[e^{-r\tau^*}|L, p_0]$$

For the low-tier firm, its payoff depends on the *actual* probability the worker it hires is of high type ($\tilde{p}_0$), *and* the initial belief (p_0) held by the high-tier firm:

$$\begin{aligned}\Sigma_L(\tilde{p}_0, p_0) &:= \tilde{p}_0 Z_H^L\Big(1 - \mathbb{E}[e^{-r\tau^*}|H, p_0]\Big) + (1-\tilde{p}_0)Z_L^L\Big(1 - \mathbb{E}[e^{-r\tau^*}|L, p_0]\Big) \\ &= \underbrace{\tilde{p}_0 Z_H^L + (1-\tilde{p}_0)Z_L^L}_{\text{Expected Match Value}} - \underbrace{\Big(\tilde{p}_0 Z_H^L \mathbb{E}[e^{-r\tau^*}|H, p_0] + (1-\tilde{p}_0)Z_L^L \mathbb{E}[e^{-r\tau^*}|L, p_0]\Big)}_{\text{Loss Due to Poaching}}\end{aligned}$$

Of particular interest is the function $\Gamma_L(p_0) := \Sigma_L(p_0, p_0)$, the expected payoff to the low-tier firm when the high-tier firm's initial belief is consistent.

Proposition 3.2 *Fix $\alpha \in (0, \infty)$. There exists $p^* \in (0,1)$ such that $p^* = arg\ \max_{p_0 \in [0,1]} \Gamma_L(p_0)$.*

The quantity p^* is the optimal induced prior in a game where the high-tier firm is committed to hiring in the secondary market *and knows* the probability that the worker hired by the low-tier firm is of high type. The optimal induced prior p^* varies naturally with α, the transparency of the secondary market. As $\alpha \to 0$ (low transparency), the low-tier firm understands that poaching is more challenging; it is more willing to hire potentially high-quality workers in the primary market. Conversely, as $\alpha \to \infty$ (high transparency), F_L seeks to hire a high-type worker with low probability to ensure it can keep the worker for a long time.

Now, the high-tier firm does not observe which worker the low-tier firm hires but forms an initial belief based on the hiring time. By interviewing at time t, the low-tier firm constrains itself to workers of high type with probability in $[M_{low}(t), M_{high}(t)]$. The equilibrium of this subgame is pinned down by the unique $p \in [M_{low}(t), M_{high}(t)]$ such that the low-tier firm does not have a profitable deviation conditional on the high-tier firm holding an initial belief of p. The low-tier firm's payoff is $\Gamma_L(p)$. It would be optimal if it could induce a subgame where its equilibrium payoff is $\Gamma_L(p^*)$.

Proposition 3.3 *Suppose the high-tier firm cannot hire in the primary market. If $p^* \in (\beta, M_{high}(0))$, the low-tier firm hires at time t^* in equilibrium, where $M_{high}(t^*) = p^*$.*

While the low-tier firm may want to hire a better worker, the only way to do so is by changing the time it matches, which will change the high-tier firm's initial belief!

4 Equilibrium and Match Quality

I now analyze the equilibrium dynamics when F_H can choose whether to hire in the primary or secondary market *as a function of the history it observes*. The decision to hire in the secondary market depends on the effectiveness of screening in the primary market. If the screening ability at the end of the primary market is such that the posterior belief is already above the poaching threshold, the high-tier firm will not hire in the secondary market. Both firms interview at $t = 0$ and hire a high-signal worker.

OBSERVATION 4.1 *If $M_{high}(0) \geq B^*$, the market does not unravel.*

If the threshold belief for poaching, B^*, is lower than the belief about a high-signal worker at the end of the primary market, the secondary market provides no value to the high-tier firm. This serves as the basis for a definition of opaqueness. Let α_{opaque} denote the transparency level at which $B^* = M_{high}(0)$.[6] As B^* is strictly increasing in α, the following observation is immediate.

[6] Formally, define $\alpha_{opaque} := sup\left\{\alpha : B^* < M_{high}(0)\right\}$. At $\alpha = 0$, B^* is equal to the high-tier firm's break-even point, and so $M_{high}(0) > B^*$ by assumption. Since B^* is increasing in α and $\lim_{\alpha \to \infty} B^* = 1$, it follows that for $M_{high}(0) < 1$, $\alpha_{opaque} \in (0, \infty)$ and solves $B^* = M_{high}(0)$.

OBSERVATION 4.2 *For $\alpha \leq \alpha_{opaque}$, the market does not unravel.*

At transparency levels $\alpha > \alpha_{opaque}$, the high-tier firm may find it beneficial to operate in the secondary market. Since the high-tier firm observes only the time the low-tier firm hires, its belief about the worker hired depends only on the hiring time t and its knowledge of the interviewing technology $M(t)$. Likewise, the low-tier firm's decision to match at a time t and hire a high or low-signal worker depends on whether it expects the high-tier firm to operate on the secondary market or at the end of the primary market. It becomes crucial, then, to pin down firms' indifference beliefs:

1. What initial belief ($\bar{p}$) does the high-tier firm need to hold to be indifferent between hiring at the end of the primary and on the secondary market?

2. If the high-tier firm is on the secondary market, what initial belief (p_{ind}) does it need to hold to make the low-tier firm indifferent between worker types?

The high-tier firm's decision to hire on the primary market is based on whether its initial belief about the low-tier firm's worker is above $\bar{p}$. If the high-tier firm chooses to hire at the end of the primary market, the low-tier firm would like to hire a high-type worker. If the high-tier firm chooses to hire in the secondary market and holds an initial belief less than p_{ind}, the low-tier firm also prefers a high-type worker. However, if the high-tier firm chooses to hire in the secondary market and holds an initial belief greater than p_{ind}, the low-tier firm prefers a low-type worker. These indifference beliefs depend on the transparency of the secondary market, and the value of $\bar{p}$ relative to p_{ind} determines the equilibria of the subgame initiated when the low-tier firm hires at time t.[7] In the equilibrium of the entire game, the low-tier firm considers the following trade-off. Interviewing early induces the high-tier firm to hire at the end of the primary market, allowing the low-tier firm to keep the worker forever. Interviewing later allows the low-tier firm to hire a better worker, but at the expense of losing her in the future, as the high-tier firm will choose to hire in the secondary market.

[7] In Appendix C, I characterize equilibrium strategies in each subgame. Then, in Appendix D, I determine the low-tier firm's overall equilibrium strategy. As $\bar{p}$ and p_{ind} are determined by the transparency level α, this characterization effectively maps the values of α to the type of equilibria that arise.

There can be equilibria where the low-tier firm wants to hire a high-signal worker even if it knows the high-tier firm will be on the secondary market. This occurs when the high-tier firm has a strong desire to screen in the secondary market, *and* the low-tier firm has strong preferences between worker types. Such equilibria can be excluded when preferences are not aligned. The condition for non-alignment is one on the preferences of the low-tier firm relative to how valuable screening in the primary market is for the high-tier firm. Specifically, it compares the ratio of the low-tier firm's ex-post match values to the ratio of the high-tier firm's *ex-ante* expected match values when hiring in the primary market at time $t = 0$. Non-alignment ensures the low-tier firm does not care "too much" about the type of worker it hires, and so if the high-tier firm chooses to operate on the secondary market, the low-tier firm prefers to hire a low-type worker.

Definition 4.3 *Firm preferences are* **not aligned** *if* $\frac{Z_H^L}{Z_L^L} < -\frac{Z_H^H}{Z_L^H} \cdot \frac{M_{high}(0)}{1 - M_{high}(0)}$.

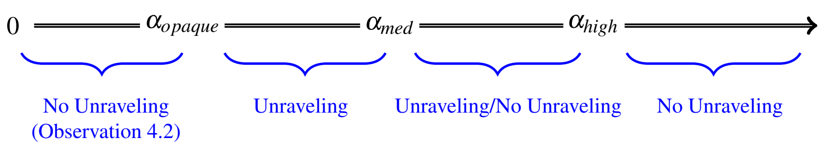
Theorem 4.4 summarizes the qualitative features of the equilibria as a function of the transparency level α when firm preferences are not aligned.

Theorem 4.4 *Suppose firm preferences are not aligned. Then there exist thresholds* $\alpha_{opaque} < \alpha_{med} < \alpha_{high}$ *such that in equilibrium:*

1. *For* $\alpha \in (\alpha_{opaque}, \alpha_{med})$, *the market unravels. The low-tier firm matches at time* $\bar{t} < 0$ *where* $M_{high}(\bar{t}) = \bar{p}$. *The high-tier firm matches at the end of the primary market. As* α *increases in this range, unraveling is exacerbated (*$\bar{t}$ *decreases).*

2. *For* $\alpha \in [\alpha_{med}, \alpha_{high}]$, *the low-tier firm mixes across times* $t \in [\underline{t}, 0]$ *where* $M_{low}(\underline{t}) = \bar{p}$. *There is multiplicity of equilibria, some with unraveling, others without. As* α *increases in this range, unraveling is diminished (*$\underline{t}$ *increases).*

3. *For* $\alpha \in [\alpha_{high}, \infty)$, *there is no unraveling. The equilibrium is unique.*



As transparency increases, the low-tier firm credibly underinvests in screening by hiring at an earlier time t. By doing so, the low-tier firm commits to not hiring a worker that is high type with probability greater than $M_{high}(t)$. Such a strategy ensures the high-tier firm does not operate in the secondary market. This is precisely the intuition behind the unraveling equilibrium that arises in the moderately transparent regime. The low-tier firm hires a worker that is high type with probability $\bar{p}$. In the moderately transparent regime, $\bar{p} \in (\beta, M_{high}(0))$, meaning the low-tier firm needs to do some screening and hire a high signal worker. However, to credibly signal that the worker hired is high type with probability $\bar{p}$, the low-tier firm must hire at time $\bar{t} < 0$ such that $M_{high}(\bar{t}) = \bar{p}$. The high-tier firm hires at the end of the primary market.

When the secondary market is highly transparent, the low-tier firm wants to induce a lower belief about the worker it hires to prevent poaching. Hiring a low-type worker with high probability requires being able to sort very well. As a result, it chooses to hire towards the end of the primary market.[8] However, if the low-tier firm is screening to hire a low-type worker with high probability, there is no incentive for the high-tier firm to operate in the secondary market. On the other hand, if the high-tier firm chooses to hire at the end of the primary market, the threat of poaching vanishes, and the low-tier firm no longer has the incentive to hire the worker with a low signal! Formally, suppose F_L interviews at time t and hires a worker that is high type with probability $p < M_{high}(t)$. If F_H hires at the end of the primary market, F_L would deviate by hiring a worker that is high type with probability $M_{high}(t)$. Such a deviation would be undetectable by F_H. Thus, to support the non-unraveling equilibrium when the secondary market is sufficiently informative, F_L hires at the end of the primary market *but mixes* between hiring a high-signal and low-signal worker. F_H mixes between operating in the secondary market and hiring at the end of the primary market.

Importantly, the transparency thresholds depend on the firms' sorting ability in the primary market. In the extreme, if firms could distinguish between types in the primary market perfectly (i.e. $M_{high}(0) \to 1$), then $\alpha_{opaque} \to \infty$ and unraveling never occurs. More generally, the thresholds are increasing in $(M_{high}(0), 1 - M_{low}(0))$.

Interpretation One may wonder whether the equilibrium strategies are realistic de-

[8]If β is small, identifying a low-type worker with high probability is easy: random selection at time $-T$ yields a low type with probability $1-\beta$. Consequently, even when the secondary market is very transparent, there is little incentive to sort to hire a low-quality worker. Formally, as $\beta \to 0$, $\alpha_{high} \to \infty$.

pictions of firm behavior. That is, do top firms condition their decisions on whether a lower-tier competitor hired a first-year law student in February? In matching markets where matching processes are public (e.g., Venture Capital funding, investment banking, and academic hiring), such strategies are realistic. In other labor markets, one should view the equilibrium strategies and outcomes as limit points of a long-run process that involves learning. Over time, a firm can observe the quality of workers at its competitors and deduce how well its competitors are screening. This interpretation echoes Green and Porter (1984), where firms cannot observe competitors' prices directly and instead rely on noisy demand estimates to infer them.

In fact, this is supported by the evolution in hiring dynamics pertaining to private equity companies (high-tier firms) poaching talent from investment banks (low-tier firms). Traditionally, private equity firms have relied entirely on poaching investment banking analysts. Over the last decade, investment banks began to hire earlier, contracting with students while they were sophomores. Private equity firms soon lamented the variance in quality of investment banking analysts. In response, private equity firms began interviewing and hiring undergraduates in their final year (hiring at the end of the primary market).[9] This is consistent with a transition to the unraveling equilibrium that arises when the secondary market is moderately transparent.

At a higher level, Theorem 4.4 highlights an important connection between secondary market transparency and screening incentives. In the model, firms use matching time as a credible way of signaling how well they will screen and the type of talent they will select. In contexts where time can not be used in such a way, unraveling may appear as a reduction in investment in screening. The effect of transparency on screening incentives is prevalent in a billion-dollar labor market: professional sports. When players are free to move across teams, small-market teams underinvest in screening for talent (Feess and Muehlheusser [2003]). This phenomenon is precisely why contracts preventing mobility exist in competitive sports.[10] I discuss the relationship between contract characteristics, unraveling, and investment in screening in Section 5.1.

[9] https://economics.virginia.edu/news/article-top-private-equity-firms-hiring-college-grads

[10] Free agency rules ensure that movement cannot be prohibited. However, some leagues have clauses allowing teams to pay their players on expiring contracts significantly more than competitors.

4.1 Match Quality

Using the equilibrium characterization in Theorem 4.4 when firm preferences are not aligned, I compare the total equilibrium match value to the benchmark where no secondary market exists. Suppose the low-tier firm can match with a low-type worker with sufficiently high probability in the primary market. Then, the total equilibrium match quality is strictly lower than the benchmark setting with no secondary market.

Theorem 4.5 *If $M_{low}(0)Z_H^H < M_{high}(0)Z_H^H + (1 - M_{high}(0))Z_L^H$, then for any $\alpha > \alpha_{opaque}$, the total ex-ante match value is strictly lower than when no secondary market exists.*

The condition in the theorem statement rules out the situation where the high-tier firm would be happy to poach from the low-tier firm even if it knew that the low-tier firm hired the applicant most likely to be of low type. It holds when the interviews at the end of the primary market are successful at identifying low types (e.g., when $M_{low}(0)$ is small). Importantly, when the secondary market is highly transparent, the low-tier firm can hire low-quality workers with a high enough probability to deter the high-tier firm from poaching. As a result, unraveling dissipates in highly transparent markets *because* the low-tier firm has an incentive to screen for lower-quality workers. This causes a reduction in total match quality relative to the benchmark.[11]

In the market design literature, reductions in match quality in unraveled markets are often seen as the product of the timing of the matches (e.g., Roth and Xing [1994]; Li and Rosen [1998]). If the timing issue is resolved, will total match quality increase?

Theorem 4.6 *Suppose firm preferences are not aligned. If $\alpha \in (\alpha_{opaque}, \alpha_{med})$, meaning the secondary market is moderately transparent, the total match value in the unraveling equilibrium is greater than in the equilibrium where interviewing and hiring are mandated to occur at $t = 0$.*

In the unraveling equilibrium, the low-tier firm hires at time $t < 0$, and the high-tier firm hires at $t = 0$. Suppose a third party could ensure that all primary market interviews

[11] If $M_{low}(0)Z_H^H > M_{high}(0)Z_H^H + (1 - M_{high}(0))Z_L^H$, high transparency leads to the high-tier firm choosing to monitor even when the low-tier firm hires a worker that is high type with minimal probability. Total match quality is $\approx M_{low}(0)Z_H^H + (1 - M_{low}(0))Z_H^L$, which may or may not be higher than the benchmark.

occur at $t = 0$. Coordinating the hiring date provides more incentives for the high-tier firm to screen. Why? Because the low-tier firm now has access to higher-quality applicants! Since interviews are more informative at $t = 0$, if the low-tier firm hired a high-signal worker now, the high-tier firm would want to operate in the secondary market. The threat of poaching is now more serious, and so the low-tier firm is incentivized to hire low-type workers. A mixed strategy equilibrium must exist with the high-tier firm poaching with non-zero probability. The low-tier firm mixes between hiring a high-signal and low-signal worker. With positive probability, only a single worker is hired by the end of the primary market. This reduces total match quality relative to the unraveled equilibrium without the mandate.

5 Applications

The table below presents a few well-known industries and their unraveling dynamics.

Markets	**Unraveling**
Corporate Law Associates	*Extensive*
Investment Banking Analysts and Private Equity Associates	*Extensive*
High-end Chefs and Line Cooks	*Extensive*
Financial Traders	*Minimal*
Assistant Professors in Economics	*Minimal*
Management Consulting	*Minimal*
Programmers and Software Engineers	*Minimal*

Table 1

Unraveling in the first three markets in the table has been documented extensively.[12] Unraveling is absent in the junior economics faculty market. Screening and hiring occur the year candidates receive their Ph.D. and only after recommendation letters, research statements, and papers are posted. In addition, until the COVID pandemic, interviews occurred at the ASSA meeting. In management consulting, interviews for full-time positions at McKinsey, Bain, and BCG occur in candidates' senior year.[13]

The equilibrium outcomes align with the observed hiring behavior in the markets in Table 1. The link I highlight between secondary market transparency and unraveling

[12]For corporate law, see Sloan (2024). For the culinary market, see Rainey (2013). For investment banking and private equity, see Cohen (2019) and UPenn's career website outlining the hiring timeline: `https://careerservices.upenn.edu/blog/2024/02/22/navigating-investment-banking-recruitment-as-a-sophomore-insights-and-tips-from-a-wharton-mba-candidate/`.

[13]See Harvard's career center outline of the hiring timeline: `https://careerservices.fas.harvard.edu/blog/2023/06/23/consulting-application-deadlines/`

suggests one could have predicted such unraveling—or its absence—in these markets ex-ante. More broadly, the qualitative characterization of the equilibrium dynamics within my model provides insights into which labor markets are prone to unraveling. Those with highly transparent or opaque secondary markets should have minimal unraveling, while those with moderately transparent secondary markets should experience unraveling. Returning to Table 1, consider the subset of labor markets that unravel: corporate law associates, investment banking and private equity analysts, and high-end line cooks. These industries have what one might consider to be moderately transparent secondary markets. Why is it reasonable to describe these industries as such?

Consider the investment banking and private equity markets. Firms in these industries generally have an understanding of the activity of their competitors. For instance, during an IPO of a company, it is publicly known which investment banks are working on the offering. Importantly, banks generally know the specific groups working on particular deals. However, observing how much an individual contributed is difficult, especially at the analyst and associate levels. Did he merely bring coffee to his bosses, akin to an intern, or was he actively engaged in the deal-structuring process? Similarly, in corporate law, while a high-tier firm can monitor associates at other firms, it is more challenging to assess associate quality compared to a market like that for academic professors, where research is published for public view. Suggestive evidence of this is law firms' approach to lateral screening and hiring. Many invest heavily in third-party recruiters specializing in identifying, monitoring, and approaching talent at other law firms. Some firms will have entire in-house divisions responsible for screening laterals (see Maloney [2023]). The difficulty of ascertaining associate quality at other firms is echoed by Ginsburg and Wolf (2004), who explicitly note "some aversion toward laterals stems from the lack of transparency of legal work". This lack of transparency leads to law firms relying on referrals (Ginsburg and Wolf [2004]).[14]

Now, consider the subset of markets listed in Table 1 that currently do not experience unraveling: assistant professors in economics, financial traders, management con-

[14]The link between transparency and lateral hiring is clear when considering that non-corporate organizations like the U.S. Attorneys' Offices recruit via poaching those with prosecutorial experience. Such a strategy is viable because court proceedings and performances are public (Ginsburg and Wolf [2004]).

sulting, and software engineers. The market for assistant professors in economics has received much attention due to the shift to virtual interviews. My model suggests that unraveling should not be considered a significant issue because the secondary market is very transparent: professors publish research in journals, share working papers online, and present research to their employer's competitors. In fact, my model highlights that it is this transparency that prevents unraveling and not the existence of a centralized system (e.g., the ASSA meeting). The centralized system is sustainable because of the secondary market transparency. Now, some may point to the difference in the timing of interviews and offers as evidence of unraveling. This is not the case. There is no reduction in available information as applicants' complete job packets are still available. The inefficiencies arising from interviews and offers occurring at slightly different times are due to a lack of coordination caused by interviewing costs. Even if one enforced a common deadline by which candidates need to make a decision, these costs remain (see Wapnir et al. [2021] for a discussion of these issues in the centralized medical match).

On the opposite side of the spectrum are the markets for software engineers and managerial consultants, which have an opaque secondary market. In the former, non-disclosure agreements are highly restrictive and comprehensive (Drange [2021]).[15] In consulting, casework is private, and employees are barred from revealing their clients.

The hiring dynamics in the market for financial traders align with my model too, although it demands additional nuance due to the large and heterogeneous set of firms involved. Within the quantitative hedge fund and proprietary trading space, firms target undergraduates. The secondary market, though, is opaque due to NDAs and, in some cases, prohibitions from publicizing places of employment. Consistent with this is the minimal unraveling in the sector, especially compared to the market for investment bankers.[16] More traditional hedge funds typically source junior hires from investment

[15]With the rise of Github, and depending on how the legal landscape around NDAs in the tech sector shifts, one may expect hiring dynamics to change in computer science markets.

[16]While high-tier prop trading firms do have internships for undergraduates, they tend to target them towards those in their penultimate year (e.g., Optiver, Hudson River Trading). Unlike in investment banking, the full-time analyst classes at such firms are not entirely made up of past summer interns. Moreover, the percentage of interns receiving such offers is significantly less than in investment banking. In this sense, their internships function much more as screening devices than contracting ones.

bank trading desks and laterally from other funds. Unlike private equity firms, which secure commitments within the first few months of analysts starting their banking jobs, hedge funds do not engage in such forward contracting. Taking an alternative frame of reference where the primary market is the analysts' tenure at an investment bank, the difference in hiring practices is rooted in the *primary market* being more informative. Trading desks at banks receive data about revenue generated by other desks across Wall Street. Some information is even reported publicly.[17] Also, there is constant communication between traders on the buy-side and sell-side due to the nature of the job. When the primary market becomes more informative, so $M_{high}(t)$ increases for all t, unraveling dissipates.

Though non-unraveling occurs when the secondary market is highly transparent or opaque, the equilibrium matches vastly differ. In the former, both firms hire at the end of the primary market, while in the latter, there is the type of mixed strategy equilibrium described in Theorem 4.4. Hence, one would expect to see differences in the frequency of junior-level lateral hiring in these industries. Industries with transparent secondary markets will have more lateral hiring than industries with opaque or inactive secondary markets. This is the case when comparing markets for managerial consultants to that for assistant professors in economics. Indeed, in the former, lateral hiring efforts tend to be focused at the senior level, as promotion to a senior position is one of the few clear signals of quality (Bhattacharya [2015]).

It is important to note that my model does not claim that the secondary market's characteristics alone determine whether unraveling occurs. Instead, it highlights another avenue by which unraveling can arise. Importantly, it illustrates how unraveling is a phenomenon that is present in markets where firms are *not* worried about whether there will be a shortage of high-quality workers at the end of the primary market. A case where these insights may not apply is the hiring of appellate court judicial clerks. There is no secondary market there, yet substantial unraveling occurs. This does not contradict my model. In my model, there is a "short side" of the market and a "long side". Unraveling does not occur in the absence of a secondary market because the firms are on the short side. In the judicial clerk market, the viable applicant pool is small; all

[17] https://www.efinancialcareers.com/news/2022/11/bonuses-traders-investment-banks

parties fear they are on the long side of the market. Thus, explanations provided by Niederle et al. (2013) and Ambuehl and Groves (2020) are better suited for this setting.

While the model is described in the context of a labor market, it applies to other two-sided matching markets with mobile assets of unknown quality. For instance, in venture capital, one can think of the primary market as the pool of early-stage startups. The secondary market consists of startups with funding looking for future series rounds. The firms finding it difficult to earn large returns are typically smaller, lesser-known ones that cannot maintain investment relations with successful startups.[18] Prominent venture firms utilize smaller ones as screening devices, poaching the "winners" in later series' rounds. As a result, the market has unraveled, with lesser-known firms investing in startups earlier in their life cycle to prevent dilution.

5.1 Potential Policy Solutions

Two crucial features of the model are the low-tier firm's ability to block off information in the primary market once it matches with a worker and the freedom of the worker to move between firms in the secondary market. Hence, two interventions may mitigate unraveling and increase efficiency:

1. Improving the Flow of Information in the Primary Market.
2. Controlling Mobility in the Secondary Market.

The first intervention corresponds to increasing $M_{high}(0)$, which reduces the high-tier firm's incentive to operate in the secondary market. The low-tier firm, then, need not hire early. Realistically, increasing $M_{high}(0)$ would require a shift of the entire $M_{high}(t)$ curve upwards and the $M_{low}(t)$ curve downwards. This could occur in two ways. If one interprets $-T$ as the point when information about candidates starts being released in the primary market, then increasing the time horizon T and extending the window of the primary market could allow for more signals about candidate quality to be revealed. Second, establishing mechanisms that directly enhance the quality and quantity of information released would also make the primary market more informative. For instance,

[18] I am grateful to Tomasz Tunguz (Partner at Redpoint Ventures) for this point.

structured assessments in medical education (USMLE) were introduced in the 1990s to provide more information about candidates to residency programs. Similarly, platforms like LinkedIn and Github allow individuals to easily showcase their skills and abilities. However, while such platforms could alleviate unraveling in the primary market, they also improve monitoring ability in the secondary market. This can increase inefficiency due to Theorems 4.5 and 4.6.

The second intervention concerns the issue of labor mobility. While public opinion on labor mobility is positive, there are several papers highlighting bidirectional effects associated with either permitting or restricting worker movement (see Shi [2023] and Jeffers [2024]). My paper points to an inefficiency caused by strategic responses to mobility: unraveling and reduced screening. A simple way to increase match quality in my model is to allow firms to offer contracts restricting workers from changing jobs. Such contracts would stipulate that a worker can only leave after some time $t_{poach} > 0$. Low-tier firms would have an incentive to offer such contracts, as it is strictly dominant for them to do so, and workers will always accept these offers if there are far more workers than slots at firms (see Observation 2.1). Including such contracts is especially significant in settings where the secondary market is highly transparent. Without them, the high-tier firm mixes between operating on the primary and secondary market, and the low-tier firm mixes between hiring high and low-signal workers. If firms could offer contracts prohibiting workers from leaving for a certain period of time, the resulting equilibrium would have both firms hiring high-signal workers at the end of the primary market. In practice, such contracts are difficult to implement due to legality issues. However, clauses that attempt to mimic their structure are utilized in some industries. For example, Burguet et al. (2002) find that in markets with a high degree of transparency, firms include clauses that confer high quitting costs on employees. Another method by which firms attempt to prevent movement is via non-compete clauses.[19] Non-competes have been viewed negatively, but this is partly because their effect has been analyzed from the perspective of workers *that are already employed*

[19]Non-competes are not necessarily enforced in the United States (e.g., California). Also, in industries such as corporate law, non-competes are generally only used at the partner level. No equivalent contract for associate-level positions exists, which is where unraveling occurs.

(e.g., Dougherty [2017]; Balasubramanian et al. [2020]). My model points out that the existence of non-competes may change the *initial* matchings: workers not hired in the old regime would be hired if long-term contracts or non-competes were permitted.[20] This is especially significant for markets with a surplus of talent and an active secondary market. Even though high-talent workers are abundant, low-tier firms often match with lesser-talented workers due to the threat of poaching. In professional sports, for instance, where the secondary market is highly transparent, incentivizing small-market teams to screen, draft, and train talented players is difficult without restrictions on player movement. Hence, there is a need for long-term, restrictive contracts. In the financial and tech industries, firms use deferred compensation, discretionary bonuses, and stock option-vesting periods to reduce movement. In areas of innovation, patents play this role.

6 Relation to the Literature

An extensive literature on market unraveling was spawned by Roth and Xing (1994), who identified the phenomenon and many markets that had experienced an unraveling of appointment dates. Along with Avery et al. (2001), they conjecture that firms "jump the gun" to acquire top talent. Niederle et al. (2013) formalize this intuition in a market with *comparable* supply and demand, where firms and workers both believe that they are on the long side of the market. My paper demonstrates that unraveling can occur *even when talent is plentiful*, and there is a supply and demand *imbalance*. This is not to say that scarcity of talent is unimportant in my model. Though talent is abundant in the primary market, it is scarce in the secondary market stage, which makes poaching in the secondary stage costly for the firm losing the worker.[21]

[20]Related is the argument non-competes can incentivize firm-sponsored general-skill provision (e.g., Aghion and Bolton [1987]; Marx et al. [2009]; Garmaise [2011]; Mukherjee and Vasconcelos [2012]). In my model, non-competes incentivize better screening at the initial stage, leading to better initial matches.

[21]Other papers propose different causes. Damiano et al. (2005) examine a search and matching model, where introducing participation costs decreases the fraction of low types searching in early periods. Firms are incentivized to match early or face a pool of workers bereft of talent. Halaburda (2010) and Echenique and Pereyra (2016) view unraveling similar to a bank run: unraveling by one firm incentivizes unraveling by others. Fainmesser (2013) highlights the effect of networks and social connections on unraveling.

A common theme in the unraveling literature is the presence of informational uncertainty. Li and Rosen (1998) and Li and Suen (2000) examine matching markets with one-sided and two-sided uncertainty, respectively. In these papers, unraveling acts as insurance against being unmatched. In my paper, unraveling is insurance against competitors poaching a hired worker. However, due to the threat of poaching, coordinating on the time of contracting does not necessarily increase match quality.

The relationship between strategic signaling incentives in labor markets and unraveling builds on Waldman (1984), Milgrom and Oster (1987), Ostrovsky and Schwarz (2010), and Ely and Siegel (2013). The first two develop models where firms gain by strategically assigning workers and placing talented ones in less visible positions to prevent wage increases from competition. In my model, firms can not limit employee visibility, but they can control the flow of information by affecting the initial signal of a worker's ability. Ostrovsky and Schwarz (2010) endogenize information revelation in the primary market to show that optimal information disclosure prevents informational unraveling. They do not consider the presence of a secondary market where more information could be revealed in the future. While unraveling in their context differs from mine, a secondary market in their setting *can counteract* the benefits of informational disclosure policies. Ely and Siegel (2013) examine a common-value labor market where firms first observe a private signal about workers and then decide whether to interview. When interview decisions are public, adverse selection arises, leading to low-tier firms never hiring. A crucial feature of my model is that it is not a common-value setting: high-tier firms are averse to hiring certain workers. This aversion, coupled with the presence of a secondary market, generates adverse signaling incentives. High-tier firms can opt out of the primary market, monitor workers at low-tier firms, and poach them in the future. The low-tier firm uses early interviewing to credibly reduce its sorting ability to disincentivize the high-tier firm from choosing to poach in the secondary market.

My paper fits into a broader literature on poaching in labor markets. Most papers in this area study how poaching affects firm investment in worker development (e.g., Moen and Rosén [2004]; Leuven [2005]). Since general skill training makes a firm's workers more attractive to outsiders, poaching reduces the firm's return from such training.

Ferreira and Nikolowa (2023) develop a model where firms choose whether to retain or hire managers externally and look at how poaching affects talent flow across and within firms. My paper differs in that I focus on how the presence of poaching impacts firms' screening incentives *before hiring occurs*; it affects the type of worker hired in the first place. In this way, the results in my model are connected to those in Terviö (2009), which finds that firms reduce investment in talent discovery when worker movement is prevalent. While there is no actual "investment cost" to discover talent, firms can use the timing of hiring as an effective way to alter their talent-screening ability.

7 Discussion of Assumptions

For poaching to be a concern, it must be easy for workers to move, and conditional on it happening, costly for the poached firm. In the model, this is assumed. There are no switching costs, and if a firm loses a worker, it loses all the value associated with her.

Self-Insurance and Replacement A key driver of the results is that while talent is abundant early on, it is scarce in the secondary market. If there is no scarcity in the secondary market and replacement costs are low, retaining talent is not a first-order concern. The assumption that there is no opportunity for rehiring in the model reflects the cost of screening and training. Furthermore, in reality, the returns to training are often only realized later in a worker's tenure, in which case, poaching is especially detrimental. Even if firms could rehire, they would need to "start over". Now, firms concerned with poaching could theoretically hire more workers in the primary market as a form of self-insurance. In the context of the model, the low-tier firm would hire a second worker, assigning her to the productive task once the present worker is poached. In the human resources literature, this is an example of "overhiring". If firms could do this, it would be an effective way to deter poaching and reduce the need for unraveling. Consistent with this is the historical lack of unraveling in tech labor markets, which are labor markets where overhiring is common. However, there are negative effects associated with overhiring: decreased productivity, lower morale, and diminished reputation.

Multiple Firms Allowing for multiple firms elucidates the importance of the *risk of poaching* to unraveling. With only a single low-tier and high-tier firm, the low-tier firm is guaranteed to be poached if the high-tier firm operates on the secondary market. Suppose a second low-tier firm is in the primary market. The high-tier firm's payoff from poaching increases because of optionality. However, the individual probability that each low-tier firm will be poached decreases. To generalize, if there are very few firms poaching relative to the number of firms available to poach from, then poaching is not a serious threat. When the individual threat of poaching reduces to ≈ 0, and talent is abundant, the market will not unravel. The lack of a threat of poaching eliminates the adverse signaling incentives, allowing the ex-ante efficient matching to be achieved. Unraveling is a phenomenon that occurs when there is a hierarchy of firms, and poaching is a credible event. Well-known markets fitting this description are the private equity space, academia, corporate law, venture capital, and professional sports.

The opposite case is also insightful: two high-tier firms, one low-tier firm. If both high-tier firms operate in the secondary market, scarcity in that stage forces them to use a poaching rule with a break-even threshold belief. The high-tier firms simultaneously rush to poach the worker. This parallels aspects of the private equity and investment bank labor markets. Banks hire at the university level, but private equity firms rush to contract with banking analysts at the start of the secondary market. The payoff from operating on the secondary market decreases due to additional competition for scarce talent. I conjecture that there will be multiple equilibria. A salient one consists of the low-tier firm hiring early and the high-tier firms mixing between hiring at the end of the primary market and on the secondary market. In this equilibrium, one observes unraveling in the primary *and* secondary markets. The latter occurs when the realization of the mixed strategies results in both high-tier firms operating on the secondary market.

Switching Costs In my model, it is costless for a worker to switch firms. If a worker incurs a cost of c when changing jobs, and a poaching firm must compensate the worker, poaching becomes more difficult. Switching costs, then, deter unraveling in talent-rich settings, akin to how opaque secondary markets mitigate unraveling. This may explain why unraveling commonly occurs in entry-level markets, where workers enjoy greater mobility and fewer age-related barriers to changing jobs.

The *talent-rich* descriptor is substantial and necessary for the argument. Suppose talent is scarce in the primary market. The logic that transparent secondary markets reduce unraveling remains valid. However, high switching costs have a different effect in this environment. With high switching costs, the threat of poaching in the secondary market vanishes, and firms focus solely on the primary market. Consequently, unraveling may arise due to competition for scarce talent.

Wages and Adverse Selection In the paper, there are no wages. One reason is for simplicity. A second is that in the entry-level markets discussed, wages are standardized with little variability. Including wages and negotiation leads to adverse selection in the secondary market. Suppose F_L initially pays a wage w. F_H must pay a wage of $\max\{w-\delta,0\}$ to successfully poach the worker. Assume F_L knows the worker's type. If F_H offers a wage $w < \max\left\{Z_H^L-\delta,0\right\}$ and successfully poaches, it knows the worker is low type. Thus, F_H will only poach once it is willing to pay $\max\left\{Z_H^L-\delta,0\right\}$. Hence, $B^* = -log\left(\frac{Z_H^H-\max\left\{Z_H^L-\delta,0\right\}}{Z_L^H-\max\left\{Z_H^L-\delta,0\right\}}\cdot\frac{R_1}{R_2}\right)$. Since B^* is independent of the initial wage, F_L offers $w=0$ in equilibrium. Theorem 4.4 still holds but with new transparency thresholds α'_{med} and α'_{high}, where $\alpha'_{med} \geq \alpha_{med}$ and $\alpha'_{high} \geq \alpha_{high}$. Adverse selection reduces the threat of poaching, mitigating unraveling.

8 Conclusion

In most industries, the initial match between an employee and a firm is not permanent. After a worker is hired, it is often the case that she will receive offers from competing firms. The addition of this secondary market, whereby firms can poach workers from other firms, introduces a new channel by which unraveling can occur. Unraveling is no longer a race to acquire top talent but a strategic decision made by low-tier firms to retain workers they do hire. While increased secondary market transparency decreases unraveling, it does so at the expense of efficiency. A highly transparent secondary market incentivizes the low-tier firm to screen workers to ensure it has *not* hired one.

Appendix: Outline and Roadmap

Appendix A: I introduce the notation used in all the proofs and the definitions of strategies and equilibrium.

Appendix B.1 and B.2: I compute the high-tier firm's optimal poaching rule. Then, I work out the payoff function for the low-tier firm (Σ_L) and high-tier firm (Γ_H) when the high-tier firm operates on the secondary market. Lastly, I include lemmas describing the properties of these payoff functions, which will be used in the subsequent analysis.

Appendix B.3 and B.4: I establish the existence and properties of the indifference beliefs, p_{ind}, and $\bar{p}$. The proofs rely on the findings in Appendix B.1 and B.2. The lemmas in this section build upon one another and culminate in two central results: Lemmas B.9 and B.10, which prove that when the preferences of the firms are not aligned, $p^* < p_{ind} < \bar{p}$ for all $\alpha \geq \alpha_{opaque}$.

Appendix C and D: These appendices characterize the equilibria of the game for all transparency levels, given *any possible values* of the indifference beliefs.

> **Step 1 (Appendix C):** I characterize the equilibria of the subgame initiated when the low-tier firm enters the primary market at time t (i.e. "t-SE"). At a given time t, the low-tier firm only has access to workers that are high type with probability in $[M_{low}(t), M_{high}(t)]$. The t-SE depends on whether $\bar{p}$ lies in the interior of $[M_{low}(t), M_{high}(t)]$ *and* if it is greater than p_{ind}. This results in multiple cases, each yielding a different set of t-SE.
>
> **Step 2 (Appendix D):** I identify which distributions over $[-T, 0]$ can be sustained in equilibrium. I need to show that there exists a collection of t-SE such that the times in the support of the distribution maximize the low-tier firm's payoff given the collection of t-SE selected.

Appendix E: This section contains the proofs to Theorems 4.4, 4.5, and 4.6. The latter two theorems are proven once Theorem 4.4 is established. To prove Theorem 4.4, I use the fact that when firm preferences are not aligned, $p^* < p_{ind} < \bar{p}$ for all $\alpha \geq \alpha_{opaque}$. Theorem 4.4 then becomes a consequence of a subset of lemmas from Appendix D.

A Notation and Definitions

Terms	Meaning
Z^i_θ	Payoff to firm F_i from hiring worker of type θ
α	Transparency of the Secondary Market; Signal-to-noise ratio ($\frac{\mu_H-\mu_L}{2\sigma}$)
$M_{high}(t)$, $M_{low}(t)$	$M_{high}(t)$ is the highest feasible probability with which a hired worker can be of high type. $M_{low}(t)$ is the lowest feasible probability. At time t, a firm can identify a worker that has a probability of $M_{high}(t)$ or $M_{low}(t)$ of being high type.
R_1, R_2	$R_1 = \dfrac{1-\sqrt{1+\frac{2r}{\alpha^2}}}{2}$, $R_2 = \dfrac{1+\sqrt{1+\frac{2r}{\alpha^2}}}{2}$
p, Q	For computational convenience, I will generally work in the log-odds space of the beliefs, $Q = log(\frac{p}{1-p})$. I will refer to both Q and p as the "belief".
$\Pi_i(p)$	The expected payoff to firm F_i from employing a worker forever that is high type with probability p. $\Pi_i(p) = pZ^i_H + (1-p)Z^i_L = \frac{1}{1+e^Q}(e^Q Z^i_H + Z^i_L)$.
$\Sigma_i(p,p')$	The expected payoff to firm F_i when F_H operates on the secondary market holding initial belief p' about the worker, and F_L has hired a worker that is high type with probability p.
$\Gamma_i(p)$	$\Gamma_i(p) := \Sigma_i(p,p)$.

Table 2: Notation

A.1 Primary Market

The primary market must be such that the earlier a firm interviews, the lower its ability to sort between high and low-type workers. To capture this, I model interviewing as a probabilistic test on each worker that returns a *high* or *low* signal (denoted by lower-case h and l) depending on the true type of the worker.

Definition A.1 *Given* $x_h, x_l \in [0,1]$*, an* $(\mathbf{x_h}, \mathbf{x_l})$**-test** *is a signal applied to each worker that returns h (high) or l (low) such that* $x_h = P(h|\theta = H)$ *and* $x_l = P(l|\theta = H)$.

Any (x_h, x_l)-test induces an ordered pair (p_h, p_l) where $p_h := P(\theta = H|h)$ and $p_l := P(\theta = H|l)$. A *partial ordering* can be defined on the space of (x_h, x_l)-tests:

Definition A.2 *An* (x_h, x_l)*-test is* **more powerful** *than an* $(\hat{x}_h, \hat{x}_l)$*-test if and only if* $p_h \geq \hat{p}_h$ *and* $p_l \leq \hat{p}_l$.

Consider any $\widetilde{M} : [-T, 0] \longrightarrow [0,1] \times [0,1]$, where $\widetilde{M}(t)$ is an (x_h, x_l)-test. To incorporate the feature that one can sort more effectively at later times, I impose the constraint that for $\widetilde{M}$ to be admissible, $\widetilde{M}(t)$ must be more powerful than $\widetilde{M}(t')$ for $t \geq t'$. Call such an $\widetilde{M}$ a testing map. It follows from Bayes' rule that any testing-map $\widetilde{M}$ generates a unique mapping M. Likewise, any mapping M corresponds to a unique testing map $\widetilde{M}$.

A.2 Strategies

Definition A.3 *A* **low-tier firm strategy** *is a distribution* λ *over matching times* $[-T,0]$ *and a set of hiring policies* $h_t \in [M_{low}(t), M_{high}(t)]$ *for each* $t \in supp(\lambda)$ *specifying the probability the worker it hires at time* t *is high type.*[22]

Definition A.4 **High-tier firm initial beliefs** *are a mapping* $\psi : [-T,0] \to [0,1]$*, where* $\psi_t := \psi(t)$ *represents the initial probability the high-tier firm attaches to the low-tier firm hiring a worker that is high type conditional on the low-tier firm hiring at time* t*.*

Definition A.5 *A* **high-tier firm strategy** *specifies a probability* $\chi_t \in [0,1]$ *of hiring at the end of the primary market and a poaching rule* τ_t *to follow on the secondary market conditional on the low-tier firm matching at time* t*.*

Consider each subgame initiated when the low-tier firm enters at time t. Call this a **t-subgame**. A t-subgame equilibrium consists of the low-tier firm's hiring policy at time t, the high-tier firm's initial belief ψ_t, and the high-tier firm's strategy conditional on the low-tier firm hiring at time t, such that each firm is behaving optimally and beliefs are consistent. The latter means $\psi_t = h_t$. As the optimal poaching rule τ_t is a static belief-threshold rule τ^* (Proposition 3.1), and the high-tier firm's initial belief must be consistent, a t-subgame equilibrium is characterized entirely by an ordered pair (ψ_t, χ_t).

Definition A.6 *Given* t*, a* **t-subgame equilibrium (t-SE)** *is a pair* (ψ_t, χ_t) *such that:*

(i) $\psi_t \in [M_{low}(t), M_{high}(t)]$

(ii) $\chi_t \in arg\max_{0\leq x\leq 1} x \cdot \Pi_H(M_{high}(0)) + (1-x)\cdot \Gamma_H(\psi_t)$

(iii) $\psi_t \in arg\max_{M_{low}(t)\leq p\leq M_{high}(t)} \chi_t \Pi_L(p) + (1-\chi_t)\Sigma_L(p, \psi_t)$

Definition A.7 *An equilibrium is a distribution* λ *over* $[-T,0]$ *and a collection* $\{(\psi_t, \chi_t)\}_{t\in[-T,0]}$ *such that* (ψ_t, χ_t) *is a t-SE for every* $t \in [-T,0]$*, and for all* $t \in supp(\lambda)$*:*

$$t \in \arg\max_{t'\in[-T,0]} \chi_{t'}\Pi_L(\psi_{t'}) + (1-\chi_{t'})\Sigma_L(\psi_{t'}, \psi_{t'})$$

[22] Hiring a high-type worker with probability in $[M_{low}(t), M_{high}(t)]$ is possible via randomization over high and low-signal workers. The randomization weights are unimportant: conditional on the high-tier firm's strategy, the low-tier firm's payoff is linear in the probability that the worker it hires is high type.

Proof of Observation 2.1: Since N is discrete, there is a non-zero probability all individuals emit the same signal in an interview. In this situation, I assume the firm randomizes over whom it hires. Receiving an offer gives the worker information about her type. She may reject it, believing she has a better chance of receiving an offer from the high-tier firm. For large N, I will show there is no gain from such "strategic rejection".

Let (x_H^t, x_L^t) denote the binary test associated with the primary market at time t. Let $\hat{\beta}(t,N)$ be the posterior probability a worker is high type given she receives an offer at time t when N workers were available. Let $\Delta_{t,\beta} := \beta(1-x_H^t) + (1-\beta)(1-x_L^t)$ be the probability a worker fails the test at time t when she is high type with probability β.

1. $\hat{\beta}(t,N+1) = (N+1)\beta \cdot \left(x_H^t \sum_{k=0}^N \binom{N}{k} \frac{\Delta_{t,\beta}^{N-k}(1-\Delta_{t,\beta})^k}{k+1} + \frac{(1-x_H^t)\Delta_{t,\beta}^N}{N+1} \right)$
2. $\hat{\beta}$ is weakly increasing in t and N, and $\lim_{N\to\infty} \hat{\beta}(t,N) = \frac{\beta x_H^t}{\beta x_H^t + (1-\beta) x_L}$

Suppose there are $N+1$ workers, and all except worker i will accept any offer they receive. From worker i's perspective, the probability of receiving an offer from F_H at $t=0$ if she rejects one from F_L at time t is:

$$\left(1-\Delta_{t,\hat{\beta}(t,N+1)}\right) \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{\Delta_{t,\beta}^{N-k-1}(1-\Delta_{t,\beta})^k}{k+1} + \frac{\Delta_{t,\hat{\beta}(t,N+1)}\Delta_{t,\beta}^{N-1}}{N}$$

$$< \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{\Delta_{t,\beta}^{N-k-1}(1-\Delta_{t,\beta})^k}{k+1} + \frac{1}{N} = \frac{1-\Delta_{t,\beta}^N}{N(1-\Delta_{t,\beta})} + \frac{1}{N}$$

The above expression approaches 0 as N grows large. ■

B Poaching and Incentives

B.1 Optimal Poaching Rule

Lemma B.1 *Given belief process $\{p_t\}$, the optimal stopping problem* $\sup_\tau \mathbb{E}[e^{-r\tau} g(p_\tau) | p_0]$ *has a solution of the form* $\tau^* := \inf\{t \colon p_t \notin (a,b)\}$.

Proof: As $\{\pi_t\}_{t\geq 0}$ is Markov, $p_t = \frac{p_0 f_t(\pi_t|\theta=H)}{p_0 f_t(\pi_t|\theta=H) + (1-p_0) f_t(\pi_t|\theta=L)}$. The innovation theorem implies that $dp_t = \frac{\mu_H - \mu_L}{\sigma}(1-p_t)p_t d\hat{B}_t$ where $\hat{B}_t = \frac{1}{\sigma}(\pi_t - (\mu_H - \mu_L)\int_0^t p_s ds)$ is a Brownian motion with respect to the filtration $\{\mathscr{F}_t^\pi\}$. Thus, $\{p_t\}_{t\geq 0}$ has the strong markov property. Let $U(p_0) := \sup_\tau \mathbb{E}[e^{-r\tau} g(p_\tau)|p_0]$ be the value function. The continuation region is given by $C{:=}\{p : U(p) > g(p)\}$ and the stopping region by $S{:=}\{p : U(p) = g(p)\}$ (see Peskir and Shiryaev [2006]). Continuity of $\{p_t\}_{t\geq 0}$ means I can restrict attention to a *connected* subset of C around p_0. The lemma follows. ■

Proof of Proposition 3.1: I will work in the log-odds space, $Q_t := log(\frac{p_t}{1-p_t})$. By Lemma B.1, the optimal poaching rule is characterized by a continuation region around Q_0. For any $\tau := \inf\{t \geq 0 : Q_t \notin (b,B)\}$ with initial condition $Q_0 \in (b,B)$:

$$Pr(Q_\tau = B|\theta = H)\mathbb{E}[e^{-r\tau}|\theta = H, Q_\tau = B] = \frac{e^{-R_1(Q_0-b)} - e^{-R_2(Q_0-b)}}{e^{-R_1(B-b)} - e^{-R_2(B-b)}}$$

$$Pr(Q_\tau = B|\theta = L)\mathbb{E}[e^{-r\tau}|\theta = L, Q_\tau = B] = \frac{e^{-R_1(Q_0-b)} - e^{-R_2(Q_0-b)}}{e^{-R_1(B-b)} - e^{-R_2(B-b)}}$$

There is no cost of observation, so there is no rejection threshold. Thus, the optimal poaching rule is of the form $\tau := \inf\{t \geq 0 : Q_t \geq B\}$, for some $B > 0$. Taking limits as $b \to -\infty$ shows that the payoff from such a poaching rule τ with initial condition Q_0 is:

$$Z_H^H \frac{e^{Q_0}}{1+e^{Q_0}} e^{R_1(B-Q_0)} + \frac{Z_L^H}{1+e^{Q_0}} e^{-R_2(B-Q_0)}$$

The maximum is attained in the interior. The first-order condition yields $B^* = -log\left(\frac{Z_H^H R_1}{Z_L^H R_2}\right)$.

B.2 Payoff Functions when High-Tier Firm is on the Secondary Market

Given the optimal poaching rule τ^*, the payoff equations on page 9 can be expressed more explicitly. The high-tier firm's payoff when operating on the secondary market with initial belief $p := \frac{e^Q}{1+e^Q}$ is:

$$\Gamma_H(Q;\alpha) = \begin{cases} \frac{e^Q}{1+e^Q} \cdot Z_H^H e^{R_1(B^*-Q)} + \frac{1}{1+e^Q} \cdot Z_L^H e^{-R_2(B^*-Q)} & \text{if } Q < B^* \\ \frac{e^Q}{1+e^Q} \cdot Z_H^H + \frac{1}{1+e^Q} \cdot Z_L^H & \text{if } Q \geq B^* \end{cases} \quad (1)$$

F_L's payoff from employing a worker that is high type with probability p when F_H operates on the secondary market with initial belief p is:

$$\Gamma_L(Q;\alpha) {:=} \begin{cases} \frac{e^Q}{1+e^Q} \cdot Z_H^L (1 - e^{R_1(B^*-Q)}) + \frac{1}{1+e^Q} \cdot Z_L^L (1 - e^{-R_2(B^*-Q)}) & \text{if } Q < B^* \\ 0 & \text{if } Q \geq B^* \end{cases} \quad (2)$$

Lemma B.2 *Fix α. $\Gamma_H(Q;\alpha)$ is strictly increasing in Q.*

Proof: Given (1), the claim is trivially true when $Q \geq B^*$. For $Q < B^*$, consider $Q' \in (Q, B^*)$. By optimality of B^*, the payoff under threshold B^* must be strictly greater than using a threshold of $B^* + Q' - Q > B^*$:

$$\frac{e^{Q'}}{1+e^{Q'}} \cdot Z_H^H e^{R_1(B^*-Q')} + \frac{1}{1+e^{Q'}} \cdot Z_L^H e^{-R_2(B^*-Q')} > \frac{e^{Q'}}{1+e^{Q'}} \cdot Z_H^H e^{R_1(B^*-Q)} + \frac{1}{1+e^{Q'}} \cdot Z_L^H e^{-R_2(B^*-Q)}$$

$$> \frac{e^{Q}}{1+e^{Q}} \cdot Z_H^H e^{R_1(B^*-Q)} + \frac{1}{1+e^{Q}} \cdot Z_L^H e^{-R_2(B^*-Q)} = \Gamma_H(Q;\alpha) \qquad \blacksquare$$

Lemma B.3 *If $Q < B^*(\hat{\alpha})$ for some $\hat{\alpha}$, $\Gamma_H(Q;\alpha)$ is strictly increasing in α for $\alpha \geq \hat{\alpha}$.*

Proof: From (1): $\Gamma_H(Q;\hat{\alpha}) = e^{R_1(\hat{\alpha})\cdot(B^*(\hat{\alpha})-Q)} \cdot \left(\frac{e^Q}{1+e^Q} \cdot Z_H^H + \frac{1}{1+e^Q} \cdot Z_L^H e^{-(B^*(\hat{\alpha})-Q)}\right)$. As $\Gamma_H(Q;\hat{\alpha}) > 0$, it must mean $\frac{e^Q}{1+e^Q} \cdot Z_H^H + \frac{1}{1+e^Q} \cdot Z_L^H e^{-(B^*(\alpha)-Q)} > 0$ for all $\alpha \geq \hat{\alpha}$. Now, consider α' and α'' such that $\alpha' > \alpha'' \geq \hat{\alpha}$. Using the fact R_1 is increasing in α:

$$\Gamma_H(Q;\alpha'') < e^{R_1(\alpha')\cdot(B^*(\alpha'')-Q)} \cdot \left(\frac{e^Q}{1+e^Q} \cdot Z_H^H + \frac{1}{1+e^Q} \cdot Z_L^H e^{-(B^*(\alpha'')-Q)}\right)$$

$$< e^{R_1(\alpha')\cdot(B^*(\alpha')-Q)} \cdot \left(\frac{e^Q}{1+e^Q} \cdot Z_H^H + \frac{1}{1+e^Q} \cdot Z_L^H e^{-(B^*(\alpha')-Q)}\right) = \Gamma_H(Q;\alpha')$$

The last inequality follows from the optimality of $B^*(\alpha')$. $\blacksquare$

Proof of Proposition 3.2: Taking the expression for Γ_L in (2) and expanding yields:

$$\Gamma_L(Q;\alpha) = \frac{1}{1+e^Q}(e^Q Z_H^L + Z_L^L) - \frac{e^{-R_2(B^*-Q)}}{1+e^Q}\left[Z_H^L e^{B^*} + Z_L^L\right]$$

Substituting in the formula for B^* and differentiating with respect to Q:

$$\frac{\partial \Gamma_L}{\partial Q} = \frac{e^Q(Z_H^L - Z_L^L)}{(1+e^Q)^2} - e^{R_2 Q}\left(\frac{R_2 + R_2 e^Q - e^Q}{(1+e^Q)^2}\right)\left(\frac{Z_H^H}{Z_L^H} \cdot \frac{R_1}{R_2}\right)^{R_2-1}\left(Z_H^L + Z_L^L \frac{R_1}{R_2}\frac{Z_H^H}{Z_L^H}\right)$$

Therefore, $\Gamma_L(Q;\alpha)$ is decreasing in Q whenever:

$$e^Q(Z_H^L - Z_L^L) - e^{R_2 Q}\left(R_2 - R_1 e^Q\right)\left(\frac{Z_H^H}{Z_L^H} \cdot \frac{R_1}{R_2}\right)^{R_2-1}\left(Z_H^L + Z_L^L \frac{R_1}{R_2}\frac{Z_H^H}{Z_L^H}\right) < 0$$

$$\iff e^{(R_2-1)Q}\left(R_2 - R_1 e^{Q}\right)\left(\frac{Z_H^H}{Z_L^H}\cdot\frac{R_1}{R_2}\right)^{R_2-1}\left(\frac{Z_H^L}{Z_L^L}+\frac{R_1}{R_2}\frac{Z_H^H}{Z_L^H}\right) > \frac{Z_H^L}{Z_L^L}-1$$

The left-hand side is strictly increasing in Q. The inequality is violated for Q sufficiently low and satisfied for Q sufficiently close to B^*. Hence, Γ_L is single-peaked with optimum at a unique $Q^* < B^*$. ■

Lemma B.4 $\lim_{\alpha\to 0} Q^* = log\left(-\frac{Z_L^H}{Z_H^H}\right)$ *and* $\lim_{\alpha\to\infty} Q^* = -\infty$.

Proof: Q^* is continuous in $\alpha \in (0,\infty)$ as it is the solution to the equation $\frac{\partial \Gamma_L}{\partial Q} = 0$. For $Q < log\left(-\frac{Z_L^H}{Z_H^H}\right)$, $\lim_{\alpha\to 0}\Gamma_L(Q;\alpha) = \frac{e^Q}{1+e^Q}Z_H^L + \frac{1}{1+e^Q}Z_L^L$.[23] This is positive and strictly increasing in Q. For $Q > log\left(-\frac{Z_L^H}{Z_H^H}\right)$, $\lim_{\alpha\to 0}\Gamma_L(Q;\alpha) = 0$. The first part of the lemma follows. From Lemma B.9, $Q^* < Q_{ind}$ for $\alpha \geq \alpha_{opaque}$. Corollary B.8 implies $\lim_{\alpha\to\infty} p_{ind} = 0$, and so $\lim_{\alpha\to\infty} Q_{ind} = -\infty$. The second part of the lemma follows. ■

B.3 Indifference Beliefs: Existence

Define p_{ind} and $\bar{p}$ to be these beliefs, respectively:

$$p_{ind} := \min\left\{p : \Sigma_L(p',p) = \Sigma_L(p'',p) \text{ for all } p',p'' \in [0,1]\right\} \tag{3}$$

$$\bar{p} := \min\left\{p : \Gamma_H(\bar{p}) \geq \Pi_H(M_{high}(0))\right\} \tag{4}$$

In log-odds space, let these quantities be denoted by Q_{ind} and $\bar{Q}$, respectively. The indifference beliefs are endogenous, depending crucially on α, the match quality values, and sorting ability at the end of the primary market ($M_{high}(0)$). For exposition, I suppress dependence on these quantities unless necessary.

Because $\Gamma_H(\cdot)$ is continuous and strictly increasing in Q (see (1) and Lemma B.2, respectively), the intermediate value theorem guarantees that $\bar{p}$ exists and is the unique solution to $\Gamma_H(\bar{p}) = \Pi_H(M_{high}(0))$.

Lemma B.5 *$\bar{p}$ is strictly decreasing in α for $\alpha \geq \alpha_{opaque}$, and* $\lim_{\alpha\to\infty}\bar{p} = \frac{\Pi_H(M_{high}(0))}{Z_H^H}$.

Proof: Recognize that $\bar{p} \leq M_{high}(0)$.[24] Since B^* is strictly increasing in α and $B^*(\alpha_{opaque}) = log\left(\frac{M_{high}(0)}{1-M_{high}(0)}\right)$, it must be that $\bar{Q}(\alpha) < B^*(\alpha)$ for all $\alpha > \alpha_{opaque}$. Given

[23] When $\alpha \to 0$, $B^* \to log\left(-\frac{Z_L^H}{Z_H^H}\right)$, $R_1 \to -\infty$, and $R_2 \to \infty$.

[24] As $\Pi_H(p)$ is also the payoff from F_H poaching immediately, $\Gamma_H(p) \geq \Pi_H(p)$.

any α and α' such that $\alpha' > \alpha > \alpha_{opaque}$:

$$\underbrace{\Gamma_H(\bar{Q}(\alpha);\alpha') > \Gamma_H(\bar{Q}(\alpha),\alpha)}_{\text{Lemma B.3}} \underbrace{= \Pi_H(M_{high}(0))}_{\text{Definition of } \bar{Q}(\alpha)}$$

As $\Gamma_H(\cdot)$ is increasing in Q, it must be that $\bar{Q}(\alpha') < \bar{Q}(\alpha)$. This proves the first part of the lemma. The last part follows from the fact that $\lim_{\alpha\to\infty}\Gamma_H(p) = pZ_H^H$ for any p. ■

To show p_{ind} is well-defined, recognize that if F_H's initial belief equals the threshold belief, F_L receives a payoff of 0. Hence, the set in (3) is non-empty, $Q_{ind}(\alpha)$ exists, and $Q_{ind}(\alpha) \leq B^*(\alpha)$ for all α.

Suppose F_H holds an initial belief $Q < B^*$ about the worker hired by F_L. F_L strictly prefers a high (low) type worker if and only if $\frac{Z_H^L}{Z_L^L} > (<) \frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}}$. F_L is indifferent between worker types only when $\frac{Z_H^L}{Z_L^L} = \frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}}$. I will show that Q_{ind} is pinned down by the solution to this equation. First, I need the following lemma:

Lemma B.6 *For any α, the function $\frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}}$ is strictly increasing in Q for $Q < B^*$.*

Proof: It suffices to show $\frac{1-e^{-ax}}{1-e^{-(a-1)x}}$ is strictly decreasing in x for $x > 0$, where $a > 1$ is a constant. It is strictly decreasing if its derivative is strictly less than 0:

$$\iff f(x) := (a-1)\cdot(1-e^{ax}) - a\cdot(1-e^{(a-1)x}) < 0$$

Now, $f'(x) = a(a-1)e^{(a-1)x}(1-e^x) < 0$ for all $x > 0$. Moreover, $f'(0) = 0$ and $f(0) = 0$. Therefore, $f(x) < 0$ for all $x > 0$. ■

With Lemma B.6 in hand, I can completely characterize Q_{ind}.

Lemma B.7 *If $\frac{Z_H^L}{Z_L^L} \geq -\frac{R_2(\alpha)}{R_1(\alpha)}$, then $Q_{ind}(\alpha) = B^*$. Otherwise, $Q_{ind}(\alpha) \in (-\infty, B^*(\alpha))$ and is the unique solution to $\frac{Z_H^L}{Z_L^L} = \frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}}$.*

Proof: By Lemma B.6, the supremum of $\frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}}$ is $\lim_{Q\to B^*}\frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}} = -\frac{R_2(\alpha)}{R_1(\alpha)}$. If $\frac{Z_H^L}{Z_L^L} \geq -\frac{R_2(\alpha)}{R_1(\alpha)}$, then $\frac{Z_H^L}{Z_L^L} > \frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}}$ for all $Q < B^*$. Thus, $Q_{ind} = B^*$. If $\frac{Z_H^L}{Z_L^L} < -\frac{R_2(\alpha)}{R_1(\alpha)}$, Lemma B.6 implies there is a unique $Q' < B^*$ such that $\frac{Z_H^L}{Z_L^L} = \frac{1-e^{-R_2(B^*-Q')}}{1-e^{R_1(B^*-Q')}}$.[25] ■

[25] $\frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}}$ is strictly increasing in Q and $\lim_{Q\to-\infty}\frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}} = 1 < \frac{Z_H^L}{Z_L^L} < \lim_{Q\to B^*}\frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}}$.

COROLLARY B.8 $\lim_{\alpha\to\infty} p_{ind} = 0$

Proof: $-\frac{R_2(\alpha)}{R_1(\alpha)}$ is strictly increasing in α and $\lim_{\alpha\to\infty} -\frac{R_2(\alpha)}{R_1(\alpha)} = \infty$. Therefore, there exists α_1 such that for $\alpha > \alpha_1$, $\frac{Z_H^L}{Z_L^L} < -\frac{R_2(\alpha)}{R_1(\alpha)}$ and $Q_{ind}(\alpha)$ satisfies $\frac{Z_H^L}{Z_L^L} = \frac{1-e^{-R_2(B^*-Q_{ind})}}{1-e^{R_1(B^*-Q_{ind})}}$.

For any $Q \in (-\infty,\infty)$, $\lim_{\alpha\to\infty} \frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}} = \infty$. Thus, for a given Q, there exists α_2 such that for $\alpha > \max\{\alpha_1,\alpha_2\}$, $\frac{Z_H^L}{Z_L^L} < \frac{1-e^{-R_2(B^*-Q)}}{1-e^{R_1(B^*-Q)}}$. By Lemmas B.6 and B.7, $Q_{ind} < Q$ for all $\alpha > \max\{\alpha_1,\alpha_2\}$. As Q was arbitrary, the lemma follows. ■

B.4 Indifference Beliefs: Ordering

Lemma B.9 *If $\alpha \geq \alpha_{opaque}$, then $p_{ind} > p^*$*

Proof: If $p_{ind} = B^*$, p^* is trivially less than p_{ind}. Therefore, consider when $p_{ind} < B^*$. Suppose for sake of contradiction that $p_{ind} \leq p^*$. Then:

$$\Gamma_L(p^*) = \Sigma_L(p^*,p^*) \leq \Sigma_L(0,p^*) < \Sigma(0,0) = \Gamma_L(0)$$

This contradicts the optimality of p^*. ■

Lemma B.10 *If $\frac{Z_H^L}{Z_L^L} < -\frac{Z_H^H}{Z_L^H} \cdot \frac{M_{high}(0)}{1-M_{high}(0)}$, then $p_{ind} < \bar{p}$ for all $\alpha > \alpha_{opaque}$.*

Proof: First recognize that $-\frac{Z_H^H}{Z_L^H} \cdot \frac{M_{high}(0)}{1-M_{high}(0)} = -\frac{R_2(\alpha_{opaque})}{R_1(\alpha_{opaque})}$. Since $-\frac{R_2(\alpha)}{R_1(\alpha)}$ is increasing in α, $\frac{Z_H^L}{Z_L^L} < -\frac{R_2(\alpha)}{R_1(\alpha)}$ for all $\alpha \geq \alpha_{opaque}$. Thus, if $p_{ind} \geq \bar{p}$ for some secondary market transparency level $\alpha > \alpha_{opaque}$, Lemmas B.6 and B.7 imply that:

$$1-e^{-R_2(B^*-\bar{Q})} < \frac{Z_H^L}{Z_L^L} \cdot (1-e^{R_1(B^*-\bar{Q})})$$

$$\implies 1-e^{-R_2(B^*-\bar{Q})} < -\frac{Z_H^H}{Z_L^H} \cdot \frac{M_{high}(0)}{1-M_{high}(0)} \cdot (1-e^{R_1(B^*-\bar{Q})})$$

$$\implies \underbrace{M_{high}(0)Z_H^H + (1-M_{high}(0))Z_L^H}_{\Pi_H(M_{high}(0))} > M_{high}(0)Z_H^H e^{R_1(B^*-\bar{Q})} + (1-M_{high}(0))Z_L^H e^{-R_2(B^*-\bar{Q})}$$

$$> \underbrace{\bar{p}Z_H^H e^{R_1(B^*-\bar{Q})} + (1-\bar{p})Z_L^H e^{-R_2(B^*-\bar{Q})}}_{\Gamma_H(\bar{p})}, \text{ contradicting the definition of } \bar{p}. \quad ■$$

B.5 Proof of Proposition 3.3

When constraining F_H to operate on the secondary market, I adjust equilibrium definitions A.6 and A.7 by setting $\chi_t = 0$ and eliminating F_H's decision problem from the equilibrium conditions. A t-SE in this constrained setting is characterized by the value $\psi_t \in [M_{low}(t), M_{high}(t)]$ such that:

$$\psi_t \in arg \max_{M_{low}(t) \leq p \leq M_{high}(t)} (1 - \chi_t)\Sigma_L(p, \psi_t) \tag{5}$$

By Lemma B.9, $p_{ind} > p^* = M_{high}(t^*) \geq \beta$, and so $p_{ind} > M_{low}(t)$ for all t. Therefore, the unique solution to (5) is $\psi_t := \min\{p_{ind}, M_{high}(t)\}$. Hence, if F_L matches at time t, its payoff is $\Sigma_L(\psi_t, \psi_t) = \Gamma_L(\psi_t)$.

As $p^* \in [\beta, M_{high}(0)]$, there exists t^* such that $M_{high}(t^*) = p^*$. Matching at time t^* gives F_L a payoff of $\Gamma_L(\min\{p_{ind}, M_{high}(t^*)\}) = \Gamma_L(M_{high}(t^*)) = \Gamma_L(p^*)$. Since $\Gamma_L(p^*) > \Gamma_L(\psi_t)$ for all $t \neq t^*$ by Proposition 3.2, it follows that the equilibrium outcome is F_L matching at time t^*. ■

C t-Subgame Equilibria

A t-SE (ψ_t, χ_t) depends entirely on the value of $\overline{p}$ and p_{ind}, and their ordering within the set $[M_{low}(t), M_{high}(t)]$. The following lemmas show that for each t, a t-SE exists, and the probability the hired worker is high type is unique. The strategy of the high-tier firm in a t-SE is unique except for some edge cases that occur when $\bar{p} \in \{M_{low}(t), M_{high}(t)\}$.

Lemma C.1 *Suppose $\bar{p} \in [M_{low}(t), M_{high}(t)]$. In a t-SE, F_L can not match with a worker that is high type with probability $p \neq \min\{\max\{\bar{p}, p_{ind}\}, M_{high}(t)\}$.*

Proof: Suppose there is a t-SE where F_L matches with a worker that is high type with probability $p \neq \min\{\max\{\bar{p}, p_{ind}\}, M_{high}(t)\}$. Let χ_t denote F_H's strategy in the t-SE. If $p < \bar{p}$, then $\chi_t = 1$. F_L can increase its payoff by hiring a worker that is high type with probability $M_{high}(t) \geq \bar{p} > p$.

If, $p > \bar{p}$, then $\chi_t = 0$. F_L's payoff is $\Sigma_L(p, p)$. Given $p \neq \min\{\max\{\bar{p}, p_{ind}\}, M_{high}(t)\}$, it must also be true that $p \neq p_{ind}$. Now, if $p < p_{ind}$, either F_L has a profitable deviation by hiring a worker that is high type with probability $p' \in (p, M_{high}(t))$ or such a deviation is infeasible. The latter only occurs when $p = M_{high}(t)$, contradicting $p \neq \min\{\max\{\bar{p}, p_{ind}\}, M_{high}(t)\}$. If $p > p_{ind}$, F_L can hire a worker that is high type with probability $M_{low}(t) \leq \bar{p} < p$, receiving a payoff of $\Sigma_L(M_{low}(t), p) > \Sigma_L(p, p)$.

If $p = \bar{p}$, then $p \neq \min\{\max\{\bar{p}, p_{ind}\}, M_{high}(t)\}$ means $\bar{p} < \min\{p_{ind}, M_{high}(t)\}$. Deviating to hiring a worker that is high type with probability $M_{high}(t)$ yields a payoff of $\chi_t \Pi_L(M_{high}(t)) + (1-\chi_t)\Sigma_L(M_{high}(t), p) > \chi_t \Pi_L(\bar{p}) + (1-\chi_t)\Sigma_L(p, p)$. ■

Lemma C.2 *Suppose $\bar{p} \notin [M_{low}(t), M_{high}(t)]$. There is a unique t-SE (ψ_t, χ_t).*

(i) If $\bar{p} < M_{low}(t)$, then $\psi_t := \min\{\max\{M_{low}(t), p_{ind}\}, M_{high}(t)\}$ and $\chi_t := 0$.

(ii) If $\bar{p} > M_{high}(t)$, then $\psi_t := M_{high}(t)$ and $\chi_t := 1$.

Proof: **(i) :** If a t-SE exists, it must be that $\chi_t = 0$ as $\bar{p} < M_{low}(t)$. Suppose F_L matches with a worker that is high type with probability p in a t-SE. Its payoff is $\Sigma_L(p, p)$. By definition of p_{ind}, $\Sigma_L(p', p)$ is strictly decreasing (increasing) in p' when $p > (<) p_{ind}$. Therefore, if $p_{ind} < M_{low}(t)$, F_L does not have a profitable deviation if and only if $p = M_{low}(t)$. If $p_{ind} > M_{high}(t)$, F_L does not have a profitable deviation if and only if $p = M_{high}(t)$. Finally, if $p_{ind} \in [M_{low}(t), M_{high}(t)]$, F_L does not have a profitable deviation if and only if $p = p_{ind}$.

(ii) : Since $\bar{p} > M_{high}(t)$, F_H hires at the end of the primary market. It is optimal for F_L to hire a worker with the highest probability of being high type. ■

Lemma C.3 *Suppose $\bar{p} \in [M_{low}(t), M_{high}(t)]$ and $\bar{p} \leq p_{ind}$. In equilibrium, F_L matches with a worker that is high type with probability $\psi_t := \min\{p_{ind}, M_{high}(t)\}$.*

(i) If $\bar{p} < M_{high}(t)$, the t-SE is unique with $\chi_t := 0$.

(ii) If $\bar{p} = M_{high}(t)$, there is a t-SE for every $\chi_t \in [0, 1]$.

Proof: By Lemma C.1, if an equilibrium exists, $\psi_t := \min\{p_{ind}, M_{high}(t)\}$.
(i): ψ_t paired with $\chi_t = 0$ is a t-SE: by deviating, F_L can only match with a worker that is high type with probability $p \leq M_{high}(t)$, yielding a payoff of $\Sigma_L(p, \psi_t) \leq \Sigma_L(\psi_t, \psi_t)$.[26]

[26] If $\psi_t = p_{ind}$, then $\Sigma_L(p, \psi_t) = \Sigma_L(\psi_t, \psi_t)$. If $\psi_t = M_{high}(t)$, then $p_{ind} \geq M_{high}(t)$, which in turn implies $\Sigma_L(p, M_{high}(t)) < \Sigma_L(M_{high}(t), M_{high}(t))$ for all $p < M_{high}(t)$.

Suppose a t-SE exists with $\chi_t > 0$. As $\chi_t > 0$, it must be that F_L matches with a worker that is high type with probability $p \leq \bar{p}$. Therefore, $\bar{p} = p_{ind}$, and so $\psi_t = \bar{p}$. F_L's payoff is $\chi_t \Pi_L(\bar{p}) + (1-\chi_t)\Sigma_L(\bar{p},\bar{p})$. Since $\bar{p} < M_{high}(t)$, deviating to a worker that is high type with probability $p \in (\bar{p}, M_{high}(t))$ is feasible and improves its payoff: $\Pi_L(p)$ is strictly increasing in p and $\Sigma_L(p,\bar{p})$ is constant in p given $\bar{p} = p_{ind}$.

(ii): For $\chi_t \in [0,1]$, F_L's payoff is $\chi_t \Pi_L(\bar{p}) + (1-\chi_t)\Sigma_L(\bar{p},\bar{p})$. The only feasible deviation is to a worker that is high type with probability $p < \bar{p} = M_{high}(t)$. Since $\Pi_L(p)$ is strictly increasing in p and $\Sigma_L(p,\bar{p})$ is weakly increasing in p given $\bar{p} \leq p_{ind}$, such a deviation reduces F_L's payoff. ■

The final possible t-subgame is when $\bar{p} \in [M_{low}(t), M_{high}(t)]$ and $\bar{p} > p_{ind}$. By Lemma C.1, if a t-SE exists, F_L must match with a worker that is high type with probability $\psi_t := \bar{p}$. To pin down F_H's strategy in this t-SE, it will be instructive to consider F_L's payoff when F_H hires at the end of the primary market with probability x:

$$x \cdot \Pi_L(p) + (1-x) \cdot \Sigma_L(p,\bar{p}) \tag{6}$$

The function in (6) is linear in p. Now, when F_H operates on the primary market, F_L prefers high-type workers, but when F_H operates on the secondary market, F_L prefers low-type workers. The latter follows from $\psi_t = \bar{p} > p_{ind}$. Therefore, there exists a cutoff $\bar{x} \in (0,1)$ such that for $x > \bar{x}$, the function in (6) is strictly increasing in p, for $x < \bar{x}$, it is strictly decreasing, and for $x = \bar{x}$, it is constant.

As a result, the following lemma is immediate.

Lemma C.4 *Suppose $\bar{p} \in [M_{low}(t), M_{high}(t)]$ and $\bar{p} > p_{ind}$. In equilibrium, F_L matches with a worker that is high type with probability $\psi_t := \bar{p}$.*

(i) If $\bar{p} \in (M_{low}(t), M_{high}(t))$, the t-SE is unique with $\chi_t := \bar{x}$.

(ii) If $\bar{p} = M_{high}(t)$, there is a t-SE for every $\chi_t \in [\bar{x}, 1]$.

(iii) If $\bar{p} = M_{low}(t)$, there is a t-SE for every $\chi_t \in [0, \bar{x}]$.

D General Characterization of Equilibria

I characterize the equilibrium for $\alpha > \alpha_{opaque}$. Note $\alpha > \alpha_{opaque}$ implies $\bar{p} < M_{high}(0)$. I split the analysis into the following cases: $\bar{p} \geq \beta$, $\bar{p} < \beta \leq p_{ind}$, and $\max\{\bar{p}, p_{ind}\} < \beta$. Recall that verifying whether a distribution over matching times $[-T, 0]$ can be sustained in equilibrium amounts to showing there exists a collection of t-SE (one for each $t \in [-T, 0]$) such that the times in the support of the distribution maximize the low-tier firm's payoff given the collection of t-SE selected.

Additional Notation: Let $\bar{t}$ and $\underline{t}$ be the times such that $M_{high}(\bar{t}) = \bar{p}$ and $M_{low}(\underline{t}) = \bar{p}$. The former exists when $\bar{p} \in [\beta, M_{high}(0)]$. The latter exists when $\bar{p} \in [M_{low}(0), \beta]$. As $\alpha > \alpha_{opaque}$, $\bar{t}$ is strictly less than 0 when it exists. Define $t^* := \arg\max_{t \in [-T,0]} \Gamma_L(M_{high}(t))$.

OBSERVATION D.1 *In equilibrium, F_L never matches with a worker that is high type with probability $p < \bar{p}$.*

Proof: By Lemmas C.2-C.4, if F_L matches at time t where $\bar{p} \leq M_{high}(t)$, then F_L matches with a worker that is high type with probability $p \geq \bar{p}$. By Lemma C.2, matching at t with $M_{high}(t) < \bar{p}$ results in a payoff of $\Pi_L(M_{high}(t))$. Such a t existing means $\bar{t}$ exists. Matching at $t' \in (t, \bar{t})$ leads to a payoff of $\Pi_L(M_{high}(t')) > \Pi_L(M_{high}(t))$. ■

Case #1: $\bar{p} \geq \beta$

As $\bar{p} \geq \beta$, time $\bar{t}$ exists. I split into two sub-cases, $\bar{p} > \beta$ and $\bar{p} = \beta$, due to the potential knife-edge situations that arise in the latter. However, across all sub-cases, the equilibrium involves F_L hiring at either time $\bar{t}$ or t^*.

Lemma D.2 *Suppose $\bar{p} > \beta$:*

(i) If $\Pi_L(\bar{p}) > \Gamma_L(M_{high}(t^*))$, the equilibrium is unique. F_L matches with a high-signal worker at $\bar{t}$. F_H matches at the end of the primary market.

(ii) If $\Gamma_L(M_{high}(t^*)) > \Pi_L(\bar{p})$, the equilibrium is unique. F_L matches with a high-signal worker at t^*. F_H operates on the secondary market.

(iii) If $\Gamma_L(p^*) = \Pi_L(\bar{p})$, any mixture over the strategies described in (i) and (ii) constitutes an equilibrium.

Proof: For $t < \bar{t}$, $\bar{p}$ is strictly less than $M_{high}(t)$, and for $t > \bar{t}$, $\bar{p} \in (M_{low}(t), M_{high}(t))$. Lemmas C.2-C.4 imply there is a unique t-SE for $t \neq \bar{t}$. By Observation D.1, there is no equilibrium where F_L matches at time $t < \bar{t}$ with positive probability.

(i): Lemmas C.3-C.4 imply that the payoff from matching at $t > \bar{t}$ is either $\bar{x}\Pi_L(\bar{p}) + (1-\bar{x})\Sigma_L(\bar{p}, \bar{p})$ for some $\bar{x} < 1$ or $\Gamma_L(\min\{p_{ind}, M_{high}(t)\})$. Either way, the payoff is below $\Pi_L(\bar{p} - \varepsilon)$ for sufficiently small $\varepsilon > 0$. Continuity implies existence of a $t' \in (-T, \bar{t})$ with $M_{high}(t') > \bar{p} - \varepsilon$. By Lemma C.2, the payoff from matching at t' is $\Pi_L(M_{high}(t')) > \Pi_L(\bar{p} - \varepsilon)$. Thus, if an equilibrium exists, F_L matches at time $\bar{t}$.

In all $\bar{t}$-SE, F_L matches with a worker that is high type with probability $\bar{p}$ (Lemmas C.3-C.4). In addition, there is some $\bar{x}$ such that F_H hiring at the end of the primary market with any probability $\chi_{\bar{t}} \in [\bar{x}, 1]$ constitutes a $\bar{t}$-SE. For any $\bar{t}$-SE with $\chi_{\bar{t}} < 1$, there is a $t' \in (-T, \bar{t})$ sufficiently close to $\bar{t}$ such that F_L matching at t' yields a payoff of $\Pi_L(M_{high}(t')) > \chi_{\bar{t}}\Pi_L(\bar{p}) + (1-\chi_{\bar{t}})\Sigma_L(\bar{p}, \bar{p})$. Therefore, the only selection of $\bar{t}$-SE that always exists and can be part of an equilibrium of the whole game is when $\chi_{\bar{t}} = 1$.

(ii): If $\Gamma_L(M_{high}(t^*)) > \Pi_L(\bar{p})$, it must be that $M_{high}(t^*) > \bar{p}$. In turn, $p_{ind} > \bar{p}$.[27] By Lemma C.3, if F_L matches at time $t > \bar{t}$, its payoff is $\Gamma_L(\min\{p_{ind}, M_{high}(t)\})$. Thus, the equilibrium must be F_L matching at time t^*.

(iii): If $\Gamma_L(M_{high}(t^*)) = \Pi_L(\bar{p})$, then $M_{high}(t^*) > \bar{p} > \beta$. As in (ii), there is a unique t-SE for each t. As F_L is indifferent between matching at $\bar{t}$ and t^*, the claim follows. ■

Lemma D.3 *Suppose $\bar{p} = \beta$. Multiple payoff-distinct equilibria exist:*

(i) If $\bar{p} \leq p_{ind}$, there is an equilibrium corresponding to any distribution over matching times $\{-T, t^*\}$.

(ii) If $\bar{p} > p_{ind}$, any distribution over matching times $[-T, 0]$ can be sustained in equilibrium.

Proof: Notice $\bar{t} = -T$ and $\bar{p} = M_{high}(-T) = M_{low}(-T)$. Thus, there is a $-T$-SE for every probability $\chi_{-T} \in [0, 1]$ of F_H matching at the end of the primary market. By

[27] $\Gamma_L(p^*) \geq \Gamma_L(M_{high}(t^*)) > \Pi_L(\bar{p})$, which means $p^* > \bar{p}$. As $p_{ind} > p^*$, it follows that $p_{ind} > \bar{p}$.

Lemmas C.3-C.4, for each $t > -T$, the t-SE is unique. Moreover, F_L either receives a payoff of $\Gamma_L(\min\{p_{ind}, M_{high}(t)\})$ in each t-SE (if $\bar{p} \leq p_{ind}$) <u>or</u> it receives a payoff of $\bar{x}\Pi_L(\beta) + (1-\bar{x})\Sigma_L(\beta,\beta)$ for some $\bar{x} \in (0,1)$ in each t-SE (if $\bar{p} > p_{ind}$). Let V denote the supremum of its t-SE payoffs over all $t > -T$.

As each t-SE is unique for $t > -T$, the selection (ψ_{-T}, χ_{-T}) of $-T$-SE determines the equilibrium. Define $\bar{\chi} := \sup\{\chi \in [0,1] : \chi\Pi_L(\beta) + (1-\chi)\Sigma_L(\beta,\beta) \leq V\}$. A different equilibrium arises depending on the relation between χ_{-T} and $\bar{\chi}$ in the selection (ψ_{-T}, χ_{-T}) of $-T$-SE. Again, uniqueness of the t-SE for each $t > -T$ means it suffices to describe F_L's on-path behavior and the $-T$-SE selection to describe an equilibrium.

(i): $V = \sup_{t\in(-T,0]} \Gamma_L(\min\{p_{ind}, M_{high}(t)\})$. By Proposition 3.2, V is attained at some $t \in [-T,0]$. Hence, $\Sigma_L(\beta,\beta) = \Gamma_L(\beta) \leq V$, and $\bar{\chi}$ is well-defined. The equilibria are:

1. F_L matches at $-T$. F_H hires at the end of the primary market with any probability $\chi_{-T} > \bar{\chi}$.

2. F_L mixes over $-T$ and t^*. If F_L matches at $-T$, F_H hires at the end of the primary market with probability $\chi_{-T} = \bar{\chi}$.

3. F_L matches at time t^*. If F_L were to match at $-T$, F_H hires at the end of the primary market with any probability $\chi_{-T} < \bar{\chi}$.

(ii): $V = \bar{x}\Pi_L(\beta) + (1-\bar{x})\Sigma_L(\beta,\beta)$ for some $\bar{x} \in (0,1)$. V is attained when matching at any $t \in (-T,0]$. Clearly, $\bar{\chi} = \bar{x}$. The equilibria are:

1. F_L matches at $-T$. F_H hires at the end of the primary market with any probability $\chi_{-T} > \bar{x}$.

2. F_L mixes over $[-T,0]$. If F_L matches at $-T$, F_H hires at the end of the primary market with probability $\bar{x}$.

3. F_L mixes over $(-T,0]$. If F_L were to match at $-T$, F_H hires at the end of the primary market with any probability $\chi_{-T} < \bar{x}$. ∎

Case #2: $\bar{p} < \beta \leq p_{ind}$

Lemma D.4 *Suppose $\bar{p} < \beta \leq p_{ind}$:*

(i) If $p_{ind} > \beta$, the equilibrium is unique. F_L matches at t^* with a worker that is high type with probability $M_{high}(t^*)$. F_H operates on the secondary market.

(ii) If $p_{ind} = \beta$, there's an equilibrium for any distribution λ over $[-T, 0]$. For $t \in supp(\lambda)$, F_L matches with a worker that is high type with probability p_{ind}.

Proof: **(i):** For any t, either $\bar{p} < M_{low}(t)$, $\bar{p} = M_{low}(t)$, or $\bar{p} \in (M_{low}(t), M_{high}(t))$. By Lemmas C.2-C.4, there is a unique t-SE for each t, and F_L's payoff is $\Gamma_L(\min\{p_{ind}, M_{high}(t)\})$ in each t-SE. As $p^* < p_{ind}$ and $\Gamma_L(\cdot)$ is single-peaked, $\Gamma_L(M_{high}(t^*)) > \Gamma_L(\min\{p_{ind}, M_{high}(t)\})$ for $t \neq t^*$. Therefore, the equilibrium must be matching at time t^*.

(ii): $p_{ind} = \beta$ implies $p_{ind} \in [M_{low}(t), M_{high}(t)]$ for all $t \in [-T, 0]$. By Lemmas C.2-C.4, there is a unique t-SE for each t. In any t-SE, F_L matches with a worker that is high type with probability p_{ind}. F_H always operates on the secondary market. ∎

Case #3: $\max\{\bar{p}, p_{ind}\} < \beta$

Lemma D.5 *Let $t_\alpha := \sup\{t : \max\{\bar{p}, p_{ind}\} \notin [M_{low}(t), M_{high}(t)]\}$. For any distribution λ over $[t_\alpha, 0]$, there is an equilibrium where F_L matches according to λ.*

Proof: First, recognize there can not be an equilibrium where F_L matches at $t < t_\alpha$ with positive probability. If such an equilibrium existed, and F_L matched at some time $t < t_\alpha$, it must hire a worker that is high type with probability $M_{low}(t)$. For any $t' \in (t, t_\alpha)$, there is a unique t'-SE, and it results in a payoff of $\Gamma_L(M_{low}(t'))$. As Γ_L attains its optimum at $p^* < p_{ind}$, and $p_{ind} < M_{low}(t') < M_{low}(t)$, it follows that $\Gamma_L(M_{low}(t')) > \Gamma_L(M_{low}(t))$. This contradicts the definition of an equilibrium.

Lemmas C.3-C.4 imply that for each $t > t_\alpha$, there is a unique t-SE, and F_L receives the same payoff in each of them. Furthermore, the t_α-SE is uniquely pinned down and yields that same payoff unless $M_{low}(t_\alpha) = \bar{p}$. When $M_{low}(t_\alpha) = \bar{p}$, multiple t_α-SE exist. However, by Lemma C.4, there is a t_α-SE which yields the same payoff as the t-SE for $t > t_\alpha$ (namely the selection where $\chi_{t_\alpha} := \bar{x}$, where $\bar{x}$ is defined as in Lemma C.4). ∎

E Proofs of Theorems 4.4-4.6

Proof of Theorem 4.4: Let $\alpha_{med} := sup\{\alpha : \bar{p} \geq \beta\}$ and $\alpha_{high} := \sup\{\alpha : \bar{p} \geq M_{low}(0)\}$. Note that $\alpha_{high} < \infty$ if and only if $M_{low}(0)Z_H^H > \Pi_H(M_{high}(0))$. By definition, for all $\alpha \in (\alpha_{opaque}, \alpha_{med})$, $\bar{p} \in (\beta, M_{high}(0))$. Likewise, for $\alpha \in (\alpha_{med}, \alpha_{high})$, $\bar{p} \in (M_{low}(0), \beta)$, and for $\alpha \in [\alpha_{high}, \infty)$, $\bar{p} \leq M_{low}(0)$.

With non-aligned firm preferences, Lemmas B.9-B.10 imply $\bar{p} > p_{ind} > p^*$. Hence, for all $\alpha > \alpha_{opaque}$, $\Pi_L(\bar{p}) > \Gamma_L(M_{high}(t^*))$. The equilibrium characterization for $\alpha \in (\alpha_{opaque}, \alpha_{med})$ follows from Lemma D.2(i). As $\bar{p}$ is strictly decreasing for $\alpha > \alpha_{opaque}$ (Lemma B.5), $\bar{t}$ is decreasing. Unraveling increases in this regime when α increases.

When $\alpha = \alpha_{med}$, $\bar{p} = \beta$ and $\underline{t} = -T$. By Lemma D.3(ii), there is an equilibrium for any distribution over $[-T, 0]$. For $\alpha \in (\alpha_{med}, \alpha_{opaque})$, $t_\alpha = \underline{t}$, and so by Lemma D.5, all distributions over $[\underline{t}, 0]$ can be sustained in equilibrium. As $\bar{p}$ is strictly decreasing for $\alpha > \alpha_{opaque}$, $\underline{t}$ is increasing. Unraveling decreases in this regime when α increases.

The equilibrium characterization when $\alpha \in [\alpha_{high}, \alpha_{opaque})$ follows from Lemma D.5, as t_α equals 0 in this situation. ■

Proof of Theorem 4.5: When $\alpha \leq \alpha_{opaque}$, the equilibrium is identical to the benchmark. For $\alpha > \alpha_{opaque}$, Theorem 4.4 implies that F_L matches with a worker that is high type with probability $\bar{p} < M_{high}(0)$ in equilibrium. F_L's payoff in equilibrium is weakly lower than $\Pi_L(\bar{p})$. On the other hand, F_H receives a payoff of $\Pi_H(M_{high}(0))$ in equilibrium. Notice that F_H's payoff is the same as its payoff in the benchmark, while F_L earns a payoff that is strictly lower than its payoff of $\Pi_L(M_{high}(0))$ in the benchmark. ■

Proof of Theorem 4.6: By Theorem 4.4, F_L matches at time $\bar{t} < 0$ in equilibrium. F_L's payoff is $\Pi_L(\bar{p})$, and F_H's payoff is $\Pi_H(M_{high}(0))$.

By Lemma C.4, if firms are constrained to interview at $t = 0$, the equilibrium is unique and entails F_L hiring a worker that is high type with probability $\bar{p}$ and F_H hiring at the end of the primary market with probability $\chi_0 \in (0, 1)$. As F_H is indifferent between hiring at the end of the primary market and operating on the secondary market, its payoff is still $\Pi_H(M_{high}(0))$. The payoff to F_L in this equilibrium is $\chi_0\Pi_L(\bar{p}) + (1-\chi_0)\Gamma_L(\bar{p})$, which is strictly lower than its equilibrium payoff of $\Pi_L(\bar{p})$ in the unraveled market. ■

F Additional Discussion of Assumptions

Information Blocking In the model, primary market hiring prevents a competing firm from learning about the hired worker before the secondary market begins. This is the reality in many two-sided matching markets. For example, in hiring at the university level, once an offer is accepted, students are not permitted to interview with other employers through the university placement office.

N **sufficiently large** When N is large, I need not consider the case of all candidates failing or passing a given test at any stage in the primary market. The probability of such an event tends rapidly to 0 as N increases. In addition, suppose F_L interviews candidates at time t, making an offer according to a known hiring rule. If F_H interviews the remaining applicants at a later date, its beliefs about the applicant hired by F_L will not be affected. This isolates the effect of the informativeness of the secondary market on firm behavior in the primary market. When the number of workers is small, one must account for strategic rejection of offers. If a worker receives an early offer from the low-tier firm, she may infer something about her type and ability to receive an offer from the high-tier firm later. This causes further unraveling as the low-tier firm must move even earlier to ensure the worker accepts the offer.

Firing Allowing for firing does not remove the low-tier firm's incentive to unravel to deter poaching. To see this, suppose firms were permitted to fire workers after some minimal retention time ε. This does not affect the strategy space of the low-tier firm, as it will never choose to fire a worker. However, it does provide a benefit to the high-tier firm. Namely, the high-tier firm can always hire at the end of the primary market at $t = 0$, fire the worker at $t = \varepsilon$ if the worker is of low type, and then monitor and poach the worker hired by the low-tier firm down the line. Such optionality, though, ensures that there is always a risk of the low-tier firm being poached in the future. This risk is present even in the extreme case where ε is sufficiently close to 0 (e.g., when firing is costless). The high-tier firm always hires at $t = 0$, but the low-tier firm must hire a worker that is high type with probability $p^* = argmax_p M_{high}(0)[pZ_H^L + (1-p)Z_L^L] + (1 - M_{high}(0))\Gamma_L(p)$. When $p^* < M_{high}(0)$, unraveling equilibria still exist.

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