Matching with Costly Interviews: The Benefits of Asynchronous Offers

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Abstract

In many matching markets, matches are formed after costly interviews. We analyze the welfare implications of costly interviewing in a model of worker-firm matching. We examine the trade-offs between a centralized matching system and a decentralized one, where matches can occur at any time. Centralized matching with a common offer date leads to coordination issues in the interview stage. Each firm must incorporate the externality imposed by the interview decisions of the firms ranked above it when deciding on its interview list. As a result, low-ranked firms often fail to interview some candidates that ex-ante have high match quality. In a decentralized setting with exploding offers, the set of candidates who receive interviews differs, but the welfare generated is weakly greater than in the centralized setting. Total welfare is highest with a system that ensures firms interview and match in sequence, clearing the market for the next firm. Such asynchronicity reduces interview congestion. This system can be implemented by encouraging top firms to interview and match early and allowing candidates to renege on offers.

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1 INTRODUCTION

The National Resident Matching Program (NRMP) was established in 1952 to match medical graduates with residency programs across the United States. Annually, the NRMP facilitates the matching of tens of thousands of aspiring doctors with suitable training positions via the Gale-Shapley Deferred Acceptance algorithm. The success of the NRMP match has led to an expanding interest in the centralization of other matching markets, ranging from the academic job market to college admissions to the market for college athletes.

Much of the theoretical analysis of centralized, two-sided matching processes has focused on settings where agents have complete information about their preferences.[1] However, a fundamental feature of labor markets is that agents are initially uncertain about their preferences and so invest in costly information acquisition — generally in the form of interviews — *prior to* submitting their rank-order lists. This process imposes significant costs on agents. For instance, in the NRMP match, medical school graduates often have to pay interview expenses out of their pocket, and the residency program doctors conducting the interviews cannot do surgeries on interview days. Moreover, due to competition for candidates, hospital interviewing decisions become calculated decisions, and much effort is devoted to thinking about which candidates are "gettable" and worth interviewing (Wapnir et al. (2021)).

We focus on how the design of matching markets affects how firms acquire information through interviews. These choices are not just about *how much* information to acquire, but *how* to acquire it: firms must not only choose how many interview invitations to issue but also which candidates will receive them. Moreover, these decisions are strategic: while a given firm cares only about the quality of the workers it is matched to, the final assignment is determined by the information that it acquires *and* the information acquired by its competitors. Consequently, the welfare produced by a matching mechanism is inextricably linked with its impact on firms' interviewing incentives.

To analyze this link, we develop a novel model of two-sided matching between firms and workers, where firms conduct costly interviews before offers are made. Interview decisions are decentralized: there is no restriction on how interviews are assigned as they are a strategic choice by firms. We show that centralized matching mechanisms where workers are matched to firms via deferred acceptance have a drawback: firms must decide which candidates to interview without knowing whether those candidates' interviews with more preferred firms will convert to offers. As a result, firms fail to interview some candidates that ex-ante have high match quality, leading to high-quality workers "falling through the cracks".

[1]The assumption of complete information of preferences is substantive. Fernandez et al. (2022) highlight the fragility of classical theorems to this assumption.

To evaluate the magnitude of this inefficiency, we contrast such a centralized system with two types of decentralized systems:

1. A decentralized system where firms can interview at any time and make binding offers.
2. A decentralized system where firms can interview at any time and make public, non-binding offers.

The first is representative of the status quo in many markets, including the market for investment banking analysts, corporate law associates, and university professors. The second is a hybrid regime of sorts. Firms are free to interview at any time and make public offers, but the offers are non-binding. Effectively, workers can hold onto offers and need not make a decision until the very end of the hiring cycle. Such a system does exist, for example, in the college athletic scholarship market: high school athletes are permitted to publicly receive scholarship offers at any point after the start of their junior year. However, they cannot officially accept them until the December of their senior year.

In the decentralized regime where firms can interview at any time and make binding offers (e.g. exploding offers), we show that all firms are *ex-ante* no worse off in equilibrium than in the centralized setting. However, ex-post, firms can be worse off. In the decentralized regime where offers are public and non-binding, worker welfare is maximized in the equilibrium where firms interview and match sequentially according to their ranking. Total firm surplus is also higher in such a setting than in the equilibrium of the centralized setting. Sequential matching removes the uncertainty about which candidates will receive offers from higher-ranked competitors *before* lower-ranked firms have to make their interview decisions. The asynchronicity of offers removes firms' incentives to skip over higher-quality applicants and leads to more efficient interview decisions and final matches.

1.1 Related Literature

There is an extensive literature on two-sided matching in labor markets, particularly on the NRMP (Roth (1984); Roth (2008); Roth and Sotomayor (1992)). In much of the theoretical analysis of such models, there is complete information about preferences and no uncertainty regarding match quality (See Kojima et al. (2020) for a comprehensive survey of this literature).

Illustrative examples of the effect of incomplete information on traditional notions of stability and other classical matching properties are provided by Roth (1989) and Fernandez et al. (2022). Liu et al. (2014) and Liu (2020) also focus on a two-sided matching market with incomplete information, developing a notion of stability to accommodate such an environment. In their setting, agents draw inferences about the value of matching with others by observing their cooperative deviations. Because of workers' common preferences, this channel does not play a role in our setting. Instead, we focus on firms' incentives to gather information through interviews *before* matching.

There is an emerging body of literature on matching with search. Chade and Smith (2006) and Ali and Shorrer (2021) analyze simultaneous search problems where a student applies to colleges but is unsure whether a particular college will admit them. Our model can be viewed as a generalization of the former in two ways: firms in our model can hire multiple workers, firms must compete with other firms for the workers, and the probability of a worker being hired is *endogenous*. These features also distinguish our paper from Ali and Shorrer (2021).

Within the search literature, our model is closest to Chade et al. (2014), who (like us) consider a model where firms compete for multiple workers of unknown value, observe a costly signal about the value of some (but not all) workers, and hire the workers with the highest signals. The key differences between our papers are the presence of private information on one side of the market, the commonality of workers' values to different firms, the nature of firms' signals, and the side of the market that decides which firms will be informed about which workers. Specifically, in their model, each worker's value is *private information*, their values to different firms are *identical*, and *workers* decide which firms to reveal *noisy signals* of that value to. In our model, there is a *public noisy signal* (application rank) of each worker's value, their values to different firms are *conditionally independent* of one another, and *firms* decide which workers to acquire *perfect information* about.

Immorlica et al. (2020) also examines a college admissions setting, but students can first acquire information about a college before applying. They develop a notion of regret-free stability, which incorporates student information acquisition decisions. In their model, agents acquiring information are part of a continuum, and so each agent's decision to gather information does not influence another agent's incentives to acquire information. This is not the case in our model: a given firm's interview decision imposes a direct externality on other firms.

A nascent literature on two-sided matching with interviews has received growing interest over the last few years. Lee and Schwarz (2017) investigates the welfare consequences of interviewing in centralized mechanisms but focuses on how interview decisions affect total matches. They find that in balanced markets, maximizing total matches is achieved when firm interview sets have perfect overlap. In our model, the market is unbalanced, and no firm must worry about not filling its capacity. Instead, each firm only cares about the match value generated less the cost of interviews. Consequently, perfect overlap in our setting is suboptimal. Echenique et al. (2022) and Manjunath and Morrill (2023) also examine a two-sided matching setting where firms want to maximize match value less interview costs. However, they assume interview assignments are determined via a many-to-many deferred acceptance algorithm, which takes as inputs the *ex-ante* rank order list. The interviews do not provide any added information. Rather, whom a firm selects to interview restricts which agents it can list in the rank order list it submits to the clearinghouse. We do not use an exogenous interview assignment protocol. Interview assignments are determined in equilibrium. It is not the case that the outcome of a many-to-many deferred-acceptance algorithm on interview preferences aligns with the outcome in a game where firms select whom to interview.

Along these lines, there are two other works, Kadam (2015) and Erlanson and Gottardi (2023), which look at centralized matching environments where firms are free to select whom to interview. These two papers investigate different questions. The former assumes a specific functional form on the interview technology and analyzes the effect of interview capacity constraints. He shows that relaxing interview capacities can reduce welfare due to over-interviewing. The latter looks at a two-firm environment where interviews are informative for workers and firms, and the interviewing technology for firms is equivalent to our "interviewing for bad news" in Example 1. Our paper differs because we use a general interview technology and focus on equilibrium outcomes *across* different matching protocols. In addition, in our centralized setting, equilibria are inefficient even when worker preferences are common and fixed due to the externality higher-ranked firms place on lower-ranked ones.

Lastly, Ferdowsian et al. (2022) presents a model similar to our decentralized setting except there are no exploding offers, workers can hold on to offers for as long as they like, and if a worker accepts an offer, they cannot renege. Notably, while private information and uncertainty exist in their model, there is no information acquisition stage.

2 MODEL

There is a finite set of F firms, indexed by $\{1,\ldots,F\}$, and a unit measure of workers. Each worker can work at one firm; each firm can hire at most $\Delta \in (0,1)$ workers. Workers have common preferences over firms: when matched with firm f, she receives a payoff of z_f, where $z_1 > z_2 > \cdots > z_F > 0$. Each worker prefers working for firm F (and thus any other firm) to remaining unmatched: If a worker does not match with any firm, she receives a payoff of $u \leq 0$.

Workers are identified by their application score $a \in [0,1]$, which is common knowledge and distributed according to a continuous distribution $W(\cdot)$. Without loss of generality, we assume $W(\cdot)$ is uniform, and so the application score is simply the application rank.[2]

Hiring a given worker yields different payoffs for different firms, and these *match values* $s \geq 0$ are not known *ex-ante* by either the workers or firms. Instead, conditional on the worker's application rank, they are i.i.d. with distribution $G(\cdot|a)$.[3] Workers with higher application ranks are more likely to yield higher match values, in the sense of first-order stochastic dominance: for all $a' > a$ and all $s \geq 0$, $G(s|a') \leq G(s|a)$.

Firms must interview workers before making offers to them. These interviews have constant marginal cost: interviewing a measure μ of workers costs $c \cdot \mu$. When a firm interviews a worker, it learns the value of matching with her.

[2]Since the application score distribution is continuous, we can always index applications by their quantile, or percentile rank, which must be uniformly distributed on the unit interval.

[3]It is without loss to consider distributions with support on a subset of $[0,\infty)$ since a firm will never hire a worker with a negative match value. Given a distribution G with negative values in its support, one can simply consider a distribution $\hat{G}$ with support on a subset of $[0,\infty)$ such that $\hat{G}(s) = G(s)$ for $s \geq 0$ and $\hat{G}(s) = 0$ for all $s < 0$.

Faced with a pool of *available* workers with known match values, a firm's *hiring policy* is defined to be a function describing which worker it will hire (or make an offer to). For tractability, we make the following assumption on matching/hiring/offers:

Assumption 2.1. *(No Discrimination) A firm can only condition offers based on match value. Moreover, if a subset of interviewed applicants have a match value of s, and a firm can only make offers to a fraction of them, the firm does so uniformly at random.*

Assumption 2.1 implies that a hiring rule is a function $x(s)$ representing the probability a firm will hire an available worker that has match value s. A hiring rule is a greedy hiring-rule if there exists $\bar{s}$ such that $x(s) = 1$ for all $s > \bar{s}$ and $x(s) = 0$ for all $s < \bar{s}$. The set of greedy hiring rules is denoted by $\mathscr{X}$. Note that $\mathscr{X}$ admits a total *permissiveness* ordering $\succ_{\mathscr{X}}$ where $\hat{x} \succ_{\mathscr{X}} x$ if and only if there exists s such that $\hat{x}(s) > x(s)$.

Each firm's payoff is separable in the values of the workers that it matches with. When firm f interviews a mass μ of workers whose match values are described by the measure Φ, and hires all of those workers with match values above s_f, its payoff is $\int_{s_f}^{\infty} v d\Phi(v) - c \cdot \mu$. We assume that $F \cdot \Delta < 1$ and $\int_0^{\infty} s dG(s|0) > c$. In other words, there are more workers than slots, and the lowest-ranked worker is worth interviewing.

2.1 Matching Regimes

We consider three types of matching mechanisms: *centralized*, *decentralized with binding offers*, and *decentralized with nonbinding offers*.

CENTRALIZED MATCHING. First, firms simultaneously choose which workers $I_f \subseteq [0,1]$ to interview. Then, both sides submit preferences to a centralized clearinghouse, which runs a worker-proposing Gale-Shapley algorithm. The algorithm is strategy-proof for the workers. Since workers have common preferences, it is optimal for each firm to report truthfully, conditional on workers' reporting truthfully. The resulting matching is the unique, stable outcome. This mechanism is equivalent to the one used in the NRMP. Furthermore, because of our independence assumption, it is also equivalent to a mechanism in which firms interview simultaneously, offers are private, and offers can be held until a common, fixed deadline.

We assume a firm can only list interviewed workers in its submitted preferences. This is what occurs in practice. We can motivate it by incorporating a small probability q that a candidate is not a good fit, and conditional on a candidate not being a good fit, a firm would never want to hire them.

DECENTRALIZED MATCHING. In a decentralized matching mechanism, there are $T \geq F$ time periods, indexed by $t \in \{1, \ldots, T\}$. In each period t, each firm f that has not already interviewed decides whether to interview in that period and, if so, chooses a set of candidates $I_f^t \subseteq [0,1]$ to interview and a hiring policy x_f^t. Then, it publicly makes employment offers to each worker it has interviewed whose match value is s with probability $x_f^t(s)$.[4]

We consider two varieties of decentralized matching mechanisms:

i. *With Binding Offers* Firms make offers that expire at the end of the period. At the end of *each* period t, each worker strategically decides whether to match with one of the firms that made her an employment offer during period t or reject each of those offers. Hence, when firms choose an interview set I_f^t, they only interview those candidates with application ranks $a \in I_f^t$ who have not accepted an offer in a previous period $t' < t$.

ii. *With Nonbinding Offers* Firms can only make non-binding offers. Equivalently, all offers expire only after the last period T, and so at the end of period T, each worker matches with the highest-ranked firm that has made her an offer.[5] Since offers are public, when firms choose a time t and an interview set I_f^t, they can base their interview decisions on a worker's application rank *and* current offer set.

3 Analysis

3.1 Centralized Regime

In the centralized setting, firms interview candidates and then submit preferences to a clearinghouse, which runs the standard deferred-acceptance algorithm to determine the matching.

Proposition 1. *Fixing the interview strategies I_f, if workers' report their preferences truthfully, truthful reporting is optimal for each firm f, independent of its competitors' reports. Moreover, the deferred-acceptance algorithm returns the unique stable matching.*

Proof. Deferred Acceptance is guaranteed to be strategy-proof for the workers. Since workers have common preference over firms, at round f of the algorithm, all workers point to firm f. Thus, firm f holds matches with the top Δ proposals of the remaining group. In other words, at round f, firm f is a dictator and chooses the best Δ candidates amongst those available. The proposition follows. □

The use of the deferred acceptance algorithm to determine the final matching implies that each firm will match with workers above a threshold and not match with any workers with a match value below that threshold. The threshold is pinned down so that the firm is matched with at most Δ workers.[6] Equivalently, since workers have common preferences over firms,

[4]No extra information is available about candidates at later periods. The reason for this assumption is to identify the inefficiencies that arise solely due to costly interviews rather than unraveling.

[5]We avoid allowing workers to accept offers before period T because doing so would not change our analysis; indeed, it would make waiting until period T to accept offers a weakly dominant strategy for the workers.

[6]Of the workers with a match value exactly equal to the threshold, the firm may only match with a subset of them if there is a positive mass of workers with that match value.

the worker-proposing deferred acceptance algorithm produces the same outcome as firm serial dictatorship. Thus, conditional on interviews $\{I_f\}_{f=1}^{F}$, the payoff to firm f is given by the following recursive rule:

$$\pi_f(\{I_h\}_{h=1}^{f}) \equiv \int_{I_f} \underbrace{\int_0^\infty s x_f(s) dG(s|a)}_{\text{Match Value from hiring policy}} \cdot \overbrace{\left(\prod_{h<f, a\in I_h} \int_0^\infty (1-x_h(s))dG(s|a)\right)}^{\substack{\text{Fraction of rank } a \text{ workers} \\ \text{not matched to firms } h<f}} - c\, da,$$

$$\text{where } x_f^*(\cdot, \{I_h\}_{h=1}^{f}) = \max_{\succ_{\mathscr{X}}} \left\{ x \in \mathscr{X} \,|\, \int_{I_f} \int_0^\infty x(s) dG(s|a) \left(\prod_{h<f, a\in I_h} \int_0^{s_h} (1-x_h^*(s))dG(s|a)\right) da \leq \Delta \right\}$$

We suppress dependence of x_f on $\{I_h\}_{h=1}^{f}$ for notational convenience. Recognize that either $x_f(s) = 1$ for all s or it is the unique greedy hiring rule that hires exactly Δ workers.[7] Moreover, when $G(\cdot|a)$ is continuous for each a, the hiring rules $x_f(\cdot)$ for each firm are "pure" greedy rules: $x_f(s) = 1$ for all $s \geq s_f$ and $x_f(s) = 0$ for all $s < s_f$. There is a one-to-one correspondence between the hiring rule and the threshold match-value set by the firms. Hence, the payoff for each firm can be rewritten as:

$$\pi_f(\{I_h\}_{h=1}^{F}) \equiv \int_{I_f} \int_{s_f^*}^\infty s dG(s|a) \cdot \left(\prod_{h<f, a\in I_h} G(s_h^*|a)\right) - c da,$$

$$\text{where } s_f^* = \min \left\{ s| \int_{I_f} (1-G(s|a)) \left(\prod_{h<f, a\in I_h} G(s_h^*|a)\right) da \leq \Delta \right\}.$$

Definition 3.1. *A (pure strategy) Nash equilibrium of a centralized matching mechanism is a strategy profile $\{I_f^*\}_{f=1}^{F}$ such that:*[8]

$$I_f^* \in \arg\max_{I_f \subseteq [0,1]} \pi_f(\{I_h\}_{h=1}^{f}) \textit{ for all } f$$

The recursively defined thresholds s_f^* are the firms' optimal thresholds for hiring, given their interview set and the interview sets of higher-ranked firms: intuitively, once they have learned the match values of the workers they interviewed, they should hire those with the highest values until they are out of open positions.[9]

Given any equilibrium, one can construct another by adding a measure 0 set of workers.

[7] By unique, we mean payoff-unique. If two greedy hiring rules both lead to exactly Δ workers being hired, they only differ on a measure zero set.

[8] It is without loss to restrict firms to pure strategies. Any mixed strategy $\sigma \in \Delta 2^{[0,1]}$ is payoff equivalent to the pure strategy $\mathbb{E}[\sigma]$.

[9] The cutoff score s_f would also arise in a setting where firms simultaneously selected a set of workers to interview *and* a cutoff score above which interviewed workers would receive an offer.

Such differences in equilibria are not substantive. We will consider two equilibria equivalent if the interview sets only differ on a measure 0 set of workers.

Proposition 2. *A pure strategy equilibrium exists.*

To provide intuition, consider the continuous case. Observe that Firm 1's decision is independent of all other firms' decisions since it is the most preferred firm. Conditional on interviewing a set of workers of size μ, it is optimal for Firm 1 to interview $I_1 = [k, 1]$, where $1-k = \mu$. In other words, Firm 1 interviews greedily (see Lemma 3 in the appendix for a formal proof). Of the set $[k, 1]$, Firm 1 will match with the top Δ workers based on post-interview match quality. Thus, Firm 1's optimization problem is:

$$a_1 \in \arg\max_k \int_k^1 \left[\int_{\bar{s}_1}^{\infty} v dG(v|a) \right] da - c(1-k)$$

$$\text{Where } \bar{s}_1(k) = \min \left\{ s | \int_k^1 (1 - G(s|a)) da \leq \Delta \right\}$$

Conditional on Firm 1 interviewing $I_1^* = [a_1, 1]$, Firm 2 must consider that some of the workers in I_1^* will be taken by Firm 1 in the matching stage. Thus, the effective match value distribution for workers $a \in I_1^*$ is $G(\bar{s}_1|a)G(\cdot|a)$.[10] Firm 2's optimization problem is:

$$I_2^* \in \arg\max_{I_2 \subset [0,1]} \int_A \psi(a) \Big(\int_{\bar{s}_2(I_2)}^{\infty} G(s|a) ds \Big) da - c\Big(\int_{I_2} da \Big)$$

$$\text{Where } \psi(a) = \begin{cases} 1 & \text{if } a \notin I_1^* \\ G(\bar{s}_1|a) & \text{if } a \in I_1^* \end{cases}, \;\; \textit{and } \bar{s}_2(I_2) = \min \left\{ s | \int_{I_2} \big(1 - G(s|a)\big) \psi(a) da \leq \Delta \right\}$$

Observe that $\psi()$ is not included in the cost of interviewing because Firm 2 does not know which workers in I_1^* will be matched to Firm 1. Therefore, it cannot discriminate between workers of a given rank $a \in I_1^*$.

We next characterize the firms' equilibrium interview sets. Unlike the top-ranked firm, lower-ranked firms do not necessarily interview greedily: even though the applicants interviewed by one or more higher-ranked firms have more favorable match value distributions than those that are not, they are less likely to be available. However, when the match value distribution satisfies two regularity conditions, we can say that interviewing greedily is optimal *on each set of candidates that are interviewed by the same set of higher-ranked firms.*

Intuitively, interviewing a set I' of higher-ranked applicants instead of an equal-mass set I of lower-ranked ones changes the firm's maximal greedy hiring policy x_f^* in two ways. First, there is a *quality effect*: the distribution of match values is shifted to the right, making x_f^* more stringent. Second, there is an *availability effect*: if the applicants in I' and I are interviewed by

[10]Here, "distribution" should not to be interpreted in the probability sense. After all, it does not integrate to 1.

the same set of higher-ranked firms, the higher-ranked applicants in I' are less likely to be available, since they are more likely to have met those other firms' hiring thresholds; consequently, interviewing I' instead of I must make x_f^* more permissive. The regularity property needed to ensure that interviewing I' is better than interviewing I — i.e., greediness — depends on which of these effects dominates.

Our first regularity condition — which is intermediate between first-order stochastic dominance and the monotone likelihood ratio property — plays this role when the quality effect dominates.

Regularity Condition #1. $G(\cdot|a)$ is increasing in a in the hazard rate order:

$$\text{For all } a' > a \text{ and } s' > s,\ (1-G(s'|a'))(1-G(s|a)) \geq (1-G(s|a'))(1-G(s'|a)).$$

Regularity Condition #1 guarantees that, holding the firm's hiring policy constant, an applicant's expected match value *conditional on being hired* is increasing in their rank a. Interviewing higher-ranked applicants under a hiring policy that is at least as stringent — as the firm does when the quality effect dominates — can only increase this conditional expectation further. Since the firm's hiring policy x_f^* is constructed to yield an equal mass of hired workers from any interview sets with the same mass, this relationship extends to the *total* value of the workers hired from higher- and lower-ranked sets of applicants.

Our second regularity condition ensures that interviewing higher-ranked applicants is more valuable when the availability effect dominates.

Regularity Condition #2. For some k, G has *increasing k-adjusted yields*: For any profile of greedy hiring policies $\{x, \{x_f\}_{f=1}^k\} \subset \mathscr{X}$,

$$\int x(s) s dG(s|a) \prod_{f=1}^{k} \left(\int (1 - x_f(s)) dG(s|a) \right) \text{ is non-decreasing in } a.$$

When G has increasing k-adjusted yields for all k, we say it has *increasing adjusted yields*.

Recall that first-order stochastic dominance ensures that, fixing a greedy hiring policy x, the value $\int x(s) s dG(s|a)$ created by interviewing a rank-a applicant is increasing in a. When the match value distribution G has increasing k-adjusted yields, the value created by interviewing an applicant is still increasing in her rank even when it is adjusted for the probability that she will be hired by one of k higher-ranked firms. Setting a more permissive hiring policy for higher-ranked applicants — as the firm does when the availability effect dominates — can only increase this value further.

We can interpret k as the *largest* number of firms whose decisions to interview a pair of applicants cannot reverse their relative attractiveness to lower-ranked firms; that is, if the property holds for k, it holds for each $k' < k$. Formally:

Lemma 1. *If G has increasing k-adjusted yields, then it has increasing k'-adjusted yields for each $k' < k$.*

All distributions satisfying first-order stochastic dominance have increasing 0-adjusted yields. For some canonical distributions, the property holds for higher k. We provide two such examples (each of these distributions also satisfy regualrity condition #1).

Example 1 (Exponential Distribution)**.** Suppose that given an applicant's rank, match values are exponentially distributed and the distribution has longer tails for higher-ranked applicants. We can write the match value distribution as $G_\lambda(s|a) = 1 - e^{s\lambda(a)}$, where $\lambda : [0,1] \to \mathbb{R}_+$ is a decreasing function. Lemma 2 shows that G_λ has increasing k-adjusted yields for some $k \geq 1$.

Example 2 (Interviewing for "Bad News")**.** Suppose interviewing an applicant either reveals that they are unacceptable ($s = 0$) with probability β or that their pre-interview rank was accurate ($s = a$). We can write the match value distribution as $G_\beta(s|a) = \begin{cases} \beta, & s < a \\ 1, & s \geq a. \end{cases}$ Such a technology appears in Chade and Smith (2006) and Erlanson and Gottardi (2023). Lemma 2 shows that G_β has increasing adjusted yields.

Proposition 3 shows that when the match value distribution is increasing in a in the hazard rate order and exhibits increasing k-adjusted yields, the highest-ranked $k+1$ firms will *overlap* greedily with the interview decisions of higher-ranked firms.

Proposition 3. *Suppose that G is increasing in a in the hazard rate order and has increasing k-adjusted yields. Then there is a Nash equilibrium $\left\{I_f^*\right\}_{f=1}^F$ in which*

i. The highest-ranked firm interviews greedily: $I_1^* = [a_1, 1]$ for some $a_1 \in [0,1]$.

ii. Each firm $f \in \{2, \ldots, k\}$

(a) Overlaps greedily with each set of firms above them: For $S \subseteq \{1, \ldots, f-1\}$,
$I_f^* \cap \bigcap_{j \in S} I_j^* = [a_f^S, 1] \cap \bigcap_{j \in S} I_j$ *for some $a_f^S \in [0,1]$.*

(b) Interviews greedily below *the firm above them: There exists a decreasing sequence $\{a_f\}_{f=1}^F$ such that $I_f^* \cap [0, a_{f-1}) = [a_f, a_{f-1})$.*

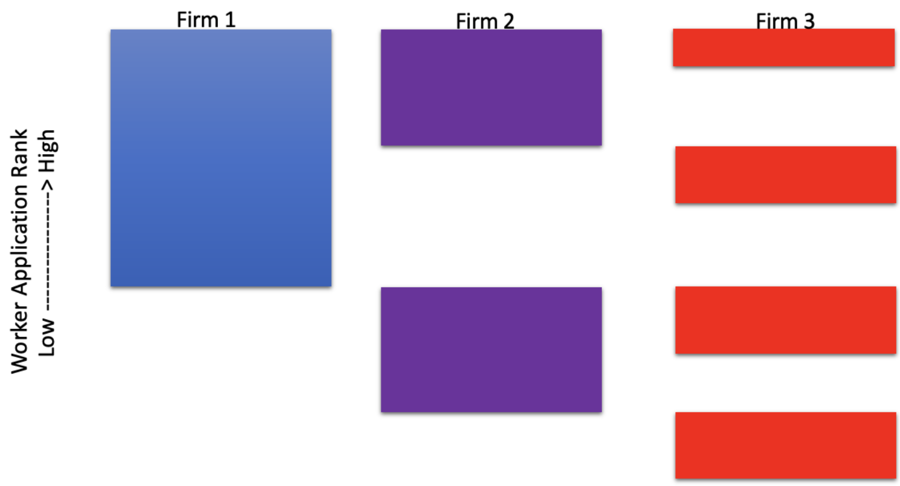
(c) Allows the lowest-ranked applicants interviewed by the firm above them to fall through the cracks: For each $S \subseteq \{1, \ldots, f-2\}, a_{f-1}^S < a_f^{S \cup \{f-1\}}$.

Our two regularity conditions on G are required for our proposition to hold due to firm uncertainty over which candidates that higher ranked firms interview will be matched with any of those firms. Suppose a given firm f knows the set of available workers (e.g. the markets for the firms ranked above it have all cleared or the results of the interviews are common knowledge). In that case, first-order stochastic dominance is sufficient to guarantee the optimal interview scheme for firm f is greedy.

Example 3. Consider three firms. There is an equilibrium where interview strategies have the following form:

- Firm 1: $[a_1, 1]$
- Firm 2: $[a_2^{(1)}, a_1] \cup [a_2^{(2)}, 1]$ for $a_2^{(1)} \leq a_1 \leq a_2^{(2)} \leq 1$.
- Firm 3: $[a_3^{(1)}, a_2^{(1)}] \cup [a_3^{(2)}, a_1] \cup [a_3^{(3)}, a_2^{(2)}] \cup [a_3^{(4)}, 1]$ for $a_3^{(1)} \leq a_2^{(1)} \leq a_3^{(2)} \leq a_1 \leq a_3^{(3)} \leq a_2^{(2)} \leq a_3^{(4)} \leq 1$.

Pictorially, the interviewing schemes look like this:



From the graphic, one can see the coordination issues that arise in the interview stage. While Firm 2 knows whom Firm 1 will interview, it does not know which workers will be matched with Firm 1 during the deferred acceptance stage. Hence, interviewing such workers carries a risk! This prompts Firm 2 to skip over a portion of ex-ante highly ranked workers. Notice, though, that applicants of low rank do have interviewing opportunities.

Each firm's interview decision imposes an externality on the firms ranked below it. This externality is the root cause of the inefficiency of the equilibrium interviewing scheme. To see why, it suffices to consider a two-firm example. Given the equilibrium interviewing scheme for Firm 1 and Firm 2, for ε sufficiently small, the bottom ε of the candidates interviewed by Firm 1 would provide a higher marginal return for Firm 2 than Firm 1. So, to increase aggregate match values, one should prohibit Firm 1 from interviewing those ε-measure of candidates. Formally, suppose a planner could select whom each firm interviews with the objective of maximizing *total* match value *less* the cost of interviewing:

$$\left\{I_f^{sp}\right\}_{f=1}^{F} \in \arg \max_{\{I_f\}_{f=1}^{F} \subseteq [0,1]} \sum_{f=1}^{F} \pi_f(\{I_h\}_{h=1}^{F})$$

Proposition 4. *The planner-optimal choice does not necessarily coincide with the equilibrium interviewing schemes:* $\left\{I_f^{sp}\right\}_{f=1}^{F} \neq \left\{I_f^{*}\right\}_{f=1}^{F}$.

It suffices to prove Proposition 4 via example. However, in the appendix there is a more formal description of why the equilibrium interviewing schemes are generally not efficient.

Example 4. There are two firms, each with a capacity of $\frac{1}{2}$ ($\Delta = \frac{1}{2}$), and interview costs are $c = \frac{12}{5}$. The pool of workers consist of two applicant types, I and II. Type I applicants provide a match value of either 1, 9, or 10 with equal probability ($\frac{1}{3}$ probability each). Type II applicants provide a match value of 0, 1, or 9 with equal probability. There is a $\frac{1}{2}$ measure of each. In the language of the model, applicants of rank $a \in [\frac{1}{2}, 1]$ are of type I and those of rank $a \in [0, \frac{1}{2}]$ are of type II.

The equilibrium in this setting is $I_1^* = [0,1]$ and $I_2^* = \emptyset$. In other words, Firm 1 interviews everyone and Firm 2 interviews no one. Total surplus is:

$$\frac{1}{6} \cdot 10 + \frac{1}{3} \cdot 9 - 1 \cdot \frac{12}{5} = \frac{34}{15}$$

This is inefficient. Firm 1's interview decisions crowd Firm 2 out of the market completely. Consider an interview assignment where Firm 1 interviews only Type I applicants and Firm 2 interviews only Type II applicants. Total surplus is:

$$\frac{1}{2}(\frac{20}{3} - \frac{12}{5}) + \frac{1}{2}(\frac{10}{3} - \frac{12}{5}) = \frac{13}{5} > \frac{34}{15}$$

3.2 Decentralized Matching with Binding Offers

The second regime is the "status quo" environment in many markets, including the academic job market. For clarity, we reiterate the description of the game:

i. At the beginning of each period t, firms that have not been matched simultaneously decide whether to interview. Firms observe which firms went on the market in previous periods, the pool of workers yet to accept an offer, and the offers that those workers hold. Once a firm decides to interview in period t, it observes which other firms are on the market and then decides whom to interview and make offers to. Offers are exploding and expire at the end of the period.[11]

[11] Given Assumption 3.2, it is without loss to consider such offers.

ii. At each t, workers who receive offers must decide whether to accept, hold, or reject any offers. At time t, a worker has the following information:

- The current offers she holds.
- The set of firms on the market in t, and the set of firms yet to be on the market.
- The pool of unmatched workers at the beginning of time t.

Our analysis considers the extreme case where workers are sufficiently risk averse in the sense that the payoff from being unmatched is sufficiently low. Risk aversion is necessary to make exploding offers viable in equilibrium: there is no point in front-running (i.e. interviewing before another firm) to make an exploding offer if said offer will never be accepted.

Assumption 3.2. $u < -\frac{1-G(\underline{s}|1)}{G(\underline{s}|1)} \cdot z_1$, *where* $\underline{s} = \inf\left\{\int_0^{1-\Delta\cdot(F-1)} 1 - G(\underline{s}|a)da \leq \Delta\right\}$

Under this assumption, workers will accept the first offer they receive.[12] In such a setting, firms will only make exploding offers: binding offers that expire immediately after the period they are made ends.

Proposition 5. *A subgame perfect equilibrium exists. In equilibrium, each firm's ex-ante expected payoff is weakly greater than its expected payoff in the centralized regime. However, firms can be strictly worse-off ex-post.*

In equilibrium, firm f randomizes over the times $t \in \{1,\ldots,f\}$ at which to interview. The probability of entering at time t is based on the observed history up until that time. While the ex-ante expected payoff for each firm is weakly higher than in the centralized setting, there may be an outcome realization such that a firm is left worse off. Furthermore, the workers the firms interview (outside of the top-ranked firm), as identified by their application rank, are *different* than in the centralized setting.

The externality imposed by higher-ranked firms on lower-ranked firms when multiple firms interview in the same period also potentially reduces worker welfare. Firms are not guaranteed to fill their slots in the equilibrium of the centralized setting nor in the equilibrium of the decentralized setting with binding offers.

Example 5. Suppose there are three firms. Let I_1^*, I_2^*, and I_3^* be each firm's equilibrium interviewing strategy in the centralized setting. In the equilibrium of the decentralized setting with binding offers, Firm 1 interviews $I_1^* = [a_1, 1]$ in period 1. Firm 2 randomizes between interviewing I_2^* in period 1 and interviewing workers $[a_2, 1]$ <u>of the available workers in period 2</u>, where a_2 is determined based on that pool. Firm 3 randomizes between interviewing $\hat{I}_3$ (not necessarily equal to I_3^*) in period 1 and interviewing workers $[a_3, 1]$ <u>of the available workers in period 3</u>, where a_3 is determined based on that pool.

[12] We discuss this assumption in detail in Section 4.1.

In equilibrium, the ex-ante expected payoff of each firm is weakly greater than in the centralized setting. However, one can see that there is a realization in which Firm 2 is worse off ex-post than in the centralized setting: when Firm 1 and Firm 3 interview and match in the first period, and Firm 2 interviews and matches in the second period.

3.3 Decentralized Matching with Nonbinding Offers

In this setting, offers are non-binding, public, and cannot be pulled. Workers can hold onto a set of offers until period T before deciding.

Observation 3.3. *Since offers are public, when firms choose an interview set I_f^t, they only interview those candidates with application ranks $a \in I_f^t$ who have not received an offer in a previous period $t' < t$ from a higher-ranked firm $f' < f$.*

Observation 3.4. *In any equilibrium, Firm i interviews after Firm i' if and only if $i \geq i'$. Moreover, firm i strictly prefers interviewing after firm i' if and only if $i' < i$.*

Since the highest-ranked firm is indifferent about the timing of its interviews, there are multiple equilibria.[13] We focus on the sequential-hiring equilibrium, where each firm f interviews and makes offers in period f.

Proposition 6. *In the sequential-hiring equilibrium:*

i. Firms interview greedily among the set of workers who have not received an offer: For each firm f, $I_f = [a_f, 1]$ for some $a_f \in [0,1]$.

ii. Firms interview monotonically: $a_f \geq a_{f'}$ whenever $f' > f$.

iii. There is maximal employment: all firms fill their slots.

Firm 1 always succeeds in hiring every worker to whom it offers a job, so its payoff is independent of the other firms' strategies. Likewise, each firm $f > 1$ always succeeds in hiring every worker that does not receive an offer from a higher-ranked firm, so its payoff is independent of the strategies of lower-ranked firms $f' > f$. When firms interview in rank order, each chooses its optimal interview set over the set of workers without a dominating offer. By Lemma 3, each firm's optimal interviewing strategy is greedy. No high-ranked applicants "fall through the cracks". Moreover, the sequential-hiring equilibrium guarantees that a total of $\Delta \cdot F$ workers are hired. Thus, total equilibrium worker payoff is weakly greater in the sequential-hiring equilibrium of the decentralized regime with non-binding offers than in the centralized regime and the decentralized regime with binding offers.[14]

[13] Conditional on all higher-ranked firms already having made offers, a given firm is indifferent about when to interview.

[14] How one would determine which setting is "better" for the workers depends on how a planner weighs each worker.

In the sequential-hiring equilibrium, where firms interview in rank order, firms avoid the cost of interviewing workers who receive a dominating offer. The asynchronicity of the offers reduces the interview congestion and coordination frictions in the centralized regime.

Proposition 7. *The sequential hiring equilibrium of the decentralized regime with nonbinding offers is more efficient than the equilibrium in the centralized regime.*

3.4 Efficiency

Consider, again, the social planner problem, with one caveat: the planner can choose the time period a firm can interview and match, and which candidates it can interview. However, the planner cannot choose with which candidates the firm can match.

Observation 3.5. *In any efficient mechanism, the planner chooses firms to interview and match sequentially.*

The planner prefers that interviews and matches occur sequentially so that interviewing is more efficient. More formally, suppose k firms are interviewing and matching in a given period. One can strictly improve welfare by assigning the same interview sets but with one firm moving after the other matches. Each firm will match with the same match-value distribution of workers but will interview less of them.

Given this, and the results from Proposition 7, one may assume that the sequential hiring equilibrium is efficient. However, that is not necessarily the case.

Proposition 8. *In any of the three regimes, equilibria may be inefficient.*

The fact that the equilibrium in the centralized regime is inefficient follows from Observation 3.5 (in fact, Proposition 4 shows it is not even necessarily constrained efficient). Any equilibrium of the decentralized regime with binding offers is inefficient if, with positive probability, at least two firms interview in the same period. If no two firms are interviewing in the same period with positive probability, it means the equilibrium coincides with the sequential hiring equilibrium of the decentralized regime with non-binding offers. Thus, it suffices to provide an example of a setting where the latter is inefficient.

Example 6. Suppose there are two firms, each in need of a measure $\frac{2}{5}$ of workers. There are two types of applicants (Type I and Type II). There are a measure of $\frac{1}{5}$ Type I applicants and a measure of $\frac{4}{5}$ Type II applicants. In other words, applicant $a \in [\frac{4}{5}, 1]$ is Type I and applicant $a \in [0, \frac{4}{5})$ is Type II. The match value distribution for each type is as follows:

- Type I provide a match value of 10 with probability $\beta_I \approx 1$, and 0 otherwise.
- Type II provide a match value of 9 with probability $\beta_{II} = 0.5$, and 0 otherwise.

If interview costs are sufficiently low ($c \to 0$), then in the sequential matching equilibrium of the decentralized setting with non-binding offers:

- Firm 1 interviews all Type I applicants and a $\frac{2}{5}$ measure of Type II applicants. It matches with a $\frac{1}{5}$ measure of 10's and $\frac{1}{5}$ measure of 9's
- Firm 2 interviews all remaining candidates and matches with a $\frac{3}{10}$ measure of 9's and $\frac{1}{10}$ measure of 0's

Total match value across both firms is $10 \cdot \frac{1}{5} + 9 \cdot \frac{1}{2} = 6.5$.

Now, consider the following interview assignment: Firm 1 interviews all the Type II applicants in the first period, and Firm 2 interviews all the remaining applicants.

- Firm 1 matches with a $\frac{2}{5}$ measure of 9's
- Firm 2 matches with a $\frac{1}{5}$ measure of 10's and a $\frac{1}{5}$ measure of 9's

Total match value across both firms is $10 \cdot \frac{1}{5} + 9 \cdot \frac{3}{5} = 7.4$.

Example 5 highlights the key reason the planner's incentives do not align with the firms'. The "10's" are likely to be "10's" everywhere: there's little variance. The Type II applicants have the most variance, so the planner prefers as many draws from this pool as possible. In other words, it is more efficient for a type II applicant to have more interviews than a Type I applicant! Firms do not internalize this particular mean-variance trade-off when they make their interview decisions.

4 DISCUSSION

4.1 Assumptions

Common Ranking of Firms Workers have a common ranking of firms unaffected by the interviews. This is partially for tractability but also to isolate the inefficiencies created by strategic interviewing *even when* firms know where they stand. To some degree, this feature resembles many of the motivating labor markets, such as medical residency and academia. One possible way to generalize our model while maintaining some tractability is to include a tiered ranking of firms. Formally, consider tiers $T_1, \ldots, T_k$, where $T_i \subset \{1, \ldots, n\}$ for all i and $T_i \cap T_j = \emptyset$ for all $i \neq j$. While each worker strictly prefers firms in T_i to firms in T_j for $i < j$, worker preferences over firms *within* a given tier are dependent on the interviews. As the results in Erlanson and Gottardi (2023) suggest, any centralization of interviews within a given tier would lead to potential multiplicity of equilibria, as well as inefficiency. In such a world, our results indicate that total surplus in a setting where each tier of firms interviews in sequence would be higher than in a centralized setting. However, firms within the same tier would have an incentive to front-run each other.

Independent Match Values In our model, the match value generated by a particular worker for a firm is independent across firms. With correlated match values, an adverse selection issue arises. To make this salient, consider a common-values environment where a worker's match value to a given firm is the same across all firms. If a candidate is available to a low-ranked firm, either that candidate was not interviewed by better firms or was unsuccessful in their interviews, meaning they have low match quality. Thus, in the centralized regime, lower-ranked firms become more averse to interviewing candidates whom top firms are interviewing.

This adverse selection issue has important implications in the decentralized regime as well. When match values were independent, the sequential interviewing procedure from Section 3.3 was an equilibrium in the decentralized regime with non-binding offers.

Observation 4.1. *The sequential-hiring mechanism is not an equilibrium of the decentralized regime with non-binding offers when match values are common across firms.*

Why? The top-ranked firm has no incentive to interview and screen first. Instead, it would prefer to wait until the end to see whom the other firms extended offers to. Such an economic force mirrors that in Ely and Siegel (2013). We conjecture that this free-riding issue from public offers leads to equilibria where all firms interview and match with the same set of workers as they would have in the centralized regime.

Risk Aversion In the decentralized regime with binding offers, we assumed workers were sufficiently risk averse. Such an environment generates the most concern amongst labor market participants and organizers, as it makes exploding offers an effective tactic. In our model, risk aversion is captured by the relative size of the workers' payoffs from being matched to different firms and the payoff associated with being unmatched. These payoffs determine workers' incentives to accept or reject early offers. To understand the crucial role of risk aversion, consider the opposite extreme, where any worker will reject an offer from a firm if they know they will be interviewed by a better firm later. Then, it can be shown that in any equilibrium of the decentralized regime with binding offers, no firm will front-run a higher-ranked one. Hence, such a regime is equivalent to the decentralized setting with non-binding offers.

4.2 Interpretation of Results

To understand the welfare consequences of any matching mechanism and the trade-offs between matching mechanisms, one must consider their effect on interviewing decisions. A common criticism of a decentralized environment with binding offers is that exploding offers prohibit workers from interviewing at more preferable firms later in the hiring cycle. Colloquially, the usual story is "Person A received an exploding offer from Firm X, and could not interview at Firm Y. We must centralize the market so that Person A can interview at X and Y, and compare offers." But it is not necessarily the case that in the centralized system, Person A would have received an interview from any of those firms! Interviewing decisions are an equilibrium object sensitive to the matching mechanism.

In particular, the nature of a centralized matching process to make the timing of decisions simultaneous generates coordination issues at the interview stage. Each firm must incorporate the externality imposed by the interview decisions of its competitors when deciding on its interview list. This is not to say that centralized mechanisms should be done away with. For instance, recall our assumption of a continuum of workers in our analysis. Such an assumption is substantive because it implies no uncertainty about yield. By the exact law of large numbers, firms can use a cutoff score and ensure they match with exactly Δ applicants. In a discrete setting, firms must be concerned about whether more than Δ candidates accept or less than Δ accept. An important property of centralized matching mechanisms is they guarantee that a firm will never be matched with more than its capacity.

Nevertheless, one benefit of a decentralized system is that the interview coordination issues can be mitigated due to matches occurring over time. Firm and worker exit can lead to more effective interviewing for the remaining firms. As an illustrative example, early-action programs in US undergraduate college admissions function to reduce application congestion in the regular admissions cycle. Now, in a completely decentralized environment where firms can make binding offers (e.g. exploding offers), the incentive of firms to front-run can reduce some of these benefits. Hence, the optimal system is a hybrid of these two systems. On the one hand, we want to allow firms to interview and make public offers at different times, but we also want to prohibit front-running. If this is achievable, we then need firms to interview in rank order by nudging top firms to interview first.

How would such a system be implemented in practice? Theoretically, one could institute a rule that offers are non-binding until a given date (e.g. ban exploding offers). This exists in the college athletic scholarship market. Unlike this market, though, most labor markets lack a governing body that can enforce such a rule. Thus, an alternative is to encourage a culture of reneging: explain to candidates that before a common date, they are free to change their mind about an accepted offer if a more preferred offer comes along.

A PROOFS

Proof of Lemma 1: Suppose G has increasing k-adjusted yields. Then for each $a' > a$ and each tuple of greedy hiring policies $\{x, \{x_f\}_{f=1}^k\} \subset \mathscr{X}$, we have

$$\int x(s)sdG(s|a') \prod_{f=1}^{k} \left(\int (1 - x_f(s))dG(s|a') \right) \geq \int x(s)sdG(s|a) \prod_{f=1}^{k} \left(\int (1 - x_f(s))dG(s|a) \right). \quad (1)$$

The claim follows from (1) by choosing $x_f(s) = 0$ for each $f > k'$ and each s. $\square$

Lemma 2. *For any $\beta \in (0,1)$ G_β has increasing adjusted yields. For any decreasing λ : $[0,1] \to \mathbb{R}_+$, G_λ has increasing k-adjusted yields for some $k \geq 1$.*

Proof. Given a hiring rule $x_f \in \mathscr{X}$, let $\bar{s}_f = \inf\{s | x_f(s) > 0\}$.

For an exponential distribution $H(t|\lambda) = 1 - e^{-\lambda t}$:

$$\left(\int (1 - x_1(s))dH(s|\lambda) \right) \cdot \int x(s)sdH(s|\lambda) = (1 - e^{-\bar{s}_1 \lambda}) \cdot e^{-\lambda \bar{s}} \cdot (\bar{s} + \frac{1}{\lambda})$$

Differentiating with respect to λ yields:

$$\frac{e^{-\lambda(\bar{s}_1 + \bar{s})}}{\lambda^2} \cdot \left[\lambda(\lambda x + 1)(\bar{s}_1 + \bar{s}) - e^{\lambda \bar{s}_1}(\lambda^2 \bar{s}^2 + \lambda \bar{s} + 1) + 1 \right]$$

This quantity is less than or equal to 0 if and only if:

$$\lambda(\lambda \bar{s} + 1)(\bar{s}_1 + \bar{s}) - e^{\lambda \bar{s}_1}(\lambda^2 \bar{s}^2 + \lambda \bar{s} + 1) + 1 \leq 0$$

$$\iff (\lambda^2 \bar{s}^2 + \lambda \bar{s} + 1)(1 - e^{-\lambda \bar{s}_1}) + \lambda \bar{s}_1(\lambda \bar{s} + 1) \leq 0 \iff 1 - e^{-\lambda \bar{s}_1} > 1 + \lambda \bar{s}_1$$

Thus, $H(\bar{s}_1|\lambda) \cdot (1 - H(\bar{s}|\lambda)) \cdot \mathbb{E}_H[t | t \geq \bar{s}]$ is decreasing in λ for all $\bar{s}$ and $\bar{s}_1$. Given a decreasing function $\lambda(\cdot) : [0,1] \to (0, \infty)$, let $G_\lambda(s|a) = H(s|\lambda(a))$. The result follows.

For the "Interviewing for bad news" environment, $G_\beta(s|a) = \begin{cases} 1 & \text{if } s \geq a \\ \beta & \text{if } s < a \end{cases}$.

$$\implies \int x(s)dG(s|a) \cdot \prod_{f=1}^{k} \left(\int (1 - x_f(s))dG(s|a) \right) = (1 - \beta) \cdot x(a) \cdot a \cdot \prod_{f=1}^{k} [\beta \cdot (1 - x_f(0)) + (1 - \beta) \cdot (1 - x_f(a))]$$

Now, $x, x_f \in \mathscr{X} \implies x, x_f$ increasing in $a \implies$ the expression above is increasing in a. $\square$

Lemma 3 (Greedy Interviewing is Optimal)**.** *Let $\underline{a}, \overline{a} \in [0,1]$ and suppose that $\phi : [0,1] \to [0,1]$ and $\psi : [0,1] \to [0,1]$ are such that either (i) $\psi(a) = 1$ on $[\underline{a}, \overline{a}]$, or (ii) $\phi(a) = 1$, $G(\cdot|a)$ is increasing in a in the hazard rate order $\succeq_{HR}$, and for each greedy hiring policy x, $\int x(s)s\psi(a)dG(s|a)$ is nondecreasing in a on $[\underline{a}, \overline{a}]$. Then for any interview set $I_f \subseteq [0,1]$ and* *greedy hiring policy x_f, there exists $a_f \in [\underline{a}, \overline{a}]$ and a greedy hiring policy x'_f such that*

$$\int_{(I_f \setminus [\underline{a},\overline{a}]) \cup [a_f,\overline{a}]} \left(\int_0^\infty x'_f(s) s \psi(a) dG(s|a) - c \right) \phi(a) da \geq \int_{I_f} \left(\int_0^\infty x_f(s) s \psi(a) dG(s|a) - c \right) \phi(a) da,$$
$$\text{and} \int_{(I_f \setminus [\underline{a},\overline{a}]) \cup [a_f,\overline{a}]} \left(\int x'_f(s) dG(s|a) \right) \psi(a) \phi(a) da \leq \int_{I_f} \left(\int x_f(s) dG(s|a) \right) \psi(a) \phi(a) da. \qquad (2)$$

Proof. Choose a_f so that $\int_{a_f}^{\overline{a}} \phi(a) da = \int_{I_f \cap [\underline{a},\overline{a}]} \phi(a) da$. Let $\overline{S}$ be a random variable with distribution $\overline{G}$, and $\underline{S}$ be a random variable with distribution $\underline{G}$, where

$$\overline{G}(s) \equiv \frac{\int_{[a_f,\overline{a}] \setminus I_f} G(s|a) \psi(a) \phi(a) da}{\int_{[a_f,\overline{a}] \setminus I_f} \psi(a) \phi(a) da}, \qquad \underline{G}(s) \equiv \frac{\int_{[\underline{a},a_f] \cap I_f} G(s|a) \psi(a) \phi(a) da}{\int_{[\underline{a},a_f] \cap I_f} \psi(a) \phi(a) da}.$$

Step 1: $\overline{S} \succeq_{FOSD} \underline{S}$. Since $G(s|a') \leq G(s|a)$ for all $a' > a$, for all $s \geq 0$, we have

$$G(s|a')\psi(a')\phi(a')\psi(a)\phi(a) \leq G(s|a)\psi(a')\phi(a')\psi(a)\phi(a) \text{ for all } a' > a$$
$$\Rightarrow \int_{[a_f,\overline{a}] \setminus I_f} G(s|a)\psi(a)\phi(a) da \int_{[\underline{a},a_f] \cap I_f} \psi(a)\phi(a) da \leq \int_{[\underline{a},a_f] \cap I_f} G(s|a)\psi(a)\phi(a) da \int_{[a_f,\overline{a}] \setminus I_f} \psi(a)\phi(a) da$$
$$\overline{G}(s) \leq \underline{G}(s),$$

as desired.

Step 2: *In case (ii),* $\overline{S} \succeq_{HR} \underline{S}$. Given $s' > s$, since $(1 - G(s'|a'))(1 - G(s|a)) \geq (1 - G(s|a'))(1 - G(s'|a))$ for all $a' > a$, we have

$$(1 - G(s'|a'))\psi(a')\phi(a')(1 - G(s|a))\psi(a)\phi(a) \geq (1 - G(s'|a))\psi(a)\phi(a)(1 - G(s|a'))\psi(a')\phi(a') \text{ for all } a' > a$$
$$\int_{[a_f,\overline{a}] \setminus I_f} (1 - G(s'|a))\psi(a)\phi(a) da \int_{[\underline{a},a_f] \cap I_f} (1 - G(s|a))\psi(a)\phi(a) da$$
$$\geq \int_{[\underline{a},a_f] \cap I_f} (1 - G(s'|a))\psi(a)\phi(a) da \int_{[a_f,\overline{a}] \setminus I_f} (1 - G(s|a))\psi(a)\phi(a) da$$
$$\left(1 - \frac{\int_{[a_f,\overline{a}] \setminus I_f} G(s'|a)\psi(a)\phi(a) da}{\int_{[a_f,\overline{a}] \setminus I_f} \psi(a)\phi(a) da}\right) \left(1 - \frac{\int_{[\underline{a},a_f] \cap I_f} G(s|a)\psi(a)\phi(a) da}{\int_{[\underline{a},a_f] \cap I_f} \psi(a)\phi(a) da}\right)$$
$$\geq \left(1 - \frac{\int_{[\underline{a},a_f] \cap I_f} G(s'|a)\psi(a)\phi(a) da}{\int_{[\underline{a},a_f] \cap I_f} \psi(a)\phi(a) da}\right) \left(1 - \frac{\int_{[a_f,\overline{a}] \setminus I_f} G(s|a)\psi(a)\phi(a) da}{\int_{[a_f,\overline{a}] \setminus I_f} \psi(a)\phi(a) da}\right)$$

$$\Longrightarrow (1 - \overline{G}(s'))(1 - \underline{G}(s)) \geq (1 - \underline{G}(s'))(1 - \overline{G}(s)) \text{ as desired}$$

Step 3: *There exists a greedy hiring policy $\overline{x_f}$ such that*

$$\int_{[a_f,\overline{a}] \setminus I_f} \int_0^\infty \overline{x_f}(s) s \psi(a)\phi(a) dG(s|a) da \geq \int_{[\underline{a},a_f] \cap I_f} \int_0^\infty x_f(s) s \psi(a)\phi(a) dG(s|a) da,$$
$$\text{and} \int_{[a_f,\overline{a}] \setminus I_f} \left(\int \overline{x_f}(s) dG(s|a) \right) \psi(a)\phi(a) da \leq \int_{[\underline{a},a_f] \cap I_f} \left(\int x_f(s) dG(s|a) \right) \psi(a)\phi(a) da. \qquad (3)$$

Let $M = \int_{[\underline{a},a_f]\cap I_f} \left(\int x_f(s)dG(s|a)\right)\psi(a)\phi(a)da$, and

$$\begin{aligned} \overline{\Psi} &= \textstyle\int_{[a_f,\overline{a}]\setminus I_f} \psi(a)\phi(a)da, & \underline{\Psi} &= \textstyle\int_{[\underline{a},a_f]\cap I_f} \psi(a)\phi(a)da, \\ \overline{s_f} &\equiv \overline{G}^{-1}\left(\max\{1-M/\overline{\Psi},0\}\right), & \underline{s_f} &\equiv \underline{G}^{-1}\left(1-M/\underline{\Psi}\right), \\ \overline{p_f} &\equiv \min\left\{\frac{M/\overline{\Psi}-(1-\overline{G}(\overline{s_f}))}{\int_{\{\overline{s_f}\}} d\overline{G}(s)}, 1\right\}, & \underline{p_f} &\equiv \frac{M/\underline{\Psi}-(1-\underline{G}(\underline{s_f}))}{\int_{\{\underline{s_f}\}} d\underline{G}(s)}, \end{aligned} \qquad \overline{x_f}(s) = \begin{cases} 1, & s > \overline{s_f}; \\ \overline{p_f}, & s = \overline{s_f}; \\ 0, & s < \overline{s_f}, \end{cases}$$

Observe that since x_f is a greedy hiring policy, we must have $x_f(s) = \begin{cases} 1, & s > \underline{s_f}; \\ \underline{p_f}, & s = \underline{s_f}; \\ 0, & s < \underline{s_f}, \end{cases}$

Moreover, by construction,

$$\int_{[a_f,\overline{a}]\setminus I_f} \left(\int \overline{x_f}(s)dG(s|a)\right)\psi(a)\phi(a)da = \min\{M,\overline{\Psi}\} \tag{4}$$

$$\int_{[a_f,\overline{a}]\setminus I_f} \left(\int \overline{x_f}(s)sdG(s|a)\right)\psi(a)\phi(a)da = \overline{\Psi}\int \overline{x_f}(s)sd\overline{G}(s) = \overline{\Psi}\int_{\max\{1-M/\overline{\Psi},0\}}^{1} \overline{G}^{-1}(q)dq; \tag{5}$$

$$\int_{[\underline{a},a_f]\cap I_f} \left(\int x_f(s)sdG(s|a)\right)\psi(a)\phi(a)da = \underline{\Psi}\int x_f(s)sd\underline{G}(s) = \underline{\Psi}\int_{1-M/\underline{\Psi}}^{1} \underline{G}^{-1}(q)dq. \tag{6}$$

Hence, (3) holds.

First consider case (i). Then by construction of a_f,

$$\overline{\Psi} = \int_{[a_f,\overline{a}]\setminus I_f} \phi(a)da = \int_{[\underline{a},a_f]\cap I_f} \phi(a)da = \underline{\Psi}.$$

Then $M/\overline{\Psi} = M/\underline{\Psi} \le 1$. By Step 1, $\overline{S} \succeq_{FOSD} \underline{S}$, and so $\overline{G}^{-1}(q) \ge \underline{G}^{-1}(q)$ for all $q \in [0,1]$. Then

$$\begin{aligned} \int_{1-M/\overline{\Psi}}^{1} \overline{G}^{-1}(q)dq &\ge \int_{1-M/\overline{\Psi}}^{1} \underline{G}^{-1}(q)dq = \int_{1-M/\underline{\Psi}}^{1} \underline{G}^{-1}(q)dq, \\ \Rightarrow \overline{\Psi}\int_{1-M/\overline{\Psi}}^{1} \overline{G}^{-1}(q)dq &\ge \underline{\Psi}\int_{1-M/\underline{\Psi}}^{1} \underline{G}^{-1}(q)dq. \end{aligned}$$

The claim then follows from (5) and (6).

Now consider case (ii). First suppose that either $\overline{s_f} > \underline{s_f}$, or $\overline{s_f} = \underline{s_f}$ and $\overline{p_f} < \underline{p_f}$. Then $\overline{x_f}(s) \le x_f(s)$ for each s. Moreover, we must have $M \le \overline{\overline{\Psi}}$: Suppose not. Then $\underline{s_f} \le \overline{s_f} = \overline{G}^{-1}(0) = 0$. So we must have $\overline{s_f} = \underline{s_f}$ and $\overline{p_f} = 1 < \underline{p_f}$, a contradiction.

Define the weighted random variables $\underline{S}_{x_f} \sim \underline{G}_{x_f}$, $\overline{S}_{x_f} \sim \overline{G}_{x_f}$, and $\overline{S}_{\overline{x_f}} \sim \overline{G}_{\overline{x_f}}$, where

$$\underline{G}_{x_f}(s) = \frac{\int_0^s x_f(t)d\underline{G}(t)}{\int_0^\infty x_f(t)d\underline{G}(t)}, \quad \overline{G}_{x_f}(s) = \frac{\int_0^s x_f(t)d\overline{G}(t)}{\int_0^\infty x_f(t)d\overline{G}(t)}, \quad \overline{G}_{\overline{x_f}}(s) = \frac{\int_0^s \overline{x_f}(t)d\overline{G}(t)}{\int_0^\infty \overline{x_f}(t)d\overline{G}(t)} = \frac{\int_0^s w(t)x_f(t)d\overline{G}(t)}{\int_0^\infty w(t)x_f(t)d\overline{G}(t)},$$

where $w(t) = \begin{cases} \overline{x_f}(t)/x_f(t), & t \ge \overline{s_f}; \\ 0, & t < \overline{s_f}. \end{cases}$

From Step 2 and Bartoszewicz and Skolimowska (2006) Theorem 9, $\overline{S}_{x_f} \succeq_{HR} \underline{S}_{x_f}$, and hence $\overline{S}_{x_f} \succeq_{FOSD} \underline{S}_{x_f}$. Since $\overline{x_f}(t) \leq x_f(t)$ for each t, w is nondecreasing; then by Theorem 1 in Błażej (2008)[15] $\overline{S}_{\overline{x_f}} \succeq_{FOSD} \overline{S}_{x_f}$. It follows that

$$\frac{\int_{[a_f,\overline{a}]\setminus I_f}\left(\int s\overline{x_f}(s)dG(s|a)\right)\psi(a)\phi(a)da}{\int_{[a_f,\overline{a}]\setminus I_f}\left(\int \overline{x_f}(s)dG(s|a)\right)\psi(a)\phi(a)da} = E[\overline{S}_{\overline{x_f}}] \geq E[\underline{S}_{x_f}] = \frac{\int_{[\underline{a},a_f]\cap I_f}\left(\int sx_f(s)dG(s|a)\right)\psi(a)\phi(a)da}{\int_{[\underline{a},a_f]\cap I_f}\left(\int x_f(s)dG(s|a)\right)\psi(a)\phi(a)da}.$$

Since $M \leq \overline{\Psi}$, the claim then follows from (4).

Alternatively, consider the case where either $\overline{s_f} < \underline{s_f}$, or $\overline{s_f} = \underline{s_f}$ and $\overline{p_f} \geq \underline{p_f}$. Then $\overline{x_f}(s) \geq x_f(s)$ for each s, and we have

$$\int_{[a_f,\overline{a}]\setminus I_f}\int \overline{x_f}(s)s\psi(a)\phi(a)dG(s|a)da \geq \int_{[a_f,\overline{a}]\setminus I_f}\int x_f(s)s\psi(a)\phi(a)dG(s|a)da \geq \int_{[\underline{a},a_f]\cap I_f} sx_f(s)\psi(a)\phi(a)dG(s|a)da,$$

where the second inequality holds since, by assumption, $\phi(a) = 1$ and $\int x_f(s)s\psi(a)dG(s|a)$ is nondecreasing in a on $[\underline{a},\overline{a}]$. The claim then follows from (4).

Step 4: $\int_{(I_f\setminus[\underline{a},\overline{a}])\cup[a_f,\overline{a}]} c\phi(a)da = \int_{I_f} c\phi(a)da$. Follows from construction of a_f.

Step 5: *Statement of Lemma 3.* By Step 3, there exists a greedy hiring policy $\overline{x_f}$ such that

$$\begin{array}{l}\int_{I_f\setminus[\underline{a},a_f]}\int x_f(s)s\psi(a)\phi(a)dG(s|a)da \\ +\int_{[a_f,\overline{a}]\setminus I_f}\int \overline{x_f}(s)s\psi(a)\phi(a)dG(s|a)da\end{array} \geq \int_{I_f}\int_{s_f}^{\infty} s\psi(a)\phi(a)dG(s|a)da, \tag{7}$$

$$\text{and } \begin{array}{l}\int_{I_f\setminus[\underline{a},a_f]}\int x_f(s)dG(s|a)\psi(a)\phi(a)da \\ +\int_{[a_f,\overline{a}]\setminus I_f}\int \overline{x_f}(s)dG(s|a)\psi(a)\phi(a)da\end{array} \leq \int x_f(s)dG(s|a)\psi(a)\phi(a)da.$$

Then the left-hand side of (7) must be no greater than the value of

$$\max_{x,y\in\mathscr{X}} \int_{I_f\setminus[\underline{a},a_f]}\int x(s)s\psi(a)\phi(a)dG(s|a)da + \int_{[a_f,\overline{a}]\setminus I_f}\int y(s)s\psi(a)\phi(a)dG(s|a)da \tag{8}$$

$$\text{s.t. } \begin{array}{l}\int_{I_f\setminus[\underline{a},a_f]}\int x(s)dG(s|a)\psi(a)\phi(a)da \\ +\int_{[a_f,\overline{a}]\setminus I_f}\int y(s)dG(s|a)\psi(a)\phi(a)da\end{array} \leq \int_{I_f}\int x_f(s)dG(s|a)\psi(a)\phi(a)da. \tag{9}$$

Clearly, it is without loss to consider $x = y$ in (8): Then let x^* be such that (x^*,x^*) solves (8), and choose $x'_f = x^*$; the statement of Lemma 3 then follows from Step 4. □

The intuition for Lemma 3 is simplest when $\phi(a) = \psi(a) = 1$ for all a — the relevant case for firm 1. Given an interview set I, we can construct a greedy interview set $[1-\mu(I),1]$ with the same mass, and keep the hiring policy the same for those workers interviewed in both sets.

[15] Since w is nondecreasing, we have (in the notation of Błażej (2008))

$$\overline{G}^*_{x_f}(u) - u = \frac{1}{\int w(s)d\overline{G}_{x_f}(s)}\left((1-u)\int_0^u w(\overline{G}_{x_f}^{-1}(z))dz - u\int_u^1 w(\overline{G}_{x_f}^{-1}(z))dz\right) \leq \frac{(1-u)uw(\overline{G}_{x_f}^{-1}(u)) - (1-u)uw(\overline{G}_{x_f}^{-1}(u))}{\int w(s)d\overline{G}_{x_f}(s)} = 0.$$

Then, for the workers that are interviewed in the greedy set but not in I, we choose a new greedy hiring policy, so that the firm hires the same mass of them as it would have hired from $I \setminus [1-\mu(I), 1]$ (the workers interviewed in I but not in the greedy set). First-order stochastic dominance then ensures that conditional on being hired, the average match value of workers only interviewed in the greedy set is higher than the average match value of workers only interviewed in the original set I.

Corollary 1. *For any $\phi : [0,1] \to [0,1]$, there exists $a^* \in [0,1]$ such that*

$$([a^*,1],x^*) \in \arg\max_{\substack{I \subseteq [0,1] \\ x \in \mathscr{X}}} \int_I \left(\int x(s) s dG(s|a) - c \right) \phi(a) da \text{ s.t. } \int_I \left(\int x(s) dG(s|a) \right) \phi(a) da \leq \Delta, \quad (10)$$

$$\text{where } x^* = \max_{\succ_{\mathscr{X}}} \left\{ x \in \mathscr{X} \mid \int_I \left(\int x(s) dG(s|a) \right) \phi(a) da \leq \Delta \right\}.$$

Proof. It is immediate from case (i) of Lemma 3 that there exists $a^* \in [0,1]$ such that for some $x \in \mathscr{X}$, $([a^*,1],x)$ solves (10). The claim follows by noting that replacing x with $x' >_{\mathscr{X}} x$ can only increase the value of the objective in (10). □

Proof of Proposition 2: Each (possibly mixed) interview strategy I_f can be represented as a function $\mu : [0,1] \to [0,1]$. Let $\mathscr{Y}$ be the set of all such functions. $\mathscr{Y}$ is compact because it is a complete metric space and totally bounded under the sup-norm. Since the payoff functions are continuous, a solution exists to the following recursive system:

$$I_f^* \in \arg\max_{I_f \in \mathscr{Y}} \int_0^1 I_f(a) \Big[\int_0^\infty s x_f^*(s; I_f) dG(s|a) \Big(\prod_{h<f} (1 - I_h^*(a)) \int_0^\infty (1 - x_h(s)) dG(s|a) \Big) - c \Big] da$$

$$\text{where } x_f^*(\cdot; I_f) = \max_{\succ_{\mathscr{X}}} \left\{ x \in \mathscr{X} \,|\, \int_{I_f} \int_0^\infty x_f(s) dG(s|a) \left(\prod_{h<f} (1 - I_h^*(a)) \int_0^\infty (1 - x_h(s)) dG(s|a) \right) da \leq \Delta \right\}$$

It is clear that any function $I_f^* \in \mathscr{Y}$ such that $I_f(W) \notin \{0,1\}$ for some measurable set W can not be optimal. Therefore, the recursive system above is equivalent to:

$$I_f^* \in \arg\max_{I_f \subseteq [0,1]} \int_{I_f} \int_0^\infty s x_f^*(s) dG(s|a) \Big(\prod_{h<f, a \in I_h^*} \int_0^\infty (1 - x_h(s)) dG(s|a) \Big) - c \, da$$

$$\text{where } x_f^*(\cdot; I_f) = \max_{\succ_{\mathscr{X}}} \left\{ x \in \mathscr{X} \,|\, \int_{I_f} \int_0^\infty x_f(s) dG(s|a) \left(\prod_{h<f, a \in I_h} \int_0^\infty (1 - x_h(s)) dG(s|a) \right) da \leq \Delta \right\} \quad \square$$

Proof of Proposition 3:

(i): Follows immediately from Corollary 1.

(iia) and (iib): We proceed by induction, beginning with $f=2$. Let

$$\psi_2(a)=\begin{cases}\int(1-x_1^*(s))dG(s|a), & a\geq a_1^*;\\ 1, & a<a_1^*.\end{cases}$$

Since G has increasing k-adjusted yields, by Lemma 1, it has increasing 1-adjusted yields, and so for any greedy hiring policy x, $\int x(s)s\psi_2(a)dG(s|a)$ is nondecreasing on $[a_1^*,1]$. Then by case (ii) of Lemma 3 (letting $\underline{a}=a_1^*$ and $\overline{a}=1$), for any $I_2\subseteq[0,1]$ and $x\in\mathscr{X}$, there exists $a_2^{\{1\}}\in[a_1^*,1]$ and $x'\in\mathscr{X}$ such that

$$\begin{gathered}\int_{(I_2\setminus I_1^*)\cup[a_2^{\{1\}},\overline{a}]}\left(\int x'(s)s\psi_2(a)dG(s|a)\right)-cda\geq\int_{I_2}\left(\int x(s)s\psi_2(a)dG(s|a)\right)-cda,\\ \text{and }\int_{(I_2\setminus I_1^*)\cup[a_2^{\{1\}},\overline{a}]}\int x'(s)dG(s|a)\psi_2(a)da\leq\int_{I_2}\int x(s)dG(s|a)\psi_2(a)da.\end{gathered}$$

Applying case (i) of Lemma 3, letting $\underline{a}=0$ and $\overline{a}=a_1^*$, shows that there exists $a_2^*\in[0,a_1^*]$ and $x''\in\mathscr{X}$ such that

$$\begin{gathered}\int_{[a_2,a_1^*)\cup[a_2^{\{1\}},\overline{a}]}\left(\int x''(s)s\psi_2(a)dG(s|a)\right)-cda\geq\int_{I_2}\left(\int x(s)s\psi_2(a)dG(s|a)\right)-cda,\\ \text{and }\int_{[a_2,a_1^*)\cup[a_2^{\{1\}},\overline{a}]}\int x''(s)dG(s|a)\psi_2(a)da\leq\int_{I_2}\int x(s)dG(s|a)\psi_2(a)da.\end{gathered}\tag{11}$$

It follows that for some greedy hiring rule x_2^* and some $a_2^{\{1\}*}\in[a_1^*,1]$ and $a_2^*\in[0,a_1^*]$,,

$$(I_2^*,x_2^*)\in\arg\max_{\substack{I\subseteq[0,1]\\ x\in\mathscr{X}}}\int_I\left(\int x(s)s\psi_2(a)dG(s|a)\right)-cda \text{ s.t. }\int_I\left(\int x(s)\psi_2(a)dG(s|a)\right)da\leq\Delta.$$

where $I_2^*=[a_2^*,a_1^*)\cup[a_2^{\{1\}*},1]$. Since replacing x'' with $\tilde{x}>_{\mathscr{X}}x''$ cannot decrease the value of the left-hand side of (11), $x_2^*=\max_{\succ_{\mathscr{X}}}\left\{x\in\mathscr{X}\mid\int_{[a_2^*,a_1^*]\cup[a_2^{\{1\}*},\overline{a}]}\left(\int x(s)\psi_2(a)dG(s|a)\right)da\leq\Delta\right\}$. Consequently, $I_2^*\in\arg\max_{I_2}\pi_2(I_1^*,I_2)$. Hence, (iia) and (iib) hold for $f=2$.

Now consider $2<f\leq k$, and suppose that (iia) and (iib) hold for each $2\leq f'<f$. Then for each $S\subseteq\{1,\ldots,f-1\}$, $\bigcap_{j\in S}I_j^*$ is an interval. Let $\psi_f(a)=\prod_{h<f,a\in I_h^*}\left(\int(1-x_h(s))dG(s|a)\right)$. Since G has increasing k-adjusted yields, by Lemma 1, it has increasing k'-adjusted yields for each $k'<k$. Then for any greedy hiring policy x and any $S\subseteq\{1,\ldots,f-1\}$, $\int x(s)s\psi_f(a)dG(s|a)$ is nondecreasing in a on $\bigcap_{j\in S}I_j^*$. Then applying case (ii) of Lemma 3 once for each $S\subseteq\{1,\ldots,f-1\}$ (letting $\underline{a}=\min\bigcap_{j\in S}I_j^*$ and $\overline{a}=\max\bigcap_{j\in S}I_j^*$), for any $I_f\subseteq[0,1]$ and $x\in\mathscr{X}$,

there exist $\{a_f^S\}_{S\subseteq\{1,\dots,f-1\}}$ and $x'\in\mathscr{X}$ such that for $I'_f=\bigcup_{S\subseteq\{1,\dots,f-1\}}\left([a_f^S,1]\cap\bigcap_{j\in S}I_j^*\right)$,

$$\int_{I'_f}\left(\int x'(s)s\psi_f(a)dG(s|a)\right)-cda\geq\int_{I_f}\left(\int x(s)s\psi_f(a)dG(s|a)\right)-cda, \tag{12}$$
$$\text{and }\int_{I'_f}\left(\int x'(s)\psi_f(a)dG(s|a)\right)da\leq\int_{I_f}\left(\int x(s)\psi_f(a)dG(s|a)\right)da.$$

It follows that for some greedy hiring rule x_f^* and some $\{a_f^{S*}\}_{S\subseteq\{1,\dots,f-1\}}$, we have

$$\left(I_f^*,x_f^*\right)\in\arg\max_{\substack{I\subseteq[0,1]\\x\in\mathscr{X}}}\int_I\left(\int x(s)s\psi_f(a)dG(s|a)\right)-cda\text{ s.t. }\int_I\left(\int x(s)\psi_f(a)dG(s|a)\right)da\leq\Delta.$$

for $I_f^*=\bigcup_{S\subseteq\{1,\dots,f-1\}}\left([a_f^{S*},1]\cap\bigcap_{j\in S}I_j^*\right)$. Since replacing x' with $\tilde{x}>_{\mathscr{X}}x'$ cannot decrease the value of the left-hand side of (12), it follows that $x_f^*=\max_{\succ_{\mathscr{X}}}\left\{x\in\mathscr{X}\mid\int_{I_f^*}\left(\int x(s)\psi_f(a)dG(s|a)\right)da\leq\Delta\right\}$. Consequently, $I_f^*\in\arg\max_{I_f}\pi_f(I_f,\{I_h^*\}_{h<f})$. The claim follows by induction. □

To prove Propositions 4-6, we will need to make use of the submodularity of a firm's payoff function. In this context, submodularity means the marginal gain from interviewing an additional applicant is higher for firms interviewing weakly fewer and worse applicants.

Definition A.1. *A function f is submodular if for any $V,Y,Z\subset[0,1]$ such that $V\subset Y$ and $Z\cap Y=\emptyset$ then:*

$$f(V\cup Z)-f(V)>f(Y\cup Z)-f(Y)$$

Lemma 4. *For any $\psi:[0,1]\to[0,1]$, consider the following function f:*

$$f(Y,\psi)=\int_Y\psi(a)\left[\int_0^\infty sx^*(s;Y,\psi)dG(s|a)-c\right]da$$

$$\textit{where } x^*(\cdot;Y,\psi)=\max\left\{x\in\mathscr{X}\,|\int_Y\int_0^\infty x_f(s)dG(s|a)\cdot\psi(a)da\leq\Delta\right\}$$

i $f(\cdot,\psi)$ and $f(X,\psi)+\int_X\psi(a)cda$ are submodular.

ii For any $Y\subset[0,1],\psi:[0,1]\to[0,1]$, $\hat{\psi}:[0,1]\to[0,1]$ such that $\hat{\psi}(a)\leq\psi(a)$ for all $a\in Y$ and $\hat{\psi}(a)=\psi(a)$ for all $a\notin Y$, then:

$$f(Y\cup Z,\hat{\psi})-f(Y,\hat{\psi})>f(Y\cup Z,\psi)-f(Y,\psi)$$

$$\textit{for all } Z\subset[0,1],\ Z\cap Y=\emptyset$$

Proof of Proposition 4: We consider a two-firm setting with G exhibiting increasing k-adjusted yields and increasing hazard rate. Thus, Firm 1 interviews $I_1^*=[a_1,1]$ and Firm 2 interviews the set $I_2^*=[a_2^{(1)},1]\cup[a_2^{(2)},a_1]$ in equilibrium, with $a_2^{(1)}>a_1$.

If Firm 1 interviews a set X, define $\psi_2(a,X) = G(s_1^*|a)$, where we suppress dependence of s_1^* on X for notational convenience.

Given an interview set X_1 and X_2 Denote the payoffs to firm 1 by $\pi_1(X_1)$ and the payoffs to firm 2 by $\pi_2(X_2, \psi_2(\cdot, X_1))$. Notice that firm 2's payoff depends on firm 1's interview set through ψ_2. Take $Y = [1-\varepsilon, 1]$. If $1 - a_1 < 1 - a_2^1 + a_1 - a_2^2$ then for sufficiently small $\varepsilon > 0$, Lemma 4 implies:

$$\pi_1(I_1^*) - \pi_1(I_1^* \setminus Y) < \pi_2\Big(I_2^* \cup Y, \psi_2(\cdot, I_1^* \setminus Y)\Big) - \pi_2\Big(I_2^*, \psi_2(\cdot, I_1^*)\Big)$$

The proposition follows. □

Proof of Proposition 5: Define the following objects:

1. For any $S \subset \{1, \dots, n\}$ define $r(S,i)$ to be the rank of firm i in S.

2. For any $\psi : [0,1] \to [0,1]$ such that $\int_0^1 \psi(a)da \leq 1$, define the candidate pool associated with ψ to be the pair $(W_\psi, \int_0^1 \psi(a)da)$, where $W_\psi(a) = \frac{a}{\int_0^a \psi(s)ds}$. In other words, there is a measure $\int_0^1 \psi(a)da$ of applicants on $[0,1]$, and the conditional distribution is given by W_ψ. Due to the one-to-one correspondence, we will refer to ψ as the applicant pool. Thus, $\psi(a) = 1$ is the original applicant pool. In an abuse of notation, we will also use ψ to refer to the actual set of applicants.

3. For any $i \in \{1, \dots, n\}$ and candidate pool ψ, let $\sigma(i, \psi)$ be the optimal interview strategy for an i-th ranked firm in a centralized setting with candidate pool ψ.

4. Given a subset of firms S and applicant pool ψ, for each $i \in S$, let $C_i(S, \psi)$ denote the equilibrium payoff in a centralized matching setting where firms S face candidate pool ψ. $C_i(S, \psi)$ is pinned down due to firm $j \in S$ using equilibrium strategy $\sigma(r(S, j), \psi)$.

5. Consider a centralized matching setting with firms $S \subset \{1, \dots, F\}$ and an applicant pool ψ, Let $\mu(S, \psi)$ be the set of applicants firms in S match with in equilibrium. Define $\Xi(S, \psi) = \psi \setminus \mu(S, \psi)$.

We proceed by induction on the number of time periods.

Base Case: $T = 2$

Firms only have two choices: interview in time period 1 or 2. Let $A \in \{0,1\}^n$ denote the choice profile for each firm, where $A_i = 0$ means firm i is choosing to interview in period 1. Notice then, that for any A, firm i's payoff is uniquely pinned down:

1. If $A_i = 0$, then firm i's payoff is $\pi_i(A) = C_i(\{j | A_j = 0\}, \psi)$

2. If $A_i = 1$, then firm i's payoff is $\pi_i(A) = C_i\Big(\left\{j|A_j = 1\right\}, \Xi(\left\{j|A_j = 0\right\}, \psi)\Big)$

A mixed strategy equilibrium exists since there are a finite number of players and actions!

Inductive Step: Assume that a subgame perfect equilibrium exists for all $T \leq k$

For any subset of firms S and applicant pool ψ, let $E_i(T, S, \psi)$ denote the equilibrium payoff to firm i in a game with T time periods.

Case $T = k+1$:

Now, consider the following game: firms can choose to interview in time period 1 or defer to a later period. Let $A \in \{0,1\}^F$ denote the choice profile for each firm, where $A_i = 0$ means firm i is choosing to interview in period 1. Notice, then, that for any A, firm i's payoff is uniquely pinned down by the inductive hypothesis:

1. If $A_i = 0$, then firm i's payoff is $\pi_i(A) = C_i\Big(\left\{j|A_j = 0\right\}, \psi\Big)$

2. If $A_i = 1$, then firm i's payoff is $\pi_i(A) = E_i\left(k, \left\{j|A_j = 1\right\}, \Xi(\left\{j|A_j = 0\right\}, \psi)\right)$

A mixed strategy equilibrium exists in this game as there are two actions and F players. Equilibrium payoff must be the subgame perfect equilibrium payoffs.

Finally, in any mixed strategy equilibrium, each firm must weakly prefer playing the strategy to interviewing in period 1. Notice, though, that by interviewing in period 1, each firm i is guaranteed a payoff of at least $C_i(\{1, \ldots, n\}, \psi)$. □

Proof of Proposition 6: Part (i) follows immediately by applying Corollary 1 recursively for each firm f, letting $\phi(a) = \prod_{h<f, I_h \ni a}\left(\int(1 - x_h^*(s))dG(s|a)\right)$.

To prove (ii), notice that firm f faces a much smaller applicant pool than the firms ranked above it. Letting $\psi^{(f)}(a) = \prod_{h<f, a \in I_h^*} \int_0^\infty (1 - x_h^*(s))dG(s|a)G(s|a)$ be the probability applicant a is available by the time firm f interviews, the optimization problem for firm f is:

$$a_f \in \arg\max_k \int_k^1 \psi^{(f)}(a)\left[\int_0^\infty s x_f(s)dG(s|a)\right]da - c\int_k^1 d\psi^{(f)}(a)$$

$$x_f = \max\left\{x|x \in \mathscr{X}, \int_k^1 \left[\int_0^\infty x(s)dG(s|a)\right]\psi^{(f)}(a)da \leq \Delta\right\}$$

Recognize the following two properties of $\psi^{(i)}(\cdot)$:

1. $\psi^{(f)}(a) \le \psi^{(f')}(a)$ for $f \ge f'$ since firm f interviews after firm f' for all $f' \le f$.
2. $\psi^{(f)}(a) < \psi^{(f)}(a')$ for $a > a'$

Letting $\pi_f(k) = \int_k^1 \psi^{(f)}(a)[\int_0^\infty sx(s)dG(v|a)]da - c\int_k^1 d\psi^{(f)}(a)$, Lemma 4 implies that for sufficiently small $\varepsilon > 0$:

$$\pi_{i+1}(a_i) - \pi_{i+1}(a_i+\varepsilon) > \pi_i(a_i) - \pi_i(a_i+\varepsilon)$$

Since interviewing $[0,a_i]$ is optimal for firm i, $\pi_i(a_i) - \pi_i(a_i+\varepsilon) \ge c\varepsilon \implies a_{i+1} \ge a_i$.

Finally, (iii) follows immediately from (i) and the fact that there are more workers than total slots. $\square$

Proof of Proposition 7: Let a firm's interview decision be given by $\mu_f : [0,1] \to [0,1]$, where $\mu_f(a)$ is the fraction of *available* applicant a's that firm f interviews. Let the firm's hiring rule be denoted by $x_f(\cdot)$. Let μ denote the profile of firm interview decisions and define $\psi_f(\mu,a)$ recursively as follows:

$$\psi_1(\mu,a) = 1$$

$$\psi_f(\mu,a) = \psi_{f-1}(\mu,a)\cdot\Big(1-\mu_{f-1}(a)\cdot\int_0^\infty x_{f-1}(s)dG(s|a)\Big)$$

Total surplus in a setting where firms interview sequentially according to μ is given by:

$$S(\mu,\psi_1) = \sum_{f=1}^{F}\int_0^1 \mu_f(a)\psi_f(\mu,a)\Big[\int_{s_f}^\infty sg(s|a)ds\Big]da - \sum_{f=1}^{F} c\cdot\int_0^1 \mu_f(a)\psi_f(\mu,a)da$$

$$\text{where } x_f = \max\left\{x|x\in\mathscr{X}, \int_0^1 \mu_f(a)\psi_f(\mu,a)\cdot\Big[\int_0^\infty x(s)dG(s|a)\Big]da \le \Delta\right\}$$

Let μ_f^* and x_f^* be firm f's interview decision and hiring rule in the sequential-hiring equilibrium.

Consider the equilibrium of the centralized setting when the pool of workers is given by ψ. Denote the equilibrium interview strategy of a firm *ranked* f amongst those interviewing by $I_f^*(\psi)$. Total surplus generated by F firms is $Cent(\left\{I_f^*\right\}_{f=1}^F,\psi)$.

Now, it is easy to see that since $\mu_1^* = I_1^*$, we have:

$$Cent(I_f^*,\psi_1) \le \int_0^1 \mu_1^*(a)\psi_1(a)\Big[\int_0^\infty sx_1^*(s)dG(s|a)\Big]da - c\cdot\int_0^1 \mu_1^*(a)\psi_1(a)da + Cent(\{I_{f-1}^*\}_{f=2}^F,\psi_2)$$

Observe that by recursion, the right-hand side is weakly less than $S(\mu^*)$. $\square$

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