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Assignment 4

Instructions:

- All python codes to be made in Jupyter scientific notebook system and generate separate cell for providing solution for each sub-question.
- All commands asked to be executed in different sub-questions, should generate results in Jupyter (i.e. by using print statements generously or by generating the plots as asked for).
- Assignment solution should be submitted in two files with filename "ME502_Assignment$_EnrolmentNumber.ipynb" and "ME502_Assignment$_EnrolmentNumber.html", where $=4 for Assignment number 4 and EnrolmentNumber should be replaced with student enrolment number. For example, student "Ankit Kumar Pandey" with Enrolment number 2019MEM1004 should submit assignment 3 solution in two files with name "ME502_Assignment4_2019MEM1004.ipynb" and ME502_Assignment4_2019MEM1004.html. *html and *.ipynb should be uploaded in the respective instance of Assignment 4 on Turnitin, as will be shared with you all.
- If the file names are not as per the format given above, it will be considered not submitted and thus will lead to deduction of marks.
- Use NumPy, Pandas and Matplotlib libraries for solving the assignment
- For Plotting 3D imagery, use standard mpl_toolkit

Q1: The general form of a three-dimensional stress field is given by a 2nd order stress tensor as

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \tag{1}$$

where the diagonal terms represent tensile or compressive stresses and the off-diagonal terms represent shear stresses. A stress field is given by

$$\boldsymbol{\sigma} = \begin{bmatrix} 10 & 14 & 25 \\ 14 & 7 & 15 \\ 25 & 15 & 16 \end{bmatrix} \times 10^6 \,. \tag{2}$$

Principal stresses ($\sigma_1, \sigma_2, \sigma_3$) are known to be the three eigen values of stress tensor $\boldsymbol{\sigma}$. For any given stress tensor $\boldsymbol{\sigma}$, it is essential to construct the following matrix

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$$\begin{bmatrix} 10-\sigma & 14 & 25 \\ 14 & 7-\sigma & 15 \\ 25 & 15 & 16-\sigma \end{bmatrix} \times 10^6.$$

Such that $\sigma_1, \sigma_2, \sigma_3$ can be solved from the equation

$$\sigma^3 - I\sigma^2 + II\sigma - III = 0 \tag{3}$$

where

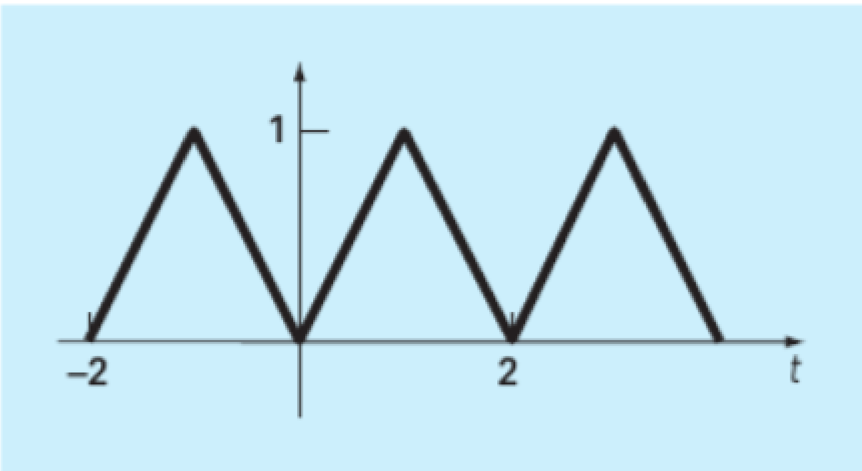
$$I = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$II = \sigma_{xx}\sigma_{yy} + \sigma_{xx}\sigma_{zz} + \sigma_{yy}\sigma_{zz} - \sigma_{xy}{}^2 - \sigma_{xz}{}^2 - \sigma_{yz}{}^2$$

$$III = \sigma_{xx}\sigma_{yy}\sigma_{zz} - \sigma_{xx}\sigma_{yz}{}^2 - \sigma_{yy}\sigma_{xz}{}^2 - \sigma_{zz}\sigma_{xy}{}^2 + 2\sigma_{xy}\sigma_{xz}\sigma_{yz}.$$

I, II, III are known as first invariant, second invariant and third invariant of stress tensor. For a given stress tensor $\boldsymbol{\sigma}$ in Equation (2), find the three roots of Equation (3) as first principal stress (σ_1), second principal stress (σ_2) and third principal stress (σ_3) using any of the root finding technique you have learnt by making a its python code. An inequality of the form $\sigma_1 > \sigma_2 > \sigma_3$ is considered to decide the first principal stress (σ_1), second principal stress (σ_2) and third principal stress (σ_3). This python code should ask the user to input value of 6 components $\sigma_{xx}, \sigma_{yy,}\sigma_{zz}, \sigma_{xy}, \sigma_{xz}$ and σ_{yz} of stress tensor, while the remaining three components can be obtained due to symmetric nature of stress tensor. **[3]**

Q2: Write a python code or use in-build python functions to use a continuous Fourier series to approximate the wave form shown in Fig. 1.



[3]

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Q3: The work equation is given as function of force *F(x) and angle θ (x) between force and the distance of movement (x) as:*

$$W = \int_{x_0}^{x_n} F(x) \cos[\theta(x)]\, dx \tag{4}$$

where x_o and x_n is taken as 0 and 30 respectively. The F(x) and angle θ (x) variation with respect to distance of movement (x) is given by following two equations as:

$$F(x) = 1.6x - 0.045x^2$$
$$\theta(x) = 0.8 + 0.125x - 0.009x^2 + 0.0002x^3$$

(a) Make a python code or use in-build python functions to compute the integral in equation (4) using 4-, 8- and 16- segment trapezoidal rules.

(b) Extend the python code developed in (a) or use in-build python functions to compute the integral in equation (4) using simpson's 1/3 rule.

(c) Extend the python code developed in (b) or use in-build python functions to compute the integral in equation (4) using Romberg integration to ε_s=0.5%.

(d) Extend the matlab code developed in (b) or use in-build python functions to compute the integral in equation (iv) using gauss quadrature.

At the end your code should print values of integral in following format:

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Trapezoidal 4: integration value

Trapezoidal 8: integration value

Trapezoidal 16: integration value

Simpson1/3: integration value

Romberg: integration value

Quadrature: integration value

[5]

Q4: Soft tissue follows an exponential deformation behavior in uniaxial tension while it is in the physiologic or normal range of elongation. This can be expressed as:

$$\sigma = \frac{E_o}{a}(e^{a\varepsilon} - 1) \tag{5}$$

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where σ = stress, ε = strain, and E_o and a are material constants that are determined experimentally. To evaluate the two material constants, the above equation is differentiated with respect to ε, which is a fundamental relationship for soft tissue

$$\frac{d\sigma}{d\varepsilon} = E_0 + a\sigma \tag{6}$$

To evaluate E_o and a, stress-strain data is used to plot $d\sigma/d\varepsilon$ versus σ and the slope and intercept of this plot are the two material constants, respectively. The following table contains stress-strain data for heart chordae tendineae (small tendons use to hold heart valves closed during contraction of the heart muscle). This is data from loading the tissue; different curves are produced on unloading.

$\sigma \times 10^3$ N/m^2	87.8	96.6	176	263	350	569	833	1227	1623	2105	2677	3378	4257
$\varepsilon \times 10^{-3}$ m/m	153	198	270	320	355	410	460	512	562	614	664	716	766

(a) Calculate the derivative $d\sigma/d\varepsilon$ using finite differences (using inbuilt function of python) that are second-order accurate. Plot the data and eliminate the data points near the zero points that appear not to follow the straight-line relationship. The error in this data comes from the inability of the instrumentation to read the small values in this region. Perform a regression analysis (using inbuilt function of python) of the remaining data points to determine the values of E_o and a. Plot the stress versus strain data points given in the above table along with the analytic curve expressed by the equation (5) using constants value E_o and a obtained after regression analysis in the same plot. This will indicate how well the analytic curve matches the data. In the plot, differentiate between two curves thus obtained by using different line type (dash --, solid -) having blue color.

[4]