Computational Physics Lab Continuous Assessment Exam

Time : 1 hour

February 04, 2016 (Session 1)

Answer all. Any deviation from instructions will lead to ***zero grade***

1. A continued fraction representation for the Golden Ratio ϕ is given by (as a sequence)

$$x_1 = 1 + \frac{1}{1}, \;\; x_2 = 1 + \frac{1}{1 + \frac{1}{1}}, \;\; x_3 = 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{1}}}, \;\; .... \;\; \phi = 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{1+......}}}$$

The sequence x_n converges to $\phi = 1.6180339887498948482045868343656...$ as the number of iterations goes to infinity.

(a) Write C++ function double mygold(int n), that evaluates the n-th order iterated value of ϕ i. e. x_n according to the above algorithm using **do...while** loop.

(b) The Golden Ratio may be used to compute the Fibonacci numbers using the relation below:

$$f_n = \frac{1}{2\phi - 1}\left(\phi^{n+1} - (1-\phi)^{n+1}\right), \text{for } n = 2, 3, ...$$

This is an amazing equation. The right-hand side involves powers and quotients of irrational numbers, but the result is a sequence of integers.
Write a C++ function double myfibo(int n) that evaluates the n-th order Fibonacci number using the above algorithm and approximating to the nearest integer value.

(c) Call double myfibo(n) in a complete C++ program and output Fibonacci numbers for $n \in [1,11]$. For you comparison, the corresponding Fibonacci numbers are: { 1,2,3,5,8,13,21,34,55,89,144 }.

[4 + 4 + 2]

2. A lunar lander is falling freely toward the surface of the moon. If $x(t)$ represents the distance of the lander from the center of the moon (in meters, with t in seconds), then $x(t)$ satisfies the initial-value problem

$$\frac{d^2x}{dt^2} = 4 - \frac{k}{x^2}$$

where $k = 4.9044 \times 10^{12}$ is a constant and and the initial values are $x(0) = 1,781,870$ m from the moon's center, $x'(0) = -450 m/s$.
Write a **Maple** procedure named **RK4** that will solve the above initial-value problem using Runge-Kutta (RK4) method. Plot x VS t and x' VS t.

[10]