Computational Physics Lab Continuous Assessment Exam

Time : 1 hour

February 04, 2016 (Session 2)

*Answer all. Any deviation from instructions will lead to **zero grade***

1. The sine function has the following infinite product representation

$$\sin x = x(1 - \frac{x^2}{\pi^2})(1 - \frac{x^2}{4\pi^2})(1 - \frac{x^2}{9\pi^2}) \ldots = x \prod_{k=1}^{\infty}(1 - \frac{x^2}{k^2\pi^2}) \tag{1}$$

(a) Write a **Maple procedure prodsine(x,n)** that evaluates the sine function from the above definition using only the first n terms in the product (i.e. $\sin x \approx x \prod_{k=1}^{k=n}(1 - \frac{x^2}{k^2\pi^2})$). **USE ITERATION FOR THE PROCEDURE. DO NOT USE THE MAPLE COMMAND THAT DIRECTLY EVALUATES PRODUCTS**.

(b) Plot the difference $|sin(x) - prodsine(x, n)|$ vs. n for $n \in [10, 100]$ at $x = \pi/4, \pi/2$ in the same plot.

[6 + 4]

2. A pair of chemical reactions, A→B and B→C take place in a batch reactor, starting with pure A at a concentration $C_A(0) = 1.00$ mol/liter. The following equations describe how the concentrations $C_A(t)$, ,$C_B(t)$, and $C_C(t)$ vary with time (sec). The notation C'_A will be used to represent the derivative dC_A/dt.

$$C'_A(t) = -0.1C_A(t),\ C_A(0) = 1.0$$
$$C'_B(t) = 0.1C_A(t) - 0.2C_B(t),\ C_B(0) = 0.0$$
$$C'_C(t) = 0.2C_B(t),\ C_C(0) = 0.0$$

Write a C++ program that will solve the above set of ODEs using Runge-Kutta (RK4) method. Plot C_A, C_B, C_C at times from $t = 0$ to $t = 40$ s on the same plot. [10]