A Project on Applications of Wavelet Analysis in Image and Signal Processing

Wavelet Analysis

Department of Mathematics and Statistics

DDU Gorakhpur University, Gorakhpur (India)

June 24, 2023

Contents

1 A Project on Applications of Wavelet Analysis in Image and Signal Processing

The most recent advancement in applied mathematics is **Wavelet Analysis**.For a lot of applications, Fourier analysis(Fourier series and Fourier transform) is not enough to provide appropriate results due to non-stationary nature of signals.
In these kind of situations **wavelet transform** can be used as a strong alternative.

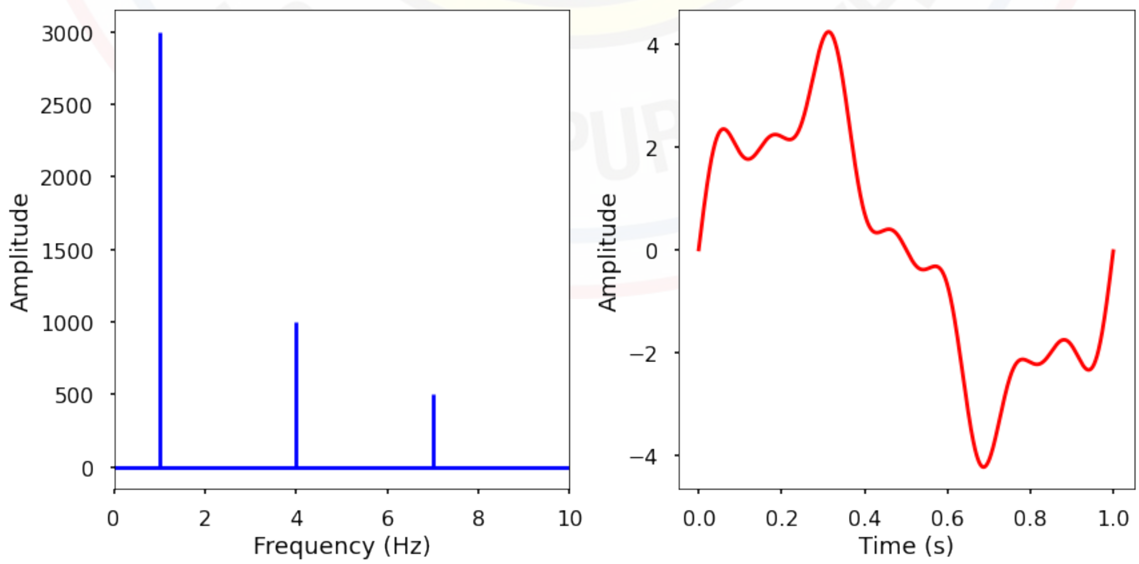
1.1 *Introduction*

We are accustomed to real-world signals like speech signal, a patient's temperature every hour, etc. Signals are typically represented as a time domain graph. In literature, the same information can be expressed in a variety of languages; similarly, signals can be represented in the frequency domain to communicate a message. Depending on the application, these signals can be processed to produce desired outputs or trigger certain actions.

1.2 *Prerequisites*

Before to move on the discussion of wavelet analysis, we need have to talk about some core concepts, that are following

1.2.1 The idea of time and frequency domain analysis



Plot represents the signal $f(t) = a\sin(2\pi nt)$ where a and n varies , expressed in time and frequency domain. The representation in frequency domain shows there is three frequency component in the waveform. This representation is simple to extract behavior of signal compared to that in time domain in majority of real-world applications.

In real world, not the signals are processed in frequency domain to arrive at desired output, since they simplify the analysis mathematically. In applications like control engineering, differential equations are used to represent systems. Frequency domain analysis converts the differential equations to algebraic equations which are relatively easy to solve. Applications related to speech, image and video also gets simplified with the use of frequency domain approach since the sensory organs interprets the signal in frequency domain.

1.2.2 Fourier Analysis

Anyy signal which satisfy Dirichlet conditions (signals with finite number of discontinuity, finite maximum or minimum magnitude) can be converted to frequency domain by Fourier analysis. For analog signals Fourier series or Fourier transform if signal is periodic or aperiodic, respectively. The corresponding counterparts for discrete signals are Discrete Fourier series and Discrete Fourier transform . Fourier analysis can be visualized as inner product of a kernel function with the signal. For example, to find the coefficient corresponding to 50 Hz, find the inner product of signal with a kernel which is a unit sine wave of 50 Hz.

The important formulas that are used to convert a time domain signal to a frequency domain signal is

$$\boxed{\hat{f}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t)dt}$$

1.2.3 Window Fourier transform

The Window Fourier transform of a function $f(t) \in L^2(\mathbb{R})$ by a window function $w(t)$ is defined as

$$\boxed{(\tilde{G}_b f)(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t)\overline{w(t-b)}dt \qquad \forall\ b \in \mathbb{R}}$$

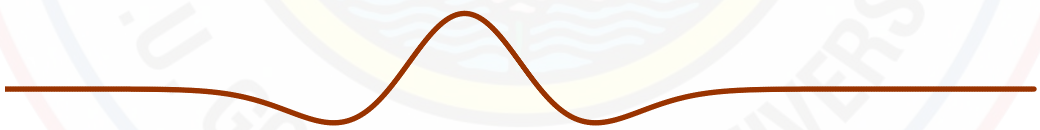
1.2.4 Wavelets

As we know the kernel of Fourier transform is not time limited (exists for all time values) and this is the bottleneck for analyzing a non-stationary signal. Wavelets are waveforms which are time limited or exists only for a given time period only(localized). Wavelets are useful for examining aperiodic, noisy signal in both time and frequency domain simultaneously. The word "wavelet" means a "small wave". There are variety of wavelets available which are selected according to the application. The short duration wavelet is superimposed to the signal under consideration for a short duration of time and decompose them to useful form. This process is called wavelet transform. The method of transforming the decomposed signal to original wave is called inverse wavelet transform.

1.2.5 Example of a wavelet(Mexican Hat)

The **Mexican Hat wavewlet/Ricker wavelet/Second derivative wavelet** is denoted by $\psi_{\mathbf{M}}(\mathbf{t})$ and defined by

$$\boxed{\psi_M(t) = -\frac{d^2}{dt^2}\left(e^{-t^2/2}\right) = \left(1 - t^2\right) e^{-t^2/2}}$$



$\psi_M(t) = (1 - t^2)e^{-t^2/2}$

The Ricker wavelet is frequently employed to model seismic data, and as a broad spectrum source term in computational electrodynamics. It is usually only referred to as the Mexican hat wavelet in the Americas, due to taking the shape of a sombrero when used as a 2D image processing kernel.

1.3 *Image and Signal Processing*

1.3.1 Processing of 2D signal using Pywavelets

PyWavelets is open source wavelet transform software for Python. It combines a simple high level interface with low level C and Cython performance.

```
import pywt
cA, cD = pywt.dwt([1, 2, 3, 4], 'db1')
```

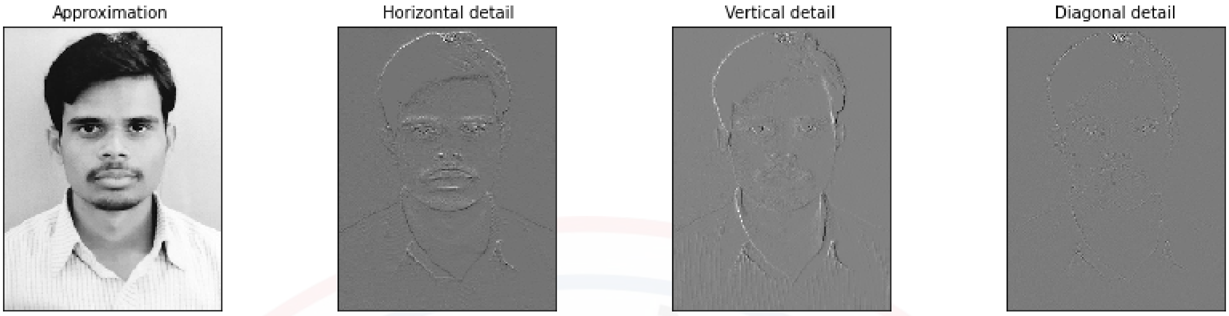
```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.image import imread

import pywt
import pywt.data


# Load image
A = imread("myimg.jpg")
original = np.mean(A, -1)

# Wavelet transform of image, and plot approximation and details
titles = ['Approximation', ' Horizontal detail',
          'Vertical detail', 'Diagonal detail']
coeffs2 = pywt.dwt2(original, 'bior1.3')
LL, (LH, HL, HH) = coeffs2
fig = plt.figure(figsize=(12, 3))
for i, a in enumerate([LL, LH, HL, HH]):
    ax = fig.add_subplot(1, 4, i + 1)
    ax.imshow(a, interpolation="nearest", cmap=plt.cm.gray)
    ax.set_title(titles[i], fontsize=10)
    ax.set_xticks([])
    ax.set_yticks([])
```

```
fig.tight_layout()
plt.show()
```



1.3.2 Applications and Conclusion

Wavelet analysis is a powerful tool for signal processing and image analysis. It has been successfully applied in many applications, including:

- Image compression: Wavelets can be used to compress images by representing them in a sparse wavelet basis. This can lead to significant compression ratios with little loss of image quality.
- Image denoising: Wavelets can be used to remove noise from images. This is done by filtering the wavelet coefficients of the image, which can effectively remove noise without affecting the underlying image structure.
- Image segmentation: Wavelets can be used to segment images into different regions. This is done by thresholding the wavelet coefficients of the image, which can effectively separate different regions of the image based on their frequency content.
- Time series analysis: Wavelets can be used to analyze time series data. This is done by decomposing the time series into its wavelet components, which can then be used to identify different features of the time series, such as trends, cycles, and outliers.
- Medical imaging: Wavelets have been used in a variety of medical imaging applications, including:
  - Magnetic resonance imaging (MRI): Wavelets can be used to improve the resolution of MRI images.

- Computed tomography (CT): Wavelets can be used to reduce noise in CT images.
- Ultrasound: Wavelets can be used to improve the contrast of ultrasound images.

These are just a few of the many applications of wavelet analysis in image and signal processing. Wavelets are a powerful tool that can be used to solve a wide variety of problems.

1.4 ***References***

1. Oppenheim A, Schafer R. Discrete-Time Signal Processing. 3rd ed. Essex: Pearson New International Edition; 2013. ISBN-10:1-292-02572-7

2. Cadence PCB Solutions. Time Domain Analysis vs Frequency Domain Analysis: A Guide and Comparison [Internet]. 2020. Available from: https://resources.pcb.cadence.com/blog/ [Accessed: 2020-08-20]

3. Robi Polikar. Wavelet tutorial [internet]. Available from: http://users.rowan.edu/polikar/WTpart [Accessed: 2020-08-28]

4. Strang G, Nguyen T. Wavelets and Filter Banks. 2nd ed. Wellesly - Cambridge Press; 1996. ISBN- 10: 0961408871

5. For more info regarding document, visit https://github.com/akhlak919

6. Google Bard AI