ASSIGNMENT: RIGID BODY DYNAMICS

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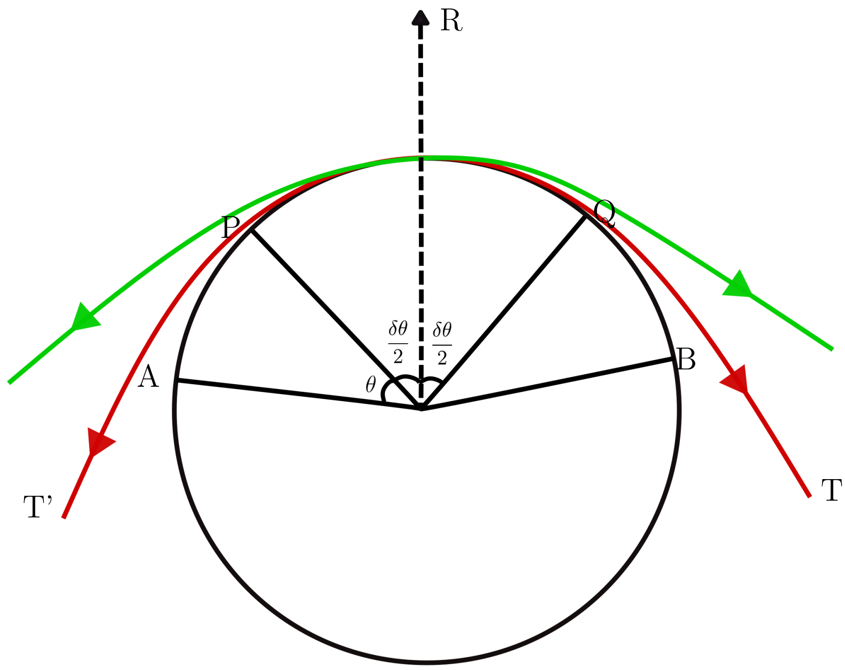
Question

A finite string has two masses M and M' tied to its end and passes over a rough pulley of mass m, whose center is fixed.Then prove that,

$$T = T'e^{\mu\pi}$$

where T and T', both are tension of string or rope according to motion and μ is the coefficient of acceleration.

Solution:



Driven Pulley System

Consider a flat belt drive in which the driven pulley is rotating in the clockwise dirrection.

Let, $T =$ Tension in belt on tight side.

$T' =$ Tension in the belt on slack side

and, $\theta =$ Angle of contact in radian, i.e. Angle suspended by arc AB along which the belt touches the pulley at the center.

Now, consider a small portion PQ of the belt AB, suspending on angle $\delta\theta$ at the center of the pulley.

The belt PQ is in equilibrium under the following forces,

- Tension T in the belt at P.

- Tension $(T + \delta T)$ in the belt at Q.
- Normal reaction, R. and,
- Frictional force, $F = \mu k$, where μ is the coefficient of friction, between the belt and pulley.

Resolving all the forces, vertically we have,

$$R = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \tag{1}$$

Since, the angle $\delta\theta$ is very small, therefore putting $\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$ in equation(1) we get as,

$$R = (T + \delta T)\cdot\frac{\delta\theta}{2} + T\cdot\frac{\delta\theta}{2}$$

$$R = T\cdot\frac{\delta\theta}{2} + \delta T\cdot\frac{\delta\theta}{2} + T\cdot\frac{\delta\theta}{2}$$

$$R = T\cdot\delta\theta \tag{2}$$

Now, Resolving all the forces horizontally, we have,

$$F = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

$$\implies \quad \mu R = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \tag{3}$$

Since, the angle $\delta\theta$ is very small, therefore putting $\cos\frac{\delta\theta}{2} = 1$ in equation(3) we get as,

$$\mu R = (T + \delta T) - T$$

$$\implies \quad R = \frac{\delta T}{\mu} \tag{4}$$

equating the value of R from equation(2) and equation(4), we get as,

$$T\cdot\delta\theta = \frac{\delta T}{\mu}$$

$$\frac{\delta T}{T} = \mu\cdot\delta\theta$$

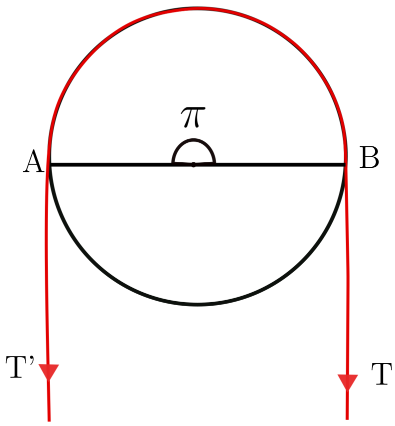
Integrating, both sides, within the limit T' and T and from 0 to θ respectively, we get as,

$$\int_{T'}^{T}\frac{dT}{T} = \int_{0}^{\theta}\mu\cdot d\theta$$

$$\implies \log\left(\frac{T}{T'}\right) = \mu\theta$$

$$\implies \frac{T}{T'} = e^{\mu\theta}$$

$$\implies \boxed{T = T'e^{\mu\theta}}$$



Figure

from this diagram $\theta = \pi$ thus,

$$\boxed{T = T'e^{\mu\pi}}$$

Hence, we have the result.

References:

For raw data of document, please visit: https://github.com/akhlak919