1 INTRODUCTON TO THE PROBLEM

1.1 List of abbreviations

EAE	Evolutionary approach to electrodynamics
EMP	Electromagnetic pulse
FD	Frequency domain
TD	Time domain
TE	Transverse electric modes
UWB	Ultra-wideband
WBO	Wave-Boundary Operators

1.2 Abstract

2 EVOLUTIONARY APPROACH TO ELECTRODYNAMICS

2.1 Statement of the problem

Initial TD problem is stated from three-dimensional Maxwell's equation set

$$\begin{cases} \nabla\times\vec{\mathbf{H}} = \dfrac{\partial}{\partial t}\vec{\mathbf{D}} + \vec{\mathbf{J}}^{\sigma} + \vec{\mathbf{J}}^{e} \\ \nabla\times\vec{\mathbf{E}} = -\dfrac{\partial}{\partial t}\vec{\mathbf{B}} - \vec{\mathbf{J}}^{h} \\ \nabla\cdot\vec{\mathbf{D}} = \rho^{\sigma} + \rho^{e} \\ \nabla\cdot\vec{\mathbf{B}} = \rho^{h} \end{cases}, \tag{1.1}$$

supplemented by constitutive relations (1.2) and equations of continuity (1.3),

$$\vec{\mathbf{D}} = \varepsilon_0\vec{\mathbf{E}} + \vec{\mathbf{P}}\left(\vec{\mathbf{E}}\right); \quad \vec{\mathbf{B}} = \mu_0\left(\vec{\mathbf{H}} + \vec{\mathbf{M}}\left(\vec{\mathbf{H}}\right)\right), \tag{1.2}$$

$$-\frac{\partial}{\partial t}\rho^{e} = \nabla\cdot\vec{\mathbf{J}}^{e}; \quad -\frac{\partial}{\partial t}\rho^{h} = \nabla\cdot\vec{\mathbf{J}}^{h}, \tag{1.3}$$

where $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$ electrical and magnetic field strength vectors, $\vec{\mathbf{D}}$ and $\vec{\mathbf{B}}$ are electric displacement field and magnetic field, $\vec{\mathbf{P}}$ and $\vec{\mathbf{M}}$ are polarization and magnetization, ε_0, μ_0 are electric and magnetic free-space constants, $\vec{\mathbf{J}}^{e,h}$ is density of electric or magnetic current, $\rho^{e,h}$ is density of electric or magnetic charges.

Equations (1.1), (1.2) and (1.3) are valid for any coordinate system expressions. So, we can solve a radiation problem in any suitable system. Cylindrical system is convenient for the EMP problems. We are going to consider this system only.

The functions $\vec{\mathbf{E}}$, $\vec{\mathbf{H}}$, $\vec{\mathbf{D}}$, $\vec{\mathbf{B}}$, $\vec{\mathbf{J}}^{\sigma}$, $\vec{\mathbf{J}}^{e}$, $\vec{\mathbf{J}}^{h}$ are depended on position vector $\vec{R} = \vec{r} + z\vec{z}_0$ and time t.The problem is completed by initial and boundary conditions that may include given sources of currents and fields.

The final solution will be found in the class of quadratically integrable vector functions that satisfy the condition

$$\int_{t_1}^{t_2} dt \int_{z_1}^{z_2} dz \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left(\varepsilon_0\vec{\mathbf{E}}\cdot\vec{\mathbf{E}}^* + \mu_0\vec{\mathbf{H}}\cdot\vec{\mathbf{H}}^*\right) < \infty\,. \tag{1.4}$$

The restriction means energy limit for EMP but nevertheless any real signal having finite energy can be considered in the EAE. The condition restricts the solution by such nonexistent waves like a plane wave.

It is necessary to separate the linear and nonlinear part in polarization and magnetization vectors as follows for nonlinear application

$$\vec{\mathbf{P}}\left(\vec{\mathbf{E}}\right)=\varepsilon_0\alpha\left(z,t\right)\vec{\mathbf{E}}+\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right),\quad \vec{\mathbf{M}}\left(\vec{\mathbf{H}}\right)=\chi\left(z,t\right)\vec{\mathbf{H}}+\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right), \tag{1.5}$$

where $\alpha\left(z,t\right)$ and $\chi\left(z,t\right)$ are electric and magnetic susceptibility. It gives possibility to rewrite the constitutive equations in the following form:

$$\vec{\mathbf{D}}\left(\vec{\mathbf{E}}\right)=\varepsilon_0\varepsilon\left(z,t\right)\vec{\mathbf{E}}+\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right);\quad \vec{\mathbf{B}}=\mu_0\mu\left(z,t\right)\vec{\mathbf{H}}+\mu_0\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right). \tag{1.6}$$

where $\varepsilon\left(z,t\right)=1+\alpha\left(z,t\right)$ is relative permittivity, $\mu\left(z,t\right)=1+\chi\left(z,t\right)$ is relative permeability.

Using (1.6) notations the Maxwell's equations can be rewritten to the form of

$$\begin{cases}\nabla\times\vec{\mathbf{H}}=\varepsilon_0\dfrac{\partial}{\partial t}\left(\varepsilon\left(z,t\right)\vec{\mathbf{E}}\right)+\left\{\dfrac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)+\vec{\mathbf{J}}^{\sigma}\left(\vec{\mathbf{E}},\vec{\mathbf{H}}\right)+\vec{\mathbf{J}}^{e}\right\}\\ -\left[\nabla\times\vec{\mathbf{E}}\right]=\mu_0\dfrac{\partial}{\partial t}\left(\mu\left(z,t\right)\vec{\mathbf{H}}\right)+\left\{\dfrac{\partial}{\partial t}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)+\vec{\mathbf{J}}^{h}\right\}\\ \varepsilon_0\left(\nabla\cdot\varepsilon\left(z,t\right)\vec{\mathbf{E}}\right)=-\left(\nabla\cdot\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)\right)+\rho^{\sigma}+\rho^{e}\\ \mu_0\left(\nabla\cdot\mu\left(z,t\right)\vec{\mathbf{H}}\right)=-\left(\nabla\cdot\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)\right)+\rho^{h}\end{cases}. \tag{1.7}$$

Derivatives $\dfrac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)$ and $\dfrac{\partial}{\partial t}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)$ have the same dimension as current densities as well as $\nabla\cdot\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)$ and $\nabla\cdot\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)$ have the dimension of charge densities. We introduce the equivalent densities of electric and magnetic currents (1.8) and charges (1.9) in right-hand sides of the equation by the following way:

$$\vec{\mathbf{J}}=\frac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)+\vec{\mathbf{J}}^{\sigma}\left(\vec{\mathbf{E}},\vec{\mathbf{H}}\right)+\vec{\mathbf{J}}^{e};\quad \vec{\mathbf{I}}=\frac{\partial}{\partial t}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)+\vec{\mathbf{J}}^{h}; \tag{1.8}$$

$$\varrho=-\left(\nabla\cdot\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)\right)+\rho^{\sigma}+\rho^{e};\quad g=-\left(\nabla\cdot\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)\right)+\rho^{h}. \tag{1.9}$$

So, the set of Maxwell's equations (1.7) acquires the form

$$
\begin{cases}
\nabla\times\vec{\mathbf{H}}=\varepsilon_0\dfrac{\partial}{\partial t}\left(\varepsilon\left(z,t\right)\vec{\mathbf{E}}\right)+\vec{\mathbf{J}} \\
\text{-}\left[\nabla\times\vec{\mathbf{E}}\right]=\mu_0\dfrac{\partial}{\partial t}\left(\mu\left(z,t\right)\vec{\mathbf{H}}\right)+\vec{\mathbf{I}} \\
\varepsilon_0\left(\nabla\cdot\varepsilon\left(z,t\right)\vec{\mathbf{E}}\right)=\varrho \\
\mu_0\left(\nabla\cdot\mu\left(z,t\right)\vec{\mathbf{H}}\right)=g
\end{cases}. \tag{1.10}
$$

All nonlinear properties of a medium are presented here in current and charge densities.

2.2 Longitudinal component extraction

The three-dimensional vectors can be presented as a sum of two-dimensional transversal vector and one-dimensional longitudinal vector

$$
\vec{\mathbf{A}}\left(\vec{R},t\right)\equiv\vec{\mathbf{A}}\left(\vec{\mathrm{r}},\mathrm{z},t\right)=\vec{A}\left(\vec{\mathrm{r}},\mathrm{z},t\right)+\vec{z}_0A_z\left(\vec{\mathrm{r}},\mathrm{z},t\right), \tag{1.11}
$$

as well as nabla operator

$$
\nabla=\nabla_\perp+\vec{z}_0\frac{\partial}{\partial z}. \tag{1.12}
$$

The expressions (1.11) and (1.12) give us possibility to transform the Maxwell's equations (1.10):

$$
\left[\nabla_\perp\times\vec{H}\right]+\left[\nabla_\perp\times\vec{z}_0\right]H_z+\frac{\partial}{\partial z}\left[\vec{z}_0\times\vec{H}\right]=\varepsilon_0\frac{\partial}{\partial t}\left(\varepsilon\vec{E}\right)+\vec{J}+\vec{z}_0\left(\varepsilon_0\frac{\partial}{\partial t}\left(\varepsilon E_z\right)+J_z\right);
$$

$$
-\left[\nabla_\perp\times\vec{E}\right]+\left[\vec{z}_0\times\nabla_\perp\right]E_z+\frac{\partial}{\partial z}\left[\vec{E}\times\vec{z}_0\right]=\mu_0\frac{\partial}{\partial t}\left(\mu\vec{H}\right)+\vec{I}+\vec{z}_0\left\{\mu_0\frac{\partial}{\partial t}\left(\mu H_z\right)+I_z\right\}.
$$

Making projections of two last equations on longitudinal axis and transversal plane one can represent the Maxwell's equations in form of two separated systems containing H_z or E_z:

$$
\left[\nabla_\perp\times\vec{z}_0\right]H_z=\varepsilon_0\frac{\partial}{\partial t}\left(\varepsilon\vec{E}\right)+\frac{\partial}{\partial z}\left[\vec{H}\times\vec{z}_0\right]+\vec{J}; \tag{1.13}
$$

$$
\mu_0\frac{\partial}{\partial z}\{\mu H_z\}=-\mu_0\mu\nabla_\perp\cdot\vec{H}+g; \tag{1.14}
$$

$$
\mu_0\frac{\partial}{\partial t}\left(\mu H_z\right)=\nabla_\perp\cdot\left[\vec{z}_0\times\vec{E}\right]-I_z \tag{1.15}
$$

$$
\left[\vec{z}_0\times\nabla_\perp\right]E_z=\mu_0\frac{\partial}{\partial t}\left(\mu\vec{H}\right)+\frac{\partial}{\partial z}\left[\vec{z}_0\times\vec{E}\right]+\vec{I} \tag{1.16}
$$

$$\varepsilon_0 \frac{\partial}{\partial t}(\varepsilon E_z) = \nabla_\perp \cdot \left[\vec{H} \times \vec{z}_0\right] - J_z \tag{1.17}$$

$$\varepsilon_0 \frac{\partial}{\partial z}\{\varepsilon E_z\} = -\varepsilon_0 \varepsilon \nabla_\perp \cdot \vec{E} + \varrho \tag{1.18}$$

2.3 Maxwell's equations of second order

Actually, topical idea to simplify Maxwell's set is incising of its order. In this case we have less number of unknown variables in each equation. Theory of EAE implies it with the help of WBO.

Equation (1.13) can be simply presented as

$$\left[\nabla_\perp \times \vec{z}_0\right] H_z = \vec{F}_H$$

where $\vec{F}_H = \varepsilon_0 \frac{\partial}{\partial t}\left(\varepsilon \vec{E}\right) + \frac{\partial}{\partial z}\left[\vec{H} \times \vec{z}_0\right] + \vec{J}$. Let us first subject this equation to operators $\mu_0 \frac{\partial}{\partial z} \mu$ and $\mu_0 \frac{\partial}{\partial t} \mu$

$$\mu_0 \left[\nabla_\perp \times \vec{z}_0\right] \frac{\partial}{\partial z} \mu H_z = \mu_0 \frac{\partial}{\partial z}\left(\mu \vec{F}_H\right); \tag{1.19}$$

$$\mu_0 \left[\nabla_\perp \times \vec{z}_0\right] \frac{\partial}{\partial t} \mu H_z = \mu_0 \frac{\partial}{\partial t}\left(\mu \vec{F}_H\right); \tag{1.20}$$

It is simply to obtain next expressions by substitution from (1.15) to (1.20) and from (1.14) to (1.19) then

$$\varepsilon_0^{-1}\left[\vec{z}_0 \times \nabla_\perp\right] \nabla_\perp \cdot \vec{H} = \mu^{-1} \frac{\partial}{\partial z}(\mu \varepsilon_0^{-1} \vec{F}_H) + \left(\varepsilon_0 \mu_0 \mu\right)^{-1} \left[\vec{z}_0 \times \nabla_\perp\right] g\,; \tag{1.21}$$

$$\mu_0^{-1} \nabla_\perp \left[\vec{z}_0 \times \nabla_\perp\right] \vec{E} = -\frac{\partial}{\partial t} \mu \left[\vec{z}_0 \times \vec{F}_H\right] - \mu_0^{-1} \nabla_\perp I_z\,. \tag{1.22}$$

Later on, we will consider a part of two-component vector equations obtained namely (1.21) and (1.22), as one four-component vector equation, namely:

$$\begin{pmatrix} \varepsilon_0^{-1}\left[\vec{z}_0 \times \nabla_\perp\right] \nabla_\perp \cdot \vec{H} \\ \mu_0^{-1} \nabla_\perp \left[\vec{z}_0 \times \nabla_\perp\right] \cdot \vec{E} \end{pmatrix} = \begin{pmatrix} \mu^{-1} \partial_z \left\{\mu \varepsilon_0^{-1} \vec{F}_H\right\} + \left(\varepsilon_0 \mu_0 \mu\right)^{-1} \left[\vec{z}_0 \times \nabla_\perp g\right] \\ -\partial_t \left\{\mu \left[\vec{z}_0 \times \vec{F}_H\right]\right\} - \mu_0^{-1} \nabla_\perp I_z \end{pmatrix}. \tag{1.23}$$

Right-hand-side of equation (1.16) is depended as

$$\vec{F}_E = \mu_0 \frac{\partial}{\partial t}\left(\mu \vec{H}\right) + \frac{\partial}{\partial z}\left[\vec{z}_0 \times \vec{E}\right] + \vec{I}\,.$$

Exclusions of E_z form equation (1.16) subject to operators $\mu_0 \frac{\partial}{\partial z}\mu$ and $\mu_0 \frac{\partial}{\partial t}\mu$ with using of equations (1.17) and (1.18) produce one more four-component vector equation.

$$\begin{pmatrix} \varepsilon_0^{-1}\nabla_\perp[\nabla_\perp\times\vec{z}_0]\cdot\vec{H} \\ \mu_0^{-1}[\nabla_\perp\times\vec{z}_0]\nabla_\perp\cdot\vec{E} \end{pmatrix} = \begin{pmatrix} -\partial_t\left\{\varepsilon\left[\vec{F}_E\times\vec{z}_0\right]\right\} - \varepsilon_0^{-1}\nabla_\perp J_z \\ \varepsilon^{-1}\partial_z\left\{\varepsilon\mu_0^{-1}\vec{F}_E\right\} + \left(\varepsilon_0\mu_0\varepsilon\right)^{-1}\left[\nabla_\perp\varrho\times\vec{z}_0\right] \end{pmatrix}. \tag{1.24}$$

Expressions (1.23) and (1.24) supplemented by boundary conditions are transversal EM field affected by WBO. In matrix operator form it could be

$$W_H\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix} = \begin{pmatrix} \mathbf{0} & \varepsilon_0^{-1}[\vec{z}_0\times\nabla_\perp]\nabla_\perp\cdot \\ \mu_0^{-1}\nabla_\perp[\vec{z}_0\times\nabla_\perp]\cdot & \mathbf{0} \end{pmatrix}\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix} = \begin{pmatrix} \varepsilon_0^{-1}[\vec{z}_0\times\nabla_\perp]\nabla_\perp\cdot\vec{H} \\ \mu_0^{-1}\nabla_\perp[\vec{z}_0\times\nabla_\perp]\cdot\vec{E} \end{pmatrix}; \tag{1.25}$$

$$W_E\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix} = \begin{pmatrix} \mathbf{0} & \varepsilon_0^{-1}\nabla_\perp[\nabla_\perp\times\vec{z}_0]\cdot \\ \mu_0^{-1}[\nabla_\perp\times\vec{z}_0]\nabla_\perp\cdot & \mathbf{0} \end{pmatrix}\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix} = \begin{pmatrix} \varepsilon_0^{-1}\nabla_\perp[\nabla_\perp\times\vec{z}_0]\cdot\vec{H} \\ \mu_0^{-1}[\nabla_\perp\times\vec{z}_0]\nabla_\perp\cdot\vec{E} \end{pmatrix}. \tag{1.26}$$

2.4 Transversal electromagnetic linear space

Introducing the element of transversal electromagnetic linear space $\mathbf{X}$. It is four dimensional vectors of electromagnetic field in the form of

$$\mathbf{X} = \begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix}. \tag{1.27}$$

There is no infinity power source, so there is no infinity power EM wave. This nature law is encapsulated in the space scalar product definition. It is determined by energy restriction (1.4) so the expression is

$$\langle\mathbf{X}_1,\mathbf{X}_2\rangle = \frac{1}{4\pi}\int_0^{2\pi} d\varphi\int_0^{\infty}\rho d\rho\left(\varepsilon_0\vec{E}_1\cdot\vec{E}_2^* + \mu_0\vec{H}_1\cdot\vec{H}_2^*\right). \tag{1.28}$$

This inner product defines a Hilbert space as the domain of the oparators W_H and W_E in the problem under consideration. Let us denote this functional space as $L_2^4(V)$ which means that we deal with L_2 involving four-component vectors which vary within finite domain (S*).

We are able to present WBOs in case of Transversal electromagnetic linear space:

$$W_H\mathbf{X} = \begin{cases} W_H\mathbf{X} \\ \frac{1}{\sqrt{\rho}}, \rho\to\infty \end{cases}; \tag{1.29}$$

$$W_E\mathbf{X}=\begin{cases} W_E\mathbf{X} \\ \dfrac{1}{\sqrt{\rho}},\rho\to\infty \end{cases}. \tag{1.30}$$

2.5 Eign functions and eign numbers

It is necessary to prove self-adjoint properties of WBO to build an expansion with expected properties.

Let X_1 and X_2 are elements of transversal EM linear space $\vec{S}^*$. Self adjoint properties of W_H and W_E implies next expressions.

$$\begin{aligned} \langle W_H X_1, X_2\rangle-\langle X_1, W_H X_2\rangle=0 \\ \langle W_E X_1, X_2\rangle-\langle X_1, W_E X_2\rangle=0 \end{aligned}. \tag{1.31}$$

To prove is we calculate a pair of inner products in accordance with definition of linear product (1.28). For W_H :

$$\begin{aligned} &\langle W_H X_1, X_2\rangle-\langle X_1, W_H X_2\rangle=\frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left\{\left[\vec{z}_0\times\vec{E}_2^*\right]\nabla_\perp\nabla_\perp\vec{H}_1+\right. \\ &\left.+\nabla_\perp\nabla_\perp\left[\vec{z}_0\times\vec{E}_1\right]\vec{H}_2^*-\left[\vec{z}_0\times\vec{E}_1\right]\nabla_\perp\nabla_\perp\vec{H}_2^*-\vec{H}_1\nabla_\perp\nabla_\perp\left[\vec{z}_0\times\vec{E}_2^*\right]\right\}. \end{aligned} \tag{1.32}$$

It is available to present $\vec{E}$ and $\vec{H}$ fields by of arbitrary function Ψ in the way

$$\begin{cases} \vec{E}=\nabla_\perp\Psi\times\vec{z}_0 \\ \vec{H}=\nabla_\perp\Psi \end{cases}. \tag{1.33}$$

These feature do not generate a contradiction with EM nature. Transversal component of electric field is always perpendicular to magnetic one. The result of applying (1.33) determination to (1.32) expression is

$$\begin{aligned} &\langle W_H X_1, X_2\rangle-\langle X_1, W_H X_2\rangle=-\frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left\{\nabla_\perp\Psi_2^*\cdot\nabla_\perp\nabla_\perp\cdot\nabla_\perp\Psi_1+\right. \\ &\left.+\nabla_\perp\Psi_2^*\cdot\nabla_\perp\nabla_\perp\cdot\nabla_\perp\Psi_1-\nabla_\perp\Psi_1\cdot\nabla_\perp\nabla_\perp\cdot\nabla_\perp\Psi_2^*-\nabla_\perp\Psi_1\cdot\nabla_\perp\nabla_\perp\cdot\nabla_\perp\Psi_2^*\right\}= \\ &=-\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left\{\nabla_\perp\Psi_2^*\cdot\nabla_\perp\nabla_\perp\cdot\nabla_\perp\Psi_1-\nabla_\perp\Psi_1\cdot\nabla_\perp\nabla_\perp\cdot\nabla_\perp\Psi_2^*\right\}. \end{aligned} \tag{1.34}$$

The form of arbitrary function Ψ is one of basis functions and will deterring class of functions that can be presented by the expansion. Next form of Ψ allows to present any function according to [source link]

$$\Psi = \sum_{m=0}^{\infty} \int_0^{\infty} \nu d\nu \Psi_m; \quad \Psi_m(\nu) = \frac{J_m(\nu\rho)}{\sqrt{\nu}} e^{im\varphi}. \tag{1.35}$$

Expression (1.34) gets next form after (1.35) substitution

$$\begin{aligned} \langle W_H \mathbf{X}_1, \mathbf{X}_2 \rangle - \langle \mathbf{X}_1, W_H \mathbf{X}_2 \rangle &= \frac{1}{2\pi} \int_0^{\infty} \chi_2 d\chi_2 \int_0^{\infty} \chi_1 d\chi_1 \times \\ &\times \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{\chi_1^2 - \chi_2^2}{\sqrt{\chi_1 \chi_2}} B_{m_1} B_{m_2} \int_0^{2\pi} d\varphi \exp\{i(m_1 - m_2)\varphi\} \times \\ &\times \int_0^{\infty} \rho d\rho \left\{ \frac{d}{d\rho} J_{m_1}(\chi_1 \rho) \cdot \frac{d}{d\rho} J_{m_2}(\chi_2 \rho) + \frac{m_1 m_2}{\rho^2} J_{m_1}(\chi_1 \rho) J_{m_2}(\chi_2 \rho) \right\}. \end{aligned} \tag{1.36}$$

It is easy to prove that (1.36) equal to zero.

$$\begin{aligned} \langle W_H \mathbf{X}_1, \mathbf{X}_2 \rangle - \langle \mathbf{X}_1, W_H \mathbf{X}_2 \rangle &= \frac{1}{2\pi} \int_0^{\infty} \chi_2 d\chi_2 \int_0^{\infty} \chi_1 d\chi_1 \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{\chi_1^2 - \chi_2^2}{\sqrt{\chi_1 \chi_2}} B_{m_1} B_{m_2} 2\pi \delta_{m_1 m_2} \times \\ &\times \int_0^{\infty} \rho d\rho \frac{\chi_1 \chi_2}{4} \left[J_{m_1 - 1}(\chi_1 \rho) - J_{m_2 - 1}(\chi_2 \rho) \right] = 0. \end{aligned} \tag{1.37}$$

Надо расписать подробнее, с 3.36, и здесь довести до ума, иначе непонятно, почему это равно нулю.

The same actions can be done for W_E. Only transversal electromagnetic vector is different.

$$\langle W_E \mathbf{X}_1, \mathbf{X}_2 \rangle - \langle \mathbf{X}_1, W_E \mathbf{X}_2 \rangle = 0 \tag{1.38}$$

$$\begin{aligned} \langle W_E \mathbf{X}_1, \mathbf{X}_2 \rangle - \langle \mathbf{X}_1, W_E \mathbf{X}_2 \rangle &= \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left(\nabla_\perp [\nabla_\perp \times \vec{z}_0] \cdot \vec{H}_1 \cdot \vec{E}_2^* + [\nabla_\perp \times \vec{z}_0] \nabla_\perp \cdot \vec{E}_1 \cdot \vec{H}_2^* \right) - \\ &- \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left(\vec{E}_1 \cdot \left(\nabla_\perp [\nabla_\perp \times \vec{z}_0] \cdot \vec{H}_2 \right)^* + \vec{H}_1 \cdot \left([\nabla_\perp \times \vec{z}_0] \nabla_\perp \cdot \vec{E}_2 \right)^* \right) = \\ &= \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left(\nabla_\perp [\nabla_\perp \times \vec{z}_0] \cdot \vec{H}_1 \cdot \vec{E}_2^* + [\nabla_\perp \times \vec{z}_0] \nabla_\perp \cdot \vec{E}_1 \cdot \vec{H}_2^* \right) - \\ &- \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left(\vec{E}_1 \cdot \nabla_\perp [\nabla_\perp \times \vec{z}_0] \cdot \vec{H}_2^* + \vec{H}_1 \cdot [\nabla_\perp \times \vec{z}_0] \nabla_\perp \cdot \vec{E}_2^* \right) = 0 \end{aligned}$$

$$\begin{cases} \vec{E} = \nabla_\perp \Phi \\ \vec{H} = \vec{z}_0 \times \nabla_\perp \Phi \end{cases}, \tag{1.39}$$

$$\langle W_E\mathbf{X}_1,\mathbf{X}_2\rangle-\langle\mathbf{X}_1,W_E\mathbf{X}_2\rangle=\frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left(\nabla_\perp[\nabla_\perp\times\vec{z}_0]\cdot\vec{H}_1\cdot\vec{E}_2^*+[\nabla_\perp\times\vec{z}_0]\nabla_\perp\cdot\vec{E}_1\cdot\vec{H}_2^*\right)-$$
$$-\frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left(\vec{E}_1\cdot\nabla_\perp[\nabla_\perp\times\vec{z}_0]\cdot\vec{H}_2^*+\vec{H}_1\cdot[\nabla_\perp\times\vec{z}_0]\nabla_\perp\cdot\vec{E}_2^*\right)=\frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\times$$
$$\times$$
$$\begin{cases}\vec{E}=\nabla_\perp\Phi\\ \vec{H}=\vec{z}_0\times\nabla_\perp\Phi\end{cases}$$

where Φ is a function like to (1.35)

$$\Phi=\sum_{n=0}^{\infty}\int_0^{\infty}\chi d\chi\Phi_n;\quad \Phi_n(\chi)=\frac{J_n(\chi\rho)}{\sqrt{\chi}}e^{in\varphi}\,. \tag{1.40}$$

Field expressions (1.33) and (1.39) are sample for future basis functions. It is possible to present them in this point:

$$Y_{\pm m}=\begin{pmatrix}\nabla_\perp\Psi_m\times\vec{z}_0\\ \pm\nabla_\perp\Psi_m\end{pmatrix};\quad Z_{\pm n}=\begin{pmatrix}\nabla_\perp\Phi_n\\ \pm\vec{z}_0\times\nabla_\perp\Phi_n\end{pmatrix}. \tag{1.41}$$

Basis functions (1.41) are eign for WBO operators according to (1.37).

$$W_HY_{\pm m}=p_mY_{\pm m};\quad W_EZ_{\pm n}=q_nZ_{\pm n} \tag{1.42}$$

where p_m and q_n are eign numbers from oscillation equations to Ψ_m and Φ_n:

$$\left(\Delta_\perp+\sqrt{\varepsilon_0\mu_0}q_n\right)\Phi_n=0;\quad \left(\Delta_\perp+\sqrt{\varepsilon_0\mu_0}p_m\right)\Psi_m=0\,. \tag{1.43}$$

Arbitrary electromagnetic field in unbounded medium can be presented by linear product of eign functions (1.41). Note, electromagnetic field in bounded medium needs linear product of three eign functions because of double rotter field existents in bounded medium (Tretyakov, 1999).

2.6 Modal expansions of EM fields

Linear product of eign functions (1.41) can constitute an expansion of transversal electromagnetic field with scalar product rule (1.28) only if orthogonal property is satisfied. Let us prove that.

$$\begin{aligned}\langle Y_m,Z_n\rangle&=\frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left\{\nabla_\perp\Phi_n^*\cdot[\nabla_\perp\Psi_m\times\vec{z}_0]+\left[\vec{z}_0\times\nabla_\perp\Phi_n^*\right]\cdot\nabla_\perp\Psi_m\right\}=\\&=\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left\{\vec{z}_0\cdot\left[\nabla_\perp\Phi_n^*\times\nabla_\perp\Psi_m\right]\right\}.\end{aligned} \tag{1.44}$$

$$\left[\nabla_\perp \Phi_n^* \times \nabla_\perp \Psi_m\right] = \frac{\vec{z}_0}{\rho}\cdot\left(\frac{\partial \Phi_n^*}{\partial \rho}\frac{\partial \Psi_m}{\partial \varphi} - \frac{\partial \Psi_m}{\partial \rho}\frac{\partial \Phi_n^*}{\partial \varphi}\right) =$$
$$= \frac{i\vec{z}_0}{\rho} e^{i(m-n)\varphi}\left\{ m\frac{\sqrt{\chi}}{\sqrt{\nu}} J_m(\nu\rho)\left(J_{m-1}(\chi\rho) - \frac{m}{\chi\rho}J_m(\chi\rho)\right) + n\frac{\sqrt{\nu}}{\sqrt{\chi}} J_n(\chi\rho)\left(\frac{n}{\nu\rho}J_m(\nu\rho) - J_{m+1}(\nu\rho)\right)\right\}$$

It is easy to prove that (1.44) can be rewrote within formula

$$\frac{d}{d\rho}J_m(\chi\rho) = \frac{\chi}{2}\left[J_{m-1}(\chi\rho) - J_{m+1}(\chi\rho)\right]$$
$$\frac{1}{\chi}\frac{d}{d\rho}J_m(\chi\rho) = J_{m-1}(\chi\rho) - \frac{m}{\chi\rho}J_m(\chi\rho) = \frac{m}{x\rho}J_m(\chi\rho) - J_{m+1}(\chi\rho),$$

$$\langle Y_m, Z_n\rangle = \frac{i}{4\pi}\int_0^{2\pi} e^{i(m-n)\varphi}d\varphi \int_0^{\infty}\frac{\rho}{\rho}d\rho\left(\vec{z}_0\cdot\vec{z}_0\right)\left\{ m\frac{\sqrt{\chi}}{\sqrt{\nu}} J_m(\nu\rho)\left(J_{m-1}(\chi\rho) - \frac{m}{\chi\rho}J_m(\chi\rho)\right) + \right.$$
$$\left. + n\frac{\sqrt{\nu}}{\sqrt{\chi}} J_n(\chi\rho)\left(\frac{n}{\nu\rho}J_m(\nu\rho) - J_{m+1}(\nu\rho)\right)\right\} = \frac{i}{2}\delta_{m,n}\int_0^{\infty} d\rho \times$$
$$\times\left\{ m\frac{\sqrt{\chi}}{\sqrt{\nu}} J_m(\nu\rho)\left(J_{m-1}(\chi\rho) - \frac{m}{\chi\rho}J_m(\chi\rho)\right) + n\frac{\sqrt{\nu}}{\sqrt{\chi}} J_n(\chi\rho)\left(\frac{n}{\nu\rho}J_m(\nu\rho) - J_{m+1}(\nu\rho)\right)\right\}$$

and gets form of

$$\langle Y_m, Z_n\rangle = \frac{im}{2}\frac{\sqrt{\chi}}{\sqrt{\nu}}\delta_{m,n}\int_0^{\infty} d\rho J_m(\nu\rho)\left(J_{m-1}(\chi\rho) - \frac{m}{\chi\rho}J_m(\chi\rho)\right) +$$
$$+ \frac{in}{2}\frac{\sqrt{\nu}}{\sqrt{\chi}}\delta_{m,n}\int_0^{\infty} d\rho J_n(\chi\rho)\left(\frac{n}{\nu\rho}J_m(\nu\rho) - J_{m+1}(\nu\rho)\right) \qquad (1.45)$$

Previous expression contains Kronecker delta. It is determined in [literature link] as

$$\delta_{m,n} = \frac{1}{2\pi}\int_0^{2\pi} e^{i(m-n)\varphi}d\varphi.$$

According to formula 2.13.31 form (Прудников, et al., 1983)

$$\int_0^{\infty} J_n(at)J_{n-1}(bt)\,dt = \begin{cases} \dfrac{b^{n-1}}{a^n} & b < a \\ \dfrac{1}{2b} & b = a \\ 0 & b > a \end{cases}.$$

extension (1.45) with little simplifications is equal to zero.

$$\left\langle Y_m, Z_n \right\rangle\Big|_{m=n=k} = \frac{ik}{2}\frac{\sqrt{\chi}}{\sqrt{\nu}}\int_0^\infty d\rho J_k(\nu\rho) J_{k-1}(\chi\rho) - \frac{ik^2}{2}\frac{1}{\sqrt{\chi\nu}}\int_0^\infty \rho d\rho J_k(\nu\rho) J_k(\chi\rho) +$$
$$+\frac{ik^2}{2}\frac{1}{\sqrt{\chi\nu}}\int_0^\infty d\rho J_k(\chi\rho) J_k(\nu\rho) - \frac{ik}{2}\frac{\sqrt{\nu}}{\sqrt{\chi}}\int_0^\infty d\rho J_k(\chi\rho) J_{k+1}(\nu\rho) =$$
$$= \frac{ik}{2}\left(\frac{\sqrt{\chi}}{\sqrt{\nu}}\int_0^\infty d\rho J_k(\nu\rho) J_{k-1}(\chi\rho) - \frac{\sqrt{\nu}}{\sqrt{\chi}}\int_0^\infty d\rho J_k(\chi\rho) J_{k+1}(\nu\rho)\right) =$$
$$= \frac{ik}{2}\left(\frac{\sqrt{\chi}}{\sqrt{\nu}}\begin{Bmatrix} \frac{\chi^k}{\nu^{k+1}}, \nu > \chi \\ \frac{1}{2\lambda}, \nu = \chi = \lambda \\ 0, \nu < \chi \end{Bmatrix} - \frac{\sqrt{\nu}}{\sqrt{\chi}}\begin{Bmatrix} 0, \nu > \chi \\ \frac{1}{2\lambda}, \nu = \chi = \lambda \\ \frac{\nu^{k-1}}{\chi^k}, \nu < \chi \end{Bmatrix}\right) = \frac{ik}{2}\begin{Bmatrix} \frac{\sqrt{\chi}}{\sqrt{\nu}}\frac{\chi^k}{\nu^{k+1}}, \nu > \chi \\ 0, \nu = \chi = \lambda \\ -\frac{\sqrt{\nu}}{\sqrt{\chi}}\frac{\nu^{k-1}}{\chi^k}, \nu < \chi \end{Bmatrix}$$

Неправильно использована предыдущая формула, внимательно

So $\left\langle Y_m, Z_n \right\rangle = 0$ and are able to present an expansion for $\mathbf{X}$ with unknown coefficients $A_m(z,t,\chi)$ and $B_n(z,t,\nu)$.

$$\mathbf{X}(\vec{r},z,t) = \sum_{m=-\infty}^{\infty}\int_0^\infty d\chi A_m(z,t,\chi)\mathbf{Y}_m(\vec{r},\chi) + \sum_{n=-\infty}^{\infty}\int_0^\infty d\nu B_n(z,t,\nu)\mathbf{Z}_n(\vec{r},\nu)\,. \tag{1.46}$$

Little manipulations to extension (1.46) by $\mathbf{X}$ vector determination (1.27) and expressions for eigen functions (1.41) give final result for electromagnetic field:

$$\begin{aligned} \vec{E} &= \sqrt[-2]{\varepsilon_0}\left\{\sum_{m=0}^{\infty}\int_0^\infty d\nu V_m^h\left[\nabla_\perp \Psi_m \times \vec{z}_0\right] + \sum_{m=1}^{\infty}\int_0^\infty d\chi V_n^e \nabla_\perp \Phi_n\right\} \\ \vec{H} &= \sqrt[-2]{\mu_0}\left\{\sum_{m=0}^{\infty}\int_0^\infty d\nu I_m^h \nabla_\perp \Psi_m + \sum_{m=1}^{\infty}\int_0^\infty d\chi I_n^e\left[\vec{z}_0 \times \nabla_\perp \Phi_n\right]\right\} \end{aligned}; \tag{1.47}$$

where coefficients $A_m(z,t,\chi)$ and $B_n(z,t,\nu)$ were renamed

$$A_m + A_{-m} = V_m^h; \quad B_n + B_{-n} = V_n^e; \quad A_m - A_{-m} = I_m^h; \quad B_n - B_{-n} = I_n^e\,. \tag{1.48}$$

Доказать что мода n=0 нулевая

It is comfortable to have simplifications for some expressions from electric and magnetic field expansions in view of Lame coefficients for cylindrical medium.

$$\nabla_\perp \Psi_m = e^{im\varphi}\left(\vec{\rho}_0\sqrt{\nu}\,\frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{2} + im\vec{\varphi}_0\,\frac{J_m(\nu\rho)}{\sqrt{\nu}\rho}\right); \tag{1.49}$$

$$\left[\nabla_\perp \Psi_m \times \vec{z}_0\right] = -e^{im\varphi}\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{2} - im\vec{\rho}_0\,\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right); \qquad (1.50)$$

Longitudinal component must satisfy boundary conditions and be the same diminution with transversal ones.

$$\begin{gathered} E_z(\rho,\phi,z,t) = \sqrt[-2]{\varepsilon_0}\sum_{n=0}^{\infty}\int_0^{\infty}\chi^2 d\chi e_n(z,t;\chi)\Phi_n(\rho,\phi;\chi) \\ H_z(\rho,\phi,z,t) = \sqrt[-2]{\mu_0}\sum_{m=0}^{\infty}\int_0^{\infty}\nu^2 d\nu h_m(z,t;\nu)\Psi_m(\rho,\phi;\nu) \end{gathered}; \qquad (1.51)$$

2.7 Evolutionary equation set

There is infinity number of unknown coefficients $\left\{V_m^h, I_m^h, V_n^e, I_n^e\right\}_{m,n=0}^{\infty}$. EAE can be applied to find the solution of radiation problem with specified accuracy which is depended of m and n numbers. Extracted Maxwell's equations (1.13)-(1.18) will get a form of evolutionary equations after substitution of (1.47) and (1.51) expansions to them with using of next formulas for simplification:

Выведем следующие свойства ортогональности для … пользуясь свойтвами ф. Бесселя в следующей строке:

$$\begin{gathered} \delta_{m,n} = \frac{1}{2\pi}\int_0^{2\pi} e^{i(m-n)\varphi}d\varphi \\ \frac{2n}{\chi\rho}J_n(\chi\rho) = J_{n-1}(\chi\rho) + J_{n+1}(\chi\rho) \\ \delta(\chi_1 - \chi_2) = \sqrt{\chi_1\chi_2}\int_0^{\infty}\rho d\rho J_m(\chi_1\rho)J_m(\chi_2\rho) \end{gathered}$$

$$\frac{\chi\chi'}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n(\chi)\Phi_{n'}^*(\chi') = \delta_{n,n'}\delta(\chi-\chi');$$

$$\begin{gathered} \frac{\chi\chi'}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n(\chi)\Phi_{n'}^*(\chi') = \frac{\chi\chi'}{2\pi}\int_0^{2\pi}e^{i(n-n')\varphi}d\varphi\int_0^{\infty}\rho d\rho\frac{J_n(\chi\rho)}{\sqrt{\chi}}\frac{J_{n'}(\chi'\rho)}{\sqrt{\chi'}} = \\ = \frac{\sqrt{\chi\chi'}}{2\pi}\int_0^{2\pi}e^{i(n-n')\varphi}d\varphi\int_0^{\infty}\rho d\rho J_n(\chi\rho)J_{n'}(\chi'\rho) = \delta_{n,n'}\delta(\chi-\chi') \end{gathered}$$

$$\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\nabla_\perp\Phi_n(\chi)\cdot\nabla_\perp\Phi_{n'}^*(\chi') = \delta_{nn'}\delta(\chi-\chi');$$

$$\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\nabla_\perp\Phi_n(\chi)\cdot\nabla_\perp\Phi_{n'}^*(\chi')=\frac{1}{2\pi}\int_0^{2\pi}e^{i(n-n')\varphi}d\varphi\int_0^{\infty}\rho d\rho\times$$

$$\left\{\vec{\rho}_0\sqrt{\chi'}\frac{J_{n'-1}(\chi'\rho)-J_{n'+1}(\chi'\rho)}{2}-\vec{\varphi}_0 in'\frac{J_{n'}(\chi'\rho)}{\rho\sqrt{\chi'}}\right\}\left\{\vec{\rho}_0\sqrt{\chi}\frac{J_{n-1}(\chi\rho)-J_{n+1}(\chi\rho)}{2}+\vec{\varphi}_0 in\frac{J_n(\chi\rho)}{\rho\sqrt{\chi}}\right\}=$$

$$=\delta_{nn'}\int_0^{\infty}\rho d\rho\left\{\vec{\rho}_0\sqrt{\chi'}\frac{J_{n'-1}(\chi'\rho)-J_{n'+1}(\chi'\rho)}{2}-\vec{\varphi}_0\frac{i\sqrt{\chi'}}{2}\left[J_{n'-1}(\chi'\rho)+J_{n'+1}(\chi'\rho)\right]\right\}\times$$

$$\times\left\{\vec{\rho}_0\sqrt{\chi}\frac{J_{n-1}(\chi\rho)-J_{n+1}(\chi\rho)}{2}+\vec{\varphi}_0\frac{i\sqrt{\chi}}{2}\left[J_{n-1}(\chi\rho)+J_{n+1}(\chi\rho)\right]\right\}=\delta_{nn'}\frac{\sqrt{\chi\chi'}}{4}\int_0^{\infty}\rho d\rho\times$$

$$\times\{J_{n'-1}(\chi'\rho)J_{n-1}(\chi\rho)-J_{n'-1}(\chi'\rho)J_{n+1}(\chi\rho)-J_{n'+1}(\chi'\rho)J_{n-1}(\chi\rho)+J_{n'+1}(\chi'\rho)J_{n+1}(\chi\rho)+$$

$$+J_{n'-1}(\chi'\rho)J_{n-1}(\chi\rho)+J_{n'-1}(\chi'\rho)J_{n+1}(\chi\rho)+J_{n'+1}(\chi'\rho)J_{n-1}(\chi\rho)+J_{n'+1}(\chi'\rho)J_{n+1}(\chi\rho)\}=$$

$$=\frac{\delta_{nn'}}{2}\left\{\int_0^{\infty}\rho d\rho J_{n'-1}(\chi'\rho)J_{n-1}(\chi\rho)+\int_0^{\infty}\rho d\rho J_{n'+1}(\chi'\rho)J_{n+1}(\chi\rho)\right\}=\delta_{nn'}\delta(\chi-\chi')$$

$$\frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left\{\nabla_\perp\Phi_n(\chi)\cdot\left[\nabla_\perp\Psi_{m'}^*(\nu')\times\vec{z}_0\right]+\left[\vec{z}_0\times\nabla_\perp\Phi_n(\chi)\right]\cdot\nabla_\perp\Psi_{m'}^*(\nu')\right\}=0\,;$$

$$\nabla_\perp\Phi_n(\chi)\cdot\left[\nabla_\perp\Psi_{m'}^*(\nu')\times\vec{z}_0\right]+\left[\vec{z}_0\times\nabla_\perp\Phi_n(\chi)\right]\cdot\nabla_\perp\Psi_{m'}^*(\nu')=$$

$$=\left[\nabla_\perp\Phi_n(\chi)=\vec{a};\quad\nabla_\perp\Psi_{m'}^*(\nu')=\vec{b}\right]=\vec{a}\cdot\left[\vec{b}\times\vec{z}_0\right]+\left[\vec{z}_0\times\vec{a}\right]\cdot\vec{b}=$$

$$\left[\vec{b}\times\vec{z}_0\right]\cdot\vec{a}-\left[\vec{z}_0\times\vec{a}\right]\cdot\vec{b}=\vec{b}\cdot\left[\vec{z}_0\times\vec{a}\right]+\left[\vec{z}_0\times\vec{a}\right]\cdot\vec{b}=\left[\vec{z}_0\times\vec{a}\right]\cdot\vec{b}+\left[\vec{z}_0\times\vec{a}\right]\cdot\vec{b}=$$

$$=2\left[\vec{z}_0\times\vec{a}\right]\cdot\vec{b}$$

$$\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left[\vec{z}_0\times\nabla_\perp\Phi_n(\chi)\right]\cdot\nabla_\perp\Psi_{m'}^*(\nu')=$$

$$\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left[\nabla_\perp\Psi_m(\nu)\times\vec{z}_0\right]\cdot\left[\nabla_\perp\Psi_{m'}^*(\nu')\times\vec{z}_0\right]=\delta_{mm'}\delta(\nu-\nu').$$

$$
\begin{aligned}
&\left[\nabla_\perp \Psi_m(\nu)\times\vec{z}_0\right] = -e^{im\varphi}\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2} - im\vec{\rho}_0\,\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right) = \\
&= -e^{im\varphi}\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2} - \vec{\rho}_0 i\sqrt{\nu}\,\frac{J_{m-1}(\nu\rho)+J_{m+1}(\nu\rho)}{2}\right) = \\
&= -e^{im\varphi}\frac{\sqrt{\nu}}{2}\left(\vec{\varphi}_0\left[J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)\right] - i\vec{\rho}_0\left[J_{m-1}(\nu\rho)+J_{m+1}(\nu\rho)\right]\right) \\
&\left[\nabla_\perp \Psi_{m'}^{*}(\nu')\times\vec{z}_0\right] = -e^{-im'\varphi}\left(\vec{\varphi}_0\sqrt{\nu'}\,\frac{J_{m'-1}(\nu'\rho)-J_{m'+1}(\nu'\rho)}{2} + im'\vec{\rho}_0\,\frac{J_{m'}(\nu'\rho)}{\rho\sqrt{\nu'}}\right) = \\
&= -e^{-im'\varphi}\left(\vec{\varphi}_0\sqrt{\nu'}\,\frac{J_{m'-1}(\nu'\rho)-J_{m'+1}(\nu'\rho)}{2} + \vec{\rho}_0 i\sqrt{\nu'}\,\frac{J_{m'-1}(\nu'\rho)+J_{m'+1}(\nu'\rho)}{2}\right) = \\
&= -e^{-im'\varphi}\frac{\sqrt{\nu'}}{2}\left(\vec{\varphi}_0\left[J_{m'-1}(\nu'\rho)-J_{m'+1}(\nu'\rho)\right] + i\vec{\rho}_0\left[J_{m'-1}(\nu'\rho)+J_{m'+1}(\nu'\rho)\right]\right) \\
&\frac{1}{2\pi}\int_0^{2\pi} d\varphi\int_0^{\infty}\rho d\rho\left[\nabla_\perp\Psi_m(\nu)\times\vec{z}_0\right]\cdot\left[\nabla_\perp\Psi_{m'}^{*}(\nu')\times\vec{z}_0\right] = -\frac{1}{2\pi}\frac{\sqrt{\nu\nu'}}{4}\int_0^{2\pi} d\varphi e^{i(m-m')\varphi}\int_0^{\infty}\rho d\rho\times \\
&\times\left\{J_{m-1}(\nu\rho)J_{m'-1}(\nu'\rho) - J_{m-1}(\nu\rho)J_{m'+1}(\nu'\rho) - J_{m+1}(\nu\rho)J_{m'-1}(\nu'\rho) + J_{m+1}(\nu\rho)J_{m'+1}(\nu'\rho) + \right. \\
&\left. + J_{m-1}(\nu\rho)J_{m'-1}(\nu'\rho) + J_{m-1}(\nu\rho)J_{m'+1}(\nu'\rho) + J_{m+1}(\nu\rho)J_{m'-1}(\nu'\rho) + J_{m+1}(\nu\rho)J_{m'+1}(\nu'\rho)\right\} = \\
&= \delta_{mm'}\frac{\sqrt{\nu\nu'}}{2}\int_0^{\infty}\rho d\rho\left\{J_{m-1}(\nu\rho)J_{m-1}(\nu'\rho) + J_{m+1}(\nu\rho)J_{m+1}(\nu'\rho)\right\} = \\
&= \delta_{mm'}\left\{\frac{\delta(\nu-\nu')}{2} + \frac{\delta(\nu-\nu')}{2}\right\} = \delta_{mm'}\delta(\nu-\nu')
\end{aligned}
$$

Finally, first two evolutionary equations can be received from (1.14) and (1.16)

$$\partial_z\{\mu h_m\} = \mu I_m^h + \sqrt[-2]{\mu_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi\int_0^{\infty}\rho d\rho\Psi_m^{*}(\nu)\,g\,; \tag{1.52}$$

$$\partial_{ct}\{\mu h_m\} = -V_m^h - \sqrt{\varepsilon_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi\int_0^{\infty}\rho d\rho\Psi_m^{*}(\nu)\,I_z\,. \tag{1.53}$$

Next two equations can be obtained from (1.13) by …

$$-\partial_{ct}\{\varepsilon V_m^h\} - \partial_z I_m^h + \nu^2 h_m = \sqrt{\mu_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi\int_0^{\infty}\rho d\rho\left[\vec{z}_0\times\vec{J}\right]\cdot\nabla_\perp\Psi_m^{*}(\nu)\,; \tag{1.54}$$

$$\partial_{ct}\{\varepsilon V_n^e\} + \partial_z I_n^e = -\sqrt{\mu_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi\int_0^{\infty}\rho d\rho\vec{J}\cdot\nabla_\perp\Phi_n^{*}(\chi)\,. \tag{1.55}$$

The same procedure leads to following results from (1.17) and (1.18):

$$\partial_{ct}\{\varepsilon e_n\} = -I_n^e - \sqrt{\mu_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi\int_0^{\infty}\rho d\rho\Phi_n^{*}(\chi)\,J_z\,; \tag{1.56}$$

$$\partial_z \left\{ \varepsilon e_n \right\} = \varepsilon V_n^e + \sqrt[-2]{\varepsilon_0} \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \Phi_n^* \left(\chi \right) \varrho \,. \tag{1.57}$$

and to these from (5):

$$-\partial_{ct} \left\{ \mu I_n^e \right\} - \partial_z V_n^e + \chi^2 e_n = \sqrt{\varepsilon_0} \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left[\vec{I} \times \vec{z}_0 \right] \cdot \nabla_\perp \Phi_n^* \left(\chi \right); \tag{1.58}$$

$$\partial_{ct} \left\{ \mu I_m^h \right\} + \partial_z V_m^h = -\sqrt{\varepsilon_0} \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \vec{I} \cdot \nabla_\perp \Psi_m^* \left(\nu \right). \tag{1.59}$$

Equations (1.52)-(1.59) are forming overloaded evolutionary equation set. One couple of them is extra. For example it can be (1.55) and (1.56) couple. It can be received from other equations with the help of equations of continuity (1.3). Decreasing of the number of equations goes with increasing of its order. Also six-equation form is more suitable for analytics: only two equations constitute mathematical problems.

First of them goes from (1.58) and have next after substitution of I_n^e from (1.56) and of V_n^e from (1.57).

$$\begin{gathered}\partial_{ct} \left\{ \mu \partial_{ct} \left(\varepsilon e_n \right) \right\} - \partial_z \left\{ \varepsilon^{-1} \partial_z \left(\varepsilon e_n \right) \right\} + \chi^2 e_n = \sqrt{\varepsilon_0} \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left[\vec{I} \times \vec{z}_0 \right] \cdot \nabla_\perp \Phi_n^* \left(\chi \right) - \\ -\partial_{ct} \left\{ \sqrt{\mu_0} \mu \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \Phi_n^* \left(\chi \right) J_z \right\} - \partial_z \left\{ \sqrt[-2]{\varepsilon_0} \varepsilon^{-1} \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \Phi_n^* \left(\chi \right) \varrho \right\}\end{gathered} \,. \tag{1.60}$$

Second one is similar. I_m^h from (1.52) and V_m^h from (1.53) substitution produces

$$\begin{gathered}\partial_{ct} \left\{ \varepsilon \partial_{ct} \left(\mu h_m \right) \right\} - \partial_z \left\{ \mu^{-1} \partial_z \left(\mu h_m \right) \right\} + \nu^2 h_m = \sqrt{\mu_0} \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left[\vec{z}_0 \times \vec{J} \right] \cdot \nabla_\perp \Psi_m^* \left(\nu \right) - \\ -\partial_{ct} \left\{ \sqrt{\varepsilon_0} \varepsilon \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \Psi_m^* \left(\nu \right) I_z \right\} - \partial_z \left\{ \sqrt[-2]{\mu_0} \mu^{-1} \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \Psi_m^* \left(\nu \right) g \right\}\end{gathered} \tag{1.61}$$

Thus, let us present a set of six scalar equations to evolutionary coefficients $\left\{ V_m^h, I_m^h, V_n^e, I_n^e \right\}_{m,n=0}^{\infty}$.

$$
\begin{cases}
\partial_{ct}\left\{\varepsilon\partial_{ct}\left(\mu h_m\right)\right\}-\partial_z\left\{\mu^{-1}\partial_z\left(\mu h_m\right)\right\}+\nu^2 h_m=\sqrt{\mu_0}\,\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\left[\vec{z}_0\times\vec{J}\right]\cdot\nabla_\perp\Psi_m^{\ *}\left(\nu\right)- \\
\qquad -\partial_{ct}\left\{\sqrt{\varepsilon_0}\,\varepsilon\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Psi_m^{\ *}\left(\nu\right)I_z\right\}-\partial_z\left\{\sqrt[-2]{\mu_0}\,\mu^{-1}\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Psi_m^{\ *}\left(\nu\right)g\right\} \\
\partial_{ct}\left\{\mu\partial_{ct}\left(\varepsilon e_n\right)\right\}-\partial_z\left\{\varepsilon^{-1}\partial_z\left(\varepsilon e_n\right)\right\}+\chi^2 e_n=\sqrt{\varepsilon_0}\,\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\left[\vec{I}\times\vec{z}_0\right]\cdot\nabla_\perp\Phi_n^{\ *}\left(\chi\right)- \\
\qquad -\partial_{ct}\left\{\sqrt{\mu_0}\,\mu\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Phi_n^{\ *}\left(\chi\right)J_z\right\}-\partial_z\left\{\sqrt[-2]{\varepsilon_0}\,\varepsilon^{-1}\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Phi_n^{\ *}\left(\chi\right)\varrho\right\} \\
I_n^e=-\partial_{ct}\left\{\varepsilon e_n\right\}-\sqrt{\mu_0}\,\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Phi_n^{\ *}\left(\chi\right)J_z \\
V_n^e=\varepsilon^{-1}\partial_z\left\{\varepsilon e_n\right\}-\sqrt[-2]{\varepsilon_0}\,\varepsilon^{-1}\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Phi_n^{\ *}\left(\chi\right)\varrho \\
I_m^h=\mu^{-1}\partial_z\left\{\mu h_m\right\}-\sqrt[-2]{\mu_0}\,\mu^{-1}\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Psi_m^{\ *}\left(\nu\right)g \\
V_m^h=-\partial_{ct}\left\{\mu h_m\right\}-\sqrt{\varepsilon_0}\,\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Psi_m^{\ *}\left(\nu\right)I_z
\end{cases}
. \qquad (1.62)
$$

$$
E_z\left(\rho,\phi,z,t\right)=\sqrt[-2]{\varepsilon_0}\sum_{n=0}^{\infty}\int\limits_0^{\infty}\chi^2 d\chi e_n\left(z,t;\chi\right)\Phi_n\left(\rho,\phi;\chi\right)
$$

$$
H_z\left(\rho,\phi,z,t\right)=\sqrt[-2]{\mu_0}\sum_{m=0}^{\infty}\int\limits_0^{\infty}\nu^2 d\nu h_m\left(z,t;\nu\right)\Psi_m\left(\rho,\phi;\nu\right)
$$

$$
\vec{E}=\sqrt[-2]{\varepsilon_0}\left\{\sum_{m=0}^{\infty}\int\limits_0^{\infty}d\nu V_m^h\left[\nabla_\perp\Psi_m\times\vec{z}_0\right]+\sum_{m=1}^{\infty}\int\limits_0^{\infty}d\chi V_n^e\nabla_\perp\Phi_n\right\}
$$

$$
\vec{H}=\sqrt[-2]{\mu_0}\left\{\sum_{m=0}^{\infty}\int\limits_0^{\infty}d\nu I_m^h\nabla_\perp\Psi_m+\sum_{m=1}^{\infty}\int\limits_0^{\infty}d\chi I_n^e\left[\vec{z}_0\times\nabla_\perp\Phi_n\right]\right\}
$$

3 DOT NONLINEAR RADIATION

4 PLANE DISK RADIATION

4.1 Statement of the problem

Radiation problem of EMP and its propagation through nonlinear medium can be considered in TD by the EAE. The field can be definitely determined by building of field expansions (1.47), (1.51) and solving set (1.62) to expansion coefficients according to the approach. We can specify the initialy conditions and field sources to solve evolutionary equation set.

Рассматриваем волну TE

Почему выбран именно этот случай?

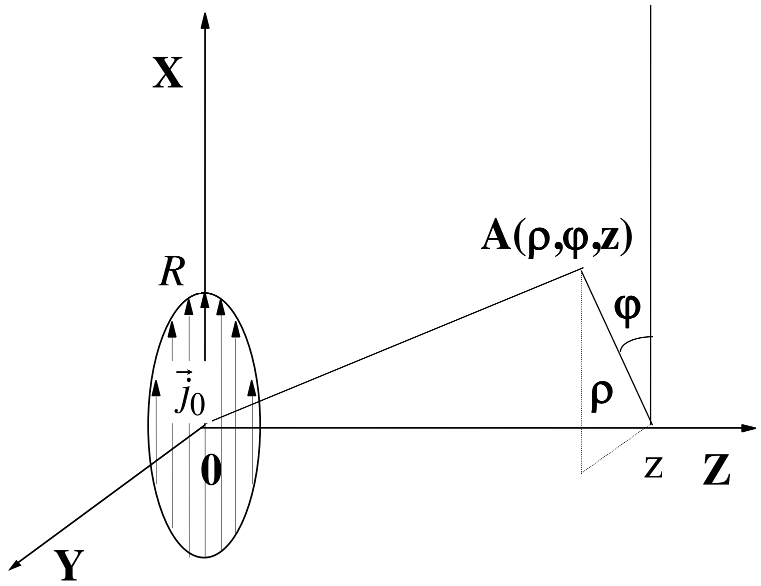

Figure 4.1. Current sources

Let us consider source of current with the form of plane disk. Disk radius is equal to R. Current $\vec{j}_0$ appears on one side of circle, streams in the plane of disk to the opposite side and dies there (Figure 4.1.). The center of the disk is located at the begging on cylindrical coordinate system $\{\vec{\rho}_0, \vec{z}_0, \vec{\varphi}_0\}$ perpendicular to $\vec{z}_0$. Disk radiated in both halfspaces, but only $z > 0$ is under consideration.

$\vec{j}_0$ is not stationary or harmonic value, its time dependence describes the moment of plugging in of a generator, so the problem of transient electromagnetic field takes place. Such behavior can be described by Heaviside's step function $H(t)$ (Волков, и др., 2002). Sure is unrealizable dependency as well as circle current density on Figure 4.1. and these facts must be taking into account – the problem is theoretical. According to following specifications

$$\overrightarrow{j_0} = \vec{J} = \overrightarrow{x_0} H(t)\delta(z)\big(H(\rho) - H(\rho - R)\big). \tag{2.1}$$

Evolutionary equations consist of components of current expansions, so current $\vec{j}_0$ must be presented in the expansion form.

$$\begin{cases} \vec{\rho}_0 = \vec{x}_0 \cos\varphi + \vec{y}_0 \sin\varphi \\ \vec{\varphi}_0 = -\vec{x}_0 \sin\varphi + \vec{y}_0 \cos\varphi \end{cases} \Rightarrow \vec{A} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \Rightarrow \vec{A}^{-1} = ?$$

$$\vec{j}_0(\vec{\rho}_0, \vec{\varphi}_0) = \vec{A} \times \vec{j}_0(\vec{x}_0, \vec{y}_0)$$

$$\vec{j}_0(\vec{x}_0, \vec{y}_0) = \vec{A}^{-1} \times \vec{j}_0(\vec{\rho}_0, \vec{\varphi}_0)$$

$$\vec{j}_0 = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} H(t)\delta(z)\big(H(\rho) - H(\rho - R)\big) \\ 0 \end{pmatrix}$$

$$= H(t)\delta(z)\big(H(\rho) - H(\rho - R)\big)\big(\vec{\rho}_0 \cos\varphi - \vec{\varphi}_0 \sin\varphi\big)$$

$$\vec{j}_0 = H(t)\delta(z)\big(H(\rho) - H(\rho - R)\big)\big(\vec{\rho}_0 \cos\varphi - \vec{\varphi}_0 \sin\varphi\big)$$

$$j_m(z,t;\nu) = \frac{\sqrt{\mu_0}}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho \, d\rho \, \overrightarrow{j_0} \left[\nabla_\perp \Psi_m^{\ *} \times \overrightarrow{z_0}\right]$$

$$J_{m-1}(\nu\rho) + J_{m+1}(\nu\rho) = \frac{2m}{\nu\rho} J_m(\nu\rho)$$

$$im\vec{\rho}_0 \frac{J_m(\nu\rho)}{\rho\sqrt{\nu}} = i\vec{\rho}_0 \frac{\sqrt{\nu}}{2} \frac{2m}{\rho\nu} J_m(\nu\rho) = i\vec{\rho}_0 \frac{\sqrt{\nu}}{2}\left[J_{m-1}(\nu\rho) + J_{m+1}(\nu\rho)\right]$$

$$\left[\nabla_\perp \Psi_m^{\ *} \times \vec{z}_0\right] = -e^{-im\varphi}\left(\vec{\varphi}_0 \sqrt{\nu} \frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{2} + im\vec{\rho}_0 \frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right) =$$
$$= -e^{-im\varphi} \frac{\sqrt{\nu}}{2}\Big(\vec{\varphi}_0\left[J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)\right] + i\vec{\rho}_0\left[J_{m-1}(\nu\rho) + J_{m+1}(\nu\rho)\right]\Big)$$

$$\vec{j}_0\left[\nabla_\perp \Psi_m^{\ *} \times \vec{z}_0\right] = -\sqrt{\nu} \frac{\cos(m\varphi) - i\sin(m\varphi)}{2} H(t)\delta(z)\left[H(\rho) - H(\rho - R)\right] \times$$
$$\times\Big(\vec{\varphi}_0\left[J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)\right] + i\vec{\rho}_0\left[J_{m-1}(\nu\rho) + J_{m+1}(\nu\rho)\right]\Big)\big(\vec{\rho}_0 \cos\varphi - \vec{\varphi}_0 \sin\varphi\big) =$$
$$= -\sqrt{\nu} \frac{\cos(m\varphi) - i\sin(m\varphi)}{2} H(t)\delta(z)\left[H(\rho) - H(\rho - R)\right] \times$$
$$\times\Big\{-\sin\varphi\left[J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)\right] + i\cos\varphi\left[J_{m-1}(\nu\rho) + J_{m+1}(\nu\rho)\right]\Big\}$$

$$j_m(z,t;\nu)=\frac{\sqrt{\mu_0}}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\,\vec{j_0}\left[\nabla_\perp\Psi_m{}^*\times\vec{z_0}\right]=$$

$$=\frac{\sqrt{\mu_0}}{2\pi}\frac{\sqrt{\nu}}{2}H(t)\delta(z)\int_0^{2\pi}\sin\varphi\left[\cos(m\varphi)-i\sin(m\varphi)\right]d\varphi\int_0^R\rho d\rho\left[J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)\right]-$$

$$-i\frac{\sqrt{\mu_0}}{2\pi}\frac{\sqrt{\nu}}{2}H(t)\delta(z)\int_0^{2\pi}\cos\varphi\left[\cos(m\varphi)-i\sin(m\varphi)\right]d\varphi\int_0^R\rho d\rho\left[J_{m-1}(\nu\rho)+J_{m+1}(\nu\rho)\right]$$

$$\cos x\cos y=\frac{1}{2}\left[\cos(x-y)+\cos(x+y)\right]$$

$$\sin x\cos y=\frac{1}{2}\left[\sin(x-y)+\sin(x+y)\right]$$

$$\sin x\sin y=\frac{1}{2}\left[\cos(x-y)-\cos(x+y)\right]$$

$$\delta_{m.n}=\frac{1}{2\pi}\int_0^{2\pi}e^{i(m-n)}d\varphi$$

$$\delta_{m,-n}=\frac{1}{2\pi}\int_0^{2\pi}e^{i(-m-n)}d\varphi$$

$$\int_0^{2\pi}\left[\cos(m\varphi)-i\sin(m\varphi)\right]\sin\varphi d\varphi=\frac{1}{2}\int_0^{2\pi}\sin(\varphi-m\varphi)d\varphi+\frac{1}{2}\int_0^{2\pi}\sin(m\varphi+\varphi)d\varphi-$$

$$-\frac{i}{2}\int_0^{2\pi}\cos(m\varphi-\varphi)d\varphi+\frac{i}{2}\int_0^{2\pi}\cos(m\varphi+\varphi)d\varphi=-\frac{i}{2}\left(\int_0^{2\pi}\cos(m\varphi-\varphi)d\varphi-i\int_0^{2\pi}\sin(m\varphi-\varphi)d\varphi\right)+$$

$$+\frac{i}{2}\left(\int_0^{2\pi}\cos(m\varphi+\varphi)d\varphi-i\int_0^{2\pi}\sin(m\varphi+\varphi)d\varphi\right)=\frac{i}{2}\left(-\int_0^{2\pi}e^{-i(m-1)\varphi}d\varphi+\int_0^{2\pi}e^{-i(m+1)\varphi}d\varphi\right)=$$

$$=i\pi\left(-\delta_{m,1}+\delta_{m,-1}\right)=-i\pi\delta_{m,1}+i\pi\delta_{m,-1}$$

$$\int_0^{2\pi}\left[\cos(m\varphi)-i\sin(m\varphi)\right]\cos\varphi d\varphi=\frac{1}{2}\int_0^{2\pi}\cos(m\varphi-\varphi)d\varphi+\frac{1}{2}\int_0^{2\pi}\cos(m\varphi+\varphi)d\varphi-$$

$$-\frac{i}{2}\int_0^{2\pi}\sin(m\varphi-\varphi)d\varphi-\frac{i}{2}\int_0^{2\pi}\sin(m\varphi+\varphi)d\varphi=\frac{1}{2}\left[\int_0^{2\pi}\cos(m\varphi-\varphi)d\varphi-i\int_0^{2\pi}\sin(m\varphi-\varphi)d\varphi\right]+$$

$$+\frac{1}{2}\left[\int_0^{2\pi}\cos(m\varphi+\varphi)d\varphi-i\int_0^{2\pi}\sin(m\varphi+\varphi)d\varphi\right]=\frac{1}{2}\left(\int_0^{2\pi}e^{-i(m-1)\varphi}d\varphi+\int_0^{2\pi}e^{-i(m+1)\varphi}d\varphi\right)=\pi\delta_{m,1}+\pi\delta_{m,-1}$$

$$m\geq 0\Rightarrow\delta_{m,-1}=0$$

Если предположить свойство симметрии значений функции $\delta_{m,-1}$ относительно плоскости излучателя, то решение 1997 и 2016 совпадет.

$$\int xJ_0(x)dx=xJ_1(x)+C$$

$$\int_0^{\infty}J_1(z)dz=1$$

$$j_m(z,t;\nu)=\frac{\sqrt{\mu_0}}{2\pi}\int_0^{2\pi}d\varphi\int_0^R\rho d\rho\,\vec{j_0}\left[\nabla_\perp\Psi_m{}^*\times\vec{z_0}\right]=$$

$$=i\frac{\sqrt{\mu_0}}{2\pi}\frac{\sqrt{\nu}}{2}H(t)\delta(z)\left(-\pi\delta_{m,1}+\pi\delta_{m,-1}\right)\int_0^R\rho d\rho\left[J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)\right]-$$

$$-i\frac{\sqrt{\mu_0}}{2\pi}\frac{\sqrt{\nu}}{2}H(t)\delta(z)\left(\pi\delta_{m,1}+\pi\delta_{m,-1}\right)\int_0^R\rho d\rho\left[J_{m-1}(\nu\rho)+J_{m+1}(\nu\rho)\right]=$$

$$=-i\frac{\sqrt{\mu_0}}{2\pi}\frac{\sqrt{\nu}}{2}H(t)\delta(z)\pi\delta_{m,1}\int_0^R\rho d\rho\left[2J_{m-1}(\nu\rho)\right]=-i\frac{\sqrt{\mu_0\nu}}{2}H(t)\delta(z)\delta_{m,1}\int_0^R\rho d\rho J_{m-1}(\nu\rho)=$$

$$=-i\frac{\sqrt{\mu_0\nu}}{2}H(t)\delta(z)\delta_{m,1}\int_0^R\rho d\rho J_0(\nu\rho)=-i\frac{\sqrt{\mu_0\nu}}{2}H(t)\delta(z)\delta_{m,1}\frac{1}{\nu^2}\int_0^R\nu\rho d\nu\rho J_0(\nu\rho)$$

$$=-i\frac{\sqrt{\mu_0\nu}}{2}H(t)\delta(z)\delta_{m,1}\frac{1}{\nu^2}\left[\nu RJ_1(\nu R)-0\cdot J_1(0)\right]=$$

$$=-i\frac{\sqrt{\mu_0\nu}}{2}H(t)\delta(z)\delta_{m,1}\frac{RJ_1(\nu R)}{\nu}=-\frac{\sqrt{\mu_0}}{2}\frac{iR}{\sqrt{\nu}}H(t)\delta(z)\delta_{m,1}J_1(\nu R)$$

$$j_m(z,t;\chi)=-\frac{iR}{\sqrt{\chi}}\delta_{m,1}J_1(\chi R)\delta(z)H(t),$$

Скорее всего в диссере 1997 потеряна 1/2

$$j_m(z,t;\nu)=\frac{\sqrt{\mu_0}}{2\pi}\int_0^{2\pi}d\varphi\int_0^\infty\rho d\rho\,\vec{j_0}\left[\nabla_\perp\Psi_m{}^*\times\vec{z_0}\right]=-\frac{\sqrt{\mu_0}}{2}\frac{iR}{\sqrt{\nu}}\delta_{m,1}J_1(\nu R)\delta(z)H(t). \tag{2.2}$$

The medium is homogeneous and stationary for simplification. It means no dependence on time or space coordinates for dielectric permeability ε and permittivity μ from polarization and magnetization vectors (1.5) definitions:

$$\varepsilon=\varepsilon(z,t)=1;\quad \mu=\mu(z,t)=1.$$

First two evolutionary equations get form of Klein-Gordon equations (Иваненко, и др., 1951), also current components $\vec{I},I_z,J_z$ are zero ones because of form of (2.1) expression.

$$
\begin{cases}
\partial_{ct}^2 h_m - \partial_z^2 h_m + \nu^2 h_m = \sqrt{\mu_0}\,\frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi \int\limits_0^{\infty} \rho d\rho \left[\vec{z}_0 \times \vec{J}\right] \cdot \nabla_\perp \Psi_m^{\ *}(\nu) \\
I_n^e = -\partial_{ct}\left\{\varepsilon e_n\right\}\big|_{\varepsilon=const} = -\frac{\varepsilon}{c}\frac{\partial e_n}{\partial t} \\
V_n^e = \varepsilon^{-1}\partial_z\left\{\varepsilon e_n\right\}\big|_{\varepsilon=const} = \frac{\partial e_n}{\partial z} \\
I_m^h = \mu^{-1}\partial_z\left\{\mu h_m\right\}\big|_{\mu=const} = \frac{\partial h_m}{\partial z} \\
V_m^h = -\partial_{ct}\left\{\mu h_m\right\}\big|_{\mu=const} = -\frac{\mu}{c}\frac{\partial h_m}{\partial t}
\end{cases}
\tag{2.3}
$$

4.2 Longitudinal coefficients

TE electromagnetic phenomena is under consideration. According to its determination $E_z = 0$. In case of EAE it can be only if coefficients for E_z form (1.51) are

$$
e_n(z,t;\chi) = 0. \tag{2.4}
$$

Second order evolutionary equations turn to Klein-Gordon equations in stationary and homogeneous medium. There is known solution by Riemann function for h_m coefficient.

$$
\int\limits_{-\infty}^{+\infty} f(x)\cdot\delta(x-a)\,dx = f(a)
$$

С чего мы взяли что это решение диф. уры???

$$
G(z',t',z,t) = \frac{c}{2} H\left(c(t-t') - (z-z')\right) J_0\left(\nu\sqrt{c^2(t-t')^2 - (z-z')^2}\right)
$$

Функция Римана с принципом причинности

$$
\begin{aligned}
&h_m(z,t;\nu)=\int_0^\infty dz'\int_0^\infty dt' H\left(c(t-t')-(z-z')\right)G(z',t',z,t)\,j_m(z',t';\chi)=\\
&=\frac{c}{2}\int_0^\infty dz'\int_0^\infty dt' H\left(c(t-t')-(z-z')\right)J_0\left(\nu\sqrt{c^2(t-t')^2-(z-z')^2}\right)j_m(z',t';\chi)=\\
&=-\frac{iR}{4\sqrt{\nu\varepsilon_0}}\delta_{m,1}J_1(\nu R)\int_0^\infty \delta(z')dz'\int_0^\infty H(t')dt' H\left(c(t-t')-(z-z')\right)J_0\left(\nu\sqrt{c^2(t-t')^2-(z-z')^2}\right)=\\
&=-\frac{iR}{4\sqrt{\nu\varepsilon_0}}\delta_{m,1}J_1(\nu R)\int_0^\infty H(t')H\left(c(t-t')-z\right)dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)=\\
&=-\frac{iR}{4\sqrt{\nu\varepsilon_0}}\delta_{m,1}J_1(\nu R)\int_0^\infty H\left(c(t-t')-z\right)dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)
\end{aligned}
\tag{2.5}
$$

$$
H(ct-ct'-z): ct-ct'-z>0\Rightarrow t'<t-\frac{z}{c}
$$

t' - блуждающее время, служит для соблюдения принципа причинности – сигнал не успевает добраться до точки наблюдения со скоростью света

$$
\begin{aligned}
&h_m(z,t;\nu)=-\frac{iR}{4\sqrt{\nu\varepsilon_0}}\delta_{m,1}J_1(\nu R)\int_0^\infty H\left(c(t-t')-z\right)dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)=\\
&=-\frac{iR}{4\sqrt{\nu\varepsilon_0}}\delta_{m,1}J_1(\nu R)\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)=\\
&=-\frac{\sqrt{\mu_0}}{4}\frac{icR}{\sqrt{\nu}}\delta_{m,1}J_1(\nu R)\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)
\end{aligned}
$$

where $H\left(c(t-t')-(z-z')\right)$ is Heaviside step function. This expression can be simplified by substitution of extended current density $j_m(z',t';\nu)$ from (2.2):

$$
h_m(z,t;\nu)=-\frac{iR}{4\sqrt{\nu\varepsilon_0}}\delta_{m,1}J_1(\nu R)\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)
\tag{2.6}
$$

The integral cannot be solved by ordinary functions, but it is close to one of Lommel function property. Let us make a variable substitutions to make it clear.

$$\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2(t-t')^2 - z^2}\right) = \left[W = \frac{c(t-t')}{z}\left|\frac{dW}{dt'} = -\frac{c}{z}\right.\right] =$$

$$= \left[W\left(t-\frac{z}{c}\right) = \frac{cz}{cz} = 1; W(0) = \frac{ct}{z}\right] = \frac{z}{c}\int_1^{\frac{ct}{z}} dW J_0\left(\nu\sqrt{W^2 z^2 - z^2}\right) =$$

$$= \frac{z}{c}\int_1^{\frac{ct}{z}} dW J_0\left(\nu z\sqrt{W^2 - 1}\right) = [s = \nu z W, ds = \nu z dW] = \frac{z}{c}\frac{1}{\nu z}\int_{\nu z}^{\nu ct} ds J_0\left(\sqrt{s^2 - \nu^2 z^2}\right) =$$

$$= \frac{1}{\nu c}\int_{\nu z}^{\nu ct} ds J_0\left(\sqrt{s^2 - \nu^2 z^2}\right)$$

$$\begin{aligned} h_m(z,t;\nu) &= -\frac{iR}{4\sqrt{\nu\varepsilon_0}}\delta_{m,1}J_1(\nu R)\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2(t-t')^2 - z^2}\right) = \\ &= -\frac{\sqrt{\mu_0}}{\sqrt{\mu_0\varepsilon_0}}\frac{iR}{4\sqrt{\nu}}\delta_{m,1}J_1(\nu R)\frac{1}{\nu c}\int_{\nu z}^{\nu ct} ds J_0\left(\sqrt{s^2 - \nu^2 z^2}\right) = \\ &= -\sqrt{\mu_0}\frac{iR}{4\sqrt{\nu^3}}\delta_{m,1}J_1(\nu R)\int_{\nu z}^{\nu ct} ds J_0\left(\sqrt{s^2 - \nu^2 z^2}\right) \end{aligned} \quad . \tag{2.7}$$

https://en.wikipedia.org/wiki/Lommel_function

https://en.wikipedia.org/wiki/Lommel_polynomial

Определения ф. Ломмеля + физ. Смысл + ссыла в литературу

$$U_n(W,Z) = \sum_{m=0}^{\infty}(-1)^m\left(\frac{W}{Z}\right)^{n+2m} J_{n+2m}(Z). \tag{2.8}$$

$$\begin{aligned} &\int_\xi^\tau ds e^{-i\gamma s} J_0\left(\sqrt{s^2 - \xi^2}\right) = \frac{e^{-i\gamma\tau}}{\sqrt{\gamma^2 - 1}}\left[U_1(W_+,Z) + iU_2(W_+,Z) - U_1(W_-,Z) - iU_2(W_-,Z)\right] \\ &W_\pm = \left(\gamma \pm \sqrt{\gamma^2 - 1}\right)(\tau - \xi); \qquad Z = \sqrt{\tau^2 - \xi^2}; \qquad \tau - \xi > 0 \end{aligned} \quad . \tag{2.9}$$

Lommel functions of two variables are ordinary in problems of acoustic wave excitation and transient electromagnetic field in LL. In our case next property can be applied.

$$\begin{aligned} &\int_{\nu z}^{\nu ct} ds e^{-i0s} J_0\left(\sqrt{s^2 - \nu^2 z^2}\right) = [\tau = \nu ct, \xi = \nu z, \gamma = 0] = \frac{e^{-i0\nu ct}}{\sqrt{0-1}} \times \\ &\times\left[U_1(W_+,Z) + iU_2(W_+,Z) - U_1(W_-,Z) - iU_2(W_-,Z)\right] = \\ &= -i\left[U_1(W_+,Z) + iU_2(W_+,Z) - U_1(W_-,Z) - iU_2(W_-,Z)\right] = \\ &= -iU_1(W_+,Z) + U_2(W_+,Z) + iU_1(W_-,Z) - U_2(W_-,Z) \end{aligned}$$

$$\left.\begin{array}{l} U_{2n}(W_+,Z)=U_{2n}(W_-,Z) \\ U_{2n+1}(W_+,Z)=-U_{2n+1}(W_-,Z) \end{array}\right| n\in\mathbb{Z}\,. \tag{2.10}$$

$$\int\limits_{\nu z}^{\nu ct} dse^{-i0s} J_0\left(\sqrt{s^2-\nu^2 z^2}\right) = -iU_1(W_+,Z)+U_2(W_+,Z)+iU_1(W_-,Z)-U_2(W_-,Z)=$$
$$=iU_1(W_-,Z)+U_2(W_-,Z)+iU_1(W_-,Z)-U_2(W_-,Z)=2iU_1(W_-,Z)=-2iU_1(W_+,Z)$$

$\tau-\xi>0 \Rightarrow \nu ct>\nu z$ выполняется благодаря принципу причинности

$$W_-=\left(\gamma\pm\sqrt{\gamma^2-1}\right)(\tau-\xi)=-i(\nu ct-\nu z)$$
$$Z=\sqrt{\tau^2-\xi^2}=\sqrt{\nu^2c^2t^2-\nu^2z^2}$$
$$\int\limits_{\nu z}^{\nu ct} dse^{-i0s} J_0\left(\sqrt{s^2-\nu^2 z^2}\right)=2iU_1(W_-,Z)=$$
$$=2iU_1\left[-i\nu(ct-z),\sqrt{\nu^2c^2t^2-\nu^2z^2}\right]$$

So expression (2.7) with substitution form (2.9) and property (2.10) can be rewrote like

$$h_m(z,t;\nu)=\sqrt{\mu_0}\,\frac{R\delta_{m,1}}{2\nu^{3/2}}J_1(\nu R)U_1\left[-i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right]$$ или так

$$h_m(z,t;\nu)=-\sqrt{\mu_0}\,\frac{R\delta_{m,1}J_1(\nu R)}{2\nu^{3/2}}U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right] \tag{2.11}$$

4.3 Transversal coefficients

Электрические равны нулю

$$I_n^e=V_n^e=0\,. \tag{2.12}$$

At first let us apply (2.13) Leibniz integral rule (Фихтенгольц Г. М., 2001) for V_m^h coefficients from (2.3).

$$\frac{\partial}{\partial\theta}\int\limits_{a(\theta)}^{b(\theta)} f(x,\theta)dx=\int\limits_{a(\theta)}^{b(\theta)}\frac{\partial f}{\partial\theta}dx+f\big(b(\theta),\theta\big)\cdot b'(\theta)-f\big(a(\theta),\theta\big)\cdot a'(\theta)\,. \tag{2.13}$$

$$\frac{\partial}{\partial t}J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)=-\nu J_1\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)\frac{\partial}{\partial t}\sqrt{c^2(t-t')^2-z^2}=$$
$$=-J_1\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)\frac{2\nu c^2(t-t')}{2\sqrt{c^2(t-t')^2-z^2}}=-\nu c^2(t-t')\frac{J_1\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)}{\sqrt{c^2(t-t')^2-z^2}}$$

$$\frac{\partial}{\partial t'} J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right) = -\nu J_1\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)\frac{\partial}{\partial t'}\sqrt{c^2(t-t')^2-z^2} =$$
$$= J_1\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)\frac{2\nu c^2(t-t')}{2\sqrt{c^2(t-t')^2-z^2}} = \nu c^2(t-t')\frac{J_1\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)}{\sqrt{c^2(t-t')^2-z^2}}$$

$$\frac{\partial}{\partial t}\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right) = \int_0^{t-\frac{z}{c}} dt'\frac{\partial}{\partial t} J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right) + J_0(0) - 0\cdot J_0\left(\nu\sqrt{c^2t^2-z^2}\right) =$$
$$= \int_0^{t-\frac{z}{c}} dt'\frac{\partial}{\partial t} J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right) + 1 = -\nu c^2\int_0^{t-\frac{z}{c}} dt'(t-t')\frac{J_1\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)}{\sqrt{c^2(t-t')^2-z^2}} + 1 =$$
$$= -\int_0^{t-\frac{z}{c}} dt'\frac{\partial}{\partial t'} J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right) + 1 = J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)\Big|_0^{t-\frac{z}{c}} + 1 =$$
$$= -J_0(0) + J_0\left(\nu\sqrt{c^2t^2-z^2}\right) + 1 = J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

$$V_m^h = -\partial_{ct}\{\mu h_m\}\big|_{\mu=const} = -\mu\frac{\partial h_m}{\partial t} = -\mu\frac{1}{c}\frac{\partial}{\partial t}\left(-\frac{iR}{4\sqrt{\nu\varepsilon_0}}\delta_{m,1}J_1(\nu R)\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)\right) =$$
$$= \mu\sqrt{\mu_0}\frac{iR\delta_{m,1}J_1(\nu R)}{4\sqrt{\nu}}\frac{\partial}{\partial t}\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right) = \mu\sqrt{\mu_0}\frac{iR\delta_{m,1}J_1(\nu R)}{4\sqrt{\nu}} J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

$$V_m^h = \mu\sqrt{\mu_0}\frac{iR\delta_{m,1}J_1(\nu R)}{4\sqrt{\nu}} J_0\left(\nu\sqrt{c^2t^2-z^2}\right). \tag{2.14}$$

I_m^h coefficients from (2.3) can be presented with **Ошибка! Источник ссылки не найден.** substitution

$$I_m^h = \frac{1}{\mu}\frac{\partial h_m}{\partial z} = -\frac{\sqrt{\mu_0}}{\mu}\frac{R\delta_{m,1}}{2\nu^{3/2}} J_1(\nu R)\frac{\partial}{\partial z} U_1\left[i\nu(ct-z), \nu\sqrt{c^2t^2-z^2}\right]. \tag{2.15}$$

According to derivative properties of Lommel function (Борисов, 1991)

$$\frac{\partial U_1}{\partial W} = \frac{1}{2}U_0(W,Z) + \frac{1}{2}\left(\frac{Z}{W}\right)^2 U_2(W,Z); \quad \frac{\partial U_1}{\partial Z} = -\frac{Z}{W}U_2(W,Z);$$

expression for

$$\left(\frac{\nu\sqrt{c^2t^2-z^2}}{i\nu(ct-z)}\right)^2 = \left(-i\frac{\nu\sqrt{c^2t^2-z^2}}{\nu(ct-z)}\right)^2 = (-i)^2\frac{ct+z}{ct-z} = -\frac{ct+z}{ct-z}$$

$$-\frac{ct+z}{ct-z}+\frac{2z}{ct-z}=\frac{-ct-z+2z}{ct-z}=\frac{-ct+z}{ct-z}=-\frac{ct-z}{ct-z}=-1$$

$$\frac{\partial}{\partial z}U_1(W_+,Z)=\frac{\partial U_1}{\partial W_+}\frac{\partial W_+}{\partial z}+\frac{\partial U_1}{\partial Z}\frac{\partial Z}{\partial z}=\left[\frac{1}{2}U_0(W_+,Z)+\frac{1}{2}\left(\frac{Z}{W_+}\right)^2U_2(W_+,Z)\right]\frac{\partial W_+}{\partial z}-$$

$$-\frac{Z}{W_+}U_2(W_+,Z)\frac{\partial Z}{\partial z}=\left[\begin{array}{c|c}\frac{\partial Z}{\partial z}=-\frac{\nu z}{\sqrt{c^2t^2-z^2}} & \frac{\partial W_+}{\partial z}=-i\nu \\ Z=\nu\sqrt{c^2t^2-z^2} & W_+=i\nu(ct-z)\end{array}\right]=$$

$$=-\frac{i\nu}{2}\left[U_0(W_+,Z)+\left(\frac{\nu\sqrt{c^2t^2-z^2}}{i\nu(ct-z)}\right)^2U_2(W_+,Z)\right]+\frac{\nu\sqrt{c^2t^2-z^2}}{i\nu(ct-z)}\frac{\nu z}{\sqrt{c^2t^2-z^2}}U_2(W_+,Z)=$$

$$=-\frac{i\nu}{2}\left[U_0(W_+,Z)-\frac{ct+z}{ct-z}U_2(W_+,Z)\right]-\frac{i\nu z}{ct-z}U_2(W_+,Z)=$$

$$=-\frac{i\nu}{2}\left[U_0(W_+,Z)+\left(-\frac{ct+z}{ct-z}+\frac{2z}{ct-z}\right)U_2(W_+,Z)\right]=-\frac{i\nu}{2}\left[U_0(W_+,Z)-U_2(W_+,Z)\right]$$

$$\frac{\partial}{\partial z}h_m(z,t;\chi)=-\frac{iR\delta_{m,1}J_1(\chi R)}{4\sqrt{\chi}}\left\{U_0\left(i\chi(ct-z),\chi\sqrt{c^2t^2-z^2}\right)-U_2\left(i\chi(ct-z),\chi\sqrt{c^2t^2-z^2}\right)\right\}.$$

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$$I_m^h=\frac{\sqrt{\mu_0}}{\mu}\frac{iR\delta_{m,1}J_1(\nu R)}{4\sqrt{\nu}}\left[U_0(W_+,Z)-U_2(W_+,Z)\right]. \tag{2.16}$$

4.4 Field expression

Evolutionary coefficients substitution to expansion form of EM field gives three dimensional expressions for electric and magnetic strength vectors dependent of radius vector $\vec{r}$ and time t.

Now we are able to substitute coefficients V_m^h form (2.14), I_m^h from (2.16) and I_n^e, V_n^e from (2.12) to (1.47) using (1.49) and (1.50) expressions.

$$V_m^h=\mu\sqrt{\mu_0}\frac{iR\delta_{m,1}J_1(\nu R)}{4\sqrt{\nu}}J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

$$\left[\nabla_\perp\Psi_m\times\vec{z}_0\right]=-e^{im\varphi}\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2}-im\vec{\rho}_0\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right)$$

$$\vec{E} = \sqrt[-2]{\varepsilon_0}\left\{\sum_{m=0}^{\infty}\int_0^{\infty} d\nu V_m^h\left[\nabla_{\perp}\Psi_m \times \vec{z}_0\right] + \sum_{m=1}^{\infty}\int_0^{\infty} d\chi V_n^e \nabla_{\perp}\Phi_n\right\} =$$
$$= -\frac{1}{\sqrt{\varepsilon_0}}\sum_{m=0}^{\infty}\int_0^{\infty} d\nu\sqrt{\mu_0}\,\mu\frac{iR\delta_{m,1}J_1(\nu R)}{2\sqrt{\nu}}J_0\left(\nu\sqrt{c^2t^2-z^2}\right)e^{im\varphi}\times$$
$$\times\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2} - im\vec{\rho}_0\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right) =$$
$$= -\mu\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{iR}{2}e^{i\varphi}\int_0^{\infty}\frac{d\nu}{\sqrt{\nu}}J_1(\nu R)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_0(\nu\rho)-J_2(\nu\rho)}{2} - i\vec{\rho}_0\frac{J_1(\nu\rho)}{\rho\sqrt{\nu}}\right)$$

$$\vec{E} = -\frac{iR}{2}\mu\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}e^{i\varphi}\int_0^{\infty}\frac{d\nu}{\sqrt{\nu}}J_1(\nu R)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_0(\nu\rho)-J_2(\nu\rho)}{2} - i\vec{\rho}_0\frac{J_1(\nu\rho)}{\rho\sqrt{\nu}}\right)$$
$$E_{\varphi} = -\frac{iR}{4}\mu\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}e^{i\varphi}\int_0^{\infty} d\nu J_1(\nu R)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_0(\nu\rho)-J_2(\nu\rho)\right]$$
$$E_{\rho} = -\frac{R}{2}\mu\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{e^{i\varphi}}{\rho}\int_0^{\infty}\frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

$$E_{\rho} = -\frac{R}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\mu\frac{e^{i\varphi}}{\rho}\int_0^{\infty}\frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right); \tag{2.17}$$

$$E_{\varphi} = -\frac{iR}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\mu e^{i\varphi}\int_0^{\infty} d\nu J_1(\nu R)\left(J_0(\nu\rho)-J_2(\nu\rho)\right)J_0\left(\nu\sqrt{c^2t^2-z^2}\right); \tag{2.18}$$

$$I_m^h = \frac{1}{\mu}\frac{\partial h_m}{\partial z} = \frac{\sqrt{\mu_0}}{\mu}\frac{iR\delta_{m,1}J_1(\nu R)}{2\sqrt{\nu}}\left[U_0(W_+,Z)-U_2(W_+,Z)\right]$$
$$\nabla_{\perp}\Psi_m = e^{im\varphi}\left(\vec{\rho}_0\sqrt{\nu}\,\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2} + im\vec{\varphi}_0\frac{J_m(\nu\rho)}{\sqrt{\nu}\rho}\right)$$

$$\vec{H} = \sqrt[-2]{\mu_0}\left\{\sum_{m=0}^{\infty}\int_0^{\infty} d\nu I_m^h\nabla_{\perp}\Psi_m + \sum_{m=1}^{\infty}\int_0^{\infty} d\chi I_n^e\left[\vec{z}_0\times\nabla_{\perp}\Phi_n\right]\right\} =$$
$$= \frac{iR}{2}\frac{e^{i\varphi}}{\mu}\int_0^{\infty}\frac{d\nu}{\sqrt{\nu}}J_1(\nu R)\left[U_0(W_+,Z)-U_2(W_+,Z)\right]\left(\vec{\rho}_0\sqrt{\nu}\,\frac{J_0(\nu\rho)-J_2(\nu\rho)}{2} + i\vec{\varphi}_0\frac{J_1(\nu\rho)}{\sqrt{\nu}\rho}\right)$$
$$H_{\rho} = \frac{iR}{4}\frac{e^{i\varphi}}{\mu}\int_0^{\infty} d\nu J_1(\nu R)\left(J_0(\nu\rho)-J_2(\nu\rho)\right)\left[U_0(W_+,Z)-U_2(W_+,Z)\right]$$
$$H_{\varphi} = \frac{1}{\mu}\frac{R}{2}\frac{e^{i\varphi}}{\rho}\int_0^{\infty}\frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)\left[U_0(W_+,Z)-U_2(W_+,Z)\right]$$

$$H_\rho = \frac{1}{\mu}\frac{iR}{4}e^{i\varphi}\int_0^\infty d\nu J_1(\nu R)\left(J_0(\nu\rho)-J_2(\nu\rho)\right)\left[U_0(W_+,Z)-U_2(W_+,Z)\right]; \quad (2.19)$$

$$H_\varphi = \frac{1}{\mu}\frac{R}{2}\frac{e^{i\varphi}}{\rho}\int_0^\infty \frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)\left[U_0(W_+,Z)-U_2(W_+,Z)\right]. \quad (2.20)$$

The same is with longitudinal ones. Substitution of **Ошибка! Источник ссылки не найден.** and (2.12) to (1.51) gives field components E_z and H_z:

$$E_z(\rho,\phi,z,t) = \sqrt[-2]{\varepsilon_0}\sum_{n=0}^{\infty}\int_0^\infty \chi^2 d\chi e_n(z,t;\chi)\Phi_n(\rho,\phi;\chi) = 0$$

$$E_z = 0; \quad (2.21)$$

$$h_m(z,t;\nu) = -\sqrt{\mu_0}\frac{R\delta_{m,1}J_1(\nu R)}{2\nu^{3/2}}U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right]; \quad \Psi_m(\nu) = \frac{J_m(\nu\rho)}{\sqrt{\nu}}e^{im\varphi}$$

$$H_z(\rho,\phi,z,t) = \sqrt[-2]{\mu_0}\sum_{m=0}^{\infty}\int_0^\infty \nu^2 d\nu h_m(z,t;\nu)\Psi_m(\rho,\phi;\nu) =$$

$$= -\frac{Re^{i\varphi}}{2}\frac{\sqrt{\mu_0}}{\sqrt{\mu_0}}\int_0^\infty \frac{\nu^2}{\nu^{3/2}\nu^{1/2}}d\nu J_1(\nu R)J_1(\nu\rho)U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right] =$$

$$= -\frac{Re^{i\varphi}}{2}\int_0^\infty d\nu J_1(\nu R)J_1(\nu\rho)U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right]$$

$$H_z = -\frac{Re^{i\varphi}}{2}\int_0^\infty d\nu J_1(\nu R)J_1(\nu\rho)U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right]. \quad (2.22)$$

(2.18) and (2.19) can be represented with using of Bessel function property

$$J_{m+1}(z) = \frac{2m}{z}J_m(z) - J_{m-1}(z); \quad J_2(\nu\rho) = \frac{2}{\nu\rho}J_1(\nu\rho) - J_0(\nu\rho), \quad (2.23)$$

so E_φ and H_ρ get the forms

$$E_\varphi = -\frac{iR}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\mu e^{i\varphi}\int_0^\infty d\nu J_1(\nu R)\left(J_0(\nu\rho) - \frac{J_1(\nu\rho)}{\nu\rho}\right)J_0\left(\nu\sqrt{c^2t^2-z^2}\right) \quad (2.24)$$

$$H_\rho = \frac{1}{\mu}\frac{iR}{2}e^{i\varphi}\int_0^\infty d\nu J_1(\nu R)\left(J_0(\nu\rho) - \frac{J_1(\nu\rho)}{\nu\rho}\right)\left[U_0(W_+,Z)-U_2(W_+,Z)\right] \quad (2.25)$$

$$E_\varphi = -\frac{iR}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\mu e^{i\varphi}\int_0^\infty d\nu J_1(\nu R)\left(J_0(\nu\rho) - \frac{J_1(\nu\rho)}{\nu\rho}\right)J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) =$$

$$= -\frac{iR}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\mu e^{i\varphi}\int_0^\infty d\nu J_0(\nu\rho)J_1(\nu R)J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) +$$

$$+\frac{iR}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\mu\frac{e^{i\varphi}}{\rho}\int_0^\infty \frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2 - z^2}\right)$$

$$E_\rho = -\frac{R}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\mu\frac{e^{i\varphi}}{\rho}\int_0^\infty \frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2 - z^2}\right)$$

$$E_\varphi = -\mu R\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{ie^{i\varphi}}{2}\left(I_2 - \frac{I_1}{\rho}\right); \quad E_\rho = -\mu R\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{e^{i\varphi}}{2}\frac{I_1}{\rho}$$

$$\vec{E} = -\mu R\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{ie^{i\varphi}}{2}\left[\vec{\varphi}_0\left(I_2 - \frac{I_1}{\rho}\right) + \frac{1}{i}\vec{\rho}_0\frac{I_1}{\rho}\right] = -\mu R\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{ie^{i\varphi}}{2}\left[\vec{\varphi}_0\left(I_2 - \frac{I_1}{\rho}\right) - i\vec{\rho}_0\frac{I_1}{\rho}\right]$$

$$\vec{E}(\rho,\phi,z,t) = \mu R\frac{ie^{i\varphi}}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\left[\vec{\varphi}_0\left(\frac{I_1}{\rho} - I_2\right) + i\vec{\rho}_0\frac{I_1}{\rho}\right] \tag{2.26}$$

$$\vec{E}(\rho,\phi,z,t) = \frac{ie^{i\phi}}{8\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left\{\vec{\phi}_0(I_1 - 2I_2) + i\vec{\rho}_0 I_1\right\};$$ поле в диссере 1997

$$\vec{H} = \vec{\varphi}_0\frac{1}{\mu}\frac{R}{2}\frac{e^{i\varphi}}{\rho}\int_0^\infty \frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)\left[U_0(W_+,Z) - U_2(W_+,Z)\right] -$$

$$-\vec{\rho}_0\frac{1}{\mu}\frac{iR}{2}\frac{e^{i\varphi}}{\rho}\int_0^\infty \frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)\left[U_0(W_+,Z) - U_2(W_+,Z)\right] +$$

$$+\frac{1}{\mu}\frac{iR}{2}e^{i\varphi}\int_0^\infty d\nu J_1(\nu R)J_0(\nu\rho)\left[U_0(W_+,Z) - U_2(W_+,Z)\right] =$$

$$= \frac{1}{\mu}\frac{R}{2}\left[i\vec{\rho}_0\left(I_2 - \frac{I_1}{\rho}\right) + \vec{\varphi}_0\frac{I_1}{\rho}\right] = \frac{R}{\mu}\frac{ie^{i\varphi}}{2}\left[\vec{\rho}_0\left(I_2 - \frac{I_1}{\rho}\right) - i\vec{\varphi}_0\frac{I_1}{\rho}\right] =$$

$$= -\frac{R}{\mu}\frac{ie^{i\varphi}}{2}\left[i\vec{\varphi}_0\frac{I_1}{\rho} + \vec{\rho}_0\left(\frac{I_1}{\rho} - I_2\right)\right]$$

$$\vec{H}(\rho,\varphi,z,t) = -\frac{R}{\mu}\frac{ie^{i\varphi}}{2}\left[i\vec{\varphi}_0\frac{I_1}{\rho} + \vec{\rho}_0\left(\frac{I_1}{\rho} - I_2\right)\right]$$

(2.17)-(2.22) consist integrals of triple Bessel function multiplications. Note, Lommel function is not an exception: it can be presented in the way of infinity summary of Bessel function with some coefficients according to (2.8).

$$H_z = -\frac{Re^{i\varphi}}{2}\int_0^\infty d\nu J_1(\nu R) J_1(\nu\rho) U_1\left[i\nu(ct-z), \nu\sqrt{c^2t^2-z^2}\right] =$$
$$= -\frac{Re^{i\varphi}}{2}\int_0^\infty d\nu J_1(\nu R) J_1(\nu\rho)\sum_{m=0}^\infty (-1)^m \left(\frac{i(ct-z)}{\sqrt{c^2t^2-z^2}}\right)^{1+2m} J_{1+2m}\left(\nu\sqrt{c^2t^2-z^2}\right) = \qquad (2.27)$$
$$= -\frac{iRe^{i\varphi}}{2}\sum_{m=0}^\infty \left(\sqrt{\frac{ct-z}{ct+z}}\right)^{2m+1}\int_0^\infty d\nu J_1(\nu R) J_1(\nu\rho) J_{1+2m}\left(\nu\sqrt{c^2t^2-z^2}\right)$$

$$U_n(W,Z) = \sum_{m=0}^\infty (-1)^m \left(\frac{W}{Z}\right)^{n+2m} J_{n+2m}(Z)$$

$$H_\rho = \frac{1}{\mu}\frac{iR}{2}e^{i\varphi}\int_0^\infty d\nu J_1(\nu R)\left(J_0(\nu\rho) - \frac{J_1(\nu\rho)}{\nu\rho}\right)\left[U_0(W_+,Z) - U_2(W_+,Z)\right] =$$
$$= \frac{1}{\mu}\frac{iR}{2}e^{i\varphi}\int_0^\infty d\nu J_1(\nu R)\left(J_0(\nu\rho) - \frac{J_1(\nu\rho)}{\nu\rho}\right)\left[U_0(W_+,Z) - U_2(W_+,Z)\right] =$$
$$= \frac{1}{\mu}\frac{iR}{2}e^{i\varphi}\sum_{m=0}^\infty (-1)^m \int_0^\infty d\nu J_1(\nu R)\left(J_0(\nu\rho) - \frac{J_1(\nu\rho)}{\nu\rho}\right)\times$$
$$\times\left[J_{2m}\left(\nu\sqrt{c^2t^2-z^2}\right) + \left(\frac{ct-z}{ct+z}\right)^{m+1} J_{2m+2}\left(\nu\sqrt{c^2t^2-z^2}\right)\right] =$$
$$= \frac{1}{\mu}\frac{iR}{2}e^{i\varphi}\sum_{m=0}^\infty (-1)^m \left(\frac{ct-z}{ct+z}\right)^m \int_0^\infty d\nu J_1(\nu R)\left(J_0(\nu\rho) - \frac{J_1(\nu\rho)}{\nu\rho}\right)\times$$
$$\times\left[J_{2m}\left(\nu\sqrt{c^2t^2-z^2}\right) + \frac{ct-z}{ct+z} J_{2m+2}\left(\nu\sqrt{c^2t^2-z^2}\right)\right]$$

$$H_\rho = -\frac{i}{8\sqrt{\mu_0}}e^{i\varphi}\sum_{m=0}^\infty (-1)^m (-i)^{2m}\left(\frac{ct-z}{ct+z}\right)^m \int_0^\infty d\nu J_1(\nu R)\times \qquad . \qquad (2.28)$$
$$\times\left[J_{2m}\left(\nu\sqrt{c^2t^2-z^2}\right) + i\frac{ct-z}{ct+z} J_{2m+2}\left(\nu\sqrt{c^2t^2-z^2}\right)\right]\left(J_0(\nu\rho) - \frac{J_1(\nu\rho)}{\nu\rho}\right)$$

$$H_\varphi = \frac{e^{i\varphi}}{4\rho\sqrt{\mu_0}}\int_0^\infty \frac{d\nu}{\nu} J_1(\nu R)(U_0 - U_2) J_1(\nu\rho) \qquad (2.29)$$

Electric strength vector components can be integrated by analytics. Let us rename integrals expressions like

$$I_1 = \int_0^\infty \frac{d\nu}{\nu} J_1(\nu R) J_1(\nu\rho) J_0\left(\nu\sqrt{c^2t^2-z^2}\right); \qquad (2.30)$$

$$I_2 = \int_0^\infty d\nu J_1(\nu R) J_0(\nu\rho) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right). \tag{2.31}$$

Later we are going to prove that I_1 is equal to

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$$\int_0^\infty \frac{dt}{t^{\lambda+\nu}} J_\mu(at) J_\nu(bt) J_\nu(ct) = \frac{(bc/2)^\nu}{\Gamma(\nu+1/2)\Gamma(1/2)} \int_0^\infty \int_0^\pi \frac{J_\mu(at) J_\nu(\omega t)}{\omega^\nu t^\lambda} \sin^{2\nu}\varphi \, d\varphi \, dt$$

$$\omega = \sqrt{b^2 + c^2 - 2bc\cdot\cos\varphi}$$

$$\int_0^\infty \frac{d\nu}{\nu} J_1(\nu R) J_1(\nu\rho) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) = \frac{\rho R}{2\Gamma(3/2)\Gamma(1/2)} \int_0^\pi \frac{\sin^2\varphi}{\sqrt{\rho^2 + R^2 - 2\rho R\cdot\cos\varphi}} \times$$

$$\times \int_0^\infty d\nu J_1(\omega\nu) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) d\varphi$$

$$\omega = \sqrt{\rho^2 + R^2 - 2\rho R\cdot\cos\varphi}$$

Прудников, Брычков, Маричев, 1983 (2.13.32, ст. 209)

$$\int_0^\infty d\nu J_n(a\nu) J_{n-1}(b\nu) = \begin{cases} b^{n-1}/a^n & ,b<a \\ 0 & ,b>a \end{cases}$$

$$\sin\frac{\varphi}{2} = \pm\sqrt{\frac{1-\cos\varphi}{2}}$$

$$1-\cos\varphi = \begin{cases} 2\sin^2\dfrac{\varphi}{2} & ,0<\varphi<\pi \\ -2\sin^2\dfrac{\varphi}{2} & ,\pi<\varphi<2\pi \end{cases}$$

$$\int_0^\infty d\nu J_n(a\nu) J_{n-1}(b\nu) = \begin{cases} b^{n-1}/a^n & ,b<a \\ 0 & ,b>a \end{cases}$$

$$\int_0^\infty d\nu J_1\left(\nu\sqrt{\rho^2 + R^2 - 2\rho R\cdot\cos\varphi}\right) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) = \frac{1}{\sqrt{\rho^2 + R^2 - 2\rho R\cdot\cos\varphi}}$$

$$\sqrt{\rho^2 + R^2 - 2\rho R\cdot\cos\varphi} > \sqrt{c^2t^2 - z^2}$$

$$\rho^2 + R^2 - 2\rho R\cdot\cos\varphi - 2\rho R + 2\rho R > c^2t^2 - z^2$$

$$(\rho - R)^2 + 2\rho R(1-\cos\varphi) > c^2t^2 - z^2$$

$$(\rho - R)^2 + 4\rho R\sin^2\frac{\varphi}{2} > c^2t^2 - z^2$$

$$\varphi > 2\arcsin\sqrt{\frac{c^2t^2 - z^2 - (\rho - R)^2}{4\rho R}}$$

$$\sqrt{\rho^2 + R^2 - 2\rho R\cdot\cos\varphi} > \sqrt{c^2t^2 - z^2}$$

$$\rho^2 + R^2 - 2\rho R\cdot\cos\varphi - 2\rho R + 2\rho R > c^2t^2 - z^2$$

$$(\rho - R)^2 + 2\rho R(1-\cos\varphi) > c^2t^2 - z^2$$

$$(\rho - R)^2 + 4\rho R\sin^2\frac{\varphi}{2} > c^2t^2 - z^2$$

$$\varphi > 2\arcsin\sqrt{\frac{c^2t^2 - z^2 - (\rho - R)^2}{4\rho R}}$$

$$\psi = 2\arcsin\sqrt{\frac{c^2t^2 - z^2 - (\rho - R)^2}{4\pi\rho}}$$

$$\psi \le \varphi \le \pi$$

$$\Gamma(3/2)\Gamma(1/2) = \left(\frac{1}{2}\sqrt{\pi}\right)\cdot\left(\sqrt{\pi}\right) = \frac{\pi}{2}$$

$$I_1 = \int_0^\infty \frac{d\nu}{\nu} J_1(\nu R) J_1(\nu\rho) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) = \frac{\rho R}{2\Gamma(3/2)\Gamma(1/2)} \times$$

$$\times \int_0^\pi \frac{\sin^2\varphi}{\sqrt{\rho^2 + R^2 - 2\rho R\cdot\cos\varphi}} \int_0^\infty d\nu J_1(\omega\nu) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) d\varphi =$$

$$= \frac{\rho R}{\pi} \int_0^\pi \frac{\sin^2\varphi}{\sqrt{\rho^2 + R^2 - 2\rho R\cdot\cos\varphi}} \int_0^\infty d\nu J_1(\omega\nu) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) d\varphi = \frac{\rho R}{\pi} \int_\psi^\pi \frac{\sin^2\varphi d\varphi}{\rho^2 + R^2 - 2\rho R\cdot\cos\varphi} =$$

$$= \frac{\rho}{\pi R} \int_\psi^\pi \frac{\sin^2\varphi d\varphi}{\frac{\rho^2}{R^2} + 1 - \frac{2\rho}{R}\cdot\cos\varphi}$$

$$\int_\psi^\pi \frac{\sin^2\varphi d\varphi}{a + b\cdot\cos\varphi} = \int_\psi^\pi \frac{(1 - \cos^2\varphi) d\varphi}{a + b\cdot\cos\varphi} = \int_\psi^\pi \frac{d\varphi}{a + b\cdot\cos\varphi} - \int_\psi^\pi \frac{\cos^2\varphi + \frac{a}{b}\cos\varphi}{a + b\cdot\cos\varphi} d\varphi + \frac{a}{b}\int_\psi^\pi \frac{\cos\varphi d\varphi}{a + b\cdot\cos\varphi} =$$

$$= \int_\psi^\pi \frac{d\varphi}{a + b\cdot\cos\varphi} - \frac{1}{b}\int_\psi^\pi \cos\varphi d\varphi + \frac{a}{b^2}\int_\psi^\pi \frac{a + b\cdot\cos\varphi}{a + b\cdot\cos\varphi} d\varphi - \frac{a^2}{b^2}\int_\psi^\pi \frac{d\varphi}{a + b\cdot\cos\varphi} =$$

$$= \left(1 - \frac{a^2}{b^2}\right)\int_\psi^\pi \frac{d\varphi}{a + b\cdot\cos\varphi} + \frac{a}{b^2}\int_\psi^\pi d\varphi - \frac{1}{b}\int_\psi^\pi \cos\varphi d\varphi$$

Прудников, Бычков, Маричев «Интегралы и ряды: Элементарные функции» ст. 181,

1.5.9.15 $$\int \frac{d\varphi}{a + b\cdot\cos\varphi} = \frac{2}{\sqrt{a^2 - b^2}} \arctan\left(\frac{\sqrt{a^2 - b^2}}{a + b}\tan\frac{\varphi}{2}\right)$$

$$\tan\frac{\pi}{2} = \infty \Rightarrow \arctan\left(\tan\frac{\pi}{2}\right) = \arctan(\infty) = \frac{\pi}{2}$$

$$\int_{\psi}^{\pi}\frac{\sin^2\varphi d\varphi}{a+b\cdot\cos\varphi}=\left(1-\frac{a^2}{b^2}\right)\int_{\psi}^{\pi}\frac{d\varphi}{a+b\cdot\cos\varphi}+\frac{a}{b^2}\int_{\psi}^{\pi}d\varphi-\frac{1}{b}\int_{\psi}^{\pi}\cos\varphi d\varphi=$$
$$=\left(1-\frac{a^2}{b^2}\right)\frac{2}{\sqrt{a^2-b^2}}\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\varphi}{2}\right)+\frac{a}{b^2}\varphi-\frac{\sin\varphi}{b}\Bigg|_{\psi}^{\pi}=$$
$$=\left(1-\frac{a^2}{b^2}\right)\frac{2}{\sqrt{a^2-b^2}}\left[\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\pi}{2}\right)-\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\psi}{2}\right)\right]+\frac{a}{b^2}\pi-\frac{a}{b^2}\psi-$$
$$-\frac{\sin\pi}{b}+-\frac{\sin\psi}{b}=-\frac{2}{b^2}\frac{a^2-b^2}{\sqrt{a^2-b^2}}\left[\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\pi}{2}\right)-\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\psi}{2}\right)\right]+\frac{a}{b^2}\pi-$$
$$-\frac{a}{b^2}\psi-\frac{\sin\pi}{b}+\frac{\sin\psi}{b}=\frac{2\sqrt{a^2-b^2}}{b^2}\left[\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\psi}{2}\right)-\frac{\pi}{2}\right]+\frac{a}{b^2}(\pi-\psi)-\frac{\sin\psi}{b}$$

$$\tan\frac{\psi}{2}=\frac{\sin\frac{\psi}{2}}{\cos\frac{\psi}{2}}=\frac{\sin\frac{\psi}{2}}{\sqrt{1-\sin^2\frac{\psi}{2}}}=\sqrt{\frac{\frac{c^2t^2-z^2-(\rho-R)^2}{4\pi\rho}}{1-\frac{c^2t^2-z^2-(\rho-R)^2}{4\pi\rho}}}=\sqrt{\frac{c^2t^2-z^2-(\rho-R)^2}{(\rho+R)^2-c^2t^2+z^2}}$$

$$\int_{\psi}^{\pi}\frac{\sin^2\varphi d\varphi}{a+b\cdot\cos\varphi}=\frac{2\sqrt{a^2-b^2}}{b^2}\left[\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\psi}{2}\right)-\frac{\pi}{2}\right]+\frac{a}{b^2}(\pi-\psi)-\frac{\sin\psi}{b}=$$
$$=2\frac{\sqrt{a^2-b^2}}{b^2}\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\sqrt{\frac{c^2t^2-z^2-(\rho-R)^2}{(\rho+R)^2-c^2t^2+z^2}}\right)-\pi\frac{\sqrt{a^2-b^2}}{b^2}+\frac{a}{b^2}(\pi-\psi)-\frac{\sin\psi}{b}=$$
$$=$$

$$I_1=\int_0^{\infty}\frac{dv}{v}J_1(vR)J_1(v\rho)J_0\left(v\sqrt{c^2t^2-z^2}\right)=\frac{\rho}{\pi R}\int_{\psi}^{\pi}\frac{\sin^2\varphi d\varphi}{\frac{\rho^2}{R^2}+1-\frac{2\rho}{R}\cdot\cos\varphi}$$

$$I_1=\frac{\rho^2+R^2}{4\pi R\rho}(\pi-2\psi)+\frac{\rho^2-R^2}{4\pi R\rho}\left(\arctan\left(\frac{\rho+R}{\rho-R}\tan\frac{\psi}{2}\right)-\arctan\left(\frac{\rho-R}{\rho+R}\tan^{-1}\frac{\psi}{2}\right)\right). \quad (2.32)$$

Integral (2.31) can be solved by the formula known form (Прудников, и др., 1983)

$$\int_0^{\infty}J_0(ax)J_0(bx)J_1(cx)dx=\frac{1}{\pi c}\arccos\frac{a^2+b^2-c^2}{2ab}$$
$$|a-b|<c<a+b\quad a,b>0$$
$$I_2=\frac{1}{\pi R}\arccos\frac{c^2t^2-z^2+\rho^2-R^2}{2\rho\sqrt{c^2t^2-z^2}}. \quad (2.33)$$

Thus it is possible to rewrite electric field components E_φ (2.24) and E_ρ(2.17) in the way of

$$E_\rho = -\frac{1}{\rho}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}e^{i\varphi}I_1; \tag{2.34}$$

$$E_\varphi = -i\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}e^{i\varphi}\left(I_2 - \frac{1}{\rho}I_1\right). \tag{2.35}$$

Complex form of a field allows to submit φ dependence of a field in the last stage of solution. The form of φ dependency determines initially conditions. Let us use real part of the exponential dependence. So, according to Euler's formula

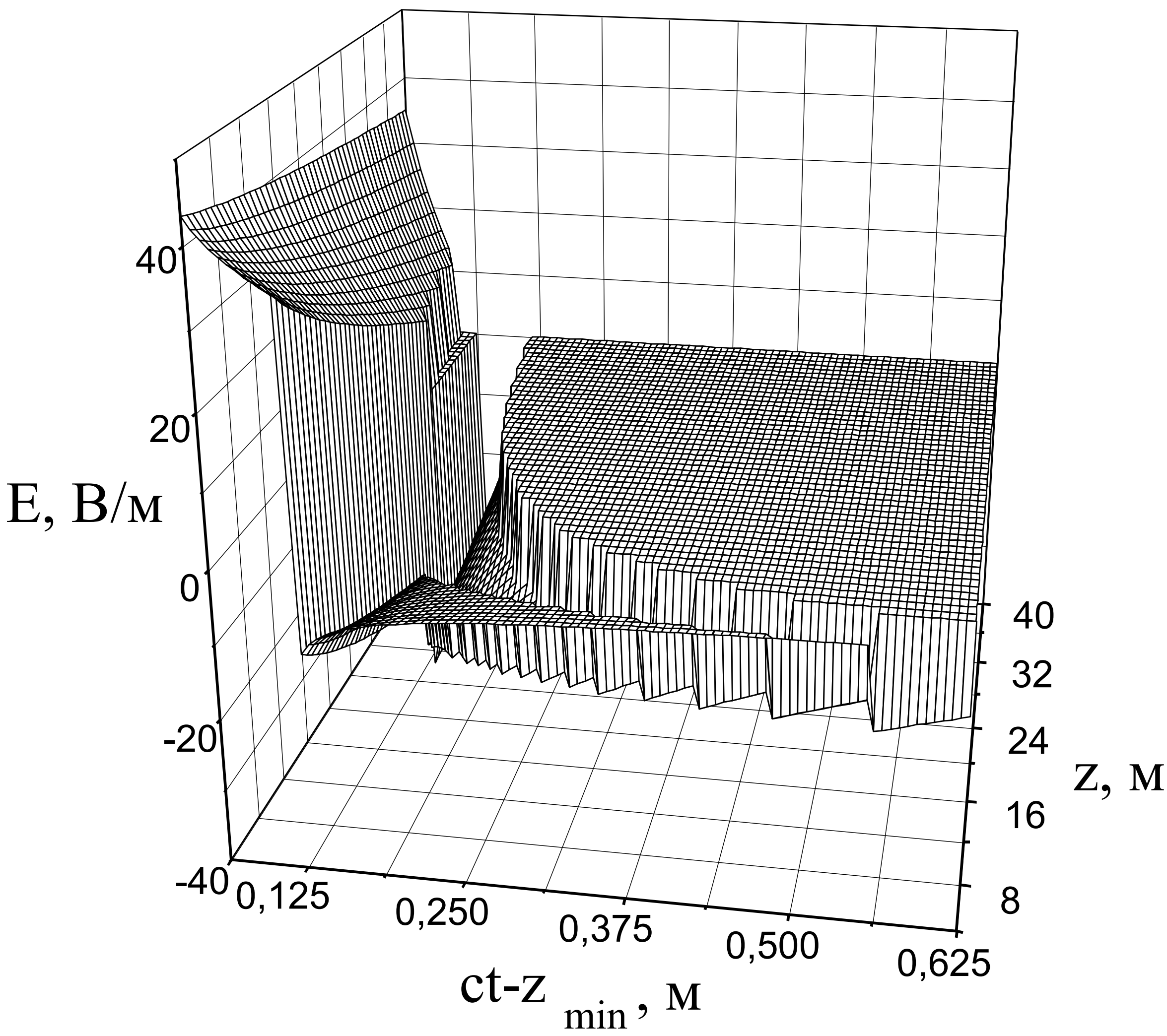
$$\begin{aligned} e^{i\varphi} &= \cos\varphi + i\sin\varphi \Rightarrow \operatorname{Re}\left(e^{i\varphi}\right) = \cos\varphi \\ ie^{i\varphi} &= -\sin\varphi + i\cos\varphi \Rightarrow \operatorname{Re}\left(e^{i\varphi}\right) = -\sin\varphi \end{aligned},$$

Field dependence can be rewritten in next form

$$E_\rho = -\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{I_1}{\rho}\cos\varphi \tag{2.36}$$

$$E_\varphi = \frac{1}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}I_2\sin\varphi - \frac{1}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{I_1}{\rho}\sin\varphi \tag{2.37}$$

The expressions which are received can be illustrated by the next plot.



If to not beat about the bush impulse radiation is shown but the solution (2.17)-(2.20) presents only for “plug in” signal. It is better to show Electromagnetic missile on the image. The observer is located out of the bound of beam array but close to it. It is clear that the field inside beam zone fades quietly than it must be according to classic theory.

4.5 On-axis field

Let us consider far linear field on ρ axis ($\rho = 0$) for simplification. It can be achieved form expressions (2.17)-(2.20). According to Bessel asymptotic property

$$\left.\frac{J_1(\chi\rho)}{\rho}\right|_{\rho\to 0} = \frac{\chi}{2},$$

field components (2.17), (2.24), (2.25) and (2.20) can be presented in next way:

$$E_\rho = -\frac{e^{i\varphi}}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) \tag{2.38}$$

$$E_\varphi = -\frac{ie^{i\varphi}}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) \tag{2.39}$$

$$H_\rho = -\frac{ie^{i\varphi}}{8\sqrt{\mu_0}}\int_0^\infty d\nu J_1(\nu R)(U_0 - U_2) \tag{2.40}$$

$$H_\varphi = \frac{e^{i\varphi}}{8\sqrt{\mu_0}}\int_0^\infty d\nu J_1(\nu R)(U_0 - U_2) \tag{2.41}$$

$$H_z = E_z = 0 \tag{2.42}$$

Double Bessel multiplication formula from (Прудников, и др., 1983) gives electric field.

$$\vec{\mathbf{E}} = -\frac{e^{i\varphi}}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\{\vec{\varphi}_0 + i\vec{\rho}_0\}\begin{cases} \frac{1}{R}, \sqrt{c^2t^2 - z^2} < R \\ \frac{1}{2R}, \sqrt{c^2t^2 - z^2} = R \\ 0, \sqrt{c^2t^2 - z^2} > R \end{cases} \tag{2.43}$$

The same is with magnetic component if consider only far field. Пояснение

В диссере посчитали что U0=J0 и U2=0 в дальней зоне – это не верно?

$$\begin{aligned} \vec{\mathbf{H}} &= \frac{e^{i\varphi}}{8\sqrt{\mu_0}}\{\vec{\varphi}_0 - i\vec{\rho}_0\}\int_0^\infty d\nu J_1(\nu R)(U_0 - U_2) = \\ &= \frac{e^{i\varphi}}{8\sqrt{\mu_0}}\{\vec{\varphi}_0 - i\vec{\rho}_0\}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) = \\ &= \frac{e^{i\varphi}}{8\sqrt{\mu_0}}\{\vec{\varphi}_0 - i\vec{\rho}_0\}\begin{cases} \frac{1}{R}, \sqrt{c^2t^2 - z^2} < R \\ \frac{1}{2R}, \sqrt{c^2t^2 - z^2} = R \\ 0, \sqrt{c^2t^2 - z^2} > R \end{cases} \end{aligned} \tag{2.44}$$

Let us present EM field by Heaviside's step functions

Здесь подогнал коэффициенты немного

$$\vec{\mathbf{H}} = \frac{ie^{i\varphi}}{8}\{i\vec{\varphi}_0 + \vec{\rho}_0\} H\left(R - \sqrt{c^2t^2 - z^2}\right) \tag{2.45}$$

$$\vec{\mathbf{E}} = \frac{ie^{i\varphi}}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\{i\vec{\rho}_0 - \vec{\varphi}_0\}H\left(R - \sqrt{c^2t^2 - z^2}\right) \tag{2.46}$$

Expressions (2.45) and (2.46) takes plays only when $c^2t^2 \geq z^2$. This restriction has physical nature. It comes from finite speed of the light transition. It can be taken into account analytically

$$\vec{\mathbf{H}} = \frac{ie^{i\varphi}}{8}\{i\vec{\varphi}_0 + \vec{\rho}_0\}H\left(R - \sqrt{c^2t^2 - z^2}\right)H\left(c^2t^2 - z^2\right); \tag{2.47}$$

$$\vec{\mathbf{E}} = \frac{ie^{i\varphi}}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\{i\vec{\rho}_0 - \vec{\varphi}_0\}H\left(R - \sqrt{c^2t^2 - z^2}\right)H\left(c^2t^2 - z^2\right). \tag{2.48}$$

Complex dependency can be removed now according to Euler's formula:

$$\begin{aligned} e^{i\varphi} &= \cos\varphi + i\sin\varphi \Rightarrow \operatorname{Re}\left(e^{i\varphi}\right) = \cos\varphi; \\ ie^{i\varphi} &= -\sin\varphi + i\cos\varphi \Rightarrow \operatorname{Re}\left(e^{i\varphi}\right) = -\sin\varphi. \end{aligned}$$

The choice of real parts is determined by initially conditions. The results are

$$\vec{\mathbf{H}} = -\frac{1}{8}\{\vec{\varphi}_0\cos\varphi + \vec{\rho}_0\sin\varphi\}H\left(R - \sqrt{c^2t^2 - z^2}\right)H\left(c^2t^2 - z^2\right); \tag{2.49}$$

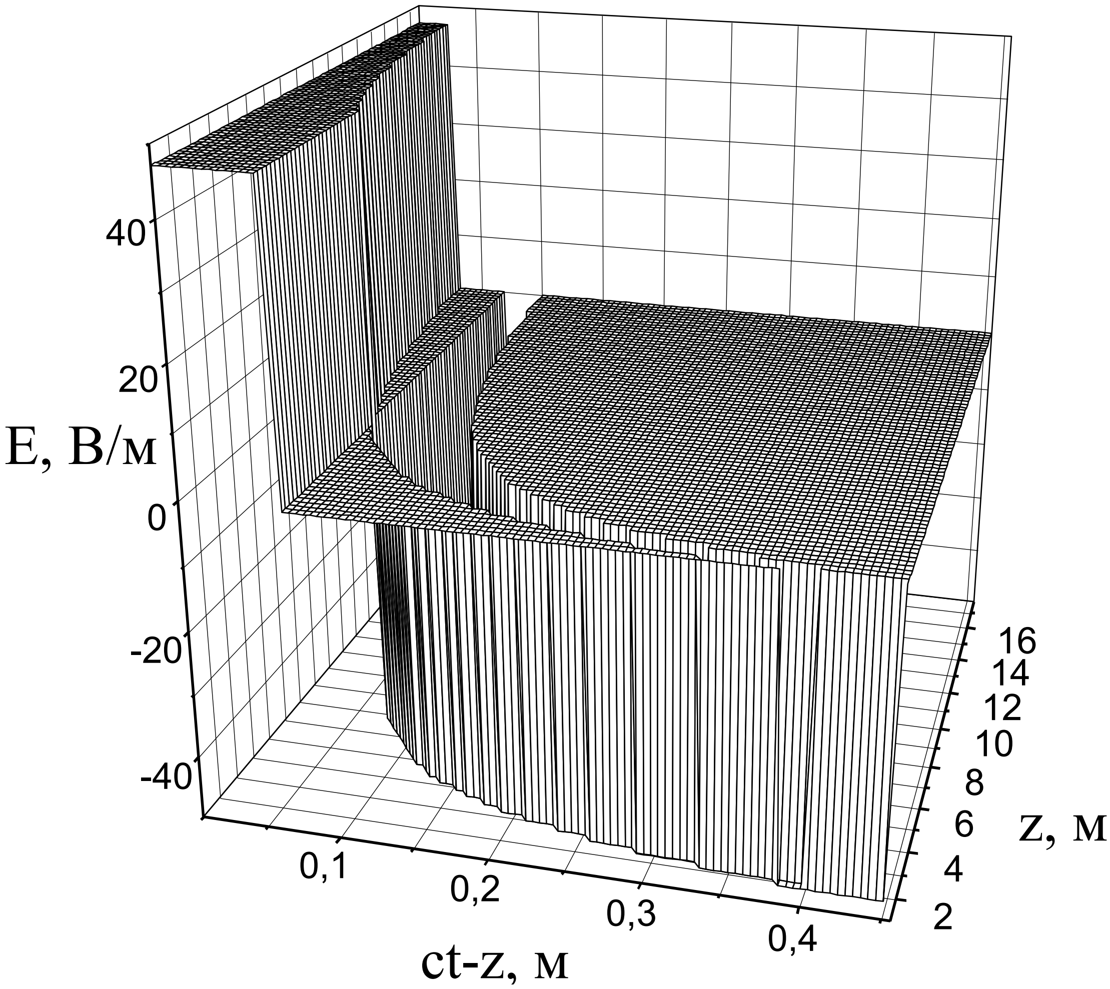
$$\vec{\mathbf{E}} = -\frac{1}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\{\vec{\rho}_0\cos\varphi - \vec{\varphi}_0\sin\varphi\}H\left(R - \sqrt{c^2t^2 - z^2}\right)H\left(c^2t^2 - z^2\right). \tag{2.50}$$

EM field (2.47) and (2.48) can be presented in Descartes coordinate system:

$$\vec{\mathbf{H}} = -\frac{\vec{y}_0}{8}H\left(R - \sqrt{c^2t^2 - z^2}\right)H\left(c^2t^2 - z^2\right); \tag{2.51}$$

$$\vec{\mathbf{E}} = -\frac{\vec{x}_0}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}H\left(R - \sqrt{c^2t^2 - z^2}\right)H\left(c^2t^2 - z^2\right). \tag{2.52}$$

There is a plot to the on-axis field solution as well as for previous one. It shows the same problem.



The difference is opened to view. On-axis field is not dependent of z coordinate – it is constant. The phenomena was firstly noted by T. T. Wu in paper (Wu, 1985).

4.6 Nonlinear sources

Nonlinear medium can be considered in EAE theory by secondary source of current. This approach was taken into account in expression for general current (1.8) by nonlinear component of polarization and magnetization vectors.

Let us consider a medium with zero nonlinear magnetization. Only nonlinear polarization exists. Also we will determinate current of conductivity to generalize a problem. Secondary electric current can be presented in next expression.

$$\vec{\mathbf{J}}'\left(\vec{\mathbf{E}},\vec{\mathbf{H}}\right)=\begin{pmatrix} J'_\rho \\ J'_\varphi \\ J'_z \end{pmatrix}=\begin{pmatrix} \overrightarrow{J'} \\ J'_z \end{pmatrix}=\begin{pmatrix} \dfrac{\partial}{\partial t}\overrightarrow{P'}\left(\vec{\mathbf{E}}\right)+\sigma\vec{E} \\ \dfrac{\partial}{\partial t}P_z{}'\left(\vec{\mathbf{E}}\right)+\sigma E_z \end{pmatrix}, \tag{2.53}$$

where $\vec{\mathbf{E}}$ is primary field generated by source $\overrightarrow{j_0}$ form linear problem (2.1)

Kerr medium implies only third power of **Ошибка! Источник ссылки не найден.** summary, so (2.53) can be rewritten.

$$\vec{\mathbf{J}}^1 = \frac{\partial}{\partial t}\left(\chi_3 \vec{\mathbf{E}}^3\right) + \sigma \vec{E} = \chi_3^E \vec{x}_0 \frac{\partial}{\partial t}\left(E_x^{\ 3}\right) + \sigma \vec{x}_0 E_x \,. \tag{2.54}$$

Expression (2.54) get next form after substitution of liner field expression (2.52).

$$\begin{aligned}
\vec{\mathbf{J}}^1 = & -\vec{x}_0 3 \frac{\chi_3^E}{512}\left(\frac{\mu_0}{\varepsilon_0}\right)^{\frac{3}{2}} H^2\left(ct-z\right) H^2\left(R-\sqrt{c^2t^2-z^2}\right)\times \\
& \times\left[H\left(ct-z\right)\delta\left(R-\sqrt{c^2t^2-z^2}\right)\frac{-c^2t}{\sqrt{c^2t^2-z^2}} + H\left(R-\sqrt{c^2t^2-z^2}\right)\delta\left(ct-z\right)c\right] - \\
& -\vec{x}_0 \frac{\sigma}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} H\left(R-\sqrt{c^2t^2-z^2}\right) H\left(ct-z\right)
\end{aligned} \tag{2.55}$$

This current can be applied by EAE and gives nonlinear correction to linear fields (2.49) and (2.50). Magnetic longitudinal evolutionary coefficient was presented by Riemann function but this expression needs electric current expansion as well as linear problem. Acording to EAE electric current is

$$j_m^{\ 1}(z,t;\nu) = \frac{\sqrt{\mu_0}}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \overrightarrow{J^1}\left[\nabla_\perp \Psi_m^{\ *} \times \overrightarrow{z_0}\right], \tag{2.56}$$

where $\overrightarrow{J^1}$ is nonlinear secondary current density and Ψ_m goes form (1.35)

The equivalent currents are valid on OZ axis, so we multiply it on delta-function

$$\begin{aligned}
\overrightarrow{J^1} = & \frac{1}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\left(\vec{\rho}_0 \cos\varphi - \vec{\varphi}_0 \sin\varphi\right)\frac{\delta(\rho)}{\rho}\left[-3\frac{\chi_3^E}{64}\left(\frac{\mu_0}{\varepsilon_0}\right) H^2\left(ct-z\right) H^2\left(R-\sqrt{c^2t^2-z^2}\right)\times\right. \\
& \times\left[H\left(ct-z\right)\delta\left(R-\sqrt{c^2t^2-z^2}\right)\frac{-c^2t}{\sqrt{c^2t^2-z^2}} + H\left(R-\sqrt{c^2t^2-z^2}\right)\delta\left(ct-z\right)c\right] - \\
& \left.-\sigma H\left(R-\sqrt{c^2t^2-z^2}\right) H\left(ct-z\right)\right].
\end{aligned} \tag{2.57}$$

It is not hard to prove that (2.57) can be rewritten in case of expansion (2.56). The result is

$$j_1^1(z,t;\nu)=\frac{-1}{8}\frac{\mu_0}{\sqrt{\varepsilon_0}}i\frac{\sqrt{\nu}}{2}\left[-3\frac{\chi_3^E}{64}\left(\frac{\mu_0}{\varepsilon_0}\right)H^2\left(ct-z\right)H^2\left(R-\sqrt{c^2t^2-z^2}\right)\times\right.$$

$$\times\left[H\left(ct-z\right)\delta\left(R-\sqrt{c^2t^2-z^2}\right)\frac{-c^2t}{\sqrt{c^2t^2-z^2}}+H\left(R-\sqrt{c^2t^2-z^2}\right)\delta\left(ct-z\right)c\right]- \tag{2.58}$$

$$\left.-\sigma H\left(R-\sqrt{c^2t^2-z^2}\right)H\left(ct-z\right)\right].$$

4.7 Evolutionary coefficients

There is known solution of the first equation of the set (2.3) by the Riemann functions. It is clearly that it is applicable to nonlinear approximation too. So longitudinal magnetic evolutionary coefficient is equal to

$$h_m=\frac{c}{2}\int_0^\infty dz'\int_0^\infty dt'H\left(c\left(t-t'\right)-\left(z-z'\right)\right)J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2-\left(z-z'\right)^2}\right)j_m\left(z',t';\nu\right). \tag{2.59}$$

After substitution of current expression (2.58) evolutionary coefficient (2.59) gets next unfriendly form.

$$h_1^{nl}\left(z,t;\nu\right)=-\sqrt{\mu_0}J_1\left(\nu R\right)\frac{1}{2\sqrt{\nu^3}}U_1\left[-i\nu\left(ct-z\right),\nu\sqrt{c^2t^2-z^2}\right]-i\frac{\sqrt{\mu_0}\sqrt{\nu}}{32\varepsilon_0}\times$$

$$\times\left\{\int_0^\infty dt'H\left(c\left(t-t'\right)-\left(z-\sqrt{c^2t'^2-R^2}\right)\right)J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2-\left(z-\sqrt{c^2t'^2-R^2}\right)^2}\right)\frac{3\chi_3^Et'}{64R{\varepsilon_0}^2}-\right.$$

$$-3\frac{\chi_3^E}{64}\left(\frac{\mu_0}{\varepsilon_0}\right)cH\left(ct-z\right)\int_0^\infty dt'J_0\left(\nu\sqrt{c^2t^2-z^2-2ct'(ct-z)}\right)- \tag{2.60}$$

$$\left.-\sigma\int_0^\infty dz'\int_{z'/c}^{\sqrt{R^2+z'^2}/c}dt'H\left(c\left(t-t'\right)-\left(z-z'\right)\right)J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2-\left(z-z'\right)^2}\right)\right\}$$

We are not able to integrate it now but it is analytical solution still and gives some results. Sure, it can be integrated numerically but the result will be not in the plane of our interest, because we need analytical solutions to implement compare analysis of linear and nonlinear solutions.

5 DIGITAL CALCULATIONS

5.1 Lommel's Function

и хотя аргументами функций Ломмеля являются мнимые числа сами значения этих функций остаются действительными.

$$U_n(W,Z)=\sum_{m=0}^{\infty}(-1)^m\left(\frac{W}{Z}\right)^{n+2m}J_{n+2m}(Z)$$

$$U_n(W,Z)=\sum_{m=0}^{\infty}(-1)^m\frac{1}{m!}\left(\frac{Z^2}{2W}\right)^m U_{n+m}(W,0)$$

$$U_{2n}(W,Z)=(-1)^n\left[\cos\frac{W}{2}-\sum_{m=0}^{n-1}(-1)^m\frac{1}{(2m)!}\left(\frac{W}{2}\right)^{2m}\right]$$

$$U_{2n+1}(W,Z)=(-1)^n\left[\sin\frac{W}{2}-\sum_{m=0}^{n-1}(-1)^m\frac{1}{(2m+1)!}\left(\frac{W}{2}\right)^{2m+1}\right]$$

$$U_0(i\pi,0)=\cos\frac{i\pi}{2}-\sum_{m=0}^{-1}(-1)^m\frac{1}{(2m)!}\left(\frac{i\pi}{2}\right)^{2m}=\cos\frac{i\pi}{2}-1+\frac{1}{2}\left(\frac{2}{i\pi}\right)^2=$$
$$=\cos\frac{i\pi}{2}-1+\frac{2}{\pi^2}(-i)^2=\cos\frac{i\pi}{2}-\frac{\pi^2-2}{\pi^2}$$

5.2 Weighted Averages Algorithm for Somerfield-type integrals

$$I=\int_a^{\infty}f(x)e^{-\gamma x}dx$$

$$\left.e^{-\gamma x}\right|_{\alpha=0}=e^{-j\beta x}=\cos(\beta x)+j\sin(\beta x)$$

$$J_n(\beta x)\sim\sqrt{\frac{2}{\pi\beta x}}\cos\left(\beta x-\frac{n\pi}{2}-\frac{\pi}{4}\right)\sim\frac{e^{-j\beta x}}{\sqrt{x}}$$

$$I=\int_a^{\infty}f(x)J_n(\beta x)\sqrt{x}dx$$

$$I\approx\frac{\sum_{i=1}^{N}\omega_i I_i}{\sum_{i=1}^{N}\omega_i};\qquad I_i=\int_a^{x_i}f(x)J_n(\beta x)\sqrt{x}dx$$

$$\omega_i=(-1)^{i+1}\binom{N-1}{i-1}e^{j\beta x_i}x_i^{N-2-\mu};\quad \mu:\lim_{x\to\infty}\left[f(x)-x^{\mu}\right]=0$$

$$x_i = \frac{ik\pi}{\beta}; \quad k \in \mathbb{Z}$$

$$\omega_i = (-1)^{i+1} \binom{N-1}{i-1} e^{j\beta x_i} x_i^{N-2-\mu} = (-1)^{i+1} \binom{N-1}{i-1} e^{-ij\pi\beta} x_i^{N-2-\mu}$$

$$\binom{n}{k} = C_n^k = \frac{n!}{k!(n-k)!}; \quad \binom{N-1}{i-1} = \frac{(N-1)!}{(i-1)!(N-i)!}$$

5.3 Weighted Averages Algorithm for double Bessel kernel

$$I = \int_a^\infty g(x) e^{-\gamma x} dx$$

$$I = \int_a^\infty f(x) J_n(bx) J_n(cx) dx$$

$$I = \int_a^\infty f_2(x) \cos(bx)\cos(cx) dx = \frac{1}{2}\int_a^\infty f_2(x)\left[\cos(|p+q|x) + \cos(|p-q|x)\right] dx =$$
$$= \frac{1}{2}\int_a^\infty f_2(x)\cos(|p+q|x) dx + \frac{1}{2}\int_a^\infty f_2(x)\cos(|p-q|x) dx$$

$$|p+q| = s = \frac{s_1}{s_2}; \quad |p-q| = d = \frac{d_1}{d_2}; \quad \{s_i, d_i\}_{i=1,2} \in \mathbb{Z}$$

$$I = \frac{1}{2}\int_a^\infty f_2(x)\cos\left(\left|\frac{s_1}{s_2} + \frac{d_1}{d_2}\right| x\right) dx + \frac{1}{2}\int_a^\infty f_2(x)\cos\left(\left|\frac{s_1}{s_2} - \frac{d_1}{d_2}\right| x\right) dx$$

$$T = \frac{2\pi}{\tau} = 2\pi \frac{LCM(s_2, d_2)}{GCD(s_1, d_1)}$$

$$g(x) = \frac{1}{\pi x\sqrt{bc}}\left[\frac{\tau}{|b+c|} f\left(\frac{\tau x}{|b+c|}\right) + \frac{\tau}{|b-c|} f\left(\frac{\tau x}{|b-c|}\right)\right]$$

5.4 Weighted Averages Algorithm for triple Bessel kernel

6 DEPRECATED CALCULATIONS

6.1 Plane disk radiation with m in range [-∞;∞]

Следующая формула верна на 100% (вывод на странице 22 и очень длинный)

$$j_m(z,t;\nu) = -i\frac{\sqrt{\mu_0\nu}}{2}H(t)\delta\left(z\right)\left(\delta_{m,1}+\delta_{m,-1}\right)\int_0^R \rho d\rho J_{m-1}\left(\nu\rho\right)$$

Формула понадобится дальше

$$J_2\left(\nu\rho\right) = \frac{2}{\nu\rho}J_1\left(\nu\rho\right) - J_0\left(\nu\rho\right)$$

Эти интегральные 2 выражения нужны в следующей формуле

$$J_1\left(\nu\rho\right) = -\frac{1}{\nu}\frac{d}{d\rho}J_0\left(\nu\rho\right)$$

$$\int_0^R d\rho J_1\left(\nu\rho\right) = -\left.\frac{J_0\left(\nu\rho\right)}{\nu}\right|_0^R = -\frac{J_0\left(\nu R\right)-1}{\nu} = \frac{1-J_0\left(\nu R\right)}{\nu}$$

$$\int xJ_0\left(x\right)dx = xJ_1\left(x\right)+C$$

$$\int_0^R \rho d\rho J_0\left(\nu\rho\right) = \frac{1}{\nu^2}\int_0^R \nu\rho d\nu\rho J_0\left(\nu\rho\right) = \frac{\nu RJ_1\left(\nu R\right)-\nu 0J_1\left(\nu 0\right)}{\nu^2} = \frac{RJ_1\left(\nu R\right)}{\nu}$$

$$\begin{aligned}
j_m(z,t;\nu) &= -i\frac{\sqrt{\mu_0\nu}}{2}H(t)\delta\left(z\right)\left(\delta_{m,1}\int_0^R \rho d\rho J_0\left(\nu\rho\right)+\delta_{m,-1}\int_0^R \rho d\rho J_2\left(\nu\rho\right)\right) = \\
&= -i\frac{\sqrt{\mu_0\nu}}{2}H(t)\delta\left(z\right)\left(\delta_{m,1}\int_0^R \rho d\rho J_0\left(\nu\rho\right)+\delta_{m,-1}\int_0^R \rho d\rho\left(\frac{2}{\nu\rho}J_1\left(\nu\rho\right)-J_0\left(\nu\rho\right)\right)\right) = \\
&= -i\frac{\sqrt{\mu_0\nu}}{2}H(t)\delta\left(z\right)\left[\frac{2\delta_{m,-1}}{\nu}\int_0^R d\rho J_1\left(\nu\rho\right)+\left(\delta_{m,1}-\delta_{m,-1}\right)\int_0^R \rho d\rho J_0\left(\nu\rho\right)\right] = \\
&= -i\frac{\sqrt{\mu_0\nu}}{2}H(t)\delta\left(z\right)\left[\frac{2\delta_{m,-1}}{\nu}\frac{1-J_0\left(\nu R\right)}{\nu}+\left(\delta_{m,1}-\delta_{m,-1}\right)\frac{RJ_1\left(\nu R\right)}{\nu}\right] = \\
&= -i\frac{\sqrt{\mu_0}}{2\sqrt{\nu}}H(t)\delta\left(z\right)\left[2\delta_{m,-1}\frac{1-J_0\left(\nu R\right)}{\nu}+\left(\delta_{m,1}-\delta_{m,-1}\right)RJ_1\left(\nu R\right)\right]
\end{aligned}$$

Приведена функция Римана

$$G\left(z',t',z,t\right) = \frac{c}{2}H\left(c\left(t-t'\right)-\left(z-z'\right)\right)J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2-\left(z-z'\right)^2}\right)$$

Вычисление продольного коэффициента

$$h_m\left(z,t;\nu\right)=\int_0^\infty dz'\int_0^\infty dt' G\left(z',t',z,t\right) j_m\left(z',t';\chi\right)=$$

$$=-\sqrt{\mu_0}\frac{ic}{2}\frac{1-J_0\left(\nu R\right)}{\nu^{3/2}}\delta_{m,-1}\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2-z^2}\right)-$$

$$-\sqrt{\mu_0}\frac{icR}{4}\frac{\delta_{m,1}-\delta_{m,-1}}{\sqrt{\nu}}J_1\left(\nu R\right)\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2-z^2}\right)=$$

$$=-\sqrt{\mu_0}\frac{ic}{2}\left[\frac{1-J_0\left(\nu R\right)}{\nu^{3/2}}\delta_{m,-1}+\frac{\delta_{m,1}-\delta_{m,-1}}{2\sqrt{\nu}}J_1\left(\nu R\right)\right]\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2-z^2}\right)$$

Вычисление интеграла

$$\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2-z^2}\right)=\frac{1}{\nu c}\int_{\nu z}^{\nu ct} ds J_0\left(\sqrt{s^2-\nu^2 z^2}\right)$$

$$\int_{\nu z}^{\nu ct} ds e^{-i0s} J_0\left(\sqrt{s^2-\nu^2 z^2}\right)=2iU_1\left[-i\nu\left(ct-z\right),\sqrt{\nu^2c^2t^2-\nu^2z^2}\right]$$

$$\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2-z^2}\right)=\frac{2i}{\nu c}U_1\left[-i\nu\left(ct-z\right),\sqrt{\nu^2c^2t^2-\nu^2z^2}\right]$$

Вторую формулу не могу проверить, да и ню надо вынести из интеграла

Продольный коэффициент через функцию Ломмеля

$$h_m\left(z,t;\nu\right)=-\sqrt{\mu_0}\frac{ic}{2}\left[\frac{1-J_0\left(\nu R\right)}{\nu^{3/2}}\delta_{m,-1}+\frac{\delta_{m,1}-\delta_{m,-1}}{2\sqrt{\nu}}J_1\left(\nu R\right)\right]\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2-z^2}\right)=$$

$$=\frac{\sqrt{\mu_0}}{\nu^{3/2}}\left[\frac{1-J_0\left(\nu R\right)}{\nu}\delta_{m,-1}+\frac{\delta_{m,1}-\delta_{m,-1}}{2}J_1\left(\nu R\right)\right]U_1\left[-i\nu\left(ct-z\right),\sqrt{\nu^2c^2t^2-\nu^2z^2}\right]$$

Ранее доказанное утверждение (пригодится в следующей формуле)

$$\frac{\partial}{\partial t}\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2-z^2}\right)=J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

$$V_m^h = -\partial_{ct}\left\{\mu h_m\right\}\Big|_{\mu=const} = -\mu\frac{\partial h_m}{\partial t} =$$

$$= -\mu\frac{\partial}{\partial t}\left(-\sqrt{\mu_0}\,\frac{i}{2}\left[\frac{1-J_0\left(\nu R\right)}{\nu^{3/2}}\delta_{m,-1} + \frac{\delta_{m,1}-\delta_{m,-1}}{2\sqrt{\nu}}J_1\left(\nu R\right)\right]\int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2 - z^2}\right)\right) =$$

$$= \mu\sqrt{\mu_0}\,\frac{i}{2}\left[\frac{1-J_0\left(\nu R\right)}{\nu^{3/2}}\delta_{m,-1} + \frac{\delta_{m,1}-\delta_{m,-1}}{2\sqrt{\nu}}J_1\left(\nu R\right)\right]J_0\left(\nu\sqrt{c^2t^2 - z^2}\right)$$

$$I_m^h = \frac{1}{\mu}\frac{\partial h_m}{\partial z} = \frac{\sqrt{\mu_0}}{\nu^{3/2}}\left[\frac{1-J_0\left(\nu R\right)}{\nu}\delta_{m,-1} + \frac{\delta_{m,1}-\delta_{m,-1}}{2}J_1\left(\nu R\right)\right]\frac{\partial}{\partial z}U_1\left[-i\nu\left(ct-z\right),\sqrt{\nu^2c^2t^2-\nu^2z^2}\right]$$

$$\frac{\partial}{\partial z}U_1\left(W_+,Z\right) = -\frac{i\nu}{2}\left[U_0\left(W_+,Z\right) - U_2\left(W_+,Z\right)\right].$$

$$I_m^h = \frac{1}{\mu}\frac{\partial h_m}{\partial z} = -\frac{i\sqrt{\mu_0}}{2\sqrt{\nu}}\left[\frac{1-J_0\left(\nu R\right)}{\nu}\delta_{m,-1} + \frac{\delta_{m,1}-\delta_{m,-1}}{2}J_1\left(\nu R\right)\right]\left[U_0\left(W_+,Z\right) - U_2\left(W_+,Z\right)\right]$$

$$\left[\nabla_\perp\Psi_m \times \vec{z}_0\right] = -e^{im\varphi}\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_{m-1}\left(\nu\rho\right) - J_{m+1}\left(\nu\rho\right)}{2} - im\vec{\rho}_0\,\frac{J_m\left(\nu\rho\right)}{\rho\sqrt{\nu}}\right)$$

Подставим базисную функцию и эволюционный коэффициент

$$\vec{E} = \sqrt[-2]{\varepsilon_0}\left\{\sum_{m=-\infty}^{\infty}\int_0^{\infty} d\nu V_m^h\left[\nabla_\perp\Psi_m\times\vec{z}_0\right] + \sum_{\substack{m=-\infty\\ m\neq 0}}^{\infty}\int_0^{\infty} d\chi V_n^e\nabla_\perp\Phi_n\right\} =$$

$$= -\frac{1}{\sqrt{\varepsilon_0}}\sum_{m=-\infty}^{\infty}\int_0^{\infty} d\nu\,\mu\sqrt{\mu_0}\,\frac{i}{2}\left[\frac{1-J_0\left(\nu R\right)}{\nu^{3/2}}\delta_{m,-1} + \frac{\delta_{m,1}-\delta_{m,-1}}{2\sqrt{\nu}}J_1\left(\nu R\right)\right]J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\times$$

$$\times e^{im\varphi}\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_{m-1}\left(\nu\rho\right) - J_{m+1}\left(\nu\rho\right)}{2} - im\vec{\rho}_0\,\frac{J_m\left(\nu\rho\right)}{\rho\sqrt{\nu}}\right) =$$

$$= -\mu\frac{i}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\sum_{m=-\infty}^{\infty}e^{im\varphi}\int_0^{\infty} d\nu\left[\frac{1-J_0\left(\nu R\right)}{\nu^{3/2}}\delta_{m,-1} + \frac{\delta_{m,1}-\delta_{m,-1}}{2\sqrt{\nu}}J_1\left(\nu R\right)\right]J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\times$$

$$\times\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_{m-1}\left(\nu\rho\right) - J_{m+1}\left(\nu\rho\right)}{2} - im\vec{\rho}_0\,\frac{J_m\left(\nu\rho\right)}{\rho\sqrt{\nu}}\right)$$

М-от минус до плюс бесконечности!Тут и далее!

$$E_\varphi = -\mu \frac{i}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} \sum_{m=-\infty}^{\infty} e^{im\varphi} \int_0^\infty d\nu \left[\frac{1-J_0(\nu R)}{\nu^{3/2}} \delta_{m,-1} + \frac{\delta_{m,1}-\delta_{m,-1}}{2\sqrt{\nu}} J_1(\nu R) \right] J_0\left(\nu\sqrt{c^2t^2-z^2}\right) \times$$

$$\times\sqrt{\nu}\,\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2} = -\mu \frac{i}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} \sum_{m=-\infty}^{\infty} e^{im\varphi} \int_0^\infty d\nu \left[\frac{1-J_0(\nu R)}{\nu} \delta_{m,-1} + \frac{\delta_{m,1}-\delta_{m,-1}}{2} J_1(\nu R) \right] \times$$

$$\times J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)\right] =$$

$$= -\mu \frac{i}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} \left(e^{-i\varphi}\int_0^\infty d\nu \frac{1-J_0(\nu R)}{\nu} J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_{-2}(\nu\rho)-J_0(\nu\rho)\right] + \right.$$

$$+\frac{e^{i\varphi}}{2}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_0(\nu\rho)-J_2(\nu\rho)\right] -$$

$$\left. -\frac{e^{-i\varphi}}{2}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_{-2}(\nu\rho)-J_0(\nu\rho)\right] \right)$$

$$E_\varphi = -\mu \frac{i}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} \left(e^{-i\varphi}\int_0^\infty \frac{d\nu}{\nu} J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_2(\nu\rho)-J_0(\nu\rho)\right] - \right.$$

$$-e^{-i\varphi}\int_0^\infty \frac{d\nu}{\nu} J_0(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_2(\nu\rho)-J_0(\nu\rho)\right] +$$

$$+\frac{e^{i\varphi}}{2}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_0(\nu\rho)-J_2(\nu\rho)\right] -$$

$$\left. -\frac{e^{-i\varphi}}{2}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_2(\nu\rho)-J_0(\nu\rho)\right] \right) =$$

$$= -\mu \frac{i}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} \left(e^{-i\varphi}\int_0^\infty \frac{d\nu}{\nu} J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_2(\nu\rho)-J_0(\nu\rho)\right] - \right.$$

$$-e^{-i\varphi}\int_0^\infty \frac{d\nu}{\nu} J_0(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_2(\nu\rho)-J_0(\nu\rho)\right] +$$

$$+\frac{e^{i\varphi}}{2}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_0(\nu\rho)-J_2(\nu\rho)\right] +$$

$$\left. +\frac{e^{-i\varphi}}{2}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_0(\nu\rho)-J_2(\nu\rho)\right] \right) =$$

$$= -\mu \frac{i}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} \left(e^{-i\varphi}\int_0^\infty \frac{d\nu}{\nu} J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_2(\nu\rho)-J_0(\nu\rho)\right] - \right.$$

$$-e^{-i\varphi}\int_0^\infty \frac{d\nu}{\nu} J_0(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_2(\nu\rho)-J_0(\nu\rho)\right] -$$

$$\left. +\frac{e^{i\varphi}+e^{-i\varphi}}{2}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_0(\nu\rho)-J_2(\nu\rho)\right] \right)$$

$$J_2(\nu\rho) = \frac{2}{\nu\rho} J_1(\nu\rho) - J_0(\nu\rho)$$

$$
\begin{aligned}
E_\varphi = -\mu \frac{i}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\Bigg(& e^{-i\varphi}\int_0^\infty \frac{d\nu}{\nu} J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[\frac{2}{\nu\rho}J_1(\nu\rho)-2J_0(\nu\rho)\right]- \\
& -e^{-i\varphi}\int_0^\infty \frac{d\nu}{\nu} J_0(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[\frac{2}{\nu\rho}J_1(\nu\rho)-2J_0(\nu\rho)\right]+ \\
& +\cos\varphi\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[2J_0(\nu\rho)-\frac{2}{\nu\rho}J_1(\nu\rho)\right]\Bigg)= \\
= -\mu\frac{i}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} & \cos\varphi\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_0(\nu\rho)-\frac{J_1(\nu\rho)}{\nu\rho}\right]- \\
-\mu\frac{ie^{-i\varphi}}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} & \int_0^\infty d\nu \frac{1-J_0(\nu R)}{\nu} J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[\frac{J_1(\nu\rho)}{\nu\rho}-J_0(\nu\rho)\right]
\end{aligned}
$$

Последнее слагаемое в конце можно было не мучать всю последнюю страницу, вы его не поменяли в результате.- упростите эти длинные без надобности выражения выше

Два последних интеграла сходятся, если не разрывать слагаемые в квадратных скобках!!Надо с ними поработать..

$$
\frac{e^{i\varphi}+e^{-i\varphi}}{2}=\cos\varphi \qquad\qquad J_{-m}(x)=(-1)^m J_m(x)
$$

$$
E_\rho = -\frac{\mu}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\sum_{m=-\infty}^{\infty} e^{im\varphi}\int_0^\infty d\nu\left[\frac{1-J_0(\nu R)}{\nu^{3/2}}\delta_{m,-1}+\frac{\delta_{m,1}-\delta_{m,-1}}{2\sqrt{\nu}}J_1(\nu R)\right]J_0\left(\nu\sqrt{c^2t^2-z^2}\right)m\vec{\rho}_0\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}
$$

В этой формуле вектор ро лишний, уберите

$$E_\rho=-\frac{\mu}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\sum_{m=-\infty}^{\infty}m\frac{e^{im\varphi}}{\rho}\int_0^\infty\frac{d\nu}{\nu}\left[\frac{1-J_0(\nu R)}{\nu}\delta_{m,-1}+\frac{\delta_{m,1}-\delta_{m,-1}}{2}J_1(\nu R)\right]J_m(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)=$$

$$=-\frac{\mu}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{e^{-i\varphi}}{\rho}\int_0^\infty\frac{d\nu}{\nu^2}\left(1-J_0(\nu R)\right)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)-$$

$$-\frac{\mu}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{e^{i\varphi}}{\rho}\int_0^\infty\frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)+$$

$$+\frac{\mu}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{e^{-i\varphi}}{\rho}\int_0^\infty\frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)=$$

$$=-\frac{\mu}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{e^{-i\varphi}}{\rho}\int_0^\infty\frac{d\nu}{\nu^2}\left(1-J_0(\nu R)\right)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)-$$

$$-\frac{i}{i}\frac{\mu}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{e^{i\varphi}-e^{-i\varphi}}{\rho}\int_0^\infty\frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)=$$

$$=-\frac{\mu}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{e^{-i\varphi}}{\rho}\int_0^\infty\frac{d\nu}{\nu^2}\left(1-J_0(\nu R)\right)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)-$$

$$-\frac{\mu}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{i\sin\varphi}{\rho}\int_0^\infty\frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

Кстати, два последних інтеграла сходяться в таком виде в 0

Компоненты поля E

$$E_\varphi=-\mu\frac{i\cos\varphi}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\int_0^\infty d\nu J_1(\nu R)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[J_0(\nu\rho)-\frac{J_1(\nu\rho)}{\nu\rho}\right]-$$

$$-\mu\frac{ie^{-i\varphi}}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\int_0^\infty d\nu\frac{1-J_0(\nu R)}{\nu}J_0\left(\nu\sqrt{c^2t^2-z^2}\right)\left[\frac{J_1(\nu\rho)}{\nu\rho}-J_0(\nu\rho)\right]$$

$$ie^{-i\varphi}=i\left(\cos\varphi-i\sin\varphi\right)=i\cos\varphi+\sin\varphi$$

$$E_\rho=-\frac{\mu}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{e^{-i\varphi}}{\rho}\int_0^\infty\frac{d\nu}{\nu^2}\left(1-J_0(\nu R)\right)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)-$$

$$-\frac{\mu}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{i\sin\varphi}{\rho}\int_0^\infty\frac{d\nu}{\nu}J_1(\nu R)J_1(\nu\rho)J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

$$e^{-i\varphi}=\cos\varphi-i\sin\varphi$$

$$\int_0^\infty d\nu J_1(\nu R) J_0(\nu\rho) J_0\left(\nu\sqrt{c^2t^2-z^2}\right) = I_2$$

$$\int_0^\infty \frac{d\nu}{\nu} J_1(\nu R) J_1(\nu\rho) J_0\left(\nu\sqrt{c^2t^2-z^2}\right) = I_1$$

$$\int_0^\infty \frac{d\nu}{\nu^2} J_1(\nu\rho) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

$$\int_0^\infty \frac{d\nu}{\nu} J_0(\nu\rho) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

$$\int_0^\infty \frac{d\nu}{\nu^2} J_1(\nu\rho) J_0(\nu R) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

$$\int_0^\infty \frac{d\nu}{\nu} J_0(\nu R) J_0(\nu\rho) J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

3,4,5,6 интегралы расходятся в нуле. Были сделаны неправильные преобразования Бесселей, надо переделать это еще раньше, чтобы избежать этого, чтобы сингулярности под интегралом вычитались.

Дальше не читаю.

6.2 Vector calculations

$$\nabla_\perp \Psi_m = \vec{\rho}_0 \frac{\partial \Psi_m}{\partial \rho} + \vec{\varphi}_0 \frac{1}{\rho}\frac{\partial \Psi_m}{\partial \varphi} = \vec{\rho}_0 \frac{\partial}{\partial \rho}\frac{J_m(\nu\rho)}{\sqrt{\nu}} e^{im\varphi} + \vec{\varphi}_0 \frac{1}{\rho}\frac{\partial}{\partial\varphi}\frac{J_m(\nu\rho)}{\sqrt{\nu}} e^{im\varphi} =$$

$$= e^{im\varphi}\left(\vec{\rho}_0\sqrt{\nu}\,\frac{J_{m-1}-J_{m+1}}{2} + \vec{\varphi}_0 im\frac{1}{\rho}\frac{J_m}{\sqrt{\nu}}\right) = e^{im\varphi}\left(\vec{\rho}_0\sqrt{\nu}\,\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2} + \vec{\varphi}_0 im\frac{J_m(\nu\rho)}{\sqrt{\nu}\rho}\right)$$

$$[\nabla_\perp \Psi_m \times \vec{z}_0] = \begin{vmatrix} \vec{\rho}_0 & \vec{\varphi}_0 & \vec{z}_0 \\ \dfrac{\partial \Psi_m}{\partial \rho} & \dfrac{1}{\rho}\dfrac{\partial \Psi_m}{\partial \varphi} & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{\rho}_0 \frac{1}{\rho}\frac{\partial \Psi_m}{\partial\varphi} - \vec{\varphi}_0\frac{\partial\Psi_m}{\partial\rho} = \vec{\rho}_0\frac{\partial}{\partial\varphi}\frac{1}{\rho}\frac{J_m(\nu\rho)}{\sqrt{\nu}}e^{im\varphi} -$$

$$-\vec{\varphi}_0\frac{\partial}{\partial\rho}\frac{J_m(\nu\rho)}{\sqrt{\nu}}e^{im\varphi} = im\vec{\rho}_0\frac{J_m}{\rho\sqrt{\nu}}e^{im\varphi} - \sqrt{\nu}\vec{\varphi}_0\frac{J_{m-1}-J_{m+1}}{2}e^{im\varphi} =$$

$$= e^{im\varphi}\left(-\sqrt{\nu}\vec{\varphi}_0\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2} + im\vec{\rho}_0\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right) =$$

$$= -e^{im\varphi}\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2} - im\vec{\rho}_0\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right)$$

$$\left[\nabla_\perp \Psi_m^* \times \vec{z}_0\right] = \begin{vmatrix} \vec{\rho}_0 & \vec{\varphi}_0 & \vec{z}_0 \\ \dfrac{\partial \Psi_m^*}{\partial \rho} & \dfrac{1}{\rho}\dfrac{\partial \Psi_m^*}{\partial \varphi} & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{\rho}_0 \frac{1}{\rho}\frac{\partial \Psi_m^*}{\partial \varphi} - \vec{\varphi}_0 \frac{\partial \Psi_m^*}{\partial \rho} = \vec{\rho}_0 \frac{\partial}{\partial \varphi}\frac{1}{\rho}\frac{J_m(\nu\rho)}{\sqrt{\nu}} e^{-im\varphi} -$$

$$-\vec{\varphi}_0 \frac{\partial}{\partial \rho}\frac{J_m(\nu\rho)}{\sqrt{\nu}} e^{-im\varphi} = -im\vec{\rho}_0 \frac{J_m}{\rho\sqrt{\nu}} e^{-im\varphi} - \sqrt{\nu}\vec{\varphi}_0 \frac{J_{m-1} - J_{m+1}}{2} e^{-im\varphi} =$$

$$= e^{-im\varphi}\left(-\sqrt{\nu}\vec{\varphi}_0 \frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{2} - im\vec{\rho}_0 \frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right) =$$

$$= -e^{-im\varphi}\left(\vec{\varphi}_0 \sqrt{\nu}\, \frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{2} + im\vec{\rho}_0 \frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right)$$

$$\nabla_\perp \Phi_n = \vec{\rho}_0 \frac{\partial \Phi_n}{\partial \rho} + \vec{\varphi}_0 \frac{1}{\rho}\frac{\partial \Phi_n}{\partial \varphi} = \vec{\rho}_0 \frac{\partial}{\partial \rho}\frac{J_n(\chi\rho)}{\sqrt{\chi}} e^{in\varphi} + \vec{\varphi}_0 \frac{1}{\rho}\frac{\partial}{\partial \varphi}\frac{J_n(\chi\rho)}{\sqrt{\chi}} e^{in\varphi} =$$

$$= e^{in\varphi}\left\{\vec{\rho}_0 \sqrt{\chi}\, \frac{J_{n-1}(\chi\rho) - J_{n+1}(\chi\rho)}{2} + \vec{\varphi}_0 in \frac{J_n(\chi\rho)}{\rho\sqrt{\chi}}\right\}$$

$$\nabla_\perp \Phi_n^* = \vec{\rho}_0 \frac{\partial \Phi_n^*}{\partial \rho} + \vec{\varphi}_0 \frac{1}{\rho}\frac{\partial \Phi_n^*}{\partial \varphi} = \vec{\rho}_0 \frac{\partial}{\partial \rho}\frac{J_n(\chi\rho)}{\sqrt{\chi}} e^{-in\varphi} + \vec{\varphi}_0 \frac{1}{\rho}\frac{\partial}{\partial \varphi}\frac{J_n(\chi\rho)}{\sqrt{\chi}} e^{-in\varphi} =$$

$$= e^{-in\varphi}\left\{\vec{\rho}_0 \sqrt{\chi}\, \frac{J_{n-1}(\chi\rho) - J_{n+1}(\chi\rho)}{2} - \vec{\varphi}_0 in \frac{J_n(\chi\rho)}{\rho\sqrt{\chi}}\right\}$$

$$\left[\vec{z}_0 \times \nabla_\perp \Phi_n^*\right] = \begin{vmatrix} \vec{\rho}_0 & \vec{\varphi}_0 & \vec{z}_0 \\ \dfrac{\partial \Phi_n^*}{\partial \rho} & \dfrac{1}{\rho}\dfrac{\partial \Phi_n^*}{\partial \varphi} & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{\rho}_0 \frac{1}{\rho}\frac{\partial \Phi_n^*}{\partial \varphi} - \vec{\varphi}_0 \frac{\partial \Phi_n^*}{\partial \rho} = \vec{\rho}_0 \frac{\partial}{\partial \varphi}\frac{1}{\rho}\frac{J_n(\chi\rho)}{\sqrt{\chi}} e^{-in\varphi} -$$

$$-\vec{\varphi}_0 \frac{\partial}{\partial \rho}\frac{J_n(\chi\rho)}{\sqrt{\chi}} e^{-in\varphi} = -in\vec{\rho}_0 \frac{J_n}{\rho\sqrt{\chi}} e^{-in\varphi} - \sqrt{\chi}\vec{\varphi}_0 \frac{J_{n-1} - J_{n+1}}{2} e^{-in\varphi} =$$

$$= -e^{-in\varphi}\left(\sqrt{\chi}\vec{\varphi}_0 \frac{J_{n-1}(\chi\rho) - J_{n+1}(\chi\rho)}{2} + in\vec{\rho}_0 \frac{J_n(\chi\rho)}{\rho\sqrt{\chi}}\right)$$

$$\nabla_\perp \Phi_n^* \cdot \left[\nabla_\perp \Psi_m \times \vec{z}_0\right] = e^{i(m-n)\varphi}\left(\vec{\rho}_0 \sqrt{\chi}\, \frac{J_{n-1}(\chi\rho) - J_{n+1}(\chi\rho)}{2} - \vec{\varphi}_0 in \frac{J_n(\chi\rho)}{\rho\sqrt{\chi}}\right) \times$$

$$\times\left(-\vec{\varphi}_0 \sqrt{\nu}\, \frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{2} + im\vec{\rho}_0 \frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right) = e^{i(m-n)\varphi}\left[im \frac{J_m(\nu\rho)}{\rho\sqrt{\nu}} \sqrt{\chi}\, \frac{J_{n-1}(\chi\rho) - J_{n+1}(\chi\rho)}{2} +\right.$$

$$\left. + in \frac{J_n(\chi\rho)}{\rho\sqrt{\chi}} \sqrt{\nu}\, \frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{2}\right] = e^{i(m-n)\varphi}\left[\frac{im}{2}\frac{\sqrt{\chi}}{\sqrt{\nu}} J_m(\nu\rho) \frac{J_{n-1}(\chi\rho) - J_{n+1}(\chi\rho)}{\rho} +\right.$$

$$\left. + \frac{in}{2}\frac{\sqrt{\nu}}{\sqrt{\chi}} J_n(\chi\rho) \frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{\rho}\right]$$

$$\left[\vec{z}_0\times\nabla_\perp\Phi_n^*\right]\cdot\nabla_\perp\Psi_m=-e^{i(m-n)\varphi}\left(\sqrt{\chi}\vec{\varphi}_0\frac{J_{n-1}(\chi\rho)-J_{n+1}(\chi\rho)}{2}+in\vec{\rho}_0\frac{J_n(\chi\rho)}{\rho\sqrt{\chi}}\right)\times$$

$$\times\left(\vec{\rho}_0\sqrt{\nu}\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2}+\vec{\varphi}_0 im\frac{J_m(\nu\rho)}{\sqrt{\nu}\rho}\right)=e^{i(m-n)\varphi}\left[-in\frac{J_n(\chi\rho)}{\rho\sqrt{\chi}}\sqrt{\nu}\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2}-\right.$$

$$\left.-im\frac{J_m(\nu\rho)}{\sqrt{\nu}\rho}\sqrt{\chi}\frac{J_{n-1}(\chi\rho)-J_{n+1}(\chi\rho)}{2}\right]=e^{i(m-n)\varphi}\left[-\frac{in}{2}\frac{\sqrt{\nu}}{\sqrt{\chi}}J_n(\chi\rho)\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{\rho}-\right.$$

$$\left.-\frac{im}{2}\frac{\sqrt{\chi}}{\sqrt{\nu}}J_m(\nu\rho)\frac{J_{n-1}(\chi\rho)-J_{n+1}(\chi\rho)}{\rho}\right]$$

6.3 Extension output of plane disk sources

$$\left[\nabla_\perp\Psi_m^*\times\vec{z}_0\right]=-e^{-im\varphi}\left(\vec{\varphi}_0\sqrt{\nu}\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2}+im\vec{\rho}_0\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right)$$

$$\vec{j}_0\cdot\left[\nabla_\perp\Psi_m^*\times\vec{z}_0\right]=-H(t)\delta(z)\big(H(\rho)-H(\rho-R)\big)\left(\vec{\rho}_0\cos\varphi-\vec{\varphi}_0\sin\varphi\right)e^{-im\varphi}\times$$

$$\times\left(\vec{\varphi}_0\sqrt{\nu}\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2}+im\vec{\rho}_0\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right)=-H(t)\delta(z)\big(H(\rho)-H(\rho-R)\big)\times$$

$$\times\left[im\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\cos\varphi-\sqrt{\nu}\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2}\sin\varphi\right]e^{-im\varphi}$$

$$j_m(z,t;\nu)=-\frac{\sqrt{\mu_0}}{2\pi}\int_0^{2\pi}d\varphi\int_0^\infty\rho d\rho H(t)\delta(z)\big(H(\rho)-H(\rho-R)\big)e^{-im\varphi}\times$$

$$\times\left[im\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\cos\varphi-\sqrt{\nu}\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2}\sin\varphi\right]=-\frac{\sqrt{\mu_0}}{2\pi}H(t)\delta(z)\int_0^{2\pi}d\varphi$$

$$\int_0^\infty\rho d\rho\big(H(\rho)-H(\rho-R)\big)e^{-im\varphi}\left[im\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\cos\varphi-\sqrt{\nu}\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2}\sin\varphi\right]=$$

$$=-\frac{\sqrt{\mu_0}}{2\pi}H(t)\delta(z)\left[\frac{im}{\sqrt{\nu}}\int_0^{2\pi}\cos\varphi e^{-im\varphi}d\varphi\int_0^R J_m(\nu\rho)d\rho-\right.$$

$$\left.-\frac{\sqrt{\nu}}{2}\int_0^{2\pi}\sin\varphi e^{-im\varphi}d\varphi\left(\int_0^R\rho J_{m-1}(\nu\rho)d\rho-\int_0^R\rho J_{m+1}(\nu\rho)d\rho\right)\right]$$

$$\delta_{m,n}=\frac{1}{2\pi}\int_0^{2\pi}e^{i(m-n)\varphi}d\varphi\Rightarrow\frac{1}{2\pi}\int_0^{2\pi}e^{-i(m\pm1)\varphi}d\varphi=\delta_{m,\mp1}$$

$$\int_0^{2\pi} \cos\varphi e^{-im\varphi} d\varphi = \left[\cos\varphi = \frac{e^{i\varphi}+e^{-i\varphi}}{2} \right] =$$

$$= \frac{1}{2}\int_0^{2\pi} \left(e^{i\varphi}+e^{-i\varphi}\right) e^{-im\varphi} d\varphi = \pi\left(\delta_{1,m}+\delta_{-1,m}\right)$$

$$\int_0^{2\pi} \sin\varphi e^{-im\varphi} d\varphi = \left[\sin\varphi = \frac{e^{i\varphi}-e^{-i\varphi}}{2i} \right] =$$

$$= \frac{1}{2i}\int_0^{2\pi} \left(e^{i\varphi}-e^{-i\varphi}\right) e^{-im\varphi} d\varphi = -i\pi\left(\delta_{1,m}-\delta_{-1,m}\right)$$

$$\int_0^{2\pi} \cos\varphi e^{-im\varphi} d\varphi \int_0^R J_m(\nu\rho)\, d\rho = \pi\int_0^R J_m(\nu\rho)\left(\delta_{1,m}+\delta_{-1,m}\right) d\rho =$$

$$= \pi\int_0^R J_1(\nu\rho)+J_{-1}(\nu\rho)\, d\rho = \pi\int_0^R \left(J_1(\nu\rho)-J_1(\nu\rho)\right) d\rho = 0$$

Symmetric property of Bessel function

$$J_{-m}(z) = (-1)^m J_m(z)$$

Derivative property of Bessel function

$$\frac{d}{d\rho} J_m(\chi\rho) = \frac{\chi}{2}\left[J_{m-1}(\chi\rho) - J_{m+1}(\chi\rho)\right]$$

$$\frac{d}{d\rho} J_1(\chi\rho) = \frac{\chi}{2}\left[J_0(\chi\rho) - J_2(\chi\rho)\right]$$

$$\frac{d}{d\rho} J_0(\chi\rho) = \frac{\chi}{2}\left[J_{-1}(\chi\rho) - J_1(\chi\rho)\right] = \frac{\chi}{2}\left[(-1)^1 J_1(\chi\rho) - J_1(\chi\rho)\right] = \chi J_1(\chi\rho)$$

$$\int_0^{2\pi} \sin\varphi e^{-im\varphi} d\varphi \left(\int_0^R \rho J_{m-1}(\nu\rho) d\rho - \int_0^R \rho J_{m+1}(\nu\rho) d\rho \right) =$$

$$= -i\pi \left(\int_0^R \rho J_{m-1}(\nu\rho)\left(\delta_{1,m} - \delta_{-1,m}\right) d\rho - \int_0^R \rho J_{m+1}(\nu\rho)\left(\delta_{1,m} - \delta_{-1,m}\right) d\rho \right) =$$

$$= -i\pi \left(\int_0^R \rho\left(J_0(\nu\rho) - J_{-2}(\nu\rho) - J_2(\nu\rho) + J_0(\nu\rho)\right) d\rho \right) =$$

$$= -2i\pi \left(\int_0^R \rho\left(J_0(\nu\rho) - J_2(\nu\rho)\right) d\rho \right) =$$

$$\left\{ \frac{d}{d\rho} J_m(\chi\rho) = \frac{\chi}{2}\left[J_{m-1}(\chi\rho) - J_{m+1}(\chi\rho)\right] \right\}$$

$$= \frac{-4i\pi}{\nu} \left(\int_0^R \rho \frac{d}{d\rho} J_1(\nu\rho) d\rho \right) =$$

$$= \frac{-4i\pi}{\nu} \left(J_1(\nu\rho)\rho\Big|_0^R - \int_0^R J_1(\nu\rho) d\rho \right) = \frac{-4i\pi}{\nu} \left(J_1(\nu R) R + \frac{1}{\nu}\int_0^R \frac{d}{d\rho} J_0(\nu\rho) d\rho \right) =$$

$$= \frac{-4i\pi}{\nu} \left(J_1(\nu R) R + \frac{1}{\nu} J_0(\nu\rho)\Big|_0^R \right) = \frac{-4i\pi}{\nu} \left(J_1(\nu R) R + \frac{1}{\nu} J_0(\nu R) - \frac{1}{\nu} \right)$$

$$j_m(z,t;\nu) = \left(-\frac{\sqrt{\mu_0}}{2\pi} H(t)\delta(z) \right)\left(-\frac{\sqrt{\nu}}{2} \right)\left(-\frac{4i\pi}{\nu} \right)\left(J_1(\nu R) R + \frac{1}{\nu} J_0(\nu R) - \frac{1}{\nu} \right) =$$

$$= -\frac{i\sqrt{\mu_0}}{\sqrt{\nu}} H(t)\delta(z)\left(J_1(\nu R) R + \frac{1}{\nu} J_0(\nu R) - \frac{1}{\nu} \right) =$$

$$= -\sqrt{\mu_0}\frac{iR}{\sqrt{\nu}} H(t)\delta(z) J_1(\nu R) - \frac{i\sqrt{\mu_0}}{\sqrt{\nu}} \frac{J_0(\nu R) - 1}{\nu} H(t)\delta(z)$$

6.4 Bessel operations for plane disk radiation

$$j_m(z,t;\nu) = -H(t)\delta(z)\frac{\sqrt{\mu_0}}{2\pi}\left(im\pi\delta_{m,1}\int_0^R \frac{J_m(\nu\rho)}{\sqrt{\nu}} d\rho + \sqrt{\nu}\, i\pi\delta_{m,1}\int_0^R \rho \frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{2} d\rho \right) =$$

$$= -\sqrt{\mu_0} H(t)\delta(z)\frac{i\delta_{m,1}}{2\sqrt{\nu}}\left(\int_0^R J_1(\nu\rho) d\rho + \nu\int_0^R \rho \frac{J_0(\nu\rho) - J_2(\nu\rho)}{2} d\rho \right) = \left[\frac{d}{d\rho} J_0(\nu\rho) = \nu J_1(\nu\rho) \right] =$$ - а

$$= -\sqrt{\mu_0} H(t)\delta(z)\frac{i\delta_{m,1}}{2\sqrt{\nu}}\left(\frac{1}{\nu}\int_0^R \frac{d}{d\rho} J_0(\nu\rho) d\rho + \nu\int_0^R \rho \frac{J_0(\nu\rho) - J_2(\nu\rho)}{2} d\rho \right)$$

минус в формуле дифференцирования не пропущен? Последняя строка не нужна, как будет видно ниже

$$
\int_0^R \rho \frac{J_0(\nu\rho)-J_2(\nu\rho)}{2} d\rho = \frac{\partial}{\partial \nu}\left\{\int_0^R J_1(\nu\rho)\, d\rho\right\} = \left[\frac{d}{d\rho} J_0(\nu\rho) = -\nu J_1(\nu\rho)\right] =
$$

$$
= \frac{\partial}{\partial \nu}\left\{-\frac{1}{\nu}\int_0^R \frac{d}{d\rho} J_0(\nu\rho)\, d\rho\right\} = -\frac{\partial}{\partial \nu}\left\{\left.\frac{J_0(\nu\rho)}{\nu}\right|_0^R\right\} = \frac{\partial}{\partial \nu}\frac{1-J_0(\nu R)}{\nu} =
$$

$$
= \frac{\nu R J_1(\nu R) - 1 + J_0(\nu R)}{\nu^2} = \frac{R J_1(\nu R)}{\nu} - \frac{1-J_0(\nu R)}{\nu^2} = \frac{R J_1(\nu R)}{\nu} - \frac{1}{\nu}\int_0^R J_1(\nu\rho)\, d\rho
$$

6.5 Approach of substitution

$$
h_m(z,t;\nu) = \sum_{k=0}^{\infty} C_{mk}^{-}(\nu)\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}} J_k\left(\nu\sqrt{c^2t^2-z^2}\right) + \sum_{k=0}^{\infty} C_{mk}^{+}(\nu)\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}} J_k\left(\nu\sqrt{c^2t^2-z^2}\right)
$$

$$
e_n(z,t;\chi) = \sum_{j=0}^{\infty} D_{nj}^{-}(\chi)\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}} J_j\left(\chi\sqrt{c^2t^2-z^2}\right) + \sum_{j=0}^{\infty} D_{nj}^{+}(\chi)\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}} J_j\left(\chi\sqrt{c^2t^2-z^2}\right)
$$

$$
\partial_{ct}(h_m) = \frac{1}{c}\frac{\partial}{\partial t}\sum_{k=0}^{\infty}\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}} + C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right] J_k = \frac{1}{c}\sum_{k=0}^{\infty}\left\{\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}} - C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right] czk\frac{J_k}{c^2t^2-z^2} -\right.
$$

$$
\left. + \left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}} + C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[\frac{\nu c^2 t}{2}\frac{J_{k-1}}{\sqrt{c^2t^2-z^2}} - \frac{\nu c^2 t}{2}\frac{J_{k+1}}{\sqrt{c^2t^2-z^2}}\right]\right\}
$$

$$\partial_{ct}^{2}\left(h_{m}\right)=\frac{1}{c^{2}}\frac{\partial}{\partial t}\left\{\sum_{k=0}^{\infty}\left\{\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}-C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]czk\frac{J_{k}}{c^{2}t^{2}-z^{2}}+\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[\frac{\nu c^{2}t}{2}\right.\right.\right.$$

$$\left.\left.\left.-\frac{\nu c^{2}t}{2}\frac{J_{k+1}}{\sqrt{c^{2}t^{2}-z^{2}}}\right]\right\}\right\}=\sum_{k=0}^{\infty}\left\{\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]z^{2}k^{2}\frac{J_{k}}{\left(c^{2}t^{2}-z^{2}\right)^{2}}+\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}-C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)\right.\right.$$

$$\left.\frac{\nu zkct}{2}\frac{J_{k+1}}{\left(c^{2}t^{2}-z^{2}\right)^{\frac{3}{2}}}-2zkct\frac{J_{k}}{\left(c^{2}t^{2}-z^{2}\right)^{2}}\right]+\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}-C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[\frac{\nu zkct}{2}\frac{J_{k-1}}{\left(c^{2}t^{2}-z^{2}\right)^{\frac{3}{2}}}-\frac{\nu zkct}{2}\frac{J}{\left(c^{2}t^{2}\right.}\right.$$

$$+\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[\frac{\nu^{2}c^{2}t^{2}}{4}\frac{J_{k-2}}{c^{2}t^{2}-z^{2}}-\frac{\nu^{2}c^{2}t^{2}}{4}\frac{J_{k}}{c^{2}t^{2}-z^{2}}-\frac{\nu c^{2}t^{2}}{2}\frac{J_{k-1}}{\left(c^{2}t^{2}-z^{2}\right)^{\frac{3}{2}}}-\frac{\nu^{2}c^{2}t^{2}}{4}\frac{J_{k}}{c^{2}t^{2}-z^{2}}\right.$$

$$\left.\left.+\frac{\nu c^{2}t^{2}}{2}\frac{J_{k+1}}{\left(c^{2}t^{2}-z^{2}\right)^{\frac{3}{2}}}+\frac{\nu}{2}\frac{J_{k-1}}{\sqrt{c^{2}t^{2}-z^{2}}}-\frac{\nu}{2}\frac{J_{k+1}}{\sqrt{c^{2}t^{2}-z^{2}}}\right]\right\}=\sum_{k=0}^{\infty}\left\{\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[z^{2}k^{2}\frac{J_{k}}{\left(c^{2}t^{2}-z^{2}\right.}\right.\right.$$

$$\left.+\frac{\nu^{2}c^{2}t^{2}}{4}\frac{J_{k-2}-2J_{k}+J_{k+2}}{c^{2}t^{2}-z^{2}}+\frac{\nu}{2}\frac{J_{k-1}-J_{k+1}}{\sqrt{c^{2}t^{2}-z^{2}}}\right]+\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}-C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[\nu zkct\frac{J_{k-1}-J_{k+1}}{\left(c^{2}t^{2}-z^{2}\right)^{\frac{3}{2}}}-2zkct\frac{}{\left(c^{2}t\right.}\right.$$

$$\partial_{z}\left(h_{m}\right)=\frac{\partial}{\partial z}\sum_{k=0}^{\infty}\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]J_{k}=\sum_{k=0}^{\infty}\left\{\left[-C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]kct\frac{J_{k}}{c^{2}t^{2}-z^{2}}+\right.$$

$$\left.+\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[-\frac{\nu z}{2}\frac{J_{k-1}}{\sqrt{c^{2}t^{2}-z^{2}}}+\frac{\nu z}{2}\frac{J_{k+1}}{\sqrt{c^{2}t^{2}-z^{2}}}\right]\right\}$$

$$\partial_{ct}^{2}\left(e_n\right)=\frac{1}{c}\frac{\partial}{\partial t}\sum_{j=0}^{\infty}\left\{\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}-D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]zj\frac{J_j}{c^2t^2-z^2}+\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[\frac{\chi ct}{2}\frac{J_{j-1}}{\sqrt{c^2t^2-}}\right.\right.$$

$$=\frac{1}{c}\sum_{j=0}^{\infty}\left\{\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]cz^2j^2\frac{J_j}{\left(c^2t^2-z^2\right)^2}+\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}-D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[\frac{\chi zjc^2t}{2}\frac{J_{j-1}}{\left(c^2t^2-z\right.}\right.\right.$$

$$\left.\left.-2zjc^2t\frac{J_j}{\left(c^2t^2-z^2\right)^2}\right]+\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}-D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[\frac{\chi zjc^2t}{2}\frac{J_{j-1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}-\frac{\chi ct}{2}\frac{J_{j+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}\right]+\left[D_{nj}^{-}\left(\frac{ct-}{ct+}\right.\right.$$

$$\times\left[\frac{\chi^2c^3t^2}{4}\frac{J_{j-2}-J_j}{c^2t^2-z^2}-\frac{\chi c^3t^2}{2}\frac{J_{j-1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}-\frac{\chi^2c^3t^2}{4}\frac{J_j-J_{j+2}}{c^2t^2-z^2}+\frac{\chi c^3t^2}{2}\frac{J_{j+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}+\frac{\chi c}{2}\frac{J_{j-1}}{\sqrt{c^2t^2-z^2}}-\frac{\chi c}{2}\frac{J_{j-}}{\sqrt{c^2t^2}}\right.$$

$$=\sum_{j=0}^{\infty}\left\{\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[z^2j^2\frac{J_j}{\left(c^2t^2-z^2\right)^2}+\frac{\chi^2c^2t^2}{4}\frac{J_{j-2}-2J_j+J_{j+2}}{c^2t^2-z^2}-\frac{\chi c^2t^2}{2}\frac{J_{j-1}-J_{j+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}+\frac{\chi}{2}\right.\right.$$

$$\left.+\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}-D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[\chi zjct\frac{J_{j-1}-J_{j+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}-2zjct\frac{J_j}{\left(c^2t^2-z^2\right)^2}\right]\right\}$$

$$\partial_z\left(e_n\right)=\frac{\partial}{\partial z}\sum_{j=0}^{\infty}\left[D_{nj}^{-}(\chi)\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}(\chi)\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]J_j=\sum_{j=0}^{\infty}\left\{\left[-D_{nj}^{-}(\chi)\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}(\chi)\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]jct\right.$$

$$\left.+\left[D_{nj}^{-}(\chi)\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}(\chi)\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[-\frac{\chi z}{2}\frac{J_{j-1}}{\sqrt{c^2t^2-z^2}}+\frac{\chi z}{2}\frac{J_{j+1}}{\sqrt{c^2t^2-z^2}}\right]\right\}$$

$$\partial_{ct}\left(e_n\right)=\frac{1}{c}\frac{\partial}{\partial t}\sum_{j=0}^{\infty}\left[D_{nj}^{-}(\chi)\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}(\chi)\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]J_j=\frac{1}{c}\sum_{j=0}^{\infty}\left\{\left[D_{nj}^{-}(\chi)\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}-D_{nj}^{+}(\chi)\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]cz\cdots\right.$$

$$\left.+\left[D_{nj}^{-}(\chi)\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}(\chi)\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[\frac{\chi c^2t}{2}\frac{J_{j-1}}{\sqrt{c^2t^2-z^2}}-\frac{\chi c^2t}{2}\frac{J_{j+1}}{\sqrt{c^2t^2-z^2}}\right]\right\}=\sum_{j=0}^{\infty}\left\{\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}-D_{nj}^{+}\left(\cdots\right.\right.\right.$$

$$\left.+\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[\frac{\chi ct}{2}\frac{J_{j-1}}{\sqrt{c^2t^2-z^2}}-\frac{\chi ct}{2}\frac{J_{j+1}}{\sqrt{c^2t^2-z^2}}\right]\right\}$$

$$\partial_z^2\left(h_m\right)=\frac{\partial}{\partial z}\sum_{k=0}^{\infty}\left\{\left[-C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]kct\frac{J_k}{c^2t^2-z^2}+\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[-\frac{\nu z}{2}\cdots\right.\right.$$

$$=\sum_{k=0}^{\infty}\left\{\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]k^2c^2t^2\frac{J_k}{\left(c^2t^2-z^2\right)^2}+\left[-C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[-\frac{\nu kzct}{2}\cdots\right.\right.$$

$$\left.+\frac{\nu kzct}{2}\frac{J_{k+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}+2kzct\frac{J_k}{\left(c^2t^2-z^2\right)^2}\right]+\left[-C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[-\frac{\nu kzct}{2}\frac{J_{k-1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}+\right.$$

$$\left.+\frac{\nu kzct}{2}\frac{J_{k+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}\right]+\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[-\frac{\nu}{2}\frac{J_{k-1}}{\sqrt{c^2t^2-z^2}}-\frac{\nu z}{2}\left(\frac{\nu z}{2}\frac{J_{k-2}}{c^2t^2-z^2}-\frac{\nu z}{2}\frac{J_k}{c^2t^2-z^2}+\cdots\right.\right.$$

$$\left.\left.+\frac{\nu z}{2}\left(\frac{\nu z}{2}\frac{J_k}{c^2t^2-z^2}-\frac{\nu z}{2}\frac{J_{k+2}}{c^2t^2-z^2}+z\frac{J_{k+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}\right)\right]=\sum_{k=0}^{\infty}\left\{\left[C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[k^2c^2t^2\frac{J_k}{\left(c^2t^2-z^2\right)^{2}}\cdots\right.\right.$$

$$\left.-\frac{\nu^2z^2}{4}\frac{J_{k-2}-2J_k+J_{k+2}}{c^2t^2-z^2}-\frac{\nu z^2}{2}\frac{J_{k-1}-J_{k+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}\right]+\left[-C_{mk}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{k}{2}}+C_{mk}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{k}{2}}\right]\left[-\nu kzct\frac{J_{k-1}-J_{k+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}+2kzc\cdots\right.$$

$$\partial_z^2\left(e_n\right)=\frac{\partial}{\partial z}\sum_{j=0}^{\infty}\left\{\left[-D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]jct\frac{J_j}{c^2t^2-z^2}+\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[-\frac{\chi z}{2}\frac{J_j}{\sqrt{c^2t^2}}\right.\right.$$

$$=\sum_{j=0}^{\infty}\left\{\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]j^2c^2t^2\frac{J_j}{\left(c^2t^2-z^2\right)^2}+\left[-D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[-\frac{\chi zjct}{2}\frac{J_{j-1}-}{\left(c^2t^2-\right.}\right.\right.$$

$$+\left[-D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[-\frac{\chi zjct}{2}\frac{J_{j-1}-J_{j+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}\right]+\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[-\frac{\chi}{2}\frac{J_{j-1}}{\sqrt{c^2t^2-z^2}}\right.$$

$$\left.\left.-\frac{\chi z^2}{2}\frac{J_{j-1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}+\frac{\chi}{2}\frac{J_{j+1}}{\sqrt{c^2t^2-z^2}}-\frac{\chi^2z^2}{4}\frac{J_j-J_{j+2}}{c^2t^2-z^2}+\frac{\chi z^2}{2}\frac{J_{j+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}\right]\right\}=\sum_{j=0}^{\infty}\left\{\left[D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right.\right.\right.$$

$$\left.-\frac{\chi}{2}\frac{J_{j-1}-J_{j+1}}{\sqrt{c^2t^2-z^2}}-\frac{\chi^2z^2}{4}\frac{J_{j-2}-2J_j+J_{j+2}}{c^2t^2-z^2}-\frac{\chi z^2}{2}\frac{J_{j-1}-J_{j+1}}{\left(c^2t^2-z^2\right)^{\frac{3}{2}}}\right]+\left[-D_{nj}^{-}\left(\frac{ct-z}{ct+z}\right)^{\frac{j}{2}}+D_{nj}^{+}\left(\frac{ct+z}{ct-z}\right)^{\frac{j}{2}}\right]\left[-\chi zjct\frac{J_{j-}}{\left(c^2t\right.}\right.$$

7 REFERENCES

Committee IEEE Aerospace and Electronic Systems Society UWB Radar IEEE P1672 TM [Article] // Ultrawideband Radar Definitions Draft. - April 4, 2006. - Version 1.5.

Crookes William Some Possibilities of Electricity [Journal] // The Fortnightly Review. - February 1, 1892. - pp. 174-176.

Dumin Olexandr [et al.] Evolutionary Approach for The Problem of Electromagnetic Field Propagation Through Nonlinear Medium [Article] // Вісник Харківського національного Університету імені В. Н. Каразіна. - Харків : [s.n.], 2014. - 24. - pp. 23-28.

Dumin Olexandr and Tretyakov Oleg Radiation of Arbitrary Signals by Plane Disk [Article]. - Lviv : MMET, 1996. - pp. 248–251.

Hertz Heinrich Rudolf Electric Waves [Book]. - London : Macmillan and Company, 1893.

Rao Sadashiva Madiraju Time Domain Electromagnetics [Book]. - Sun Diego : Academic Press, 1999.

Schantz Hans The Art and Science of Ultra-Wideband Antennas [Book]. - Norwood : Artech House, 2005.

Tretyakov Oleg Alexandrovich [et al.] Evolutionary Approach for the Problems of Transient Electromagnatic Field Propagation in Nonlinear Medium [Article]. - Dnipropetrovsk : MMET, 2014.

Tretyakov Oleg Alexandrovich and Hashimoto M Evolutionary Approach For Electrodynamics [Article] // Essentials of Nonstationary and Nonlinear Electromagnetic Field Theory. - Tokyo : Science House Co, 1993. - p. 572.

Tretyakov Oleg Alexandrovich Evolutionary Approach For Electrodynamics [Book]. - Kharkov : Kharkov State Universite, 1999. - Vol. II.

Wu T. T. Electromagnetic missiles [Article] // J. Appl. Phys. - 1985. - 7 : Vol. 57. - pp. 2370–2373.

Борисов Виктор Васильевич Неустановившиеся поля в волноводах [Book]. - Ленинград : Издательство ленинградского университета, 1991. - p. 41.

Волков И. К. и Канатников А. Н. Интегральные преобразования и операционное исчисление [Книга]. - Москва : Изд-во МГТУ им. Н. Э. Баумана, 2002. - стр. 228.

Иваненко Д. и Соколов А. Классичекая терия поля [Книга]. - Москва : Государственное издательство физико-технической литературы, 1951.

Прудников А П, Брычков Ю А and Маричев О И Интегралы и ряды: Специальные функции [Book]. - Москва : Наука, 1983. - p. 228.

Прудников А П, Брычков Ю А and Маричев О И Интегралы и ряды: Специальные функции [Book]. - Москва : Наука, 1983. - p. 209.

Прудников А П, Брычков Ю А и Маричев О И Интегралы и ряды: Специальные функции [Книга]. - Москва : Наука, 1983. - стр. 440.

Фихтенгольц Г. М. Основы математического анализа [Book]. - Москва : Физматлит, 2001. - Vol. I : p. 172.

Черногор Леонид Феоктистович Введение в нелинейную радиофизику [Книга]. - Харьков : [б.н.], 2012.

Шубарин Юрий Васильевич Антенны сверхвысоких частот [Книга]. - Харьков : Харьковский Государственный Университет, 1960.