Излучение нестационарных полей и их распространение в нелинейном пространстве

В данной теоретической работе получены нелинейные свойства электромагнитного поля путем сравнения аналитических решений классических задач нестационарной электродинамики для линейных и нелинейных случаев распространения.

Решения модельных задач получены без перехода в частотную область с применением метода эволюционных уравнений. Нелинейная модель построена на гипотезе появления вторичных источников и предложен итеративный метод их учета.

Пространство распространения неограниченно, поэтому метод модового базиса не может быть применен напрямую, а упомянутый метод эволюционных уравнений, фактически, является его расширением на бесконечное или полубесконечное пространство. На рассмотрение предложена нестационарная слоисто-неоднородная среда.

1 Введение

Equation Section (Next)

1.1 Основные определения

1.2 Теоретическое обоснование

1.3 Область применения

2 Функция Бесселя первого рода

Equation Section (Next)

2.1 Определение и линейные свойства

$$J_{-n}(z) = (-1)^n J_n(z) \tag{2.1}$$

$$J_{n+1}(z) + J_{n-1}(z) = \frac{2n}{z} J_n(z) \tag{2.2}$$

2.2 Интегро-дифференциальные свойства

$$2\frac{d}{dz} J_n(z) = J_{n-1}(z) - J_{n+1}(z) \tag{2.3}$$

$$\frac{d}{dz} J_n(z) = J_{n-1}(z) - \frac{n}{z} J_n(z) \tag{2.4}$$

$$\frac{d}{dz} J_n(z) = \frac{n}{z} J_n(z) - J_{n+1}(z) \tag{2.5}$$

$$\frac{d}{dz}\frac{J_n(z)}{z^n} = -\frac{J_{n+1}(z)}{z^n} \tag{2.6}$$

$$\frac{d}{dz}\left[z^n J_n(z)\right] = z^n J_{n-1}(z) \tag{2.7}$$

2.3 Интеграл 1

$$I_1 = \int_0^\infty \frac{d\nu}{\nu} J_1(\nu R) J_1(\nu\rho) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) \tag{2.8}$$

Ватсон «Теория бессолевых функций» (п. 13.46, ст. 450)

$$\int_0^\infty \frac{dt}{t^{\lambda+\nu}} J_\mu(at) J_\nu(bt) J_\nu(ct) = \frac{(bc/2)^\nu}{\Gamma(\nu+1/2)\Gamma(1/2)} \int_0^\infty \int_0^\pi \frac{J_\mu(at) J_\nu(\omega t)}{\omega^\nu t^\lambda} \sin^{2\nu}\varphi \, d\varphi \, dt$$

$$\omega = \sqrt{b^2 + c^2 - 2bc\cdot\cos\varphi}$$

$$\int_0^\infty \frac{d\nu}{\nu} J_1(\nu R) J_1(\nu\rho) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) = \frac{\rho R}{2\Gamma(3/2)\Gamma(1/2)} \int_0^\pi \frac{\sin^2\varphi}{\sqrt{\rho^2 + R^2 - 2\rho R\cdot\cos\varphi}} \times$$

$$\times \int_0^\infty d\nu J_1(\omega\nu) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) d\varphi$$

$$\omega = \sqrt{\rho^2 + R^2 - 2\rho R\cdot\cos\varphi}$$

Прудников, Брычков, Маричев, 1983 (2.13.32, ст. 209)

$$\int_0^\infty d\nu J_n \; a\nu \; J_{n-1} \; b\nu \; = \begin{cases} b^{n-1}/a^n & ,b<a \\ 0 & ,b>a \end{cases}$$

$$\sin\frac{\varphi}{2} = \pm\sqrt{\frac{1-\cos\varphi}{2}}$$

$$1-\cos\varphi = \begin{cases} 2\sin^2\dfrac{\varphi}{2} & ,0<\varphi<\pi \\ -2\sin^2\dfrac{\varphi}{2} & ,\pi<\varphi<2\pi \end{cases}$$

$$\int_0^\infty d\nu J_1 \; \nu\sqrt{\rho^2+R^2-2\rho R\cdot\cos\varphi} \; J_0 \; \nu\sqrt{c^2t^2-z^2} \; = \frac{1}{\sqrt{\rho^2+R^2-2\rho R\cdot\cos\varphi}}$$

$$\sqrt{\rho^2+R^2-2\rho R\cdot\cos\varphi} > \sqrt{c^2t^2-z^2}$$

$$\rho^2+R^2-2\rho R\cdot\cos\varphi-2\rho R+2\rho R > c^2t^2-z^2$$

$$\rho-R \;^2 + 2\rho R \; 1-\cos\varphi \; > c^2t^2-z^2$$

$$\rho-R \;^2 + 4\rho R\sin^2\frac{\varphi}{2} > c^2t^2-z^2$$

$$\varphi > 2\arcsin\sqrt{\frac{c^2t^2-z^2- \; \rho-R \;^2}{4\rho R}}$$

$$\psi = 2\arcsin\sqrt{\frac{c^2t^2-z^2- \; \rho-R \;^2}{4\pi\rho}}; \quad \psi\le\varphi\le\pi$$

$$\Gamma \; 3/2 \; \Gamma \; 1/2 \; = \left(\frac{1}{2}\sqrt{\pi}\right)\cdot \; \sqrt{\pi} \; = \frac{\pi}{2}$$

$$\int_0^\infty \frac{d\nu}{\nu} J_1 \; \nu R \; J_1 \; \nu\rho \; J_0 \; \nu\sqrt{c^2t^2-z^2} \; = \frac{\rho R}{2\Gamma \; 3/2 \; \Gamma \; 1/2} \times$$

$$\times\int_0^\pi \frac{\sin^2\varphi}{\sqrt{\rho^2+R^2-2\rho R\cdot\cos\varphi}} \int_0^\infty d\nu J_1 \; \omega\nu \; J_0 \; \nu\sqrt{c^2t^2-z^2} \; d\varphi =$$

$$= \frac{\rho R}{\pi}\int_0^\pi \frac{\sin^2\varphi}{\sqrt{\rho^2+R^2-2\rho R\cdot\cos\varphi}} \int_0^\infty d\nu J_1 \; \omega\nu \; J_0 \; \nu\sqrt{c^2t^2-z^2} \; d\varphi = \frac{\rho R}{\pi}\int_\psi^\pi \frac{\sin^2\varphi d\varphi}{\rho^2+R^2-2\rho R\cdot\cos\varphi} =$$

$$= \frac{\rho}{\pi R}\int_\psi^\pi \frac{\sin^2\varphi d\varphi}{\dfrac{\rho^2}{R^2}+1-\dfrac{2\rho}{R}\cdot\cos\varphi}$$

$$\int_{\psi}^{\pi}\frac{\sin^2\varphi d\varphi}{a+b\cdot\cos\varphi}=\int_{\psi}^{\pi}\frac{1-\cos^2\varphi\ \ d\varphi}{a+b\cdot\cos\varphi}=\int_{\psi}^{\pi}\frac{d\varphi}{a+b\cdot\cos\varphi}-\int_{\psi}^{\pi}\frac{\cos^2\varphi+\frac{a}{b}\cos\varphi}{a+b\cdot\cos\varphi}d\varphi+\frac{a}{b}\int_{\psi}^{\pi}\frac{\cos\varphi d\varphi}{a+b\cdot\cos\varphi}=$$
$$=\int_{\psi}^{\pi}\frac{d\varphi}{a+b\cdot\cos\varphi}-\frac{1}{b}\int_{\psi}^{\pi}\cos\varphi d\varphi+\frac{a}{b^2}\int_{\psi}^{\pi}\frac{a+b\cdot\cos\varphi}{a+b\cdot\cos\varphi}d\varphi-\frac{a^2}{b^2}\int_{\psi}^{\pi}\frac{d\varphi}{a+b\cdot\cos\varphi}=$$
$$=\left(1-\frac{a^2}{b^2}\right)\int_{\psi}^{\pi}\frac{d\varphi}{a+b\cdot\cos\varphi}+\frac{a}{b^2}\int_{\psi}^{\pi}d\varphi-\frac{1}{b}\int_{\psi}^{\pi}\cos\varphi d\varphi$$

Прудников, Бычков, Маричев «Интегралы и ряды: Элементарные функции» ст. 181,

$$\int\frac{d\varphi}{a+b\cdot\cos\varphi}=\frac{2}{\sqrt{a^2-b^2}}\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\varphi}{2}\right)$$

$$\tan\frac{\pi}{2}=\infty\Rightarrow\arctan\left(\tan\frac{\pi}{2}\right)=\arctan\ \ \infty\ \ =\frac{\pi}{2}$$

$$\int_{\psi}^{\pi}\frac{\sin^2\varphi d\varphi}{a+b\cdot\cos\varphi}=\left(1-\frac{a^2}{b^2}\right)\int_{\psi}^{\pi}\frac{d\varphi}{a+b\cdot\cos\varphi}+\frac{a}{b^2}\int_{\psi}^{\pi}d\varphi-\frac{1}{b}\int_{\psi}^{\pi}\cos\varphi d\varphi=$$
$$=\left(1-\frac{a^2}{b^2}\right)\frac{2}{\sqrt{a^2-b^2}}\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\varphi}{2}\right)+\frac{a}{b^2}\varphi-\frac{\sin\varphi}{b}\Bigg|_{\psi}^{\pi}=$$
$$=\left(1-\frac{a^2}{b^2}\right)\frac{2}{\sqrt{a^2-b^2}}\left[\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\pi}{2}\right)-\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\psi}{2}\right)\right]+\frac{a}{b^2}\pi-\frac{a}{b^2}\psi-$$
$$-\frac{\sin\pi}{b}+-\frac{\sin\psi}{b}=-\frac{2}{b^2}\frac{a^2-b^2}{\sqrt{a^2-b^2}}\left[\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\pi}{2}\right)-\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\psi}{2}\right)\right]+\frac{a}{b^2}\pi-$$
$$-\frac{a}{b^2}\psi-\frac{\sin\pi}{b}+\frac{\sin\psi}{b}=\frac{2\sqrt{a^2-b^2}}{b^2}\left[\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\psi}{2}\right)-\frac{\pi}{2}\right]+\frac{a}{b^2}(\pi-\psi)\frac{\sin\psi}{b}$$

$$\tan\frac{\psi}{2}=\frac{\sin\frac{\psi}{2}}{\cos\frac{\psi}{2}}=\frac{\sin\frac{\psi}{2}}{\sqrt{1-\sin^2\frac{\psi}{2}}}=\sqrt{\frac{\frac{c^2t^2-z^2-\ \ \rho-R\ \ ^2}{4\pi\rho}}{1-\frac{c^2t^2-z^2-\ \ \rho-R\ \ ^2}{4\pi\rho}}}=\sqrt{\frac{c^2t^2-z^2-\ \ \rho-R\ \ ^2}{\ \rho+R\ \ ^2-c^2t^2+z^2}}$$

$$\int_{\psi}^{\pi}\frac{\sin^2\varphi d\varphi}{a+b\cdot\cos\varphi}=\frac{2\sqrt{a^2-b^2}}{b^2}\left[\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{\psi}{2}\right)-\frac{\pi}{2}\right]+\frac{a}{b^2}\ \pi-\psi\ -\frac{\sin\psi}{b}=$$

$$=2\frac{\sqrt{a^2-b^2}}{b^2}\arctan\left(\frac{\sqrt{a^2-b^2}}{a+b}\sqrt{\frac{c^2t^2-z^2-\ \rho-R\ ^2}{\rho+R\ ^2-c^2t^2+z^2}}\right)-\pi\frac{\sqrt{a^2-b^2}}{b^2}+\frac{a}{b^2}\ \pi-\psi\ -\frac{\sin\psi}{b}$$

$$I_1=\int_0^{\infty}\frac{d\nu}{\nu}J_1\ \nu R\ J_1\ \nu\rho\ J_0\ \nu\sqrt{c^2t^2-z^2}\ =\frac{\rho}{\pi R}\int_{\psi}^{\pi}\frac{\sin^2\varphi d\varphi}{\frac{\rho^2}{R^2}+1-\frac{2\rho}{R}\cdot\cos\varphi}$$

$$I_1=\frac{\rho^2+R^2}{4\pi R\rho}\ \pi-2\psi\ +\frac{\rho^2-R^2}{4\pi R\rho}\left(\arctan\left(\frac{\rho+R}{\rho-R}\tan\frac{\psi}{2}\right)-\arctan\left(\frac{\rho-R}{\rho+R}\tan^{-1}\frac{\psi}{2}\right)\right)\quad(2.9)$$

2.4 Интеграл 2

$$I_2=\int_0^{\infty}d\nu J_1\ \nu R\ J_0\ \nu\rho\ J_0\ \nu\sqrt{c^2t^2-z^2}\quad(2.10)$$

Прудников

$$\int_0^{\infty}J_0\ ax\ J_0\ bx\ J_1\ cx\ dx=\frac{1}{\pi c}\arccos\frac{a^2+b^2-c^2}{2ab}$$

$$|a-b|<c<a+b\quad a,b>0\quad(2.11)$$

$$I_2=\frac{1}{\pi R}\arccos\frac{c^2t^2-z^2+\rho^2-R^2}{2\rho\sqrt{c^2t^2-z^2}}$$

$$I_2=\quad(2.12)$$

3 Функция Ломмеля двух переменных

3.1 Определение и линейные свойства

3.2 Интегро-дифференциальные свойства

3.3 Интеграл 3

3.4 Интеграл 4

3.5 Интеграл 5

4 Метод эволюционных уравнений для цилиндрикой системы

$$\begin{cases}
\partial_{ct}\left(\varepsilon\partial_{ct}\left(\mu h_m\right)\right)-\partial_z\left(\mu^{-1}\partial_z\left(\mu h_m\right)\right)+\nu^2 h_m=\sqrt{\mu_0}\,\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\left[\vec{z}_0\times\vec{J}\right]\cdot\nabla_\perp\Psi_m{}^*\left(\nu\right)-\\
\qquad-\partial_{ct}\left\{\sqrt{\varepsilon_0}\,\varepsilon\,\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)I_z\right\}-\partial_z\left\{\sqrt[-2]{\mu_0}\,\mu^{-1}\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)g\right\}\\
\partial_{ct}\left(\mu\partial_{ct}\left(\varepsilon e_n\right)\right)-\partial_z\left(\varepsilon^{-1}\partial_z\left(\varepsilon e_n\right)\right)+\chi^2 e_n=\sqrt{\varepsilon_0}\,\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\left[\vec{I}\times\vec{z}_0\right]\cdot\nabla_\perp\Phi_n{}^*\left(\chi\right)-\\
\qquad-\partial_{ct}\left\{\sqrt{\mu_0}\,\mu\,\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)J_z\right\}-\partial_z\left\{\sqrt[-2]{\varepsilon_0}\,\varepsilon^{-1}\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)g\right\}\\
I_n^e=-\partial_{ct}\left(\varepsilon e_n\right)-\sqrt{\mu_0}\,\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)J_z\\
V_n^e=\varepsilon^{-1}\partial_z\left(\varepsilon e_n\right)-\sqrt[-2]{\varepsilon_0}\,\varepsilon^{-1}\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)g\\
I_m^h=\mu^{-1}\partial_z\left(\mu h_m\right)-\sqrt[-2]{\mu_0}\,\mu^{-1}\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)g\\
V_m^h=-\partial_{ct}\left(\mu h_m\right)-\sqrt{\varepsilon_0}\,\frac{1}{2\pi}\int\limits_0^{2\pi}d\varphi\int\limits_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)I_z
\end{cases}$$

$$\vec{E}=\sqrt[-2]{\varepsilon_0}\left\{\sum_{m=-\infty}^{\infty}\int\limits_0^{\infty}d\nu V_m^h\left(\nabla_\perp\Psi_m\times\vec{z}_0\right)+\sum_{m=1}^{\infty}\int\limits_0^{\infty}d\chi V_n^e\nabla_\perp\Phi_n\right\}$$

$$\vec{H}=\sqrt[-2]{\mu_0}\left\{\sum_{m=-\infty}^{\infty}\int\limits_0^{\infty}d\nu I_m^h\nabla_\perp\Psi_m+\sum_{m=1}^{\infty}\int\limits_0^{\infty}d\chi I_n^e\left(\vec{z}_0\times\nabla_\perp\Phi_n\right)\right\}$$

$$E_z\left(\rho,\phi,z,t\right)=\sqrt[-2]{\varepsilon_0}\sum_{n=0}^{\infty}\int\limits_0^{\infty}\chi^2 d\chi e_n\left(z,t;\chi\right)\Phi_n\left(\rho,\phi;\chi\right)$$

$$H_z\left(\rho,\phi,z,t\right)=\sqrt[-2]{\mu_0}\sum_{m=0}^{\infty}\int\limits_0^{\infty}\nu^2 d\nu h_m\left(z,t;\nu\right)\Psi_m\left(\rho,\phi;\nu\right)$$

5 Нелинейное приложение эволюционного подхода

6 Методы учета нелинейные свойства слоистых сред

7 Излучение точечного источника

8 Импульсное излучение плоского диска с током

8.1 Постановка задачи

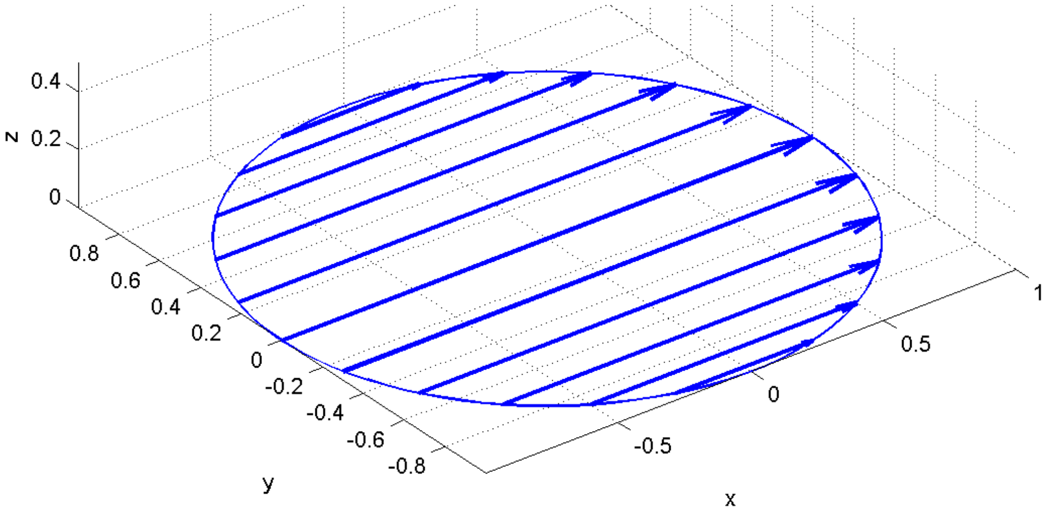

Рисунок 1. Распределение электрического тока

$$\vec{j_0} = \vec{J} = \vec{x_0} H(t)\delta(z)\left(H(\rho) - H(\rho - R)\right) \qquad (2.13)$$

8.2 Источник в базисной форме

$$\begin{cases} \vec{\rho}_0 = \vec{x}_0 \cos\varphi + \vec{y}_0 \sin\varphi \\ \vec{\varphi}_0 = -\vec{x}_0 \sin\varphi + \vec{y}_0 \cos\varphi \end{cases} \Rightarrow \vec{A} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$$

$$\vec{j}_0(\vec{\rho}_0, \vec{\varphi}_0) = \vec{A} \times \vec{j}_0(\vec{x}_0, \vec{y}_0)$$

$$\vec{j}_0(\vec{x}_0, \vec{y}_0) = \vec{A}^{-1} \times \vec{j}_0(\vec{\rho}_0, \vec{\varphi}_0)$$

$$\vec{j}_0 = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} H(t)\delta(z)\left(H(\rho) - H(\rho - R)\right) \\ 0 \end{pmatrix}$$

$$= H(t)\delta(z)\left(H(\rho) - H(\rho - R)\right)\left(\vec{\rho}_0 \cos\varphi - \vec{\varphi}_0 \sin\varphi\right)$$

$$\vec{j}_0 = H(t)\delta(z)\left(H(\rho) - H(\rho - R)\right)\left(\vec{\rho}_0 \cos\varphi - \vec{\varphi}_0 \sin\varphi\right)$$

$$j_m(z,t;\nu) = \frac{\sqrt{\mu_0}}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \, \vec{j_0} \left[\nabla_\perp \Psi_m^* \times \vec{z_0} \right]$$

$$J_{m-1}(\nu\rho) + J_{m+1}(\nu\rho) = \frac{2m}{\nu\rho} J_m(\nu\rho)$$

$$im\vec{\rho}_0 \frac{J_m(\nu\rho)}{\rho\sqrt{\nu}} = i\vec{\rho}_0 \frac{\sqrt{\nu}}{2} \frac{2m}{\rho\nu} J_m(\nu\rho) = i\vec{\rho}_0 \frac{\sqrt{\nu}}{2} \left[J_{m-1}(\nu\rho) + J_{m+1}(\nu\rho) \right]$$

$$\left[\nabla_{\perp}\Psi_{m}^{\ *}\times\vec{z}_0\right]=-e^{-im\varphi}\left(\vec{\varphi}_0\sqrt{\nu}\,\frac{J_{m-1}\;\nu\rho\;-J_{m+1}\;\nu\rho}{2}+im\vec{\rho}_0\,\frac{J_{m}\;\nu\rho}{\rho\sqrt{\nu}}\right)=$$

$$=-e^{-im\varphi}\,\frac{\sqrt{\nu}}{2}\;\;\vec{\varphi}_0\left[J_{m-1}\;\nu\rho\;-J_{m+1}\;\nu\rho\;\right]+i\vec{\rho}_0\left[J_{m-1}\;\nu\rho\;+J_{m+1}\;\nu\rho\;\right]$$

$$\vec{j}_0\left[\nabla_{\perp}\Psi_{m}^{\ *}\times\vec{z}_0\right]=-\sqrt{\nu}\,\frac{\cos\;m\varphi\;-i\sin\;m\varphi}{2}H(t)\delta\;\;z\;\left[H\;\;\rho\;-H\;\;\rho-R\;\right]\times$$
$$\times\;\;\vec{\varphi}_0\left[J_{m-1}\;\nu\rho\;-J_{m+1}\;\nu\rho\;\right]+i\vec{\rho}_0\left[J_{m-1}\;\nu\rho\;+J_{m+1}\;\nu\rho\;\right]\;\;\;\vec{\rho}_0\cos\varphi-\vec{\varphi}_0\sin\varphi\;\;=$$
$$=-\sqrt{\nu}\,\frac{\cos\;m\varphi\;-i\sin\;m\varphi}{2}H(t)\delta\;\;z\;\left[H\;\;\rho\;-H\;\;\rho-R\;\right]\times$$
$$\times\;\;-\sin\varphi\left[J_{m-1}\;\nu\rho\;-J_{m+1}\;\nu\rho\;\right]+i\cos\varphi\left[J_{m-1}\;\nu\rho\;+J_{m+1}\;\nu\rho\;\right]$$

$$j_m(z,t;\nu)=\frac{\sqrt{\mu_0}}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\,\vec{j}_0\left[\nabla_{\perp}\Psi_{m}^{\ *}\times\vec{z}_0\right]=$$
$$=\frac{\sqrt{\mu_0}}{2\pi}\frac{\sqrt{\nu}}{2}H(t)\delta\;\;z\;\;\int_0^{2\pi}\sin\varphi\left[\cos\;m\varphi\;-i\sin\;m\varphi\;\right]d\varphi\int_0^{R}\rho d\rho\left[J_{m-1}\;\nu\rho\;-J_{m+1}\;\nu\rho\;\right]-$$
$$-i\frac{\sqrt{\mu_0}}{2\pi}\frac{\sqrt{\nu}}{2}H(t)\delta\;\;z\;\;\int_0^{2\pi}\cos\varphi\left[\cos\;m\varphi\;-i\sin\;m\varphi\;\right]d\varphi\int_0^{R}\rho d\rho\left[J_{m-1}\;\nu\rho\;+J_{m+1}\;\nu\rho\;\right]$$

$$\cos x\cos y=\frac{1}{2}\left[\cos\;\;x-y\;\;+\cos\;\;x+y\;\;\right]$$
$$\sin x\cos y=\frac{1}{2}\left[\sin\;\;x-y\;\;+\sin\;\;x+y\;\;\right]$$
$$\sin x\sin y=\frac{1}{2}\left[\cos\;\;x-y\;\;-\cos\;\;x+y\;\;\right]$$

$$\delta_{m.n}=\frac{1}{2\pi}\int_0^{2\pi}e^{i(m-n)}d\varphi$$
$$\delta_{m,-n}=\frac{1}{2\pi}\int_0^{2\pi}e^{i(-m-n)}d\varphi$$

$$\int_0^{2\pi}\left[\cos\;\;m\varphi\;\;-i\sin\;\;m\varphi\;\;\right]\sin\varphi d\varphi=\frac{1}{2}\int_0^{2\pi}\sin\;\;\varphi-m\varphi\;\;d\varphi+\frac{1}{2}\int_0^{2\pi}\sin\;\;m\varphi+\varphi\;\;d\varphi-$$
$$-\frac{i}{2}\int_0^{2\pi}\cos\;\;m\varphi-\varphi\;\;d\varphi+\frac{i}{2}\int_0^{2\pi}\cos\;\;m\varphi+\varphi\;\;d\varphi=-\frac{i}{2}\left(\int_0^{2\pi}\cos\;\;m\varphi-\varphi\;\;d\varphi-i\int_0^{2\pi}\sin\;\;m\varphi-\varphi\;\;d\varphi\right)+$$
$$+\frac{i}{2}\left(\int_0^{2\pi}\cos\;\;m\varphi+\varphi\;\;d\varphi-i\int_0^{2\pi}\sin\;\;m\varphi+\varphi\;\;d\varphi\right)=\frac{i}{2}\left(-\int_0^{2\pi}e^{-i\;m-1\;\varphi}d\varphi+\int_0^{2\pi}e^{-i\;m+1\;\varphi}d\varphi\right)=$$
$$=i\pi\left(-\delta_{m,1}+\delta_{m,-1}\right)-i\pi\delta_{m,1}+i\pi\delta_{m,-1}$$

$$\int_0^{2\pi}\left[\cos(m\varphi)-i\sin(m\varphi)\right]\cos\varphi d\varphi=\frac{1}{2}\int_0^{2\pi}\cos(m\varphi-\varphi)d\varphi+\frac{1}{2}\int_0^{2\pi}\cos(m\varphi+\varphi)d\varphi-$$
$$-\frac{i}{2}\int_0^{2\pi}\sin(m\varphi-\varphi)d\varphi-\frac{i}{2}\int_0^{2\pi}\sin(m\varphi+\varphi)d\varphi=\frac{1}{2}\left[\int_0^{2\pi}\cos(m\varphi-\varphi)d\varphi-i\int_0^{2\pi}\sin(m\varphi-\varphi)d\varphi\right]+$$
$$+\frac{1}{2}\left[\int_0^{2\pi}\cos(m\varphi+\varphi)d\varphi-i\int_0^{2\pi}\sin(m\varphi+\varphi)d\varphi\right]=\frac{1}{2}\left(\int_0^{2\pi}e^{-i(m-1)\varphi}d\varphi+\int_0^{2\pi}e^{-i(m+1)\varphi}d\varphi\right)=\pi\delta_{m,1}+\pi\delta_{m,-1}$$

$$j_m(z,t;\nu)=i\frac{\sqrt{\mu_0}}{2\pi}\frac{\sqrt{\nu}}{2}H(t)\delta(z)\left\{\left[\delta_{m,-1}-\delta_{m,1}\right]\int_0^R\rho d\rho\left[J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)\right]-\right.$$
$$\left.-\left[\delta_{m,-1}+\delta_{m,1}\right]\int_0^R\rho d\rho\left[J_{m-1}(\nu\rho)+J_{m+1}(\nu\rho)\right]\right\}=$$
$$=i\frac{\sqrt{\mu_0}}{2}\frac{\sqrt{\nu}}{2}H(t)\delta(z)\left\{\delta_{m,-1}\int_0^R\rho d\rho\left[J_2(\nu\rho)-J_0(\nu\rho)\right]-\delta_{m,1}\int_0^R\rho d\rho\left[J_0(\nu\rho)-J_2(\nu\rho)\right]-\right.$$
$$\left.-\delta_{m,-1}\int_0^R\rho d\rho\left[J_2(\nu\rho)+J_0(\nu\rho)\right]-\delta_{m,1}\int_0^R\rho d\rho\left[J_0(\nu\rho)+J_2(\nu\rho)\right]\right\}=$$
$$=i\frac{\sqrt{\mu_0}}{2}\frac{\sqrt{\nu}}{2}H(t)\delta(z)\left\{\delta_{m,-1}\int_0^R\rho d\rho\left[J_2(\nu\rho)-J_0(\nu\rho)-J_2(\nu\rho)-J_0(\nu\rho)\right]-\right.$$
$$\left.-\delta_{m,1}\int_0^R\rho d\rho\left[J_0(\nu\rho)-J_2(\nu\rho)+J_0(\nu\rho)+J_2(\nu\rho)\right]\right\}=$$
$$=-\sqrt{\mu_0}\frac{i}{2}\sqrt{\nu}H(t)\delta(z)\left[\delta_{m,1}+\delta_{m,-1}\right]\int_0^R\rho d\rho J_0(\nu\rho)$$

$$\int zJ_0(z)dx=zJ_1(z)+C$$
$$\int_0^R\rho d\rho J_0(\nu\rho)=\frac{1}{\nu^2}\int_0^R\nu\rho d\nu\rho J_0(\nu\rho)=\frac{\nu RJ_1(\nu R)-\nu 0J_1(\nu 0)}{\nu^2}=\frac{RJ_1(\nu R)}{\nu}$$

$$j_m(z,t;\nu)=-\sqrt{\mu_0}\frac{i}{2}\sqrt{\nu}H(t)\delta(z)\left[\delta_{m,1}+\delta_{m,-1}\right]\int_0^R\rho d\rho J_0(\nu\rho)=$$
$$=-\sqrt{\mu_0}\frac{i}{2}\sqrt{\nu}H(t)\delta(z)\left[\delta_{m,1}+\delta_{m,-1}\right]\frac{RJ_1(\nu R)}{\nu}=-\sqrt{\mu_0}\frac{iR}{2\sqrt{\nu}}H(t)J_1(\nu R)\delta(z)\left[\delta_{m,1}+\delta_{m,-1}\right]$$

8.3 Продольные эволюционные коэффициенты

$$G(z',t',z,t)=\frac{c}{2}H\left(c(t-t')-|z-z'|\right)J_0\left(\nu\sqrt{c^2(t-t')^2-(z-z')^2}\right)$$

$$h_m(z,t;\nu)=-\sqrt{\mu_0}\frac{icR}{4}\frac{\delta_{m,1}+\delta_{m,-1}}{\sqrt{\nu}}J_1(\nu R)\int\limits_0^{\infty}\delta(z')\,dz'\int\limits_0^{t-\frac{z}{c}}dt' J_0\left(\nu\sqrt{c^2(t-t')^2-(z-z')^2}\right)=$$
$$=-\sqrt{\mu_0}\frac{icR}{4}\frac{\delta_{m,1}+\delta_{m,-1}}{\sqrt{\nu}}J_1(\nu R)\int\limits_0^{t-\frac{z}{c}}dt' J_0\left(\nu\sqrt{c^2(t-t')^2-(z-z')^2}\right)$$

$$\int\limits_0^{t-\frac{z}{c}}dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)=\frac{1}{\nu c}\int\limits_{\nu z}^{\nu ct}ds J_0\left(\sqrt{s^2-\nu^2z^2}\right)$$
$$\int\limits_{\nu z}^{\nu ct}ds e^{-i0s}J_0\left(\sqrt{s^2-\nu^2z^2}\right)=2iU_1\left[-i\nu(ct-z),\sqrt{\nu^2c^2t^2-\nu^2z^2}\right]$$
$$\int\limits_0^{t-\frac{z}{c}}dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)=\frac{2i}{\nu c}U_1\left[-i\nu(ct-z),\sqrt{\nu^2c^2t^2-\nu^2z^2}\right]$$

$$h_m(z,t;\nu)=-\sqrt{\mu_0}\frac{ic}{4}\frac{\delta_{m,1}+\delta_{m,-1}}{\sqrt{\nu}}J_1(\nu R)\int\limits_0^{t-\frac{z}{c}}dt' J_0\left(\nu\sqrt{c^2(t-t')^2-(z-z')^2}\right)=$$
$$=\sqrt{\mu_0}\frac{R}{2}\frac{\delta_{m,1}+\delta_{m,-1}}{\nu^{3/2}}J_1(\nu R)\,U_1\left[-i\nu(ct-z),\sqrt{\nu^2c^2t^2-\nu^2z^2}\right]=$$
$$=-\sqrt{\mu_0}\frac{R}{2}\frac{\delta_{m,1}+\delta_{m,-1}}{\nu^{3/2}}J_1(\nu R)\,U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right]$$

8.4 Поперечные электрические эволюционные коэффициенты

$$\frac{\partial}{\partial t}\int\limits_0^{t-\frac{z}{c}}dt' J_0\left(\nu\sqrt{c^2(t-t')^2-z^2}\right)=J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

$$V_m^h=-\partial_{ct}(\mu h_m)\big|_{\mu=const}=-\mu\frac{\partial h_m}{\partial t}=$$
$$=-\mu\frac{\partial}{\partial t}\left(-\sqrt{\mu_0}\frac{iR}{4}\frac{\delta_{m,1}+\delta_{m,-1}}{\sqrt{\nu}}J_1(\nu R)\int\limits_0^{t-\frac{z}{c}}dt' J_0\left(\nu\sqrt{c^2(t-t')^2-(z-z')^2}\right)\right)=$$
$$=\mu\sqrt{\mu_0}\frac{iR}{4}\frac{\delta_{m,1}+\delta_{m,-1}}{\sqrt{\nu}}J_1(\nu R)\,J_0\left(\nu\sqrt{c^2t^2-z^2}\right)$$

8.5 Поперечные магнитные эволюционные коэффициенты

$$I_m^h=\frac{1}{\mu}\frac{\partial h_m}{\partial z}=-\frac{\sqrt{\mu_0}}{\mu}\frac{R}{2}\frac{\delta_{m,1}+\delta_{m,-1}}{\nu^{3/2}}J_1(\nu R)\frac{\partial}{\partial z}U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right]$$

$$\frac{\partial}{\partial z}U_1(W_+,Z)=-\frac{i\nu}{2}\left[U_0(W_+,Z)-U_2(W_+,Z)\right]$$

$$I_m^h = \frac{1}{\mu}\frac{\partial h_m}{\partial z} = \frac{\sqrt{\mu_0}}{\mu}\frac{iR}{4}\frac{\delta_{m,1}+\delta_{m,-1}}{\sqrt{\nu}} J_1\ \nu R\ \left[U_0\ W_+, Z\ - U_2\ W_+, Z\ \right]$$

8.6 Компоненты электрического поля

$$\nabla_\perp \Psi_m \times \vec{z}_0\ = -e^{im\varphi}\left(\vec{\varphi}_0\sqrt{\nu}\frac{J_{m-1}\ \nu\rho\ - J_{m+1}\ \nu\rho}{2} - im\vec{\rho}_0\frac{J_m\ \nu\rho}{\rho\sqrt{\nu}}\right)$$

$$\vec{E} = -\sqrt[2]{\varepsilon_0}\left\{\sum_{m=-\infty}^{\infty}\int_0^\infty d\nu V_m^h\ \nabla_\perp\Psi_m \times \vec{z}_0\ + \sum_{m=-\infty}^{\infty}\int_0^\infty d\chi V_n^e \nabla_\perp \Phi_n\right\} =$$

$$= -\frac{1}{\sqrt{\varepsilon_0}}\sum_{m=-\infty}^{\infty}\int_0^\infty d\nu \mu\sqrt{\mu_0}\frac{iR}{4}\frac{\delta_{m,1}+\delta_{m,-1}}{\sqrt{\nu}} J_1\ \nu R\ J_0\ \nu\sqrt{c^2t^2 - z^2}\ \times$$

$$\times e^{im\varphi}\left(\vec{\varphi}_0\sqrt{\nu}\frac{J_{m-1}\ \nu\rho\ - J_{m+1}\ \nu\rho}{2} - im\vec{\rho}_0\frac{J_m\ \nu\rho}{\rho\sqrt{\nu}}\right) =$$

$$= -\mu\frac{iR}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\ \delta_{m,1}+\delta_{m,-1}\ \sum_{m=-\infty}^{\infty} e^{im\varphi}\int_0^\infty \frac{d\nu}{\sqrt{\nu}} J_1\ \nu R\ J_0\ \nu\sqrt{c^2t^2 - z^2}$$

$$\times\left(\vec{\varphi}_0\sqrt{\nu}\frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{2} - im\vec{\rho}_0\frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right)$$

$$E_\varphi = -\mu\frac{iR}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\ \delta_{m,1}+\delta_{m,-1}\ \sum_{m=-\infty}^{\infty} e^{im\varphi}\int_0^\infty \frac{d\nu}{\sqrt{\nu}} J_1\ \nu R\ J_0\ \nu\sqrt{c^2t^2 - z^2}\ \times$$

$$\times\sqrt{\nu}\frac{J_{m-1}\ \nu\rho\ - J_{m+1}\ \nu\rho}{2} = -\mu\frac{iR}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\left(e^{i\varphi}\int_0^\infty d\nu J_1\ \nu R\ \ J_0\ \nu\rho\ - J_2\ \nu\rho\ \ J_0\ \nu\sqrt{c^2t^2 - z^2}\ -\right.$$

$$\left. -e^{-i\varphi}\int_0^\infty d\nu J_1\ \nu R\ \ J_0\ \nu\rho\ - J_2\ \nu\rho\ \ J_0\ \nu\sqrt{c^2t^2 - z^2}\ \right) =$$

$$= \mu\frac{R}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{e^{i\varphi} - e^{-i\varphi}}{2i}\int_0^\infty d\nu J_1\ \nu R\ \ J_0\ \nu\rho\ - J_2\ \nu\rho\ \ J_0\ \nu\sqrt{c^2t^2 - z^2}\ =$$

$$= \mu\frac{R}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\sin\varphi\int_0^\infty d\nu J_1(\nu R)\left(J_0(\nu\rho) - J_2(\nu\rho)\right)J_0\left(\nu\sqrt{c^2t^2 - z^2}\right)$$

$$J_{m+1}\ z\ = \frac{2m}{z}J_m\ z\ - J_{m-1}\ z\ ;\quad J_2\ \nu\rho\ = \frac{2}{\nu\rho}J_1\ \nu\rho\ - J_0\ \nu\rho$$

$$E_\varphi = \mu\frac{R}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\sin\varphi\int_0^\infty d\nu J_1\ \nu R\ \left(J_0\ \nu\rho\ - \frac{2}{\nu\rho}J_1\ \nu\rho\ + J_0\ \nu\rho\ \right)J_0\ \nu\sqrt{c^2t^2 - z^2}\ =$$

$$= \mu\frac{R}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\sin\varphi\int_0^\infty d\nu J_1\ \nu R\ \left(J_0\ \nu\rho\ - \frac{J_1\ \nu\rho}{\nu\rho}\right)J_0\ \nu\sqrt{c^2t^2 - z^2}\ = \mu\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{R\sin\varphi}{2}\left(I_2 - \frac{I_1}{\rho}\right)$$

$$E_\rho = \mu \frac{iR}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\;\; \delta_{m,1}+\delta_{m,-1}\;\; \sum_{m=-\infty}^{\infty} e^{im\varphi}\int_0^\infty \frac{d\nu}{\sqrt{\nu}} J_1\;\; \nu R\;\; J_0\;\; \nu\sqrt{c^2t^2-z^2}\;\; im\frac{J_m\;\; \nu\rho}{\rho\sqrt{\nu}} =$$

$$= -\mu\frac{R}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{\delta_{m,1}+\delta_{m,-1}}{\rho}\sum_{m=-\infty}^{\infty} m e^{im\varphi}\int_0^\infty \frac{d\nu}{\nu} J_1\;\; \nu R\;\; J_m\;\; \nu\rho\;\; J_0\;\; \nu\sqrt{c^2t^2-z^2} \;\; =$$

$$= -\mu\frac{R}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{e^{i\varphi}+e^{-i\varphi}}{\rho}\int_0^\infty \frac{d\nu}{\nu} J_1\;\; \nu R\;\; J_1\;\; \nu\rho\;\; J_0\;\; \nu\sqrt{c^2t^2-z^2} \;\; =$$

$$= -\frac{\mu}{4}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{R\cos\varphi}{\rho}\int_0^\infty \frac{d\nu}{\nu} J_1\;\; \nu R\;\; J_1\;\; \nu\rho\;\; J_0\;\; \nu\sqrt{c^2t^2-z^2} \;\; = -\mu\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{R\cos\varphi}{2\rho} I_1$$

$$\vec{E}\;\; \rho,\phi,z,t\;\; = \mu\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{R}{2}\left[\vec{\varphi}_0 \sin\varphi\left(I_2 - \frac{I_1}{\rho}\right) - \vec{\rho}_0\cos\varphi\frac{I_1}{\rho}\right] \qquad (2.14)$$

8.7 Компоненты магнитного поля

$$\vec{H} = \sqrt[-2]{\mu_0}\left\{\sum_{m=0}^{\infty}\int_0^\infty d\nu I_m^h \nabla_\perp \Psi_m + \sum_{m=1}^{\infty}\int_0^\infty d\chi I_n^e\;\; \vec{z}_0 \times \nabla_\perp \Phi_n\right\}$$

$$\nabla_\perp \Psi_m = e^{im\varphi}\left(\vec{\rho}_0\sqrt{\nu}\frac{J_{m-1}\;\; \nu\rho\;\; - J_{m+1}\;\; \nu\rho}{2} + \vec{\varphi}_0 im\frac{J_m\;\; \nu\rho}{\sqrt{\nu}\rho}\right)$$

$$I_m^h = \frac{1}{\mu}\frac{\partial h_m}{\partial z} = \frac{\sqrt{\mu_0}}{\mu}\frac{iR}{4}\int_0^\infty d\nu\frac{\delta_{m,1}+\delta_{m,-1}}{\sqrt{\nu}} J_1\;\; \nu R\;\; \left[U_0\;\; W_+,Z\;\; - U_2\;\; W_+,Z\;\;\right]$$

$$\vec{H} = \frac{1}{\sqrt{\mu_0}}\sum_{m=-\infty}^{\infty}\int_0^\infty d\nu I_m^h \nabla_\perp\Psi_m = \frac{1}{\mu}\frac{iR}{4}\int_0^\infty d\nu\frac{\delta_{m,1}+\delta_{m,-1}}{\sqrt{\nu}} J_1\;\; \nu R\;\; \left[U_0\;\; W_+,Z\;\; - U_2\;\; W_+,Z\;\;\right]\times$$
$$\times e^{im\varphi}\left(\vec{\rho}_0\sqrt{\nu}\frac{J_{m-1}\;\; \nu\rho\;\; - J_{m+1}\;\; \nu\rho}{2} + \vec{\varphi}_0 im\frac{J_m\;\; \nu\rho}{\sqrt{\nu}\rho}\right)$$

$$H_\varphi = \frac{iR}{\mu}\frac{\delta_{m,1}+\delta_{m,-1}}{4} J_1\;\; \nu R\;\; \int_0^\infty d\nu\frac{U_0\;\; W_+,Z\;\; - U_2\;\; W_+,Z}{\sqrt{\nu}} im\frac{J_m\;\; \nu\rho}{\sqrt{\nu}\rho} =$$

$$= -\frac{R}{\mu}\frac{\delta_{m,1}+\delta_{m,-1}}{4}\int_0^\infty d\nu J_1\;\; \nu R\;\; \frac{U_0\;\; W_+,Z\;\; - U_2\;\; W_+,Z}{\sqrt{\nu}} m\frac{J_m\;\; \nu\rho}{\sqrt{\nu}\rho} =$$

$$= -\frac{R}{\mu}\frac{\delta_{m,1}+\delta_{m,-1}}{4\rho} e^{im\varphi} m\int_0^\infty \frac{d\nu}{\nu} J_1\;\; \nu R\;\; J_m\;\; \nu\rho\;\; \left[U_0\;\; W_+,Z\;\; - U_2\;\; W_+,Z\;\;\right] =$$

$$= -\frac{R}{\mu}\frac{e^{i\varphi}+e^{-i\varphi}}{4\rho}\int_0^\infty \frac{d\nu}{\nu} J_1\;\; \nu R\;\; J_1\;\; \nu\rho\;\; \left[U_0\;\; W_+,Z\;\; - U_2\;\; W_+,Z\;\;\right] =$$

$$= -\frac{R}{\mu}\frac{\cos\varphi}{2\rho}\int_0^\infty \frac{d\nu}{\nu} J_1(\nu R) J_1(\nu\rho)\left[U_0(W_+,Z) - U_2(W_+,Z)\right]$$

$$H_\rho=\frac{1}{\sqrt{\mu_0}}\sum_{m=-\infty}^{\infty}\int_0^\infty d\nu I_m^h\nabla_\perp\Psi_m=\frac{1}{\mu}\frac{iR}{4}\int_0^\infty d\nu\frac{\delta_{m,1}+\delta_{m,-1}}{\sqrt{\nu}}J_1(\nu R)\left[U_0(W_+,Z)-U_2(W_+,Z)\right]\times$$
$$\times e^{im\varphi}\sqrt{\nu}\frac{J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)}{2}=\frac{iR}{\mu}\frac{\delta_{m,1}+\delta_{m,-1}}{8}e^{im\varphi}\int_0^\infty d\nu J_1(\nu R)\begin{Bmatrix}\times U_0(W_+,Z)-U_2(W_+,Z)\\ \times\ J_{m-1}(\nu\rho)-J_{m+1}(\nu\rho)\end{Bmatrix}=$$
$$=\frac{iR}{\mu}\frac{e^{i\varphi}-e^{-i\varphi}}{8}\int_0^\infty d\nu J_1(\nu R)\left[U_0(W_+,Z)-U_2(W_+,Z)\right]\left[J_0(\nu\rho)-J_2(\nu\rho)\right]$$

$$J_2(\nu\rho)=\frac{2}{\nu\rho}J_1(\nu\rho)-J_0(\nu\rho)$$

$$H_\rho=-\frac{R}{\mu}\frac{\sin\varphi}{2}\int_0^\infty d\nu J_1(\nu R)\left[U_0(W_+,Z)-U_2(W_+,Z)\right]\left[J_0(\nu\rho)-\frac{J_1(\nu\rho)}{\nu\rho}\right]$$

$$H_\rho=-\frac{R}{\mu}\frac{\sin\varphi}{2}\left\{I_3+\frac{I_4}{\rho}\right\}$$

$$\vec{H}(\rho,\phi,z,t)=\vec{\varphi}_0\frac{R}{\mu}\frac{\cos\varphi}{2}\frac{I_4}{\rho}-\vec{\rho}_0\frac{R}{\mu}\frac{\sin\varphi}{2}\left\{I_3+\frac{I_4}{\rho}\right\}=\frac{R}{2\mu}\left\{\vec{\varphi}_0\frac{I_4\cos\varphi}{\rho}-\vec{\rho}_0\sin\varphi\left(I_3+\frac{I_4}{\rho}\right)\right\}$$

$$\vec{H}(\rho,\phi,z,t)=\frac{R}{2\mu}\left\{\vec{\varphi}_0\frac{I_4\cos\varphi}{\rho}-\vec{\rho}_0\sin\varphi\left(I_3+\frac{I_4}{\rho}\right)\right\}\qquad(2.15)$$

$$h_m=-\sqrt{\mu_0}\frac{R}{2}\frac{\delta_{m,1}+\delta_{m,-1}}{\nu^{3/2}}J_1(\nu R)U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right]$$

$$\Psi_m(\nu)=\frac{J_m(\nu\rho)}{\sqrt{\nu}}e^{im\varphi}$$

$$H_z(\rho,\phi,z,t)=-\sqrt{\mu_0}\sum_{m=0}^{\infty}\int_0^\infty\nu^2d\nu h_m(z,t;\nu)\Psi_m(\rho,\phi;\nu)=$$
$$=-\frac{R}{2}\int_0^\infty\nu^2\frac{\delta_{m,1}+\delta_{m,-1}}{\nu^{3/2}}J_1(\nu R)\frac{J_m(\nu\rho)}{\sqrt{\nu}}e^{im\varphi}U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right]d\nu=$$
$$=-\frac{R}{2}(\delta_{m,1}+\delta_{m,-1})e^{im\varphi}\int_0^\infty d\nu J_1(\nu R)J_m(\nu\rho)U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right]=$$
$$=-iR\frac{e^{i\varphi}-e^{-i\varphi}}{2i}\int_0^\infty d\nu J_1(\nu R)J_1(\nu\rho)U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right]=$$
$$=-iR\sin\varphi\int_0^\infty d\nu J_1(\nu R)J_1(\nu\rho)U_1\left[i\nu(ct-z)\nu\sqrt{c^2t^2-z^2}\right]$$

$$H_z(\rho,\phi,z,t)=-iR\sin\varphi\int_0^\infty d\nu J_1(\nu R)J_1(\nu\rho)U_1\left[i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right]\qquad(2.16)$$

8.8 Декартовы координаты

9 Нелинейная поправка для задачи плоского диска

$$\vec{\mathbf{J}} = \frac{\partial}{\partial t}\vec{\mathbf{P}}' \;\; \vec{\mathbf{E}} \;\; + \vec{\mathbf{J}}^{\sigma} \;\; \vec{\mathbf{E}}, \vec{\mathbf{H}} \;\; + \vec{\mathbf{J}}^{e}; \quad \vec{\mathbf{I}} = \frac{\partial}{\partial t}\vec{\mathbf{M}}' \;\; \vec{\mathbf{H}} \;\; + \vec{\mathbf{J}}^{h};$$

$$\vec{\mathbf{J}} = \frac{\partial}{\partial t}\vec{\mathbf{P}}' \;\; \vec{\mathbf{E}} \;\; + \vec{\mathbf{J}}^{\sigma} \;\; \vec{\mathbf{E}}, \vec{\mathbf{H}} \;\; + \vec{\mathbf{J}}^{e} = \frac{\partial}{\partial t}\vec{\mathbf{P}}' \;\; \vec{\mathbf{E}}$$

$$\vec{\mathbf{J}}' \;\; \vec{\mathbf{E}}, \vec{\mathbf{H}} \;\; = \begin{pmatrix} J'_{\rho} \\ J'_{\varphi} \\ J'_{z} \end{pmatrix} = \begin{pmatrix} \overrightarrow{J'} \\ J'_{z} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial t}\overrightarrow{P'} \;\; \vec{\mathbf{E}} \\ \frac{\partial}{\partial t}P_{z}' \;\; \vec{\mathbf{E}} \end{pmatrix}$$

$$P_{z}' \;\; \vec{\mathbf{E}} \;\; = ?$$

$$\vec{\mathbf{J}}^{1} = \frac{\partial}{\partial t} \;\; \chi_{3}\vec{\mathbf{E}}^{3} \;\; + \sigma\vec{E} = \chi_{3}^{E}\vec{x}_{0}\frac{\partial}{\partial t} \;\; E_{x}^{\;3} \;\; + \sigma\vec{x}_{0}E_{x}$$