

Electromagnetic Waves Generated by Line Current Pulses

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1. Introduction

Solving electromagnetic problems in which both the source current and the emanated wave have complicated, essentially nonsinusoidal structure is of paramount interest for many real-world applications including weaponry, communications, energy transportation, radar, and medicine (Harmuth, 1986; Fowler et al., 1990; Harmuth et al., 1999; Hernández-Figueroa et al., 2008). In this chapter we will focus on electromagnetic fields produced by source-current pulses moving along a straight line. The explicit space-time representation of these fields is important for investigation of man-made (Chen, 1988; Zhan & Qin, 1989) and natural (Master & Uman, 1984) travelling-wave radiators, such as line antennas and lightning strokes.

Traditional methods of solving the electromagnetic problems imply passing to the frequency domain via the temporal Fourier (Laplace) transform or introducing retarded potentials. However, the resulted spectra do not provide adequate description of the essentially finite-energy, space-time limited source-current pulses and radiated transient waves. Distributing jumps and singularities over the entire frequency domain, the spectral representations cannot depict explicitly the propagation of leading/trailing edges of the pulses and designate the electromagnetic-pulse support (the spatiotemporal region in which the wavefunction is nonzero). Using the retarded potentials is not an easy and straightforward technique even for the extremely simple cases, such as the wave generation by the rectangular current pulse — see, e.g., the analysis by Master & Uman (1983), re-examined by Rubinstein & Uman (1991). In the general case of the sources of non-trivial space-time structure, the integrand characterizing the entire field via retarded inputs can be derived relatively easily. In contrast, the definition of the limits of integration is intricate for any moving source: one must obtain these limits as solutions of a set of simultaneous inequalities, in which the observation time is bounded with the space coordinates and the radiator's parameters. The explicit solutions are thus difficult to obtain.

In the present analysis, another approach, named *incomplete separation of variables in the wave equation*, is introduced. It can be generally characterized by the following stages:

- The system of Maxwell's equations is reduced to a second-order partial differential equation (PDE) for the electric/magnetic field components, or potentials, or their derivatives.
- Then one or two spatial variables are separated using the expansions in terms of eigenfunctions or integral transforms, while one spatial variable and the temporal variable remain bounded, resulting in a second-order PDE of the hyperbolic type, which, in its turn, is solved using the Riemann method.

- Sometimes these solutions, being multiplied by known functions of the previously separated variables, result in the expressions of a clear physical meaning (nonsteady-state modes), and for these cases we have explicit description of the field in the space-time representation. When it is possible, we find the explicit solution harnessing the procedure that is inverse with respect to the separation of variables, summing up the expansions or doing the inverse integral transform. In this case the solution yields the space-time structure of the entire transient field rather than its modal expansion or integral representation.

2. Electromagnetic problem

As far as the line of the current motion is the axis of symmetry, it is convenient to consider the problem of wave generation in the cylindrical coordinate system ρ, φ, z , for which the direction of the z -axis coincides with the direction of the current-density vector, $\mathbf{j} = j\mathbf{e}_z$. Following the concept discussed above, we suppose that the space-time structure of the source corresponds to a finite-energy pulse turned on in some fixed moment of time. Introduction of the time variable in the form $\tau = ct$, where t is time reckoned from this moment and c is the speed of light, results in the conditions

$$j_z \equiv 0, \quad \mathbf{E} \equiv 0, \quad \mathbf{B} \equiv 0 \quad \text{for } \tau < 0. \quad (1)$$

Here \mathbf{E} and \mathbf{B} conventionally denote the force-related electromagnetic field vectors – the electric field intensity and the magnetic induction. The current pulse is supposed to be generated at one of the radiator's ends, $z = 0$, to travel with constant front and back velocity $v = \beta c$ ($0 < \beta \leq 1$) along the radiator and to be completely absorbed at the other end, $z = l$, as illustrated in Fig. 1.

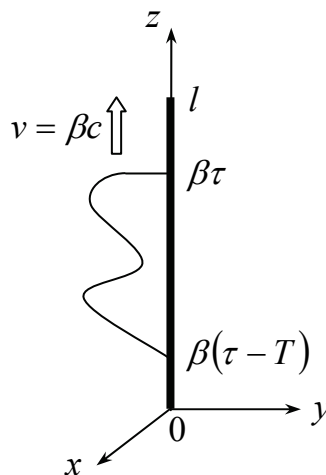


Fig. 1. Space-time structure of the source current.

Introducing, along with the finite radiator length l , the finite current pulse duration T , one can express the current density using the Dirac delta function $\delta(\rho)$ and the Heaviside step

function $h(z) = \begin{cases} 1 & \text{for } z > 0 \\ 0 & \text{for } z < 0 \end{cases}$ as

$$j(\rho, z, \tau) = \frac{\delta(\rho)}{2\pi\rho} J(z, \tau) h\left(\tau - \frac{z}{\beta}\right) h\left(\frac{z}{\beta} - \tau + T\right) h(z) h(l - z), \quad (2)$$

where $J(z, \tau)$ is an arbitrary continuous function describing the current distribution.

Bearing in mind the axial symmetry of the problem, let us seek the solution in the form of a TM wave whose components can be expressed via the Borgnis-Bromwich potential W (Whittaker, 1904; Bromwich, 1919) as

$$E_\rho = \frac{1}{\varepsilon_0 c} \frac{\partial^2 W}{\partial \rho \partial z}, \quad E_z = \frac{1}{\varepsilon_0 c} \left(-\frac{\partial^2 W}{\partial \tau^2} + \frac{\partial^2 W}{\partial z^2} \right), \quad B_\phi = -\mu_0 \frac{\partial^2 W}{\partial \rho \partial \tau}, \quad (3)$$

where ε_0 and μ_0 are the electric and magnetic constants. Substitution of representation (3) into the system of Maxwell's equations yields the scalar problem

$$\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial z^2} - \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} \right) \right) \Psi(\rho, z, \tau) = j(\rho, z, \tau), \quad (4)$$

$$\Psi \equiv 0 \quad \text{for } \tau < 0$$

with respect to the function $\Psi(\rho, z, \tau) = \frac{\partial W}{\partial \tau}$.

3. Solving algorithm

3.1 Transverse coordinate separation

Let us separate ρ by the Fourier-Bessel transform

$$\begin{pmatrix} \tilde{\Psi}(s, z, \tau) \\ \tilde{j}(s, z, \tau) \end{pmatrix} = \int_0^\infty \begin{pmatrix} \Psi(\rho, z, \tau) \\ j(\rho, z, \tau) \end{pmatrix} J_0(s\rho) \rho d\rho, \quad \begin{pmatrix} \Psi(\rho, z, \tau) \\ j(\rho, z, \tau) \end{pmatrix} = \int_0^\infty \begin{pmatrix} \tilde{\Psi}(s, z, \tau) \\ \tilde{j}(s, z, \tau) \end{pmatrix} J_0(s\rho) s ds, \quad (5)$$

(J_0 is the Bessel function of the first kind of order zero) which turns problem (4) into one for the 1D Klein-Gordon equation

$$\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial z^2} + s^2 \right) \tilde{\Psi}(s, z, \tau) = \tilde{j}(s, z, \tau), \quad (6)$$

with the initial conditions

$$\tilde{\Psi} \equiv 0 \quad \text{for } \tau < 0, \quad (7)$$

where, in accordance with representation (5),

$$\tilde{j}(s, z, \tau) = \tilde{j}(z, \tau) = \frac{1}{2\pi} J(z, \tau) h\left(\tau - \frac{z}{\beta}\right) h\left(\frac{z}{\beta} - \tau + T\right) h(z) h(l - z). \quad (8)$$

3.2 Riemann (Riemann–Volterra) method

Problem (6) can easily be solved for arbitrary source function by the Riemann (also known as Riemann–Volterra) method. Although being very powerful, this method is scarcely discussed in the textbooks; a few considerations (see, for example, Courant & Hilbert, 1989) treat one and the same case related to the first canonical form of a more general equation

$$\hat{L}(a, b, c; u) = \left(\frac{\partial^2}{\partial \xi \partial \eta} + a(\xi, \eta) \frac{\partial}{\partial \xi} + b(\xi, \eta) \frac{\partial}{\partial \eta} + c(\xi, \eta) \right) u(\xi, \eta) = f(\xi, \eta), \quad a, b, c \in C^1, \quad (9)$$

aiming to represent the solution at a point $P(\xi_0, \eta_0)$ in terms of f and the values of u and its normal derivative $\frac{\partial u}{\partial n}$ on the initial-data curve Σ as depicted in Fig. 2(a).

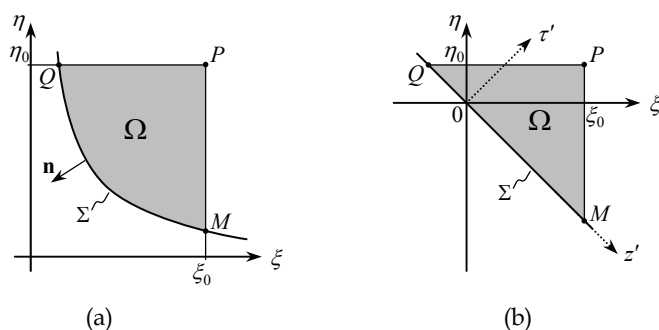


Fig. 2. Characteristic ξ, η diagrams representing the initial-data curve Σ and the integration domain Ω for the standard (a) and *ad hoc* (b) Riemann-method procedures.

As far as our objectives are limited to solving problem (6), (7), we will consider simplified *ad hoc* Riemann-method procedure involving the differential operator

$$\hat{L}(u) = \left(\frac{\partial^2}{\partial \xi \partial \eta} + s^2 \right) u(\xi, \eta), \quad s = \text{const}, \quad (10)$$

and the extension of this procedure to the case of the second canonical form of the 1D Klein-Gordon equation (6). Corresponding diagram on the ξ, η plane is represented in Fig. 2(b); the initial data are defined on the straight line $\eta = -\xi$. The procedure is based on the fact that for any two functions u and R the difference $R\hat{L}(u) - u\hat{L}(R)$ is a divergence expression

$$R\hat{L}(u) - u\hat{L}(R) = R \frac{\partial^2 u}{\partial \xi \partial \eta} - u \frac{\partial^2 R}{\partial \xi \partial \eta} = \frac{1}{2} \left(\frac{\partial A_\eta}{\partial \xi} - \frac{\partial A_\xi}{\partial \eta} \right), \quad (11)$$

$$A_\xi = u \frac{\partial R}{\partial \xi} - R \frac{\partial u}{\partial \xi}, \quad A_\eta = R \frac{\partial u}{\partial \eta} - u \frac{\partial R}{\partial \eta}.$$

Thus, integrating over the domain Ω with boundary $\partial\Omega$, one obtains by the Gauss-Ostrogradski formula

$$I_{\Omega} \stackrel{\text{def}}{=} \iint_{\Omega} [R\hat{L}(u) - u\hat{L}(R)] d\xi d\eta = \frac{1}{2} \oint_{\partial\Omega} (A_{\xi} d\xi + A_{\eta} d\eta), \quad (12)$$

where the contour integration must be performed counterclockwise. Applying formula (12) to the particular case in which:

- the integration domain Ω corresponds to that of Fig. 2(b);
- the function u is the desired solution of the inhomogeneous equation

$$\hat{L}(u) = \left(\frac{\partial^2}{\partial \xi \partial \eta} + s^2 \right) u(\xi, \eta) = f(\xi, \eta); \quad (13)$$

- the function R is the Riemann function corresponding to the linear differential operator (10) and the observation point $P(\xi_0, \eta_0)$, that is

$$\hat{L}(R) = 0, \quad R|_{QP} = R|_{MP} = 1; \quad (14)$$

we have

$$I_{\Omega} = \iint_{\Omega} [R\hat{L}(u) - u\hat{L}(R)] d\xi d\eta = \iint_{\Omega} Rf d\xi d\eta. \quad (15)$$

On the other hand

$$\begin{aligned} I_{\Omega} &= \frac{1}{2} \oint_{\partial\Omega} \left[u \frac{\partial R}{\partial \xi} - R \frac{\partial u}{\partial \xi} \right] d\xi + \left[R \frac{\partial u}{\partial \eta} - u \frac{\partial R}{\partial \eta} \right] d\eta \\ &= \frac{1}{2} \int_{QM+MP+PQ} \left[u \frac{\partial R}{\partial \xi} - R \frac{\partial u}{\partial \xi} \right] d\xi + \left[R \frac{\partial u}{\partial \eta} - u \frac{\partial R}{\partial \eta} \right] d\eta. \end{aligned} \quad (16)$$

For the contour $\partial\Omega$ of Fig. 2(b) $d\xi = 0$ on MP while $d\eta = 0$ on PQ and $d\eta = -d\xi$ on QM , which reduces the integral to

$$\begin{aligned} I_{\Omega} &= \frac{1}{2} \int_{QM} \left(u \frac{\partial R}{\partial \xi} - R \frac{\partial u}{\partial \xi} - R \frac{\partial u}{\partial \eta} + u \frac{\partial R}{\partial \eta} \right) d\xi \\ &\quad + \frac{1}{2} \int_{MP} \left(R \frac{\partial u}{\partial \eta} - u \frac{\partial R}{\partial \eta} \right) d\eta + \frac{1}{2} \int_{PQ} \left(u \frac{\partial R}{\partial \xi} - R \frac{\partial u}{\partial \xi} \right) d\xi. \end{aligned} \quad (17)$$

Noticing that

$$\frac{1}{2} \int_{MP} \left(R \frac{\partial u}{\partial \eta} - u \frac{\partial R}{\partial \eta} \right) d\eta = \frac{1}{2} \int_{MP} \left[\frac{\partial}{\partial \eta} (Ru) - 2u \frac{\partial R}{\partial \eta} \right] d\eta = \frac{1}{2} Ru \Big|_M^P - \int_{MP} u \frac{\partial R}{\partial \eta} d\eta, \quad (18)$$

$$\begin{aligned} \frac{1}{2} \int_{PQ} \left(u \frac{\partial R}{\partial \xi} - R \frac{\partial u}{\partial \xi} \right) d\xi &= \frac{1}{2} \int_{QP} \left(R \frac{\partial u}{\partial \xi} - u \frac{\partial R}{\partial \xi} \right) d\xi \\ &= \frac{1}{2} \int_{QP} \left[\frac{\partial}{\partial \xi} (Ru) - 2u \frac{\partial R}{\partial \xi} \right] d\xi = \frac{1}{2} Ru \Big|_Q^P - \int_{QP} u \frac{\partial R}{\partial \xi} d\xi \end{aligned} \quad (19)$$

and, due to the second of properties (14),

$$\frac{\partial R}{\partial \eta} \Big|_{MP} = \frac{\partial R}{\partial \xi} \Big|_{QP} = 0, \quad R|_P = 1, \quad (20)$$

one has

$$I_\Omega = -\frac{1}{2} \int_{QM} \left(R \frac{\partial u}{\partial \xi} + R \frac{\partial u}{\partial \eta} - u \frac{\partial R}{\partial \xi} - u \frac{\partial R}{\partial \eta} \right) d\xi + u|_P - \frac{1}{2} (Ru|_Q + Ru|_M). \quad (21)$$

Substituting the LHS of Eq. (21) by the RHS of Eq. (15) and solving the resulting equation with respect to $u|_P$ yield the Riemann formula corresponding to operator (10)

$$u|_P = \frac{1}{2} (Ru|_Q + Ru|_M) + \frac{1}{2} \int_{QM} \left[R \left(\frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial \eta} \right) - u \left(\frac{\partial R}{\partial \xi} + \frac{\partial R}{\partial \eta} \right) \right] d\xi + \iint_{\Omega} Rf d\xi d\eta, \quad (22)$$

with the Riemann function (Courant & Hilbert, 1989)

$$R(\xi_0, \eta_0; \xi, \eta) = J_0 \left(s \sqrt{4(\xi_0 - \xi)(\eta_0 - \eta)} \right). \quad (23)$$

To apply this result to problem (4), let us postulate that the variables ξ, ξ_0 and η, η_0 are related to the longitudinal-coordinate z, z' and time τ, τ' variables via the expressions

$$\xi = \frac{1}{2}(\tau' + z'), \quad \xi_0 = \frac{1}{2}(\tau + z), \quad \eta = \frac{1}{2}(\tau' - z'), \quad \eta_0 = \frac{1}{2}(\tau - z). \quad (24)$$

Axes corresponding to the variables z' and τ' are shown in Fig. 2(b) as dotted lines while the entire z', τ' diagram of the Riemann-method procedure is represented in Fig. 3.

In the new variables

$$R(z, \tau; z', \tau') = J_0 \left(s \sqrt{(\tau - \tau')^2 - (z - z')^2} \right), \quad (25)$$

$$d\xi d\eta = \frac{\partial(\xi, \eta)}{\partial(z', \tau')} dz' d\tau' = \frac{1}{2} dz' d\tau', \quad (26)$$

$$\frac{\partial}{\partial \xi} = \frac{\partial}{\partial z'} + \frac{\partial}{\partial \tau'}, \quad \frac{\partial}{\partial \eta} = -\frac{\partial}{\partial z'} + \frac{\partial}{\partial \tau'} \Rightarrow \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} = 2 \frac{\partial}{\partial \tau'} \quad (27)$$

and the differential operator (10) takes the second canonical form

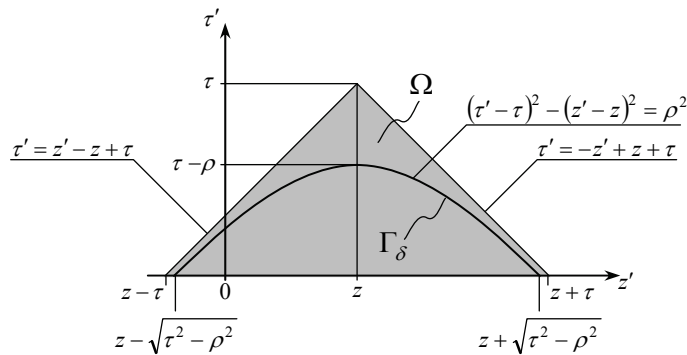


Fig. 3. A z', τ' plane diagram representing the initial 2D integration domain Ω eventually reduced to the segment of the hyperbola $(\tau' - \tau)^2 - (z' - z)^2 = \rho^2$, the support of kernel (36). while on the integration segment QM

$$\hat{L}(\tilde{u}) = \left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial z^2} + s^2 \right) \tilde{u}(z, \tau), \quad \tilde{u}(z, \tau) = u(\xi_0, \tau_0) \Big|_{\xi_0 = \frac{1}{2}(\tau+z), \tau_0 = \frac{1}{2}(\tau-z)}, \quad (28)$$

$$d\xi = \frac{1}{2} dz'. \quad (29)$$

In view of (28)-(29), the Riemann formula for the second canonical form of the 1D Klein-Gordon equation reduces to

$$\begin{aligned} \Psi(z, \tau) = & \frac{1}{2} [R(z, \tau; z - \tau, 0) \tilde{u}(z - \tau, 0) + R(z, \tau; z + \tau, 0) \tilde{u}(z + \tau, 0)] \\ & + \frac{1}{2} \int_{QM} \left[2R(z, \tau; z', 0) \frac{\partial \tilde{u}(z', \tau')}{\partial \tau'} \Big|_{\tau'=0} - 2\tilde{u}(z', 0) \frac{\partial R(z, \tau; z', \tau')}{\partial \tau'} \Big|_{\tau'=0} \right] \frac{1}{2} dz' \\ & + \frac{1}{2} \iint_{\Omega} R(z, \tau; z', \tau') f(z', \tau') dz' d\tau', \end{aligned} \quad (30)$$

whose explicit representation for the problem

$$\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial z^2} + s^2 \right) \tilde{u}(z, \tau) = f(z, \tau), \quad \tilde{u}(z, 0) = u_0(z), \quad \frac{\partial \tilde{u}}{\partial \tau}(z, 0) = u_\tau(z) \quad (31)$$

is

$$\begin{aligned} \tilde{u}(z, \tau) = & \frac{1}{2} [R(z, \tau; z - \tau, 0) u_0(z - \tau) + R(z, \tau; z + \tau, 0) u_0(z + \tau)] \\ & + \frac{1}{2} \int_{z-\tau}^{z+\tau} \left[R(z, \tau; z', 0) u_\tau(z') - u_0(z') \frac{\partial R}{\partial \tau'}(z, \tau; z', 0) \right] dz' + \frac{1}{2} \int_{z-\tau}^{z+\tau} \int_0^{\tau-|z-z'|} R(z, \tau; z', \tau') f(z', \tau') d\tau' dz', \end{aligned} \quad (32)$$

where the Riemann function is defined by Eq. (25).

3.3 Space-time domain solution

In the particular case of problem (6)-(8) with the homogeneous initial conditions, the Riemann method yields

$$\tilde{\Psi}(s, z, \tau) = \frac{1}{2} \int_{z-\tau}^{z+\tau} \int_0^{\tau-|z-z'|} J_0 \left(s \sqrt{(\tau-\tau')^2 - (z-z')^2} \right) \tilde{j}(z', \tau') d\tau' dz'. \quad (33)$$

To obtain the explicit representation of the solution to the original problem (4), let us perform the inverse Fourier-Bessel transform (5)

$$\begin{aligned} \Psi(\rho, z, \tau) &= \int_0^\infty \tilde{\Psi}(s, z, \tau) J_0(s\rho) s ds \\ &= \frac{1}{2} \int_0^\infty \left(\int_{z-\tau}^{z+\tau} \int_0^{\tau-|z-z'|} J_0 \left(s \sqrt{(\tau-\tau')^2 - (z-z')^2} \right) \tilde{j}(z', \tau') d\tau' dz' \right) J_0(s\rho) s ds. \end{aligned} \quad (34)$$

Changing the order of integration, one gets the source-to-wave integral transform

$$\Psi(\rho, z, \tau) = \frac{1}{2} \int_{z-\tau}^{z+\tau} \int_0^{\tau-|z-z'|} K(\rho, z, \tau, z', \tau') \tilde{j}(z', \tau') d\tau' dz', \quad (35)$$

where

$$K(\rho, z, \tau, z', \tau') \stackrel{\text{def}}{=} \int_0^\infty J_0 \left(s \sqrt{(\tau-\tau')^2 - (z-z')^2} \right) J_0(s\rho) s ds. \quad (36)$$

Crucial reduction of the integral wavefunction representation (35) can be achieved using the closure equation (Arfken & Weber, 2001, p. 691)

$$\int_0^\infty J_0(s\rho) J_0(s\rho') s ds = \frac{1}{\rho} \delta(\rho - \rho'), \quad (37)$$

which enables kernel (36) to be represented in the form

$$K(\rho, z, \tau, z', \tau') = \frac{1}{\rho} \delta \left(\rho - \sqrt{(\tau-\tau')^2 - (z-z')^2} \right). \quad (38)$$

A more explicit relationship can be obtained treating the kernel as a function of τ' and using the representation of the delta function with simple zeros $\{\tau_i\}$ on the real axis (Arfken & Weber, 2001, p. 87)

$$\delta(g(\tau')) = \sum_i \frac{\delta(\tau' - \tau_i)}{\left| \frac{\partial g}{\partial \tau'}(\tau_i) \right|}. \quad (39)$$

Two zeros must be taken into account

$$\tau_{1,2} = \tau \mp \sqrt{\rho^2 + (z-z')^2}, \quad (40)$$

but $\tau_2 = \tau + \sqrt{\rho^2 + (z - z')^2} > \tau$ results in the delta function whose support always lies outside the integration domain and therefore corresponds to zero input. Thus we can write

$$K(\rho, z, \tau, z', \tau') = \frac{\delta\left(\tau' - \tau + \sqrt{\rho^2 + (z - z')^2}\right)}{\sqrt{\rho^2 + (z - z')^2}}, \quad (41)$$

which, together with Eq. (8), yields

$$\Psi(\rho, z, \tau) = \frac{1}{4\pi} \int_{z-\tau}^{z+\tau} \int_0^{\tau-|z-z'|} \frac{\delta(\tau' - \tau + r')}{r'} J(z', \tau') h\left(\tau' - \frac{z'}{\beta}\right) h\left(\frac{z'}{\beta} - \tau' + T\right) h(z') h(l - z') d\tau' dz', \quad (42)$$

where

$$r' = \sqrt{\rho^2 + (z - z')^2} \quad (43)$$

denotes the distance between the observation point ρ, z and the source location $0, z'$. The integration domain, now reduced to the inlying support of the delta function, a segment of the hyperbolic curve

$$\begin{aligned} \Gamma_\delta &\stackrel{\text{def}}{=} \Omega \cap \left\{ z', \tau': (\tau' - \tau)^2 - (z' - z)^2 = \rho^2 \right\} \\ &= \left\{ z', \tau': (\tau' - \tau)^2 - (z' - z)^2 = \rho^2, (\tau' - \tau) + (z' - z) < 0, \tau' > 0 \right\}, \end{aligned} \quad (44)$$

is shown in Fig. 3.

4. Explicit representations

4.1 Preliminary considerations

Formula (42) requires further examination in order to resolve inequalities implicitly introduced by the step functions in the integrand and obtain analyzable expressions.

Although it is possible to consider one-dimensional inequalities that bound only the longitudinal variable z' — just using the property of the delta function while performing integration with respect to τ' and passing to the single-integral relation

$$\Psi(\rho, z, \tau) = \frac{1}{4\pi} \int_{z-\sqrt{\tau^2-\rho^2}}^{z+\sqrt{\tau^2-\rho^2}} \frac{J(z', \tau - r')}{r'} h\left(\tau - r' - \frac{z'}{\beta}\right) h\left(\frac{z'}{\beta} - \tau + r' + T\right) h(z') h(l - z') dz' \quad (45)$$

— a more convenient study can be done using basic expression (42) and the two-dimensional z', τ' plane diagrams, in which the inequalities bound both z' and τ' , have a linear form, and admit illustrative graphical representation.

This study results in a set of particular expressions for certain interrelations between the spatiotemporal coordinates ρ, z, τ , the radiator length l , and the current pulse duration T .

For each observation point ρ, z , the set of expression may have one of two distinct forms, depending on what information reaches the observer first: one concerning the finiteness of the current pulse or one about the radiator finiteness. The finiteness of the current pulse comes into the scene at the spatiotemporal point $\rho=0, z=0, \tau=T$ (see Fig. 4). Corresponding information is carried by the back of the electromagnetic pulse with the speed of light and arrives at the point ρ, z

$$r = \sqrt{\rho^2 + z^2} \quad (46)$$

time units after, that is, at the moment $T+r$. The stopping of the source-current motion along the z axis due to the finiteness of the radiator is first manifested at $\rho=0, z=l, \tau=l/\beta$. Related information is propagated through the distance

$$r_l = \sqrt{\rho^2 + (z-l)^2} \quad (47)$$

with the speed of light and reaches the point ρ, z at the moment $l/\beta + r_l$. From here on the source current pulse will be called *short* provided that

$$r + T < r_l + \frac{l}{\beta} \quad (48)$$

and *long* in the opposite case. This definition depends on ρ, z , so a current pulse considered to be short for one observation point may appear as long for another, and vice versa.

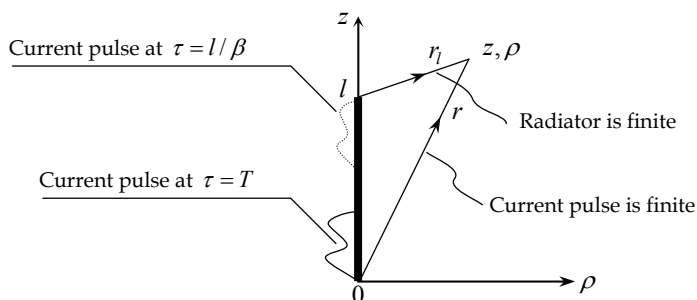


Fig. 4. On definition of the short and long pulse types.

4.2 Definition of the integration limits

The z', τ' plane diagrams for the case of a short source-current pulse are shown in Fig. 5. The step-function factors in formula (42) define the parallelogram area Ω_h within which the integrand differs from zero, and the eventual integration domain is the intersection of Ω_h and the segment of hyperbola Γ_δ defined by Eq. (44). Progression of the observation time τ unfolds the following concretization of the general formula:

- Case aS: $-\infty < \tau < r$, Fig. 5(a).

$\Gamma_\delta = \emptyset$ (for $\tau < \rho$ the hyperbola branch resides below $\tau' = 0$) or there is no intersection between Ω_h and Γ_δ , $\Omega_h \cap \Gamma_\delta = \emptyset$, so

$$\Psi(\rho, z, \tau) = \Psi_{aS}(\rho, z, \tau) = 0. \quad (49)$$

This is in a complete accord with the causality principle, as any effect of the light-speed-limited process initiated at the spatiotemporal point $\rho=0, z=0, \tau=0$ cannot reach the point ρ, z prior to $\tau=r$.

- Case bS: $r < \tau < r+T$, Fig. 5(b).

$\Omega_h \cap \Gamma_\delta$ is a segment of Γ_δ limited by $z=0$ and $z=z_0$, where

$$z_0 = \beta \frac{\tau^2 - r^2}{\tau - \beta z + \sqrt{(1 - \beta^2)\rho^2 + (z - \beta\tau)^2}} \quad (50)$$

is defined by the intersection of Γ_δ and the line $\tau' - z' / \beta = 0$.

- Case cS: $r+T < \tau < r_l + l / \beta$, Fig. 5(c).

$\Omega_h \cap \Gamma_\delta$ is a segment of Γ_δ limited by $z=z_T$ and $z=z_0$, where

$$z_T = z_0|_{\tau \mapsto \tau-T} = \beta \frac{(\tau-T)^2 - r^2}{\tau-T - \beta z + \sqrt{(1 - \beta^2)\rho^2 + [z - \beta(\tau-T)]^2}} \quad (51)$$

is defined by the intersection of Γ_δ and the line $z' / \beta - \tau' + T = 0$.

- Case dS: $r_l + l / \beta < \tau < r_l + l / \beta + T$, Fig. 5(d).

$\Omega_h \cap \Gamma_\delta$ is a segment of Γ_δ limited by $z=z_T$ and $z=l$.

- Case eS: $r_l + l / \beta + T < \tau < \infty$, Fig. 5(e).

The hyperbola branch resides above Ω_h , $\Omega_h \cap \Gamma_\delta = \emptyset$, and as in Case aS

$$\Psi(\rho, z, \tau) = \Psi_{eS}(\rho, z, \tau) = 0. \quad (52)$$

This situation relates to the epoch after passing of the electromagnetic-pulse back, corresponding to complete disappearance of the source current pulse at the spatiotemporal point $\rho=0, z=l, \tau=l / \beta + T$, which manifests itself r_l units of time later, at $\tau=r_l + l / \beta + T$.

Diagrams for a long source-current pulse are shown in Fig. 6. They correspond to the following set of cases:

- Case aL: $-\infty < \tau < r$, Fig. 6(a).

This case is identical to Case aS: $\Omega_h \cap \Gamma_\delta = \emptyset$, and

$$\Psi(\rho, z, \tau) = \Psi_{aL}(\rho, z, \tau) = 0. \quad (53)$$

- Case bL: $r < \tau < r_l + l / \beta$, Fig. 6(b).

Apart from the condition imposed on τ , this case is identical to Case bS; the limits are $z=0$ and $z=z_0$.

- Case cL: $r_l + l / \beta < \tau < r+T$, Fig. 6(c).

$\Omega_h \cap \Gamma_\delta$ is a segment of Γ_δ limited by $z=0$ and $z=l$.

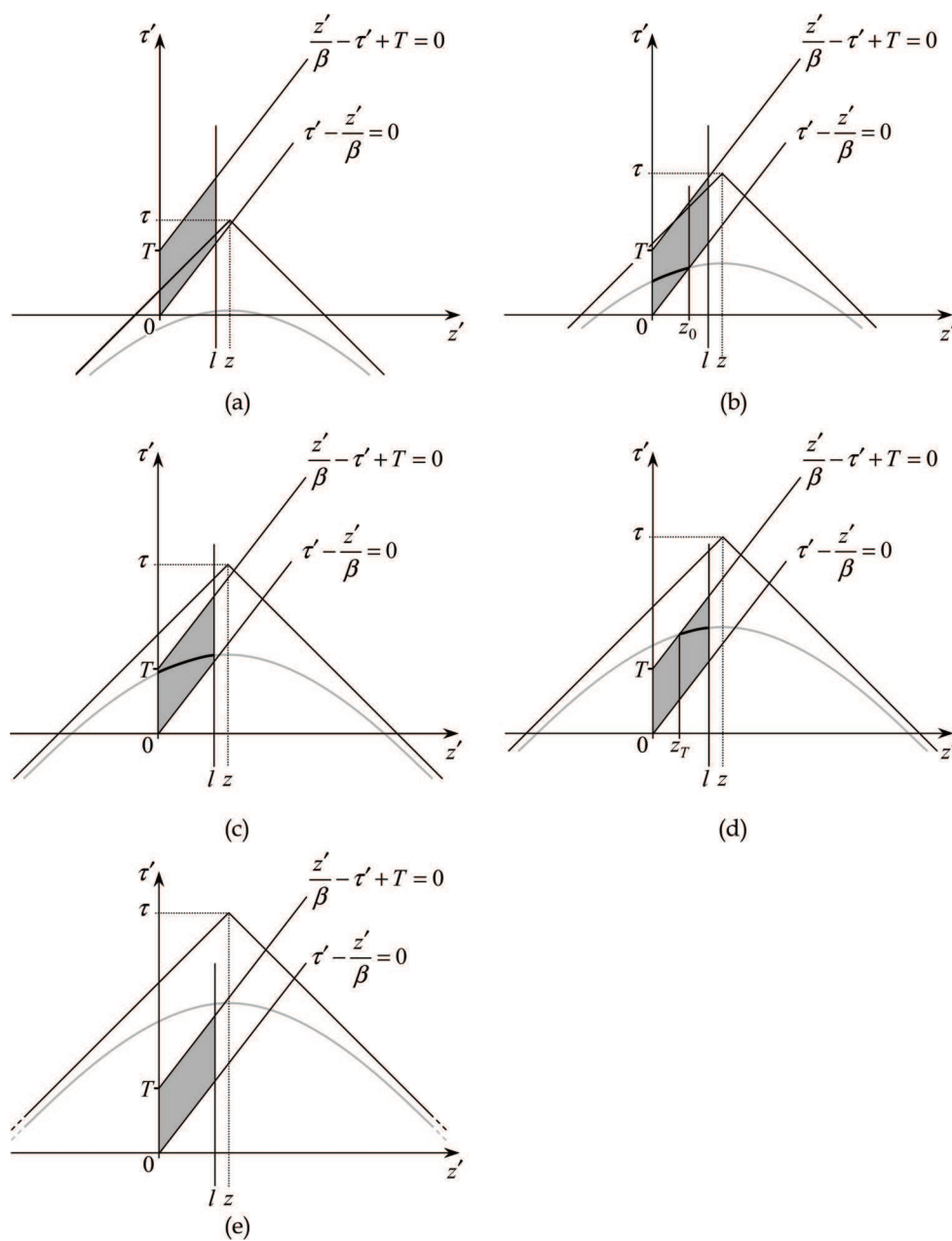


Fig. 6. Definition of the integration limits a long source-current pulse.

- Case dL: $r + T < \tau < r_l + l / \beta + T$, Fig. 6(d).

$\Omega_h \cap \Gamma_\delta$ is a segment of Γ_δ limited by $z = z_T$ and $z = l$. Apart from the condition imposed on τ , this case is identical to Case dS.

- Case eL: $r_l + l / \beta + T < \tau < \infty$, Fig. 6(e).

This case is identical to Case eS: $\Omega_h \cap \Gamma_\delta = \emptyset$ and

$$\Psi(\rho, z, \tau) = \Psi_{eL}(\rho, z, \tau) = 0. \quad (54)$$

4.3 General solutions

The results obtained for the integration limits are summarized in Table 1. With all the integration limits defined, for cases corresponding to nonvanishing solution, the explicit representation of the wavefunction takes the general form akin to (45)

$$\Psi(\rho, z, \tau) = \begin{cases} 0 & \text{case } \lambda = a, e \\ \Psi_{\lambda C}(\rho, z, \tau) = \frac{1}{4\pi} \int_{z_{1\lambda C}}^{z_{2\lambda C}} \frac{J(z', \tau - r')}{r'} dz' & \text{case } \lambda = b, c, d \end{cases}, \quad C = \begin{cases} S & \text{for short pulse} \\ L & \text{for long pulse} \end{cases} \quad (55)$$

where the case choice λ depends on the current value of time variable, see the column $\text{Cond}(\lambda C)$ of Table 1.

One can notice from the diagrams or proof by direct calculations that the piecewise solution (55) provides continuous joining, that is,

$$\begin{aligned} \Psi_{bS,L}|_{\tau=r} = \Psi_{aS,L}|_{\tau=r} = 0, \quad \Psi_{cS}|_{\tau=r+T} = \Psi_{bS}|_{\tau=r+T}, \quad \Psi_{cL}|_{\tau=r_l+\frac{l}{\beta}} = \Psi_{bL}|_{\tau=r_l+\frac{l}{\beta}}, \\ \Psi_{dS}|_{\tau=r_l+\frac{l}{\beta}} = \Psi_{cS}|_{\tau=r_l+\frac{l}{\beta}}, \quad \Psi_{dL}|_{\tau=r+T} = \Psi_{cL}|_{\tau=r+T}, \quad \Psi_{eS,L}|_{\tau=r_l+\frac{l}{\beta}} = \Psi_{dS,L}|_{\tau=r_l+\frac{l}{\beta}} = 0. \end{aligned} \quad (56)$$

Relation (55) represents the solution of the scalar problem (4). With this solution constructed, one can readily find the magnetic induction using relation (3)

$$\mathbf{B} = B \mathbf{e}_\phi, \quad B = B_\phi = -\mu_0 \frac{\partial^2 W}{\partial \rho \partial \tau} = -\mu_0 \frac{\partial \Psi}{\partial \rho} \quad (57)$$

while the definition of the electric field components in the near zone requires calculation of the Borgnis-Bromwich potential itself, which leads to integration with respect to the time variable. Due to the initial conditions for which the charge distribution must be specified, such a procedure requires consideration that is specific to physical realization of the model (wire antenna, lightning, macroscopic current pulse accompanying absorption of hard radiation by a medium, etc.) and will not be discussed in the scope of the present work. Notably, \mathbf{E} (and, consequently, the entire electromagnetic field and the electromagnetic energy density) in the far field, $r \gg l$, can be found from the known magnetic induction \mathbf{B} (Dlugosz & Trzaska 2010; Stratton, 2007).

Solutions (55), (57) describe emanation of finite transient electromagnetic pulses by line source-current pulses of arbitrary shape $J(z, \tau)$. They constitute the most practical and

illustrative concretization of general solution (35), (41) for the pulsed sources whose front and back propagate with the same constant velocity v . The z', τ' plane diagrams admit definition of the actual integration limits in (35) for arbitrary temporal dependence of the velocities of the current-pulse front and back. In this case the limiting straight lines $\tau' - z' / \beta = 0$ and $z' / \beta - \tau' + T = 0$ must be replaced by curves $z' = z_f(\tau')$ and $z' = z_b(\tau')$ characterizing the front/back motion.

Models based on infinitely long source-current pulses, $T \rightarrow \infty$, results into the set of cases aL, bL, and cL. Electromagnetic problems describing waves generated by exponentially decaying current pulses are discussed in (Utkin 2007, 2008).

Case	Condition	Limits of integration
λC	Cond(λC)	$z_{1\lambda C}, z_{2\lambda C}$
aS,L	$-\infty < \tau < r$	not applicable, $\Psi_{aS,L}(\rho, z, \tau) = 0$
bS	$r < \tau < r + T$	$z_{1bS} = z_{1bL} = 0$
bL	$r < \tau < r_l + \frac{l}{\beta}$	$z_{2bS} = z_{2bL} = z_0 = \beta \frac{\tau^2 - r^2}{\tau - \beta z + \sqrt{(1 - \beta^2)\rho^2 + (z - \beta\tau)^2}}$
cS	$r + T < \tau < r_l + \frac{l}{\beta}$	$z_{1cS} = z_T = \beta \frac{(\tau - T)^2 - r^2}{\tau - T - \beta z + \sqrt{(1 - \beta^2)\rho^2 + [z - \beta(\tau - T)]^2}}$ $z_{2cS} = z_0 = \beta \frac{\tau^2 - r^2}{\tau - \beta z + \sqrt{(1 - \beta^2)\rho^2 + (z - \beta\tau)^2}}$
cL	$r_l + \frac{l}{\beta} < \tau < r + T$	$z_{1cL} = 0$ $z_{2cL} = l$
dS	$r_l + \frac{l}{\beta} < \tau < r_l + \frac{l}{\beta} + T$	$z_{1dS} = z_{1dL} = z_T = \beta \frac{(\tau - T)^2 - r^2}{\tau - T - \beta z + \sqrt{(1 - \beta^2)\rho^2 + [z - \beta(\tau - T)]^2}}$
dL	$r + T < \tau < r_l + \frac{l}{\beta} + T$	$z_{2dS} = z_{2dL} = l$
eS,L	$r_l + \frac{l}{\beta} + T < \tau < \infty$	not applicable, $\Psi_{eS,L}(\rho, z, \tau) = 0$

Table 1. Conditions and parameters of the wavefunction representation via explicit formula (55).

5. Current pulse with high-frequency filling

Of special interest is investigation of waves launched by a pulse with high-frequency filling, which was stimulated by the problem of launching directional scalar and electromagnetic waves (missiles) as well as by results of experimental investigation of superradiation waveforms (Egorov et al., 1986). The model in question can roughly describe a number of traditional artificial as well as natural line radiators and, being characterized by two different velocities -- the phase velocity of the carrier wave and the source-pulse velocity, -- explains

characteristic features observed in the laboratory and natural conditions for waves emanated by sources with high-frequency filling: their directionality, frequency transform, and beats.

5.1 Specific solutions

The problem of wave generation by the current pulse with high-frequency filling corresponds to a particular case of the electromagnetic problem discussed in Section 2, for which the continuous function describing the current distribution can be expressed in the form

$$J(z, \tau) = \tilde{U}(z, \tau) M_{\pm}(z, \tau), \quad (58)$$

where a differentiable function $\tilde{U}(z, \tau)$ represents the source-current envelope and

$$M_{\pm} = \exp\left(ik(v_{ph}t \pm z)\right) = \exp\left(ik(\beta_{ph}\tau \pm z)\right) \quad (59)$$

the factor corresponding to the cosinusoidal (the real part) and sinusoidal (the imaginary part) modulating wave with the spatial period $2\pi/k$ and the phase velocity $v_{ph} = \beta_{ph}c$. The minus sign corresponds to propagation of the modulating wave in the positive z direction (in the same direction as the source-current pulse front and back, case of copropagation) while the plus sign describes the situation in which the modulating wave propagates in the negative z direction, opposite to the direction of propagation of the pulse front and back (case of counterpropagation).

As far as superluminal phase velocity v_{ph} is readily admissible for a much wider range of real-world models than the superluminal front velocity v , the values of β_{ph} are supposed to vary from 0 to infinity. To be able to pass to spatiotemporal coordinates in the frame moving with velocity v , we will assume that $v < c$, that is, $\beta < 1$. All the final results obtained in this chapter are easy to extend to the case of luminal source-current pulse taking the limit $\beta \rightarrow 1$.

To make the solutions easier to analyze, let us express $\tilde{U}(z, \tau)$ as a function of $\tau \pm z$

$$U(\tau - z, \tau + z) \stackrel{\text{def}}{=} \tilde{U}(z, \tau). \quad (60)$$

Then, substituting (58) into general solution (55), one has

$$\Psi_{\lambda C}^{(\pm)}(\rho, z, \tau) = \frac{1}{4\pi} \int_{z_{1\lambda C}}^{z_{2\lambda C}} \frac{U(\tau - r' - z', \tau - r' + z')}{r'} \exp\left(ik[\beta_{ph}(\tau - r') \pm z']\right) dz'. \quad (61)$$

In the case of copropagation of the modulating wave (M_-), changing the integration variable in representation (61) to $\zeta = \tau - r' - z'$ yields

$$\begin{aligned} \Psi_{\lambda C}^{(-)} &= \frac{1}{4\pi} \int_{\zeta_{1\lambda C}}^{\zeta_{2\lambda C}} \frac{U(\zeta, S)}{\tau - z - \zeta} \exp\left(ik[\zeta + \varepsilon S(\zeta)]\right) d\zeta \\ &= \frac{1}{4\pi iK} \int_{\zeta_{1\lambda C}}^{\zeta_{2\lambda C}} U(\zeta, S) \frac{\tau - z - \zeta}{(\tau - z - \zeta)^2 - \varepsilon \rho^2} \frac{\partial}{\partial \zeta} \exp\left(ik[\zeta + \varepsilon S(\zeta)]\right) d\zeta, \end{aligned} \quad (62)$$

where

$$\zeta_{1\lambda C} = \zeta|_{z'=z_{2\lambda C}}, \quad \zeta_{2\lambda C} = \zeta|_{z'=z_{1\lambda C}},$$

$$S(\zeta) = \tau - r' + z' = \tau + z - \frac{\rho^2}{\tau - z - \zeta}, \quad \varepsilon = \frac{\beta_{ph} - 1}{\beta_{ph} + 1}, \quad K = \frac{\beta_{ph} + 1}{2}k. \quad (63)$$

Assuming that $U(\zeta, S(\zeta))$ is a slowly varying function of ζ , $q(\zeta) = U(\zeta, S) \frac{\tau - z - \zeta}{(\tau - z - \zeta)^2 - \varepsilon \rho^2}$ is the continuous function and $q'(\zeta)$ is the absolutely integrable function, one gets by integration by parts the following estimation

$$\Psi_{\lambda C}^{(-)} = \frac{1}{4\pi i K} \left\{ U(\zeta_{2\lambda C}, S(\zeta_{2\lambda C})) \frac{\tau - z - \zeta_{2\lambda C}}{(\tau - z - \zeta_{2\lambda C})^2 - \varepsilon \rho^2} \exp(iK[\zeta_{2\lambda C} + \varepsilon S(\zeta_{2\lambda C})]) \right. \\ \left. - U(\zeta_{1\lambda C}, S(\zeta_{1\lambda C})) \frac{\tau - z - \zeta_{1\lambda C}}{(\tau - z - \zeta_{1\lambda C})^2 - \varepsilon \rho^2} \exp(iK[\zeta_{1\lambda C} + \varepsilon S(\zeta_{1\lambda C})]) \right\} + o\left(\frac{1}{K}\right). \quad (64)$$

Neglecting terms of order $(K)^{-1}$ and higher, we readily get the following magnetic induction approximation

$$B_{\lambda C}^{(-)} = -\mu_0 \frac{\partial \Psi_{\lambda C}^{(-)}}{\partial \rho} \\ \cong \frac{\mu_0}{4\pi} \left\{ \frac{U(\zeta_{2\lambda C}, S(\zeta_{2\lambda C}))(\tau - z - \zeta_{2\lambda C})}{(\tau - z - \zeta_{2\lambda C})^2 - \varepsilon \rho^2} \left[\frac{\partial \zeta_{2\lambda C}}{\partial \rho} + \varepsilon \frac{\partial}{\partial \rho}(S(\zeta_{2\lambda C})) \right] \exp(iK[\zeta_{2\lambda C} + \varepsilon S(\zeta_{2\lambda C})]) \right. \\ \left. + \frac{U(\zeta_{1\lambda C}, S(\zeta_{1\lambda C}))(\tau - z - \zeta_{1\lambda C})}{(\tau - z - \zeta_{1\lambda C})^2 - \varepsilon \rho^2} \left[\frac{\partial \zeta_{1\lambda C}}{\partial \rho} + \varepsilon \frac{\partial}{\partial \rho}(S(\zeta_{1\lambda C})) \right] \exp(iK[\zeta_{1\lambda C} + \varepsilon S(\zeta_{1\lambda C})]) \right\}. \quad (65)$$

Finally, using the explicit representation of $S(\zeta_{1,2\lambda C})$ and making the differentiation, Eq. (65) can be reduced to

$$B_{\lambda C}^{(-)} \cong B^{(-)}(\zeta_{2\lambda C}) - B^{(-)}(\zeta_{1\lambda C}) \quad (66)$$

where

$$B^{(-)}(\Phi) \stackrel{\text{def}}{=} \frac{\mu_0}{4\pi} U(\Phi, S(\Phi)) \left[\frac{1}{\tau - z - \Phi} \left(-\frac{\partial \Phi}{\partial \rho} \right) + \varepsilon \frac{2\rho}{(\tau - z - \Phi)^2 - \varepsilon \rho^2} \right] \exp(iK[\Phi + \varepsilon S(\Phi)]). \quad (67)$$

Application of a similar procedure in the case of counterpropagation of the modulating wave (M_+), which harness a new integration variable $\rho^2 / (z' - z - r')$, yield approximations for the wavefunction $\Psi_{\lambda C}^{(+)}$ and the magnetic induction $B_{\lambda C}^{(+)}$.

Making routine calculations for each case, one can express the final result in the form

$$B(\rho, z, \tau) = \begin{cases} \text{Case bS, bL: } B_{bS,L}^{(\pm)} = B_0^{(\pm)} - B_\beta^{(\pm)} \\ \text{Case cS: } B_{cS}^{(\pm)} = B_T^{(\pm)} - B_\beta^{(\pm)} \\ \text{Case cL: } B_{cL}^{(\pm)} = B_0^{(\pm)} - B_l^{(\pm)} \\ \text{Case dS, dL: } B_{dS,L}^{(\pm)} = B_T^{(\pm)} - B_l^{(\pm)} \\ \text{otherwise: } 0 \end{cases} \quad (68)$$

where all RHS terms are of the structure

$$B_\alpha^{(\pm)} = \frac{\mu_0}{4\pi} \frac{U_\alpha}{r_\alpha} \chi_\alpha^{(\pm)} R_\alpha^{(\pm)}, \quad \alpha = 0, \beta, T, l. \quad (69)$$

The part depending on the source-current shape is represented by the factors U_α and r_α^{-1} , which are the same for both modulation types:

$$\begin{aligned} U_0 &= U_\beta|_{\beta=0} = U(\tau - r, \tau - r), \\ U_\beta &= U\left(\sqrt{\frac{1-\beta}{1+\beta}}(\tau_\beta - r_\beta), \sqrt{\frac{1+\beta}{1-\beta}}(\tau_\beta - r_\beta)\right), \\ U_T &= U\left(\sqrt{\frac{1-\beta}{1+\beta}}(\tau_T - r_T) + T, \sqrt{\frac{1+\beta}{1-\beta}}(\tau_T - r_T) + T\right), \\ U_l &= U(\tau - r_l - l, \tau - r_l + l), \end{aligned} \quad (70)$$

$$\begin{aligned} r_0 &= r = \sqrt{\rho^2 + z^2}, \\ \tau_\beta &= \frac{\tau - \beta z}{\sqrt{1 - \beta^2}}, \quad r_\beta = \sqrt{\rho^2 + z_\beta^2}, \quad z_\beta = \frac{z - \beta \tau}{\sqrt{1 - \beta^2}}, \\ \tau_T &= \frac{\tau - T - \beta z}{\sqrt{1 - \beta^2}}, \quad r_T = \sqrt{\rho^2 + z_T^2}, \quad z_T = \frac{z - \beta(\tau - T)}{\sqrt{1 - \beta^2}}, \\ r_l &= \sqrt{\rho^2 + z_l^2}, \quad z_l = z - l. \end{aligned} \quad (71)$$

The factor $R_\alpha^{(\pm)}$ defines the characteristic angular structure of the emanated wave due to given parameters of the wave excitation β, β_{ph}, T and l

$$R_\lambda^{(\pm)} = \begin{cases} \frac{\beta_{ph} \sin \theta_\alpha}{1 \pm \beta_{ph} \cos \theta_\alpha} & \alpha = 0, l \\ \frac{(\beta_{ph} \pm \beta) \sin \theta_\alpha}{1 \pm \beta_{ph} \beta \pm (\beta_{ph} \pm \beta) \cos \theta_\alpha} & \alpha = \beta, T, \end{cases} \quad (72)$$

where the angle θ_α is a part of spheric-coordinate representation of ρ, z_α via r_α :

$$\rho = r_\alpha \sin \theta_\alpha, \quad z_\alpha = r_\alpha \cos \theta_\alpha. \quad (73)$$

The modulation factor $\chi_\alpha^{(\pm)}$,

$$\begin{aligned}\chi_0^{(\pm)} &= \exp\left(\frac{i}{c}\omega_0(\tau - r)\right), \\ \chi_\beta^{(\pm)} &= \exp\left(\frac{i}{c}\left[\omega_0\left(1 \pm \frac{\beta}{\beta_{ph}}\right)\right]\frac{\tau_\beta - r_\beta}{\sqrt{1 - \beta^2}}\right), \\ \chi_T^{(\pm)} &= \exp\left(\frac{i}{c}\omega_0 T\right) \exp\left(\frac{i}{c}\left[\omega_0\left(1 \pm \frac{\beta}{\beta_{ph}}\right)\right]\frac{\tau_T - r_T}{\sqrt{1 - \beta^2}}\right), \\ \chi_l^{(\pm)} &= \exp(\pm ikl) \exp\left(\frac{i}{c}\omega_0(\tau - r_l)\right),\end{aligned}\quad (74)$$

where $\omega_0 = kv_{ph} = k\beta_{ph}c$ is the source modulation frequency, characterize the local high-frequency modulation, so the product $\frac{\mu_0}{4\pi} \frac{U_\alpha}{r_\alpha} R_\alpha^{(\pm)}$ may be treated as the wave envelope.

5.2 Directionality of the emanated waves

Inherent directionality of the waves produced by the current pulse with high-frequency filling in question is defined by the terms of the type $\frac{\beta_{ph} \sin \theta_\alpha}{1 \pm \beta_{ph} \cos \theta_\alpha}$ in the case of $B_0^{(\pm)}$, $B_l^{(\pm)}$

and those of the type $\frac{(\beta_{ph} \pm \beta) \sin \theta_\alpha}{1 \pm \beta_{ph} \beta \pm (\beta_{ph} \pm \beta) \cos \theta_\alpha}$ in the case of $B_\beta^{(\pm)}$, $B_T^{(\pm)}$.

In the case of the luminal phase velocity, $v_{ph} = c$, $\beta_{ph} = 1$ the factors $R_\alpha^{(\pm)}$ get much simpler forms (Borisov et al., 2005)

$$R_\alpha^{(-)} \Big|_{\beta_{ph}=1} = \frac{\sin \theta_\alpha}{1 - \cos \theta_\alpha} = \cot \frac{\theta_\alpha}{2}, \quad R_\alpha^{(+)} \Big|_{\beta_{ph}=1} = \frac{\sin \theta_\alpha}{1 + \cos \theta_\alpha} = \tan \frac{\theta_\alpha}{2}, \quad (75)$$

indicating that in the case of M_- modulation the electromagnetic wave is predominantly emanated along the direction of the source-current pulse propagation, $\theta_\alpha \cong 0$, while the case of M_+ is characterized by the opposite wave directionality, $\theta_\alpha \cong \pi$. That is, it is the direction of propagation of the modulating wave, rather than the carrier, that defines the angular localization of the emanated radiation. In spite of the apparent divergence of the terms due to the presence of the tangent/cotangent factors, their sums composing the magnetic induction remains finite everywhere except for the source domain. For example, in the case of $r + T < \tau < r_l + l/\beta$ the radiation intensity for a short source-current pulse $I_{cs}^{(\pm)}$ in the far field is given by

$$\begin{aligned}I_{cs}^{(-)} &\cong I_0 \left(\frac{U_\beta}{r_\beta}\right)^2 \cot^2 \frac{\theta_\beta}{2} \sin^2 \left(kT \frac{\beta}{1 + \beta} \sin^2 \frac{\theta_\beta}{2}\right), \\ I_{cs}^{(+)} &\cong I_0 \left(\frac{U_\beta}{r_\beta}\right)^2 \tan^2 \frac{\theta_\beta}{2} \sin^2 \left(kT \frac{\beta}{1 + \beta} \cos^2 \frac{\theta_\beta}{2}\right),\end{aligned}\quad (76)$$

while for a long pulse we have

$$\begin{aligned} I_{cl}^{(-)} &\cong I_0 \left(\frac{U_0}{r} \right)^2 \cot^2 \frac{\theta_0}{2} \sin^2 \left(kl \sin^2 \frac{\theta_0}{2} \right), \\ I_{cl}^{(+)} &\cong I_0 \left(\frac{U_0}{r} \right)^2 \tan^2 \frac{\theta_0}{2} \sin^2 \left(kl \cos^2 \frac{\theta_0}{2} \right), \end{aligned} \quad (77)$$

where $I_0 = (8\pi^2 \varepsilon_m c)^{-1}$, ε_m is the permittivity of the medium.

For subluminal and superluminal phase velocities, $\beta_{ph} \neq 1$, the angular factors of the field components $R_\alpha^{(\pm)}$ have more complicated form (72); their dependence on β_{ph} and $\theta_{0,l}$ is illustrated in Fig. 7.

As seen from the figure, for subluminal phase velocity the tendency of $B_{0,l}^{(-)}$ to be directed at $\theta_{0,l} = 0$ and $B_{0,l}^{(+)}$ at $\theta_{0,l} = \pi$, clearly manifested at $\beta_{ph} = 1$, holds down to $\beta_{ph} \approx 0.7$. For superluminal phase velocities the angular factors demonstrate lateral (towards $\theta_{0,l} = \pi/2$) shift of the propagation directionality, which is observed in the vicinity of the singularity curves, $\theta_{0,l} = \arccos \beta_{ph}^{-1}$ for $R_{0,l}^{(-)}$ and $\theta_{0,l} = \pi - \arccos \beta_{ph}^{-1}$ for $R_{0,l}^{(+)}$.

For analysis of more complicated factors $R_{\beta,T}^{(\pm)}$ let us represent them in a two-parameter form

$$R_{\beta,T}^{(\pm)} = \frac{\sin \theta_{\beta,T}}{\Theta^{(\pm)} \pm \cos \theta_{\beta,T}}, \quad \Theta^{(\pm)} \stackrel{\text{def}}{=} \frac{1 \pm \beta_{ph} \beta}{\beta_{ph} \pm \beta}. \quad (78)$$

As functions of β and β_{ph} , the parameters $\Theta^{(-)}$ and $\Theta^{(+)}$ demonstrate different behaviour:

$\Theta^{(-)}$ varies from $-\infty$ to ∞ and has the area of singularity $\beta_{ph} = \beta$ while the sign of $\Theta^{(+)}$ is always positive and the area of singularity is limited to the point $\beta_{ph} = \beta = 0$, see Fig. 8.

Note that $\lim_{\beta \rightarrow 1} \Theta^{(\pm)} = \pm 1$, $\lim_{\beta_{ph} \rightarrow +\infty} \Theta^{(\pm)} = \pm \beta$, and $\Theta^{(\pm)} \Big|_{\beta_{ph}=1} = 1$. The angular factors $R_{\beta,T}^{(\pm)}$ are finally plotted as functions of $\Theta^{(\pm)}$ and $\theta_{\beta,T}$ in Fig. 9.

5.3 Frequency transform

The modulation frequencies corresponding to the terms $B_{0,\beta,T,l}^{(\pm)}$ composing the magnetic field strength are defined by the arguments of the modulation factors $\chi_{0,\beta,T,l}^{(\pm)}$. As

$\chi_0^{(\pm)} = \exp\left(\frac{i}{c} \omega_0 (\tau - r)\right)$ and $\chi_l^{(\pm)} = \exp(\pm ikl) \exp\left(\frac{i}{c} \omega_0 (\tau - r_l)\right)$ always oscillate with the

initial modulation frequency ω_0 , the frequency transform is observed only in $B_{\beta,T}^{(\pm)}$. The frequency transform range can be found considering wave propagation in the two limiting directions: parallel ($\theta_{\beta,T} = 0$) and anti-parallel ($\theta_{\beta,T} = \pi$) to the direction of propagation of the source current. In the case $\theta_{\beta,T} = 0$ one has $\tau_{\beta,T} - r_{\beta,T} = \tau_{\beta,T} - z_{\beta,T}$ and, coming back to the initial frame of reference τ, z , one can express the modulation factors as

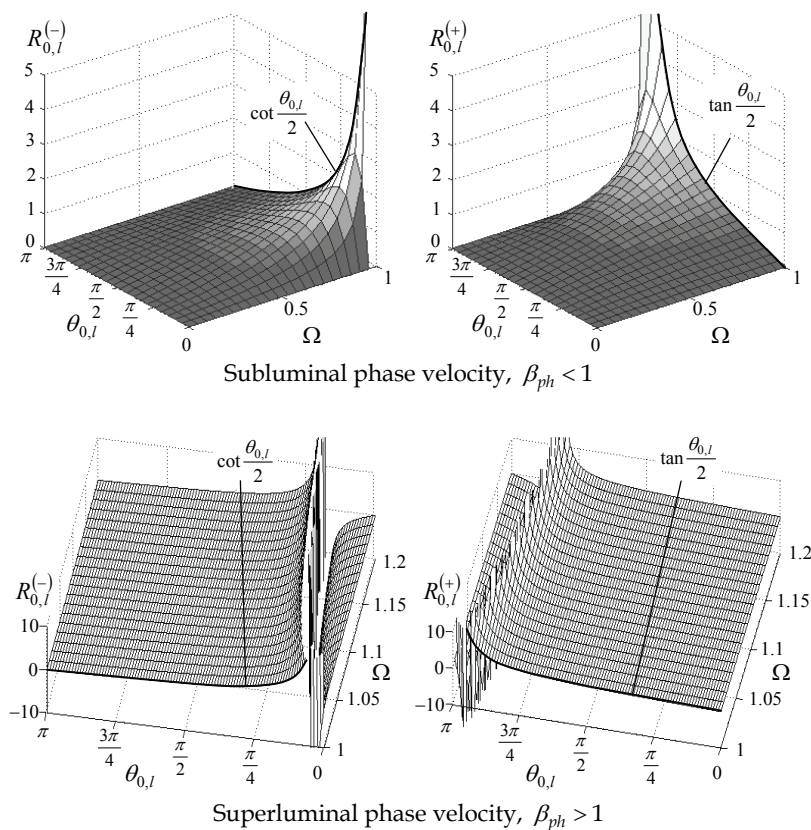


Fig. 7. Dependence of the factors $R_{0,l}^{(\pm)}$ on the dimensionless modulation phase velocity β_{ph} and the angular parameter $\theta_{0,l}$.

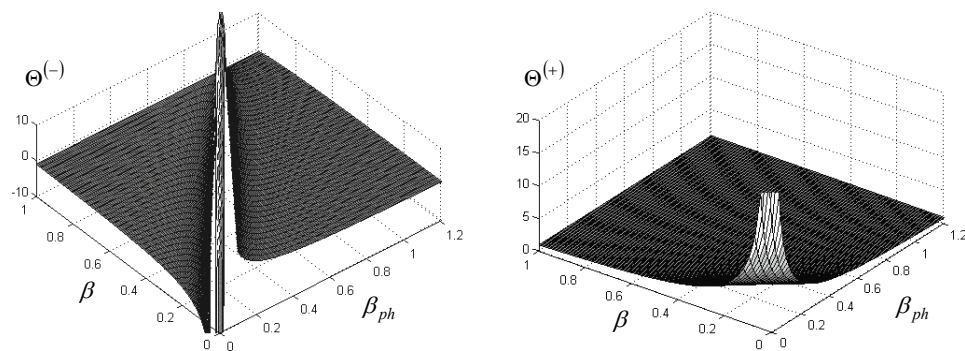


Fig. 8. Parameters $\Theta^{(\pm)}$ plotted versus the dimensionless velocities β_{ph} and β .

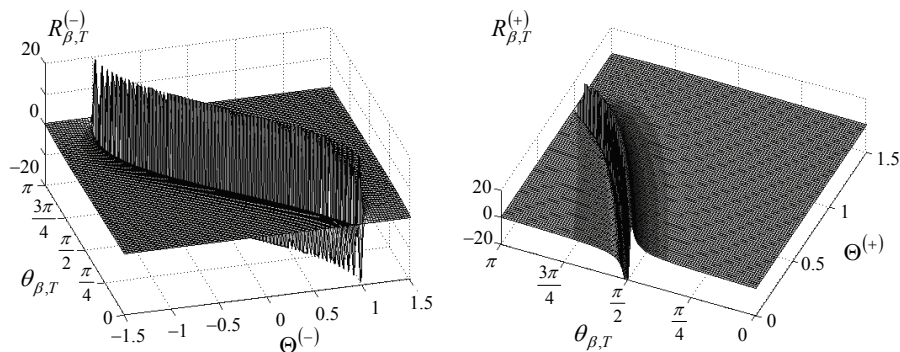


Fig. 9. The angular factors $R_{\beta,T}^{(\pm)}$ plotted as functions of $\Theta^{(\pm)}$ and $\theta_{\beta,T}$.

$$\chi_{\beta,T}^{(\pm)} = \text{const} \times \exp \left(\frac{i}{c} \left[\omega_0 \frac{|1 \pm \beta / \beta_{ph}|}{1 - \beta} \right] (\tau - r) \right), \quad \cos \theta_{\beta,T} = 1. \quad (79)$$

The other case $\theta_{\beta,T} = \pi$, corresponding to $\tau_{\beta,T} - r_{\beta,T} = \tau_{\beta,T} + z_{\beta,T}$, results in

$$\chi_{\beta,T}^{(\pm)} = \text{const} \times \exp \left(\frac{i}{c} \left[\omega_0 \frac{|1 \pm \beta / \beta_{ph}|}{1 + \beta} \right] (\tau - r) \right), \quad \cos \theta_{\beta,T} = -1. \quad (80)$$

This shows that for different directions of wave propagation θ the magnetic-induction components $B_{\beta,T}^{(\pm)}$ are subjected to the modulation-frequency transform with respect to the initial modulation frequency of the source current ω_0 : $\omega_0 \rightarrow \omega_{\beta,T}^{(\pm)}(\theta)$. The range of this transformation is defined by the inequality

$$\frac{|1 \pm \beta / \beta_{ph}|}{1 + \beta} \omega_0 = \omega_{\beta,T}^{(\pm)}(\pi) \leq \omega_{\beta,T}^{(\pm)}(\theta) \leq \omega_{\beta,T}^{(\pm)}(0) = \frac{|1 \pm \beta / \beta_{ph}|}{1 - \beta} \omega_0. \quad (81)$$

In the particular case of a nearly luminal or luminal phase velocity,

$$\beta_{ph} = 1 - \varepsilon_{ph}, \quad 0 \leq \varepsilon_{ph} \ll 1, \quad (82)$$

inequality (81) takes the form

$$\left(\frac{1 - \beta}{1 + \beta} - \varepsilon_{ph} \frac{\beta}{1 + \beta} \right) \omega_0 \leq \omega_{\beta,T}^{(-)}(\theta) \leq \left(1 - \varepsilon_{ph} \frac{\beta}{1 - \beta} \right) \omega_0 \leq \omega_0 \quad (83)$$

for $\omega_{\beta,T}^{(-)}$, describing a *red shift* at copropagation of the modulating wave, and the form

$$\omega_0 \leq \omega_0 \left(1 + \varepsilon_{ph} \frac{\beta}{1 + \beta} \right) \leq \omega_{\beta,T}^{(+)}(\theta) \leq \omega_0 \left(\frac{1 + \beta}{1 - \beta} + \varepsilon_{ph} \frac{\beta}{1 - \beta} \right), \quad (84)$$

demonstrating a *blue shift* at counterpropagation of the modulating wave, phenomena described for the particular case of luminal phase velocity ($\varepsilon_{ph}=0$) by Borisov et al. (2005).

5.4 Beats

One more phenomenon discussed by Borisov et al. (2005) for the case of luminal phase velocity is the appearance of low-frequency modulated envelopes (beats) due to the excitation of two wave components modulated with slightly different frequencies. As seen from Eqs. (68), (69) and (74) this situation may occur in Cases bS, bL and dS, dL and results in frequency subtraction $\Delta\beta_{ph} \sim |\omega_0 - \omega_{\beta,T}^{(\pm)}|$ at mixing the wave components $B_0^{(\pm)}$ and $B_\beta^{(\pm)}$ (Cases bS, bL) or $B_l^{(\pm)}$ and $B_T^{(\pm)}$ (Cases dS, dL). Remarkably, for $\beta_{ph} < 1$ one more type of beats can be observed in Cases bS, bL, cS, and dS, dL for $\beta_{ph} \sim \beta$ in a single component, $B_\beta^{(-)}$ or $B_T^{(-)}$, due to interference of the carrier and modulating waves propagating with nearly the same speed in the same direction: as seen from Eq. (74), $\lim_{\beta_{ph} \rightarrow \beta} \omega_{\beta,T}^{(-)}(\theta) = 0$.

6. Conclusion

The theoretical basics of incomplete separation of variables in the wave equation discussed in this chapter can be applied for a wide range of problems involving scalar and electromagnetic wave generation, propagation and diffraction. The use of Riemann method and the z', τ' plane diagrams provides rigorously substantiated and easy-to-follow procedures resulting in construction of analyzable *signal solutions* (Harmuth et al., 1999) of essentially nonsinusoidal nature. Concretization of the general solutions for source currents of particular shape often leads to analytical expressions. Practically important analytical solutions describing waves generated by a linear combination of exponentially decaying current pulses propagating in lossy media are constructed in (Utkin, 2008).

Although the present discussion is constrained to the line sources, its extension to the more complicated source configurations is straightforward: for instance, the multipole expansion and introduction of the Debye potential result, in the spherical coordinate system, to the Euler-Poisson-Darboux equation of known Riemann function with respect to the transient spherical-harmonic expansion coefficients of the desired wavefunction (Borisov et al., 1996). Less complex solutions were obtained by Borisov and Simonenko for sources located on moving and expanding circles (Borisov & Simonenko, 1994, 1997, 2000). An example of application of the method in the case of superluminal source pulses can be found in (Borisov, 2001).

The procedure of constructing the Riemann function $R(z, \tau; z', \tau')$ to the equation

$$\left[\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial \tau^2} + p_1(z) + p_2(\tau) \right] \Psi(z, \tau) = 0 \quad (85)$$

on the basis of known Riemann functions $R_{1,2}(z, \tau; z', \tau')$ for the reduced equations

$$\left[\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial \tau^2} + p_1(z) \right] \Psi(z, \tau) = 0 \quad \text{and} \quad \left[\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial \tau^2} + p_2(\tau) \right] \Psi(z, \tau) = 0 \quad (86)$$

via the integral formula

$$R(z, \tau; z', \tau') = R_1(z, \tau; z', \tau') + \int_{\tau-\tau'}^{z-z'} R_1(z, \zeta; z', 0) \frac{\partial}{\partial \zeta} R_2(\zeta, \tau; 0, \tau') d\zeta \quad (87)$$

proposed by Olevskii (1952) enables the applicability of the method of incomplete separation of variables to be extended to more complicated conditions. In particular, some processes of wave generation in dispersive media can be described via integral formulas (Borisov, 2002, 2003, 2008).

More specific results obtained for the current pulses with high-frequency filling demonstrate how some non-stationary phenomena can be investigated even within the framework of the high-frequency asymptotic approach. Abandoning the immediate separation of the time variable, that is, the enforcement of the temporal dependence $\exp(-i\omega_0 t)$ – a well grounded cornerstone of the physical theory of diffraction and stealth technology (Ufimtsev, 2007), but not at all a universal technique – results in direct spatiotemporal representation of the emanated electromagnetic pulses. Using this representation, one can analytically describe transformation of the frequency of the electromagnetic wave carrier with respect to the initial frequency of the source current modulation, which is manifested as the red or ultraviolet shift for the modulation factors M_- and M_+ correspondingly. In certain space-time domains, the waves of two different frequencies, fundamental and shifted, are excited, which leads to low-frequency modulated envelopes (beats). One more type of beats can be observed in the case of M_- modulation when the modulating-wave velocity comes close to the source-current pulse propagation velocity.

Spatiotemporal description of transients also gains increasing importance for localized wave generation, anti-stealth radar applications, electronic warfare and radio-frequency weaponization (Fowler et al., 1990). Formation of localized waves by source pulses with Gaussian transverse variation is discussed in (Borisov & Utkin, 1994). Having the space-time structure akin to Brittingham's focus wave modes (Brittingham, 1993), such waves can be expressed via Lommel's functions of two variables.

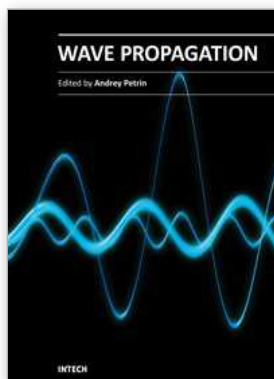
The first steps towards creating the theory of diffraction and guided propagation of transient waves were made in monographs by Harmuth (1986), Harmuth et al. (1999) and Borisov (1987). The practical needs of the ultra-wideband technology are believed to give rise to further advances in this area.

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