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ЕВОЛЮЦІЯ НЕСТАЦІОНАРНИХ ЕЛЕКТРОМАГНІТНИХ ПОЛІВ В НЕОБМЕЖЕНОМУ НЕЛІНІЙНОМУ СЕРЕДОВИЩІ

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Аннотация

Текущая работа откосится к области нелинейной нестационарной электродинамики. Задачей исследования явления выявления закономерностей, лежащих в основе нелинейных явлений, а также нахождение точного аналитического решения для класса нелинейных задач электродинамики.

В работе удалось представить метод модового базиса, который также называют методом эволюционных уравнений и его применение на задаче нестационарного излучения плоским диском. Предложен последовательный метод учета нелинейности по средствам вторичных нелинейных источников энергии и продемонстрирован на примере слабой Керровской нелинейности.

Показаны причины предпочтения методам временной области над классическими. Наряду с этим подчеркивается тесная связь частотных и временных подходов к решению задач электродинамики.

Анотація

Подана робота відноситься до області нелінійної нестаціонарної електродинаміки. Завданням дослідження є пошук закономірностей, що визначають нелінійні явища, а також побудова точного аналітичного розв'язання класу нелінійних задачі електродинаміки в часовій області.

В роботі вдалося відобразити метод модового базису, який також відомій як метод еволюційних рівнянь, а також показати його на прикладі задачі нестаціонарного випромінення пласким диском. Запропоновано послідовний метод розглядання нелінійності за рахунок вторинних джерел енергії та продемонстровано на прикладі слабкої Керрівської нелінійності.

Відмічено причини віддання переваги методам в часовій області над класичними. Поряд з цим підкреслено тісний зв'язок частотних та часових підходів до розв'язання задач електродинаміки.

Abstract

This writing belongs to transient nonlinear electrodynamics. The behaviour of nonlinear effects is under consideration. Also accurate analytical solution for external radiation problem is under research.

It was tern up well to introduce modal basis method also known as evolutionary approach to electrodynamics as well as to apply it to transient radiation of plane disk. A serial method of taking nonlinear nature into account was offered. The method implies nonlinear application as secondary source of energy. Also Kerr nonlinear medium was considered under the method as an example.

Advantages of time domain approach over classical methods were opened. Nevertheless, deep links betting time and frequency domain methods were shown.

2 LIST OF ABBREVIATIONS

UWB	Ultra-wideband
EMP	Electromagnetic pulse
EAE	Evolutionary approach to electrodynamics
TD	Time domain
WBO	Wave-Boundary Operators
TE	Transverse electric modes
FD	Frequency domain
LL	Long Transmission Line

3 INTRODUCTION

3.1 Importance and current interest

Time domain modeling in electrodynamics has been of interest since the invention of Maxwell's equations. Despite the fact that, historically, most analysis and experimentation was performed in the frequency domain, EM fields are dynamic phenomena, and even FD results contain an explicit time-harmonic variation [1]. Unfortunately, it is rare that frequency domain solutions are examined as a function of time by the simple expedient of determining the real components of the fields as the time phasor rotators. Observing the time behavior of frequency domain fields could add greatly to our physical understanding as is demonstrably the case when a time domain result is available. Although the transient response of an object can be obtained directly in the time domain, or from transformed frequency domain data, the emphasis here is on the former, so the results presented are called time domain, rather than transient responses.

H. Hertz demonstrated the existence of what we would now call radio waves in 1888 [2]. Within a few years, Crookes and others were speculating on the possibility of wireless telegraphy [3]. However now allowed for communication radio spectrum is going to overload because of data amount and user number. Ultra-wideband (UWB) electrodynamics is solving this problem. Time domain application is the most common way to provide UWB theory now [4]. Also penetrating power of UWB signal makes it significant for remote sensing.

All of that means a little transcendence of time domain electrodynamics in some topics and an importance of its development. To generalize the topic layered inhomogeneous transient nonlinear medium is under consideration. Ultra-wide spectrum η correspondents to section $0,25 < \eta < 1$ according to [5] and can be determined by next formula.

$$\eta = \frac{f_{\max} - f_{\min}}{f_{\max} + f_{\min}}.$$

3.2 Advantages and disadvantages of EAE

Necessary to note that TD approach is good in bounded spectrum of problems only. The most types of workday problems can be solved in frequency domain which is much more simpler and productive. However, presented approach can be applied to the majority of nonlinear nonstationary inhomogeneous radiation problems because of the necessity of Fourier transformation of sources.

Evolutionary approach implies the taking into account the nonlinearity as a part of the transient equation system. As opposed to frequency domain solution that is convenient for research steady-state problems, TD approach is naturally used to transient processes in electrodynamic volumes.

3.3 Direction of investigation

In electromagnetics, there are three canonical boundary value problem with given initial conditions for the electromagnetic field sought, namely: cavity problem, waveguide problem, and external problem.

The EAE developed in [6] for the cavity problem is oriented on study of the electromagnetic fields in time domain. As for the time dependent amplitudes, a system of ordinary differential equations has been obtained for them which may be linear or nonlinear as dictated by the constitutive relations for a medium within the cavity. This old and fruitful field of mathematics opens wide possibilities for fast future progress in electromagnetics in time domain.

Second canonical problem in electromagnetics is the waveguide problem that dealt with a study of electromagnetic field in a subdomain of the Euclidean background space which is infinite in one direction. The problem was considered in [6] by changing linear matrix operator determination.

The third canonical external problem of electromagnetics regards to study of electromagnetic field in the unbounded Euclidean background space was presented in [7]. It may be considered as the case of the waveguide problem when the contour of waveguide transverse section increases and tends to infinity.

Maxwell's partial differential equation of electrodynamics, formulated circa 1870, represent equations in vector quantities [1]. Frequency domain simplifies the solution by reducing time dependences to harmonic ones [8]. However, in case of time domain simplifications the scalar equations are received by modal basis eliminating the two transversal coordinate dependences [9]. This method is named modal basis method or evolutionary equation method. It implies the expansion of electric and magnetic strength vector components in the basis. Then the equation set in expansion coefficients must be solved.

Evolutionary approach contain the coefficients that have time and space variable in unseparated form because of causality principle. For stationary and homogeneous medium the method gives Klein-Gordon equations with known solutions [10] received by the Riemann function method and the method of separation of variables.

The nonlinear medium can cause the self-action and interaction types of nonlinear effects [11]. These phenomena can be taken into account in electrodynamic theory by constitutive relations with nonlinear polarization $\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)$ and magnetization $\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)$ vectors. Then it is possible present them like a component of electromagnetic field sources with two equivalent variables: current density or charge density. They are connected by equations of continuity for electric and magnetic currents.

Weak nonlinearity can be presented by nonlinear components of polarization and magnetization in form of series with linear coefficients ${\chi^E}_k$ and ${\chi^H}_k$ [source link].

$$\begin{gathered}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)=\sum_{k=2}^{\infty}{\chi^E}_k\vec{\mathbf{E}}^k\\ \vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)=\sum_{k=2}^{\infty}{\chi^H}_k\vec{\mathbf{H}}^k\end{gathered}. \tag{3.1}$$

Only third power of (3.1) summary is under consideration in the work. This kind of weak nonlinear property is named by Kerr [source link]. Also only polarization nonlinear property is under consideration to simplify the problem. Physical nature of Kerr nonlinearity is determined by rotation of polarization.

4 EVOLUTIONARY APPROACH

4.1 Statement of the problem

Initial TD problem is stated from three-dimensional Maxwell's equation set

$$\begin{cases} \nabla \times \vec{\mathbf{H}} = \dfrac{\partial}{\partial t}\vec{\mathbf{D}} + \vec{\mathbf{J}}^{\sigma} + \vec{\mathbf{J}}^{e} \\ \nabla \times \vec{\mathbf{E}} = -\dfrac{\partial}{\partial t}\vec{\mathbf{B}} - \vec{\mathbf{J}}^{h} \\ \nabla \cdot \vec{\mathbf{D}} = \rho^{\sigma} + \rho^{e} \\ \nabla \cdot \vec{\mathbf{B}} = \rho^{h} \end{cases}, \tag{4.1}$$

supplemented by constitutive relations (4.2) and equations of continuity (4.3),

$$\vec{\mathbf{D}} = \varepsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}}\left(\vec{\mathbf{E}}\right); \quad \vec{\mathbf{B}} = \mu_0 \left(\vec{\mathbf{H}} + \vec{\mathbf{M}}\left(\vec{\mathbf{H}}\right)\right), \tag{4.2}$$

$$-\frac{\partial}{\partial t}\rho^{e} = \nabla \cdot \vec{\mathbf{J}}^{e}; \quad -\frac{\partial}{\partial t}\rho^{h} = \nabla \cdot \vec{\mathbf{J}}^{h}, \tag{4.3}$$

where $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$ electrical and magnetic field strength vectors, $\vec{\mathbf{D}}$ and $\vec{\mathbf{B}}$ are electric displacement field and magnetic field, $\vec{\mathbf{P}}$ and $\vec{\mathbf{M}}$ are polarization and magnetization, ε_0, μ_0 are electric and magnetic free-space constants, $\vec{\mathbf{J}}^{e,h}$ is density of electric or magnetic current, $\rho^{e,h}$ is density of electric or magnetic charges.

Equations (4.1), (4.2) and (4.3) are valid for any coordinate system expressions. So, we can solve a radiation problem in any suitable system. Cylindrical system is convenient for the EMP problems. We are going to consider this system only.

The functions $\vec{\mathbf{E}}$, $\vec{\mathbf{H}}$, $\vec{\mathbf{D}}$, $\vec{\mathbf{B}}$, $\vec{\mathbf{J}}^{\sigma}$, $\vec{\mathbf{J}}^{e}$, $\vec{\mathbf{J}}^{h}$ are depended on position vector $\vec{R} = \vec{r} + z\vec{z}_0$ and time t.The problem is completed by initial and boundary conditions that may include given sources of currents and fields.

The final solution will be found in the class of quadratically integrable vector functions that satisfy the condition

$$\int_{t_1}^{t_2} dt \int_{z_1}^{z_2} dz \int_{0}^{2\pi} d\varphi \int_{0}^{\infty} \rho d\rho \left(\varepsilon_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{E}}^{*} + \mu_0 \vec{\mathbf{H}} \cdot \vec{\mathbf{H}}^{*}\right) < \infty . \tag{4.4}$$

The restriction means energy limit for EMP but nevertheless any real signal having finite energy can be considered in the EAE. The condition restricts the solution by such nonexistent waves like a plane wave.

It is necessary to separate the linear and nonlinear part in polarization and magnetization vectors as follows for nonlinear application

$$\vec{\mathbf{P}}\left(\vec{\mathbf{E}}\right)=\varepsilon_0\alpha\left(z,t\right)\vec{\mathbf{E}}+\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right),\quad \vec{\mathbf{M}}\left(\vec{\mathbf{H}}\right)=\chi\left(z,t\right)\vec{\mathbf{H}}+\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right), \tag{4.5}$$

where $\alpha\left(z,t\right)$ and $\chi\left(z,t\right)$ are electric and magnetic susceptibility. It gives possibility to rewrite the constitutive equations in the following form:

$$\vec{\mathbf{D}}\left(\vec{\mathbf{E}}\right)=\varepsilon_0\varepsilon\left(z,t\right)\vec{\mathbf{E}}+\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right);\quad \vec{\mathbf{B}}=\mu_0\mu\left(z,t\right)\vec{\mathbf{H}}+\mu_0\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right). \tag{4.6}$$

where $\varepsilon\left(z,t\right)=1+\alpha\left(z,t\right)$ is relative permittivity, $\mu\left(z,t\right)=1+\chi\left(z,t\right)$ is relative permeability.

Using (4.6) notations the Maxwell's equations can be rewritten to the form of

$$\begin{cases}\nabla\times\vec{\mathbf{H}}=\varepsilon_0\dfrac{\partial}{\partial t}\left(\varepsilon\left(z,t\right)\vec{\mathbf{E}}\right)+\left\{\dfrac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)+\vec{\mathbf{J}}^{\sigma}\left(\vec{\mathbf{E}},\vec{\mathbf{H}}\right)+\vec{\mathbf{J}}^{e}\right\}\\ \text{-}\left[\nabla\times\vec{\mathbf{E}}\right]=\mu_0\dfrac{\partial}{\partial t}\left(\mu\left(z,t\right)\vec{\mathbf{H}}\right)+\left\{\dfrac{\partial}{\partial t}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)+\vec{\mathbf{J}}^{h}\right\}\\ \varepsilon_0\left(\nabla\cdot\varepsilon\left(z,t\right)\vec{\mathbf{E}}\right)=-\left(\nabla\cdot\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)\right)+\rho^{\sigma}+\rho^{e}\\ \mu_0\left(\nabla\cdot\mu\left(z,t\right)\vec{\mathbf{H}}\right)=-\left(\nabla\cdot\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)\right)+\rho^{h}\end{cases}. \tag{4.7}$$

Derivatives $\dfrac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)$ and $\dfrac{\partial}{\partial t}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)$ have the same dimension as current densities as well as $\nabla\cdot\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)$ and $\nabla\cdot\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)$ have the dimension of charge densities. We introduce the equivalent densities of electric and magnetic currents (4.8) and charges (4.9) in right-hand sides of the equation by the following way:

$$\vec{\mathbf{J}}=\frac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)+\vec{\mathbf{J}}^{\sigma}\left(\vec{\mathbf{E}},\vec{\mathbf{H}}\right)+\vec{\mathbf{J}}^{e};\quad \vec{\mathbf{I}}=\frac{\partial}{\partial t}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)+\vec{\mathbf{J}}^{h}; \tag{4.8}$$

$$\varrho=-\left(\nabla\cdot\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right)\right)+\rho^{\sigma}+\rho^{e};\quad g=-\left(\nabla\cdot\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right)\right)+\rho^{h}. \tag{4.9}$$

So, the set of Maxwell's equations (4.7) acquires the form

$$\begin{cases} \nabla\times\vec{\mathbf{H}} = \varepsilon_0 \dfrac{\partial}{\partial t}\left(\varepsilon(z,t)\vec{\mathbf{E}}\right)+\vec{\mathbf{J}} \\ \text{-}\left[\nabla\times\vec{\mathbf{E}}\right] = \mu_0 \dfrac{\partial}{\partial t}\left(\mu(z,t)\vec{\mathbf{H}}\right)+\vec{\mathbf{I}} \\ \varepsilon_0\left(\nabla\cdot\varepsilon(z,t)\vec{\mathbf{E}}\right)=\varrho \\ \mu_0\left(\nabla\cdot\mu(z,t)\vec{\mathbf{H}}\right)=g \end{cases}. \tag{4.10}$$

All nonlinear properties of a medium are presented here in current and charge densities.

4.2 Longitudinal component extraction

The three-dimensional vectors can be presented as a sum of two-dimensional transversal vector and one-dimensional longitudinal vector

$$\vec{\mathbf{A}}\left(\vec{R},t\right)\equiv\vec{\mathbf{A}}\left(\vec{\mathrm{r}},\mathrm{z},t\right)=\vec{A}\left(\vec{\mathrm{r}},\mathrm{z},t\right)+\vec{z}_0 A_z\left(\vec{\mathrm{r}},\mathrm{z},t\right), \tag{4.11}$$

as well as nabla operator

$$\nabla=\nabla_\perp+\vec{z}_0\frac{\partial}{\partial z}. \tag{4.12}$$

The expressions (4.11) and (4.12) give us possibility to transform the Maxwell's equations (4.10):

$$\left[\nabla_\perp\times\vec{H}\right]+\left[\nabla_\perp\times\vec{z}_0\right]H_z+\frac{\partial}{\partial z}\left[\vec{z}_0\times\vec{H}\right]=\varepsilon_0\frac{\partial}{\partial t}\left(\varepsilon\vec{E}\right)+\vec{J}+\vec{z}_0\left(\varepsilon_0\frac{\partial}{\partial t}\left(\varepsilon E_z\right)+J_z\right);$$

$$-\left[\nabla_\perp\times\vec{E}\right]+\left[\vec{z}_0\times\nabla_\perp\right]E_z+\frac{\partial}{\partial z}\left[\vec{E}\times\vec{z}_0\right]=\mu_0\frac{\partial}{\partial t}\left(\mu\vec{H}\right)+\vec{I}+\vec{z}_0\left\{\mu_0\frac{\partial}{\partial t}\left(\mu H_z\right)+I_z\right\}.$$

Making projections of two last equations on longitudinal axis and transversal plane one can represent the Maxwell's equations in form of two separated systems containing H_z or E_z:

$$\left[\nabla_\perp\times\vec{z}_0\right]H_z=\varepsilon_0\frac{\partial}{\partial t}\left(\varepsilon\vec{E}\right)+\frac{\partial}{\partial z}\left[\vec{H}\times\vec{z}_0\right]+\vec{J}; \tag{4.13}$$

$$\mu_0\frac{\partial}{\partial z}\left\{\mu H_z\right\}=-\mu_0\mu\nabla_\perp\cdot\vec{H}+g; \tag{4.14}$$

$$\mu_0\frac{\partial}{\partial t}\left(\mu H_z\right)=\nabla_\perp\cdot\left[\vec{z}_0\times\vec{E}\right]-I_z \tag{4.15}$$

$$\left[\vec{z}_0\times\nabla_\perp\right]E_z=\mu_0\frac{\partial}{\partial t}\left(\mu\vec{H}\right)+\frac{\partial}{\partial z}\left[\vec{z}_0\times\vec{E}\right]+\vec{I} \tag{4.16}$$

$$\varepsilon_0 \frac{\partial}{\partial t}\left(\varepsilon E_z\right) = \nabla_\perp \cdot \left[\vec{H} \times \vec{z}_0\right] - J_z \tag{4.17}$$

$$\varepsilon_0 \frac{\partial}{\partial z}\left\{\varepsilon E_z\right\} = -\varepsilon_0 \varepsilon \nabla_\perp \cdot \vec{E} + \varrho \tag{4.18}$$

4.3 Maxwell's equations of second order

Actually, topical idea to simplify Maxwell's set is incising of its order. In this case we have less number of unknown variables in each equation. Theory of EAE implies it with the help of WBO.

Equation (4.13) can be simply presented as

$$\left[\nabla_\perp \times \vec{z}_0\right] H_z = \vec{F}_H$$

where $\vec{F}_H = \varepsilon_0 \frac{\partial}{\partial t}\left(\varepsilon \vec{E}\right) + \frac{\partial}{\partial z}\left[\vec{H} \times \vec{z}_0\right] + \vec{J}$. Let us first subject this equation to operators $\mu_0 \frac{\partial}{\partial z}\mu$ and $\mu_0 \frac{\partial}{\partial t}\mu$

$$\mu_0 \left[\nabla_\perp \times \vec{z}_0\right] \frac{\partial}{\partial z} \mu H_z = \mu_0 \frac{\partial}{\partial z}\left(\mu \vec{F}_H\right); \tag{4.19}$$

$$\mu_0 \left[\nabla_\perp \times \vec{z}_0\right] \frac{\partial}{\partial t} \mu H_z = \mu_0 \frac{\partial}{\partial t}\left(\mu \vec{F}_H\right); \tag{4.20}$$

It is simply to obtain next expressions by substitution from (4.15) to (4.20) and from (4.14) to (4.19) then

$$\varepsilon_0^{-1} \left[\vec{z}_0 \times \nabla_\perp\right] \nabla_\perp \cdot \vec{H} = \mu^{-1} \frac{\partial}{\partial z} (\mu \varepsilon_0^{-1} \vec{F}_H) + \left(\varepsilon_0 \mu_0 \mu\right)^{-1} \left[\vec{z}_0 \times \nabla_\perp\right] g; \tag{4.21}$$

$$\mu_0^{-1} \nabla_\perp \left[\vec{z}_0 \times \nabla_\perp\right] \vec{E} = -\frac{\partial}{\partial t} \mu \left[\vec{z}_0 \times \vec{F}_H\right] - \mu_0^{-1} \nabla_\perp I_z. \tag{4.22}$$

Later on, we will consider a part of two-component vector equations obtained namely (4.21) and (4.22), as one four-component vector equation, namely:

$$\begin{pmatrix} \varepsilon_0^{-1} \left[\vec{z}_0 \times \nabla_\perp\right] \nabla_\perp \cdot \vec{H} \\ \mu_0^{-1} \nabla_\perp \left[\vec{z}_0 \times \nabla_\perp\right] \cdot \vec{E} \end{pmatrix} = \begin{pmatrix} \mu^{-1} \partial_z \left\{\mu \varepsilon_0^{-1} \vec{F}_H\right\} + \left(\varepsilon_0 \mu_0 \mu\right)^{-1} \left[\vec{z}_0 \times \nabla_\perp g\right] \\ -\partial_t \left\{\mu \left[\vec{z}_0 \times \vec{F}_H\right]\right\} - \mu_0^{-1} \nabla_\perp I_z \end{pmatrix}. \tag{4.23}$$

Right-hand-side of equation (4.16) is depended as

$$\vec{F}_E = \mu_0 \frac{\partial}{\partial t}\left(\mu \vec{H}\right) + \frac{\partial}{\partial z}\left[\vec{z}_0 \times \vec{E}\right] + \vec{I}.$$

Exclusions of E_z form equation (4.16) subject to operators $\mu_0 \frac{\partial}{\partial z}\mu$ and $\mu_0 \frac{\partial}{\partial t}\mu$ with using of equations (4.17) and (4.18) produce one more four-component vector equation.

$$\begin{pmatrix} \varepsilon_0^{-1}\nabla_\perp\left[\nabla_\perp\times\vec{z}_0\right]\cdot\vec{H} \\ \mu_0^{-1}\left[\nabla_\perp\times\vec{z}_0\right]\nabla_\perp\cdot\vec{E} \end{pmatrix} = \begin{pmatrix} -\partial_t\left\{\varepsilon\left[\vec{F}_E\times\vec{z}_0\right]\right\}-\varepsilon_0^{-1}\nabla_\perp J_z \\ \varepsilon^{-1}\partial_z\left\{\varepsilon\mu_0^{-1}\vec{F}_E\right\}+\left(\varepsilon_0\mu_0\varepsilon\right)^{-1}\left[\nabla_\perp\varrho\times\vec{z}_0\right] \end{pmatrix}. \tag{4.24}$$

Expressions (4.23) and (4.24) supplemented by boundary conditions are transversal EM field affected by WBO. In matrix operator form it could be

$$W_H\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix} = \begin{pmatrix} \mathbf{0} & \varepsilon_0^{-1}\left[\vec{z}_0\times\nabla_\perp\right]\nabla_\perp\cdot \\ \mu_0^{-1}\nabla_\perp\left[\vec{z}_0\times\nabla_\perp\right]\cdot & \mathbf{0}\end{pmatrix}\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix} = \begin{pmatrix} \varepsilon_0^{-1}\left[\vec{z}_0\times\nabla_\perp\right]\nabla_\perp\cdot\vec{H} \\ \mu_0^{-1}\nabla_\perp\left[\vec{z}_0\times\nabla_\perp\right]\cdot\vec{E}\end{pmatrix}; \tag{4.25}$$

$$W_E\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix} = \begin{pmatrix} \mathbf{0} & \varepsilon_0^{-1}\nabla_\perp\left[\nabla_\perp\times\vec{z}_0\right]\cdot \\ \mu_0^{-1}\left[\nabla_\perp\times\vec{z}_0\right]\nabla_\perp\cdot & \mathbf{0}\end{pmatrix}\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix} = \begin{pmatrix} \varepsilon_0^{-1}\nabla_\perp\left[\nabla_\perp\times\vec{z}_0\right]\cdot\vec{H} \\ \mu_0^{-1}\left[\nabla_\perp\times\vec{z}_0\right]\nabla_\perp\cdot\vec{E}\end{pmatrix}. \tag{4.26}$$

4.4 Transversal electromagnetic linear space

Introducing the element of transversal electromagnetic linear space $\mathbf{X}$. It is four dimensional vectors of electromagnetic field in the form of

$$\mathbf{X} = \begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix}. \tag{4.27}$$

There is no infinity power source, so there is no infinity power EM wave. This nature law is encapsulated in the space scalar product definition. It is determined by energy restriction (4.4) so the expression is

$$\langle \mathbf{X}_1, \mathbf{X}_2\rangle = \frac{1}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\infty}\rho d\rho\left(\varepsilon_0\vec{E}_1\cdot\vec{E}_2^* + \mu_0\vec{H}_1\cdot\vec{H}_2^*\right). \tag{4.28}$$

This inner product defines a Hilbert space as the domain of the oparators W_H and W_E in the problem under consideration. Let us denote this functional space as $L_2^4(V)$ which means that we deal with L_2 involving four-component vectors which vary within finite domain (S*).

We are able to present WBOs in case of Transversal electromagnetic linear space:

$$W_H\mathbf{X} = \begin{cases} W_H\mathbf{X} \\ \frac{1}{\sqrt{\rho}}, \rho\to\infty \end{cases}; \tag{4.29}$$

$$W_E\mathbf{X} = \begin{cases} W_E\mathbf{X} \\ \dfrac{1}{\sqrt{\rho}}, \rho \to \infty \end{cases}. \quad (4.30)$$

4.5 Eign functions and eign numbers

It is necessary to prove self-adjoint properties of WBO to build an expansion with expected properties.

Let X_1 and X_2 are elements of transversal EM linear space $\bar{S}^*$. Self adjoint properties of W_H and W_E implies next expressions.

$$\begin{aligned} &\langle W_H X_1, X_2\rangle - \langle X_1, W_H X_2\rangle = 0 \\ &\langle W_E X_1, X_2\rangle - \langle X_1, W_E X_2\rangle = 0 \end{aligned}. \quad (4.31)$$

To prove is we calculate a pair of inner products in accordance with definition of linear product (4.28). For W_H :

$$\begin{aligned} &\langle W_H X_1, X_2\rangle - \langle X_1, W_H X_2\rangle = \frac{1}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \Big\{ \big[\vec{z}_0 \times \vec{E}_2^*\big] \nabla_\perp \nabla_\perp \vec{H}_1 + \\ &+\nabla_\perp \nabla_\perp \big[\vec{z}_0 \times \vec{E}_1\big] \vec{H}_2^* - \big[\vec{z}_0 \times \vec{E}_1\big] \nabla_\perp \nabla_\perp \vec{H}_2^* - \vec{H}_1 \nabla_\perp \nabla_\perp \big[\vec{z}_0 \times \vec{E}_2^*\big] \Big\}. \end{aligned} \quad (4.32)$$

It is available to present $\vec{E}$ and $\vec{H}$ fields by of arbitrary function Ψ in the way

$$\begin{cases} \vec{E} = \nabla_\perp \Psi \times \vec{z}_0 \\ \vec{H} = \nabla_\perp \Psi \end{cases}. \quad (4.33)$$

These feature do not generate a contradiction with EM nature. Transversal component of electric field is always perpendicular to magnetic one. The result of applying (4.33) determination to (4.32) expression is

$$\begin{aligned} &\langle W_H X_1, X_2\rangle - \langle X_1, W_H X_2\rangle = -\frac{1}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \big\{ \nabla_\perp \Psi_2^* \cdot \nabla_\perp \nabla_\perp \cdot \nabla_\perp \Psi_1 + \\ &+\nabla_\perp \Psi_2^* \cdot \nabla_\perp \nabla_\perp \cdot \nabla_\perp \Psi_1 - \nabla_\perp \Psi_1 \cdot \nabla_\perp \nabla_\perp \cdot \nabla_\perp \Psi_2^* - \nabla_\perp \Psi_1 \cdot \nabla_\perp \nabla_\perp \cdot \nabla_\perp \Psi_2^* \big\} = \\ &= -\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \big\{ \nabla_\perp \Psi_2^* \cdot \nabla_\perp \nabla_\perp \cdot \nabla_\perp \Psi_1 - \nabla_\perp \Psi_1 \cdot \nabla_\perp \nabla_\perp \cdot \nabla_\perp \Psi_2^* \big\}. \end{aligned} \quad (4.34)$$

The form of arbitrary function Ψ is one of basis functions and will deterring class of functions that can be presented by the expansion. Next form of Ψ allows to present any function according to [source link]

$$\Psi = \sum_{m=0}^{\infty} \int_0^{\infty} \xi d\xi \Psi_m; \quad \Psi_m = \frac{J_m(\nu\rho)}{\sqrt{\nu}} e^{im\varphi} . \tag{4.35}$$

Expression (4.34) gets next form after (4.35) substitution

$$\begin{gathered} \langle W_H X_1, X_2 \rangle - \langle X_1, W_H X_2 \rangle = \frac{1}{2\pi} \int_0^{\infty} \chi_2 d\chi_2 \int_0^{\infty} \chi_1 d\chi_1 \times \\ \times \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{\chi_1{}^2 - \chi_2{}^2}{\sqrt{\chi_1 \chi_2}} B_{m_1} B_{m_2} \int_0^{2\pi} d\varphi \exp\left\{ i(m_1 - m_2)\varphi \right\} \times \\ \times \int_0^{\infty} \rho d\rho \left\{ \frac{d}{d\rho} J_{m_1}(\chi_1\rho) \cdot \frac{d}{d\rho} J_{m_2}(\chi_2\rho) + \frac{m_1 m_2}{\rho^2} J_{m_1}(\chi_1\rho) J_{m_2}(\chi_2\rho) \right\}. \end{gathered} \tag{4.36}$$

It is easy to prove that (4.36) equal to zero.

$$\begin{gathered} \langle W_H X_1, X_2 \rangle - \langle X_1, W_H X_2 \rangle = \frac{1}{2\pi} \int_0^{\infty} \chi_2 d\chi_2 \int_0^{\infty} \chi_1 d\chi_1 \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{\chi_1{}^2 - \chi_2{}^2}{\sqrt{\chi_1 \chi_2}} B_{m_1} B_{m_2} 2\pi \delta_{m_1 m_2} \times \\ \times \int_0^{\infty} \rho d\rho \frac{\chi_1 \chi_2}{4} \left[J_{m_1-1}(\chi_1\rho) - J_{m_2-1}(\chi_2\rho) \right] = 0. \end{gathered} \tag{4.37}$$

The same actions can be done for W_E. Only transversal electromagnetic vector is different.

$$\begin{cases} \vec{E} = \nabla_\perp \Phi \\ \vec{H} = \vec{z}_0 \times \nabla_\perp \Phi \end{cases}, \tag{4.38}$$

where Φ is a function like to (4.35)

$$\Phi = \sum_{n=0}^{\infty} \int_0^{\infty} \xi d\xi \Phi_n; \quad \Phi_n = \frac{J_n(\chi\rho)}{\sqrt{\chi}} e^{in\varphi} . \tag{4.39}$$

Field expressions (4.33) and (4.38) are sample for future basis functions. It is possible to present them in this point:

$$Y_{\pm m} = \begin{pmatrix} \nabla_\perp \Psi_m \times \vec{z}_0 \\ \pm \nabla_\perp \Psi_m \end{pmatrix}; \quad Z_{\pm n} = \begin{pmatrix} \nabla_\perp \Phi_n \\ \pm \vec{z}_0 \times \nabla_\perp \Phi_n \end{pmatrix}. \tag{4.40}$$

Basis functions (4.40) are eign for WBO operators according to (4.37).

$$W_H Y_{\pm m} = p_m Y_{\pm m}; \quad W_E Z_{\pm n} = q_n Z_{\pm n} \tag{4.41}$$

where p_m and q_n are eign numbers from oscillation equations to Ψ_m and Φ_n:

$$\left(\Delta_\perp + \sqrt{\varepsilon_0 \mu_0} q_n \right) \Phi_n = 0; \quad \left(\Delta_\perp + \sqrt{\varepsilon_0 \mu_0} p_m \right) \Psi_m = 0 . \tag{4.42}$$

Arbitrary electromagnetic field in unbounded medium can be presented by linear product of eign functions (4.40). Note, electromagnetic field in bounded medium needs linear product of three eign functions because of double rotter field existents in bounded medium [12].

4.6 Modal expansions of EM fields

Linear product of eign functions (4.40) can constitute an expansion of transversal electromagnetic field with scalar product rule (4.28) only if orthogonal property is satisfied. Let us prove that.

$$\langle Y_m, Z_n\rangle = \frac{1}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \nabla_\perp \Phi_n^{\ *} \cdot \left[\nabla_\perp \Psi_m + \vec{z}_0\right] + \left[\vec{z}_0 \times \nabla_\perp \Phi_n^*\right] \cdot \nabla_\perp \Psi_m . \tag{4.43}$$

It is easy to prove that (4.43) can be rewrote within formula

$$\frac{1}{x}\frac{d}{d\rho} J_m(x\rho) = J_{m-1}(x\rho) - \frac{m}{x\rho} J_m(x\rho) = \frac{m}{x\rho} J_m(x\rho) - J_{m+1}(x\rho),$$

and gets form of

$$\langle Y_m, Z_n\rangle = \frac{i}{2\pi}\int_0^{2\pi} d\varphi e^{i(m-n)\varphi} \int_0^{\infty} d\rho \left[m\sqrt{\frac{\nu}{\chi}}\left(J_{n-1}(\nu\rho) - \frac{n}{\nu\rho} J_n(\nu\rho)\right) J_m(\chi\rho) + \right.$$
$$\left. + n\sqrt{\frac{\chi}{\nu}} J_n(\nu\rho)\left(\frac{m}{\chi\rho} J_m(\chi\rho) - J_{m+1}(\chi\rho)\right)\right] \tag{4.44}$$

According to formula 2.13.31 form [13]

$$\int_0^{\infty} J_n(at) J_{n-1}(bt) dt = \begin{cases} \dfrac{b^{n-1}}{a^n} & b < a \\ \dfrac{1}{2b} & b = a \\ 0 & b > a \end{cases}.$$

extension (4.44) with little simplifications is equal to zero.

So $\langle Y_m, Z_n\rangle = 0$ and are able to present an expansion for $\mathbf{X}$ with unknown coefficients $A_m(z,t,\chi)$ and $B_n(z,t,\nu)$.

$$\mathbf{X}(\vec{r}, z, t) = \sum_{m=-\infty}^{\infty}\int_0^{\infty} d\chi A_m(z,t,\chi) \mathbf{Y}_m(\vec{r}, \chi) + \sum_{n=-\infty}^{\infty}\int_0^{\infty} d\nu B_n(z,t,\nu) \mathbf{Z}_n(\vec{r}, \nu) . \tag{4.45}$$

Little manipulations to extension (4.45) by **X** vector determination (4.27) and expressions for eign functions (4.40) give final result for electromagnetic field:

$$\begin{aligned}\vec{E} &= \sqrt[-2]{\varepsilon_0}\left\{\sum_{m=0}^{\infty}\int_0^{\infty} d\nu V_m^h\left[\nabla_\perp \Psi_m \times \vec{z}_0\right] + \sum_{m=1}^{\infty}\int_0^{\infty} d\chi V_n^e \nabla_\perp \Phi_n\right\} \\ \vec{H} &= \sqrt[-2]{\mu_0}\left\{\sum_{m=0}^{\infty}\int_0^{\infty} d\nu I_m^h \nabla_\perp \Psi_m + \sum_{m=1}^{\infty}\int_0^{\infty} d\chi I_n^e\left[\vec{z}_0 \times \nabla_\perp \Phi_n\right]\right\}\end{aligned}; \qquad (4.46)$$

where coefficients $A_m(z,t,\chi)$ and $B_n(z,t,\nu)$ were renamed

$$A_m + A_{-m} = V_m^h; \quad B_n + B_{-n} = V_n^e; \quad A_m - A_{-m} = I_m^h; \quad B_n - B_{-n} = I_n^e. \qquad (4.47)$$

It is comfortable to have simplifications for some expressions from electric and magnetic field expansions in view of Lame coefficients for cylindrical medium.

$$\nabla_\perp \Psi_m = e^{im\varphi}\left(\vec{\rho}_0 \sqrt{\nu}\,\frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{2} + im\vec{\varphi}_0 \frac{J_m(\nu\rho)}{\sqrt{\nu}\rho}\right); \qquad (4.48)$$

$$\left[\nabla_\perp \Psi_m \times \vec{z}_0\right] = -e^{im\varphi}\left(\vec{\varphi}_0 \sqrt{\nu}\,\frac{J_{m-1}(\nu\rho) - J_{m+1}(\nu\rho)}{2} - im\vec{\rho}_0 \frac{J_m(\nu\rho)}{\rho\sqrt{\nu}}\right); \qquad (4.49)$$

Получение продольных компонент

$$\begin{aligned}E_z(\rho,\phi,z,t) &= \sqrt[-2]{\varepsilon_0}\sum_{n=0}^{\infty}\int_0^{\infty}\chi^2 d\chi e_n(z,t;\chi)\Phi_n(\rho,\phi;\chi) \\ H_z(\rho,\phi,z,t) &= \sqrt[-2]{\mu_0}\sum_{m=0}^{\infty}\int_0^{\infty}\nu^2 d\nu h_m(z,t;\nu)\Psi_m(\rho,\phi;\nu)\end{aligned}; \qquad (4.50)$$

4.7 Evolutionary equation set

There is infinity number of unknown coefficients $\left\{V_m^h, I_m^h, V_n^e, I_n^e\right\}_{m,n=0}^{\infty}$. EAE can be applied to find the solution of radiation problem with specified accuracy which is depended of m and n numbers. Extracted Maxwell's equations (4.13)-(4.18) will get a form of evolutionary equations after substitution of (4.46) and (4.50) expansions to them with using of next formulas for simplification:

$$\frac{\chi\chi'}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty}\rho d\rho \Phi_n(\chi)\Phi_{n'}^{*}(\chi') = \delta_{nn'}\delta(\chi - \chi');$$

$$\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \nabla_\perp \Phi_n(\chi)\cdot\nabla_\perp \Phi_{n'}^*(\chi') = \delta_{nn'}\delta(\chi-\chi');$$

$$\frac{1}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \nabla_\perp \Phi_n(\chi)\cdot\left[\nabla_\perp \Psi_{m'}^*(\nu')\times\vec{z}_0\right]+\left[\vec{z}_0\times\nabla_\perp\Phi_n(\chi)\right]\cdot\nabla_\perp\Psi_{m'}^*(\nu') = 0;$$

$$\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left[\nabla_\perp \Psi_m(\nu)\times\vec{z}_0\right]\cdot\left[\nabla_\perp \Psi_{m'}^*(\nu')\times\vec{z}_0\right] = \delta_{mm'}\delta(\nu-\nu').$$

Finally, first two evolutionary equations can be received from (4.14) and (4.16)

$$\partial_z\{\mu h_m\} = \mu I_m^h + \sqrt[-2]{\mu_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \Psi_m^*(\nu)\, g; \tag{4.51}$$

$$\partial_{ct}\{\mu h_m\} = -V_m^h - \sqrt{\varepsilon_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \Psi_m^*(\nu)\, I_z. \tag{4.52}$$

Next two equations can be obtained from (4.13) by …

$$-\partial_{ct}\left\{\varepsilon V_m^h\right\} - \partial_z I_m^h + \nu^2 h_m = \sqrt{\mu_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left[\vec{z}_0\times\vec{J}\right]\cdot\nabla_\perp\Psi_m^*(\nu); \tag{4.53}$$

$$\partial_{ct}\left\{\varepsilon V_n^e\right\} + \partial_z I_n^e = -\sqrt{\mu_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \vec{J}\cdot\nabla_\perp\Phi_n^*(\chi). \tag{4.54}$$

The same procedure leads to following results from (4.17) and (4.18):

$$\partial_{ct}\{\varepsilon e_n\} = -I_n^e - \sqrt{\mu_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \Phi_n^*(\chi)\, J_z; \tag{4.55}$$

$$\partial_z\{\varepsilon e_n\} = \varepsilon V_n^e + \sqrt[-2]{\varepsilon_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \Phi_n^*(\chi)\varrho. \tag{4.56}$$

and to these from (5):

$$-\partial_{ct}\left\{\mu I_n^e\right\} - \partial_z V_n^e + \chi^2 e_n = \sqrt{\varepsilon_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left[\vec{I}\times\vec{z}_0\right]\cdot\nabla_\perp\Phi_n^*(\chi); \tag{4.57}$$

$$\partial_{ct}\left\{\mu I_m^h\right\} + \partial_z V_m^h = -\sqrt{\varepsilon_0}\,\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \vec{I}\cdot\nabla_\perp\Psi_m^*(\nu). \tag{4.58}$$

Equations (4.51)-(4.58) are forming overloaded evolutionary equation set. One couple of them is extra. For example it can be (4.54) and (4.55) couple. It can be received from other equations with the help of equations of continuity (4.3). Decreasing of the number of equations goes with increasing of its order. Also six-equation form is more suitable for analytics: only two equations constitute mathematical problems.

First of them goes from (4.57) and have next after substitution of I_n^e from (4.55) and of V_n^e from (4.56).

$$\begin{gathered}\partial_{ct}\left\{\mu\partial_{ct}\left(\varepsilon e_n\right)\right\}-\partial_z\left\{\varepsilon^{-1}\partial_z\left(\varepsilon e_n\right)\right\}+\chi^2 e_n=\sqrt{\varepsilon_0}\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left[\vec{I}\times\vec{z}_0\right]\cdot\nabla_\perp\Phi_n{}^*\left(\chi\right)-\\-\partial_{ct}\left\{\sqrt{\mu_0}\,\mu\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)J_z\right\}-\partial_z\left\{\sqrt[-2]{\varepsilon_0}\,\varepsilon^{-1}\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)\varrho\right\}\end{gathered}. \quad (4.59)$$

Second one is similar. I_m^h from (4.51) and V_m^h from (4.52) substitution produces

$$\begin{gathered}\partial_{ct}\left\{\varepsilon\partial_{ct}\left(\mu h_m\right)\right\}-\partial_z\left\{\mu^{-1}\partial_z\left(\mu h_m\right)\right\}+\nu^2 h_m=\sqrt{\mu_0}\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left[\vec{z}_0\times\vec{J}\right]\cdot\nabla_\perp\Psi_m{}^*\left(\nu\right)-\\-\partial_{ct}\left\{\sqrt{\varepsilon_0}\,\varepsilon\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)I_z\right\}-\partial_z\left\{\sqrt[-2]{\mu_0}\,\mu^{-1}\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)g\right\}\end{gathered} \quad (4.60)$$

Thus, let us present a set of six scalar equations to evolutionary coefficients $\left\{V_m^h,I_m^h,V_n^e,I_n^e\right\}_{m,n=0}^{\infty}$.

$$\begin{cases}\partial_{ct}\left\{\varepsilon\partial_{ct}\left(\mu h_m\right)\right\}-\partial_z\left\{\mu^{-1}\partial_z\left(\mu h_m\right)\right\}+\nu^2 h_m=\sqrt{\mu_0}\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left[\vec{z}_0\times\vec{J}\right]\cdot\nabla_\perp\Psi_m{}^*\left(\nu\right)-\\ \qquad -\partial_{ct}\left\{\sqrt{\varepsilon_0}\,\varepsilon\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)I_z\right\}-\partial_z\left\{\sqrt[-2]{\mu_0}\,\mu^{-1}\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)g\right\}\\ \partial_{ct}\left\{\mu\partial_{ct}\left(\varepsilon e_n\right)\right\}-\partial_z\left\{\varepsilon^{-1}\partial_z\left(\varepsilon e_n\right)\right\}+\chi^2 e_n=\sqrt{\varepsilon_0}\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left[\vec{I}\times\vec{z}_0\right]\cdot\nabla_\perp\Phi_n{}^*\left(\chi\right)-\\ \qquad -\partial_{ct}\left\{\sqrt{\mu_0}\,\mu\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)J_z\right\}-\partial_z\left\{\sqrt[-2]{\varepsilon_0}\,\varepsilon^{-1}\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)\varrho\right\}\\ I_n^e=-\partial_{ct}\left\{\varepsilon e_n\right\}-\sqrt{\mu_0}\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)J_z\\ V_n^e=\varepsilon^{-1}\partial_z\left\{\varepsilon e_n\right\}-\sqrt[-2]{\varepsilon_0}\,\varepsilon^{-1}\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)\varrho\\ I_m^h=\mu^{-1}\partial_z\left\{\mu h_m\right\}-\sqrt[-2]{\mu_0}\,\mu^{-1}\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)g\\ V_m^h=-\partial_{ct}\left\{\mu h_m\right\}-\sqrt{\varepsilon_0}\,\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)I_z\end{cases}. \quad (4.61)$$

5 LINEAR SOLUTION

5.1 Transient TE problem

Radiation problem of EMP and its propagation through nonlinear medium can be considered in TD by the EAE. The field can be definitely determined by building of field expansions (4.46), (4.50) and solving set (4.61) to expansion coefficients according to the approach. We can specify the initialy conditions and field sources to solve evolutionary equation set.

Рассматриваем волну TE

Почему выбран именно этот случай?

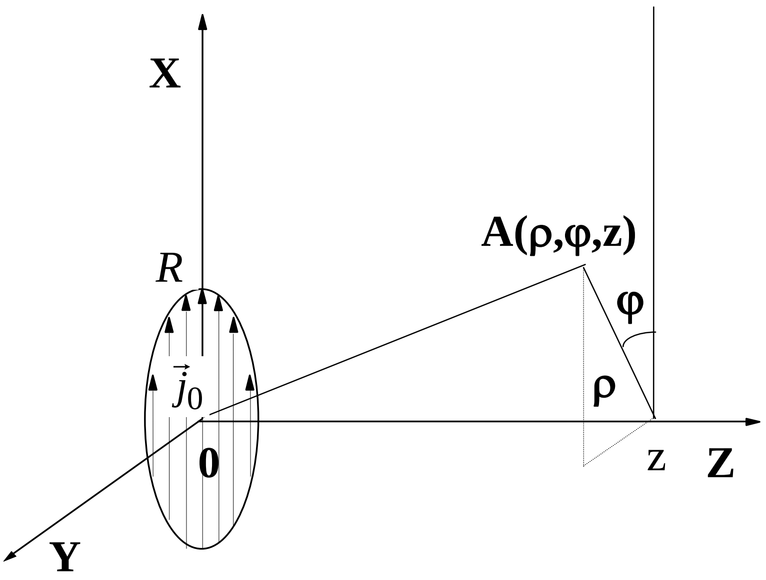


Figure 5.1. Current sources

Let us consider source of current with the form of plane disk. Disk radius is equal to R. Current $\vec{j}_0$ appears on one side of circle, streams in the plane of disk to the opposite side and dies there (Figure 5.1.). The center of the disk is located at the begging on cylindrical coordinate system $\{\vec{\rho}_0, \vec{z}_0, \vec{\varphi}_0\}$ perpendicular to $\vec{z}_0$. Disk radiated in both halfspaces, but only $z > 0$ is under consideration.

$\vec{j}_0$ is not stationary or harmonic value, its time dependence describes the moment of plugging in of a generator, so the problem of transient electromagnetic field takes place. Such behavior can be described by Heaviside's step function $H(t)$ [14]. Sure is unrealizable dependency as well as circle current density on Figure 5.1. and these facts must be taking into account – the problem is theoretical. According to following specifications

$$\vec{j}_0 = \vec{x}_0 H(t)\delta(z)\big(H(\rho) - H(\rho - R)\big). \tag{5.1}$$

Evolutionary equations consist components of current expansions, so current $\vec{j}_0$ must be presented in the expansion form. Откуда взята формула?

$$j_m(z,t;\nu) = \frac{\sqrt{\mu_0}}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho\, \vec{j}_0 \left[\nabla_\perp \Psi_m^{\ *} \times \vec{z}_0\right] = -\frac{\sqrt{\mu_0}}{2\pi}\frac{iR}{\sqrt{\chi}}\delta_{m,1} J_1(\nu R)\delta(z) H(t). \tag{5.2}$$

The medium is homogeneous and stationary for simplification. It means no dependence on time or space coordinates for dielectric permeability ε and permittivity μ from polarization and magnetization vectors (4.5) definitions: Константы или единицы?

$$\varepsilon = \varepsilon(z,t) = 1; \quad \mu = \mu(z,t) = 1.$$

First two evolutionary equations get form of Klein-Gordon equations [10], also current components $\vec{I}, I_z, J_z$ are zero ones because of form of (5.1) expression. Источники заряда должны преобразовываться в источники тока, а не исчезать?

$$\begin{cases} \partial_{ct}^2 h_m - \partial_z^2 h_m + \nu^2 h_m = \sqrt{\mu_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left[\vec{z}_0 \times \vec{J}\right] \cdot \nabla_\perp \Psi_m^{\ *}(\nu) \\ I_n^e = -\partial_{ct}\{\varepsilon e_n\}\big|_{\varepsilon=const} = -\dfrac{\varepsilon}{c}\dfrac{\partial e_n}{\partial t} \\ V_n^e = \varepsilon^{-1}\partial_z\{\varepsilon e_n\}\big|_{\varepsilon=const} = \dfrac{\partial e_n}{\partial z} \\ I_m^h = \mu^{-1}\partial_z\{\mu h_m\}\big|_{\mu=const} = \dfrac{\partial h_m}{\partial z} \\ V_m^h = -\partial_{ct}\{\mu h_m\}\big|_{\mu=const} = \dfrac{\mu}{c}\dfrac{\partial h_m}{\partial t} \end{cases} \tag{5.3}$$

5.2 Longitudinal coefficients

TE electromagnetic phenomena is under consideration. According to its determination $E_z = 0$. In case of EAE it can be only if coefficients for E_z form (4.50) are

$$e_n(z,t;\chi) = 0. \tag{5.4}$$

Second order evolutionary equations turn to Klein-Gordon equations in stationary and homogeneous medium. There is known solution by Green function for h_m coefficient.

$$\begin{aligned} h_m &= \int_0^\infty dz' \int_{f(z')}^\infty dt' G(t,z,t',z')\, j_m = \int_0^\infty dz' \int_0^\infty dt' H\left(c(t-t')-(z-z')\right) G(t,z,t',z')\, j_m = \\ &= \frac{c}{2}\int_0^\infty dz' \int_0^\infty dt' H\left(c(t-t')-(z-z')\right) J_0\left(\nu\sqrt{c^2(t-t')^2-(z-z')^2}\right) j_m(z',t';\nu) \end{aligned}, \qquad (5.5)$$

where $H\left(c(t-t')-(z-z')\right)$ is Heaviside step function. This expression can be simplified by substitution of extended current density $j_m(z',t';\nu)$ from (5.2):

$$h_m(z,t;\nu) = \sqrt{\mu_0}\, J_1(\nu R) \frac{ic\delta_{m,1}}{4\sqrt{\nu}} \int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2(t-t')^2 - z^2}\right). \qquad (5.6)$$

The integral cannot be solved by ordinary functions, but it is close to one of Lommel function property. Let us make a variable substitutions to make it clear.

$$h_m(z,t;\nu) = \sqrt{\mu_0}\, J_1(\nu R) \frac{i\delta_{m,1}}{4\sqrt{\nu^3}} \int_{\nu z}^{\nu ct} ds J_0\left(\sqrt{s^2 - \nu^2 z^2}\right). \qquad (5.7)$$

https://en.wikipedia.org/wiki/Lommel_function

https://en.wikipedia.org/wiki/Lommel_polynomial

Определения ф. Ломмеля + физ. Смысл + ссыла в литературу

$$U_n(W,Z) = \sum_{m=0}^{\infty} (-1)^m \left(\frac{W}{Z}\right)^{n+2m} J_{n+2m}(Z). \qquad (5.8)$$

$$\begin{aligned} &\int_\xi^\tau ds e^{-i\gamma s} J_0\left(\sqrt{s^2-\xi^2}\right) = \frac{e^{-i\gamma\tau}}{\sqrt{\gamma^2-1}}\left[U_1(W_+,Z) + iU_2(W_+,Z) - U_1(W_-,Z) - iU_2(W_-,Z)\right] \\ &W_\pm = \left(\gamma \pm \sqrt{\gamma^2-1}\right)(\tau-\xi); \qquad Z = \sqrt{\tau^2-\xi^2}; \qquad \tau-\xi>0 \end{aligned}. \qquad (5.9)$$

Lommel functions of two variables are ordinary in problems of acoustic wave excitation and transient electromagnetic field in LL. In our case next property can be applied.

$$\left.\begin{aligned} U_{2n}(W_+,Z) &= U_{2n}(W_-,Z) \\ U_{2n+1}(W_+,Z) &= -U_{2n+1}(W_-,Z) \end{aligned}\right| n \in \mathbb{Z}. \qquad (5.10)$$

So expression (5.7) with substitution form (5.9) and property (5.10) can be rewrote like

$$. \qquad (5.11)$$

5.3 Transversal coefficients

Электрические равны нулю

$$I_n^e = V_n^e = 0\,. \tag{5.12}$$

At first let us apply (5.13) Leibniz integral rule [15] for V_m^h coefficients from (5.3).

$$\frac{\partial}{\partial\theta}\int\limits_{a(\theta)}^{b(\theta)} f(x,\theta)\,dx = \int\limits_{a(\theta)}^{b(\theta)} \frac{\partial f}{\partial\theta}dx + f\big(b(\theta),\theta\big)\cdot b'(\theta) - f\big(a(\theta),\theta\big)\cdot a'(\theta)\,. \tag{5.13}$$

$$V_m^h = \mu\sqrt{\mu_0}\,J_1(\nu R)\frac{i\delta_{m,1}}{4\sqrt{\nu}}\,J_0\left(\nu\sqrt{c^2(t-t')^2 - z^2}\right). \tag{5.14}$$

I_m^h coefficients from (5.3) can be presented with (5.11) substitution

$$I_m^h = \frac{\partial h_m}{\partial z} = -\sqrt{\mu_0}\,J_1(\nu R)\frac{\delta_{m,1}}{2\sqrt{\nu^3}}\frac{\partial}{\partial z}U_1\left[-i\nu(ct-z), \nu\sqrt{c^2t^2 - z^2}\right]. \tag{5.15}$$

According to derivative properties of Lommel function [16]

$$\frac{\partial U_1}{\partial W} = \frac{1}{2}U_0(W,Z) + \frac{1}{2}\left(\frac{Z}{W}\right)^2 U_2(W,Z); \quad \frac{\partial U_1}{\partial Z} = -\frac{Z}{W}U_2(W,Z);$$

expression for

$$I_m^h = \frac{\partial h_m}{\partial z} = -\sqrt{\mu_0}\,J_1(\nu R)\frac{\delta_{m,1}}{2\sqrt{\nu^3}}\frac{\partial}{\partial z}U_1 = -\sqrt{\mu_0}\,J_1(\nu R)\frac{i\delta_{m,1}}{4\sqrt{\nu}}(U_0 - U_2)\,. \tag{5.16}$$

5.4 Filed expression

Evolutionary coefficients substitution to expansion form of EM field gives three dimensional expressions for electric and magnetic strength vectors dependent of radius vector $\vec{r}$ and time t.

Now we are able to substitute coefficients V_m^h form (5.14), I_m^h from (5.16) and I_n^e, V_n^e from (5.12) to (4.46) using (4.48) and (4.49) expressions.

$$E_\rho = -\frac{1}{\rho}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}e^{i\varphi}\int\limits_0^\infty \frac{d\nu}{\nu}J_1(\nu R)\,J_1(\nu\rho)\,J_0\left(\nu\sqrt{c^2t^2 - z^2}\right); \tag{5.17}$$

$$E_\varphi = -\frac{i}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}e^{i\varphi}\int\limits_0^\infty d\nu\,J_1(\nu R)\big(J_0(\nu\rho) - J_2(\nu\rho)\big)\,J_0\left(\nu\sqrt{c^2t^2 - z^2}\right); \tag{5.18}$$

$$H_\rho = -\frac{i}{8\sqrt{\mu_0}} e^{i\varphi} \int_0^\infty d\nu J_1(\nu R)(U_0 - U_2)(J_0(\nu\rho) - J_2(\nu\rho)); \tag{5.19}$$

$$H_\varphi = \frac{e^{i\varphi}}{4\rho\sqrt{\mu_0}} \int_0^\infty \frac{d\nu}{\nu} J_1(\nu R)(U_0 - U_2) J_1(\nu\rho). \tag{5.20}$$

The same is with longitudinal ones. Substitution of (5.11) and (5.12) to (4.50) gives field components E_z and H_z:

$$E_z = 0; \tag{5.21}$$

$$H_z = \frac{e^{i\varphi}}{2} \int_0^\infty d\nu J_1(\nu R) J_1(\nu\rho) U_1\left[-i\nu(ct - z), \nu\sqrt{c^2t^2 - z^2}\right]. \tag{5.22}$$

(5.18) and (5.19) can be represented with using of Bessel function property

$$J_{m+1}(z) = \frac{2m}{z} J_m(z) - J_{m-1}(z), \tag{5.23}$$

so E_φ and H_ρ get the forms

$$E_\varphi = -i\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} e^{i\varphi} \int_0^\infty d\nu J_1(\nu R)\left(J_0(\nu\rho) - \frac{J_1(\nu\rho)}{\nu\rho}\right) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right); \tag{5.24}$$

$$H_\rho = -\frac{i}{4\sqrt{\mu_0}} e^{i\varphi} \int_0^\infty d\nu J_1(\nu R)(U_0 - U_2)\left(J_0(\nu\rho) - \frac{J_1(\nu\rho)}{\nu\rho}\right). \tag{5.25}$$

В (5.24) и (5.25)была ошибка

Сравнить с дисером

$$\vec{E} = i\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} e^{i\varphi}\left[\vec{\varphi}_0\left(\frac{I_1}{\rho} - I_2\right) + i\vec{\rho}_0\frac{I_1}{\rho}\right] \tag{5.26}$$

(5.17)-(5.22) consist integrals of triple Bessel function multiplications. Note, Lommel function is not an exception: it can be presented in the way of infinity summary of Bessel function with some coefficients according to (5.8).

$$H_z = -\frac{e^{i\varphi}}{2}\sum_{m=0}^\infty i^{2m+1}\left(\sqrt{\frac{ct - z}{ct + z}}\right)^{2m+1} \int_0^\infty d\nu J_1(\nu R) J_1(\nu\rho) J_{2m+1}\left(\nu\sqrt{c^2t^2 - z^2}\right); \tag{5.27}$$

$$H_\rho = -\frac{i}{8\sqrt{\mu_0}} e^{i\varphi} \sum_{m=0}^{\infty} (-1)^m (-i)^{2m} \left(\frac{ct-z}{ct+z}\right)^m \int_0^\infty d\nu J_1(\nu R) \times$$
$$\times \left[J_{2m}\left(\nu\sqrt{c^2t^2-z^2}\right) + i\frac{ct-z}{ct+z} J_{2m+2}\left(\nu\sqrt{c^2t^2-z^2}\right)\right]\left(J_0(\nu\rho) - \frac{J_1(\nu\rho)}{\nu\rho}\right). \quad (5.28)$$

$$H_\varphi = \frac{e^{i\varphi}}{4\rho\sqrt{\mu_0}} \int_0^\infty \frac{d\nu}{\nu} J_1(\nu R)(U_0 - U_2) J_1(\nu\rho) \quad (5.29)$$

Electric strength vector components can be integrated by analytics. Let us rename integrals expressions like

$$I_1 = \int_0^\infty \frac{d\nu}{\nu} J_1(\nu R) J_1(\nu\rho) J_0\left(\nu\sqrt{c^2t^2-z^2}\right); \quad (5.30)$$

$$I_2 = \int_0^\infty d\nu J_1(\nu R) J_0(\nu\rho) J_0\left(\nu\sqrt{c^2t^2-z^2}\right). \quad (5.31)$$

Later we are going to prove that I_1 is equal to

$$I_1 = \frac{\rho^2+R^2}{4\pi R\rho}(\pi - 2\psi) + \frac{\rho^2-R^2}{4\pi R\rho}\left(\arctan\left(\frac{\rho+R}{\rho-R}\tan\frac{\psi}{2}\right) - \arctan\left(\frac{\rho-R}{\rho+R}\tan^{-1}\frac{\psi}{2}\right)\right)$$
$$\psi = 2\arcsin\sqrt{\frac{c^2t^2-z^2-(\rho-R)^2}{4\pi\rho}}. \quad (5.32)$$

Integral (5.31) can be solved by the formula known form [17]

$$\int_0^\infty J_0(ax) J_0(bx) J_1(cx)\,dx = \frac{1}{\pi c}\arccos\frac{a^2+b^2-c^2}{2ab}$$
$$|a-b| < c < a+b \quad a,b > 0$$
$$I_2 = \frac{1}{\pi R}\arccos\frac{c^2t^2-z^2+\rho^2-R^2}{2\rho\sqrt{c^2t^2-z^2}}. \quad (5.33)$$

Thus it is possible to rewrite electric field components E_φ (5.24) and E_ρ(5.17) in the way of

$$E_\rho = -\frac{1}{\rho}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} e^{i\varphi} I_1; \quad (5.34)$$

$$E_\varphi = -i\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} e^{i\varphi}\left(I_2 - \frac{1}{\rho} I_1\right). \quad (5.35)$$

Complex form of a field allows to submit φ dependence of a field in the last stage of solution. The form of φ dependency determines initially conditions. Let us use real part of the exponential dependence. So, according to Euler's formula

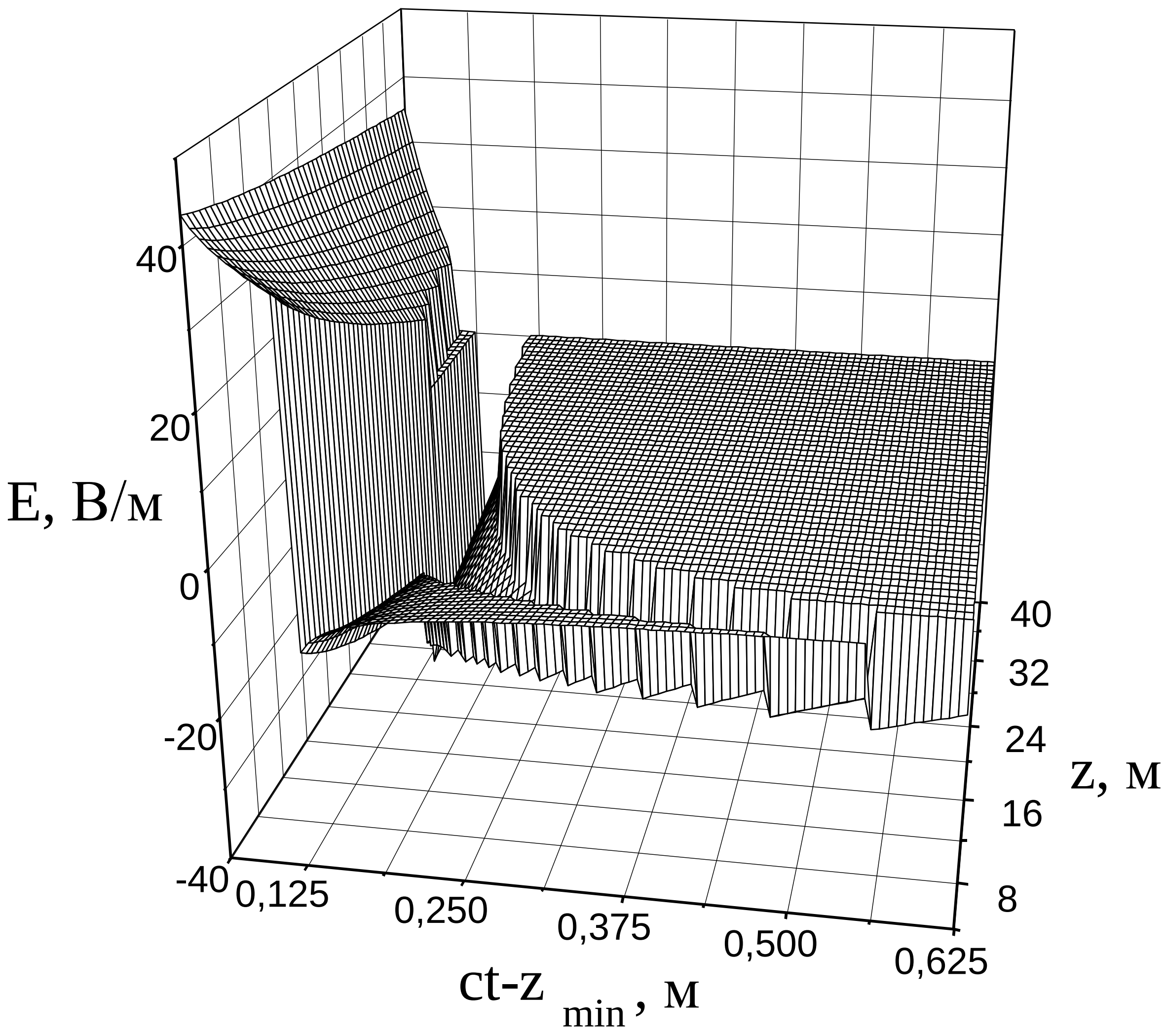
$$e^{i\varphi} = \cos\varphi + i\sin\varphi \Rightarrow \mathrm{Re}\left(e^{i\varphi}\right) = \cos\varphi$$
$$ie^{i\varphi} = -\sin\varphi + i\cos\varphi \Rightarrow \mathrm{Re}\left(e^{i\varphi}\right) = -\sin\varphi,$$

Field dependence can be rewritten in next form

$$E_\rho = -\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{I_1}{\rho}\cos\varphi \tag{5.36}$$

$$E_\varphi = \frac{1}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}I_2\sin\varphi - \frac{1}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\frac{I_1}{\rho}\sin\varphi \tag{5.37}$$

The expressions which are received can be illustrated by the next plot.



If to not beat about the bush impulse radiation is shown but the solution (5.17)-(5.20) presents only for “plug in” signal. It is better to show Electromagnetic missile on the image. The observer is located out of the bound of beam array but close to it. It is clear that the field inside beam zone fades quietly than it must be according to classic theory.

5.5 On-axis field

Let us consider far linear field on ρ axis ($\rho = 0$) for simplification. It can be achieved form expressions (5.17)-(5.20). According to Bessel asymptotic property

$$\left.\frac{J_1(\chi\rho)}{\rho}\right|_{\rho\to 0} = \frac{\chi}{2},$$

field components (5.17), (5.24), (5.25) and (5.20) can be presented in next way:

$$E_\rho = -\frac{e^{i\varphi}}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) \tag{5.38}$$

$$E_\varphi = -\frac{ie^{i\varphi}}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) \tag{5.39}$$

$$H_\rho = -\frac{ie^{i\varphi}}{8\sqrt{\mu_0}}\int_0^\infty d\nu J_1(\nu R)(U_0 - U_2) \tag{5.40}$$

$$H_\varphi = \frac{e^{i\varphi}}{8\sqrt{\mu_0}}\int_0^\infty d\nu J_1(\nu R)(U_0 - U_2) \tag{5.41}$$

$$H_z = E_z = 0 \tag{5.42}$$

Double Bessel multiplication formula from [18] gives electric field.

$$\vec{\mathbf{E}} = -\frac{e^{i\varphi}}{2}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\{\vec{\varphi}_0 + i\vec{\rho}_0\}\begin{cases}\dfrac{1}{R}, \sqrt{c^2t^2 - z^2} < R\\ \dfrac{1}{2R}, \sqrt{c^2t^2 - z^2} = R\\ 0, \sqrt{c^2t^2 - z^2} > R\end{cases} \tag{5.43}$$

The same is with magnetic component if consider only far field. Пояснение

В диссере посчитали что U0=J0 и U2=0 в дальней зоне – это не верно?

$$\vec{\mathbf{H}} = \frac{e^{i\varphi}}{8\sqrt{\mu_0}}\{\vec{\varphi}_0 - i\vec{\rho}_0\}\int_0^\infty d\nu J_1(\nu R)(U_0 - U_2) =$$
$$= \frac{e^{i\varphi}}{8\sqrt{\mu_0}}\{\vec{\varphi}_0 - i\vec{\rho}_0\}\int_0^\infty d\nu J_1(\nu R) J_0\left(\nu\sqrt{c^2t^2 - z^2}\right) = \qquad (5.44)$$
$$= \frac{e^{i\varphi}}{8\sqrt{\mu_0}}\{\vec{\varphi}_0 - i\vec{\rho}_0\}\begin{cases} \frac{1}{R}, \sqrt{c^2t^2 - z^2} < R \\ \frac{1}{2R}, \sqrt{c^2t^2 - z^2} = R \\ 0, \sqrt{c^2t^2 - z^2} > R \end{cases}$$

Let us present EM field by Heaviside's step functions

Здесь подогнал коэффициенты немного

$$\vec{\mathbf{H}} = \frac{ie^{i\varphi}}{8}\{i\vec{\varphi}_0 + \vec{\rho}_0\} H\left(R - \sqrt{c^2t^2 - z^2}\right) \qquad (5.45)$$

$$\vec{\mathbf{E}} = \frac{ie^{i\varphi}}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\{i\vec{\rho}_0 - \vec{\varphi}_0\} H\left(R - \sqrt{c^2t^2 - z^2}\right) \qquad (5.46)$$

Expressions (5.45) and (5.46) takes plays only when $c^2t^2 \geq z^2$. This restriction has physical nature. It comes from finite speed of the light transition. It can be taken into account analytically

$$\vec{\mathbf{H}} = \frac{ie^{i\varphi}}{8}\{i\vec{\varphi}_0 + \vec{\rho}_0\} H\left(R - \sqrt{c^2t^2 - z^2}\right) H\left(c^2t^2 - z^2\right); \qquad (5.47)$$

$$\vec{\mathbf{E}} = \frac{ie^{i\varphi}}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\{i\vec{\rho}_0 - \vec{\varphi}_0\} H\left(R - \sqrt{c^2t^2 - z^2}\right) H\left(c^2t^2 - z^2\right). \qquad (5.48)$$

Complex dependency can be removed now according to Euler's formula:

$$e^{i\varphi} = \cos\varphi + i\sin\varphi \Rightarrow \mathrm{Re}\left(e^{i\varphi}\right) = \cos\varphi;$$
$$ie^{i\varphi} = -\sin\varphi + i\cos\varphi \Rightarrow \mathrm{Re}\left(e^{i\varphi}\right) = -\sin\varphi.$$

The choice of real parts is determined by initially conditions. The results are

$$\vec{\mathbf{H}} = -\frac{1}{8}\{\vec{\varphi}_0\cos\varphi + \vec{\rho}_0\sin\varphi\} H\left(R - \sqrt{c^2t^2 - z^2}\right) H\left(c^2t^2 - z^2\right); \qquad (5.49)$$

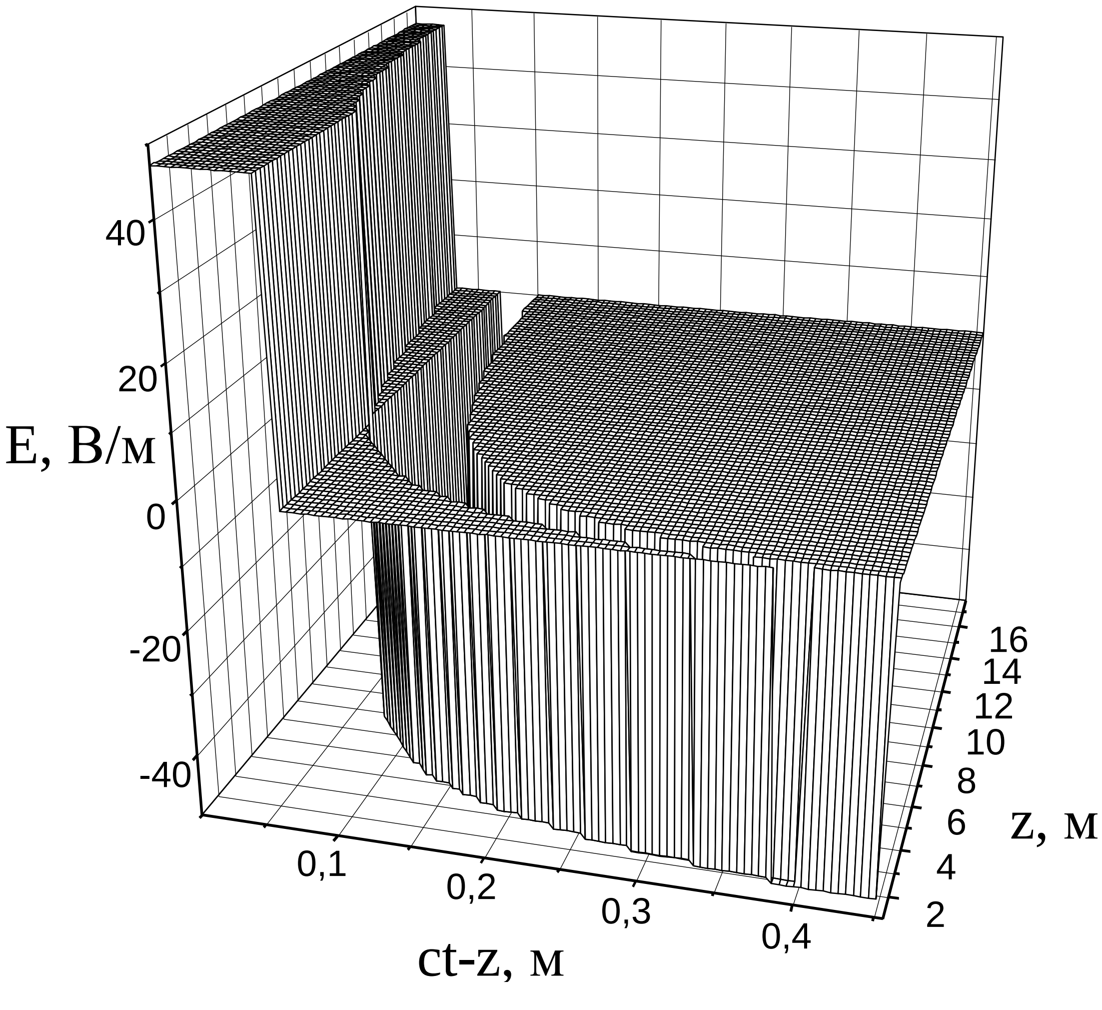
$$\vec{\mathbf{E}} = -\frac{1}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\{\vec{\rho}_0\cos\varphi - \vec{\varphi}_0\sin\varphi\} H\left(R - \sqrt{c^2t^2 - z^2}\right) H\left(c^2t^2 - z^2\right). \qquad (5.50)$$

EM field (5.47) and (5.48) can be presented in Descartes coordinate system:

$$\vec{\mathbf{H}} = -\frac{\vec{y}_0}{8} H\left(R - \sqrt{c^2t^2 - z^2}\right) H\left(c^2t^2 - z^2\right); \tag{5.51}$$

$$\vec{\mathbf{E}} = -\frac{\vec{x}_0}{8} \frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}} H\left(R - \sqrt{c^2t^2 - z^2}\right) H\left(c^2t^2 - z^2\right). \tag{5.52}$$

There is a plot to the on-axis field solution as well as for previous one. It shows the same problem.



The difference is opened to view. On-axis field is not dependent of z coordinate – it is constant. The phenomena was firstly noted by T. T. Wu in paper [19].

6 NONLINEAR SOLUTION

6.1 Nonlinear sources

Nonlinear medium can be considered in EAE theory by secondary source of current. This approach was taken into account in expression for general current (4.8) by nonlinear component of polarization and magnetization vectors.

Let us consider a medium with zero nonlinear magnetization. Only nonlinear polarization exists. Also we will determinate current of conductivity to generalize a problem. Secondary electric current can be presented in next expression.

$$\vec{\mathbf{J}}'\left(\vec{\mathbf{E}},\vec{\mathbf{H}}\right)=\begin{pmatrix} J'_\rho \\ J'_\varphi \\ J'_z \end{pmatrix}=\begin{pmatrix} \overrightarrow{J'} \\ J'_z \end{pmatrix}=\begin{pmatrix} \frac{\partial}{\partial t}\overrightarrow{P'}\left(\vec{\mathbf{E}}\right)+\sigma\vec{E} \\ \frac{\partial}{\partial t}P_z'\left(\vec{\mathbf{E}}\right)+\sigma E_z \end{pmatrix}, \tag{6.1}$$

where $\vec{\mathbf{E}}$ is primary field generated by source $\overrightarrow{j_0}$ form linear problem (5.1)

Kerr medium implies only third power of (3.1) summary, so (6.1) can be rewritten.

$$\vec{\mathbf{J}}^1=\frac{\partial}{\partial t}\left(\chi_3\vec{\mathbf{E}}^3\right)+\sigma\vec{E}=\chi_3^E\vec{x}_0\frac{\partial}{\partial t}\left({E_x}^3\right)+\sigma\vec{x}_0E_x\,. \tag{6.2}$$

Expression (6.2) get next form after substitution of liner field expression (5.52).

$$\begin{gathered}\vec{\mathbf{J}}^1=-\vec{x}_0 3\frac{\chi_3^E}{512}\left(\frac{\mu_0}{\varepsilon_0}\right)^{\frac{3}{2}}H^2\left(ct-z\right)H^2\left(R-\sqrt{c^2t^2-z^2}\right)\times \\ \times\left[H\left(ct-z\right)\delta\left(R-\sqrt{c^2t^2-z^2}\right)\frac{-c^2t}{\sqrt{c^2t^2-z^2}}+H\left(R-\sqrt{c^2t^2-z^2}\right)\delta\left(ct-z\right)c\right]- \\ -\vec{x}_0\frac{\sigma}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}H\left(R-\sqrt{c^2t^2-z^2}\right)H\left(ct-z\right)\end{gathered} \tag{6.3}$$

This current can be applied by EAE and gives nonlinear correction to linear fields (5.49) and (5.50). Magnetic longitudinal evolutionary coefficient was presented by Riemann function but this expression needs electric current expansion as well as linear problem. Acording to EAE electric current is

$$j_m{}^1(z,t;\nu)=\frac{\sqrt{\mu_0}}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\overrightarrow{J^1}\left[\nabla_\perp{\Psi_m}^*\times\overrightarrow{z_0}\right], \tag{6.4}$$

where $\overrightarrow{J^1}$ is nonlinear secondary current density and Ψ_m goes form (4.35)

The equivalent currents are valid on OZ axis, so we multiply it on delta-function

$$\overrightarrow{J^1} = \frac{1}{8}\frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_0}}\left(\vec{\rho}_0\cos\varphi - \vec{\varphi}_0\sin\varphi\right)\frac{\delta(\rho)}{\rho}\left[-3\frac{\chi_3^E}{64}\left(\frac{\mu_0}{\varepsilon_0}\right)H^2(ct-z)H^2\left(R-\sqrt{c^2t^2-z^2}\right)\times\right.$$
$$\times\left[H(ct-z)\delta\left(R-\sqrt{c^2t^2-z^2}\right)\frac{-c^2t}{\sqrt{c^2t^2-z^2}} + H\left(R-\sqrt{c^2t^2-z^2}\right)\delta(ct-z)c\right]- \tag{6.5}$$
$$\left.-\sigma H\left(R-\sqrt{c^2t^2-z^2}\right)H(ct-z)\right].$$

It is not hard to prove that (6.5) can be rewritten in case of expansion (6.4). The result is

$$j_1^1(z,t;\nu) = \frac{-1}{8}\frac{\mu_0}{\sqrt{\varepsilon_0}}i\frac{\sqrt{\nu}}{2}\left[-3\frac{\chi_3^E}{64}\left(\frac{\mu_0}{\varepsilon_0}\right)H^2(ct-z)H^2\left(R-\sqrt{c^2t^2-z^2}\right)\times\right.$$
$$\times\left[H(ct-z)\delta\left(R-\sqrt{c^2t^2-z^2}\right)\frac{-c^2t}{\sqrt{c^2t^2-z^2}} + H\left(R-\sqrt{c^2t^2-z^2}\right)\delta(ct-z)c\right]- \tag{6.6}$$
$$\left.-\sigma H\left(R-\sqrt{c^2t^2-z^2}\right)H(ct-z)\right].$$

6.2 Evolutionary coefficients

There is known solution of the first equation of the set (5.3) by the Riemann functions. It is clearly that it is applicable to nonlinear approximation too. So longitudinal magnetic evolutionary coefficient is equal to

$$h_m = \frac{c}{2}\int_0^\infty dz'\int_0^\infty dt' H\left(c(t-t')-(z-z')\right)J_0\left(\nu\sqrt{c^2(t-t')^2-(z-z')^2}\right)j_m(z',t';\nu). \tag{6.7}$$

After substitution of current expression (6.6) evolutionary coefficient (6.7) gets next unfriendly form.

$$h_1^{nl}(z,t;\nu) = -\sqrt{\mu_0}J_1(\nu R)\frac{1}{2\sqrt{\nu^3}}U_1\left[-i\nu(ct-z),\nu\sqrt{c^2t^2-z^2}\right] - i\frac{\sqrt{\mu_0}\sqrt{\nu}}{32\varepsilon_0}\times$$
$$\times\left\{\int_0^\infty dt' H\left(c(t-t')-\left(z-\sqrt{c^2t'^2-R^2}\right)\right)J_0\left(\nu\sqrt{c^2(t-t')^2-\left(z-\sqrt{c^2t'^2-R^2}\right)^2}\right)\frac{3\chi_3^E t'}{64R{\varepsilon_0}^2}-\right.$$
$$-3\frac{\chi_3^E}{64}\left(\frac{\mu_0}{\varepsilon_0}\right)cH(ct-z)\int_0^\infty dt' J_0\left(\nu\sqrt{c^2t^2-z^2-2ct'(ct-z)}\right)- \tag{6.8}$$
$$\left.-\sigma\int_0^\infty dz'\int_{z'/c}^{\sqrt{R^2+z'^2}/c} dt' H\left(c(t-t')-(z-z')\right)J_0\left(\nu\sqrt{c^2(t-t')^2-(z-z')^2}\right)\right\}$$

We are not able to integrate it now but it is analytical solution still and gives some results. Sure, it can be integrated numerically but the result will be not in the plane of our interest, because we need analytical solutions to implement compare analysis of linear and nonlinear solutions.

7 CONCLUSIONS

7.1 EAE for unbounded nonlinear medium

This writing shows the impotence of the EAE. The system of evolutionary equations is received. Also proved its applicability for very strong filed. Final view of evolutionary set is

$$
\begin{cases}
\partial_{ct}\left\{\varepsilon\partial_{ct}\left(\mu h_m\right)\right\}-\partial_z\left\{\mu^{-1}\partial_z\left(\mu h_m\right)\right\}+\nu^2 h_m=\sqrt{\mu_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left[\vec{z}_0\times\vec{J}\right]\cdot\nabla_\perp\Psi_m{}^*\left(\nu\right)- \\
\qquad -\partial_{ct}\left\{\sqrt{\varepsilon_0}\,\varepsilon\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)I_z\right\}-\partial_z\left\{\sqrt[-2]{\mu_0}\,\mu^{-1}\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)g\right\} \\
\partial_{ct}\left\{\mu\partial_{ct}\left(\varepsilon e_n\right)\right\}-\partial_z\left\{\varepsilon^{-1}\partial_z\left(\varepsilon e_n\right)\right\}+\chi^2 e_n=\sqrt{\varepsilon_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left[\vec{I}\times\vec{z}_0\right]\cdot\nabla_\perp\Phi_n{}^*\left(\chi\right)- \\
\qquad -\partial_{ct}\left\{\sqrt{\mu_0}\,\mu\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)J_z\right\}-\partial_z\left\{\sqrt[-2]{\varepsilon_0}\,\varepsilon^{-1}\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)\varrho\right\} \\
I_n^e=-\partial_{ct}\left\{\varepsilon e_n\right\}-\sqrt{\mu_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)J_z \\
V_n^e=\varepsilon^{-1}\partial_z\left\{\varepsilon e_n\right\}-\sqrt[-2]{\varepsilon_0}\,\varepsilon^{-1}\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n{}^*\left(\chi\right)\varrho \\
I_m^h=\mu^{-1}\partial_z\left\{\mu h_m\right\}-\sqrt[-2]{\mu_0}\,\mu^{-1}\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)g \\
V_m^h=-\partial_{ct}\left\{\mu h_m\right\}-\sqrt{\varepsilon_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m{}^*\left(\nu\right)I_z
\end{cases}
$$

Variables $\left\{V_m^h, I_m^h, V_n^e, I_n^e, e_n, h_m\right\}_{m,n=0}^{\infty}$ are unknown values of expended field

$$
\begin{aligned}
\vec{E}&=\sqrt[-2]{\varepsilon_0}\left\{\sum_{m=0}^{\infty}\int_0^{\infty}d\nu V_m^h\left[\nabla_\perp\Psi_m\times\vec{z}_0\right]+\sum_{m=1}^{\infty}\int_0^{\infty}d\chi V_n^e\nabla_\perp\Phi_n\right\} \\
\vec{H}&=\sqrt[-2]{\mu_0}\left\{\sum_{m=0}^{\infty}\int_0^{\infty}d\nu I_m^h\nabla_\perp\Psi_m+\sum_{m=1}^{\infty}\int_0^{\infty}d\chi I_n^e\left[\vec{z}_0\times\nabla_\perp\Phi_n\right]\right\}. \\
E_z\left(\rho,\phi,z,t\right)&=\sqrt[-2]{\varepsilon_0}\sum_{n=0}^{\infty}\int_0^{\infty}\chi^2 d\chi e_n\left(z,t;\chi\right)\Phi_n\left(\rho,\phi;\chi\right) \\
H_z\left(\rho,\phi,z,t\right)&=\sqrt[-2]{\mu_0}\sum_{m=0}^{\infty}\int_0^{\infty}\nu^2 d\nu h_m\left(z,t;\nu\right)\Psi_m\left(\rho,\phi;\nu\right)
\end{aligned}
$$

7.2 Electromagnetic missile solution

EAE is time domain method, so it is clearly now that there are a lot of reasons to apply it to electromagnetic missile theory. Also the light bullet solution was achived in the writing for classical task of plane disk radiation.

There are some new steps to nonlinear application too. New method of strong field electrodynamics is designed. EAE was modified for this too. The method is based on taking nonlinear nature into account by secondary sources. Nonlinear approach was shown on the plane disk radiation problem too. Only Kerr nonlinear nature was under consideration. More idears about nonlinear iterative corrections are available in [20].

Physical nature of the affect can by accounted for nonlinear property of the medium: the nonlinear medium is secondary source of energy. Also we are able to note that nonlinear effect get more strings with deep propagation into the medium. Weak nonlinear solution coefficients are liner dependent to the nonlinear effect strength. Solution (6.8) can be transformed to linear problem with zero nonlinear coefficients and non-conducting medium ($\sigma = 0$).

8 REFERENCES

1. Rao Sadashiva Madiraju Time Domain Electromagnetics. — Sun Diego : Academic Press, 1999.

2. Hertz Heinrich Rudolf Electric Waves. — London : Macmillan and Company, 1893.

3. Crookes William Some Possibilities of Electricity // The Fortnightly Review. — 1892. — P. 174-176.

4. Schantz Hans The Art and Science of Ultra-Wideband Antennas. — Norwood : Artech House, 2005.

5. Committee IEEE Aerospace and Electronic Systems Society UWB Radar IEEE P1672 TM // Ultrawideband Radar Definitions Draft. — April 4, 2006. — Version 1.5.

6. Tretyakov Oleg Alexandrovich, Hashimoto M Evolutionary Approach For Electrodynamics // Essentials of Nonstationary and Nonlinear Electromagnetic Field Theory. — Tokyo : Science House Co, 1993. — P. 572.

7. Dumin Olexandr, Tretyakov Oleg Radiation of Arbitrary Signals by Plane Disk. — Lviv : MMET, 1996. — P. 248–251.

8. Шубарин Юрий Васильевич Антенны сверхвысоких частот. — Харьков : Харьковский Государственный Университет, 1960.

9. Tretyakov Oleg Alexandrovich, Dumin Oleksandr, Akhmedov Rolan Javadovich, Katrich Victor Alexandrovich, Dumina Olga Olexandrivna Evolutionary Approach for the Problems of Transient Electromagnatic Field Propagation in Nonlinear Medium. — Dnipropetrovsk : MMET, 2014.

10. Иваненко Д., Соколов А. Классичекая терия поля. — Москва : Государственное издательство физико-технической литературы, 1951.

11. Черногор Леонид Феоктистович Введение в нелинейную радиофизику. — Харьков, 2012.

12. Tretyakov Oleg Alexandrovich Evolutionary Approach For Electrodynamics. — Kharkov : Kharkov State Universite, 1999. — Vol. II.

13. Прудников А П, Брычков Ю А, Маричев О И Интегралы и ряды: Специальные функции. — Москва : Наука, 1983. — 209 p.

14. Волков И. К., Канатников А. Н. Интегральные преобразования и операционное исчисление. — Москва : Изд-во МГТУ им. Н. Э. Баумана, 2002. — 228 с.

15. Фихтенгольц Г. М. Основы математического анализа. — Москва : Физматлит, 2001. — Vol. I : 172 p.

16. Борисов Виктор Васильевич Неустановившиеся поля в волноводах. — Ленинград : Издательство ленинградского университета, 1991. — 41 p.

17. Прудников А П, Брычков Ю А, Маричев О И Интегралы и ряды: Специальные функции. — Москва : Наука, 1983. — 228 p.

18. Прудников А П, Брычков Ю А, Маричев О И Интегралы и ряды: Специальные функции. — Москва : Наука, 1983. — 440 с.

19. Wu T. T. Electromagnetic missiles // J. Appl. Phys. — 1985. — Vol. 57, 7. — P. 2370–2373.

20. Dumin Olexandr, Tretyakov Oleg, Akhmedov Rolan, Dumina Olga Evolutionary Approach for The Problem of Electromagnetic Field Propagation Through Nonlinear Medium // Вісник Харківського національного Університету імені В. Н. Каразіна. — Харків, 2014. — 24. — P. 23-28.