Assignment 2

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Clean the environment and import essentials

```
rm(list = ls())
cat('\014')
```

```
library(psych)
library(corrplot)
```

```
## corrplot 0.92 loaded
```

Here,

$$log(Q_1) = A_1 - 2.5 \cdot log(P_1) + 1.5 \cdot log(P_2) + 1.2 \cdot log(P_3)$$

$$log(Q_2) = A_2 + 0.7 \cdot log(P_1) - 1.5 \cdot log(P_2) + 0.5 \cdot log(P_3)$$

$$log(Q_3) = A_3 + 0.7 \cdot log(P_1) + 0.4 \cdot log(P_2) - 1.2 \cdot log(P_3)$$

$$\beta_{11} = -2.5, \beta_{12} = 1.5, \beta_{13} = 1.2$$
$$\beta_{21} = 0.7, \beta_{22} = -1.5, \beta_{23} = 0.5$$
$$\beta_{31} = 0.7, \beta_{32} = 0.4, \beta_{33} = -1.2$$

$$P_1 = 2, P_2 = 1.75, P_3 = 1.5$$

$$C_1 = 0.5, C_2 = 0.75, C_3 = 0.9$$

$$r = 0.1$$

Question 1

The own price elasticites of the product 1, 2 and 3 are -2.5, -1.5 and -1.2 respectively.

Question 2

The price elasticity matrix is as follows:

$$\begin{bmatrix} -2.5 & 1.5 & 1.2 \\ 0.7 & -1.5 & 0.5 \\ 0.7 & 0.4 & -1.2 \end{bmatrix}$$

Question 3

$$\frac{Q_{i1}}{Q_{i0}} = (1+\gamma_1)^{\beta_{i1}} * (1+\gamma_2)^{\beta_{i2}} * (1+\gamma_3)^{\beta_{i3}}$$

Here,

$$\gamma_1 = 0.05, \gamma_2 = 0, \gamma_3 = -0.15$$

$$\frac{Q_{11}}{Q_{10}} = (1+0.05)^{-2.5} * (1+0)^{1.5} * (1-0.15)^{1.2} = (1.05)^{-2.5} * (1)^{1.5} * (0.85)^{1.2} = 0.728$$

$$\frac{Q_{21}}{Q_{20}} = (1+0.05)^{0.7} * (1+0)^{-1.5} * (1-0.15)^{0.5} = (1.05)^{0.7} * (1)^{-1.5} * (0.85)^{0.5} = 0.954$$

$$\frac{Q_{31}}{Q_{30}} = (1+0.05)^{0.7} * (1+0)^{0.4} * (1-0.15)^{-1.2} = (1.05)^{0.7} * (1)^{0.4} * (0.85)^{-1.2} = 1.257$$

We observe that two ratios are less than 1 (demand after is less than demand before) and one ratio is greater than 1 (demand after is greater than demand before). For product 1, the ratio is 0.728. Product 1 sees a decrease in demand of 27.2%. For product 2, the ratio is 0.954. Product 2 sees a decrease in demand of 4.6%. For product 3, the ratio is 1.257. Product 3 sees an increase in demand of 25.7%.

The above outcome can be explained as follows: 1. Product 1 sees a decrease in demand because it has the highest elasticity (-2.5) among the products, making it more susceptible to price changes. Therefore, its price increase affects negatively its demand. 2. Product 2 sees a decrease in demand because there is an effect of the price of product 3 on the sales of product 2 (price of product 3 decreases affecting negatively the sales of product 2). 3. Product 3 sees an increase in demand because it decreases its price, product 2 has zero change in its price and product 1 price increase affects positively product 3 sales.

```r
# Create a function to calculate above ratios given the price changes, own and cross elasticities
demand_change <- function(price_change_1, price_change_2, price_change_3,
                          elasticity_1, elasticity_2, elasticity_3,
                          cross_elasticity_1_2, cross_elasticity_1_3,
                          cross_elasticity_2_1, cross_elasticity_2_3,
                          cross_elasticity_3_1, cross_elasticity_3_2) {

  # Ratio for product 1
  ratio_1 = ((1+price_change_1)^elasticity_1)*((1+price_change_2)^cross_elasticity_1_2)*
    ((1+price_change_3)^cross_elasticity_1_3)

  # Ratio for product 2
  ratio_2 = ((1+price_change_1)^cross_elasticity_2_1)*((1+price_change_2)^elasticity_2)*
    ((1+price_change_3)^cross_elasticity_2_3)

  # Ratio for product 3
  ratio_3 = ((1+price_change_1)^cross_elasticity_3_1)*((1+price_change_2)^cross_elasticity_3_2)*
    ((1+price_change_3)^elasticity_3)

  # Return results as a list
  return(list(ratio_1 = ratio_1, ratio_2 = ratio_2, ratio_3 = ratio_3))
}
```

```
quantitychange = demand_change(0.05, 0, -0.15, -2.5,-1.5, -1.2, 1.5, 1.2, 0.7, 0.5, 0.7, 0.4)
quantitychange
```

```
## $ratio_1
## [1] 0.7283321
##
## $ratio_2
## [1] 0.9539859
##
## $ratio_3
## [1] 1.257563
```

Question 4

$$\frac{\pi_{i1}}{\pi_{i0}} = \frac{Q_{i1} * (P_{i1} * (1 - r_i) - C_{i1})}{Q_{i0} * (P_{i0} * (1 - r_i) - C_{i0})}$$

Profit ratio for Product 1

$$\frac{\pi_{11}}{\pi_{10}} = \frac{Q_{11} * (P_{11} * (1 - r) - C_1)}{Q_{10} * (P_{10} * (1 - r) - C_1)} = \frac{\alpha * Q_{10} * (P_{10} * (1 + \gamma_1) * (1 - r) - C_1)}{Q_{10} * (P_{10} * (1 - r) - C_1)} = \alpha * \frac{(P_{10} * (1 + \gamma_1) * (1 - r) - C_1)}{(P_{10} * (1 - r) - C_1)} =$$

$$= 0.728 * \frac{(2 * (1 + 0.05) * (1 - 0.10) - 0.5)}{2 * (1 - 0.10) - 0.5)} = 0.728 * \frac{1.39}{1.3} = 0.778$$

Profit ratio for Product 2

$$\frac{\pi_{21}}{\pi_{20}} = \frac{Q_{21} * (P_{21} * (1 - r) - C_2)}{Q_{20} * (P_{20} * (1 - r) - C_2)} = \frac{\alpha * Q_{20} * (P_{20} * (1 + \gamma_2) * (1 - r) - C_2)}{Q_{20} * (P_{20} * (1 - r) - C_2)} = \alpha * \frac{(P_{20} * (1 + \gamma_2) * (1 - r) - C_2)}{(P_{20} * (1 - r) - C_2)} =$$

$$= 0.954 * \frac{(1.75 * (1 + 0) * (1 - 0.10) - 0.75)}{1.75 * (1 - 0.10) - 0.75)} = 0.954 * \frac{0.825}{0.825} = 0.954$$

Profit ratio for Product 3

$$\frac{\pi_{31}}{\pi_{30}} = \frac{Q_{31} * (P_{31} * (1 - r) - C_3)}{Q_{30} * (P_{30} * (1 - r) - C_3)} = \frac{\alpha * Q_{30} * (P_{30} * (1 + \gamma_3) * (1 - r) - C_3)}{Q_{30} * (P_{30} * (1 - r) - C_3)} = \alpha * \frac{(P_{30} * (1 + \gamma_3) * (1 - r) - C_3)}{(P_{30} * (1 - r) - C_3)} =$$

$$= 1.257 * \frac{(1.5 * (1 - 0.15) * (1 - 0.10) - 0.9)}{1.5 * (1 - 0.10) - 0.9)} = 1.257 * \frac{0.247}{0.45} = 0.689$$

We observe that profits after the price adjustments are less than the profits before the price adjustments for all products. More specifically: Product 1: Profits_after/Profits_before = 0.778 Product 2: Profits_after/Profits_before = 0.954 Product 3: Profits_after/Profits_before = 0.689

For product 1, the profit decreases due to the high elasticity of the product that drives the demand downwards after the price increase. For product 2, the profit decreases slightly by 5% due to the non-change to its price and the decrease in price of product 3. For product 3, the profit decreases significantly due to its price decrease and its relatively higher marginal cost. Based on that, the price cut is not justified.

```r
# Create a function to calculate profit ratios given the price changes, the demand ratios, the prices o

profit_change <- function(price_change_1, price_change_2, price_change_3, price_1, price_2, price_3,
                          retail_margin, cost_1, cost_2, cost_3) {

  # Ratio for product 1
  profit_ratio_1 = (quantitychange$ratio_1*(price_1*(1+price_change_1)*(1-retail_margin)-cost_1))/
    (price_1*(1-retail_margin)-cost_1)

  # Ratio for product 2
  profit_ratio_2 = (quantitychange$ratio_2*(price_2*(1+price_change_2)*(1-retail_margin)-cost_2))/
    (price_2*(1-retail_margin)-cost_2)

  # Ratio for product 3
  profit_ratio_3 = (quantitychange$ratio_3*(price_3*(1+price_change_3)*(1-retail_margin)-cost_3))/
    (price_3*(1-retail_margin)-cost_3)

  # Return results as a list
  return(list(profit_ratio_1 = profit_ratio_1, profit_ratio_2 = profit_ratio_2,
              profit_ratio_3 = profit_ratio_3))
}
```

```r
profitchange = profit_change(0.05, 0, -0.15, 2, 1.75, 1.5, 0.1, 0.5, 0.75, 0.9)
profitchange
```

```
## $profit_ratio_1
## [1] 0.7787551
##
## $profit_ratio_2
## [1] 0.9539859
##
## $profit_ratio_3
## [1] 0.6916597
```

Question 5

The highest marginal cost at which the price cut is justified for product 3 should satisfy the following:

$$\pi_1 > \pi_0 => \frac{Q_1}{Q_0} > \frac{\frac{P_0 - C}{P_0}}{\frac{(1+\gamma)P_0 - C}{P_0}}$$

```r
price_3 = 1.5
price_change_1 = 0.05
price_change_2 = 0
price_change_3 = -0.15
cost_3 = 0.4
elasticity_3 = -1.2
cross_elasticity_3_1 = 0.7
cross_elasticity_3_2 = 0.4

gross_margin = (price_3-cost_3)/price_3
```

```r
B = gross_margin/(gross_margin+price_change_3)
A = ((1+price_change_1)^cross_elasticity_3_1)*((1+price_change_2)^cross_elasticity_3_2)*
    ((1+price_change_3)^elasticity_3)
A>B
```

```
## [1] TRUE
```

The highest marginal cost at which the price cut is justified for product 3 is 0.4.