Assignment 2: Demand and Profitability Analysis for a Multi-Product Category

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Overview

You are a marketing analytic consultant doing category analysis for a multi-product category. Your are asked to perform pricing and profitability analysis for this category. The market has three products sold by different firms: 1, 2, and 3. You have been informed that the marginal cost of the products are $C_1 = 0.5, C_2 = 0.75, C_3 = 0.9$, and the retail margin for all the products is 10%. Denote the prices of the three products as P_1, P_2, P_3. Based on the demand models that your data science team has run on the sales and pricing data for this category, you know that the own and cross price elasticities for the three products can be specified using the following demand system:

$$Q_1 = A_1 P_1^{-2.5} P_2^{1.5} P_3^{1.2}$$
$$Q_2 = A_2 P_1^{0.7} P_2^{-1.5} P_3^{0.5}$$
$$Q_3 = A_3 P_1^{0.7} P_2^{0.4} P_3^{-1.2}$$

Further, the base prices of three products are given as: $P_1 = \$2$, $P_2 = \$1.75$ and $P_3 = \$1.5$. Please answer the following questions based on this information. (Note: You are not given information on A_1, A_2, and A_3.)

Questions

1. What are the own price elasticites of the three products? (0.5 points)
2. Write down a matrix of price elasticities. (1 point)
3. Now assume that product 1 increases its price by $\gamma_1 = 5\%$, product 2 keeps its price constant ($\gamma_2 = 0$) and product 3 reduces its price by $\gamma_3 = 15\%$. Calculate the ratio of new quantity to the old quantity for each of the three products, i.e., $\frac{Q_{11}}{Q_{10}}, \frac{Q_{21}}{Q_{20}}, \frac{Q_{31}}{Q_{30}}$.
    a. First, do this using a calculator. Please show the formulae used and the steps in your calculation. (1.5 points)
    b. Interpret and discuss why each of these changes is happening. Which products see an increase in demand and which see a decrease (and why)? (1.5 points)
    c. Write a function that takes as input – (1) the price changes for the three products $(\gamma_1, \gamma_2, \gamma_3)$ and (2) price elasticities and cross price elasticties for all the products, and gives as output the the ratio of the new to old quantities for the three products: $\frac{Q_{11}}{Q_{10}}, \frac{Q_{21}}{Q_{20}}, \frac{Q_{31}}{Q_{30}}$. (2 points)
    d. Evaluate the function at the values of $\gamma_1, \gamma_2, \gamma_3$ specified earlier and store the results in an array named `quantitychange`. Check if the results from the function are the same as that in Step 3a. (1 point)
4. Using the same price changes as in the previous question, calculate what happens to the profits of the three products. Specifically: a. Calculate (using a calculator) the ratio of the new profit to old profit for all three products, i.e., $\frac{\pi_{11}}{\pi_{10}}, \frac{\pi_{21}}{\pi_{20}}, \frac{\pi_{31}}{\pi_{30}}$, and derive the percentage increase or decrease in profit for all three products. Please show the formulae used and the steps in your calculation. (1.5 points)

b. Interpret and discuss why each of these changes is happening. Which products benefit from the price changes and why (or why not)? Is product 3's price cut justified? (1.5 points) c. Now write a function that takes as input the price change percentages $(\gamma_1, \gamma_2, \gamma_3)$, the price and price elasticities for the three products, the retail margin (r), and the marginal costs C_1, C_2, C_3 and gives as output the ratios of new to old profits: $\frac{\pi_{11}}{\pi_{10}}, \frac{\pi_{21}}{\pi_{20}}, \frac{\pi_{31}}{\pi_{30}}$. (**Note:** You can also simplify the function by simply using the vector `quantitychange` that you stored in Question 3 and thereby avoid using price elasticities in this function. This is not necessary, but you may find that this simplifies your function specification significantly. (2 points) d. Evaluate the function at the values specified earlier and store the results in an array named `profitchange`. Check if the results from the function are the same as that from in Step 4a. (1 point)

5. Can you calculate the higest marginal cost at which this price cut is justified for product 3? (1.5 point)

Instructions

1. Late assignments will automatically receive a zero.
2. The points for each question are shown next to it. The total is 15 points.
3. You are expected to write up the assignment using R Markdown (in the style of the handouts you were given for the in-class work). Please answer each question one by one. For each question, show the R commands you used and then briefly present any discussion/text that you want below it. Please submit the .Rmd file and the 'knitted' word document. You do not need to submit any data files.
4. If you have any questions or need any clarifications on the assignment, please post on your question on the discussion board for this assignment. I or the TA will try to answer it as soon as possible. If you see a question from another student that you know the answer to, please feel free to chip in.