INFERENTIAL STATISTICS

CODED PROJECT

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Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	**145**
Players Not Injured	32	38	11	9	**90**
Total	**77**	**94**	**35**	**29**	**235**

Based on the above data, answer the following questions.

1.1 <u>What is the probability that a randomly chosen player would suffer an injury?</u>

With the use of below formula, we can calculate the probability required

Probability of injury = Total players injured/Total Number of players

Total players injured = **145**
Total Number of players = **235**

$$\text{Probability of Injury} = \frac{145}{235}$$

Probability of randomly chosen player suffers an injury is <u>0.61702</u>

1.2 <u>What is the probability that a player is a forward or a winger?</u>

Players can either be forward or winger at a time therefore, it is a mutually exclusive event. We can add the probabilities of forward and wingers

$$\text{Probability of forward or winger } = \frac{\text{Total Forwards}}{\text{Total number of players}} + \frac{\text{Total wingers}}{\text{Total number of Players}}$$

Total Number of Forwards = **94**
Total Number of Wingers = **29**
Total Number of players = **235**

$$\text{Probability of Forward or Winger } = \frac{94}{235} + \frac{29}{235}$$

Probability that a player is a Forward or a Winger is <u>0.5234</u>

1.3 <u>What is the probability that a randomly chosen player plays in a striker position and has a foot injury?</u>

We can find the total number players who is a striker forward and has foot injury from the table.

$$\text{Probability of striker with foot injury} = \frac{\text{Total striker with foot injury}}{\text{Total number of players}}$$

Total Number of strikers with foot injury = **45**
Total Number of players = **235**

$$\text{Probability of striker with foot injury } = \frac{45}{235}$$

Probability of a randomly chosen player plays in a striker position with foot injury is <u>0.19149</u>

1.4 <u>What is the probability that a randomly chosen injured player is a striker?</u>

Given subset of players is injured and we can find the probability of player to be striker per below:

$$\text{Probability of a striker is injured} = \frac{\text{Total strikers}}{\text{Total number of injured players}}$$

Total injured strikers = **45**
Total players injured = **145**

$$\text{Probability of striker Injured } = \frac{45}{145}$$

Probability that a randomly chosen injured player will be a striker is <u>0.31034</u>

<u>Problem 2</u>

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimetre and a standard deviation of 1.5 kg per sq. centimetre. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information; **(Provide an appropriate visual representation of your answers, without which marks will be deducted)**

Mean = 5

Standard deviation = 1.5

2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm?

We shall use Norm.pdf() function with x = 3.17 μ = 5 σ= 1.5 to find the probability of having breaking strength less 3.17.

The proportion of gunny bags with a breaking strength less than 3.17 kg per sq. cm is approximately **0.11123 (11.12%.)**

We can create a plot with x axis having breaking strength using np.linspace() function taking values start= mu - 4*sigma, stop=mu + 4*sigma, with number of samples 100.

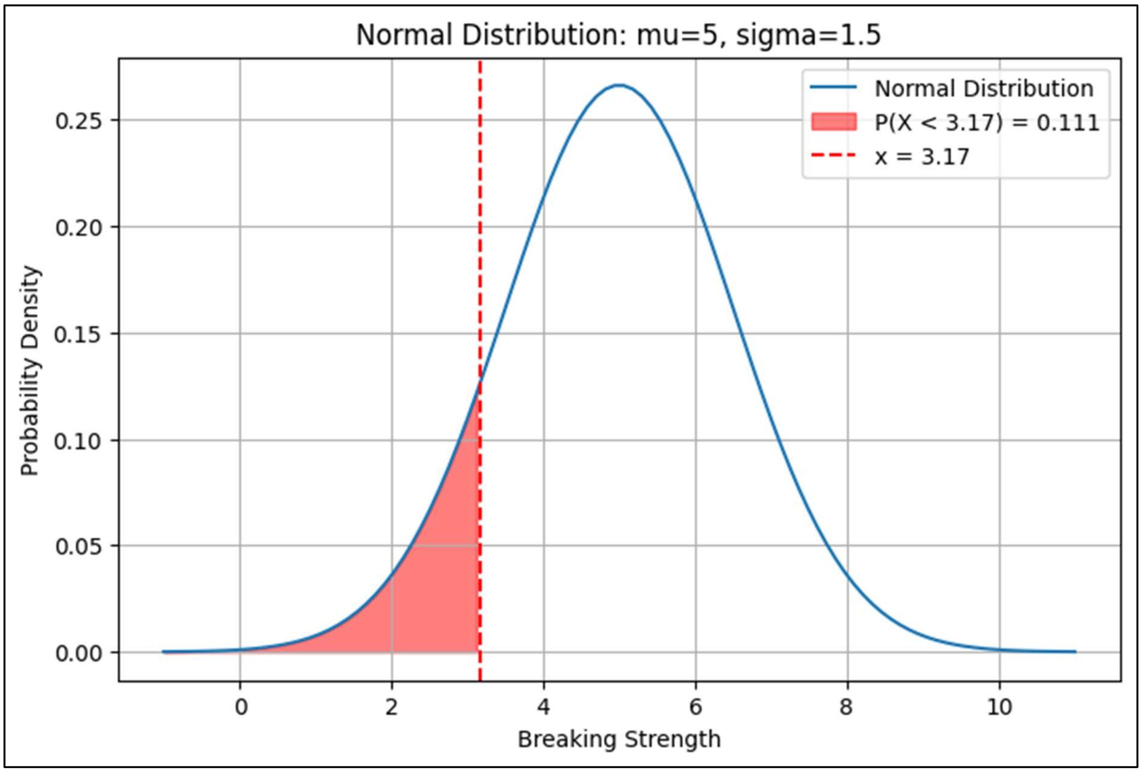

Figure 1 Probability of breaking strength of less than 3.17

2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq cm.?

We can use Norm.cdf() function with x = 3.6, μ = 5, σ= 1.5 to find the probability of having breaking strength of at least 3.6.

Cumulative probability of breaking strength up to 3.6 is 0.1753

We calculate the probability of values being 3.6 or more by subtracting the cumulative distribution function (CDF) value of 3.6 from 1, representing the entire area under the normal curve.

Hence 1 – 0.1753 = 0.82467

The proportion of gunny bags with a breaking strength at least 3.6 kg per sq. cm is approximately 0.82467 **(82.46%.)**

We can create a plot with x axis having breaking strength using np.linspace() function taking values start= mu - 4*sigma, stop=mu + 4*sigma, with number of samples 100.

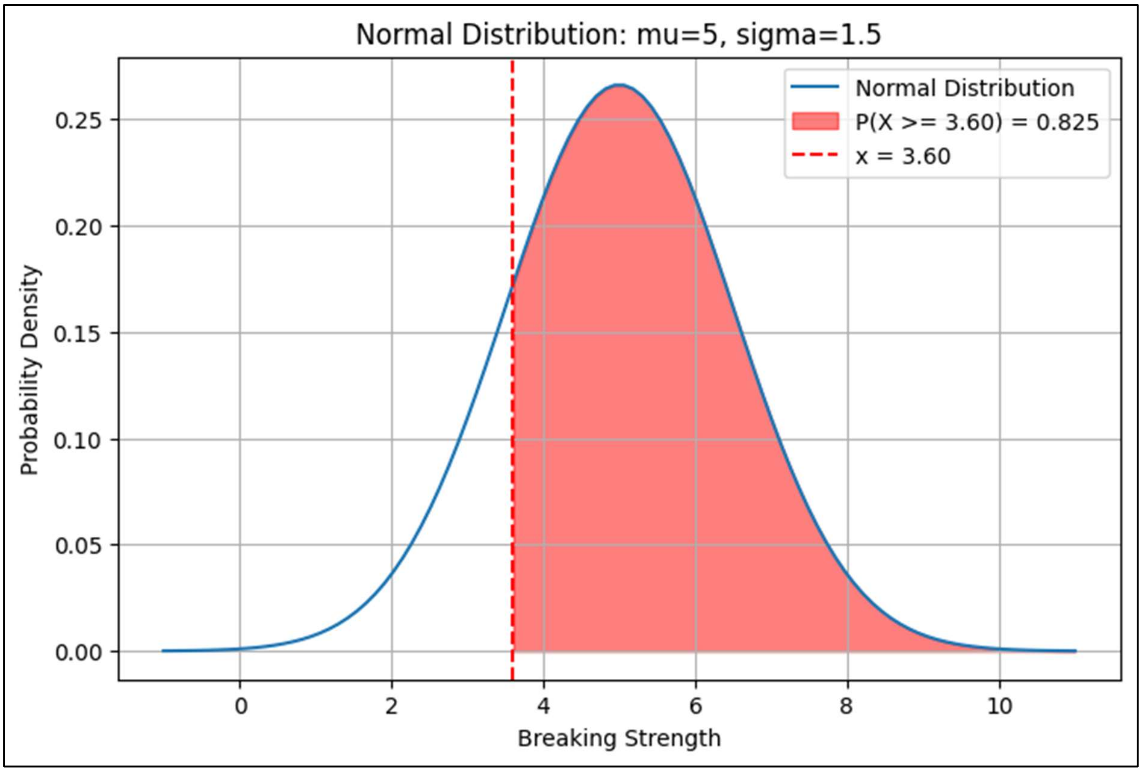

Figure 2 probability of breaking strength of *at least 3.6*

2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

We require probability of breaking strength between 5 and 5.5

First, we should find the cumulative probability up to 5.5 using norm cdf function which is 0.630558

Then, Cumulative probability up to 5 with cdf function, which is 0.5

Subtracting probability of 5 from probability of 5.5

0.630558 - 0.5 = 0.1305

Therefore, the probability of breaking strength between 5 and 5.5 is **0.1305 (13.05%)**

Plot showing the probability of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm

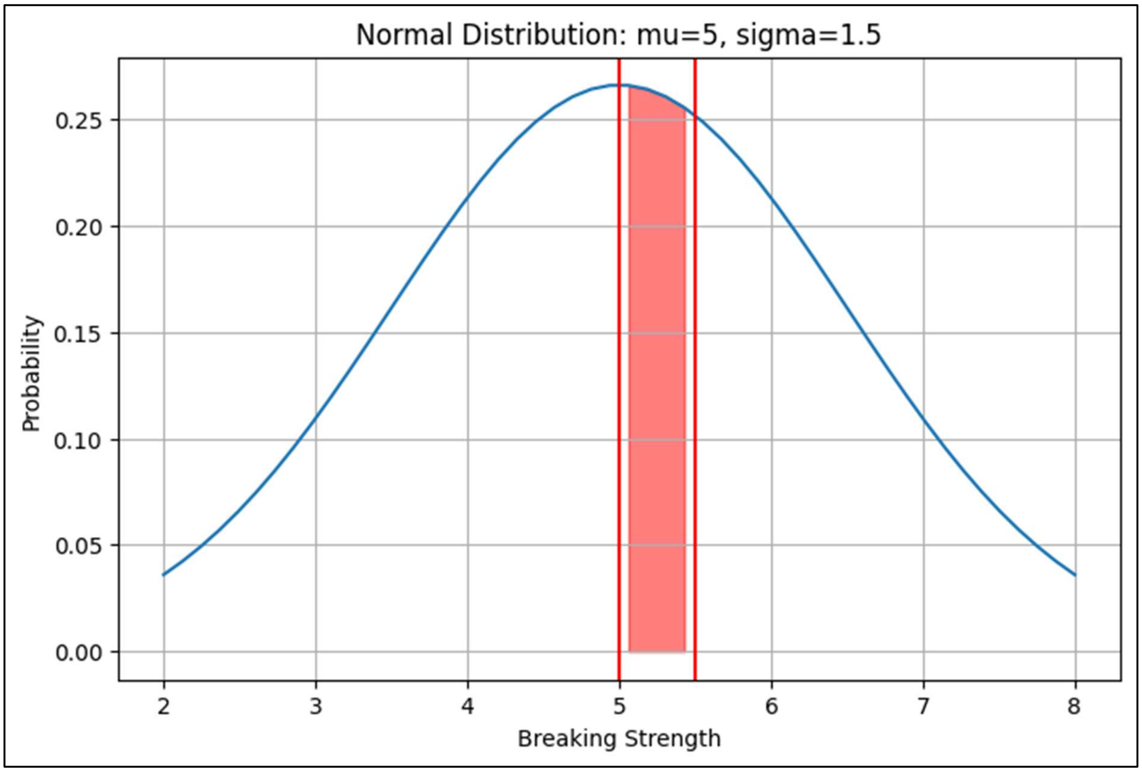

Figure 3 Probability of breaking strength between 5 and 5.5

2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm.?

To find the probability of breaking strength not between 3 and 7.5kg

First, we need to find the cumulative probability of 3 using norm.cdf function which is 0.0912

Next, let us find the cumulative probability of 7.5 using norm.cdf function 0.9522

Then, 1 – (Cumulative probability of 7.5 - Cumulative probability of 3) = 0.139001

Therefore, the probability of breaking strength NOT between 3 and 7.5 kg per sq. cm is approximately **0.139001 (13.90%)**

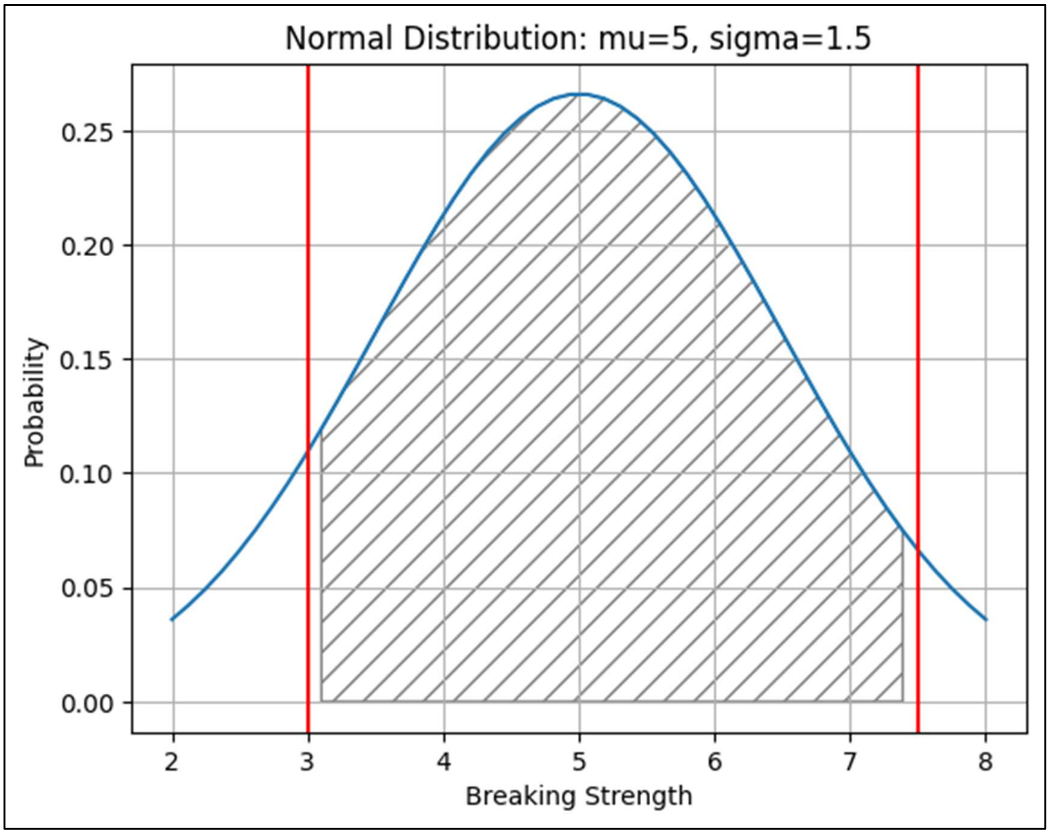

Figure 4 Probability of breaking strength NOT between 3 and 7.5

Problem 3

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

State the null and alternate hypotheses

Let μ be the mean hardness of unpolished stones

H_0: μ < 150

H_1: μ >= 150

Conduct the hypothesis test and compute the p-value

Select Appropriate test

The formulated hypotheses have unknown population standard deviation. Hence, one sample T test will be used to analyse the hypotheses and draw a conclusion.

Let's test whether the T-test assumptions are satisfied or not

- Continuous data - Yes, the hardness of unpolished is measured on a continuous scale.
- Normally distributed population and Sample size > 30 - Yes, it is assumed that the population is normal and the sample size is 75 which is greater than 30.
- Observations are from a simple random sample - Yes, we are informed that the collected sample a simple random sample.
- Population standard deviation is known - No

With help of ttest_1samp function with population mean as 150 and alternative is greater, we computed P-Value as 0.9999

Conclusions from the test results

P value 0.9999 is greater than significance level of 0.05. Hence, we accept the null hypothesis that hardness of unpolished stone is less than 150.

Hence, we have enough statistical evidence to say that hardness of unpolished stone is less than 150.

3.2 Is the mean hardness of the polished and unpolished stones the same?

State the null and alternate hypotheses

Let µ1 be the mean hardness of unpolished stones and µ2 be the mean hardness of Treated and polished stones

Null hypothesis (H_0) : $\mu_1 = \mu_2$

Alternate hypothesis (H_1) : $\mu_1 \neq \mu_2$

Conduct the hypothesis test and compute the p-value

Select Appropriate test

The formulated hypotheses have two sample mean to compare with unknown population standard deviation. Hence, two sample independent T test will be used to analyse the hypotheses and draw a conclusion.

Let's test whether the T-test assumptions are satisfied or not

- Continuous data - Yes, the hardness of unpolished is measured on a continuous scale.
- Normally distributed population and Sample size > 30 - Yes, it is assumed that the population is normal and the sample size is 75 which is greater than 30.
- Observations are from a simple random sample - Yes, we are informed that the collected sample is a simple random sample.
- Population standard deviation is known – No

With help of ttest_ind function setting equal var as false and alternative as two sided, we computed P-Value as 0.001588

Conclusions from the test results

P value 0.001588 is less than significance level of 0.05. Hence, we reject the null hypothesis that the mean hardness of unpolished stones and treated & polished stones are same.

Hence, we have enough statistical evidence to say that mean hardness of unpolished stones and treated and polished stones are not same.

Problem 4

Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favour one method above another and may work better in his/her favourite method. The response is the variable of interest.

Exploratory data analysis

Let us check the data

	Dentist	Method	Alloy	Temp	Response
0	1	1	1	1500	813
1	1	1	1	1600	792
2	1	1	1	1700	792
3	1	1	2	1500	907
4	1	1	2	1600	792

```
 #   Column    Non-Null Count  Dtype
---  ------    --------------  -----
 0   Dentist   90 non-null     int64
 1   Method    90 non-null     int64
 2   Alloy     90 non-null     int64
 3   Temp      90 non-null     int64
 4   Response  90 non-null     int64
dtypes: int64(5)
```

Figure 5 Data preview and information

We have 4 columns of data type as integer. However, Dentist, Method, Alloys are levels/ factors and response dependant variable assumed as Dental hardness.

Statistical Summary

	Dentist	Method	Alloy	Temp	Response
count	90.000000	90.000000	90.000000	90.000000	90.000000
mean	3.000000	2.000000	1.500000	1600.000000	741.777778
std	1.422136	0.821071	0.502801	82.107083	145.767845
min	1.000000	1.000000	1.000000	1500.000000	289.000000
25%	2.000000	1.000000	1.000000	1500.000000	698.000000
50%	3.000000	2.000000	1.500000	1600.000000	767.000000
75%	4.000000	3.000000	2.000000	1700.000000	824.000000
max	5.000000	3.000000	2.000000	1700.000000	1115.000000

Figure 6 Statistical Summary

In the response column we see that mean hardness is 741.77 with Standard deviation of 145.76. We should analyse what causes this standard deviation and which factor affects it.

Let us analyse the level present in each factor

Dentist	count
1	18
2	18
3	18
4	18
5	18

Method	count
1	30
2	30
3	30

Alloy	count
1	45
2	45

Figure 7 Value counts of dentist, method, Alloy

Dentist – 5 levels i.e. 5 groups of dentists.
Method – 3 levels of Dental implant methods
Alloy – 2 types of alloys used for implant

We can create separate data frame for each Alloys as Alloy1 and Alloy2

4.1 How does the hardness of implants vary depending on dentists?

Problem statement mentions separate analysis of each alloy. Hence, we proceed with Alloy 1. We will consider the level of significance as 0.05

Visual inspection for hardness and dentist for both Alloys

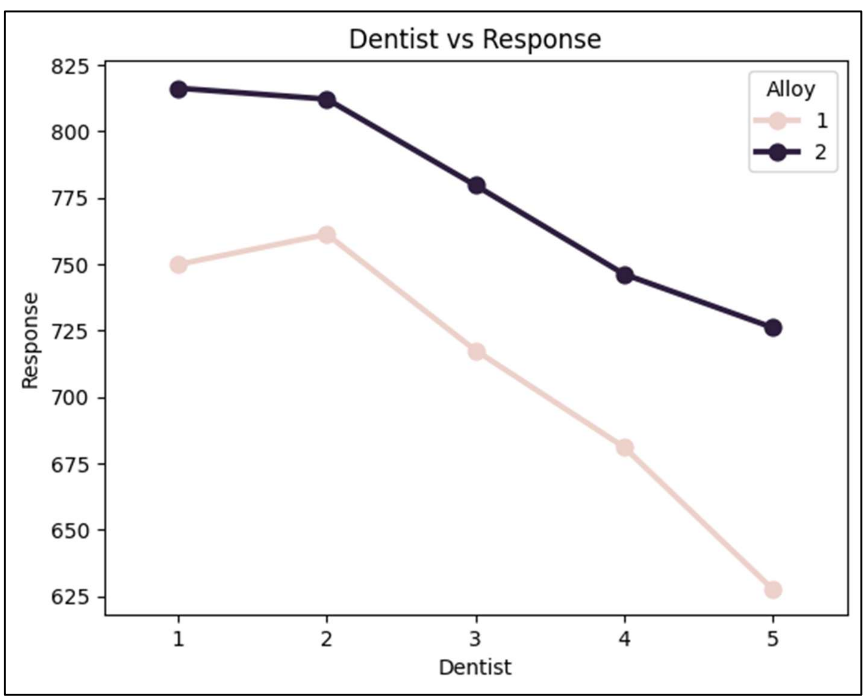

Figure 8 Dentist Vs Response

We find the dentist have effect on hardness, where hardness decreases with increase in dentist level for both alloys. We will procced with statistical testing to find out the statistical evidence for the effect.

Let us preform the test for each alloy separately as required per the problem statement.

Alloy-1

State the null and alternate hypotheses

Null Hypothesis (H_0): The mean hardness of implants is the same across all dentists
Alternative Hypothesis (H_1): The mean hardness of implants differs among dentists

This is a problem, concerning 5 population means of dentist level. One-way ANOVA is an appropriate test here, provided normality and equality of variance assumptions are verified.

One-way ANOVA test

In a one-way ANOVA test, we compare mean values from several populations to test if there is any significance difference between them. The results from an ANOVA test are most reliable when the assumptions of normality and equality of variances are satisfied.

- For testing of normality, Shapiro-Wilk's test is applied to the response variable.
- For equality of variance, Levene test is applied to the response variable.

Check the assumptions of the hypothesis test.

Shapiro-Wilk's test

This test is performed with function stats. Shapiro to find the P -Value

H_0: The dental hardness follows a normal distribution
H_1: The dental hardness does not follow a normal distribution

Obtained P value is 1.1945308699072215e-05 which is less than level of significance. Hence Null hypothesis is rejected. Hardness do not follow normal distribution. Though test failed, we will continue with further testing.

Levene's test

H_0: All the population variances are equal
H_1: At least one variance is different from the rest

With Stats.levene function obtained P-value is 0.256553 which is greater than the significance level. We accept the null hypothesis of homogeneity of variances.

Conduct the hypothesis test and compute the p-value

One-Way ANOVA Testing

We will set response as dependant variable and dentist level as independent variable with the help of Ols model we have obtained the below Anova table

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	1.977112	0.116567
Residual	40.0	539593.555556	13489.838889	NaN	NaN

Figure 8 Alloy1 Dentist ANOVA Table

P- Value is 0.116567 which is higher than significance level of 0.05 hence accept the null hypothesis

Conclusions from the test results

We do not have enough statistical evidence to prove that hardness of implant is affected by dentist for Alloy-1.

Alloy 2

State the null and alternate hypotheses

Null Hypothesis (H_0): The mean hardness of implants is the same across all dentists.
Alternative Hypothesis (H_1): The mean hardness of implants differs among dentists.

Check the assumptions of the hypothesis test.

Shapiro-Wilk's test

This test is performed with the function stats.shapiro to find the P -Value

H_0: The dental hardness follows a normal distribution
H_1: The dental hardness does not follow a normal distribution

Obtained P value is 0.000402 is less than the level of significance. Hence Null hypothesis is rejected. Hardness do not follow normal distribution. Though normality assumption failed, we will continue with further testing.

Levene's test

H_0: All the population variances are equal
H_1: At least one variance is different from the rest

With Stats.levene function obtained P-value is 0.236867 which is greater than the significance level. We accept the null hypothesis of homogeneity of variances.

Conduct the hypothesis test and compute the p-value

One-Way ANOVA Testing

We will set response as a dependant variable and dentist level as an independent variable with the help of Ols model, we obtained below Anova table

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	5.679791e+04	14199.477778	0.524835	0.718031
Residual	40.0	1.082205e+06	27055.122222	NaN	NaN

Figure 9 Alloy2 Dentist ANOVA Table

P- Value is 0.718031 which is higher than significance level of 0.05 hence accept the null hypothesis

Conclusions from the test results

We do not have enough statistical evidence to prove that hardness of the implant is affected by the dentist for Alloy-2

Summary

Hence for both Alloy 1 and 2 , we do not have enough statistical evidence to prove that the hardness of the implant is affected by dentists.

4.2 How does the hardness of implants vary depending on methods?

Let us first do Visual inspection for effect of methods on hardness by creating point plot

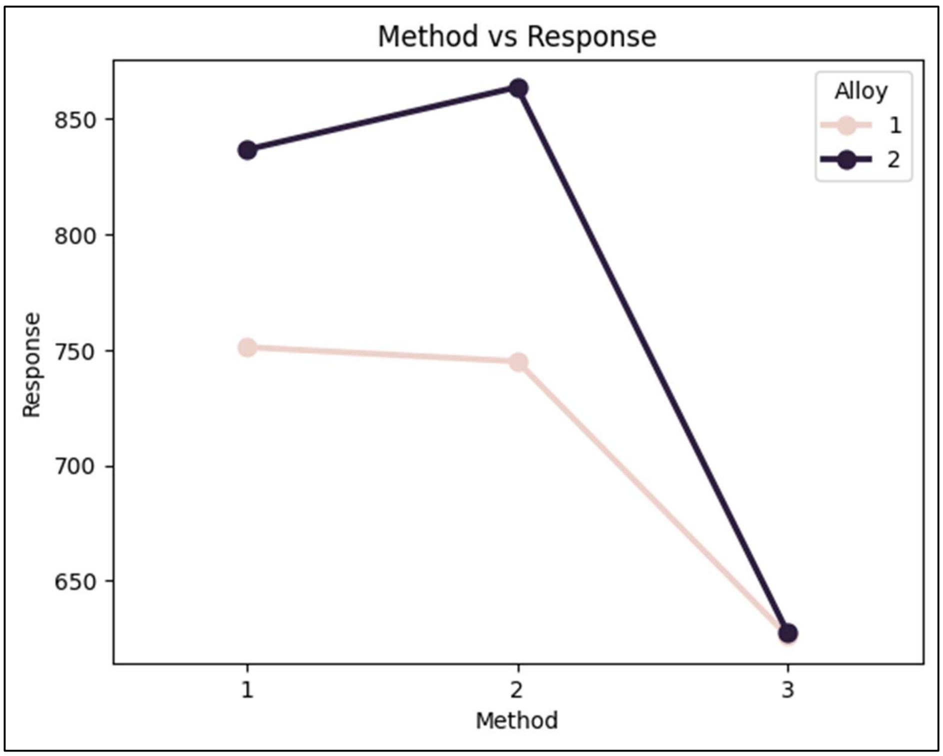

Figure 10 Method Vs Response

We can see that hardness varies for each method significantly. We will procced with statistical testing to find out the statistical evidence for the effect.

Let us preform the test for each alloy separately as required per the problem statement.

Alloy-1

State the null and alternate hypotheses

Null Hypothesis (H_0): The mean hardness of implants is the same across all Methods.
Alternative Hypothesis (H_1): The mean hardness of implants differs for among Methods

Check the assumptions of the hypothesis test.

We have already performed Shapiro-Wilk's test for dental hardness for question 4.1. Hence, result stays same i.e. P value is 0.000402, which is less than level of significance. Hence Null hypothesis is rejected. Hardness do not follow normal distribution. Though test failed, we will continue with further testing.

Levene's test

H_0: All the population variances are equal
H_1: At least one variance is different from the rest

With Stats.levene function obtained P-value is 0.00341 which is less than the significance level, we reject the null hypothesis of homogeneity of variances.

Let's test whether the assumptions are satisfied or not

- The populations are normally distributed – No, the normality assumption failed using the Shapiro-Wilk's test.
- Samples are independent simple random samples - Yes, we are informed that the collected sample is a simple random sample.
- Population variances are equal - No, the homogeneity of variance assumption is rejected using the Levene's test

Though normality and variance assumption are not satisfied, we will continue with further testing.

Conduct the hypothesis test and compute the p-value

One-Way ANOVA Testing

We shall set response as dependant variable and methods level as independent variable with help of Ols model. We have obtained the below Anova table

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	148472.177778	74236.088889	6.263327	0.004163
Residual	42.0	497805.066667	11852.501587	NaN	NaN

Figure 11 Alloy1 Method ANOVA Table

P- Value is 0.004163 which is less than significance level of 0.05 hence reject the null hypothesis

Conclusions from the test results

We have enough statistical evidence to prove that hardness of implant is affected by Methods for Alloy-1

Multiple Comparison test (Tukey HSD)

We can find out which method mean is affecting the hardness using Multiple Comparison test (Tukey HSD)

State the null and alternate hypotheses

$$H_0: \mu_1=\mu_2 \text{ and } \mu_2=\mu_3 \text{ and } \mu_2=\mu_3$$
$$H_1: \mu_1\neq\mu_2 \text{ or } \mu_2\neq\mu_3 \text{ or } \mu_2\neq\mu_3$$

Conduct the hypothesis test and compute the p-value

We computed the below test results using pairwise_tukeyhsd function

```
 Multiple Comparison of Means - Tukey HSD, FWER=0.05
=====================================================
group1 group2  meandiff p-adj   lower    upper   reject
-----------------------------------------------------
     1      2   -6.1333  0.987  -102.714  90.4473  False
     1      3    -124.8 0.0085 -221.3807 -28.2193   True
     2      3 -118.6667 0.0128 -215.2473  -22.086   True
-----------------------------------------------------
```

Figure 12 Alloy1 Method Turkey HSD

P-Value of group 1 and 2 is above level of significance hence null hypothesis is accepted.
P-Value of group 2 & 3 and 1&3 is below level of significance, hence null hypothesis is rejected.

Conclusions from the test results

As the p-values for comparing the mean dental hardness for the Method 1 & 3 and Method 2 & 3 is less than the significance level, the null hypothesis of equality of all population means can be rejected.

Thus, we can say that the mean dental hardness for methods 1 and 2 is similar but dental hardness for Method 3 is significantly different from Method 1 and 2.

Alloy-2

State the null and alternate hypotheses

Null Hypothesis (H_0): The mean hardness of implants is the same across all Methods.
Alternative Hypothesis (H_1): The mean hardness of implants differs for among Methods

Check the assumptions of the hypothesis test.

We have already performed Shapiro-Wilk's test for dental hardness for question 4.1. Hence, result stays same i.e. P value is 0.000402 which is less than level of significance. Therefore, Null hypothesis is rejected. Hardness do not follow normal distribution. Though test failed, we will continue with further testing.

Levene's test

H_0: All the population variances are equal
H_1: At least one variance is different from the rest

With Stats.levene function obtained P-value is 0.04469 which is less than significance level we reject the null hypothesis of homogeneity of variances.

Let's check whether the assumptions are satisfied or not

- The populations are normally distributed – No, the normality assumption failed using the Shapiro-Wilk's test.

- Samples are independent simple random samples - Yes, we are informed that the collected sample is a simple random sample.
- Population variances are equal - No, the homogeneity of variance assumption is rejected using the Levene's test

Though normality and variance assumption are not satisfied, we will continue with further testing.

Conduct the hypothesis test and compute the p-value

One-Way ANOVA Testing

We shall set response as dependant variable and methods level as independent variable with help of Ols model. We have obtained the below Anova table

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	499640.4	249820.200000	16.4108	0.000005
Residual	42.0	639362.4	15222.914286	NaN	NaN

Figure 13 Alloy2 Method ANOVA Table

P- Value is 0.000005 which is less than significance level of 0.05 hence reject the null hypothesis

Conclusions from the test results

We have enough statistical evidence to prove that hardness of implant is affected by Methods for Alloy-2

Multiple Comparison test (Tukey HSD)

We will find out which method mean is affecting the hardness using Multiple Comparison test (Tukey HSD)

State the null and alternate hypotheses

$$H_0: \mu_1=\mu_2 \text{ and } \mu_2=\mu_3 \text{ and } \mu_2=\mu_3$$
$$H_1: \mu_1\neq\mu_2 \text{ or } \mu_2\neq\mu_3 \text{ or } \mu_2\neq\mu_3$$

Conduct the hypothesis test and compute the p-value

We computed the below test results using pairwise_tukeyhsd function

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1	2	27.0	0.8212	-82.4546	136.4546	False
1	3	-208.8	0.0001	-318.2546	-99.3454	True
2	3	-235.8	0.0	-345.2546	-126.3454	True

Figure 14 Alloy2 Method Turkey HSD

P-Value of group 1 and 2 is above level of significance hence null hypothesis is accepted.
P-Value of group 2 & 3 and 1&3 is below level of significance, hence null hypothesis is rejected.

Conclusions from the test results

As the p-values for comparing the mean dental hardness for the Method 1 & 3 and Method 2 & 3 is less than the significance level, the null hypothesis of equality of all population means can be rejected.

Thus, we can say that the mean dental hardness for methods 1 and 2 is similar but dental hardness for Method 3 is significantly different from Method 1 and 2.

Summary

Hence for Alloy1 and Alloy2 the mean dental hardness for methods 1 and 2 is similar but dental hardness for Method 3 is significantly different from Method 1 and 2.

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?

Create an Interaction Plot

We will use interaction plot function for alloy 1 & 2 separately

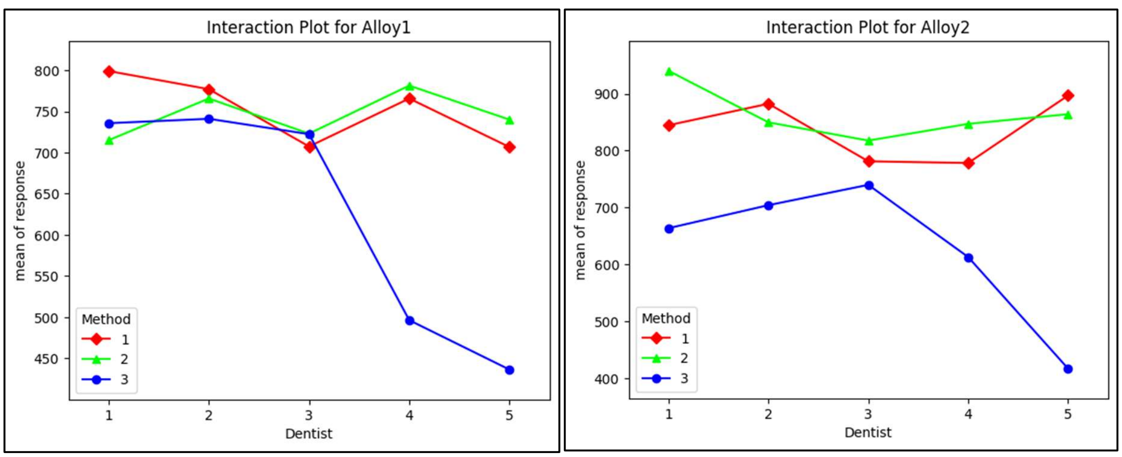

Figure 15 Interaction plot Alloy1 and Alloy 2 for Dentists and Method

Analysis of plot

Alloy 1

- Hardness is almost same for all methods up to dentists 3 implying no significant difference.
- Hardness varies for method 3 of dentist 4 and 5 compared to method 1 & 2

Alloy 2

- Hardness is almost same for methods 1 and 2 for all dentists implying no significant difference.
- Hardness varies for method 3 in all level of dentist compared to method 1 & 2

4.4 How does the hardness of implants vary depending on dentists and methods together?

Alloy-1

State the null and alternate hypotheses

Null Hypothesis (H_0): There is no interaction effect between dentist and method together on the hardness of implants

Alternative Hypothesis (H_1): There is a significant interaction between dentist and method together on the hardness of implants.

Check the assumptions of the hypothesis test.

We have already performed Shapiro-Wilk's test for dental hardness for question 4.1. Hence, result stays same i.e. P value is 0.000402 which is less than level of significance. Hence Null hypothesis is rejected. Hardness do not follow normal distribution. Though test failed, we will continue with further testing.

Levene's test

H_0: All the population variances are equal
H_1: At least one variance is different from the rest

With Stats.levene function obtained P-value is 0.31281 which is greater than significance level we accept the null hypothesis of homogeneity of variances.

Let's test whether the assumptions are satisfied or not

- The populations are normally distributed – No , the normality assumption failed using the Shapiro-Wilk's test.
- Population variances are equal - yes, the homogeneity of variance assumption is verified using the Levene's test

Though normality assumption satisfied and variance assumption is satisfied, we will continue with further testing.

Conduct the hypothesis test and compute the p-value

Two-Way ANOVA Testing

We shall set response as dependant variable and methods , dentist, interaction dentist and method as independent variable with help of ols model. we obtained below anova table

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist):C(Method)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

Figure 16 Alloy1 Interaction of dentist and Methd ANOVA Table

P- Value is 0.006793 which is less than significance level of 0.05 hence reject the null hypothesis.

P-Value of dentist is 0.011484 which explain that dentist level has effect on hardness due interaction of dentist and methods however individually we are unable to find the dentist impact of hardness.

Conclusions from the test results

We have enough statistical evidence to prove that hardness of implant is affected by interaction of dentist and methods for Alloy-1

Multiple Comparison test (Tukey HSD)

We shall find out which method mean is affecting the hardness using Multiple Comparison test (Tukey HSD)

State the null and alternate hypotheses

H_0: There is no significant interaction between method and dentist on the dental hardness
H_1: There is a significant interaction between method and dentist on the mean response

Conduct the hypothesis test and compute the p-value

We computed below test results using pairwise_tukeyhsd function. Below is the table of interaction combination that have significant difference between them.

	group1	group2	meandiff	p-adj	lower	upper	reject
10	1_1	4_3	-302.6667	0.0070	-551.4950	-53.8383	True
13	1_1	5_3	-362.6667	0.0007	-611.4950	-113.8383	True
26	1_2	5_3	-278.6667	0.0173	-527.4950	-29.8383	True
38	1_3	5_3	-299.3333	0.0079	-548.1617	-50.5050	True
46	2_1	4_3	-280.6667	0.0160	-529.4950	-31.8383	True
49	2_1	5_3	-340.6667	0.0016	-589.4950	-91.8383	True
56	2_2	4_3	-269.3333	0.0243	-518.1617	-20.5050	True
59	2_2	5_3	-329.3333	0.0025	-578.1617	-80.5050	True
68	2_3	5_3	-304.6667	0.0065	-553.4950	-55.8383	True
76	3_1	5_3	-271.0000	0.0229	-519.8283	-22.1717	True
83	3_2	5_3	-286.6667	0.0128	-535.4950	-37.8383	True
89	3_3	5_3	-286.0000	0.0131	-534.8283	-37.1717	True
91	4_1	4_3	-269.3333	0.0243	-518.1617	-20.5050	True
94	4_1	5_3	-329.3333	0.0025	-578.1617	-80.5050	True
95	4_2	4_3	-285.0000	0.0137	-533.8283	-36.1717	True
98	4_2	5_3	-345.0000	0.0013	-593.8283	-96.1717	True
103	5_1	5_3	-270.3333	0.0234	-519.1617	-21.5050	True
104	5_2	5_3	-303.6667	0.0067	-552.4950	-54.8383	True

Figure 17 Alloy1 Turkey HSD for interaction dentist and Methd

P-Value of above combination is below level of significance, hence null hypothesis is rejected.

Conclusion of test results

Since the p-values for comparing the mean dental hardness for the combination 5_3 with rest of combination are less than the significance level, the null hypothesis of equality of all population means can be rejected except combination 4_3.

P-values for comparing the mean of dental hardness for the combination 4_3 with rest of combination are less than the significance level, the null hypothesis of equality of all population means can be rejected except combination 1_2 , 1_3, 2_3 ,3_1, 3_2,3_3, 5_3 having p-value higher than the level of significance.

Thus, we can say that the interaction effect of combination 5_3 and 4_3 affects the mean hardness with certain exceptions.

Alloy-2

State the null and alternate hypotheses

Null Hypothesis (H_0): The mean hardness of implants is the same across all Methods.
Alternative Hypothesis (H_1): The mean hardness of implants differs for among Methods

Check the assumptions of the hypothesis test.

We have already performed the Shapiro-Wilk's test for dental hardness for question 4.1. Hence, result stays same i.e. P value is 0.000402 which is less than level of significance. Hence Null hypothesis is rejected. Hardness do not follow normal distribution. Though test failed, we will continue with further testing.

Levene's test

H_0: All the population variances are equal
H_1: At least one variance is different from the rest

With Stats.levene function obtained P-value is 0.783173 which is greater than significance level we accept the null hypothesis of homogeneity of variances.

Let's test whether the assumptions are satisfied or not

- The populations are normally distributed – No , the normality assumption failed using the Shapiro-Wilk's test.
- Population variances are equal - Yes, the homogeneity of variance assumption is satisfied using the Levene's test

Though normality and variance assumption are not satisfied, we will continue with further testing.

Conduct the hypothesis test and compute the p-value

Two-Way ANOVA Testing

We shall set response as dependant variable and methods level as independent variable with help of Ols model. we obtained below Anova table

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist):C(Method)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

Figure 18 Alloy2 Interaction of dentist and Methd ANOVA Table

P- Value is 0.093234 which is greater than significance level of 0.05 hence accept the null hypothesis .

Conclusions from the test results

We do not have enough statistical evidence to prove that hardness of implant is affected by interaction of dentist and methods for Alloy-2

Summary

Interaction of dentist and method have impact on hardness for alloy 1 only with interaction effect of combination 5_3 and 4_3 affecting the mean hardness with certain exceptions.