Errors in Numerical Method

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What is Error in Numerical Method?

- The difference between **the true value** of a quantity and **the approximate value** computed or obtained by measurement is called **Error**.
- So if **XT** and **XA** be the true and approximate value of the solution in solving a problem, then the quantity gives the error in **Xa**. The absolute error **Ea** involved in x is given by **Ea = XT-XA.**
- The relative error Er in Xa is defined by: $$Er = \frac{|XT-XA|}{XT} \quad \text{Provided } XT \neq 0$$
- The percentage error Ep is 100 times the relative error:

$$Ep = Er * 100 = \frac{|XT-XA|}{XT} * 100 \quad \text{Provided } XT \neq 0$$

true value = approximation + error

Absolute, Relative, Percentage Errors

$$\text{Absolute error} = |V_{true} - V_{observed}|$$

$$\text{Relative error} = \frac{|V_{true} - V_{observed}|}{V_{true}}$$

$$\text{Percentage error} = \frac{|V_{true} - V_{observed}|}{V_{true}} \times 100$$

Absolute and Relative Errors

- To illustrate, suppose the number 4.6285 is rounded to 4.628 which is correct into four important numbers.Then we have:

  XT= 4.6285, **XA** = 4.628

- The absolute error is: **Ea**= | 4.6285 - 4.628 | = 0.0005
- The relative error is given by: **Er** = 0.0005 / 4.6285 = 10804 × 10^{-4}
- The percentage error is: **Ep** = 10804 × 10^{-4} x 100= 1.0804 x 10^{-2}

Rounding Error

- This is a one type of computation error which arise due to the process of rounding off the number during the computation. Roundoff errors occur because computers have a limited ability to represent numbers. For example, π has infinite digits, but due to precision limitations, only 16 digits may be stored in MATLAB.
- While this roundoff error may seem insignificant, if your process involves multiple iterations that are dependent on one another, these small errors may accumulate over time and result in a significant deviation from the expected value.

- For example, when a result 2.01536 is rounded off to for decimal places, then:

  XT = 2.01536

  XA = 2.0154

  So in this case the rounded off error is given by:

  Ea = XT - XA = | 2.01536-2.0154 |

  = 0.00004

Example of Rounding Error

The quadratic formula is represented as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Using a = 0.2, b = – 47.91, c = 6 and if we carry out rounding to two decimal places at every intermediate step:

$$x = \frac{-47.91 \pm \sqrt{(-47.91)^2 - 4(0.2)(6)}}{2(0.2)} = \frac{-47.91 \pm \sqrt{2295.36 - 4.8}}{0.4} = \frac{-47.91 \pm 47.86}{0.4}$$

$$x_1 = 239.425$$

$$x_2 = 0.125$$

Example of Rounding Error (cont.)

The error between our approximations and true values can be found as follows:

$$Absolute\ Error_{x1} = \left|\frac{actual - approximation}{actual}\right| * 100 = \frac{239.4246996 - 239.425}{239.4246996} * 100$$
$$= 1.25 \times 10^{-4}\ \%$$

$$Absolute\ Error_{x2} = \left|\frac{0.1253003556 - 0.125}{0.1253003556}\right| * 100 = 24\%$$

As can be seen, the smaller root has a larger error associated with it because deviations will be more apparent with smaller numbers than larger numbers.

Example of Rounding Error (cont.)

If you have the insight to see that your computation will involve operations with numbers of differing magnitudes, the equations can sometimes be cleverly manipulated to reduce roundoff error. In our example, if the quadratic formula equation is rationalized, the resulting absolute error is much smaller because fewer operations are required and numbers of similar magnitudes are being multiplied and added together:

$$x = \frac{2c}{-b \pm \sqrt{b^2 - 4ac}} = \frac{2(6)}{47.91 \pm \sqrt{(-47.91)^2 - 4(0.2)(6)}} = \frac{12}{47.91 \pm 47.86}$$

$$x_1 = 240$$

$$x_2 = 0.1253001984$$

$$Absolute\ Error_{x1} = \left|\frac{actual - approximation}{actual}\right| * 100 = \frac{239.4246996 - 240}{239.4246996} * 100$$
$$= 0.24\%$$

$$Absolute\ Error_{x2} = \left|\frac{0.1253003556 - 0.1253001984}{0.1253003556}\right| * 100 = 1.25 \times 10^{-4}\ \%$$

Post Test

1. Give 3 examples of errors due to cutting process!
2. Give 2 examples of errors due to the rounding process!
3. It is known that the exact value of the calculation of the building area is 403.1476 m^2, while the estimated value is 403 m^2. Calculate absolute errors, relative errors and percentage of errors!
4. Measurements Bridge and pencil lengths of 9999 cm and 9 cm. When the true (exact) length is 10000 cm and 10 cm respectively. Calculate absolute errors, relative errors and percentage of errors of both object mentioned!
5. Give each of the 2 examples of numbers that have 4 and 5 significant figures!