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ORIGINAL ARTICLE

Granular computing-driven two-stage consensus model for large-scale group decision-making

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Abstract
As new technical paradigms like electronic democracy and e-governance have emerged quickly, large-scale group decision-making (LSGDM) has become a significant area of research. In LSGDM, conflicting interests and divergent viewpoints have grown especially widespread, making it challenging to bring individual preferences into line with a productive group consensus. This paper uses the concept of information granules to design a granular LSGDM consensus framework that addresses two core aspects of LSGDM: the clustering process and the consensus reaching process. First, granular hierarchical clustering is designed based on the principle of justifiable granularity, with a novel division index introduced to determine the optimal number of subgroups. Next, the fuzzy consensus measure is defined by the specificity and coverage of information granule, and a two-stage granule consensus model is proposed by integrating the maximum consensus rule and minimum consensus cost to optimize individual opinions and achieve an efficient group consensus. Finally, an illustrative example with detailed experiments is conducted to demonstrate the practicality and effectiveness of the granular LSGDM consensus model in enhancing consensus and group division among DMs.

Keywords Granular aggregation method · Two-stage granular consensus model · Large-scale group decision-making · Principle of justifiable granularity

Introduction

Group decision-making (GDM) is crucial for addressing practical problems, as it incorporates the diverse perspectives of different decision-makers (DMs) to arrive at reasonable decision results. GDM considers the preferences of DMs to identify the ideal solution from a set of possible alternatives [1], with the final solution derived from the adjusted preference information of the group members. Currently, GDM has been widely applied in fields such as connected autonomous vehicles [2], supply chain investment [3], renewable energy [4], medical diagnosis [5], and emergency plans making [6].

The traditional method of having DMs participate in live activities and numerically express their choices has evolved into more diverse and complex forms of interaction. With advancements in network technology and the widespread use of social media, DMs can now engage in the decision-making process through multiple channels, leading to the emergence of large-scale group decision-making (LSGDM). If the number of DMs involved in solving a decision problem exceeds 20, it may be classified as an LSGDM problem [7]. However, there are two challenges associated with the implementation of LSGDM. One challenges is managing a large number of participants and their inputs, which can be quite intricate and time-consuming. Another is that the effectiveness of decision-making in LSGDM can be influenced by various factors, including the level of expertise and knowledge of the participants, as well as the potential for biases and groupthink.

To address LSGDM issues, it is crucial to establish a decision-making model that considers both individual perspectives and group opinions. The ultimate aim is to enhance

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the effectiveness of GDM and overall group satisfaction by identifying potential problems and achieving consensus or agreement. Currently, numerous approaches have been established to address LSGDM issues [8–12]. The LSGDM resolution typically consists of two significant processes [13]: (1) The group clustering process seeks to divide numerous DMs into smaller groups to reduce the complexity and cost of LSGDM. (2) The consensus reaching process (CRP) enables DMs to adjust their preferences or those of their groups, contributing to higher satisfaction with the final outcome.

In the clustering process, DMs can be categorized into several subgroups based on their individual preferences. The goal is to group DMs by their similarities, ensuring that those with similar opinions are placed in the same group. Various clustering algorithms have been designed to categorize DMs in LSGDM problems. Liang et al. [14] proposed a novel clustering model based on opinion similarity and trust relationships among DMs, maximizing the combined cohesion of the subgroups. Ding et al. [15] developed a conflict-relationship-based clustering method to effectively address opinion conflicts in LSGDM. Liang et al. [16] presented a clustering method based on the similarity-trust score between DMs to reduce the complexity of decision-making. Wang et al. [17] designed a dual-attribute affinity propagation algorithm to divide the large group into manageable subgroups. Peng and Chen [18] proposed a clustering method based on the picture fuzzy Chi-square similarity measure to reduce the complexity and costs associated with the decision-making process. Tang et al. [19] applied the K-medoids clustering algorithm to cluster DMs into multiple subgroups, aiming to reduce the dimension of the problem. Yang et al. [20] offered a two-stage clustering approach that groups DMs by objectively selecting initial cluster centers based on the number of supporters and the separations between DMs. The aforementioned works highlight how existing clustering methods simplify the LSGDM problems and offer a direct approach to identifying similar preference within the LSGDM framework.

Differences in cognition, background, experience and expertise among DMs lead to variations in their evaluation of alternatives, often resulting in a lack of consensus among DMs and hindering the achievement of a universally accepted solution. Consequently, the initial preference aggregation may only be acceptable to a subset of DMs [21]. To address this, CRP must be implemented to enhance consensus among DMs, ensuring broader acceptance of the final result [22]. CRP is a dynamic, iterative group negotiation process that guides DMs towards consensus [23]. By adjusting DMs' preferences, their perspectives can be better aligned, facilitating the establishment of a high-level consensus before making a group decision.

However, consensus adjustment is both time-consuming and resource-intensive, requiring low-cost adjustment strategy. Many studies have focused on narrowing the preference gaps among DMs and achieving consensus, considering the challenges of time and resource limitations. Xing et al. [24] established an incentive-based minimum adjustment consensus model by developing a trust-relationship-driven incentive mechanism to generate recommendations for individual preference. Yuan et al. [25] designed a budget-constrained consensus framework to reduce group conflict to address non-cooperative behaviors. Meng et al. [26] used the Gini coefficient to measure the fairness of consensus adjustment and proposed a two-stage consensus mechanism to minimize both consensus adjustment and ensure fairness while respecting minority opinions. Wang et al. [27] established a feedback adjustment model that minimizes individual adjustment cost and maximizes group consensus. Ma et al. [28] utilized a visual adjustment path in the maximum expert consensus model to avoid over-adjusted individual opinions and facilitate consensus formation. Feng et al. [29] offered the minimum cost consensus model, considering altruism-fairness preferences within a social trust network to enhance DMs' satisfaction. Qin et al. [30] developed a social network-driven bi-level minimum cost consensus model, focusing on the interactions between the moderator and DMs, as well as among DMs themselves.

The aforementioned analysis suggests that the clustering process and CRP offer valuable insights. However, several unsolved problems persist in practical LSGDM, especially when the number of DMs reaches the hundreds or thousands. These issues can be summarized as follows:

1) In the clustering process, methods that rely on a predefined number of subgroups often struggle with large-scale data. While traditional techniques, such as the elbow method and silhouette coefficient, can help determine the number of subgroups, they primarily focus on data patterns and lack the ability to quantify trade-offs among multiple objectives, such as consensus levels among DMs and conflicts of interest. Additionally, existing clustering methods are primarily designed for decision-making groups of up to 50 individuals and have not yet been validated for decision problems involving thousands of participants.
2) In consensus measurement, distance functions are commonly used to measure the consensus level. However, different distance functions can produce varying results during the CRP and can influence the convergence speed in distinct ways. Currently, there is no benchmark to determine which distance function performs best in practical applications, creating challenges in selecting the

appropriate distance function and ensuring the effectiveness and reliability of consensus measurement.

Granular computing (GrC) is a new paradigm extracting knowledge for complex data, providing a highly abstract data modeling strategy. Recently, integrating GrC with group decision-making problems offers a potential way for constructing CRP models [31, 32]. Research in references [31, 33, 34] indicates that information granules, as entries for preference information, contribute to achieving group consensus. Additionally, the group opinion derived from traditional aggregation operators is often susceptible to the influence of outliers. To mitigate this influence and to describe the consensus level among groups more flexibly, this study proposes the granular LSGDM (G-LSGDM) consensus model for aggregating preference information, measuring the consensus degree, and implementing the consensus feedback mechanism. Specifically, information granules are generated by introducing coverage and specificity, guided by the principle of justifiable granularity [35], and then the aggregation method and the consensus measurement function based on information granularity are designed. This method allows for the description of group opinions and illustrate the proximity between individual opinions and the group opinion. The main contributions of the G-LSGDM consensus model can be categorized into three aspects.

1.) During the clustering process, we develop a granular hierarchical clustering method that leverages local and global consensus levels to create a division index for determining the number of subgroups. Additionally, we introduce a granular aggregation method that generates valuable information granules representing group opinions and reduces outliers.
2.) We create a fuzzy consensus measurement method that prioritizes the quality of the consensus granule from a GrC perspective. This method describes the level of consensus attained and provides flexible control over the number of DMs needed to reach consensus.
3.) A two-stage consensus model is developed for balancing moderator costs with consensus level, resulting in an efficient and cost-effective consensus.

The organization of this study is below. Section Preliminaries provides an overview of granular computing and the consensus model. Section Granular large-scale group decision making consensus model presents a detailed explanation of the G-LSGDM consensus model, including clustering process, information aggregation process, and consensus reaching process. In Section Example, a case study on awarding honours to outstanding graduates is used to illustrate the presented granular LSGDM model. Section Comparison analysis provides a detailed comparative discussion of the proposed consensus model. Section Conclusion concludes the study with a summary of the research findings and suggestions for future research.

Preliminaries

This section reviews some basic concepts and definitions associated with the design of G-LSGDM.

Interval fuzzy number

Fuzzy interval numbers are a valuable tool for reflecting the uncertainty in DMs' preferences. By using interval fuzzy numbers, DMs can articulate their preferences in a way that considers the uncertainty associated with the evaluation objectives.

Definitoin 1 [36] *Interval number is defined as* $p = \left[p^-, p^+\right] = \{y \mid p^- \leq y \leq p^+\}$, *where* p^- *and* p^+ *represent the lower and upper boundaries of the interval defined by* p.

Definitoin 2 [37] *Let* $I(\cdot)$ *represent the length of an interval, the priority* $Pr\,(p_1, p_2)$ *of the interval number* p_1 *relative to* p_2 *is calculated below,*

$$Pr\,(p_1, p_2) = \frac{\min\left\{I(p_1) + I(p_2), \max\left\{p_2^+ - p_1^-, 0\right\}\right\}}{I(p_1) + I(p_2)},$$

where $p_1 = \left[p_1^-, p_1^+\right]$ *and* $p_2 = \left[p_2^-, p_2^+\right]$.

Definitoin 3 [37] *The similarity* $S\,(p_1, p_2)$ *between two interval numbers* p_1 *and* p_2 *is calculated below,*

$$S\,(p_1, p_2) = 1 - \frac{\left|p_1^- - p_1^-\right| + \left|p_2^+ - p_2^+\right|}{p_1^+ + p_1^- + p_2^+ + p_2^-}.$$

Principle of justifiable granularity

Information granule is element factors in granular computing, which can take on different forms, such as numerical, symbolic, linguistic, or perceptual. The local structures of complex data can be represented and comprehended by breaking them down into meaningful information granules, which can then be combined to form a higher-level hierarchical structure. This process creates a global and more abstract description that reveals the data structures. Pedrycz [38] proposed the principle of justifiable granularity, which offers guidelines on how to generate information granules. The focus of this principle is on two conflicting requirements: coverage and specificity, and seeks to balance them in order to achieve optimal results.

1) Coverage, denoted by Cov, indicates the coverage degree of an information granule. The value of Cov increases with the size of the information granule.
2) Specificity Sp is a measure of the compactness of an information granule, where its value decreases as the size of the information granule increases.

In practical applications, the determination of Cov and Sp can take various forms. Cov is typically treated as a function that increases with the cardinality of information granules, while Sp is defined as a function that decreases with the size of the information granules. As coverage increases, specificity generally decreases. Higher values of both specificity and coverage suggest better quality of the information granules. Therefore, we need to find a compromise in the granule generation. The given information granule should not only cover the preferences of the DMs as much as possible, but also comprehensively reflect the preference information of each DM. The preference information granule can be obtained by maximizing the product of its specificity and coverage,

$$Q = Cov \cdot Sp. \tag{1}$$

As the value of Q increases, the information granules become more effective in terms of coverage and specificity. The principle of justifiable granularity can guide the granulation of preference information at different abstract levels, providing a range for adjustment [32]. In this study, the generated information granules offer a more comprehensive description and interpretation of subgroup preference information.

Consensus model

In the LSGDM problem, achieving a consensus frequently requires engaging in discussions and employing persuasive strategies. The linear consistent cost function is expressed below [39, 40],

$$\sum_{i=1}^{n} c_i \left| p_i - \overline{p_i} \right|,$$

where p_i is the individual preference of DM e_i, c_i represents the unit cost of changing the individual preference of e_i, and $\overline{p_i}$ denotes the modified individual preference.

Minimum cost consensus model

Let σ be the deviation threshold, which is satisfied when $|\overline{p_i} - \overline{cp}| \leq \sigma$, where $\overline{cp}$ represents the adjusted group preference information. Typically, if the condition $|\overline{p_i} - \overline{cp}| \leq \sigma$ is true, it indicates that the group has achieved consensus. If this condition does not hold for a specific issue, DMs are required to adjust their individual preferences or opinions to reach a higher consensus level. The minimum cost consensus model is proposed below,

$$\min \sum_{i=1}^{n} c_i \left| p_i - \overline{p_i} \right|$$
$$\text{s.t.} \begin{cases} |\overline{p_i} - \overline{cp}| \leq \sigma \\ \overline{cp} = \sum_{i=1}^{n} w_i \overline{p_i} \end{cases},$$

where w_i represents the weight assigned to the e_i.

Maximum expert consensus model

The objective of the maximum expert consensus model (MECM) is to determine the optimal number of DMs that can reach a consensus while remaining within a specific cost threshold. Similarly, the MECM [41] is extended and expressed below,

$$\max \sum_{i=1}^{n} x_i$$
$$\text{s.t.} \begin{cases} \sum_{i=1}^{n} c_i \left| p_i - \overline{p_i} \right| \leq B \\ \overline{cp} = \sum_{i=1}^{n} w_i \overline{p_i} \\ x_i = \begin{cases} 1, & \text{if } |\overline{p_i} - \overline{cp}| \leq \sigma \\ 0, & \text{else} \end{cases} \end{cases},$$

where B denotes the given cost budget, and x_i is the consensus indicator for p_i.

Granular large-scale group decision making consensus model

The developed G-LSGDM consensus model will be discussed in detail in this section. Fig. 1 illustrates the flowchart of the consensus model, which includes the aggregation process, clustering process and consensus reaching process.

When addressing LSGDM problems, several factors and their corresponding symbols must typically be considered, as follows,

- $E = \{e_1, e_2, \ldots, e_n\}$ is the set of n decision-makers, where e_k denotes the k-th decision-maker, $k \in \{1, 2, \cdots, n\}$.
- $A = \{a_1, a_2, \ldots, a_s\}$ represents the set of s alternatives, where a_i denotes the i-th alternative, $i \in \{1, 2, \cdots, s\}$.
- $B = \{b_1, b_2, \ldots, b_t\}$ denotes the set of t attributes, where b_j denotes the j-th attribute, $j \in \{1, 2, \cdots, t\}$.

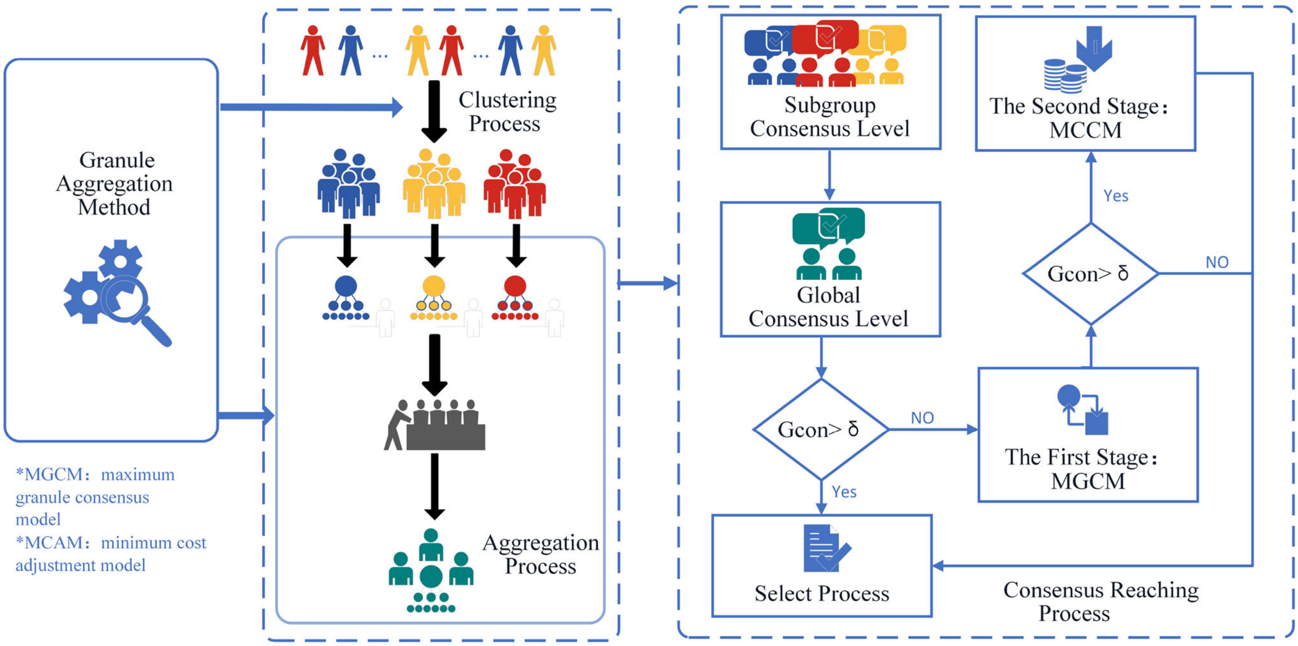


Fig. 1 The Flowchart of Granular Large Scale Group Decision Making Consensus Model

- $C = \{c_1, c_2, \ldots, c_m\}$ is the set of m subgroups, where c_h denotes the h-th subgroup and the number of members in c_h is n_h, $h \in \{1, 2, \cdots, m\}$.
- $p^k = (p^k_{ij})_{s\times t}$ means the individual preference provided by the k-th decision-maker, where p^k_{ij} denotes the preference information of the alternative a_i on the attribute b_j denoted by interval numbers.
- $p^{hk} = (p^{hk}_{ij})_{s\times t}$ means the individual preference information provided by the k-th decision-maker of the h-th subgroup.
- $q^h = (q^h_{ij})_{s\times t}$ represents the subgroup preference information for subgroup c_h.
- $g = (g_{ij})_{s\times t}$ denotes the global preference information.

Granular aggregation method

In this study, the individual preferences of DMs are granulated to describe subgroups and assist in aggregating information from different subgroups. Rather than relying on single values, the designed granular aggregation method leverages information granules to represent the essential information regarding DMs' preferences within subgroups.

In this study, preference information granule $q^h_{ij} = \left[c^h_{ij} - r^h_{ij}, c^h_{ij} + r^h_{ij}\right]$ is created to represent the aggregated preference information of alternative a_i regarding attribute b_j in subgroup c_h. The terms c^h_{ij} and r^h_{ij} represent the midpoint and radius of the interval granule, respectively. To ensure that $q^h_{ij} \subseteq [0, 1]$, the value of r^h_{ij} is limited to the range of $(0, \alpha \cdot \min\left\{1 - c^h_{ij}, c^h_{ij} - 0\right\})$, where α is a parameter used to control the radius range.

For determining the optimal midpoint and radius $\{r^h_{ij}, c^h_{ij}\}$ of q^h_{ij}, we take the following formula,

$$\{r^h_{ij}, c^h_{ij}\}_{opt} = \arg\max \ Q(r^h_{ij}, c^h_{ij}),$$

where the term $Q(r^h_{ij}, c^h_{ij})$ refers to the quality of the preference information granule q^h_{ij},

$$Q(r^h_{ij}, c^h_{ij}) = Cov^h_{ij} \cdot Sp^h_{ij},$$

where Cov^h_{ij} and Sp^h_{ij} represent the coverage and specificity of q^h_{ij}, respectively.

The Cov^h_{ij} represents the coverage of the preference information encapsulated within the given preference information granule. Its form is taken as follows,

$$Cov^h_{ij} = \frac{1}{n_h} \cdot \sum_{k=1}^{n_h} I\left(q^h_{ij} \cap p^{hk}_{ij}\right).$$

The Sp^h_{ij} of interval information granule is calculated as follows,

$$Sp^h_{ij} = \frac{n_h}{\sum_{k=1}^{n_h} I\left(q^{hl}_{ij} \cup p^{hk}_{ij}\right)}.$$

Granular hierarchical clustering

The hierarchical clustering method (HCM) is a widely used approach for clustering data without requiring the number of clusters to be determined beforehand. It relies on distance measures and connection types to generate various clustering results. The use of HCM helps break down LSGDM problems into smaller, more manageable subgroups, facilitating a more effective decision-making process. By addressing each subproblem separately, we can achieve local solutions that can be combined to derive the overall solution.

This study introduces the granular hierarchical clustering method (G-HCM), which combines granular computing with hierarchical clustering to investigate group division and aggregate preference information in LSGDM. The principle of justifiable granularity is suitable for guiding the granulation of preference information at different abstract levels, thereby enabling the exploration of a two-stage consensus-reaching model that incorporates granular computing.

G-HCM

G-HCM measures the distance between two subgroups based on aggregation information. The distance $d(u, v)$ between the preference information granule $q_{ij}^u = [q_{ij}^{u-}, q_{ij}^{u+}]$ and $q_{ij}^v = [q_{ij}^{v-}, q_{ij}^{v+}]$ is defined in the following way,

$$d(u, v) = 1 - \frac{\sum_{i=1}^{s} \sum_{j=1}^{t} S(q_{ij}^u, q_{ij}^v)}{s \cdot t}, \tag{2}$$

where $S(q_{ij}^u, q_{ij}^v)$ denotes the similarity between interval numbers as described in Definition 3.

Algorithm 1: Granular Hierarchical Clustering

Input: Decision-makers' preference information $p^k = (p_{ij}^k)_{s\times t}$, $k \in \{1, 2, \cdots, n\}$

Output: Hierarchical clustering results

for $i = 1$ *to* n **do**

  Create the i-th cluster containing the i-th decision-maker's preference information.

end

Calculate the distances between clusters, respectively.

while $m > 1$ **do**

  Find the nearest pair of clusters based on the distance matrix.

  Merge the closest pair of clusters to form a new cluster.

  Calculate the distance between the new cluster and all other clusters using Eq. (2).

  Obtain the preference information granule for the new cluster using the granular aggregation method proposed in Section Granular aggregation method.

end

Selecting the number of clusters

Considering the ultimate goal of achieving consensus among subgroup members in LSGDM, a new division index is proposed to determine the number of subgroups based on two essential components: the global consensus level $Gcon$, derived from subgroup clustering outcomes, and the consensus level within subgroups $in\_con^h (h = 1, 2, \cdots, N-l)$, as given in Eq. (10) and Eq. (6), respectively. By comparing the division index across different potential subgroup partitions, the optimal number of subgroups can be determined.

Let N represent the number of layers of pedigree chart of clustering, $CI(l)$ represent the division index corresponding to the l-th layer ($l = 1, 2, \cdots, N$),

$$CI(l) = \beta \cdot \frac{Gcon(l)}{\sum_{l=1}^{N} Gcon(l)} + (1-\beta) \cdot \frac{\frac{1}{N-l}\sum_{h=1}^{N-l} in\_con^h(l)}{\sum_{l=1}^{N} \frac{1}{N-l}\sum_{h=1}^{N-l} in\_con^h(l)} + \lambda \cdot \ln \frac{1}{N-l}, \tag{3}$$

where $Gcon(l)$ represents the global consensus level of the clustering result on the l-th layer, and $in\_con^h(l)$ represents the consensus level within subgroup c_h on the l-th layer, λ is the penalty coefficient. Larger subgroups are more capable of capturing common opinions and preferences, thereby minimizing the impact of different individual views on group preference information. In smaller subgroups, the opinions and information of DMs can significantly influence decision results, increasing the complexity of the CRP. By maintaining larger subgroups, the influence of information among different DMs is balanced, which helps reduce the influence of information bias on clustering results.

The weight adjustment parameter β aims to balance the influence of global consensus level and the consensus level within subgroups on the division index in solving LSGDM problems. By adjusting the value of β, the weights of $Gcon$ and in_con^h in clustering results can be controlled. When β has a smaller value, in_con^h has a greater impact on the division index, resulting in clustering results that prioritize the preservation of distinct characteristics among subgroups. Conversely, when β has a larger value, $Gcon$ has a significantly impact on the division index, highlighting the importance of group consensus levels among all DMs. By selecting and adjusting the value of β based on the requirements of LSGDM, a better balance between $Gcon$ and in_con^h can be achieved to obtain more reasonable group division results. We choose the one with the highest value by calculating the division index $CI(l)$ for the clustering results in the pedigree chart.

Consensus reaching process

The CRP encompasses both consensus measurement and preference adjustment. The former evaluates the group consensus level, while the latter promotes the attainment of consensus, both play a significant role in achieving while assessing consensus.

Consensus measuring process

This study assesses DMs' agreement on collective preference information form two perspectives: the representativeness of collective preference with respect to DMs' preferences and the reliability of aggregated preference information.

The consensus level within subgroups and the global consensus level serve as indicators of the agreement and consent among DMs regarding the aggregated subgroup and global preference information, which impacts the entire decision-making process. The steps for measuring consensus level are outlined below.

Individual consensus level of decision-makers: Let in_con^{hk} represent the consensus level of DM e_k within subgroup c_h, which is calculated as follows,

$$in\_con_{ij}^{hk} = \frac{I\left(q_{ij}^{h} \cap p_{ij}^{hk}\right)}{I\left(q_{ij}^{h}\right)}, \tag{4}$$

$$in\_con^{hk} = \frac{\sum_{i=1}^{s}\sum_{j=1}^{t} in\_con_{ij}^{hk}}{s \cdot t}. \tag{5}$$

Consensus level within subgroups: Let in_con^{h} reflect the consensus level reached by each subgroup members for its subgroup c_h. It can be calculated as follows,

$$in\_con^{h} = \frac{\sum_{k=1}^{n_h} in\_con^{hk}}{n_h}. \tag{6}$$

Consensus level between subgroups: Let con^{h} represent the consensus level of subgroup c_h, and g_{ij} represent the global preference information of the alternative a_i concerning the attribute b_j. The corresponding calculation methods are shown as follows,

$$con_{ij}^{h} = \frac{I\left(g_{ij} \cap q_{ij}^{h}\right)}{I\left(g_{ij}\right)}, \tag{7}$$

$$con^{h} = \frac{\sum_{i=1}^{s}\sum_{j=1}^{t} con_{ij}^{h}}{s \cdot t}. \tag{8}$$

Consensus level of global: The global consensus level $Gcon$ can be defined as follows.

$$Gcon_{ij} = \frac{\sum_{h=1}^{m} con_{ij}^{h}}{m}, \tag{9}$$

$$Gcon = \frac{\sum_{i=1}^{s}\sum_{j=1}^{t} Gcon_{ij}}{s \cdot t}, \tag{10}$$

where m represents the number of subgroups.

Given a predefined consensus threshold δ, two scenarios can be distinguished:

① $Gcon \geq \delta$ indicates that DMs reach an agreement without changing the preference information.

② $Gcon < \delta$ indicates that DMs do not have an acceptable agreement, hence consensus adjustment process should be implemented.

Consensus adjustment process

During the consensus adjustment process, DMs strive to minimize their adjustment expenses while achieving their goals. This stems from the rational belief that lower costs are preferable. Consequently, this study proposes a two-stage consensus adjustment model based on granular computing. The first stage is to determine whether a predefined consensus threshold can be reached, while the second stage focuses on identifying the adjustment strategy to minimize adjustment costs.

The first stage: *maximum granular consensus model (MGCM)*

In the two-stage consensus model, prioritizing the local adjustment strategy over the global adjustment can improve efficiency. The key is to initially create identification rules to select the specific alternatives and attributes requiring adjustment. The set of local positions to be adjusted is expressed as SNA,

$$SNA = \left\{(i, j) \mid Gcon_{ij} < Gcon\right\}. \tag{11}$$

The preferences for certain specific locations are modified based on the set SNA to improve the consensus level during the first stage of the two-stage consensus adjustment model.

Let θ_{ij}^{h} denote the adjustment parameter, $\overline{q}_{ij}^{h} = [\overline{q}_{ij}^{h-}, \overline{q}_{ij}^{h+}]$ represent the adjusted preference information of subgroup c_h, and $\overline{g}_{ij} = [\overline{g}_{ij}^{-}, \overline{g}_{ij}^{+}]$ represent the adjusted global preference information. The MGCM is developed as Model 1,

$$\text{(Model 1)} \quad \max \overline{Gcon}$$
$$s.t. \begin{cases} \overline{Gcon}_{ij} = \frac{\sum_{h=1}^{m} \overline{con}_{ij}^{h}}{m} \\ \overline{Gcon} = \frac{\sum_{i=1}^{s}\sum_{j=1}^{t} \overline{Gcon}_{ij}}{s \cdot t} \\ \overline{q}_{ij}^{h-} = \theta_{ij}^{h} \cdot g_{ij}^{-} + \left(1-\theta_{ij}^{h}\right) \cdot q_{ij}^{h-} \\ \overline{q}_{ij}^{h+} = \theta_{ij}^{h} \cdot g_{ij}^{+} + \left(1-\theta_{ij}^{h}\right) \cdot q_{ij}^{h+} \\ \overline{con}_{ij}^{h} = \frac{I\left(\overline{g}_{ij} \cap \overline{q}_{ij}^{h}\right)}{I\left(\overline{g}_{ij}\right)} \\ \theta_{ij}^{h} \in (0,1) \\ (i,j) \in SNA \end{cases}, \tag{12}$$

where $\overline{Gcon}$ stands for the global consensus level of the adjusted preference information for subgroup c_h.

There are two possible results after completing the first adjustment step:

① $\overline{Gcon} < \delta$ means that the minimum consensus cost adjustment is disregarded, and the adjusted preference information is aggregated and sorted to obtain the final decision result.

② $\overline{Gcon} \geq \delta$ means that an adjustment strategy is required to reduce costs and proceed to the second stage.

The second stage: *minimum cost adjustment model (MCAM)*

In the two-stage adjustment model, the composition of unit adjustment costs includes variable costs and fixed costs. The variable costs are closely related to the consensus level within subgroups. As the consensus level increases, variable costs exhibit a downward trend.

Definitoin 4 *Let AC^h denote the unit adjustment cost of subgroup c_h, and its calculation is shown as follows,*

$$AC^h = (1 - in\_con^h) \cdot n_h + FC,$$

where in_con^h represents the consensus level within c_h, $(1 - in\_con^h) \cdot n_h$ means the variable unit adjustment cost of c_h, FC represents the fixed cost required to adjust the subgroup preference information.

The MCAM is utilized to determine the most effective adjustment strategy, while ensuring low adjustment costs and meeting the consensus requirements. The MCAM is developed as Model 2,

$$\text{(Model 2)} \quad \min \sum AC^h \left(\left| q_{ij}^{h-} - \overline{q}_{ij}^{h-} \right| + \left| q_{ij}^{h+} - \overline{q}_{ij}^{h+} \right| \right)$$
$$s.t. \begin{cases} AC^h = (1 - in\_con^h) \cdot n_h + FC \\ \overline{Gcon} \geq \delta \\ \overline{Gcon} = \frac{\sum_{i=1}^{s}\sum_{j=1}^{t} \overline{Gcon}_{ij}}{s \cdot t} \\ \overline{q}_{ij}^{h-} = \theta_{ij}^{h} \cdot g_{ij}^{-} + \left(1-\theta_{ij}^{h}\right) \cdot q_{ij}^{h-} \\ \overline{q}_{ij}^{h+} = \theta_{ij}^{h} \cdot g_{ij}^{+} + \left(1-\theta_{ij}^{h}\right) \cdot q_{ij}^{h+} \\ \overline{Gcon}_{ij} = \frac{\sum_{h=1}^{m} \overline{con}_{ij}^{h}}{m} \\ \overline{con}_{ij}^{h} = \frac{I\left(\overline{g}_{ij} \cap \overline{q}_{ij}^{h}\right)}{I\left(\overline{g}_{ij}\right)} \\ (i,j) \in SNA, \quad \theta_{ij}^{h} \in (0,1) \end{cases}. \tag{13}$$

After obtaining the adjusted results of the two-stage adjustment model, the alternatives undergo a process to determine their priority degree, which are then used for ranking and determining the final choice.

Definitoin 5 *Let AD_i represent the priority degree of alternative a_i. The priority degree of alternative a_i is computed as follows,*

$$AD_i = \sum_{j=1}^{t} \frac{\sum_{i=1, i \neq l}^{s} Pr(g_{ij}, g_{lj})}{s-1},$$

where $Pr(g_{ij}, g_{lj})$ represents the priority of g_{ij} relative to g_{lj} defined in Definition 2.

The algorithm for the proposed G-LSGDM consensus model is presented in Algorithm 2.

Example

During the selection process for outstanding graduates at ABC University, one student distinguished himself/herself among seven candidates and was awarded the honor of Outstanding Graduate. The event involves 1,000 participants, including teachers, students and alumni, garnering significant attention and support. The evaluation process involves a comprehensive assessment of candidates' scientific research capabilities (b_1), social practice experiences (b_2), and ideological character (b_3). However, each participant has their own standards for evaluating a candidate's excellence, which introduced a degree of subjectivity into the evaluation process. To ensure fairness and objectivity, the evaluation also required that each candidate receive corresponding objective scores.

The teachers and students involved in the evaluation are denoted by the set $\{e_1, e_2, e_3, \cdots, e_{1000}\}$. The seven candidates who advanced to the final selection are denoted as $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$. Furthermore, the three evaluation

Algorithm 2: Granular Large-Scale Group Decision-Making Consensus Model

Input: Initial information $p^k = (p^k_{ij})_{s\times t}$, the consensus threshold δ, granulation parameter α, adjustment parameter β, penalty coefficient λ, and the fixed cost FC

Output: The priority degree AD_i

Apply the G-HCM to classify decision-makers into subgroups.

Calculate the division index $CI(l)$ and select the group division result $C = \{c_1, c_2, \cdots, c_m\}$.

Aggregate the preference information of each subgroup $q^h = (q^h_{ij})_{s\times t}$ $(h = 1, 2, \cdots, m)$.

Aggregate the global preference information $g = (g_{ij})_{s\times t}$.

Calculate the global consensus level $Gcon$.

if $Gcon < \delta$ **then**

  Use Model 1 to calculate the maximum consensus level $\overline{Gcon}$.

  if $\overline{Gcon} \geq \delta$ **then**

    Calculate the individual unit adjustment cost AC^h.

    Use Model 2 to adjust the initial preference information of subgroups.

  end

  Aggregate the global preference information $\overline{g} = (\overline{g}_{ij})_{s\times t}$ based on the adjusted subgroup information.

end

Calculate the priority degree AD_i for alternative a_i and determine the final ranking.

criteria are denoted by the set $\{b_1, b_2, b_3\}$. In light of the principle of fairness, it is essential to emphasize that all DMs are regarded as equally significant in this evaluation process. The preference information provided by the DMs is expressed in the form of interval values, which are presented in Table 1.

In addition to the criteria and alternatives, it is essential to assign values to the following parameters, consensus threshold: $\delta = 0.82$, control parameter: $\alpha = 1$, adjustment parameter: $\beta = 0.75$, penalty coefficient: $\lambda = 10^{-4}$, fixed cost: $FC = 2$.

The following are the specific steps for deriving the optimal alternative using the proposed method.

Step 1: The clustering process

Using the division index $CI(l)$, the optimal number of subgroups for clustering the 1,000 DMs is determined to be 36, as shown in Fig. 2. Table 2 lists the number of DMs in each subgroup.

Step 2: Consensus degree measurement

The global consensus level is calculated using Eq. (10), resulting in a value of $Gcon = 0.765$. Since $Gcon < \delta$, the two-stage feedback adjustment is applied to adjust the initial preference information.

Step 3: Two-stage feedback adjustment process

First, the positions that need adjustment are identified using Eq. (11), with SNA specified as,

$$SNA = \Big\{(2, 1), (2, 3), (4, 2), (4, 3), (5, 1), (6, 1), (7, 2), (8, 1), (8, 2), (8, 3)\Big\}.$$

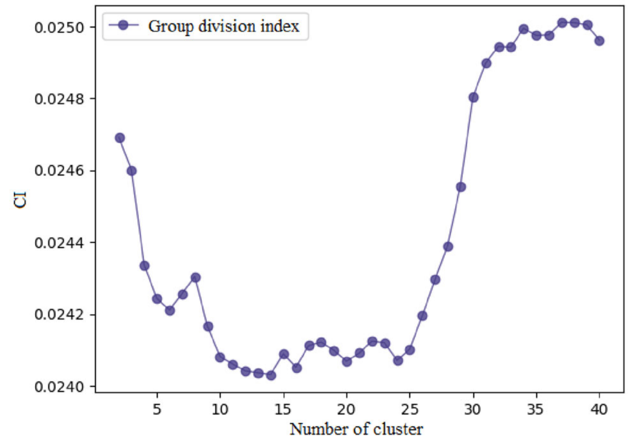


Fig. 2 Group Division Index

Second, Model 1 is employed to adjust the preference information for the specified positions, resulting in an updated group consensus level of $\overline{Gcon} = 0.85$. Since $\overline{Gcon} > \delta$, we focus on minimizing adjustment costs while ensuring that the group consensus level remains within an acceptable range, thereby entering the second adjustment stage.

In the second adjustment stage, the cost of adjusting the subgroups is calculated using Definition 4, as shown in Table 3. Meanwhile, Model 2 is employed to adjust the initial preference information of the subgroups. The updated consensus level is $\overline{Gcon} = 0.822 > \delta$, and the adjustment cost is 34.4.

Step 4: Selection process

First, we employ the granular aggregation method described in Section Granular aggregation method to aggregate the final preference information obtained from each DMs within the subgroups, as illustrated in Table 4.

Second, the overall preference information is aggregated, as shown in Table 5.

Finally, the comparison matrix CM and the priority degree AD_i for alternative a_i can be calculated as follows: $AD_1 = 1.063$, $AD_2 = 1.469$, $AD_3 = 1.268$, $AD_4 = 1.259$, $AD_5 = 1.321$, $AD_6 = 1.334$, $AD_7 = 2.785$. The ranking is determined as follows: $a_7 \succ a_2 \succ a_6 \succ a_5 \succ a_3 \succ a_4 \succ a_1$.

$$CM = \begin{pmatrix} 0.4125 & 0.3575 & 0.2929 \\ 0.5312 & 0.3446 & 0.5931 \\ 0.4339 & 0.3310 & 0.5025 \\ 0.4766 & 0.4352 & 0.3472 \\ 0.5506 & 0.2771 & 0.4929 \\ 0.3472 & 0.6203 & 0.3671 \\ 0.9293 & 0.9323 & 0.9239 \end{pmatrix}$$

Table 1 Preference Information Provided by Decision-Makers

		a_1	a_2	a_3	a_4	a_5	a_6	a_7
e_1	b_1	[0.4,0.6]	[0.6,0.8]	[0.4,0.7]	[0.6,0.7]	[0.4,0.7]	[0.5,0.8]	[0.7,0.9]
	b_2	[0.4,0.5]	[0.6,0.8]	[0.4,0.7]	[0.6,0.7]	[0.4,0.7]	[0.4,0.5]	[0.7,0.9]
	b_3	[0.4,0.5]	[0.5,0.8]	[0.3,0.5]	[0.6,0.7]	[0.5,0.7]	[0.3,0.5]	[0.6,0.8]
e_2	b_1	[0.6,0.7]	[0.6,0.8]	[0.4,0.7]	[0.6,0.7]	[0.2,0.5]	[0.4,0.5]	[0.7,0.9]
	b_2	[0.5,0.6]	[0.5,0.7]	[0.4,0.7]	[0.6,0.7]	[0.4,0.7]	[0.4,0.5]	[0.4,0.8]
	b_3	[0.2,0.3]	[0.6,0.8]	[0.4,0.7]	[0.6,0.7]	[0.6,0.7]	[0.4,0.5]	[0.7,0.9]
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
e_{999}	b_1	[0.4,0.6]	[0.6,0.8]	[0.3,0.6]	[0.6,0.7]	[0.5,0.7]	[0.4,0.5]	[0.7,0.9]
	b_2	[0.5,0.6]	[0.6,0.8]	[0.5,0.7]	[0.6,0.7]	[0.4,0.6]	[0.4,0.5]	[0.7,1.0]
	b_3	[0.5,0.6]	[0.6,0.8]	[0.4,0.6]	[0.6,0.7]	[0.4,0.7]	[0.4,0.5]	[0.8,0.9]
e_{1000}	b_1	[0.2,0.5]	[0.6,0.8]	[0.4,0.7]	[0.6,0.7]	[0.4,0.7]	[0.4,0.5]	[0.7,0.9]
	b_2	[0.4,0.6]	[0.6,0.8]	[0.3,0.5]	[0.3,0.7]	[0.5,0.8]	[0.4,0.5]	[0.5,0.7]
	b_3	[0.5,0.7]	[0.5,0.8]	[0.4,0.7]	[0.5,0.7]	[0.6,0.7]	[0.4,0.5]	[0.4,0.6]

Table 2 Number of Cluster Members

Clusters	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}
Number of members	10	16	26	15	8	6	25	19	22	16	26
Clusters	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}	c_{21}	c_{22}
Number of members	13	13	13	17	17	26	33	46	30	30	34
Clusters	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}	c_{31}	c_{32}	c_{33}
Number of members	29	32	32	31	32	48	39	32	27	44	48
Clusters	c_{34}	c_{35}	c_{36}								
Number of members	54	51	40								

Table 3 The Unit Cost of Each Subgroup

Clusters	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}
Unit adjustment cost	2.31	2.43	2.67	2.43	2.27	2.22	2.67	2.54	2.62	2.45	2.66
Clusters	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}	c_{21}	c_{22}
Unit adjustment cost	2.41	2.39	2.41	2.52	2.54	2.68	2.86	3.15	2.77	2.77	2.87
Clusters	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}	c_{31}	c_{32}	c_{33}
Unit adjustment cost	2.74	2.89	2.88	2.86	2.82	3.16	2.99	2.87	2.75	3.14	3.2
Clusters	c_{34}	c_{35}	c_{36}								
Unit adjustment cost	3.44	3.39	3.07								

Comparison analysis

This section explores the advantages and roles of various parameters in the decision-making process within the G-LSGDM consensus model. We conducted two simulation experiments utilizing randomly generated preference information, as well as a comparative analysis experiment based on the example presented in Sect. Example.

Simulation analysis

In this section, we conduct simulation experiments using 110 randomly generated preference information sets, each consisting of a single alternative and its attributes, to analyze the impact of different parameter values on the proposed granular aggregation method.

The size of the interval information granule is significantly influenced by the parameter α. As illustrated in Fig. 3, raising α within a particular range results in a higher level of group consensus, showing that members' viewpoints are more in line with others. However, when aggregating group preference information, it is essential to adhere to the principle of justifiable granularity. Once the parameter α exceeds a certain threshold, the consensus level remains stable, even as α continues to rise. The δ represents the specified consensus threshold, which plays a crucial role in the LSGDM. Changes

Table 4 Preference Information of Subgroups

		a_1	a_2	a_3	a_4	a_5	a_6	a_7
c_1	b_1	[0.16,0.36]	[0.20,0.44]	[0.23,0.35]	[0.24,0.36]	[0.18,0.32]	[0.18,0.32]	[0.72,0.91]
	b_2	[0.18,0.37]	[0.19,0.39]	[0.17,0.31]	[0.26,0.54]	[0.12,0.30]	[0.22,0.56]	[0.73,0.95]
	b_3	[0.14,0.28]	[0.26,0.42]	[0.22,0.41]	[0.43,0.59]	[0.21,0.37]	[0.17,0.35]	[0.64,0.80]
c_2	b_1	[0.17,0.33]	[0.22,0.34]	[0.28,0.39]	[0.16,0.30]	[0.23,0.37]	[0.09,0.28]	[0.71,0.91]
	b_2	[0.22,0.34]	[0.09,0.24]	[0.13,0.31]	[0.22,0.36]	[0.12,0.24]	[0.265,0.385]	[0.68,0.87]
	b_3	[0.15,0.35]	[0.20,0.32]	[0.19,0.49]	[0.16,0.32]	[0.21,0.37]	[0.14,0.28]	[0.72,0.88]
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
c_{35}	b_1	[0.12,0.30]	[0.26,0.48]	[0.19,0.37]	[0.17,0.35]	[0.14,0.32]	[0.44,0.56]	[0.78,0.94]
	b_2	[0.18,0.36]	[0.15,0.29]	[0.12,0.26]	[0.20,0.36]	[0.15,0.27]	[0.30,0.48]	[0.71,0.89]
	b_3	[0.13,0.25]	[0.13,0.27]	[0.15,0.49]	[0.39,0.51]	[0.22,0.40]	[0.13,0.27]	[0.68,0.88]
c_{36}	b_1	[0.20,0.38]	[0.23,0.41]	[0.13,0.31]	[0.19,0.83]	[0.19,0.37]	[0.13,0.27]	[0.81,0.95]
	b_2	[0.15,0.33]	[0.11,0.39]	[0.16,0.30]	[0.13,0.31]	[0.16,0.28]	[0.23,0.35]	[0.75,0.95]
	b_3	[0.13,0.39]	[0.15,0.33]	[0.17,0.37]	[0.12,0.30]	[0.12,0.66]	[0.17,0.25]	[0.60,0.76]

Table 5 Global Granulation Preference

	a_1	a_2	a_3	a_4	a_5	a_6	a_7
b_1	[0.34,0.51]	[0.56,0.72]	[0.33,0.57]	[0.56,0.66]	[0.49,0.64]	[0.42,0.59]	[0.76,0.83]
b_2	[0.46,0.57]	[0.61,0.77]	[0.52,0.66]	[0.45,0.56]	[0.47,0.71]	[0.61,0.75]	[0.64,0.81]
b_3	[0.38,0.61]	[0.60,0.71]	[0.34,0.47]	[0.37,0.51]	[0.51,0.63]	[0.42,0.61]	[0.77,0.97]

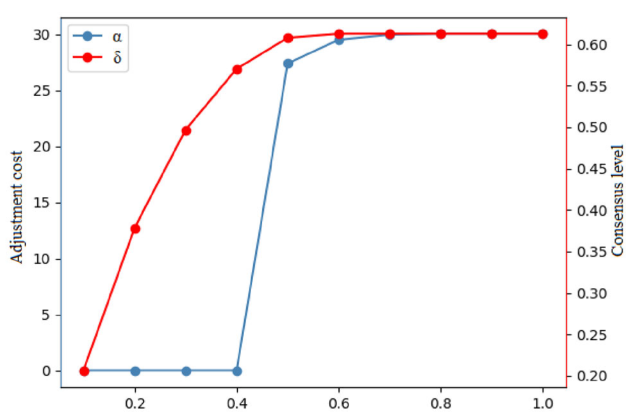


Fig. 3 The influence of α and δ

in δ directly affect the group's consensus adjustment cost, as illustrated in Fig. 3. When the consensus threshold is set to a relatively low value (e.g., $\delta = 0.2$), there is no need to modify any preference information within the group. Consequently, in this scenario, an acceptable consensus level is achieved among group members regarding the existing preference information, resulting in an adjustment cost of 0.

In this section, we employ a traditional information aggregation method, the weighted average (WA) operator, to aggregate global preferences for comparative analysis. The expression of the WA operator [41] is provided below,

$$WA(q^1, q^2, \cdots, q^m) = \sum_{i=1}^{m} \omega_i q^i,$$

where $\omega_i = \frac{1}{m}$ is the weight of subgroup c_i. In Fig. 4, there are two intervals: **interval I** and **interval II**, representing two different results of preference information aggregation methods. The former is generated using the granular aggregation method, while the latter employs the traditional WA aggregation operator. The similarity between these two intervals and their corresponding initial preference information can be determined using Definition 3. The similarity degree for **interval I** is 0.52, whereas that for **interval II** is 0.5. These findings indicate that the aggregation result produced by the proposed method exhibits a suitable representative pattern, providing a solid foundation for future research and decision-making.

Figure 5 shows that when the group contains DMs with extreme preferences, their preference information (**Interval II**) can influence the aggregated result, skewing it in favor of those with extreme preferences. Specifically, the similarity degree for **Interval I** remains steady at 0.52, while the similarity degree for **Interval II** experiences a slight decline to 0.48. The similarity degree between the aggregated preference information and the initial preference information for **Interval I** remains constant at 0.52, indicating the stability

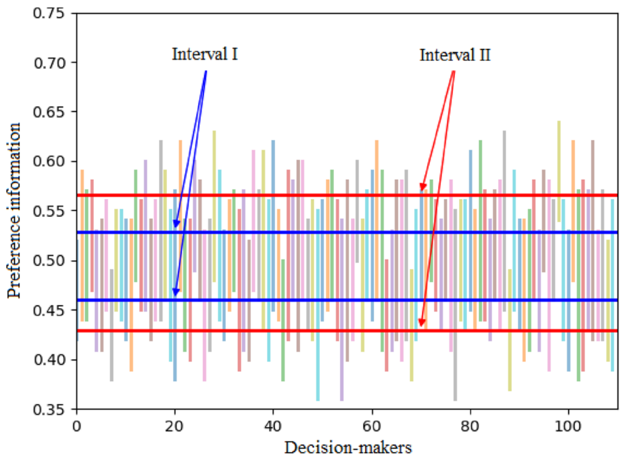


Fig. 4 Aggregation Result without Outliers

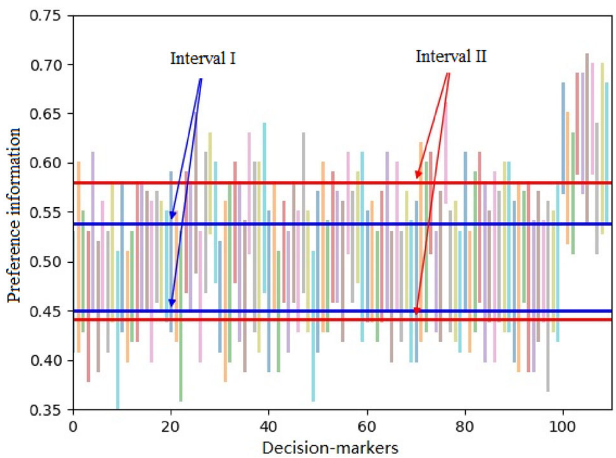


Fig. 5 Aggregation Result with Outliers

of the proposed method. In contrast, the similarity degree between the aggregated preference information generated using the WA method and the initial preference information for **Interval II** decreases from 0.50 to 0.48, reflecting its sensitivity to extreme preferences. Overall, these results indicate that the granular aggregation method effectively filters out extreme preferences and reduces the impact of outliers on decision-making results, thereby ensuring the robustness and reliability of the proposed method.

Comparison analysis

To examine the effectiveness of the G-LSGDM consensus model in the clustering process, we utilize the experimental data provided in Section Example to compare the G-HCM clustering algorithm with the traditional HCM clustering algorithm.

The effectiveness of granular aggregation methods in managing extreme preference information has been analyzed through simulation studies. To further highlight the superiority of the clustering method proposed in this paper, we conducted a comparative analysis of the G-HCM and traditional HCM methods, examining the division indices and consensus levels for different subgroup numbers. The results are presented in Fig. 6.

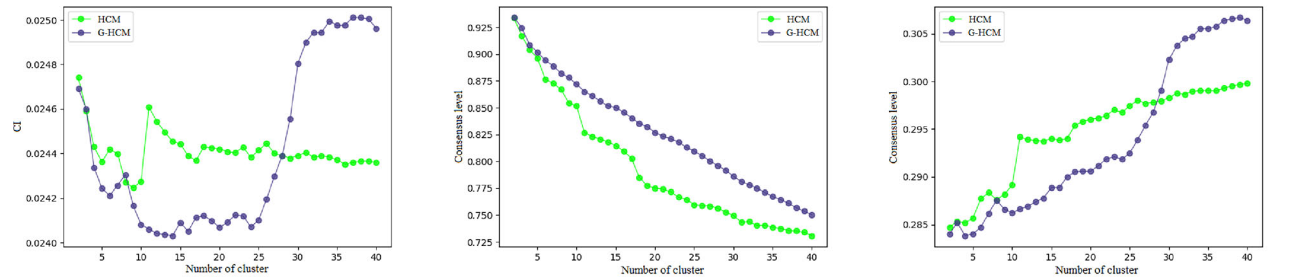
The results shown in Fig. 6(a) indicate that the HCM method performs better when the number of clusters ranges from 10 to 27. However, when the number of participants in the decision-making process exceeds 1000, this range of clusters does not significantly reduce the complexity of the decision-making problem, as each cluster still contains a relatively large number of DMs. In contrast, when the number of clusters exceeds 27, the effectiveness of the G-HCM method becomes more apparent. Furthermore, the G-HCM method considers aggregated preference information, allowing for the integration of diverse perspectives. This helps mitigate the influence of extreme individual opinions, ultimately leading to more objective and reliable group division outcomes. Fig. 6(b) illustrates that the overall consensus level obtained by the G-HCM method is higher, indicating that the decision results from this method will be more widely accepted and yield greater satisfaction. Additionally, Fig. 6(c) shows that when the number of DMs exceeds 1000, the G-HCM method can significantly enhance the consensus level within each subgroup.

Discussion

This section presents a comparative analysis of various decision-making models. By comparing the G-LSGDM consensus model with existing methods, the unique advantages and efficacy of the G-LSGDM consensus model are assessed. Table 6 offers a detailed comparison.

For solving LSGDM problems, clustering algorithms simplify the decision-making process for complex groups. However, some algorithms [20, 44] require predetermined cluster numbers, which have significant limitations that make it challenging to accurately reflect group structures and may introduce subjective bias, potentially reducing the reliability of the results. This study presents a granular hierarchical clustering method that utilizes preference distribution information and consensus levels to determine the optimal number of subgroups, thereby minimizing the influence of individual preferences and enhancing the rationality of the results.

The weighted average operator [17, 42–44] is the most commonly used technique for aggregating preference information in addressing LSGDM problems. However, this method may be sensitive to outliers in practical applications. In contrast, the granular aggregation method proposed in this study offers a more flexible approach that effectively summarizes and describes collective opinions at a higher level of abstraction, while also helping to mitigate the influence



(a) Division index (b) Consensus level of global (c) Consensus level within subgroups

Fig. 6 Comparison Results of G-HCM and HCM

Table 6 Comparison with Existing Method

Reference	Clustering method	Aggregated methods	Consensus measurement method	Scope of adjustment	Strategy of adjustment
Liang et al. [16]	Clustering method based on the similarity-trust score of DMs	Weighted average operator	Distance-based measurement method	Global adjustment	Adjustment strategy utilizing the DeGroot model
Wang et al. [17]	Affinity propagation algorithm	Weighted average operator	Distance-based measurement method	Local adjustment	Bi-directional feedback mechanism
Yang et al. [20]	Two-stage clustering approach	–	–	–	–
Yang et al. [42]	–	Weighted average operator	Distance-based measurement method	Global adjustment	Considering the acceptance probability of DMs
Cao et al. [43]	–	Weighted average operator	Distance-based measurement method	Global adjustment	A bilateral negotiation consensus mechanism
Liao et al. [44]	K-means clustering algorithm	Weighted average operator	Distance-based measurement method	Local adjustment	Minimum cost consensus-based feedback mechanism
This paper	G-HCM	Granular aggregation method	Fuzzy consensus measurement method	Local adjustment	Two-stage adjustment strategy

The indicator "–" indicates that the corresponding item is not considered

of outliers, thereby enhancing the stability of the decision-making process.

In the CRP, the process is generally divided into two stages: the consensus measurement stage and the feedback adjustment stage. On one hand, existing methods [17, 42–44] typically assess the consensus level by measuring the distance between DMs' preference information; however, different distance functions may yield varying results. On the other hand, the feedback adjustment stage involves modifying individual preferences. Some studies [16, 42, 43] have adopted a global adjustment strategy, which can lead to the loss of initial information and make the adjustment process more time-consuming. This study designs a fuzzy consensus measurement method within a granular computing framework that offers flexibility and applicability, as group consensus levels can be derived from the diverse specificity and coverage of information granules. Furthermore, the proposed two-stage consensus model provides more precise local information adjustments, ensuring that DMs achieve satisfactory results while preserving their initial perspectives as much as possible.

Compared to the feedback adjustment strategy proposed in references [16, 17, 42–44], the developed two-stage adjustment model integrates the principles of maximum consensus and minimum cost adjustment, with the goal of achieving an effective group consensus, thereby improving the acceptance and satisfaction of the decision-making results.

In summary, our method offers significant advantages over existing approaches and provides potential solutions for practical applications. However, it also has certain limitations. First, with the growth of social media, the number of participants in decision-making problems has increased exponentially, leading to greater divergence of opinions among stakeholders, as well as more pronounced competitive and non-cooperative behaviors. Future research should incorporate these behaviors into consensus models. Second, given the differences among DMs in terms of knowledge structure, individual preferences, and information representation (such as numerical, interval, and linguistic formats) within LSGDM, establishing a consensus model that effectively integrates heterogeneous evaluation information becomes crucial for addressing the practical challenges of LSGDM.

Conclusion

This study introduces the concept of information granules to design the G-LSGDM consensus framework, aiming to achieve more satisfactory decision-making results in LSGDM. Additionally, we provide an example to illustrate the specific application of the proposed method and compare it with traditional methods to demonstrate the superiority of the proposed model in terms of consensus and clustering, especially in decision-making processes involving thousands of participants. The main innovations and theoretical contributions of this study are summarized as follows,

- 1) For the clustering process, we construct a granular hierarchical clustering approach based on the principle of justifiable granularity, where the division index is used to determine the optimal number of subgroups. Additionally, we develop a granular aggregation method that generates information granules representing group opinions and helps eliminate outliers.
- 2) For the group consensus measurement process, we define the fuzzy consensus measure function by considering both the coverage and specificity of information granules. This function offers flexibility and applicability due to the diverse expression functions used for the specificity and coverage of the consensus granules.
- 3) For the information adjustment process, we incorporate the moderator's decision cost into the feedback mechanism and propose a two-stage granule consensus model that reduces adjustment costs while ensuring the DMs' satisfaction with the final solution.

In LSGDM, the individual behaviors of DMs, including non-cooperative and manipulative behaviors, directly impact both the decision-making results and the efficiency of the CRP. Therefore, constructing a consensus feedback mechanism that considers the individual behavior of DMs in the CRP is promising. Additionally, exploring the allocation of information granularity in LSGDM presents another intriguing direction. In our proposed model, we primarily focus on interval-based information granules. In the future, we will explore other forms of information granules, such as type-2 fuzzy sets, to characterize the uncertainty in the process of decision making activities and examine their impact on decision results. Furthermore, the proposed model can be applied to other relevant fields, such as Hospital-based Health Technology Assessment, to help hospitals evaluate the value and effectiveness of new technologies, thereby enhancing the quality and efficiency of medical services.

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Declarations

Conflicts of Interest The authors declare that they have no conflicts of interest.

Ethical Approval This article does not contain any studies with human or animals performed by any of the authors.

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