



Confidence consensus-based model for large-scale group decision making: A novel approach to managing non-cooperative behaviors

Xuan-hua Xu^a, Zhi-jiao Du^{a,b,*}, Xiao-hong Chen^{a,c}, Chen-guang Cai^d

^a School of Business, Central South University, Changsha 410083, China

^b Business School, Sun Yat-Sen University, Guangzhou 510275, China

^c School of Accounting, Hunan University of Commerce, Changsha 410205, China

^d College of Logistics and Transportation, Central South University of Forestry and Technology, Changsha, 410004, China

ARTICLE INFO

Article history:

Received 23 March 2016

Revised 21 September 2018

Accepted 28 October 2018

Available online 30 October 2018

Keywords:

Large-scale group decision making (LSGDM)

Non-cooperative behaviors

Confidence consensus-based model

Rationality degree

ABSTRACT

Because of the complexity of real-world problems, large-scale group decision making has become a research topic of great interest in the field of decision science. Differences of opinion in a large group are highly likely. Sometimes, decision makers are unwilling to adjust their opinions to promote consensus. It is hence necessary to establish a consensus model for the effective management of opinion differences and non-cooperative behaviors. More importantly, the credibility of the adjustment information must be ensured. In this paper, we present a confidence consensus-based model for large-scale group decision making that provides a novel approach to addressing non-cooperative behaviors. First, some new concepts are proposed, including the collective adjustment suggestion and rationality degree. Then, we combine the rationality and non-cooperation of the adjustment information to construct the concept of a confidence level. This confidence level measures the impartiality and objectivity of the adjustment information and is the basis for managing non-cooperative behaviors. We then establish a mechanism for addressing non-cooperative behaviors. Finally, we present a case study that illustrates that the proposed model is feasible and effective. A comparative analysis reveals the features and advantages of this model for managing large-scale group decision making.

© 2018 Elsevier Inc. All rights reserved.

1. Introduction

Group decision making (GDM) aims to achieve a common solution for a decision problem in which two or more decision makers (DMs) participate, and it has been paid an increasing amount of attention [1,3,9,23,28]. Traditional GDM is a type of group discussion and decision process in which the number of DMs is small (e.g., three to five people), and the complexity is often not very high (e.g., a car purchase decision for a family). In GDM for public-interest decisions, however, a large number of DMs with different interests and knowledge structures must be involved. Research into large-scale GDM (LSGDM) is a new area in the field of decision making. LSGDM problems are generally characterized by some or all the following four features [4,7,10,20,30,41]: (a) The group usually involves a large number of DMs from different sectors and professional fields. (b) Obtaining a high-consensus solution necessitates the implementation of a consensus reaching process (CRP). (c) The social

* Corresponding author.

E-mail addresses: xuxh@csu.edu.cn (X.-h. Xu), duzhj3@mail2.sysu.edu.cn, 131611141@csu.edu.cn (Z.-j. Du).

networks of the DMs and the evolution of their opinions are considered. (d) DMs often have individual concerns about alternatives, and thus they adopt heterogeneous preference representation structures to express their preferences. Note that the number of DMs in current LSGDM problems varies from dozens to thousands [32]. Xu et al. [34] and Zhang et al. [41] believe that if the number of DMs in a group exceeds 11, the group is a large group, whereas Liu et al. [17] insists that this number should be more than 20. In this study, to include all the research on consensus measures and mechanisms for managing non-cooperative behaviors in the CRP, we employ the first and second characteristics above to define LSGDM.

Clustering is an effective approach to organizing and managing a large group because it can divide a large group into small-scale subgroups. Many clustering methods have been proposed such as *K*-means algorithm [14,32], fuzzy clustering algorithm [20], grey clustering algorithm [16], and vector space-based [33] clustering method. Clustering in LSGDM is typically based on the similarity of DMs' opinions. If the opinions of certain DMs are sufficiently similar (i.e., over a certain threshold), these DMs are assigned to the same subgroup. This classification produces a hierarchical structure. The outer layer represents the interaction between subgroups and the inner layer represents the discussion among the members of each subgroup. The coordination among subgroups does not need to consider the internal members of the subgroup; the members in a subgroup form a unified subgroup opinion through discussion and coordination. Generally, the clustering structure does not change once it has been determined [34,35] because this helps the members in each subgroup to establish trust feelings and form a unified opinion. In contrast, Wu and Xu [32] and Zhang et al. [41] studied consensus models in which the clusters are allowed to change. This study follows Xu et al.'s research [34] on classifying large-scale DMs and uses the cluster as a basic unit once the clustering structure has been determined.

Generally, both of the two processes are used to address LSGDM problems: the consensus process and the selection process [34]. Reaching a consensus is a dynamic and iterative group discussion process in which some DMs must make compromises. Unanimity is difficult to attain, and "soft" consensus is widely used in CRPs [13]. To date, various consensus reaching models (CRMs) have been proposed that address different decision-making problems: (1) GDM with linguistic assessments [2,5,11,12,18,21,25,31,40,42,43], (2) LSGDM [15,19,20,24,32,34,35,41], (3) GDM that considers the non-cooperative behaviors [8,10,20,34], and (4) GDM under a social network environment [6,26,27,29]. With respect to non-cooperative behaviors in particular, Palomares et al. [20] proposed a weight penalizing method that aims to reduce the weights of non-cooperating experts' preferences, Xu et al. [34] developed a consensus model based on the level of non-cooperation to address non-cooperative behaviors, and Dong et al. [8] introduced three types of non-cooperative behaviors, in which some DMs express their opinions dishonestly or refuse to change their opinions to further their own interests. Based on the above literature, we observe the following information about the management of non-cooperative behaviors.

- (1) Research on non-cooperative behaviors is still in its inception stage, and there are few studies in the literature, particularly those that consider the LSGDM environment.
- (2) Few studies have considered the rationality of the adjustment information provided by non-cooperative DMs (or subgroups) and used it as a reference for managing non-cooperative behaviors. Hence, this is a research issue that should be addressed.

An LSGDM problem often involves multiple DMs, and they may have different attitudes toward compromise to achieve a consensus. Some DMs are in favor of adjusting their opinions, whereas others refuse to compromise or make very small compromises. Non-cooperative behaviors may seriously affect the decision process and the final decision result; hence, it is necessary to respond appropriately. In addition to the CRMs for managing non-cooperative behaviors listed above, Pelta and Yager [22] and Yager [38,39] considered non-cooperative behaviors in the selection process of GDM problems, but not in the CRP. Managing non-cooperative behaviors in the CRP helps to better adjust the opinions of non-cooperative DMs and further influence the decision result. We prefer to manage non-cooperative behaviors and achieve agreement among the DMs' opinions before applying the selection process. Therefore, in this study, we continue the approach of [34] and propose a novel consensus model for managing non-cooperative behaviors in LSGDM that combines the non-cooperation and rationality of the adjustment information.

The CRP consists of two core elements: a consensus measure and feedback adjustment. There are usually two ways to measure consensus: the first is based on the distance to the group opinion [34,40] and the second is based on the distances among DMs' opinions [8,31,42]. Different consensus measures lead to different consensus processes and decision results. Therefore, it is necessary to analyze the suitability of each measure for a particular application.

The contributions of this paper are as follows.

- Different consensus measures are defined and their advantages and disadvantages are compared.
- A mechanism for addressing non-cooperative behaviors is proposed. First, we insert an interaction step in the management of non-cooperative behaviors that effectively enables DMs to provide better adjustment coefficients. We combine the levels of rationality and non-cooperation to construct the confidence level of a cluster's adjustment coefficient. By referring to the confidence level, non-cooperative behaviors can be better managed.
- An algorithm for the confidence consensus-based model for LSGDM is presented that incorporates the proposed mechanism.
- A detailed comparative analysis of the confidence consensus-based model with other models and a simulation that summarizes the characteristics of the CRP are presented.

The remainder of this paper is organized as follows: In [Section 2](#), we review the basic concepts of non-cooperative behaviors. In [Section 3](#), we propose the confidence consensus-based model to address non-cooperative behaviors. A case study is presented in [Section 4](#) to illustrate the feasibility and validity of the proposed model. In [Section 5](#), we compare our model with existing consensus models, discuss the results obtained using different consensus measures, and analyze the results of setting consensus thresholds and other parameters to different values. We draw our conclusions in [Section 6](#).

2. Concepts related to non-cooperative behaviors

Non-cooperative behaviors exist widely in LSGDM because DMs come from different professions and industries and hence may represent different interests. Non-cooperative behaviors can be categorized as follows: (a) an expert or an experienced person who insists that his/her opinion is right but has no private interests; (b) a leader who thinks his/her opinion is that of authority, where there may be private interests; (c) a noteworthy and independent person who provides a view that is often out of the ordinary and insists that his/her opinion is innovative; and (d) a stakeholder with private interests. In this study, we focus on the management of the first type of DM. In future research, we will study the other three types of non-cooperative DMs.

There are typically two approaches to addressing non-cooperative behaviors [\[34\]](#): a weight adjustment, used to decrease a DM's influence to achieve a greater consensus, and an opinion adjustment, used to make a DM's opinion closer to the group's opinion.

3. Confidence consensus-based model that considers non-cooperative behaviors

In [Section 3.1](#), we introduce different rules for computing consensus measures. In [Section 3.2](#), we propose a mechanism for addressing non-cooperative behaviors in LSGDM. We present the algorithm for the confidence consensus-based model in [Section 3.3](#).

Given an LSGDM problem, let $X = \{x_1, x_2, \dots, x_m\}$ be a finite set of m alternatives, $A = \{a_1, a_2, \dots, a_n\}$ be a set of n attributes, and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of the attributes, where $\sum_{j=1}^n \omega_j = 1$, $0 \leq \omega_j \leq 1$ ($j = 1, 2, \dots, n$). Let $E = \{e_1, e_2, \dots, e_q\}$ ($q \geq 20$) be a set of q DMs. Each DM expresses its opinion using numerical decision matrix $V^l = [v_{ij}^l]_{m \times n}$, where v_{ij}^l represents the opinion of DM e_l on alternative $x_i \in X$ with respect to attribute $a_j \in A$. Before entering the CRP, individual decision matrix V^l should be normalized to standardized decision matrix $R^l = [r_{ij}^l]_{m \times n}$, where [\[36\]](#)

$$r_{ij}^l = \frac{v_{ij}^l - \min_l \{v_{ij}^l\}}{\max_l \{v_{ij}^l\} - \min_l \{v_{ij}^l\}} \quad \text{for benefit attributes and}$$

$$r_{ij}^l = \frac{\max_l \{v_{ij}^l\} - v_{ij}^l}{\max_l \{v_{ij}^l\} - \min_l \{v_{ij}^l\}} \quad \text{for cost attributes.}$$

Clustering the DMs' opinions is the basis for analyzing a large group. In this study, we adopt the clustering method in [\[33\]](#) to classify a large number of DMs into K ($1 \leq K \leq q$) subgroups (the clustering method is described in the supplementary materials). The weight vector of the clusters $\lambda = (\lambda^1, \lambda^2, \dots, \lambda^K)^T$ is obtained such that

$$\lambda^k = \frac{(n_k)^2}{\sum_{k=1}^K (n_k)^2}, \quad (1)$$

where n_k is the number of DMs in cluster g_k . Clearly, $0 \leq \lambda^k \leq 1$ ($k = 1, 2, \dots, K$) and $\sum_{k=1}^K \lambda^k = 1$. The cluster decision matrix can be obtained, i.e., $R^k = (r_{ij}^k)_{m \times n}$, where $r_{ij}^k = (1/n_k) \sum_{l=1}^{n_k} r_{ij}^l$, $e_l \in g_k$. The group decision matrix is represented by $R^c = (r_{ij}^c)_{m \times n}$, where $r_{ij}^c = \sum_{k=1}^K r_{ij}^k \lambda^k$.

3.1. Consensus measures

Two methods for calculating consensus measures exist: the first is based on the distance to the group opinion and the second is based on the distances between individual opinions. In practice, consensus measures can be classified into the following three categories.

(1) Consensus measure based on the distance to the group opinion using an average operator (CM-I):

The individual consensus level is calculated as

$$CI_{CM-I}^{k,t} = 1 - d(R^{k,t}, R^{c,t}) = 1 - \frac{1}{m \times n} \sum_{i=1}^m \sum_{j=1}^n d(r_{ij}^{k,t}, r_{ij}^{c,t}), \quad (2)$$

where $d(r_{ij}^{k,t}, r_{ij}^{c,t})$ is the distance between $r_{ij}^{k,t}$ and $r_{ij}^{c,t}$, and the group consensus level is calculated as

$$GCI_{CM-I}^t = \frac{1}{K} \sum_{k=1}^K CI_{CM-I}^{k,t}. \quad (3)$$

(2) Consensus measure based on the distance to the group opinion using a minimum operator (CM-II):

In this rule, the individual consensus level measure is calculated using Eq. (2) and the group consensus level is defined as

$$GCI_{CM-II}^t = \min_k \{CI_{CM-II}^{k,t}\}. \quad (4)$$

(3) Consensus measure based on the distance between DMs' opinions using a minimum operator (CM-III):

Here, the individual consensus level is

$$\begin{aligned} CI_{CM-III}^{k,t} &= 1 - \frac{1}{K-1} \sum_{g=1, k \neq g}^{K-1} d(R^{k,t}, R^{g,t}) \\ &= 1 - \frac{1}{K-1} \sum_{g=1, k \neq g}^{K-1} \frac{1}{m \times n} \sum_{i=1}^m \sum_{j=1}^n d(r_{ij}^{k,t}, r_{ij}^{g,t}), \end{aligned} \quad (5)$$

where $d(r_{ij}^{k,t}, r_{ij}^{g,t})$ is the distance between $r_{ij}^{k,t}$ and $r_{ij}^{g,t}$, and the group consensus level is

$$GCI_{CM-III}^t = \min_k \{CI_{CM-III}^{k,t}\}. \quad (6)$$

Clearly, $0 \leq GCI_{CM-I}^t \leq 1$, $0 \leq GCI_{CM-II}^t \leq 1$, and $0 \leq GCI_{CM-III}^t \leq 1$. If the group consensus level is sufficiently high, then the selection process can be followed; otherwise, the CRP should be applied to manage the opinion differences. In Section 5.4, we discuss the advantages and disadvantages of the above three consensus measures.

3.2. Mechanism for managing non-cooperative behaviors

In LSGDM after clustering, it is common for some clusters to adopt a non-cooperative attitude toward opinion adjustment. Managing these non-cooperative behaviors is a challenge but also an important part of the process of consensus building. Inspired by the research in [20,34], this study employs three phases to address non-cooperative behaviors: identification, interaction, and proper modification.

3.2.1. Identification rule for non-cooperative behaviors

Step 1. Detect and determine the most deviant individual opinion.

Suppose that the opinion of cluster g_{k^*} deviates most from those of the other clusters. This cluster can be determined using the following formulas:

$$\begin{aligned} CI_{CM-I}^{k^*,t} &= \min_k \{CI_{CM-I}^{k,t}\} \quad \text{for CM - I,} \\ CI_{CM-II}^{k^*,t} &= \min_k \{CI_{CM-II}^{k,t}\} \quad \text{for CM - II,} \\ CI_{CM-III}^{k^*,t} &= \min_k \{CI_{CM-III}^{k,t}\} \quad \text{for CM - III.} \end{aligned}$$

Step 2. Cluster g_{k^*} provides the adjustment coefficient and the other clusters provide the related adjustment suggestions.

Because of the complexity of decision making, the adjustment coefficient and adjustment suggestions are given as interval numbers. Suppose that the adjustment coefficient is $\tilde{\eta}^{k^*,t} = [\eta^{Lk^*,t}, \eta^{Uk^*,t}]$, where $\eta^{Uk^*,t}$ and $\eta^{Lk^*,t}$ are the upper and lower limits of $\tilde{\eta}^{k^*,t}$, respectively, such that $0 \leq \eta^{Lk^*,t} \leq \eta^{Uk^*,t} \leq 1$. $\tilde{\eta}^{k^*,t}$ represents the degree of respect that cluster g_{k^*} has for the group opinion. The adjustment suggestion $\tilde{\eta}^{k' \rightarrow k^*,t}$ ($k' = 1, 2, \dots, K; k^* \neq k'$) is also an interval number, i.e., $\tilde{\eta}^{k' \rightarrow k^*,t} = [\eta^{Lk' \rightarrow k^*,t}, \eta^{Uk' \rightarrow k^*,t}]$, where $\eta^{Uk' \rightarrow k^*,t}$ and $\eta^{Lk' \rightarrow k^*,t}$ are the upper and lower limits of $\tilde{\eta}^{k' \rightarrow k^*,t}$, respectively, such that $0 \leq \eta^{Lk' \rightarrow k^*,t} \leq \eta^{Uk' \rightarrow k^*,t} \leq 1$.

Step 3. Calculate the collective adjustment suggestion.

Definition 1. By aggregating all adjustment suggestions, the collective adjustment suggestion $\hat{\eta}^{k^*,t}$ can be obtained such that

$$\hat{\eta}^{k^*,t} = \sum_{k'=1, k' \neq k^*}^K \tilde{\eta}^{k' \rightarrow k^*,t} \lambda^{k',t}, \quad (7)$$

where $\lambda^{k',t}$ is the remaining cluster's weight, $0 \leq \lambda^{k',t} \leq 1$ and $\sum_{k'=1, k' \neq k^*}^K \lambda^{k',t} = 1$.

Step 4. Measure the degree of non-cooperation and identify non-cooperative behaviors.

Use the probability degree of two interval numbers to compute the degree of non-cooperation of adjusting coefficient $\bar{\eta}^{k^*,t}$ [34]. If $p(\bar{\eta}^{k^*,t} \geq \bar{\eta}^{k' \rightarrow k^*,t}) = 1$, then this indicates that g_{k^*} is a completely cooperative subgroup; otherwise, g_{k^*} is regarded as a non-cooperative subgroup.

Note that only using the degree of non-cooperation to punish non-cooperative behaviors is likely to induce the cluster to deliberately reduce it and reduce the rationality of the adjustment coefficient. Therefore, we should further measure the rationality of the adjustment coefficient.

Based on the majority principle, an adjustment coefficient that is closer to the collective adjustment suggestion is more reasonable. Thus, using the similarity [37] to compute the proximity of the adjustment coefficient to the collective adjustment suggestion, we can measure the rationality $T^{k^*,t}$ ($0 \leq T^{k^*,t} \leq 1$) of the given adjustment coefficient. Appendix A presents the definition of the similarity of two interval numbers. Values of $T^{k^*,t}$ closer to one indicate an adjustment coefficient that is more similar to the collective adjustment suggestion.

3.2.2. Interaction and discussion

If cluster g_{k^*} is determined to be a non-cooperative subgroup, then more in-depth discussions can be conducted between g_{k^*} and the other clusters. Two scenarios are presented: (a) cluster g_{k^*} is urged to make its adjustment coefficient closer to the collective adjustment suggestion and (b) cluster g_{k^*} is impelled to reduce its degree of non-cooperation to obtain greater consensus. After discussion and interaction, cluster g_{k^*} changes its adjustment coefficient $\bar{\eta}^{k^*,t}$ to ${}^z\bar{\eta}^{k^*,t}$, where ${}^z\bar{\eta}^{k^*,t}$ is an interval number. Then, the non-cooperation and rationality are recalculated, denoted by ${}^z\tau^{k^*,t}$ and ${}^zT^{k^*,t}$, respectively. It is noted that cluster g_{k^*} may have the option of not changing its adjustment coefficient.

3.2.3. Proper modification

Combining the value of rationality with that of non-cooperation, we can construct a new concept called the confidence level to measure quantitatively the extent to which cluster g_{k^*} can provide a rational adjustment coefficient. The confidence level represents three aspects: (a) other clusters' feelings that the given adjustment coefficient can be trusted; (b) the reflection of honesty and emotional expression of cluster g_{k^*} in terms of opinion adjustment; and (c) an indication of the understanding of cluster g_{k^*} of the decision problem and its impartiality in the decision-making process.

Definition 2. The confidence level of the given adjustment coefficient is the mapping $f: {}^z\tau^{k^*,t}, {}^zT^{k^*,t} \rightarrow \varsigma^{k^*,t}$ ($\varsigma^{k^*,t} \in [0, 1]$).

Definition 3. A weighted geometric averaging operator is used to describe the mapping f and obtain $f_{WG}: {}^z\tau^{k^*,t}, {}^zT^{k^*,t} \rightarrow \varsigma^{k^*,t}$, such that

$$\varsigma^{k^*,t} = f_{WG}({}^z\tau^{k^*,t}, {}^zT^{k^*,t}) = (1 - {}^z\tau^{k^*,t})^{\mu_{WG}} \times ({}^zT^{k^*,t})^{\nu_{WG}}, \quad (8)$$

where μ_{WG} and ν_{WG} are control coefficients that define the relative weight of the non-cooperation and rationality, respectively, such that $\mu_{WG}, \nu_{WG} \in [0, 1]$ and $\mu_{WG} + \nu_{WG} = 1$.

The following theorem can easily be determined.

Theorem 1. If $\mu_{WG}, \nu_{WG} \neq 0$, then $\varsigma^{k^*,t} \in [0, 1]$.

Theorem 2. If $\mu_{WG} = 0$, then Eq. (8) changes to $\varsigma^{k^*,t} = {}^zT^{k^*,t}$. In this case, the confidence level only depends on the rationality of the adjustment coefficient.

Theorem 3. If $\nu_{WG} = 0$, then Eq. (8) changes to $\varsigma^{k^*,t} = {}^z\tau^{k^*,t}$. In this case, the confidence level only relies on the non-cooperation of the adjustment coefficient.

Commonly, we set $0 < \mu_{WG}, \nu_{WG} < 1$. Specifically,

- (1) if $\varsigma^{k^*,t} = 0$, then the confidence level of the adjustment coefficient is zero. This can result from the following:
 - a) ${}^z\tau^{k^*,t} = 1$: this indicates that the adjustment coefficient cannot satisfy the minimum requirement of the collective adjustment suggestion; thus, cluster g_{k^*} is regarded as a completely non-cooperative subgroup.
 - b) ${}^zT^{k^*,t} = 0$: This indicates that the adjustment coefficient deviates far from the collective adjustment suggestion.
 Whether cluster g_{k^*} is regarded as a completely non-cooperative subgroup or the adjustment is too deviant, the DMs in cluster g_{k^*} should be advised to exit the decision process to ensure decision quality and decision-making efficiency. In this study, we focus on the category of the non-cooperating experts; hence, the cause of non-cooperative behaviors or very deviant adjustments is a poor understanding of the decision problem or subjectively persistent with few private interests.
- (2) If $\varsigma^{k^*,t} \in (0, 1)$, then the confidence level can be used as the basis of the weight adjustment.

A DM's weights reflect its influence on the group opinion. By modifying the weights, the influence is adjusted. When confronted with non-cooperative clusters, we can reduce their weight to reduce their impact so as to promote consensus.

Definition 4. The weight penalty function is defined as

$${}^z\lambda^{k^*,t+1} = \chi^t \cdot \varsigma^{k^*,t} \cdot \lambda^{k^*,t}, \quad (9)$$

where $z\lambda^{k^*,t+1}$ is the modified weight of g_{k^*} and χ^t is used to regulate the weight penalty and is calculated as

$$\chi^t = \sum_{k'=1, k^* \neq k'}^K \chi^{k',t} \bar{\lambda}^{k',t}. \quad (10)$$

The symbol $\bar{\lambda}^{k',t}$ is the normalized weight of the remaining clusters such that $\bar{\lambda}^{k',t} \geq 0 (k' = 1, 2, \dots, K; k^* \neq k')$ and $\sum_{k'=1, k^* \neq k'}^K \bar{\lambda}^{k',t} = 1$. The remaining clusters provide parameters $\chi^{k',t} (k' = 1, 2, \dots, K; k^* \neq k')$ such that they meet the conditions $\chi^{k',t} \geq 1$ and $\chi^{k',t} \cdot \varsigma^{k^*,t} < 1$. The first condition ensures that each DM's opinion can be effectively expressed, including the DMs of the non-cooperative subgroup, which is in accordance with the principle of cautious handling of and appropriate respect for non-cooperative behaviors. The second condition ensures that non-cooperative behaviors are efficiently managed. Clearly, $\chi^t \geq 1$ and $\chi^t \cdot \varsigma^{k^*,t} < 1$.

Then, we use the following to adjust the decision matrix of cluster g_{k^*} as follows:

$$R^{k^*,t+1} = \frac{\eta^{Lk^*,t} + \eta^{Uk^*,t}}{2} R^{c,t} + \left(1 - \frac{\eta^{Lk^*,t} + \eta^{Uk^*,t}}{2}\right) R^{k^*,t}, \quad (11)$$

where the adjustment coefficient is set as the midpoint of $\bar{\eta}^{k^*,t}$. That is, $0 \leq (\eta^{Lk^*,t} + \eta^{Uk^*,t})/2 \leq 1$. Here, $R^{k^*,t}$ and $R^{k^*,t+1}$ represent the opinions of cluster g_{k^*} before and after the $t+1$ th iteration, respectively, and $R^{c,t}$ represents the group opinion in the $t+1$ th iteration.

Remark 1. Let $\varsigma^{k^*,t} = f_{WG}(z\tau^{k^*,t}, zT^{k^*,t}) = (1 - z\tau^{k^*,t})^{\mu_{WG}} \times (zT^{k^*,t})^{\nu_{WG}}$ be a common mathematical function that represents the mapping $z\tau^{k^*,t}, zT^{k^*,t} \rightarrow \varsigma^{k^*,t}$. Here, $z\tau^{k^*,t}$ and $zT^{k^*,t}$ are two independent variables, and $\varsigma^{k^*,t}$ is a dependent variable. Let $0 \leq z\tau^{k^*,t}, zT^{k^*,t} \leq 1$ and $0 < \mu_{WG}, \nu_{WG} < 1$. We can calculate the partial derivative of this function as

$$\begin{aligned} \frac{\partial \varsigma^{k^*,t}}{\partial z\tau^{k^*,t}} &= -\mu_{WG} \cdot (1 - z\tau^{k^*,t})^{\mu_{WG}-1} \cdot (zT^{k^*,t})^{\nu_{WG}}, \\ \frac{\partial \varsigma^{k^*,t}}{\partial zT^{k^*,t}} &= (1 - z\tau^{k^*,t})^{\mu_{WG}} \cdot \nu_{WG} \cdot (zT^{k^*,t})^{\nu_{WG}-1}. \end{aligned}$$

Clearly, $\frac{\partial \varsigma^{k^*,t}}{\partial z\tau^{k^*,t}} < 0$, $\frac{\partial \varsigma^{k^*,t}}{\partial zT^{k^*,t}} > 0$. Thus, $\varsigma^{k^*,t}$ is a strictly monotonic decreasing function of $z\tau^{k^*,t}$, whereas $\varsigma^{k^*,t}$ is a strictly monotonic increasing function of $zT^{k^*,t}$.

Note that in accordance with Definition 2, the values $z\tau^{k^*,t}$ and $zT^{k^*,t}$ depend on $z\bar{\eta}^{k^*,t}$ and $\hat{\eta}^{k^*,t}$. Let $z\bar{\eta}^{k^*,t}$ and $\hat{\eta}^{k^*,t}$ be two interval numbers such that $0 \leq z\eta^{Lk^*,t} \leq z\eta^{Uk^*,t} \leq 1$ and $0 \leq \hat{\eta}^{Lk^*,t} \leq \hat{\eta}^{Uk^*,t} \leq 1$. We calculated the confidence level of these two interval numbers 1,000 times using MATLAB software, and obtained the simulation results (see Fig. 1). Fig. 1(a)–(e) show five independent sub-graphs, and Fig. 1(f) shows these five graphs overlapped as a single graph.

From the results in Fig. 1, we make the following observations:

- (1) The simulation results presented in Fig. 1(b)–(d) are similar in shape.
- (2) Each graph in Fig. 1(a)–(e) has a maximum point. This finding conforms to Theorem 1. Further, in Fig. 1(b)–(d), the three maximum values correspond to the case in which all the values of non-cooperation are close to 0.5 and all the values of rationality are close to one (points (0.492, 0.969, 0.798) in Fig 1(b), (0.503, 0.989, 0.701) in Fig 1(c), and (0.489, 0.959, 0.617) in Fig 1(d)).
- (3) When we set $0 < \mu_{WG}, \nu_{WG} < 1$, the relationship between the confidence level and degrees of rationality and non-cooperation can be described as one of two cases.
 - a) *Partially positive relationship*: The rationality is strongly positively correlated with the confidence level, whereas the non-cooperation is strongly negatively correlated with the confidence level; that is, the confidence level increases when the rationality increases and the non-cooperation decreases.
 - b) *Completely positive relationship*: The confidence level increases with the increase in rationality and the increase in non-cooperation.
- (4) Figs. 1(a) and (e) are two special scenarios that correspond to Theorems 2 and 3, respectively.

Note 1. If the adjustment coefficient and collective adjustment suggestion are expressed as interval numbers, then a completely negative relationship may emerge when different methods are used to calculate the rationality and non-cooperation. The completely negative relationship is defined as follows: the confidence level decreases when the rationality increases and non-cooperation increases. In future research, we will study the influence of different measurement formulas for rationality and non-cooperation on the relationship between the adjustment coefficient, collective adjustment suggestion, and confidence level.

3.3. Algorithm for the confidence consensus-based model in LSGDM

The proposed consensus model in this study aims to obtain a common decision result with a sufficiently high level of group consensus among DMs. The algorithm for this confidence consensus model is summarized as follows (also see Fig. 2).

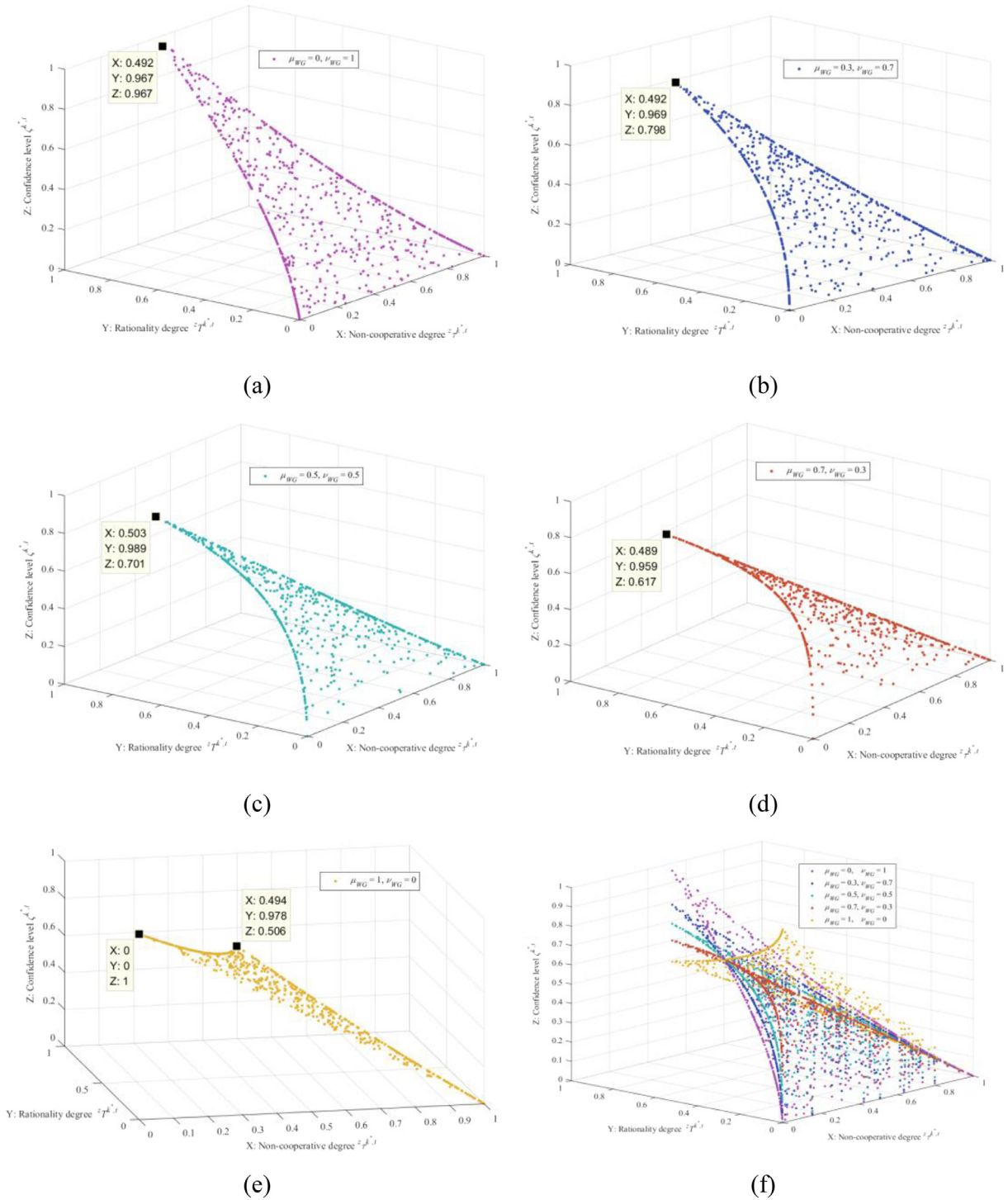


Fig. 1. Simulation results of the confidence level under different parameters μ_{WG} and ν_{WG} .

Input: initial normalized individual decision matrices $R^{l,0} (l = 1, 2, \dots, q)$, weight vector of the attributes ω , and acceptable threshold of the consensus level \overline{GCI} .

Output: final number of iterations, final group matrix, and alternative ranking.

Step 1. Classify the DMs into several subgroups.

Divide the large group into K clusters and calculate the initial weight vector of the clusters $\lambda^0 = (\lambda^{1,0}, \lambda^{2,0}, \dots, \lambda^{K,0})^T$. The initial clusters' decision matrices $R^{k,0} (k = 1, 2, \dots, K)$ can be obtained using the weighted averaging operator. Let $t = 0$.

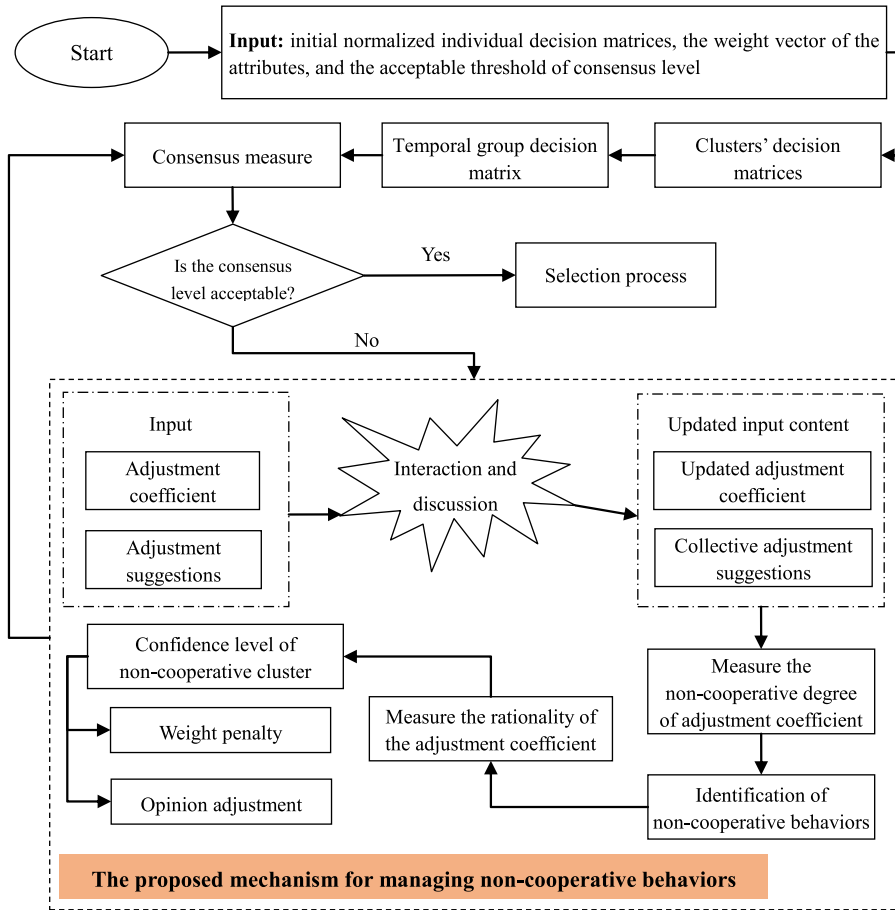


Fig. 2. Confidence consensus-based model for LSGDM.

Step 2. Compute the temporal group decision matrix.

Aggregate the clusters' decision matrices into temporal group decision matrices $R^{c,t} = (r_{ij}^{c,t})_{m \times n}$, where $r_{ij}^{c,t} = \sum_{k=1}^K r_{ij}^{k,t} \lambda^{k,t}$.

Step 3. Consensus measure.

Select the appropriate rule of consensus measures to calculate the clusters' consensus levels and group consensus level. If the group consensus level is greater than the predefined consensus threshold, then proceed to Step 5; otherwise, proceed to the next step.

Step 4. Consensus reaching process.

Apply the consensus model presented in Section 3 to manage both the CRPs and non-cooperative behaviors. Let $t = t + 1$ and return to Step 2.

Step 5. Output the related decision information.

Output the final iterative time t^* and final group decision matrix R^{c*} .

Step 6. Select the best alternative(s).

Using the obtained final group matrix, calculate the overall evaluation values of all alternatives and then select the best alternative(s).

4. Case study

We use the case in [35] to illustrate the effectiveness of the proposed model. The decision information is summarized as follows: Emergency shelter construction is a priority task for post-earthquake recovery. Four types of materials can be selected to construct emergency shelters: tents (x_1), plastic sheeting (x_2), waste wood (x_3), and light steel (x_4). Three aspects are considered: availability (a_1), convenience (a_2), and perceived safety (a_3). These attribute values are typically given on a 0–100 scale and the weight vector of the attributes is $\omega = (0.25, 0.5, 0.25)^T$. Twenty-five experts $e_l (l = 1, 2, \dots, 25)$ are invited to participate in the emergency decision process. Assume that the acceptable consensus threshold is $\overline{GCI} = 0.75$. The 25 experts offer their preferences over the alternatives with respect to attributes. The normalized individual decision matrices are shown in Appendix B in [35].

Table 1Clustering results with threshold $\gamma = 0.73$.

g_k	n_k	e_l	$R^{k,0}$			g_k	n_k	e_l	$R^{k,0}$		
g_1	5	$e_1, e_5, e_7, e_9, e_{12}$	0.501	0.571	1.000	g_2	3	e_2, e_4, e_{10}	0.000	0.173	0.430
			0.800	0.075	0.210				0.868	1.000	0.767
			0.033	0.132	0.470				1.000	0.000	1.000
			0.538	1.000	0.137				0.331	0.834	0.000
g_3	8	$e_3, e_8, e_{13}, e_{15}, e_{18}, e_{20}, e_{22}, e_{25}$	1.000	0.608	0.560	g_4	1	e_6	0.460	1.000	0.640
			0.431	0.625	1.000				0.745	0.245	0.000
			0.347	0.281	0.365				1.000	0.450	1.000
			0.000	0.407	0.105				0.000	0.000	0.320
g_5	8	$e_{11}, e_{14}, e_{16}, e_{17}, e_{19}, e_{21}, e_{23}, e_{24}$	0.000	0.000	0.000	–	–	–	–	–	–
			1.000	0.413	0.510						
			0.513	1.000	0.482						
			0.443	0.496	1.000						

Below, we apply the algorithm presented in Section 3.3 to select the best alternative. The experts decide to use CM-I to measure the consensus levels and set $\mu_{WG} = \nu_{WG} = 0.5$. The reason why CM-I is chosen will be explained in Section 5.4.

Input: normalized individual decision matrices R^l ($l = 1, 2, \dots, 25$), weight vector of the attributes ω , and acceptable consensus threshold $\overline{GCI} = 0.75$.

Output: final number of iterations, final group decision matrix, and alternative ranking.

Step 1. Classify the 25 experts into several subgroups.

Using the clustering method presented in [33], the large group is divided into five smaller clusters (see Table 1). The initial weight vector of the clusters is obtained as $\lambda^0 = (0.153, 0.055, 0.393, 0.006, 0.393)^T$. Let $t = 0$.

Step 2. Compute the temporal group decision matrix.

Aggregate the temporal clusters' decision matrices to obtain the temporal group matrix

$$R^{c,0} = \begin{pmatrix} 0.472 & 0.342 & 0.401 \\ 0.737 & 0.476 & 0.668 \\ 0.404 & 0.526 & 0.466 \\ 0.274 & 0.553 & 0.457 \end{pmatrix}.$$

Step 3. Consensus measure.

Use CM-I to calculate the individual consensus levels: $CI_{CM-I}^{1,0} = 0.702$, $CI_{CM-I}^{2,0} = 0.677$, $CI_{CM-I}^{3,0} = 0.757$, $CI_{CM-I}^{4,0} = 0.668$, and $CI_{CM-I}^{5,0} = 0.745$. The group consensus level is obtained as $GCI_{CM-I}^0 = 0.710$ using Eq. (3). Because $GCI_{CM-I}^0 < \overline{GCI}$, this indicates that there are large differences among the opinions of the clusters. Hence, CRP should be used.

Step 4. Consensus reaching process.

(i) First consensus iteration.

Because $CI_{CM-I}^{4,0} = \min_k \{CI_{CM-I}^{k,0}\}$, g_4 is the furthest from the group opinion. Cluster g_4 provides the adjustment coefficient of $\tilde{\eta}^{4,0} = [0.1, 0.15]$. Other clusters give their suggestions regarding the adjustment of its opinion, i.e., $\tilde{\eta}^{1 \rightarrow 4,0} = [0.4, 0.6]$, $\tilde{\eta}^{2 \rightarrow 4,0} = [0.5, 0.8]$, $\tilde{\eta}^{3 \rightarrow 4,0} = [0.3, 0.4]$, and $\tilde{\eta}^{5 \rightarrow 4,0} = [0.2, 0.4]$. The collective adjustment suggestion is $\hat{\eta}^{4,0} = [0.287, 0.453]$. The mechanism for managing non-cooperative behaviors is hence employed.

- Identification of non-cooperative behaviors:** Because $\tau^{4,0} = 1 > 0$, cluster g_4 can be regarded as a completely non-cooperative subgroup.
- Interaction and discussion:** Cluster g_4 changes its adjustment coefficient to ${}^z\tilde{\eta}^{4,0} = [0.2, 0.25]$.
- Proper modification:** The rationality is calculated as ${}^zT^{4,0} = 0$. Thus, the confidence level is $\varsigma^{4,0} = 0$. To ensure the quality and efficiency of decision making, the single member that makes up cluster g_4 is advised to exit the decision process.

The weight vector of clusters is updated to $\lambda^1 = (0.154, 0.056, 0.395, 0.395)^T$. The updated consensus levels become $CI_{CM-I}^{1,1} = 0.701$, $CI_{CM-I}^{2,1} = 0.677$, $CI_{CM-I}^{3,1} = 0.757$, and $CI_{CM-I}^{5,1} = 0.744$, and the group consensus level is $GCI_{CM-I}^1 = 0.720 < \overline{GCI}$. Thus, after the first iteration, differences remain among the clusters' opinions.

(ii) Second consensus iteration.

Because $CI_{CM-I}^{2,1} = \min_k \{CI_{CM-I}^{k,1}\}$, this means that g_2 has an opinion with the largest difference from the group opinion. Cluster g_2 provides the adjustment coefficient $\tilde{\eta}^{2,1} = [0.3, 0.5]$. Other clusters provide their suggestions regarding the adjustment of the opinion of g_2 as $\tilde{\eta}^{1 \rightarrow 2,1} = [0.4, 0.6]$, $\tilde{\eta}^{3 \rightarrow 2,1} = [0.5, 0.8]$, and $\tilde{\eta}^{5 \rightarrow 2,1} = [0.6, 0.9]$. The collective adjustment suggestion is hence $\hat{\eta}^{2,1} = [0.526, 0.809]$. The mechanism for managing non-cooperative behaviors proceeds as follows.

- Identification of non-cooperative behaviors:** because $\tau^{2,1} = 0.872 > 0$, cluster g_2 can be regarded as a non-cooperative subgroup.

- b) *Interaction and discussion*: Cluster g_2 changes its adjustment coefficient to $z\tilde{\eta}^{4,1} = [0.3, 0.55]$.
- c) *Proper modification*. The rationality degree is computed as $zT^{2,1} = 0.048$. Thus, the confidence level is $\zeta^{2,1} = 0.047$. Suppose that $\chi^1 = 1$. The weight of cluster g_2 is modified to $z\lambda^{2,2} = \chi^1 \cdot \zeta^{2,1} \cdot \lambda^{2,1} = 0.03$ using Eq. (9). The weight vector of the clusters is updated to $\lambda^2 = (0.163, 0.03, 0.417, 0.417)^T$. Eq. (11) is used to modify the matrix of cluster g_2 . The temporal group decision matrix becomes

$$R^{c,2} = \begin{pmatrix} 0.499 & 0.347 & 0.397 \\ 0.729 & 0.447 & 0.666 \\ 0.365 & 0.557 & 0.431 \\ 0.273 & 0.540 & 0.484 \end{pmatrix}.$$

After the second iteration, the cluster consensus levels are $CI_{CM-I}^{1,2} = 0.700$, $CI_{CM-I}^{2,2} = 0.807$, $CI_{CM-I}^{3,2} = 0.760$, and $CI_{CM-I}^{5,2} = 0.744$. The group consensus level is $GCI_{CM-I}^2 = 0.753 \geq \overline{GCI}$; that is, the group consensus level has reached an acceptable level (above the threshold) after two rounds of CRP.

Step 5. Output the related decision information: final group decision matrix $R^c = R^{c,2}$.

Step 6. Select the best alternative(s).

The overall evaluation value of each alternative is

$$EV(x_1) = 0.398, \quad EV(x_2) = 0.572, \quad EV(x_3) = 0.477, \quad \text{and} \quad EV(x_4) = 0.459.$$

The derived ranking of alternatives is $x_2 > x_3 > x_4 > x_1$, and thus x_2 is the best alternative. Hence, plastic sheeting is the most desirable material for constructing emergency shelters.

5. Comparative analysis

This section comprises five parts. We analyze the effect of the proposed model on managing non-cooperative behaviors. In Section 5.2, we formulate the influence of different consensus thresholds on the CRP. In Section 5.3, we present a comparison of the proposed model with other models [34,41]. We discuss the comparison results for different consensus measures and parameters in Sections 5.4 and 5.5, respectively. Note that the data used in the comparative analysis come from the case study in Section 4.

5.1. Effect of the confidence consensus-based model on managing non-cooperative behaviors

In the first iteration of the case study, cluster g_4 was regarded as a completely non-cooperative subgroup. In addition, $\tau^{4,0} = 1$ indicated that the given adjustment coefficient could not satisfy the minimum requirement of the collective adjustment suggestion. After an interaction with the other clusters, cluster g_4 was still very reluctant to modify its opinion to improve consensus (i.e., $z\tau^{4,0} = 1$). Hence, the DM in cluster g_4 was advised to exit the decision process. We can see that the group consensus level clearly increased from $GCI^0 = 0.710$ to $GCI^1 = 0.720$ after the DM in cluster g_4 exited the decision. This shows that a cluster who has very large difference of opinion from the group and is not willing to modify its opinion should be advised to exit the decision process in a timely manner. This is useful for enabling the remaining DMs to reach an agreement quickly.

In the second iteration, cluster g_2 was regarded as a partially non-cooperative subgroup. Using weight penalties and opinion adjustment, the individual consensus level was increased from $CI_{CM-I}^{2,1} = 0.677$ to $CI_{CM-I}^{2,2} = 0.807$, and eventually the group consensus level reached the consensus threshold. This suggests that to reduce the weight and modify an opinion that is substantially different from the group opinion, it is useful to reduce the differences among the DMs and reach an agreement.

5.2. Analysis of different consensus thresholds

In the following, we analyze the effect of setting different consensus thresholds. Table 2 and Fig. 3 present the decision results for different consensus thresholds. Fig. 3 shows the number of iterations and group consensus levels. Table 2 lists the final group decision matrices, overall evaluation values of each alternative, and alternative rankings. To better reflect the impact of different consensus thresholds on the decision, in this section, we adopt the objective adjustment coefficient presented in [34] to simulate the CRP and do not consider non-cooperative behaviors.

Based on the results in Table 2 and Fig. 3, the following observations can be obtained.

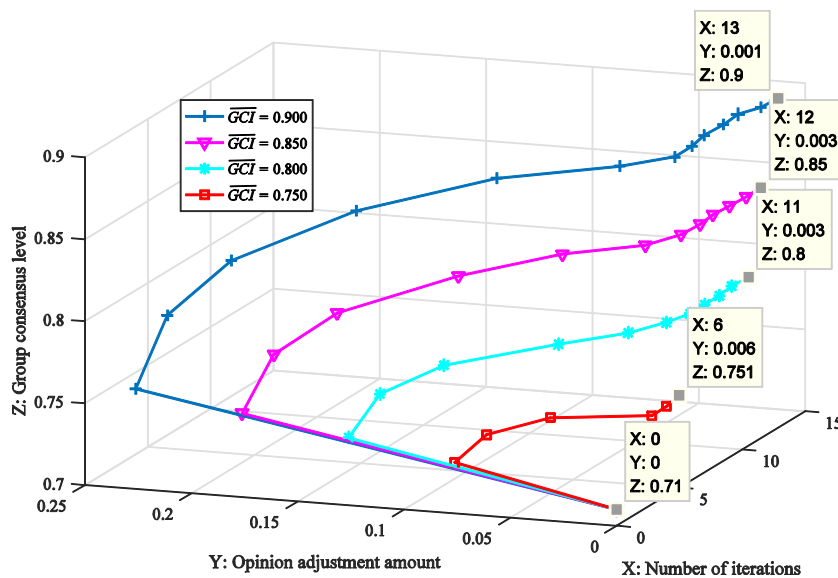
- (1) The number of iterations decreases as the consensus threshold decreases, and the final alternative rankings do not change. No change in alternative rankings can happen because the attribute weights are fixed.
- (2) The final rankings are the same; however, the final group decision matrices and overall evaluation values of the alternatives are different.
- (3) As the number of iterations increases, the group consensus level eventually meets the threshold.
- (4) The closer the group consensus level to the consensus threshold, the lower the opinion adjustment amount.

Table 2

Comparison results for different consensus thresholds.

Consensus threshold	Total adjustment amount	Final group decision matrix	Overall evaluation values of alternatives	Alternative ranking
0.900	0.980	$\begin{pmatrix} 0.507 & 0.347 & 0.382 \\ 0.717 & 0.488 & 0.702 \\ 0.401 & 0.544 & 0.447 \\ 0.251 & 0.523 & 0.465 \end{pmatrix}$	$EV(x_1) = 0.396$ $EV(x_2) = 0.599$ $EV(x_3) = 0.484$ $EV(x_4) = 0.441$	$x_2 > x_3 > x_4 > x_1$
0.850	0.719	$\begin{pmatrix} 0.504 & 0.349 & 0.390 \\ 0.719 & 0.486 & 0.696 \\ 0.401 & 0.534 & 0.451 \\ 0.254 & 0.531 & 0.457 \end{pmatrix}$	$EV(x_1) = 0.398$ $EV(x_2) = 0.597$ $EV(x_3) = 0.480$ $EV(x_4) = 0.443$	$x_2 > x_3 > x_4 > x_1$
0.800	0.465	$\begin{pmatrix} 0.504 & 0.353 & 0.397 \\ 0.717 & 0.487 & 0.695 \\ 0.400 & 0.527 & 0.453 \\ 0.254 & 0.534 & 0.450 \end{pmatrix}$	$EV(x_1) = 0.403$ $EV(x_2) = 0.596$ $EV(x_3) = 0.477$ $EV(x_4) = 0.443$	$x_2 > x_3 > x_4 > x_1$
0.750	0.223	$\begin{pmatrix} 0.478 & 0.337 & 0.382 \\ 0.733 & 0.480 & 0.681 \\ 0.405 & 0.545 & 0.457 \\ 0.267 & 0.538 & 0.473 \end{pmatrix}$	$EV(x_1) = 0.383$ $EV(x_2) = 0.594$ $EV(x_3) = 0.488$ $EV(x_4) = 0.454$	$x_2 > x_3 > x_4 > x_1$
0.700	0.000	$\begin{pmatrix} 0.472 & 0.342 & 0.401 \\ 0.737 & 0.476 & 0.668 \\ 0.404 & 0.526 & 0.466 \\ 0.275 & 0.553 & 0.457 \end{pmatrix}$	$EV(x_1) = 0.389$ $EV(x_2) = 0.589$ $EV(x_3) = 0.480$ $EV(x_4) = 0.460$	$x_2 > x_3 > x_4 > x_1$

Note: The total adjustment amount can be calculated using Definition 3 presented in [34], which is also reviewed in Appendix B.

**Fig. 3.** Simulation results based on the objective adjustment coefficient for different consensus thresholds.

- (5) A higher consensus threshold requires a greater adjustment amount, which leads to a more likely distortion of opinions. Therefore, the threshold needs to be set reasonably.
- (6) An available method to confirm the two important parameters (i.e., the number of iterations and acceptable consensus threshold) is to simulate the CRP using the objective adjustment coefficient presented in [34]. The simulation results demonstrate the relationship between the number of iterations and consensus threshold, which can be used as a reference to determine these two parameters.

5.3. Comparison with Xu et al.'s model [34] and Zhang et al.'s model [41]

To demonstrate the advantages of the proposed method, a comparative analysis is conducted using another technique [34]. Xu et al. [34] utilized the degree of non-cooperation to manage non-cooperative behaviors. In our study, the proposed

Table 3

Comparison of Xu et al.'s model and our proposed model.

Consensus model	Updated adjustment coefficient	Collective adjustment suggestion	Non-cooperative degree	Rationality degree	Confidence level	Modified weight	Updated group consensus level
Xu et al.'s model [34]	–	[0.225, 0.900]	0.686	–	–	0.022	0.757
Proposed model	[0.3, 0.55]	[0.526, 0.809]	0.954	0.048	0.047	0.003	0.753

Note: To ensure the effectiveness of the comparison results, we suppose that The two models had the same adjustment coefficient (i.e., $\bar{\eta}^{2,1} = [0.3, 0.5]$) and adjustment suggestions (i.e., $\bar{\eta}^{1 \rightarrow 2,1} = [0.4, 0.6]$, $\bar{\eta}^{3 \rightarrow 2,1} = [0.5, 0.8]$, $\bar{\eta}^{5 \rightarrow 2,1} = [0.6, 0.9]$).

Table 4

Comparison of Zhang et al.'s model and our proposed model.

Consensus model	Changeable clusters?	Expression form of satisfaction	Object of preference adjustment	Management of non-cooperative behaviors	Composition of the CRP
Zhang et al.'s model [41]	Yes	Explicit	All the preferences can be modified	No	(1) Consensus measure (2) Satisfaction measure (3) Feedback adjustment
Proposed model	No	Implicit	Preference which has the lowest consensus level should be modified	Yes	(1) Consensus measure (2) Feedback adjustment (Including addressing non-cooperative behaviors)

Note: The item “Object of preference adjustment” refers to which preferences may be adjusted during each iteration of CRPs.

consensus model includes not only the degree of non-cooperation but also the newly defined degree rationality. We compare these two models in the second iteration of the case study in Section 4.

The results in Table 3 show the following differences between our proposed model and Xu et al.'s model [34]:

- (1) The updated group consensus levels obtained by the two models both exceed the consensus threshold. This finding implies that both models are effective at promoting consensus by managing non-cooperative behaviors.
- (2) The modified weights are obviously different. This is for two reasons: (a) The two models calculate the collective adjustment suggestions differently. Xu et al.'s model uses the max-min operator to combine the objective adjustment coefficient and adjustment suggestions, whereas the proposed model directly aggregates the adjustment suggestions. This causes the different results for the non-cooperation. (b) The proposed model introduces the rationality and confidence level, which not only considers the non-cooperation of the adjustment but, more importantly, the rationality of the adjustment coefficient.
- (3) The proposed model adds the interaction step and a discussion that could help DMs to provide more favorable adjustment coefficients as well as reducing the distrust and emotional expression of DMs.

Further considering the reasons for the existence of non-cooperative behaviors in the CRP, we recognize that the varying levels of DMs' satisfaction with respect to the consensus level can lead to non-cooperative behaviors. For example, in a CRP under a GDM environment, the DM Andy is asked to adjust his opinion to promote consensus. Suppose that Andy's consensus level is 0.7, the group consensus level is 0.75, and the consensus threshold is set as 0.8. Andy thinks his consensus level is close to the threshold; that is, he is somewhat satisfied with this level and hence is unwilling to make a noticeable adjustment to his opinion. Other DMs, however, do not agree with Andy's satisfaction regarding the consensus level, and view Andy's adjustment action as a non-cooperative behavior. Zhang et al. [41] investigated the individual satisfaction with consensus level under a heterogeneous LSGDM environment. To better demonstrate the contribution of our proposed model to managing consensus in LSGDM, we compare Zhang et al.'s model and our proposed model (Table 4).

A detailed explanation of each of these differences are presented below.

5.3.1. Explicit satisfaction versus implicit satisfaction

“Explicit satisfaction” refers to a scenario in which DMs provide their own satisfaction with the consensus level. In Zhang et al.'s research [41], the collective satisfaction index is obtained by aggregating the individual satisfactions. This collective satisfaction is compared with the established satisfaction index to determine whether a feedback mechanism is needed. An implicit expression of satisfaction is thought to be a clue to non-cooperative behaviors. Generally, non-cooperative behaviors occur for three reasons: (1) a DM (or cluster) thinks the current level of consensus is high enough (that is, he/she is satisfied with the current level of consensus), so there is no need to adjust the preference significantly; (2) a DM (or cluster) sticks to his/her own preferences and is reluctant to adjust them; and (3) a mixture of the first two reasons.

The implicit expression of satisfaction is hard to measure. An explicit expression can intuitively quantify a DM's satisfaction with the consensus level. Unlike our model, Zhang et al.'s model [41] adds a “satisfaction measure” to the CRP. However, dishonesty may exist and affect the accuracy of such expressions. For example, a non-cooperative DM can avoid being forced

Table 5
Simulation results for different consensus measures ($\overline{GCI} = 0.75$).

Consensus measure rules	Total adjustment amount	Final group decision matrix	Overall evaluation values of alternatives	Alternative ranking
CM-I	0.223	$\begin{pmatrix} 0.507 & 0.347 & 0.382 \\ 0.717 & 0.488 & 0.702 \\ 0.401 & 0.544 & 0.447 \\ 0.251 & 0.523 & 0.465 \end{pmatrix}$	$EV(x_1) = 0.396$ $EV(x_2) = 0.599$ $EV(x_3) = 0.484$ $EV(x_4) = 0.441$	$x_2 > x_3 > x_4 > x_1$
CM-II	0.227	$\begin{pmatrix} 0.504 & 0.349 & 0.390 \\ 0.719 & 0.486 & 0.696 \\ 0.401 & 0.534 & 0.451 \\ 0.254 & 0.531 & 0.457 \end{pmatrix}$	$EV(x_1) = 0.398$ $EV(x_2) = 0.597$ $EV(x_3) = 0.480$ $EV(x_4) = 0.443$	$x_2 > x_3 > x_4 > x_1$
CM-III	0.811	$\begin{pmatrix} 0.504 & 0.353 & 0.397 \\ 0.717 & 0.487 & 0.695 \\ 0.400 & 0.527 & 0.453 \\ 0.254 & 0.534 & 0.450 \end{pmatrix}$	$EV(x_1) = 0.403$ $EV(x_2) = 0.596$ $EV(x_3) = 0.477$ $EV(x_4) = 0.443$	$x_2 > x_3 > x_4 > x_1$

Note: We use the objective adjustment coefficient presented in [34] to simulate the consensus process for different consensus measures rules in this subsection.

to adjust his/her opinion by falsely reporting a high level of satisfaction. In this sense, dishonesty and non-cooperative behaviors are two issues that must be addressed. Our model proposes a mechanism for managing non-cooperative behaviors that could be extended to consensus building where satisfaction is expressed explicitly.

5.3.2. The management of non-cooperative behaviors

In LSGDM problems, non-cooperative behaviors always exist [10,20,34]. For example, some individuals may refuse to modify their preferences or move their preferences to resist consensus to further their private interests. Hence, adding the management of non-cooperative behaviors to the CRP is necessary in many decision scenarios. Compared with Zhang et al.' model [41], our proposed model presents a confidence-consensus model that contains a novel mechanism for addressing non-cooperative behaviors.

5.3.3. Changeable cluster structure versus unalterable cluster structure

Generally, the cluster structure is one of two types: (1) it does not change once it has been determined [34,35] because a stable clustering structure prompts the members in each subgroup to establish trust feelings and form a unified opinion and (2) it changes with the dynamic changes in similarity of DM preferences caused by preference adjustment (see [32,41]). This study uses the cluster as a basic unit once the clustering structure has been determined.

Additionally, in Zhang et al.'s research [41], when the collective satisfaction degree is lower than the predefined threshold, all the DMs modify their own preferences. In our model, only the preference of the cluster that is the most different from the group preference should be modified. If multiple DMs adjust their preferences simultaneously, the decision-making process may quickly reach a consensus level with high satisfaction, but it could also become more complex and uncertain.

5.4. Comparison with different consensus measures

In terms of the consensus measure, CM-II is much stricter than CM-I. Because it uses a minimum operator, which is a more rigorous criterion, its use can avoid the situation in which some clusters' consensus levels are still lower than the consensus threshold even though the group consensus level is sufficiently high in the final decision results. In contrast to CM-I and CM-II, CM-III directly measures the difference between clusters and can avoid the deviation caused by the weighted average operation when aggregating cluster opinions. Table 5 and Fig. 4 both describe the CRP and results of using the objective adjustment coefficient to simulate the consensus process for different consensus measures.

Clearly, there are differences in the adjustment amount, final decision matrix, final alternative evaluation value, and number of iterations when different consensus measures are used. Based on the simulation results in Table 6 and Fig. 4, we conclude the following.

- (1) CM-II and CM-III have more stringent requirements than CM-I. Different consensus measures may apply to different decision problems.
- (2) Given the differences in consensus measures, different consensus thresholds should be set for the three measures.

In the case study presented in Section 4, the DMs believed that the construction of emergency shelters was the most important part of post-earthquake recovery and the most appropriate solution was to follow the majority of opinion. Therefore, the DMs with relatively large weights should have a significant impact on the group opinion and consensus measure. Moreover, just as importantly, because of the timeliness of constructing emergency shelters, it is preferable that the solution be acceptable to most DMs, rather than recognizing all opinions. Eventually, the DMs decided to use CM-I to measure the consensus levels.

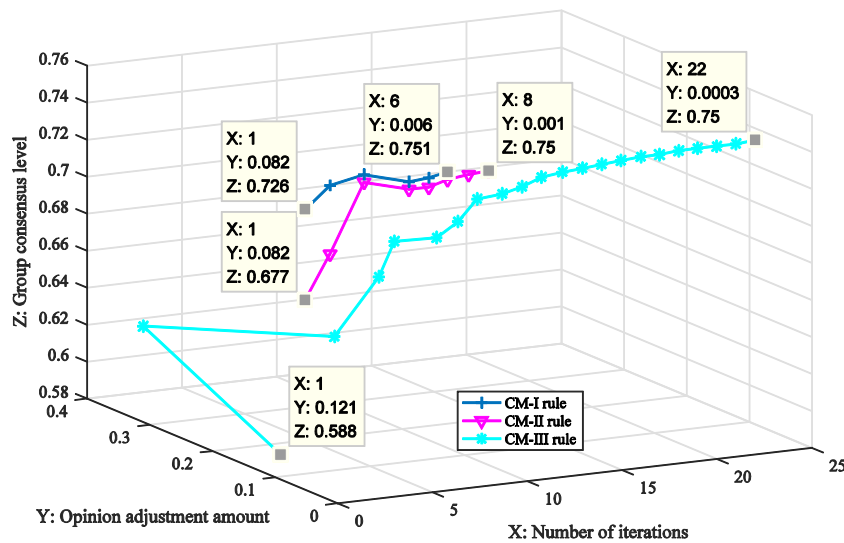


Fig. 4. Simulation results based on the objective adjustment coefficient for different consensus measures.

Table 6

Comparison of the advantages and disadvantages of different measure rules.

Consensus measure rules	Detailed description
CM-I	Advantages: <ul style="list-style-type: none"> The DMs with relatively large weights have a significant impact on the consensus measure. The greater the weight of a DM, the greater the contribution of its opinion to the group opinion, and thus the higher its individual consensus level. Only the average value of individual consensus levels needs to be higher than the predefined consensus threshold without all individual consensus levels reaching the threshold. It may not need too many iterations and the consensus process may not be difficult to implement. Disadvantages: <ul style="list-style-type: none"> DMs' weights need to be calculated in advance. The consensus process is generally aimed at the lower-weight DMs' opinions because their opinions contribute less to the aggregated group opinion and thus are relatively far from the group opinion. A compromise appears between some DMs' opinions with very high consensus levels and those with low consensus levels when the group consensus level is sufficiently high.
CM-II	Advantages: <ul style="list-style-type: none"> DMs with relatively large weights have obvious advantages. Using a min operator to calculate consensus allows for a more rigorous criterion that can avoid the compromise presented in CM-I rule. Disadvantages: <p>A tighter measure rule leads to more iterations and a greater adjustment of opinions when the consensus threshold is fixed.</p>
CM-III	Advantages: <p>There is no need to consider the weights of DMs when measuring the consensus. All the concerns regard calculating the differences between DM's opinions.</p> Disadvantages: <p>The advantages of DMs with relatively large weights cannot be effectively embodied in a consensus measure.</p>

5.5. Comparison with different parameters

In this part, we analyze the influence of parameters μ_{WG} , ν_{WG} , and χ^t on the decision results. Considering the first iteration in the case study, Table 7 and Fig. 5 show the simulation results when different input parameters μ_{WG} , ν_{WG} , and χ^1 are set. The simulation results in Table 7 use five sets of μ_{WG} and ν_{WG} and three sets of χ^1 and illustrate the change in the confidence level, group consensus level, and adjusted weight. Fig. 5 shows the results when we randomly generated input parameters μ_{WG} , ν_{WG} , and χ^1 and run the simulation 1,000 times to obtain the confidence level and adjusted weight values. To compare the data more intuitively, we present four significant figures for the weights $z\lambda^{2,2}$ and updated group consensus levels in Table 7 and Fig. 5.

The following observations are obtained.

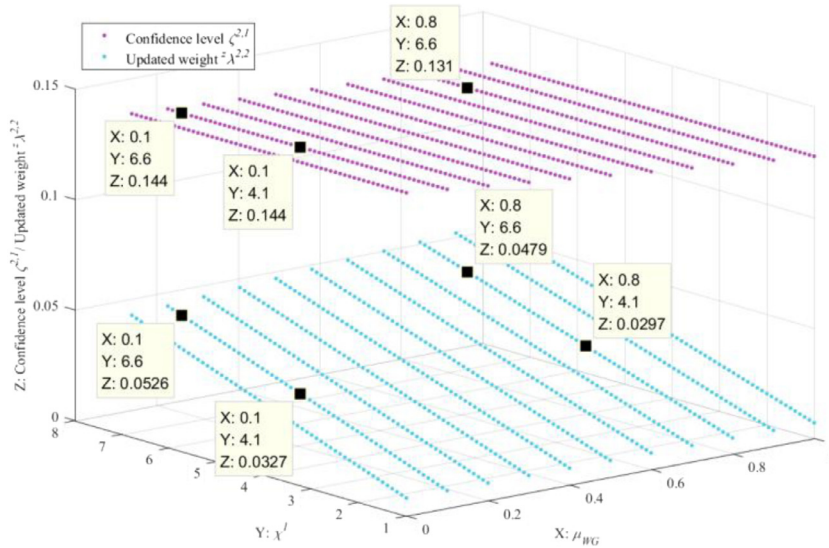
- (1) Let the auxiliary coefficient χ^1 be fixed. When setting different parameters μ_{WG} and ν_{WG} , there are differences in the confidence level, adjusted weight, and updated group consensus level. For example, in Table 7, for $\chi^1 = 5$, when

Table 7Comparison results based on the data in the case study for different parameters μ_{WG} , ν_{WG} , and χ^1 .

μ_{WG}, ν_{WG}	$\zeta^{2,1}$	$z\lambda^{2,2}$			Updated alternative ranking	Updated group consensus level
		$\chi^1 = 5$	$\chi^1 = 2$	$\chi^1 = 1$		
0.0, 1.0	0.1462	0.0405	0.0162	0.0081	2341, 2341, 2341	0.7511, 0.7498, 0.7494
0.3, 0.7	0.1404	0.0388	0.0155	0.0078	2341, 2341, 2341	0.7511, 0.7498, 0.7493
0.5, 0.5	0.1366	0.0378	0.0151	0.0076	2341, 2341, 2341	0.7510, 0.7498, 0.7493
0.7, 0.3	0.1329	0.0368	0.0147	0.0074	2341, 2341, 2341	0.7509, 0.7497, 0.7493
1.0, 0.0	0.1276	0.0353	0.0141	0.0071	2341, 2341, 2341	0.7509, 0.7497, 0.7493

Notes: (a) The data in the item "Updated alternative ranking" represent three groups of alternative rankings when parameter χ^1 is set to 5, 2 and 1, respectively. "2341" indicates that the alternative ranking is $x_2 > x_3 > x_4 > x_1$.

(b) The data in the item "Updated group consensus level" represent the different group consensus levels when parameter χ^1 is set to 5, 2, and 1, respectively.

**Fig. 5.** Simulation results of confidence level $\zeta^{2,1}$ and adjusted weight $z\lambda^{2,2}$ for different parameters μ_{WG} , ν_{WG} , and χ^1 .

$\mu_{WG} = 0$ and $\nu_{WG} = 1$, the updated group consensus level is 0.7511 and $z\lambda^{2,2} = 0.0405$; when $\mu_{WG} = 0.7$ and $\nu_{WG} = 0.3$, the group consensus level is 0.7509 and $z\lambda^{2,2} = 0.0368$. Furthermore, Fig. 5 shows our comparison of the two sets of coordinates, that is, the pair (0.1, 6.6, 0.144) and (0.8, 6.6, 0.131) and the pair (0.1, 6.6, 0.0526) and (0.8, 6.6, 0.0479). The difference in the confidence level and adjusted weight is obvious.

- (2) Let μ_{WG} and ν_{WG} be fixed. When χ^1 is set to different values, then the adjusted weight of cluster g_2 may also be different. For example, in Table 7, when $\mu = 0.3$, $\nu = 0.7$, and χ^1 is set to 5 or 2, the corresponding adjusted weight of g_2 is 0.0388 or 0.0155. In accordance with Eq. (9), when the confidence level is fixed, the adjusted weight is proportional to parameter χ^1 . The two coordinates shown in Fig. 5, i.e., (0.1, 6.6, 0.0526) and (0.1, 4.1, 0.0327), also demonstrate this proportional relation.

6. Conclusions

Because LSGDM can model many real-world decisions, a confidence consensus-based model that considers non-cooperative behaviors was proposed. The major contributions of this study are as follows:

- New concepts were proposed: the collective adjustment suggestion, degree of rationality, and confidence level.
- A novel mechanism for addressing non-cooperative behaviors was presented. We divide the management of non-cooperative behaviors into three phases. The interaction and discussion effectively make DMs provide more suitable adjustment coefficients. In this process, we combine the degrees of rationality and non-cooperation to construct the confidence level of a cluster's adjustment coefficient. It is more accurate and objective to use the confidence level to measure the willingness of a cluster to adjust its opinion to promote agreement. Further, it ensures that the cluster has an accurate and reasonable understanding of the decision problem.
- An algorithm for the confidence consensus-based model in LSGDM was presented. Its aim is to better address non-cooperative behaviors and obtain a common decision result with a sufficiently high level of group consensus.

- We summarized different consensus measures and used simulation to compare them. The simulation results reveal the advantages and disadvantages of these different measures.
- A comparison with Zhang et al.'s model [41] leads to the concepts of “explicit satisfaction” and “implicit satisfaction.” These concepts lead to a significant research issue: how to measure satisfaction that is implicitly expressed.

Meanwhile, there still exist some limitations:

- (1) In some real-life LSGDM, trust relationships among DMs defined by a social network play a key role. As a consequence, the research on CRMs in social network-based GDM has been paid increasing attention. It could be meaningful to build an improved CRM for addressing non-cooperative behaviors within a social network environment.
- (2) Zhang et al.'s research [41] indicates that in a LSGDM, DMs often have individual concerns about alternatives and satisfactions regarding the degree of consensus in CRPs. Thus, they prefer to use heterogeneous preference representation structures to express their opinions. It is important to extend the traditional CRMs to manage non-cooperative behaviors in heterogeneous LSGDM with individual concerns and satisfactions.
- (3) How to more reasonably modify DMs' weights based on the confidence level is also an important issue to address. Solving this matter depends on multiple simulation analyses with real data from different sources.

Acknowledgements

The authors would like to thank the editors and anonymous reviewers for their insightful comments and suggestions. This research was supported by the [National Natural Science Foundation of China](#) (No. 71671189), the Major Project of the [Natural Science Foundation of China](#) (No. 71790615), and the Key Project of the [Natural Science Foundation of China](#) (No. 71431006).

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:[10.1016/j.ins.2018.10.058](https://doi.org/10.1016/j.ins.2018.10.058).

Appendix A. Definition of the similarity of two interval numbers

Definition A.1. [37]. The similarity of the adjustment coefficient $\tilde{\eta}^{k^*,t}$ and collective adjustment suggestion $\hat{\eta}^{k^*,t}$ can be obtained as

$$T(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t}) = \begin{cases} \frac{1-d(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t})}{1+d(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t})}, & 0 \leq d(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t}) \leq 1 \\ 0, & d(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t}) \geq 1 \end{cases}, \quad (12)$$

where $d(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t})$ is the distance between $\tilde{\eta}^{k^*,t}$ and $\hat{\eta}^{k^*,t}$; that is,

$$d(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t}) = \frac{|\eta^{Lk^*,t} - \hat{\eta}^{Lk^*,t}| + |\eta^{Uk^*,t} - \hat{\eta}^{Uk^*,t}|}{l(\tilde{\eta}^{k^*,t}) + l(\hat{\eta}^{k^*,t})}, \quad (13)$$

where $l(\tilde{\eta}^{k^*,t}) = \eta^{Uk^*,t} - \eta^{Lk^*,t}$, $l(\hat{\eta}^{k^*,t}) = \hat{\eta}^{Uk^*,t} - \hat{\eta}^{Lk^*,t}$. If $\tilde{\eta}^{k^*,t}$ and $\hat{\eta}^{k^*,t}$ are both crisp numbers, then the distance between them is

$$d(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t}) = |\eta^{Uk^*,t} - \hat{\eta}^{Uk^*,t}|. \quad (14)$$

Yang et al. [37] proved that the similarity in [Definition A.1](#) could meet the axiomatic definition of similarity. Clearly, $T(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t})$ has the following properties:

- (1) $0 \leq T(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t}) \leq 1$;
- (2) $T(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t}) = T(\hat{\eta}^{k^*,t}, \tilde{\eta}^{k^*,t})$;
- (3) $T(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t}) = 1$, if and only if $\tilde{\eta}^{k^*,t} = \hat{\eta}^{k^*,t}$;
- (4) If the three interval numbers meet the condition that $\bar{c} \subseteq \hat{\eta}^{k^*,t} \subseteq \tilde{\eta}^{k^*,t}$, then $T(\tilde{\eta}^{k^*,t}, \bar{c}) \leq T(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t}) + T(\hat{\eta}^{k^*,t}, \bar{c})$, where \bar{c} is an interval number.

Definition A.2. Using the similarity of two interval numbers presented in [Definition A.1](#), the rationality of the adjustment coefficient provided by cluster g_{k^*} is defined as

$$T^{k^*,t} = T(\tilde{\eta}^{k^*,t}, \hat{\eta}^{k^*,t}). \quad (15)$$

Clearly, $0 \leq T^{k^*,t} \leq 1$.

Appendix B. Definition of the total amount of adjustment

Definition B. [34] The amount of opinion adjustment between $R^{k,t+1}$ and $R^{k,t}$ is defined as

$$AD(R^{k,t+1}, R^{k,t}) = d(R^{k,t+1}, R^{k,t}) = \frac{1}{m \times n} \sum_{i=1}^m \sum_{j=1}^n d(r_{ij}^{k,t+1}, r_{ij}^{k,t}), \quad (16)$$

where $d(R^{k,t+1}, R^{k,t})$ is the Manhattan distance between $R^{k,t+1}$ and $R^{k,t}$. The total adjustment amount is the sum of all the opinion adjustment amounts in the CRP.

References

- [1] S. Alonso, A web based consensus support system for group decision making problems and incomplete preferences, *Inf. Sci.* 180 (2010) 4477–4495.
- [2] F.J. Cabrerizo, I.J. Pérez, E. Herrera-Viedma, Managing the consensus in group decision making in an unbalanced fuzzy linguistic context with incomplete information, *Knowl. Based Syst.* 23 (2010) 169–181.
- [3] X. Chen, H.J. Zhang, Y.C. Dong, The fusion process with heterogeneous preference structures in group decision making: a survey, *Inf. Fusion* 24 (2015) 72–83.
- [4] Y.C. Dong, Z.G. Ding, L. Martínez, F. Herrera, Managing consensus based on leadership in opinion dynamics, *Inf. Sci.* 397–398 (2017) 187–205.
- [5] Y.C. Dong, C.C. Li, Y.F. Xu, X. Gu, Consensus-based group decision making under multi-granular unbalanced 2-tuple linguistic preference relations, *Group Decis. Negot.* 24 (2015) 217–242.
- [6] Y.C. Dong, Q.B. Zha, H.J. Zhang, G. Kou, H. Fujita, F. Chiclana, E. Herrera-Viedma, Consensus reaching in social network group decision making: Research paradigms and challenges, *Knowl. Based Syst.* (2018). <https://doi.org/10.1016/j.knosys.2018.06.036>.
- [7] Y.C. Dong, M. Zhan, G. Kou, Z.G. Ding, H.M. Liang, A survey on the fusion process in opinion dynamics, *Inf. Fusion* 43 (2018) 57–65.
- [8] Y.C. Dong, H.J. Zhang, E. Herrera-Viedma, Integrating experts' weights generated dynamically into the consensus reaching process and its applications in managing non-cooperative behaviors, *Decis. Support Syst.* 84 (2016) 1–15.
- [9] Y.C. Dong, H.J. Zhang, E. Herrera-Viedma, Consensus reaching model in the complex and dynamic MAGDM problem, *Knowl. Based Syst.* 106 (2016) 206–219.
- [10] Y.C. Dong, S.H. Zhao, H.J. Zhang, F. Chiclana, E. Herrera-Viedma, A self-management mechanism for non-cooperative behaviors in large-scale group consensus reaching processes, *IEEE Trans. Fuzzy Syst.* (2018). <https://doi.org/10.1109/tfuzz.2018.2818078>.
- [11] F. Herrera, E. Herrera-Viedma, J.L. Verdegay, A model of consensus in group decision making under linguistic assessments, *Fuzzy Sets Syst.* 78 (1996) 73–87.
- [12] E. Herrera-Viedma, L. Martínez, F. Mata F, F. Chiclana, A consensus support system model for group decision-making problems with multigranular linguistic preference relations, *IEEE Trans. Fuzzy Syst.* 13 (2005) 644–658.
- [13] J. Kacprzyk, M. Fedrizzi, A “soft” measure of consensus in the setting of partial (fuzzy) preferences, *Eur. J. Oper. Res.* 34 (1988) 316–325.
- [14] T. Kanungo, D.M. Mount, N.S. Netanyahu, C.D. Piatko, R. Silverman, A.Y. Wu, An efficient k -means clustering algorithm: analysis and implementation, *IEEE Trans. Pattern Anal. Mach. Intell.* 24 (2002) 881–892.
- [15] Á. Labella, Y. Liu, R.M. Rodríguez, L. Martínez, Analyzing the performance of classical consensus models in large scale group decision making: a comparative study, *Appl. Soft Comput.* (2017). <https://doi.org/10.1016/j.asoc.2017.05.045>.
- [16] C.H. Lin, C.H. Wu, P.Z. Huang, Grey clustering analysis for incipient fault diagnosis in oil-immersed transformers, *Expert Syst. Appl.* 36 (2009) 1371–1379.
- [17] B.S. Liu, Y.H. Shen, X.H. Chen, Y. Chen, X.Q. Wang, A partial binary tree DEA-DA cyclic classification model for decision makers in complex multi-attribute large-group interval-valued intuitionistic fuzzy decision-making problems, *Inf. Fusion* 18 (2014) 119–130.
- [18] F. Mata, L. Martínez, E. Herrera-Viedma, An adaptive consensus support model for group decision-making problems in a multigranular fuzzy linguistic context, *IEEE Trans. Fuzzy Syst.* 17 (2009) 279–290.
- [19] I. Palomares, Consensus model for large-scale group decision support in IT services management, *Intell. Decis. Technol.* 8 (2014) 81–94.
- [20] I. Palomares, L. Martínez, F. Herrera, A consensus model to detect and manage noncooperative behaviors in large-scale group decision making, *IEEE Trans. Fuzzy Syst.* 22 (2014) 516–530.
- [21] R.O. Parreiras, P.Y. Ekel, J.S.C. Martini, R.M. Palhares, A flexible consensus scheme for multicriteria group decision making under linguistic assessments, *Inf. Sci.* 180 (2010) 1075–1089.
- [22] D.A. Pelta, R.R. Yager, Decision strategies in mediated multiagent negotiations: an optimization approach, *IEEE Trans. Syst. Man Cybern. Part A* 40 (2010) 635–640.
- [23] L.G. Pérez, F. Mata, F. Chiclana, Social network decision making with linguistic trustworthiness-based induced OWA operators, *Int. J. Intell. Syst.* 29 (2015) 1117–1137.
- [24] F.J. Quesada, I. Palomares, L. Martínez, Managing experts behavior in large-scale consensus reaching processes with uninorm aggregation operators, *Appl. Soft Comput.* 35 (2015) 873–887.
- [25] L. Roselló, M. Sánchez, N. Agell, F. Prats, F.A. Mazaira, Using consensus and distances between generalized multi-attribute linguistic assessments for group decision-making, *Inf. Fusion* 17 (2014) 83–92.
- [26] J. Wu, F. Chiclana, A social network analysis trust-consensus based approach to group decision-making problems with interval-valued fuzzy reciprocal preference relations, *Knowl. Based Syst.* 59 (2014) 97–107.
- [27] J. Wu, F. Chiclana, E. Herrera-Viedma, Trust based consensus model for social network in an incomplete linguistic information context, *Appl. Soft Comput.* 35 (2015) 827–839.
- [28] J. Wu, R.Y. Xiong, F. Chiclana, Uninorm trust propagation and aggregation methods for group decision making in social network with four tuple information, *Knowl. Based Syst.* 96 (2016) 29–39.
- [29] J. Wu, L.F. Dai, F. Chiclana, H. Fujita, E. Herrera-Viedma, A minimum adjustment cost feedback mechanism based consensus model for group decision making under social network with distributed linguistic trust, *Inf. Fusion* 41 (2018) 232–242.
- [30] T. Wu, X.W. Liu, F. Liu, An interval type-2 fuzzy TOPSIS model for large scale group decision making problems with social network information, *Inf. Sci.* 432 (2018) 392–410.
- [31] Z.B. Wu, J.P. Xu, Managing consistency and consensus in group decision making with hesitant fuzzy linguistic preference relations, *Omega* 65 (2016) 28–40.
- [32] Z.B. Wu, J.P. Xu, A consensus model for large-scale group decision making with hesitant fuzzy information and changeable clusters, *Inf. Fusion* 41 (2018) 217–231.
- [33] X.H. Xu, X.H. Chen, Research on the group clustering method based on vector space, *Syst. Eng. Electron.* 27 (2005) 1034–1037.
- [34] X.H. Xu, Z.J. Du, X.H. Chen, Consensus model for multi-criteria large-group emergency decision making considering non-cooperative behaviors and minority opinions, *Decis. Support Syst.* 79 (2015) 150–160.
- [35] Y.J. Xu, X.W. Wen, W.C. Zhang, A two-stage consensus method for large-scale multi-attribute group decision making with an application to earthquake shelter selection, *Comput. Ind. Eng.* 116 (2018) 113–129.

- [36] Z.S. Xu, An automatic approach to reaching consensus in multiple attribute group decision making, *Comput. Ind. Eng.* 56 (2009) 1369–1374.
- [37] C.L. Yang, C.F. Zhang, W.C. Xu, A model based on similarity degree of interval number for uncertain multi-attribute decision making, *Math. Pract. Theory* 40 (2010) 148–154.
- [38] R.R. Yager, Penalizing strategic preference manipulation in multi-agent decision making, *IEEE Trans. Fuzzy Syst.* 9 (2001) 393–403.
- [39] R.R. Yager, Defending against strategic manipulation in uninorm-based multi-agent decision making, *Eur. J. Oper. Res.* 141 (2002) 217–232.
- [40] B.W. Zhang, H.M. Liang, G.Q. Zhang, Reaching a consensus with minimum adjustment in MAGDM with hesitant fuzzy linguistic term sets, *Inf. Fusion* 42 (2018) 12–23.
- [41] H.J. Zhang, Y.C. Dong, E. Herrera-Viedma, Consensus building for the heterogeneous large-scale GDM with the individual concerns and satisfactions, *IEEE Trans. Fuzzy Syst.* 26 (2018) 884–898.
- [42] Y.X. Zhang, Z.S. Xu, H.C. Liao, A consensus process for group decision making with probabilistic linguistic preference relations, *Inf. Sci.* 414 (2017) 260–275.
- [43] M. Zhao, X.Y. Ma, D.W. Wei, A method considering and adjusting individual consistency and group consensus for group decision making with incomplete linguistic preference relations, *Appl. Soft Comput.* 54 (2017) 322–346.