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Heat transfer in packed beds

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Abstract—The heat transfer between a fluid flowing through a packed bed and the wall is interpreted according to a one-dimensional model characterized by an overall heat transfer coefficient, h_w and by two-dimensional models with one parameter, the effective thermal conductivity, k_e or two parameters, the effective thermal conductivity λ_e and the wall heat transfer coefficient α_w. The experimental results for h_w, k_e, λ_c and α_w are correlated as a function of Reynolds number, packing and tube diameter. The correlations with respect to the Reynolds number are all linear. Correlations between the parameters of the various models are also given.

1. INTRODUCTION

THE SITUATION considered in this paper is the heating or cooling of a fluid flowing through a packed bed and exchanging heat with the wall. Until a few years ago this phenomenon was described by a one dimensional continuum model which considers the temperature of the fluid to be uniform in a section perpendicular to flow. The heat exchange with the wall is then characterized by means of a so called overall heat transfer coefficient, h_w (Max Leva[9–12]; Maeda[7, 8]; Schlünder[13, 14]).

The condition of radial uniformity is generally not fulfilled, especially in chemical reactors with important heat effects. In such cases it may be of importance to take the radial gradients into account. This more detailed approach may be required, e.g. when it is necessary to check if the temperatures in the axis do not exceed a certain value which would cause the reactor to be oversensitive. It has been shown that the 'mean' temperature predicted by the one-dimensional model may differ significantly from the true radial mean (Froment[5, 6]). These considerations led to the use of a two-dimensional model. In such a model heat transfer in radial direction is superposed upon the heat transfer by convection in the flow direction. Several mechanisms are involved in the heat transfer in radial direction. In order to limit the complexity of design calculations the packing and the fluid are taken as a continuum and all heat transfer in radial direction is considered to occur by 'effective conduction'. Evidently, the effective conductivity characterising this mode of heat transfer is not a conductivity in the true sense: it contains contributions of conduction in both fluid and solid, convection in radial direction and radiation, again in both phases.

Bunnell, Irving, Olson and Smith[15] have interpreted their heat transfer data according to such a model. When this effective conductivity is calculated in various points of a section perpendicular to flow it is found that it is decreasing strongly in the vicinity of the wall: it is as if a supplementary resistance were experienced near the wall. Two attitudes are possible: either take a mean value over the whole section, k_e, or introduce an additional coefficient accounting for the wall effect. With the latter point of view the heat transfer in radial direction is characterized by the effective thermal conductivity λ_e and the wall heat transfer coefficient, α_w. Most of the two dimensional heat transfer measurements were interpreted along the latter lines—(Coberly and Marshall[1], Yagi and Kunii[2, 3], Calderband and Pogorsky[16], Kunii and Smith[4]). The data show considerable scatter however, especially those on the wall heat transfer coefficients. It was shown recently that the precision of the available data is insufficient for a satisfactory prediction of reactor operation for more or less drastic conditions (Froment[5]). This conclusion led to the work reported here. The purpose of this work was to determine precise parameter values for the three models discussed above and to correlate them as a function of the flow rate and the tube to particle dia. ratio.

2. APPARATUS AND EXPERIMENTAL TECHNIQUE

The experimental equipment is similar to that used by previous investigators[1, 16]. The packed bed itself was contained in a copper cylinder surrounded by a concentric cylinder also made of copper. Saturated steam condensed in the annulus at 100°C. The construction permitted easy interchange of the inner cylinder in order to investigate the influence of packing to tube dia.

The fluid, air, was delivered by a two-stage blower with a capacity of 160 m^3/hr at zero pressure drop and 40 m^3/hr with a packed bed of 1 m height. Flow rates larger than 30 m^3/hr were measured with an orifice, British norm B.S. 726, having a pressure drop of 1 m water at the maximum flow rate of 100 m^3/hr.

Flow rates lower than 30 m^3/hr were measured by means of a rotameter. At high flow rates the air was heated considerably by the blower. Therefore, a heat exchanger was installed before the packed bed, so as to maintain the inlet temperature of the air to the bed at 30°C $\pm$ 0·1°C in all cases.

The experiment consists of measuring the radial temperature profile in the fluid just above the packing. This is done for several bed heights. For each bed height a wide range of fluid velocities is used. The thermocouples were chromel alumel Philips Thermocoax AB–AC 10 with diameter 1 mm. The emf was read on a Pye potentiometer thermocouple testing set.

Twelve such thermocouples were mounted on a cross which has springs guiding it along the inner cylinder wall. The center of the cross is attached to a vertical rod extending to the top of the cylinder, where another cross helps keeping the rod in the center axis of the inner cylinder. The thermocouples are connected to the compensating cables in the lower cross. They extend 18 cm down from this cross. It was found necessary to hold the junction of the couple exactly in the desired radial position. Therefore, a very thin third cross was attached to the above mentioned lower cross at 1 cm above the thermocouple junction.

The radial location of the couples can be seen from Fig. 1. The thermocouples were located slightly above the packing, at a distance of one particle diameter. From this distance onwards the radial temperature profile was completely smooth.

The wall-temperature was taken equal to the temperature of the steam condensing on the outer surface of the copper wall.

For every bed height and flow rate eight profiles were measured by turning the crosses over 45°. This was done in order to minimize deviations from the radial position and effects of irregularities in the bed surface.

Steady state was attained after about 1 hr.

3. EXPERIMENTAL RESULTS

The experimental program and conditions are shown in Table 1.

Figure 1 shows a typical radial temperature profile.

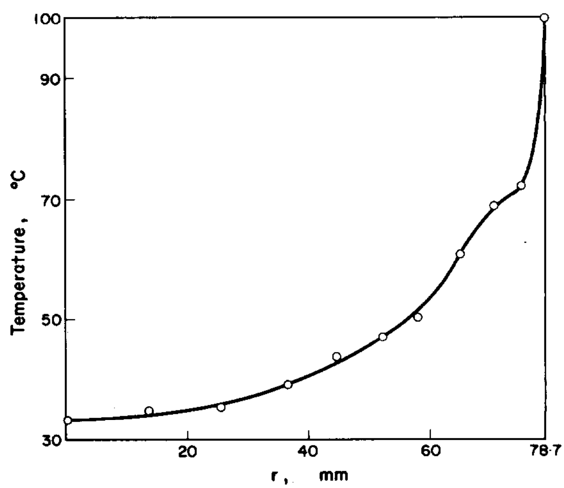

Fig. 1. Radial temperature profiles; V_2O_5-catalyst. $d_t = 0{\cdot}1575$ m; $d_p = 0{\cdot}0057$ m; $z = 0{\cdot}293$ m; $Re = 252$.

Table 1. Experimental program and conditions

d_t (m)	d_p (m)	d_t/d_p	Re	Bed height (m)
Iron oxide catalyst for ammonia synthesis				
0·099	0·0095	10·4	200–900	0·3–1·0
0·1575	0·0095	16·6	90–400	0·4–1·4
Vanadium pentoxide catalyst for sulfuric acid production				
0·099	0·0059	16·7	60–600	0·4–1·0
0·1575	0·0059	26·7	25–300	0·4–0·8
Vanadium pentoxide catalyst for phthalic anhydride synthesis				
0·099	0·0057	17·4	150–400	0·3–1·0
0·1575	0·0057	27·6	50–260	0·2–1·1

All three catalysts are of cylindrical shape with height = dia.

In all cases the fluid is air.

The hump in the profile is typical. It is located at a distance of 1 to 1·5 d_p of the wall and its magnitude increases as d_p/d_t increases. It probably corresponds to a non uniformity in the velocity profile, due to the packing. Indeed, Schwartz and Smith[24] found a maximum in the velocity at a distance from the wall of about 1·5 d_p. Schertz and Bischoff[22] came to the same conclusion.

At greater bed lengths the hump is less pronounced, due to the decreasing difference in temperature between the bed and the wall.

4. MATHEMATICAL MODELS AND COMPUTATION OF PARAMETERS

As mentioned in the Introduction three models may be considered for heat transfer in packed beds:

(a) *One dimensional model*

A heat balance on a volume element of the bed leads to the following differential equation:

$$\frac{RGc_p}{2}\frac{dt}{dz} = h_w(t_w - t) \tag{1}$$

with

$$z = 0 \quad t = t_0$$

$$z = L \quad t = t_u$$

$$t_w \text{ constant for all } z.$$

After integration the wall heat transfer coefficient is obtained from

$$R_w = \frac{1}{2}\frac{Gc_pR}{L}\ln\frac{t_w - t_0}{t_w - t_u}.$$

The mean outlet temperature t_u is obtained from the radial outlet profile from the formula:

$$t_u = \frac{2\int_0^R rt\,dr}{R^2}.$$

(b) *Two-dimensional models*

(1) With one parameter, k_e

In this case a heat balance on a volume element of the bed leads to

$$Gc_p\frac{\partial t}{\partial z} = k_e\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r}\right) \tag{2}$$

with B.C.:

$$t = t_0 \quad z = 0$$

$$\frac{\partial t}{\partial r} = 0 \quad r = 0$$

$$t = t_w \quad r = R.$$

When k_e is invariant with r and z integration over the whole bed leads to:

$$\frac{t_w - t}{t_w - t_0} = 2\sum_1^\infty \frac{1}{\lambda_n J_1(\lambda_n)} e^{-a\lambda_n^2 z} J_0(\lambda_n \rho) \tag{3}$$

where λ_n are the roots of $J_0(x) = 0$;

$$a = \frac{k_e}{Gc_pR^2} \text{ and } \rho = r/R$$

(2) With two parameters λ_e and α_w:

The differential heat balance is identical to (2); with the effective thermal conductivity now being represented by λ_e, however. The boundary conditions are different:

$$t = t_0 \qquad x = 0$$

$$\frac{\partial t}{\partial r} = 0 \qquad r = 0$$

$$\alpha_w(t_w - t_r) = \lambda_e\left(\frac{\partial t}{\partial r}\right)_R$$

Integration now leads to:

$$\frac{t_w - t}{t_w - t_0} = 2\sum_{1}^{\infty} \frac{J_0(b_n\rho)\,\mathrm{e}^{-ab_n^2 z}}{b_n J_1(b_n)[1 + (b_n/m)^2]} \tag{4}$$

where

$$a = \frac{\lambda_e}{Gc_pR^2} \quad m = \frac{\alpha_w R}{\lambda_e}$$

and b_n are the roots of

$$b_n = m\frac{J_0(b_n)}{J_1(b_n)}. \tag{5}$$

Coberly & Marshall determined λ_e from the differential Eq. (2) by determining graphically first and second derivatives from the experimental profiles both in z and r direction.

α_w is then calculated from the integrated Eq. (4) by plotting the logarithm of the left hand side versus the logarithm of the first term of the right hand side.

The drawbacks of this method are obvious: the graphical differentiation introduces considerable scattering—as high as 40 per cent depending upon the individual—and taking the derivative in z-direction requires experiments at different bed heights but with exactly the same flow rate, which is not always easy to realize.

In this work an objective method was used. It is based on the integrated version of the basic differential equation and minimizes the sum of squares of residuals on the outlet temperatures by the choice of λ_e and α_w.

$$F = \sum_{1}^{N} (t_{\text{exp}} - t_{\text{calc}})^2 \tag{6}$$

where N is the number of temperature measurements on a radial profile. Evidently, from the concept itself λ_e and α_w are invariant with the radius and the bed height considered.

Since no differentials are involved each bed height can be treated independently. The method therefore does not require rigorously identical flow rates for the different bed heights. The procedure is as follows. The group m is calculated from first estimates of the parameters λ_e and α_w. Then b_n is calculated from the transcendental equation (5) by iteration. Upon this the right hand side of (4) may be calculated and from this $t(r, z)$. The series converges very rapidly; only three terms were necessary, even in the most unfavorable cases. The estimated temperature was then compared with the experimental. These calculations were performed for all the radial positions.

Several methods were used to reach the minimum of (6) by successive improvement of the parameters λ_e and α_w. The shape of the F-surface is such that neither the method of steepest descent nor the method of Fletcher–Powell [23] is the most efficient, especially as the gradient has to be calculated numerically. Preference was given to the relaxation method. The only advantage of both methods first mentioned with respect to relaxation lies in the initial step, leading to the first minimum, which may be closer to the final minimum than the one obtained by relaxation.

The relaxation method does not involve gradients. One parameter is kept constant while the other is being varied until F is minimum. Then, starting from this first minimum the other parameter is varied and so on, until the difference between two successive F-minima is smaller than 0·01. Advantage was taken from the observation that extrapolation of the curves through the minima arrived at by varying each parameter permits a very accurate determination of the real minimum of the F-function and therefore of the optimum values of λ_e and α_w. This is illustrated in Fig. 2.

The calculation of one set of the parameters for a given flow rate and one height took about 15 min on the I.B.M. 360-30.

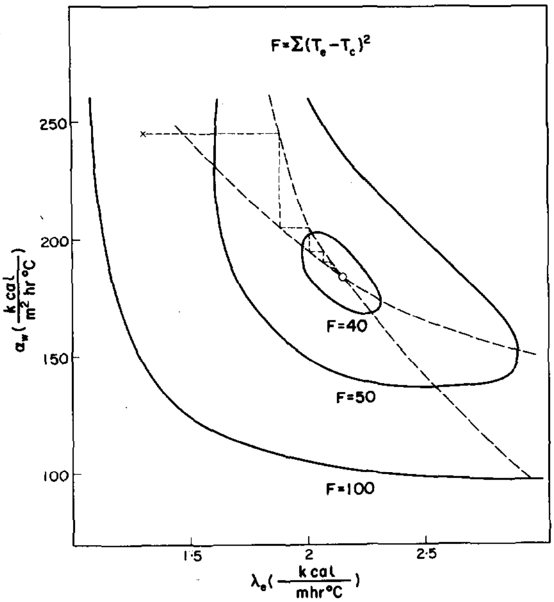

Fig. 2. Determination of λ_e and α_w by a relaxation procedure. F-contours in the $\lambda_e - \alpha_w$ plane. V_2O_5–catalyst; $d_t = 0{\cdot}099$ m; $d_p = 0{\cdot}0095$ m; $z = 0{\cdot}291$ m; $Re = 903$.

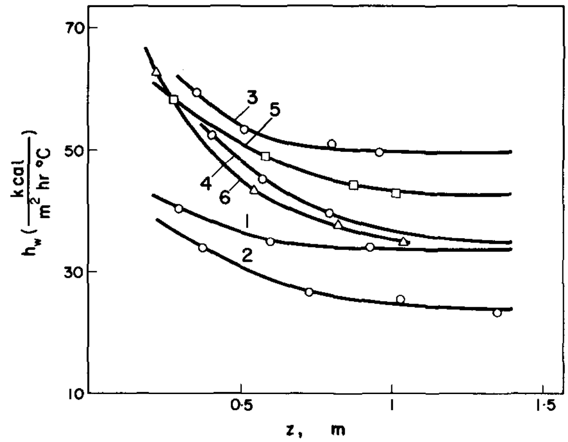

Fig. 3. h_w vs. bed height for $Re = 300$.

	d_t(m)	d_p(m)	
curve 1	0·099	0·0095	V_2O_5–cat for SO_3–synthesis
curve 2	0·1575	0·0095	
curve 3	0·099	0·0059	Iron–cat for NH_3–synthesis
curve 4	0·1575	0·0059	
curve 5	0·099	0·0057	V_2O_5–cat for PA–synthesis.
curve 6	0·1575	0·0057	

In the calculation of k_e, the parameter of the two dimensional model with one parameter the three dimensional error-surface (F, λ_e, α_w) is reduced to a curve in a plane with coordinate axes F and k_e. The best value of k_e corresponds to the minimum of this curve.

5. CORRELATIONS

(a) *One dimensional model*

Figure 3 shows how for a given Reynolds number, h_w depends on the bed height, due to the development of the temperature and velocity profiles. It tends to a limiting value, which depends on both d_p and d_t. Given d_p the height required to attain this limit increases with the tube diameter. Given d_t the height required to attain the limit decreases with d_p. It follows that the limit height increases with d_t/d_p.

Figure 4 shows the relation between h_w and the Reynolds number, based on the particle diameter for the V_2O_5 catalyst for phthalic anhydride production. The relationship is linear, in contrast with what has been published to date. The intersection with the ordinate may be considered as a static contribution, $h_w{}^0$. Except for the lowest bed height of 0·218 m, at which the profile is not yet fully developed, all straight lines extrapolate to the same $h_w{}^0$. The slope depends on the bed height but tends to a limiting value.

Similar results are obtained with the other tube diameter and the other catalysts.

The final correlation for the three catalysts and the two tube diameters is as follows:

$$h_w = h_w{}^0 + \frac{0{\cdot}0005924}{d_p} Re \quad \text{for } 30 < Re < 1000$$

or in dimensionless form:

$$\frac{h_w d_p}{\lambda_g} = \frac{h_w{}^0 d_p}{\lambda_g} + 0{\cdot}0240\, Re$$

where the $h_w{}^0$ for the different catalysts are given in Table 2. $h_w{}^0$ is a function of the tube diameter and the catalyst geometry and properties. The slope of $h_w d_p/\lambda_g$ vs. Re is identical for the three catalysts, however.

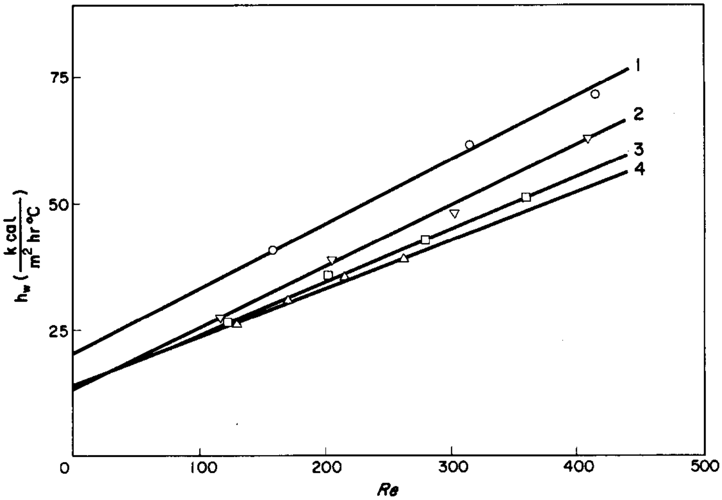

Fig. 4. h_w as a function of Reynolds number. V_2O_5–catalyst; d_t: 0·099 m; d_p = 0·0057 m. Bed height

line 1 : 0·284 m
line 2 : 0·582 m
line 3 : 0·875 m
line 4 : 1·016 m

Table 2. Static contributions to the parameters of the different models

	h_w^0		k_e^0		λ_e^0		α_w^0	
	d_{t_1}	d_{t_2}	d_{t_1}	d_{t_2}	d_{t_1}	d_{t_2}	d_{t_1}	d_{t_2}
SO_3 cat	5·2	14·0	0·138	0·223	0·176	0·240	17·0	51·0
PA cat	6·6	13·3	0·173	0·219	0·241	0·224	16·2	70·0
NH_3 cat	10·0	20·5	0·270	0·320	0·360	0·350	31·0	85·0

d_{t_1} = 0·1575 m.
d_{t_2} = 0·099 m.

The mean deviation between calculated and experimental values is 3·75 per cent.

(b) *Two dimensional model with one parameter*

The same trends as for h_w were found for k_e: given d_p the height required to attain the limit value of k_e increases with d_t; given d_t the height required to attain the limit decreases with d_p. The relation $k_e = f(Re)$ is shown in Fig. 5 for the phthalic anhydride synthesis catalyst and a tube diameter of 0·099 m. Again the relation is linear, while static and dynamic contributions are apparent. All straight lines corresponding to different bed heights extrapolate to the same static contribution, k_e^0, except again that of 0·218 m. The slope tends to a limiting value which depends on d_p/d_t. In the limit the following correlation is obtained:

$$k_e = k_e^0 + \frac{0{\cdot}0022}{1 + 120\,(d_p/d_t)^2} Re \tag{8}$$

where the k_e^0 for the different catalysts are given in Table 2. The mean deviation between calculated and experimental values is 3·75 per cent. From the analogy between heat and mass transfer the dynamic contribution to k_e may be written

$$k_e^t = \lambda_g \frac{PrRe}{Pe}.$$

The d_p/d_t dependence of the dynamic contribution in (8) is inspired from Fahien and Smith's[25] correlation for the Peclet number for radial effective transport:

$$\frac{Pe}{1 + 19{\cdot}4\,(d_p/d_t)^2} = 10 \quad (Re > 50).$$

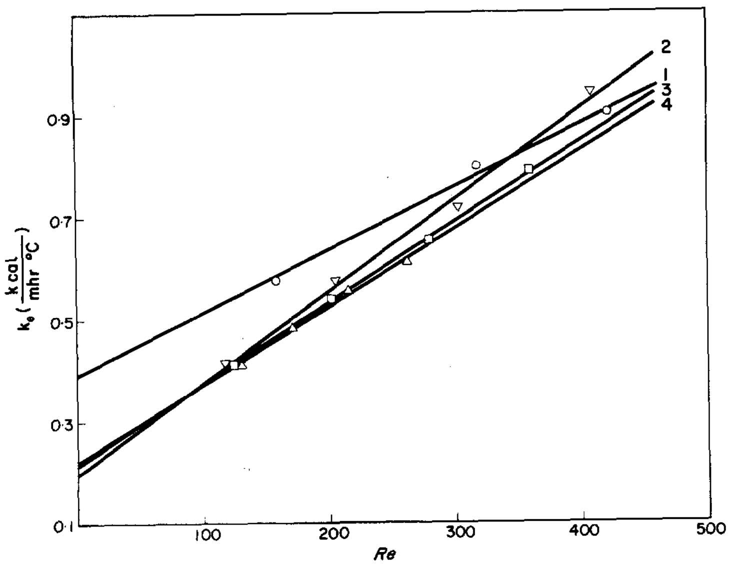

Fig. 5. k_e as a function of Reynolds number. Same catalyst, d_t, d_p and z as in Fig. 4.

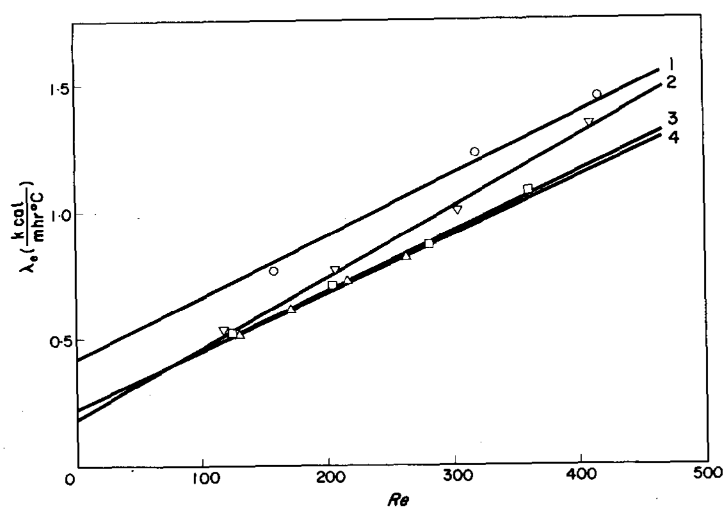

Fig. 6. λ_e as a function of Reynolds number. Same catalyst d_p, d_t and z as in Figs. 4 and 5.

Bunnell *et al.* correlated their data according to this model[15]. If their d_p and d_t values are substituted in (8) a value of 0·00153 is found for the slope, in excellent agreement with their value of 0·0015.

(c) *Two dimensional model with two parameters*

(1) *Radial effective thermal conductivity*. The trends are analogous to those observed for k_e: the higher d_t/d_p the longer the bed has to be, to reach the limit for λ_e.

Figure 6 shows the relation $\lambda_e = f(Re)$ is linear, with the slope tending to a limit value as the bed height increases.

The correlation for the limit value is as follows:

$$\lambda_e = \lambda_e^0 + \frac{0{\cdot}0025}{1+46(d_p/d_t)^2} Re \tag{9}$$

where the λ_e^0 for the different catalysts are given in Table 2.

The mean deviation between calculated and experimental values is only 1·1 per cent. This linear relationship was found already by previous investigators using this model (Coberly and Marshall[1]; Campbell and Huntington[17, 18]; Calderbank and Pogorsky[16], Yagi and Kunii [2, 3]) and is predicted by theoretical models (Yagi and Kunii[2, 3]; Kunii and Smith[4]).

The slope of the λ_e vs. Re line is dependent on d_p/d_t, as can be seen from (9). When the slopes obtained by previous investigators are compared with those predicted by Eq. (9) for corresponding d_p and d_t, excellent agreement is found for those presented by Plautz and Johnstone[19] and Quinton and Storrow[20].

(2) *Wall heat transfer coefficient.* α_w also varies up to a certain extent with bed height. The tendency is analogous to that obtained for h_w, λ_e: the higher d_t/d_p the longer the bed has to be to reach the limit value.

Figure 7 shows the relation $\alpha_w = f(Re)$ for the phthalic anhydride-catalyst. Again a linear relation is found, in contrast with Coberly and Marshall and Calderbank and Pogorsky, but in agreement with Yagi and Kunii. In the limit the following correlation has been obtained:

$$\alpha_w = \alpha_w^0 + 0{\cdot}01152 \frac{d_t}{d_p} Re \tag{10}$$

where the α_w^0 are given in Table 2.

The correlation for α_w is far superior to those published until now. The mean deviation between calculated and experimental values is only 6 per cent.

Figure 8 finally shows the agreement between

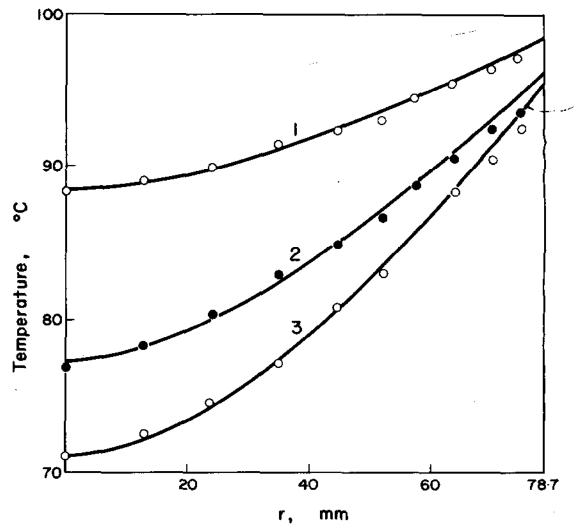

Fig. 8. Calculated and experimental radial temperature profiles. V_2O_5–catalyst.
$d_t = 0{\cdot}1575$ m; $d_p = 0{\cdot}0095$ m; $z = 1{\cdot}345$ m
curves 1, 2, 3: calculated profiles. Two-dimensional models with 2 parameters λ_e and α_w. Correlations (9 and 10).
Curve 1: $Re = 81$
Curve 2: $Re = 188$
Curve 3: $Re = 358$
Points: corresponding experimental measurements.

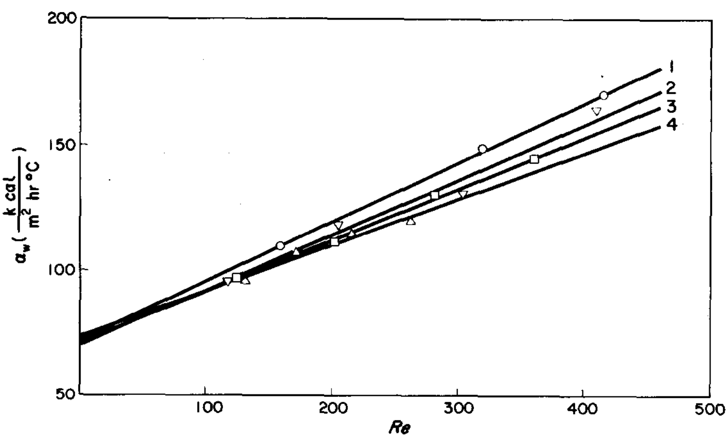

Fig. 7. α_w as a function of Reynolds number. Same catalyst, d_p, d_t and z as in Figs. 4, 5 and 6.

the experimental profiles and those calculated on the basis of the two-dimensional model with two parameters using the correlations (9) and (10). There is only a slight difference in the immediate vicinity of the wall.

6. RELATIONS BETWEEN THE PARAMETERS

Some relations between the static contributions of the different models were established:

$$h_w{}^0 = 6\cdot15\frac{k_e{}^0}{d_t}$$

with the dimensions used in this paper. The form of this expression is analogous to that observed by Verschoor and Schuit[21]. Further:

$$\lambda_e{}^0 = \frac{k_e{}^0 d_t{}^{1/3}}{0\cdot42}$$

$$\alpha_w{}^0 = 20\, k_e{}^0/d_t$$

The accuracy of these relations is illustrated by the excellent agreement between experimental values for $\lambda_c{}^0$, $\alpha_w{}^0$ and $h_w{}^0$ and those calculated from $k_e{}^0$ e.g. the mean deviation does not exceed 2·8 per cent for $\lambda_e{}^0$, 10·5 per cent for $\alpha_w{}^0$ and 2·7 per cent for $h_w{}^0$.

Finally, h_w may be calculated from λ_e and α_w using the relation (Froment[5]):

$$h_w = \frac{1}{(R/4\lambda_e) + (1/\alpha_w)}$$

The h_w calculated from λ_e and α_w are in excellent agreement with those directly measured: the deviation never exceeds 4 per cent.

7. GENERAL CONCLUSIONS

In this work heat transfer data in packed beds were interpreted according to various models. The heat transfer parameters of these models were correlated in function of flow rate, tube and particle dia. For a given Reynolds number the values of the parameters tend to a limit as the bed height is increased. The height at which this limit is attained increases with d_t/d_p.

All correlations of the parameters in function of the Reynolds number lead to straight lines which intersect the heat transfer coefficient axis, indicating there is a static and a dynamic contribution. The static contribution is independent of bed height, except for very shallow beds, but clearly dependent on the conductivity of the particle. Correlations were also set up between the parameters of the various models. The mean deviation on the correlations is very low: the data show very little scatter. This is believed to be due to the great precision with which the thermocouples were located and to the optimization procedure used for determining the parameters.

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NOTATION

- b_n nth root of $x - m(J_0(x)/J_1(x)) = 0$
- c_p specific heat, kcal/kg°C
- d_p equivalent particle dia., dia. of a sphere with equal volume, m
- d_t tube dia., m
- G mass flow velocity, superficial, kg/m² hr
- J_0 zero-th order Bessel function, first kind
- J_1 first order Bessel function, first kind
- h_w overall heat transfer coefficient, kcal/m² hr °C
- $h_w{}^0$ static contribution to above coefficient, kcal/m² hr °C
- k_e overall effective conductivity, kcal/m hr °C
- $k_e{}^0$ static contribution to above coefficient, kcal/m hr °C
- L total height of bed, m
- r radial coordinate, m
- R radius, m
- t temperature, °C
- t_0 inlet temperature, °C
- t_u mean outlet temperature, °C

t_w wall temperature, °C
z axial coordinate, m

Greek symbols

α_w wall heat transfer coefficient, kcal/m² hr °C
$\alpha_w{}^0$ static contribution to above coefficient, kcal/m² hr °C
λ_e effective thermal conductivity of the bed, kcal/m hr °C
$\lambda_e{}^0$ static contribution to above coefficient, kcal/m² hr °C
λ_g thermal conductivity of the gas, kcal/m hr °C
λ_n nth root of $J_0(x) = 0$
ρ dimensionless radial coordinate
Pe Peclet number
Re Reynolds number

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Résumé—Les auteurs interprètent le transfert de chaleur entre la paroi d'un tube et un fluide coulant à travers un lit garni d'après un modèle à une dimension caractérisé par un coefficient du transfert global de température, h_w, et par des modèles à deux dimensions et à un paramètre, la conductivité thermique effective K_e, ou à deux paramètres, la conductivité thermique effective λ_e et le coefficient α_w du transfert de chaleur dans la paroi. Les résultats expérimentaux pour h_w, k_e, λ_e et α_w sont des fonctions du nombre de Reynolds, et du diamètre du tube et des grains. Les corrélations par rapport au nombre de Reynolds sont toutes linaires. Let auteurs donnent aussi les corrélations entre les paramètres des différents modeles.

Zusammenfassung—Die Wärmeübertragung zwischen einer durch eine Füllkörperschicht strömenden Flüssigkeit und der Wand wird gemäss einem eindimensionalem Modell, gekennzeichnet durch einen Gesamtwärmeübertragungskoeffizienten h_w, und durch zweidimensionale Modelle mit einem Parameter, die effektive Wärmeleitung k_e, oder zwei Parameter, die effektive Wärmeleitung λ_e und den Wandübertragungskoeffizienten α_w ausgedrückt. Die Versuchsergebnisse für h_w, k_e, λ_c und α_w werden als Funktion der Reynoldsschen Zahl, der Füllung und des Rohrdurchmessers in Korrelation gebracht. Die Korrelationen in bezug auf die Reynoldssche Zahl sind alle linear. Es werden ferner Korrelationen unter den Parametern der verschiedenen Modelle angeführt.