PW1 – Computational Methods – M1 E3A International Track

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Computational Methods

The objective of the practical work is to implement and compare different methods to solve non-linear equations of the type: f(x) = 0, the function f being able to be scalar or vectorial. We will then look for a root noted s.
The student is then made aware of algorithms, numerical precision and the interpretation of results. The programs can be commented on.

A PW1 report must be provided by each student individually, in digital format in a single PDF file, at the end of each session. The PW1 report should include : a general introduction, a reminder of the objectives, the methodology implemented, the calculations if requested, an observation and analysis of the simulated results, partial conclusions for each part of the practical work, a general conclusion. The programs carried out can be provided as an annex to the PW1 report.

I- 1-variable Functions

1. Write a function with MATLAB giving the value of the function for which we are searching for the roots at any point *x*. This function will have to accept vectorial variables. It will be used for the function : $f(x) = \cos(x) \,.\, \cosh(x) + 1$
2. Write a function with MATLAB giving the value of the derivative of the function for which we are searching for the roots at any point *x*. : *Function [y] = df(x)*
3. Programm the four following methods :

   function [] = dich(a; b; epsilon)
   function [] = reg_fal(a; b; epsilon)
   function [] = secant(a; b; epsilon)
   function [] = newton(a; epsilon)

   These different fucntions will wall the created functions *f.m* and *df.m*
4. Which is the number of iterations *k* needed for each method if we desire to stop the algorithm at the time we verify : $\left|f(x^{(k)})\right| \leq 10^{-9}$?
5. Which is the length of the last segment ?

II- Non-linear systems of equations

Let to be solved the system of equations :

$$e^{xy} + x^2 + y - 1.2 = 0$$
$$x^2 + y^2 + x - 0.55 = 0$$

6. Write a function with MATLAB giving the value of a function for which we are searching for the roots at any point u = [x ; y] : *function [F] = sys(u)* where *u* and *F* are vectors of dimension 2x1.
7. Write a function with MATLAB giving the value of the Jacobian under a matrix 2x2 : *function [J] = Jac(u)*
8. Programm the Newton method for this case. It will be chosen as initial point, vector :

$$u^{(0)} = \left[x^{(0)}; y^{(0)}\right] = [0.6\,;\,0.5]$$

function [] = new(u; epsilon)

9. Which is the number of iterations *k* needed for each method if we desire to stop the algorithm at the time we verify : $\left\| f(x^{(k)}) \right\| \leq 10^{-9}$?

III- Zeros of polynomials

Polynomial : $P(x) = x^3 + 4x^2 - 55x + 50$ admits {1 ; -10 ; 5} as roots.

10. Programm the method presented in the course. It will be chosen as initial point a near value of the searched root. Algorithm will be stopped when criterium : $P(s) \leq 1e^{-6}$ will be verified.
11. Propose a programm improvement permitting to calculate all the roots.