M1 Computational Methods
PW4

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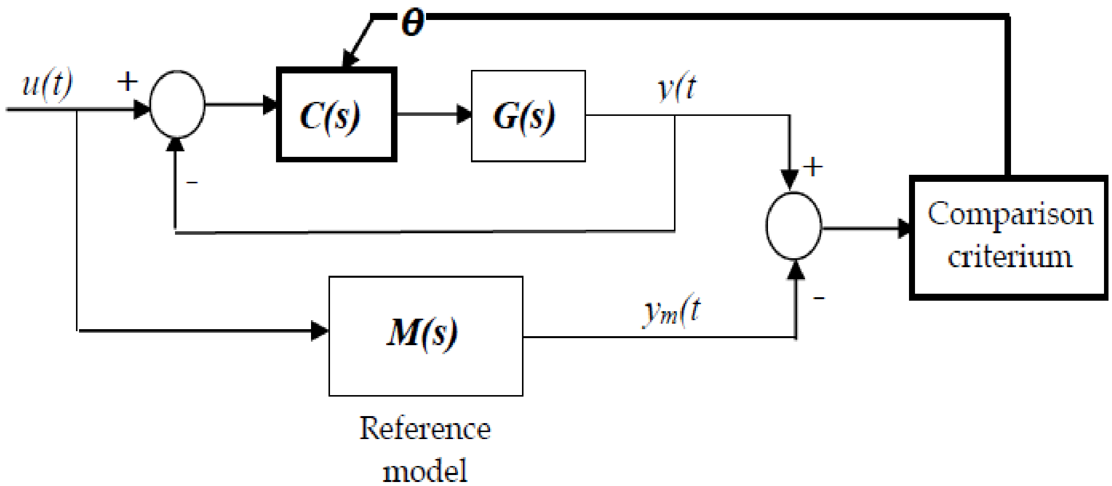
PW4 – COMPUTATIONAL METHODS

Optimization for Control

Control tuning

The objective of this study is to establish a method for parameters tuning of a controller intended to improve the performance of a process whose transfer function is of the 3-order G(s) = $1/(1+s)^2$.

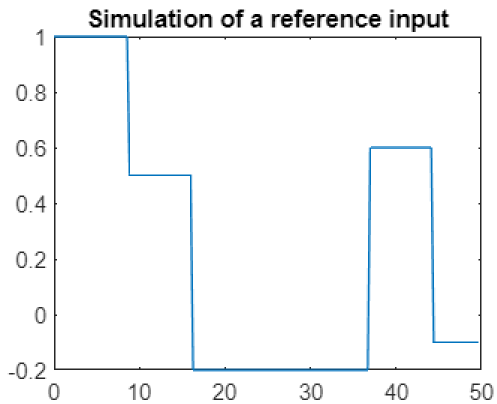
The controller is a PID controller written as 2 C(s) = (a + bs + cs^2)/ s whose parameters vector is θ= $(\mathrm{abc})^T$.



PART I – Model formulation and performance study

1 - Simulation of a reference input

```
 x1 = ones(1,30);
 x2(1:1,1:25) = 0.5;
 x3(1:1,1:35) = -0.2;
 x4(1:1,1:35) = -0.2;
 x5(1:1,1:25) = 0.6;
 x6(1:1,1:19) = -0.1;
 u = [x1 x2 x3 x4 x5 x6];
 total_time=50; % Explaination
 tt=0:total_time/(size(u,2)):total_time-0.00001; %Starting time, steps and
finishing time.
 plot(tt,u)
 title("Simulation of a reference input")
```



2 – non-Controlled system study

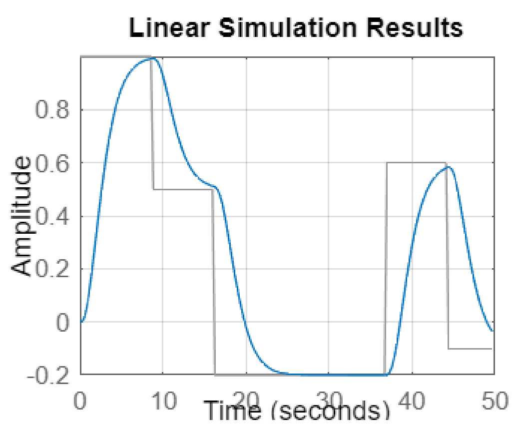
```
numerator = 1;
denominator = [1 3 3 1];
G = tf(numerator,denominator)
```

```
G =

          1
  ---------------------
  s^3 + 3 s^2 + 3 s + 1

Continuous-time transfer function.
```

```
lsim(G,u,tt)
grid on
```



Here, the response time can be seen around 10 and no overshoot has been noted. Moreover, it is not precise or accurate.

3 – Reference model conception

3.1- Calculate the coefficients ξ and ω_n of reference model M(s) .

ξ = 0.6362

ω_n = 2.715

3.2- Simulate the response ym(t) of this model thanks to the reference input. Represent on the same graph input u(t) and output ym(t) and then y(t) and ym(t).

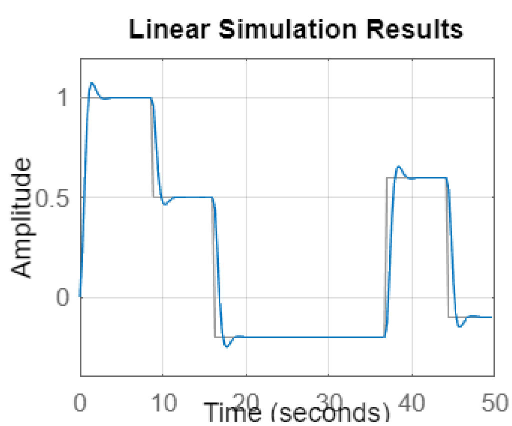
```
numerator = 7.371225;
denominator = [1 3.4545 7.371225];
M = tf(numerator,denominator)
```

```
M =

          7.371
  ---------------------
  s^2 + 3.454 s + 7.371

Continuous-time transfer function.
```

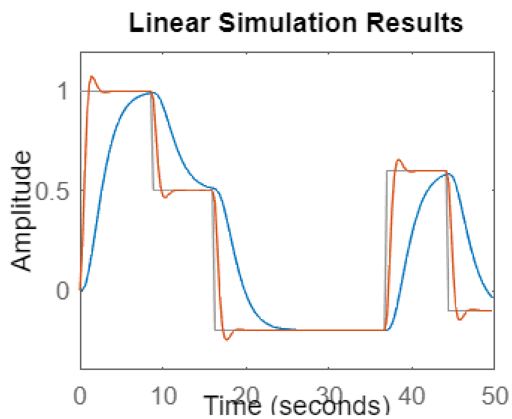
```
lsim(M,u,tt)
grid on
```



3.3- Does the model seem to have any good performances?

```
lsim(G,u,tt)
hold on

lsim(M,u,tt)
hold off
```



At the start, we can observe some fluctuation for ym but still it is more accurate and precise. So, it can be said that it has better performance.

PART II – Controller optimal tuning

1 – System sensitivity

1.1- Show that these sensitivities are solution of linear differential equations.

$$\sigma_a(t) = \frac{\partial y(t)}{\partial a}, \sigma_b(t) = \frac{\partial y(t)}{\partial b}, \sigma_c(t) = \frac{\partial y(t)}{\partial c}$$

```
syms a b c s t
Y = (a+b*s+c*s^2)/(a+(b+1)*s+(c+3)*s^2+3*s^3+s^4);
y=ilaplace(Y,s,t);
a_diff = diff(y,a);
b_diff = diff(y,b);
c_diff = diff(y,c);
```

a_diff =

$$b\left(\sum_{k=1}^{4} \frac{e^{\sigma_2 t}\,\sigma_2}{6\,\sigma_2 + b + \sigma_2\, c\, 2 + 9\,\sigma_2^2 + 4\,\sigma_2^3 + 1}\right) + c\left(\sum_{k=1}^{4} \frac{e^{t\,\sigma_2}\,\sigma_2^2}{\sigma_1}\right) + a\left(\sum_{k=1}^{4} \frac{e^{t\,\sigma_2}}{\sigma_1}\right)$$

where

$$\sigma_1 = b + 2\,c\,\sigma_2 + 9\,\sigma_2^2 + 4\,\sigma_2^3 + 6\,\sigma_2 + 1$$

$$\sigma_2 = \mathrm{root}(z^4 + 3\,z^3 + c\,z^2 + 3\,z^2 + b\,z + z + a, z, k)$$

```
b_diff =
```

$$b\left(\sum_{k=1}^{4}\frac{e^{\sigma_2 t}\,\sigma_2}{6\,\sigma_2+b+\sigma_2\,c\,2+9\,\sigma_2{}^2+4\,\sigma_2{}^3+1}\right)+c\,\left(\sum_{k=1}^{4}\frac{e^{t\,\sigma_2}\,\sigma_2{}^2}{\sigma_1}\right)+a\,\left(\sum_{k=1}^{4}\frac{e^{t\,\sigma_2}}{\sigma_1}\right)$$

where

$$\sigma_1=b+2\,c\,\sigma_2+9\,\sigma_2{}^2+4\,\sigma_2{}^3+6\,\sigma_2+1$$

$$\sigma_2=\mathrm{root}(z^4+3\,z^3+c\,z^2+3\,z^2+b\,z+z+a,z,k)$$

```
c_diff =
```


$$b\left(\sum_{k=1}^{4}\frac{e^{\sigma_2 t}\,\sigma_2}{6\,\sigma_2+b+\sigma_2\,c\,2+9\,\sigma_2{}^2+4\,\sigma_2{}^3+1}\right)+c\,\left(\sum_{k=1}^{4}\frac{e^{t\,\sigma_2}\,\sigma_2{}^2}{\sigma_1}\right)+a\,\left(\sum_{k=1}^{4}\frac{e^{t\,\sigma_2}}{\sigma_1}\right)$$

where

$$\sigma_1=b+2\,c\,\sigma_2+9\,\sigma_2{}^2+4\,\sigma_2{}^3+6\,\sigma_2+1$$

$$\sigma_2=\mathrm{root}(z^4+3\,z^3+c\,z^2+3\,z^2+b\,z+z+a,z,k)$$

1.2- Write the transfer functions de transfer defining these sensitivities.

```
a_lap = laplace(a_diff);
b_lap = laplace(b_diff);
c_lap = laplace(c_diff);
```

2 – Controller parameters tuning: Gradient method

2.1- Write a MATLAB code for this algorithm

$$\frac{\partial\phi}{\partial\theta} = \int_0^\infty (y(t) - y_m(t))\frac{\partial y}{\partial\theta}dt, \quad \theta = (a,b,c)^T$$

```
Y_m = 7.371 /(s^2 + 3.454*s + 7.371);
y_m=ilaplace(Y_m,s,t);
a0 = 1;
b0 = 1;
c0 = 1;
alpha = 0.5; % step size
for n = 1:5 % Iteration number
y_t = subs(y,{a,b,c},{a0,b0,c0});
a_diff = subs(a_diff,{a,b,c},{a0,b0,c0});
b_diff = subs(b_diff,{a,b,c},{a0,b0,c0});
c_diff = subs(c_diff,{a,b,c},{a0,b0,c0});

fi_diff_1 = matlabFunction(vpa((y_t-y_m)*a_diff));
fi_diff_2 = matlabFunction(vpa((y_t-y_m)*b_diff));
fi_diff_3 = matlabFunction(vpa((y_t-y_m)*c_diff));

grad_a = integral(fi_diff_1,0, Inf);
grad_b = integral(fi_diff_2,0, Inf);
grad_c = integral(fi_diff_3,0, Inf);


a0 = a0 - alpha*grad_a;
b0 = b0 - alpha*grad_b;
c0 = c0 - alpha*grad_c;
end
```

2.2- On the same graph, compare respectively input u(t) and output ym(t), input u(t) output y(t),

Outputs y(t) and ym(t).

```
Sys_new = tf([c0 b0 a0], [1 3 c0+3 b0+1 a0])
```

```
Sys_new =

        1.441 s^2 + 1.149 s + 0.6917
  ------------------------------------------
  s^4 + 3 s^3 + 4.441 s^2 + 2.149 s + 0.6917
```

```
Continuous-time transfer function.
```

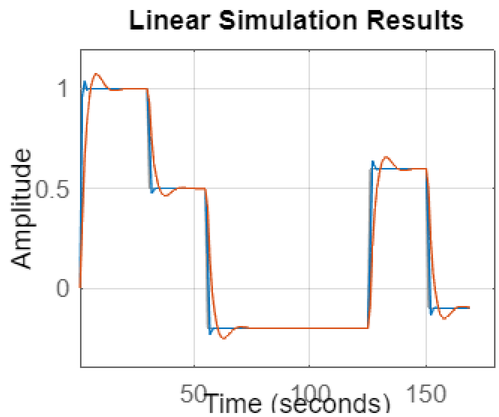
```
t=1:169;
lsim(M,u,t);
```

```
Warning: Simulation will start at a nonzero initial time.
```

```
hold on;
lsim(sys_new,u,t);
```

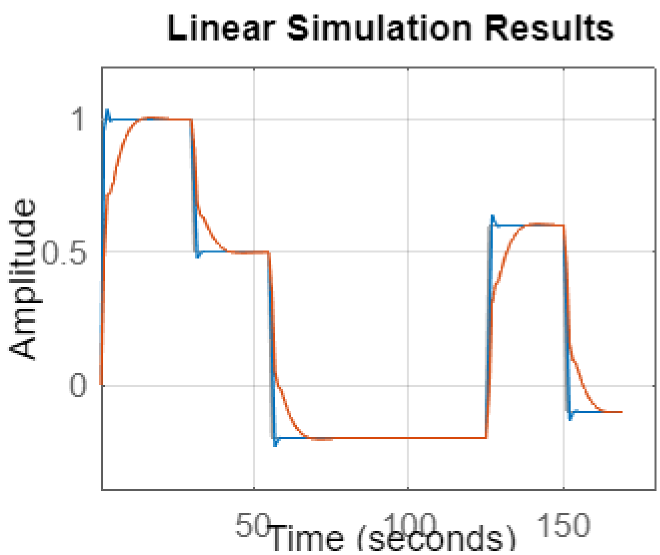
```
Warning: Simulation will start at a nonzero initial time.
```

```
hold off;
grid on;
```



In the beginning the poor result can be noticed for y(t). But with increase of the iteration, better result can be noticed and in case of changing the step size, inadequate result will be noticed.

When the step size is increased 1 and the iteration is around 10, it can be seen that the expected result is not accurate and precise.



When the step size is around 0.3 and iteration is around 10, the output is almost accurate.