→ we have ⊗
hence Avg ⊗ diff$\left(\frac{dy}{dx}\right)$.

EX2:

1. $[-1 \;\; 0 \;\; 2 \;\; 0 \;\; -1] = [-1 \;\; 0 \;\; 1] \circledast [1 \;\; 0 \;\; -1]$.

$$\frac{\delta^2 f}{\delta x^2} = \left(Av(1\times 2) * \frac{\delta f}{\delta x}(1\times 2)\right) * \left(Avg(1\times 2) \circledast \frac{df}{dx}\right)$$

2. $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} = \frac{1}{2}[1 \;\; 1] * \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$Avg.(1\times 2) * \frac{\delta f}{\delta x}$$

$$\nabla = \frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2}$$

3. $[-1 \;\; 0 \;\; 2 \;\; 0 \;\; -1] + \begin{bmatrix} -1 \\ 0 \\ 2 \\ 0 \\ -1 \end{bmatrix}$

⊛ Laplacian is addition of second derivatives.

EX 3: Filter syntax

1.

$$f * h = f$$

$$h = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad L = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 4 & -1 \\ 1 & 1 & 0 \end{bmatrix}.$$

→ basic laplacia

$$h + L = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

→ Image & highlight of edges.

$$\underbrace{f * h}_{\text{Image}} + \underbrace{f * L}_{\text{edge}} = f * (h + L)$$

→ Photoshop

→ Edge detection.

Image Enhancement & Segmentation

→ enhance quality of image → Using relevant values of points/nbr neighbors.

Why image quality is lost:

✓ shooting problems.

✓ optic problems

✓ Sensing problems — sensor — noisy pixels

✓ Discretization problems.

Level contrast

→ low contrast → s.d. is close. relevant values are close. & vice versa.

→ **Histogram**

→ Give statistical distribution of relevant values (pdf)

relevant value → random variable

normalize

Histogram $P(x) = \lim \frac{P(\Delta x)}{\Delta x} = F(x).$

C. Histogram $= \int_0^{x_{max}} F'(x)\,dx = F(x_{max})$

Example

→ All relevant values on one side. Hist: visual inspection

Point operations

→ Modification of relevant value using a fnctn.

→ New grey values stored in lookup table.

$$f(u) = \begin{cases} u^{1-\beta} & \text{if } \beta > 0 \end{cases}$$

→ constant ; increase or decrease contrast

→ Diff. of two relevant value is increased → Contrast increase. and vice versa.

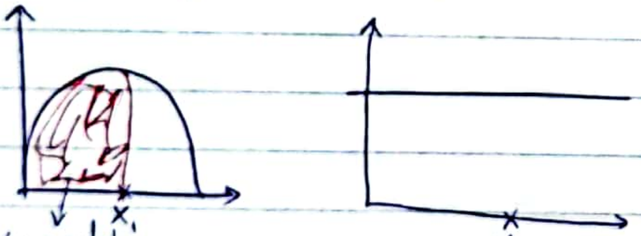
→ Contrast stretching: Histogram concentrated btwn Gmin - Gmax

$$f(u) = au + b$$

$$\left.\begin{aligned} aG_{min} + b &= 0 \\ aG_{max} + b &= L \end{aligned}\right\} \text{we have } a \,\&\, b$$

Histogram equalization.

→ Modify shape of histogram. (flat



cummulate this

$\int_0 P_1(m)\,dm$

Dest.

summation → nearest existing rel. value

Introduces errors

n_k → occurence.

S_k → relevant values.

P_s → Dest. probabilities

Conc. Histogram enables us to see the image quality.

Histogram modification

→ to reach a specific shape of histogram.
→ equalise both shapes & find correspondence.

Thresholding

$$f(u) = \begin{cases} L & \text{if } u > T \\ 0 & \text{else.} \end{cases}$$

→ Thresholding (T stg) → first step towards segmentation
→ We can separate objects in image by choosing T
e.g background & image.

→ Homogenous image = One dirac
→ One Perfect image + background → two "
→ Imperfect → a gaussian → homogenous real image. 
→ first step of segmentation.

→ More noise, difficult to segment. Mostly we choose gaussian noise mostly.

→ Connectivity ; we segment with labels because of objects with same rel. value.

EX 4 : Binomial Filters

① - pascal

- Convolution with avg. filter

1×7

Pascal.

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1 .

Convolve.

[1 1] * [1 1]. → 1 2 1

[1 2 1] ⊛ [1 2 1]

[1 4 6 4 1] ⊛ [1 2 1]

[1 6 15 20 15 6 1]

or $\underbrace{[1\ 1] \circledast [1\ 1]}_{1,2,1} * \underbrace{[1\ 1] * [1,1]}_{1,2,1} * \underbrace{[1\ 1] * [1 * 1]}_{1\ 2\ 1}$

② Deduce Binomial filter of size 7×7

→ use separability

$$[1\ 6\ 15\ 20\ 15\ 6\ 1] * \begin{bmatrix} 1 \\ 6 \\ 15 \\ 20 \\ 15 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 6 & 15 & 20 & 15 & 6 & 1 \\ 6 & 36 & 90 & 120 & 90 & 36 & 6 \\ 15 & 90 & 225 & 300 & 225 & 90 & 15 \\ 20 & 120 & 300 & 400 & 300 & 120 & 20 \\ 15 & 90 & 225 & 300 & 225 & 90 & 15 \\ 6 & 36 & 90 & 120 & 90 & 36 & 6 \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{bmatrix}$$

③ $B = \dfrac{B}{\sum\sum b_{ij}} = \dfrac{\text{each value}}{\text{sum of all values}}$

btwn [0, 1] in binom.

4) → To keep the dynamic range [0 255] after Conv we still have values btw [0, 255]

5) Use of binomial filter.
① easy to generate than gaussian.
② we have integral values.

uses
used for smoothening of image.

Ex 5: Gaussian Differences.

1. G.F. of 1x5. Variance $\sigma^2 = 4$ $\sigma = 2$.

Variance is 1 & 4.

$$G(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(\frac{-x^2}{2\sigma^2}\right)$$

x	-2	-1	0	1	2.

$G(x) = [0.12 \quad 0.17 \quad 0.2 \quad 0.17 \quad 0.12$

② $\sigma = 1$
$G(x) = [0.05 \quad 0.24 \quad 0.38 \quad 0.24 \quad 0.05]$

3 ~~Smoothing LPF~~.

B

③. → smoothening

④ Difference of ① & ② $[0.07 \quad -0.07 \quad -0.19 \quad -0.07 \quad 0.07]$.
- shape of Laplacian (second derivative)
- Two Gaussians = Second derivative filter
↓
difference of gaussian (DOG)

6)→ HPF → edge detection. $ noise removal

EX 6. Median Filter.

1 2 5
3 3 4
5 4 5

(*) choose an odd no.
– we don't create new value.

① 1 2 3 3 4 4 5 5 5
– replace the pixel in the middle by 4.

2. Majority filter.
– replace middle by most frequent (5)
1 : 1
2 : 1
3 : 2
4 : 2
$\boxed{5 : 3}$

3. It is kept.
4. Density of noise after which it doesn't work.

$n = 9$ in above.

$n_{critical\ (noise)} = \left[\frac{n}{2}\right]$ → more than half → it is no longer efficient.

If 5 is more than $\frac{9}{2}$ then we have noise.

– We should know the density of noise.

Scanned with CamScanner

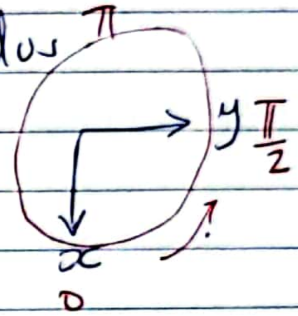
tg^{-1} Argtan

EX 7. Derivative filters.

1. $I = \begin{bmatrix} 255 & 255 & 255 & 0 & 0 \\ 255 & 255 & 252 & & \\ & & & & \\ & & & & \end{bmatrix}$ $\circledast \; [-1 \;\; 0 \;\; 1]$ (f_y) $\circledast \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ (F_x)

$$\begin{cases} G_x = I * F_x \\ G_y = I * f_y . \end{cases}$$

$|\vec{G}|$ → modulus



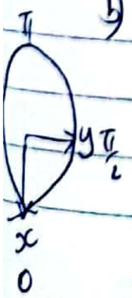
a) Vertical.

$G_y) \begin{bmatrix} 0 & -255 & -255 & 0 \\ 0 & -255 & -255 & 0 \\ 0 & & & 0 \\ \vdots & & & \end{bmatrix}$ → $\underset{G_x}{[0]}_x$ →

$|\vec{G}| = \begin{bmatrix} 0 & 255 & 255 & 0 & 0 \\ 0 & 255 & 255 & 0 & 0 \\ 0 & 255 & & & \\ 0 & 255 & & & \\ \cdot & & & & \end{bmatrix}$

$G \; \text{Artg} \; \frac{\cdot}{0} = \frac{\pi}{2}$ $\Rightarrow$ $\text{Artg} \; \frac{G_y}{G_x} = \begin{bmatrix} 0 & \frac{\pi}{2} & \frac{\pi}{2} & 0 \\ 0 & & & \vdots \\ \vdots & & & \vdots \\ \vdots & & & 0 \end{bmatrix}$

b) horizontal



255	255	255	255	255	
255	255	255	255	255	$* [-1 \;\; 0 \;\; 1] \Rightarrow G_y = \begin{bmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix}$
255	255	255	255	255	→ slide in I_y
255	255	255	255	255	
0	0	0	0	0	$* \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $G_x = \begin{bmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 255&-&-&-&- \\ 255&-&-&-&- \\ 0&0&&& \end{bmatrix}$
~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	flip the filter in conv.

$$|\vec{G}| = \sqrt{G_x^2 + G_y^2} = \begin{vmatrix} 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & - \\ 255 & - & - & & - \\ 255 & - & - & & - \\ 0 & & & & \end{vmatrix}$$

~~Indeterminate~~.

(+). Undetermined.

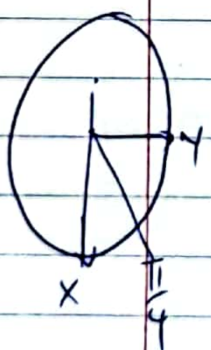
$$\theta = tg^{-1}\left(\frac{G_y}{G_x}\right) = \begin{vmatrix} Ind & - & - & - \\ Ind & - & - & - \\ 0 & & & \\ 0 & & & \\ Ind & - & - & - \end{vmatrix}$$

(3)

$$\begin{matrix} 255 & 255 & 255 & 255 & 255 \\ 255 & 255 & 255 & 255 & 0 \\ 255 & 255 & 255 & 0 & 0 \\ 255 & 255 & 0 & 0 & 0 \\ 255 & 0 & 0 & 0 & 0 \end{matrix} * [-1 \; 0 \; 1] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 255 & 255 \\ 0 & 0 & 255 & 255 & 0 \\ 0 & 255 & 255 & 0 & 0 \\ 255 & 255 & 0 & 0 & 0 \end{bmatrix}$$

$$* \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} =$$

$$|\vec{G}| = \sqrt{G_x^2 + G_y^2} = \begin{vmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 255 \\ 255 & 255 \end{vmatrix}$$



$$\theta = tg^{-1}\left(\frac{G_y}{G_x}\right) = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{\pi}{2} \\ & & & \frac{\pi}{4} & \frac{\pi}{4} \\ \frac{\pi}{4} & & & & \end{bmatrix}$$

normalize > Probability.

2. Zero-Crossing – Change of signs

Vertical

$$\begin{bmatrix} 255 & 255 & 255 & 0 & 0 \\ 255 & 255 & 255 & 0 & 0 \\ 255 & 255 & 0 & 0 & 0 \\ 255 & 255 & 0 & 0 & 0 \\ 255 & 255 & 0 & 0 & 0 \end{bmatrix} \circledast \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 255 & -255 & 0 & 0 \\ 0 & 255 & -255 & 0 & 0 \end{bmatrix}$$

→ symmetric, no flipping.

zero crossing

→ Zero crossings

Thresholding Cont'

Global thresholding – Adaptive thresholding.

- Choose global T → Avg. of gray levels.
- Segment image using T into M_1 & M_2.
- new $T = \frac{1}{2}(m_1 + m_2)$.
- Repeat until diff btwn T_k & T_{k+1} is very small (δT)

Otsu Method.

- each class each object each threshold.
- Maximize variance of two classes to separate them well.
- Have histogram.
- Choose T_k & probabilities.

P_1 – prob of region 1., M_2 – aver. relevant of

Region Definition → labelling = Partitioning

- attribute (predicate)
-

$P(R_i) = \text{True}$

If

① vary around a given value.

Large Partition.

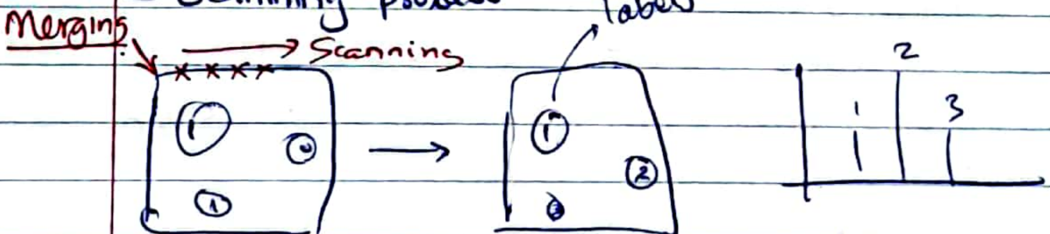
✓ cardinal - lower no of regions (Possibility that over the...)
✓ Minimize size of smallest region.
✓ close objects ignored.

Mostly used.

✓ $P(R_i \cup R_j) = \text{False}$ → fusion of 2 regions
✓ All pixels are connected inside the region.

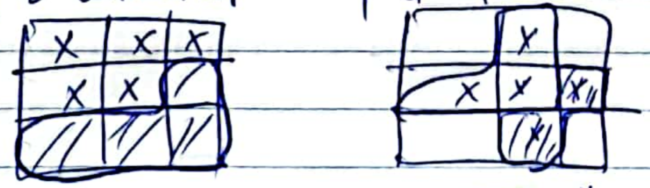
Blob Coloring

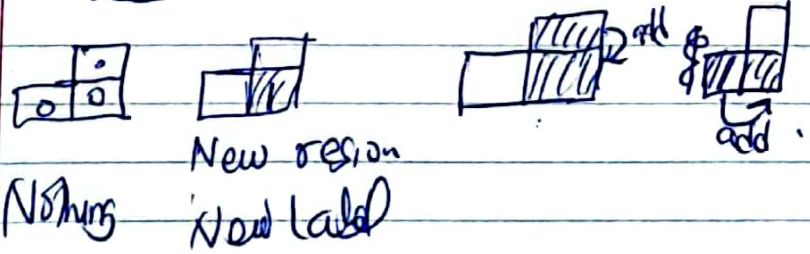
- Scanning process



Past 8-Connectivity

Past 4-Connectivity





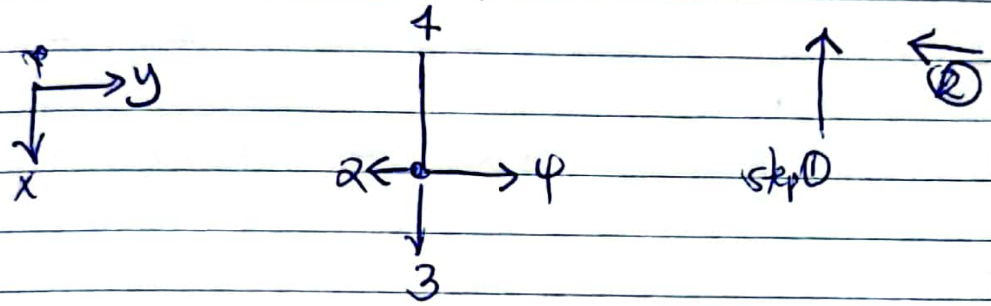
Growing.

→ start with seedpoint. → mean value of 11×11 neighbourhood.
→ to avoid noise & starting at boundary

→ Consider neighbours → same relevant value → same region

→ To extract one region

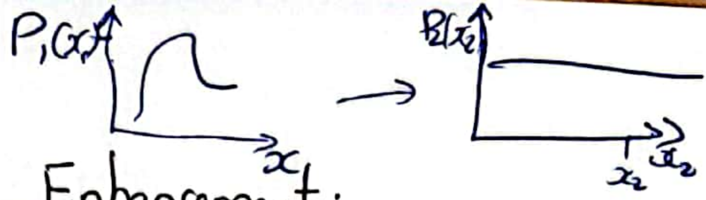
→ Another seed not explored. (not in the labelled picture).



Region split

- homogenous? no → split into 4 else stop. & assign label

Repeat Criterion homogeneous for regions to merge same regions.



TD: Image Enhancement.

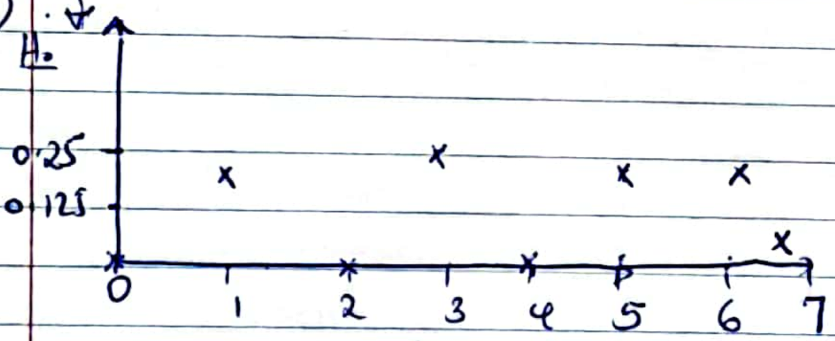
EX1: Histogram Equalization

	0	0.142 ($\frac{1}{7}$)	0.286 ($\frac{2}{7}$)	0.429	0.571	0.714	0.85	1
class	0	1	2	3	4	5	6	7
$\frac{\text{occur}}{\text{Total Pixel}}$ P.K. ①	0.193 ($\frac{790}{4096}$)	0.25	0.207	0.16	0.08	0.06	0.03	0.02 → $\frac{\text{occu}}{4096}$
(commulative) SK ②	0.193	0.443	0.65	0.81	0.89	0.95	0.98	1
classes ③ new r. value	1 (nearest to existing)	3	5	6	6	7	7	7.
	0	1	2	3	4	5	6	7
New Probab ④	0	0.193	0	0.25	0	0.207	0.24	0.11
Commu. value ⑤	0	0.193	0.193	0.443	0.443	0.065	0.89	1

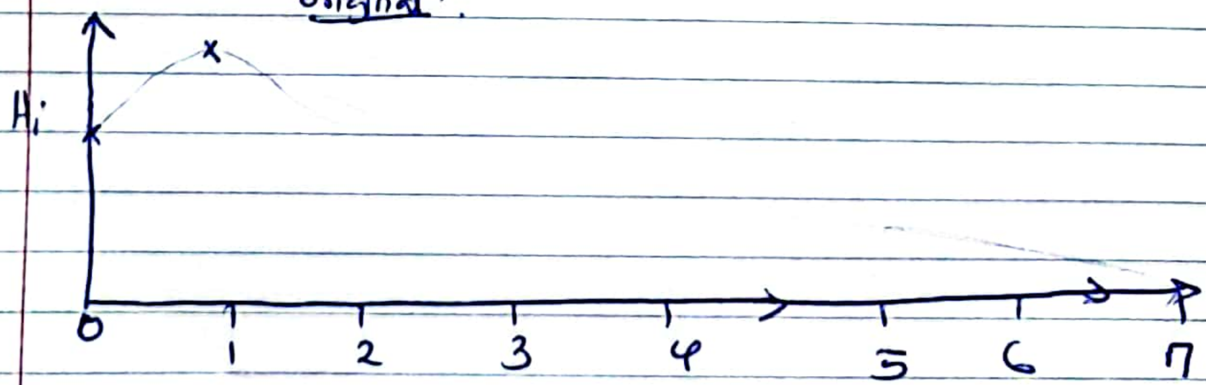
one replaced zero (take original).

because 4 is not available

⑥



original.



6. flat → No → because of error during merging of classes.
(taking close to.

EX2: Ottsu Method.

(*) index of Pixels start from 0

1. Maximum relevant value = 5
Bits = 3 bits

Dynamic range [0, 5].

Average gray level image (MG).
using histogram
$$MG = \frac{(0\times8)+(1\times7)+(2\times2)+(6\times3)+(4\times9)+(1\times4)}{36}$$
$$MG = 2.36$$

Pixels for P_1 are strictly lows than threshold.

k = 0.

Threshold K	0 (select everything)	1 (no select 0)	2 (not select 0 & 1)	3	4	5
$<k\ P_1(k)$	0	$\frac{8}{36} = 0.22$	$0.416\ (\frac{15}{36})$	0.472	0.638	0.888
$\geq k\ P_2(k)$	1	0.78	0.584	0.528	0.362	0.112.
$M_1(k)$	$\frac{0\times8+1\times7}{8+7}$ 0	0	$\frac{0\times8+1\times7}{8+7} = 0.466$	0.65	1.26	
$M_2(k)$	2.36 (MG)	3.03	$\frac{2\times2+3\times6+4\times9+5\times4}{2+6+9+4}$ 3.71	3.89	4.307	5
σ_B^2 ($m_1 - M_G$)	0	1.575	2.56	2.62	2.14	0.87

$\frac{1\times7+0\times8}{7}$

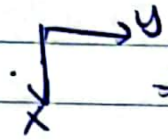
	Occurance
0	8
1	7 → m_1
2	2
3	6
4	9 ↓ m_2
5	4

K=2

$$\sigma_B = (M_1 - M_G)^2 P_1$$

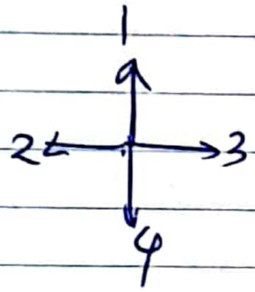
$$\sigma_B = (m_1 - m_G)^2 P_1 + (M_2 - M_G)^2 P_2$$

Max variance is at $K = 3$
- Good separation in histo & pixels.
- to separate well the two regions
- Efficient method.

(3) 

→ begin at 7 → not possible

→ Diff lower than thr grow if not try other direction



(2) (4,3) → 22
include 2.
- Doesn't work go back to previous

→ Answer used

(Ex 4). Blob Coloring.

Mathematical Morphilogy

→ A, B → Binary Image is a set of Coordinates.

$\subseteq$ → always included

→ X
↓ Y

translation → add x to set A.

⊕ Dilation → makes image bigger
⊖ Erosion → Completly included. → thins the remaining objects.

no symmetry of B

$\tilde{B}$ → Symmetric of B.

(0,0)
(0,1)

$-b\left\{\begin{pmatrix}-0, & -0 \\ -0, & -1\end{pmatrix}\right\}$. ✓

A^c → Complementary of A.
0 → 1
1 → 0

→ Used to enhance the shapes after segmentation
→ Play with Shapes

- Opening → fill holes of background (seperate objects).
  → erosion then
  → Dilation to recover original size.
- Closing → fill holes of object
  → Dilation then → size is bigger (holes were filled).
  → then erosion.

→ Idempotency: not necessary to itterate opening operation

Skeleton
→ Basic signature (summary) of the object after segmentation
→ Recoverable. (using structuring element).

S_n = erosion − erosion of opening.
→ erode until we have S_n empty set.

$\oplus$

$\ominus$

Using neighb
→ until v doesnt change.

Morph. Edges
→ Diff of Dilation & Erosion → we get the edges
→ we get Complete/continuous edges

Symmetric Diff Δ.
→ reveals what is moving & its edges

Hit & Miss
↓ object
↓ background.

B_1 & B_2 → Completely Complementary. ϕ
→ Detect corners.

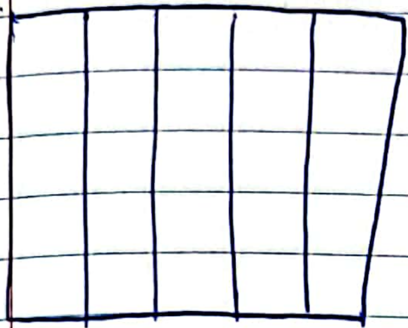
Shape filling

Dilation → Maximum relevant Value Considered.
Erosion → Minimum

Scanned with CamScanner

TD6: Mathematical Morphology.

EX1:



* Opening $(A \cdot B) = (A \ominus B) \oplus B$ → zero $\xrightarrow{\text{erosion}}$ remains 0 → Background.

  erosion $\xrightarrow{\text{result}}$ dilation

* Closing $(A \bullet B) = (A \oplus B) \ominus B$

  = Dilation $\xrightarrow{\text{result}}$ erosion.

* Boundary Extraction.

  $= A - (A \ominus B)$