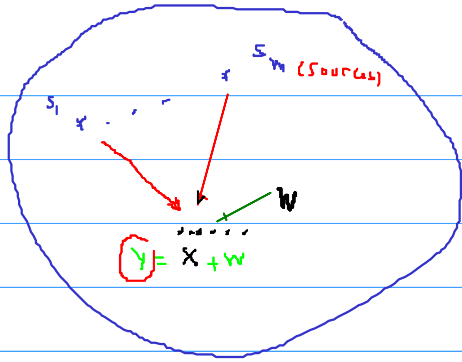
APPLICATION TO ANTENNA PROCESSING (Music Algorithm)

VAN TREES : Estimation, Detection & Modulation
(2 VOL.)
LIBGEN



$$\vec{X} = \sum_{i=1}^{m} \theta_i \underline{S}_i$$

θ_i: Random, $\underline{S}_i$: Deterministic

Antenna $= \underset{\sim}{S} \cdot \underline{\theta}$

$$\vec{Y} = \vec{X} + \vec{W} \quad \text{Noise.}$$

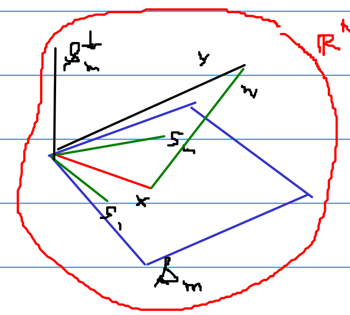
$\overline{W} = 0 \qquad \overline{WW^T} = R_W \qquad ; \; R_{XW} = E[XW^T] = 0$

Optimal Receiver

$$\boxed{L = R_{XY} R_Y^{-1}} = \overline{X(X+W)^T} \left\{ \overline{(X+W)(X+W)^T} \right\}^{-1}$$

$$= \underbrace{R_X}\underbrace{[R_X + R_W]^{-1}}_{R_Y} = \underset{\sim}{S} \underbrace{\overline{\theta\theta^T}}_{C_\theta} \underset{\sim}{S} \left[\underset{\sim}{S} C_\theta \underset{\sim}{S}^T + R_W \right]^{-1}$$

$\underset{\sim}{S} = [\underline{S}_1, \ldots, \underline{S}_m]$. $\quad \theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_m \end{bmatrix}$



* Hypothesis: Noise is 'white', isotropic $\rightarrow R_W = \beta I$

$$\rightarrow \quad L = R_X R_Y^{-1} = (R_Y - R_W) R_Y^{-1} = I - R_W R_Y^{-1} = I - \beta R_Y^{-1}$$

* $R_Y = R_X + \beta I \quad \rightarrow$ 1/ Eigenvectors of R_X are eigenvectors of R_Y
2/ $\mathcal{S}$ and $\mathcal{S}^\perp$ are eigenspaces of R_Y

- $R_Y = \beta I + \underset{\sim}{S} C_\theta \underset{\sim}{S}^T$
- $R_Y \underline{S}_i = \beta \underline{S}_i + \underbrace{\underset{\sim}{S} C_\theta \underset{\sim}{S}^T \cdot \underline{S}_i}_{\in \mathcal{S}} \in \mathcal{S} \quad \therefore \; R_Y \mathcal{S} \subseteq \mathcal{S}$
- If $\vec{V} \in \mathcal{S}^\perp$, $R_Y \vec{V} = \beta \vec{V} + \underset{\sim}{S} C_\theta \underbrace{\underset{\sim}{S}^T \vec{V}}_{0} = \beta \vec{V} \in \mathcal{S}^\perp$
- Let $R_X = E\, XX^T = \underbrace{\sum_{i=1}^{m}}\lambda_i \underline{u}_i \underline{u}_i^T$

$u_i \perp u_j \quad i \neq j$
$\{u_i\}$ span $\mathcal{S}$

: $\{u_i\}$ are orthogonal for $R_X = R_X^T$
$\lambda_i \geq 0$ (R_X is positive definite)

$$R = \underset{\sim}{U} \Lambda \underset{\sim}{U}^{-1} = \underset{\sim}{U} \Lambda \underset{\sim}{U}^{+} = [\underline{u}_1, \ldots, \underline{u}_N] \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_N \end{bmatrix} \begin{bmatrix} \underline{u}_1^+ \\ \cdots \\ \underline{u}_N^+ \end{bmatrix}$$

* Let $u_{m+1}, \ldots, u_N$ span $\mathcal{S}^\perp$ and $u_i \perp u_j = 0 \quad i \neq j, \; i,j = m+1 : N$

$$I = \sum_{i=1}^{N} u_i u_i^+ \Rightarrow \underbrace{R_W} = \beta I = \beta \underbrace{\sum_{i=1}^{m} \underline{u}_i \underline{u}_i^+}_{\text{Proj on } \mathcal{S}} + \beta \underbrace{\sum_{i=m+1} \underline{u}_i \underline{u}_i^+}_{\text{Proj on } \mathcal{S}^\perp}$$

$\underline{u}_i \underline{u}_i^+$: Projector in direction u_i

$$R_Y = \sum_{i=1}^{m} \mu_i \underline{u}_i \underline{u}_i^+ + \beta \sum_{m+1}^{N} u_i u_i^+$$