TD : Filtering in the spatial domain

M1 E3A international track, Evry site

UE "Image and signal processing", Upsay / UEVE

Answer of exercise 1

1. $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$ Binomial Filter of size 1×3. Low pass.
2. $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$ Derivative Filter including averaging. High-Pass.
3. $\begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$ Second Derivative $\frac{\partial f}{\partial y}$. High-Pass.
4. $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Prewitt (derivative filter including a first derivative on x and an averaging, then an addition with a second derivative on y : $\frac{\partial f}{\partial x} + \frac{\partial^2 f}{\partial y^2}$, useful to detect corners).
5. $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ Variante of a binomial filter. Low-Pass.
6. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ Diagonal derivative Filter and diagonal averaging (other diagonal). High-Pass. Same principle than ROBERTS but with an integrated smoothing.
7. $\begin{bmatrix} -1 & -1 & 0 & 1 & 1 \end{bmatrix}$ Derivation and averaging in the same direction. The derivative filter includes by itself an average. High-Pass.

Answer of exercise 2

1. $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} = \frac{\partial f}{\partial y} * -\frac{\partial f}{\partial y} = -\frac{\partial^2 f}{\partial y^2}$
2. $\frac{1}{2}\begin{bmatrix} 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
3. This filter could be deduced from question 1. It is than equal to: $\frac{\partial f}{\partial y} * -\frac{\partial f}{\partial y} = -(\frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial x^2})$

Answer of exercise 3

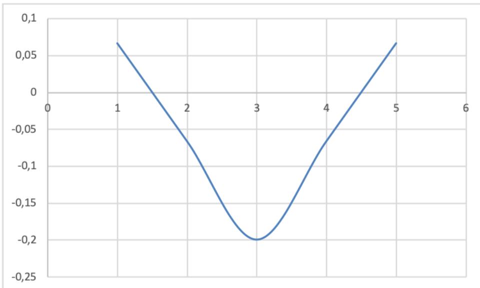
1. $h = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
2. $h + L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$
3. This filter makes it possible to highlight the transitions (role of the Laplacian) while maintaining the gray levels of the original image on the pixels which do not correspond to transitions (role of h).

Answer of exercise 4

1. i/ With the binomial coefficients (Pascal's triangle) or ii/ by convolving the average filter 1×2 with itself (see lecture slides).
   $\begin{bmatrix}1 & 1\end{bmatrix} * \begin{bmatrix}1 & 1\end{bmatrix} * \begin{bmatrix}1 & 1\end{bmatrix} * \begin{bmatrix}1 & 1\end{bmatrix} * \begin{bmatrix}1 & 1\end{bmatrix} = \begin{bmatrix}1 & 6 & 15 & 20 & 15 & 6 & 1\end{bmatrix}$
2. We use the separability property:
$$\begin{bmatrix}1 & 6 & 15 & 20 & 15 & 6 & 1\end{bmatrix} * \begin{bmatrix}1\\6\\15\\20\\15\\6\\1\end{bmatrix} = \begin{bmatrix}1 & 6 & 15 & 20 & 15 & 6 & 1\\6 & 36 & 90 & 120 & 90 & 36 & 6\\15 & 90 & 225 & 300 & 225 & 90 & 15\\20 & 120 & 300 & 400 & 300 & 120 & 20\\15 & 90 & 225 & 300 & 225 & 90 & 15\\6 & 36 & 90 & 120 & 90 & 36 & 6\\1 & 6 & 15 & 20 & 15 & 6 & 1\end{bmatrix}$$
3. In order to normalizer, we divide by the sum of all the filter values.
4. Normalization keeps the dynamic range.
5. Binomial filters are alternatives to Gaussian filters, more easy to calculate with integer coefficients.

Answer of exercise 5

1. $\begin{bmatrix}0.120985362 & 0,176032663 & 0.19947114 & 0.176032663 & 0.120985362\end{bmatrix}$
2. $\begin{bmatrix}0.053990967 & 0.241970725 & 0.39894228 & 0.241970725 & 0.0539909672\end{bmatrix}$
3. Smoothing. Low-Pass.
4. $\begin{bmatrix}-0.066994396 & 0.065938061 & 0.19947114 & 0.065938061 & -0.066994396\end{bmatrix}$
5. The profile correspond to a DoG filter which is an approximation of a LoG.



6. Second derivative filter, High pass.

Answer of exercise 6

1. After sorting the levels, the median value is 4.
2. The majority filter gives 5. The median avoids extreme values and the majority favors the levels with the highest occurrence.
3. Yes.
4. Density > n div 2 + 1.