3/ X Poisson $X \in \{0,1,2,..\} \equiv \mathbb{N}$ $\quad \mathbb{P}(X=n) = e^{-\lambda} \frac{\lambda^n}{n!}$; $\lambda = \mathbb{E}(X)$ (parameter)

$$\varphi_X(u) = \mathbb{E}\, e^{iuX} = \sum_n e^{iun} \cdot \mathbb{P}[X=n]$$

$$= \sum_{n \geq 0} \frac{\lambda^n}{n!} e^{-\lambda + inu} = e^{\lambda(e^{iu}-1)}$$

$Z = X + Y$

X, Y indep & Poisson

$$\Big\} \; \varphi_Z(u) = \varphi_X(u) \cdot \varphi_Y(u) = e^{(\lambda+\mu)(e^{iu}-1)}$$

* $X_1, \dots, X_n$: $X_k \sim N(0, \sigma_k^2)$ & indep

$$Z = \alpha_1 X_1 + \dots + \alpha_N X_N \qquad \varphi_Z(u) = \mathbb{E}\left[e^{iu(\alpha_1 X_1 + \dots + \alpha_n X_n)}\right] = \varphi_{X_1}(\alpha_1 u) \cdots \varphi_{X_n}(\alpha_n u)$$

$$= \exp -\frac{1}{2}(\alpha_1^2 \sigma_1^2 + \dots + \alpha_n^2 \sigma_n^2) u^2 \Rightarrow Z \sim \text{Gaussian}$$

4/ $Z = X + Y$, X, Y Poisson indep. λ_1, λ_2

$$\mathbb{P}[X=k \mid Z=n] \overset{?}{=} \text{Binomial}$$

$X \in \{0, 1, 2, .., N\}$

$$\mathbb{P}(X=k) = \binom{N}{k} \alpha^k (1-\alpha)^{N-k}$$

$0 \leq \alpha \leq 1$

$$\frac{N!}{k!(N-k)!}$$

$$= \mathbb{P}[A \mid B] \overset{?}{=} \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}$$

$$= \frac{\mathbb{P}[Z_n = n \mid X=k]\, \mathbb{P}(X=k)}{\mathbb{P}[Z=n]} = \frac{\mathbb{P}[Y=n-k]\, \mathbb{P}[X=k]}{\mathbb{P}[Z=n]} = \frac{e^{\lambda_2} \lambda_2^{n-k} e^{\lambda_1} \lambda_1^k}{(n-k)!\, k!\, (\lambda_1+\lambda_2)^n} n!$$

$X \sim \lambda_1$

$Y \sim \lambda_2$

$$= \binom{n}{k} q^k (1-q)^{n-k}, \quad q \doteq \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

* Conditional expectations : (X, Y) $\quad \mathbb{P}_{X,Y}(x,y)$

observable

$$\mathbb{E}[X \mid Y=y] = \int x\, p_{X|Y}(x/y)\, dx$$

$$= \int x \frac{p_{XY}(x,y)}{p_Y(y)} dx = \frac{1}{p_Y(y)} \int x\, p_{XY}(x,y)\, dx$$

x