TD: Image enhancement

M1 E3A international track, Evry

UE "Image and signal processing", Upsay / UEVE

Exercise 1 : Histogram equalization (from Gonzales et Woods)

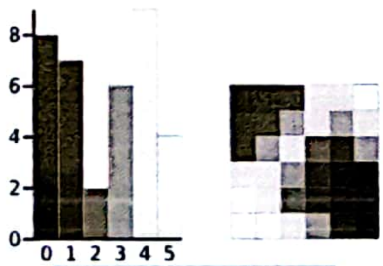
Let be the histogram of an image of size 64×64 coded according to 8 classes also distributed between 0 and 1 according to the following figure:

Class	0	1	2	3	4	5	6	7
Level	0	1/7	2/7	3/7	4/7	5/7	6/7	1
Number of occurrences	790	1023	850	656	329	245	122	81

1. Calculate the probability of each class and plot the normalized histogram (probability density function) before equalization.
2. Calculate the cumulative distribution function.
3. Apply the equalization algorithm. Deduce the classes that were merged.
4. Plot the new probability density function.
5. Plot the new cumulative distribution function, compare it with that of an equally likely histogram.
6. Is the new histogram flat? → because error.

Exercise 2 : Thresholding using Otsu method

1. Consider the histogram below. What is the number of bits necessary for the encoding of the image? Find the dynamic range as well as the average of the gray levels of the image from the histogram.



A 6-level greyscale image and its histogram

0	0	1	4	4	5	→13
0	1	3	4	3	4
1	3	4	2	1	3
4	4	3	1	0	0
5	4	2	1	0	0
5	5	4	3	1	0

2. In the so-called Otsu thresholding method, the optimal threshold is that which maximizes the inter-class variance. We then consider two classes (regions) in the image: the background and the object (foreground). The goal is to find the threshold that allows the best separation. Complete the table below and find the optimal threshold.

k	0	1	2	3	4	5
$P_1(k)$	0					
$P_2(k)$	1					
$m_1(k)$						
$m_2(k)$						
σ_B^2						

Exercise 3 : Region growing

	0	1	2	3	4	5	6	7
0	2	16	14	16	14	4	14	12
1	14	15	13	15	12	15	13	15
2	16	16	13	21	22	21	22	13
3	14	17	21	22	24	23	23	13
4	7	16	21	22	30	23	27	13
5	16	16	20	22	7	24	12	16
6	17	15	16	20	22	22	13	15
7	14	17	13	13	13	18	12	14
8	14	15	17	15	14	15	14	15

1. Achieve the 4-connected region growing of the image considering as the starting point the point (x = 5, y = 4) and a threshold of 4. → pas possible
2. Carry out growth with the point (4,3) with first a threshold of 2 then a threshold of 6.

Exercise 4 : Blob coloring

1. Carry out a a connected component labeling using 4-connected neighbors then 8-connected neighbors of the image below. Compare and conclude.

new label

1	1	1	0	1	1	0	1
1	1	0	1	1	0	1	1
1	0	1	1	0	1	1	1
1	1	1	0	1	1	1	1
0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

labels

~~0~~	1	2	3	4	5	6	7	8	9
~~0~~	1	2	3	2	3	2	3		

erosion → all points (ones) of structure covered (Center everywhere).
Dilation → Atleast any point of structure is satisfied (covered)

TD 6 : Mathematical morphology

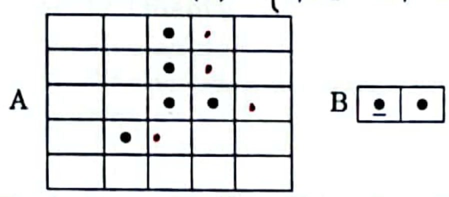
M1 E3A international Track

"Signal and image processing"

Exercise 1 : Basic morphological operations

1. For each of the morphological operations defined below, find the transformed image using the proposed images A and the structuring elements B (the line indicates the center of the structuring element).
   — Dilation : $D_B(A) = \left\{x / \widetilde{B_x} \cap A \neq \emptyset\right\} = A \oplus B$

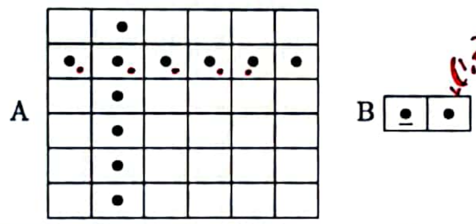
   - If intersection yes ✓ take the point



2. Check that we find the same results using the following definition : $A \oplus B = \bigcup (A)_b$ (union of the translations of A with the elements of B).

   → translate A ? → then make Union with A

3. Erosion : $E_B(A) = \{x / B_x \subset A\} = A \ominus B$



(1,0) → trans → (-1,0)

- translate original ← -1,0 (0,0)
  then
- Intersection

4. Check that we find the same results using the following definition : $A \ominus B = \bigcap (A)_{-b}$ (intersection of translations of A with the elements of -B).

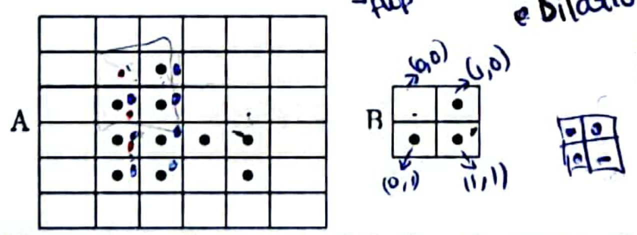
① erosion

② Put on the image that is the result after erosion.

5. Opening : $A \circ B = (A \ominus B) \oplus B$

   flip

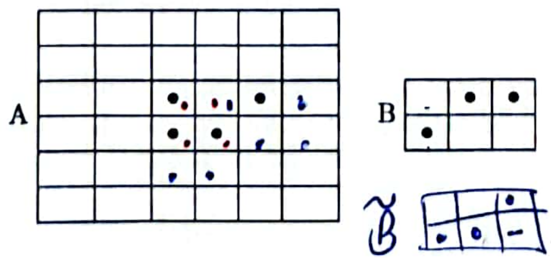
   • erosion
   • Dilation



→ If intersection anywhere → take center.

6. What is the result of opening ? Perform the opening of the open. What do you notice ? → Same result opening

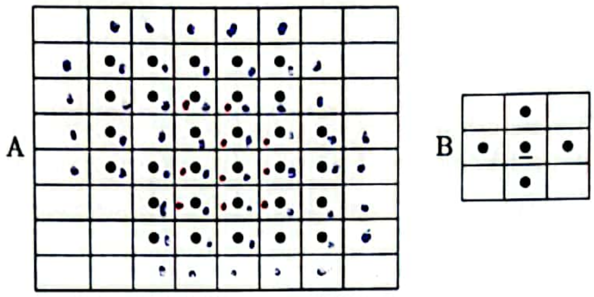
7. Closing : $A \bullet B = (A \oplus B) \ominus B$



• e
• D

8. What is the result of closing? Perform the closure of the closed image. What do you notice?
9. Combinations of dilations and erosions :

- $D_B A$
- $E_B A$



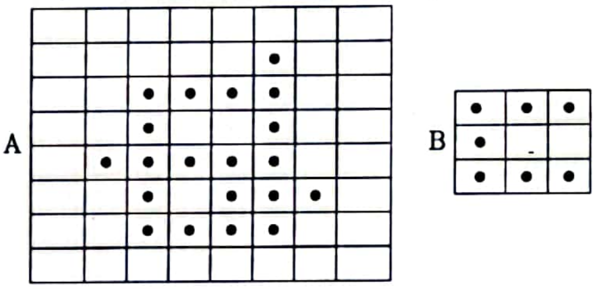
10. Determine the dilated image A as well as the eroded one using the proposed structuring element. What is the result of $D_B(A) - E_B(A)$? What is the result of $D_B(A) - A$ and $A - E_B(A)$?

edges

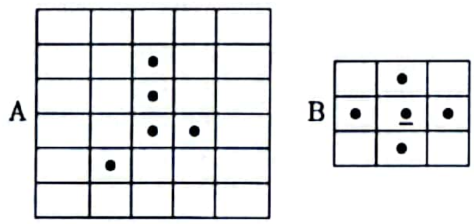
Exercise 2 : Duality

Check the following properties on the given images and structuring elements.

1. Duality of dilation and erosion : $(A \ominus B)^c = A^c \oplus \widetilde{B}$
2. Duality of opening and closing : $(A \bullet B)^c = A^c \circ \widetilde{B}$.



Exercise 3 : Union of structuring elements



B_1 : $\underline{\bullet}\ \bullet$ $\qquad$ B_2 : $\bullet\ \underline{\bullet}$ $\qquad$ B_3 $\begin{matrix}\bullet\\ \underline{\bullet}\end{matrix}$: $\qquad$ B_4 : $\begin{matrix}\underline{\bullet}\\ \bullet\end{matrix}$

1. Compare the erosions and dilations of the various structuring elements B_i with the given image A.
2. Calculate dilation and erosion of image A with the structuring element B (noting that it is the union of the previous structuring elements). Reminder : $D_{B \cup C}(A) = D_B(A) \cup D_C(A)$ and $E_{B \cup C}(A) = E_B(A) \cap E_C(A)$.

Exercise 4 : Skeleton