* Cond. Expectation as a information processing tool

X, Y

$$\min_{\forall h} \; \mathbb{E}\,\|X-h(Y)\|^2 \Rightarrow h(Y)=\mathbb{E}[X|Y]$$

$$\mathbb{E}\Big\{\mathbb{E}\big[\underbrace{\|X-h(Y)\|^2}\,\big|\,Y\big]\Big\}$$

$$X^2 + h(Y)^2 - X\,h(Y)$$

$$\mathbb{E}\Big\{\mathbb{E}[X^2|Y] - \mathbb{E}\big(X-h(Y)\big)\,|\,Y\big).h(Y)\Big\}$$

$\Rightarrow$ Min. for $\boxed{h(Y)=\mathbb{E}(X|Y)}$

* Exercise: $\underline{X}, \underline{Y}$ gaussian Random vectors.

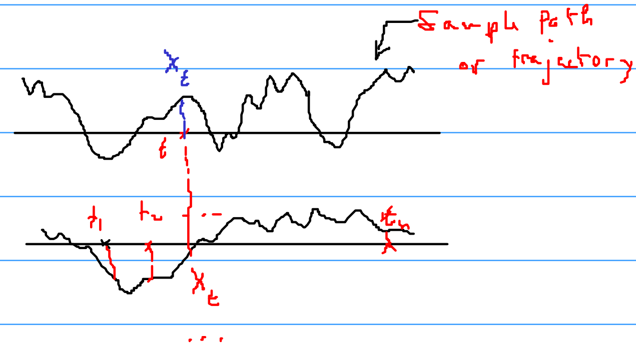
Show that $\mathbb{E}[\underline{X}|\underline{Y}] = L\underline{Y}$ $\quad \Big\{ L = \underbrace{\mathbb{E}[\underline{X}\cdot\underline{Y}^T]}_{R_{XY}} \cdot \underbrace{\mathbb{E}[\underline{Y}\,\underline{Y}^T]}_{R_Y}{}^{-1}$

(X,Y) gaussian

$$\underline{Z} = \begin{bmatrix}\underline{X}\\ \underline{Y}\end{bmatrix}, \quad R_Z = \left[\begin{array}{c|c} R_X & R_{XY} \\ \hline R_{YX} & R_Y \end{array}\right] \;; \quad R_{YX} = R_{XY}^T$$

$$\mathbb{E}[X|y] = \int x \,\frac{P_{XY}(x,y)}{P_Y(y)}\,dx$$

10/ Random Process: $\{X_t,\; t\in\mathbb{R}\}$



Independent increment process:

$\forall n,\; t_1,\dots,t_n$

$$(X_{t_2}-X_{t_1}),\; (X_{t_3}-X_{t_2}),\; \dots,\; (X_{t_n}-X_{t_{n-1}}) \quad \text{Independent}$$

Ex: Brownian process