* Now minimize directly

$$\min_{L} \mathbb{E}\{\| \underbrace{\underline{x} - L\underline{y}}_{\varepsilon} \|^2\}$$

for any (non necessarily joint gaussian)

random vectors x, y

Reminder: Many square matrix M $\quad tr(M) = \sum_{k=1}^{n} M_{kk} = \sum \text{eigen values} = \sum_{k=1}^{n} \lambda_k$

$\underline{x}, \underline{y}$ vectors $\quad \underbrace{\underline{x}^T \cdot \underline{y}}_{\sum x_k y_k} = tr(\underline{y} \cdot \underline{x}^T)$

$$Var(\underline{\varepsilon}) = \mathbb{E}\{\|\varepsilon\|^2\} = \mathbb{E}(\varepsilon^T \cdot \varepsilon) = \mathbb{E}[tr(\varepsilon \cdot \varepsilon^T)] = tr\, \mathbb{E}[\underline{\varepsilon} \cdot \underline{\varepsilon}^T] \qquad \mathbb{E}[W] = \overline{W}$$

$$= tr\{\underbrace{\mathbb{E}[x \cdot x^T]}_{R_x} - L\underbrace{\overline{YX^T}}_{R_{Yx}} - \underbrace{\overline{XY^T}}_{R_{xY}} L^T + L\underbrace{\overline{YY^T}}_{R_Y} L^T\}$$

$$= tr\{(L - R_{xY} R_Y^{-1}) R_Y (L - R_{xY} R_Y^{-1})^T\} + tr\{R_x - R_{xY} R_Y^{-1} R_{Yx}\}$$

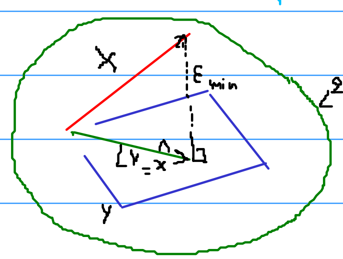
positive definite : $\forall \vec{w}$: $\vec{w}^T R_Y w > 0$

$Var(\varepsilon)$ is minimized by

$$L = R_{xY} \cdot R_Y^{-1}$$

$$\min_{L} Var(\varepsilon) = tr\{R_x - \underbrace{R_{xY} R_Y^{-1} R_{Yx}}_{L R_Y L^T}\}$$

geometrical Interpretation :



$$\mathbb{E}[\varepsilon^T \cdot Y] = 0$$

$$tr\, \mathbb{E}[(X - LY) Y^T] = 0$$

$$\underbrace{\mathbb{E}[x \cdot Y^T]}_{R_{xY}} - L \underbrace{\mathbb{E}[YY^T]}_{R_Y} = 0 \qquad \text{(Normal Equation)}$$

$$Cov(\varepsilon) = Cov(X) - Cov(LY)$$