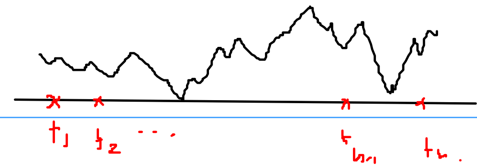
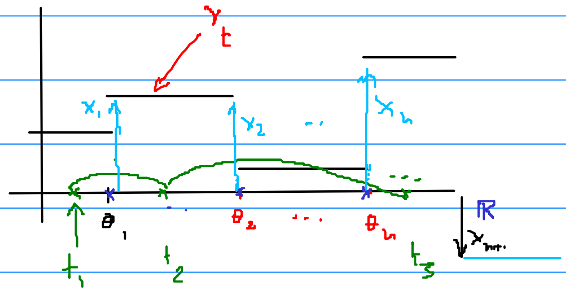
Exercise on correlation functions

A Random Process is defined by all dist. functions



$$P(x_{t_1}, \ldots, x_{t_n}) \quad \forall n, \quad \forall\ t_1 < t_2 < \ldots < t_n$$

Corr. function $\quad \gamma_X(t_1,t_2) = \mathbb{E}[X_{t_1} \cdot X_{t_2}] \quad = \gamma_X(t_2 - t_1)$ $\boxed{\text{in stationary case}}$



$\{\theta_n\}$ is Poisson Process if:

1/ For any times $t-\tau$, t, $\tau \geq 0$

$N_{t,t-\tau}$ = nb. of random pulses θ_i occuring in $]t-\tau, t]$.

is a poisson R.V. with parameter $\lambda\tau$

$$\mathbb{P}[N_{t,t-\tau} = n] = e^{-\lambda\tau} \frac{(\lambda\tau)^n}{n!}$$

2/ for any times $t_1, t_2, \ldots, t_N$.

$N_{t_i, t_{i+1}}$, are indep.

3/ $\{X_n\}$ independent, 0 mean, gaussian with variance σ^2

Ex: compute $\gamma_Y(t, t-\tau) = \mathbb{E}[X_t \cdot X_{t-\tau}]$, $\tau \geq 0$

$$\sim e^{-\sigma^2 \cdot \tau}$$

Hint:
$$\left\{ \begin{array}{l} \mathbb{E}[Y_t \cdot Y_{t-\tau} \mid N_{t,t-\tau} = n] = ? \\ \\ \underbrace{\mathbb{E}[Z]}_{} = \mathbb{E}\{\mathbb{E}[Z \mid U]\} = \sum_n \underbrace{\mathbb{E}[Z \mid U = n]}_{} \cdot \mathbb{P}[U = n] \end{array} \right.$$

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