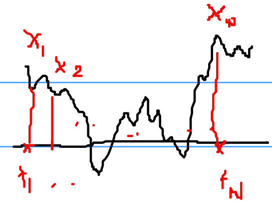
1/

$$\overline{(X+Y)^2} = \overline{X^2} + \overline{Y^2} + 2\,\overline{XY}$$

$$\mathrm{Var}(X) = \overline{X^2} - \bar{X}^2 = \mathbb{E}\,(X-\bar{X})^2$$

$\underline{X} : \Omega \to \mathbb{R}^N$ $\qquad \underline{X} = \begin{bmatrix} x_1 \\ \cdots \\ x_N \end{bmatrix}$



$\underline{Y} : \Omega \to \mathbb{R}^N$

$$\mathbb{E}\left[\underline{X}\cdot\underline{Y}^T\right] = \begin{array}{c c} & m \\ n & \begin{bmatrix} & \vdots \\ \cdots & \overline{x_n y_m} \end{bmatrix} \end{array}$$

$$\overline{(X+Y)^2} = \overline{X^2} + \overline{Y^2} + 2\,\overline{XY}$$

$$= (\overline{X^2} - \bar{X}^2) + (\overline{Y^2} - \bar{Y}^2) + 2\,\overline{XY} - 2\bar{X}\bar{Y}$$

$$+ \underbrace{\bar{X}^2 + \bar{Y}^2 + 2\bar{X}\bar{Y}}_{(\bar{X}+\bar{Y})^2}$$

* Weak law of large Numbers

$Z_n = \frac{1}{n}\underbrace{(X_1 + \dots + X_n)}$ $\qquad X_n$ indep. identically distributed iid

$\bar{X} = \bar{X}_n \qquad \sigma^2 = \overline{X_n^2}$

$\mathbb{E} Z_n = \frac{1}{n}\sum \mathbb{E} X_i = \frac{1}{n} \times n\bar{X} = \bar{X}$

Chebyshev. Th.

$$\mathbb{P}\left\{ |Z_n - \bar{X}| > \varepsilon \right\} = \mathbb{P}\left\{ |Z_n - \bar{X}|^2 > \varepsilon^2 \right\} \leq \frac{1}{\varepsilon^2}\,\underbrace{\mathbb{E}\left[|Z_n - \bar{X}|^2 \right]} \longrightarrow 0$$

$\varepsilon > 0$

But: $\mathbb{E}\,|Z_n - \bar{X}|^2 = \mathrm{Var}(Z_n) = \frac{1}{n^2}(n \times \mathrm{Var}(X_i)) = \frac{n\cdot\sigma^2}{n^2} = \frac{\sigma^2}{n} \xrightarrow{n\to\infty} 0$

Proof of Chebyshev.

Let $X \geq 0$, $\mathbb{E} X < \infty$

$$\mathbb{E} X = \int_0^\infty x\, P_X(x)\, dx = \int_0^\infty G_X(x)\, dx \qquad G_X(x) = \mathbb{P}[X > x]$$

$$\geq \varepsilon\, G(\varepsilon)$$