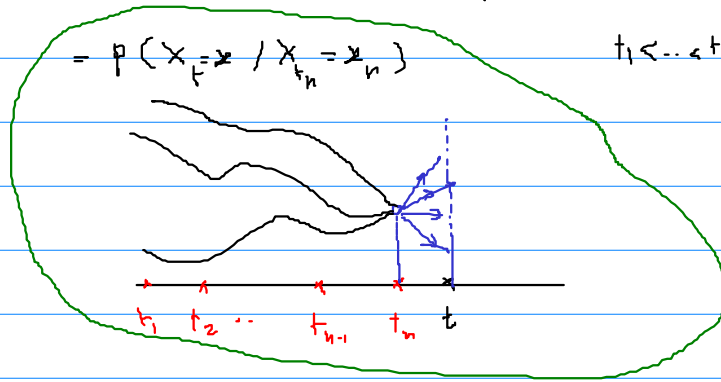


10/ Markov. processes:

$$P(X_t = x \mid X_{t_1} = x_1, \dots, X_{t_n} = x_n)$$

$$= P(X_t = x \mid X_{t_n} = x_n)$$

$$t_1 < \dots < t_n$$



Given any X_t with indep increments:

$$P[X_t = x \mid X_{t_1} = x_1, \dots, X_{t_n} = x_n] = P[X_t - X_{t_n} = x - x_n \mid X_{t_n} = x_n, X_{t_i} - X_{t_{i-1}} = x_i - x_{i-1}, i=2:n]$$

$$= P[X_t - X_{t_n} = x - x_n \mid X_{t_n} = x_n]$$

$$= P[X_t = x \mid X_{t_n} = x_n]$$

$\rightarrow \{X_t\}$ is Markovian

11/ $Y_{1n}, Y_{2n}, \dots, Y_{nn}$ triangular array of R.V.

iid, uniformly distributed on $(0, t_n)$

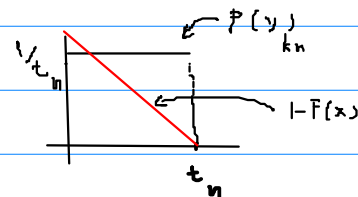
$$Z_n = \min(Y_{1n}, \dots, Y_{nn})$$

$$a) P(Z_n > x) = P(Y_{1n} > x, \dots, Y_{nn} > x)$$

$$= P(Y_{1n} > x) \dots P(Y_{nn} > x)$$

$$= [1 - F(x)]^n \quad F(x) = P[Y_{kn} \leq x] \quad \forall k$$

$$= \exp n \log(1 - F(x))$$



$$b) \log P(Z_n > x) = n \log(1 - x/t_n) = n \left\{ -\frac{x}{t_n} + o\left(\frac{x}{t_n}\right) \right\} = -x \left(\frac{n}{t_n} + o\left(\frac{n}{t_n}\right) \right)$$

$$\left\{ P(Z_n > x) \xrightarrow[n \rightarrow \infty]{} e^{-\lambda x} \right.$$