TD : Filtering in the spatial domain

M1 E3A international track, Evry site

UE "Image and signal processing", Upsay / UEVE

Exercise 1 : Spatial Convolution

Calculate and name the following filters:

1. $\begin{bmatrix} 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 \end{bmatrix}$
2. $\begin{bmatrix} 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 1 \end{bmatrix}$
3. $\begin{bmatrix} 1 & -1 \end{bmatrix} * \begin{bmatrix} 1 & -1 \end{bmatrix}$
4. $\left(\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right) + \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$
5. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$
6. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
7. $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

Exercise 2 : Filter Decomposition

Give the usefulness of the filters below by studying their answer on simple images. For each of them, break down the filter into several filters if possible. Otherwise express the filter continuously using the classical differential operators ($\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial^2}{\partial x^2}, etc.$). Give images of your choice allowing you to highlight the responses of the filters.

1. $\begin{bmatrix} -1 & 0 & 2 & 0 & -1 \end{bmatrix}$
2. $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$
3. $\begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 4 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}$

Exercise 3 : Filter synthesis

1. Give the filter h of size 3×3 that correspond to the identity filter for the convolution with an image f ($f * h = f$)
2. Compute the sum of the resulting filter h with a Laplacian of size 3×3.
3. Explain the utility of this kind of resulting filter.

Exercise 4 : Binomial filters

1. Give two ways to generate the binomial filter of size 1×7.

2. Deduce the binomial filter of size 7×7.
3. Give its normalized form.
4. In the general case, what is the point of normalizing the filters?
5. What is the use of binomial filtering?

Exercise 5 : Gaussian Differences

1. Calculate the Gaussian filter of size 1×5 with a variance equal to 4.
2. Calculate the Gaussian filter of size 1×5 with a variance equal to 1.
3. What is the use of Gaussian filtering?
4. Calculate the difference between the two filters of question 1 and 2.
5. Display the intensity profile of the difference. What do you deduce?
6. Give the utility and use of the new filter thus generated.

Exercise 6 : Median Filter

We consider a small neighborhood around a pixel of size 3×3:

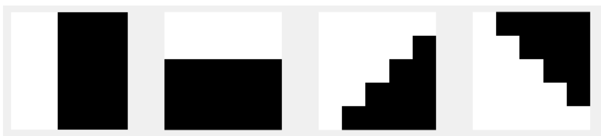
1	2	5
3	3	4
5	4	5

1. Calculate the result of applying a median filter on this neighborhood.
2. Calculate the result of applying a majority filter on this same neighborhood.
3. Is the median value kept when the image is multiplied by a scalar?
4. Give the density of the noise in the neighborhood of size $n \times n$ to from which the median filter is no longer effective.

Exercise 7 : Derivative Filters

Let's consider the following filters: $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

1. Calculate the modulus and direction of the gradient for small binary images of sizes 5×5 that correspond to ideal edges (vertical, horizontal, left diagonal, right diagonal).



2. Determine for each of the 4 considered ideal edges the zero crossing of the Laplacian (convolve with $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$).