Frequency Approach in IProcessing

- 2D DFT
- FT of basic images
- FFT
- Frequency domain filtering.

Recall:

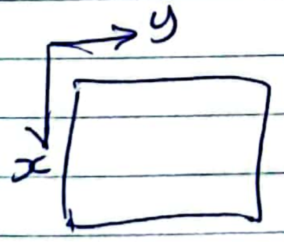
→ Variation in space (x) not t (Spacial domain).
→ u → frequency domain

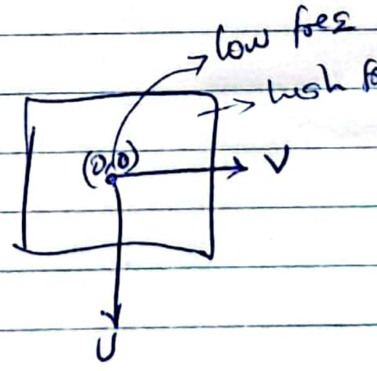
$$\int_{-\infty}^{+\infty} f(x)\,\delta(x-x_0)\,dx = f(x_0)$$ → dirac shifted by x_0

FT of Rectangular pulse.

σ → spread (width).

M → rows
N → columns





white → high values.

black →

variation → high

FT use.
- Remove noise
- Review edges

$a\,f(x,y)$ → brightness

$f(ax, by)$ → zoom

TD1&2: Fourier Transform of 2D

EX1: Properties of FT

1. Linearity of FT

$$FT\{f(x)\} = F(u) = \int f(x)\exp(-2\pi j u x)\,dx.$$

$$F\{af(x)+bg(x)\} = aF\{f(x)\} + bF\{g(x)\}.$$

$$\int (af(x)+bg(x))\cdot e^{-2\pi j u x}\,dx$$

$$= \int af(x)e^{-2\pi j u x}\,dx + \int bg(x)e^{-2\pi j u x}\,dx$$

$$= aF(u) + bG(u) \rightarrow \text{Superposition}$$

2. FT of the transposed function $f(-x)$. $\Rightarrow dy = -dx$

3. Scaling. FT of $f(ax)$.

$$F\{f(ax)\} = \int f(ax)\,e^{-2\pi j u x}\,dx$$

$f(x) \rightarrow F(u)$
$f(ax) \rightarrow \frac{1}{a}F\left(\frac{u}{a}\right)$

$y = ax \Rightarrow dy = a\,dx$

Case $a>0$: $x \rightarrow -\infty \Rightarrow y \rightarrow -\infty$
$x \rightarrow +\infty \Rightarrow y \rightarrow +\infty$

F

$$F = \int_{-\infty}^{+\infty} f(y)\cdot e^{\frac{-2\pi j u y}{a}}\,\frac{dy}{a}$$

$$= \frac{1}{a}\cdot\int_{-\infty}^{+\infty} f(y)e^{-2\pi j\left(\frac{u}{a}\right)\cdot y}\,dy = \frac{1}{a}F\left(\frac{u}{a}\right)$$

Case $a < 0$

$x \to -\infty \Rightarrow y \to +\infty$

$x \to +\infty \Rightarrow y \to -\infty$

$$F = \int_{+\infty}^{-\infty} f(y)\, e^{-2\pi j \frac{u}{a} y}\, \frac{dy}{a}$$

$$F = -\frac{1}{a} F\left(\frac{u}{a}\right) \Rightarrow \frac{1}{|a|} F\left(\frac{u}{a}\right)$$

4. FT of translated function

$$= \int f(x-a)\, e^{-2\pi j u x}$$

$y = x - a$

$dx = dy$

$$\int F(y) \cdot e^{-2\pi u j(y+a)} = \int \left(F(y)\, e^{-2\pi u y j}\right) e^{-2\pi u a j}$$

$$= F(y)\, e^{-2\pi u a j}$$

$$= f(x-a)\, e^{-2\pi u a j}$$

hence $F\{f(x-a)\} \to F(u)\, e^{-2\pi j u a}$

if $F\{f(x)\} = F(u)$

5. FT of modulated function $f(x) \exp(2\pi j u_0 x)$

$$FT\left\{f(x)\, e^{2\pi j u_0 x}\right\} = \int f(x)\, e^{2\pi j u_0 x}\, e^{-2\pi j u x}\, dx$$

$uv - \int v\,du$

$$= \int f(x)\, e^{-2\pi j x(u-u_0)}\, dx = F(u-u_0)$$

6. FT of the derivative function $f'(x)$

$$\int_{-\infty}^{\infty} \frac{df(x)}{du} e^{-2\pi j u x}\, dx = \left[f(x) \cdot e^{-2\pi j u x} \right]_{-\infty}^{\infty} +$$

$$2\pi j \cdot u \underbrace{\int_{-\infty}^{\infty} f(x) \cdot e^{-2\pi j u x}\, dx}_{F(u)}$$

$\int u\,dv = uv - \int v\,du$

Another way

$$\boxed{\begin{aligned} f'(t) &= \frac{d}{dt}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{i\omega t}\, d\omega\right) \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} i\omega F(\omega) e^{i\omega t}\, d\omega \\ \text{FT of } f'(t) &= i\omega F(\omega) \end{aligned}}$$

$$= \underbrace{f(-\infty) \cdot e^{\infty}}_{0} + 2\pi j u\, F(u)$$

$$= 2\pi j u\, F(u)$$

7. $$\frac{\delta F(u)}{\delta u} = \int_{-\infty}^{\infty} g(x) \cdot e^{-2\pi j u x}\, dx \Rightarrow u \text{ and } x \text{ are independent.}$$

$$F(u) = \int_{-\infty}^{\infty} g(x) \left(\int e^{-2\pi j u} \cdot du \right) dx = \int_{-\infty}^{\infty} \underbrace{\boxed{\frac{g(x)}{-2\pi j u}}}_{f(x)} e^{-2\pi j u x}\, dx$$

$$\Rightarrow g(x) = -2\pi j u\, f(x).$$

$$\Downarrow$$

$$FT\{g(x)\}\ G(u) = 2\pi j u\, F(u)$$

τ τ

Ex2: Fourier T & Conv.

1 Show FT of $f * g$ is $= F(\omega) G(\omega)$

Conv. property $f(t) * g(t) \longleftrightarrow F(\omega) G(\omega)$

let $y(t) = \int_{-\infty}^{\infty} f(\tau) g(t-\tau) \, d\tau = f(t) * g(t)$

FT of $y(t)$ and plug conv. Integral.

$$Y(\omega) = \int_{-\infty, dt}^{\infty} y(t) e^{-j\omega t} dt = \int_{-\infty, dt}^{\infty} \int_{-\infty, d\tau}^{\infty} f(\tau) g(t-\tau) d\tau \, e^{-j\omega t} dt$$

We can switch the order of two integrals and let $v = t - \tau$

$t = v + \tau$

$dt =$

$$Y(\omega) = \int_{-\infty, d\tau}^{\infty} f(\tau) \int_{-\infty, dt}^{\infty} g(t-\tau) \, e^{-j\omega t} dt \, d\tau$$

$$= \int_{-\infty, d\tau}^{\infty} f(\tau) \int_{-\infty, dv} g(v) \, e^{-j\omega(v+\tau)} dv \, d\tau$$

$$= \underbrace{\int_{-\infty, d\tau}^{\infty} f(\tau) e^{-j\omega\tau} d\tau}_{F(\omega)} \cdot \underbrace{\int_{-\infty, dv}^{\infty} g(v) \, e^{-j\omega v} dv}_{G(\omega)}$$

$\cancel{Y(\omega) = F(\omega) \cdot G(\omega)}$

$Y(\omega) = F(\omega) \cdot G(\omega)$

EX2:

$$F(u) = F\{f(x)\} = \int f(x) \exp(-2\pi j u x)\, dx.$$

$$F\{f*g\} = \iint f(t)\, g(x-t)\, dt \cdot e^{-2\pi j v y}\, dy$$

$$= \int f(t) \int g(x-t)\, e^{-2\pi j v x}\, dx\, dt \quad \text{let } u = x-t$$

$$du = dx$$

$$= \int f(t) \int g(u)\, e^{-2\pi j v (u+t)}\, du\, dt.$$

$$= \int f t\, e^{-2\pi j v t}\, dt \int g(u)\, e^{-2\pi j v u}\, du = F(v) \cdot G(v)$$

$$F\{f * g(u)\} = F\left\{\int f(t)\, g(x-t)\, dt\right\},$$

EX3: FT $F(u)$ of $f(x) = \exp(-\pi x^2)$ → use property 6 & 7

Method 1.

$du \cdot d/dx$

$F'(x) = ---$

use property 6 & 7

$F\{f'(x)\} = 2\pi j u F(u)$

$F\{-2\pi j x f(u)\} = \frac{\delta F}{\delta u}$

Method 2.

$F(u) = \int \exp(-\pi x^2) \exp(-2\pi j u x)\, dx$ → $\exp(-\pi u^2)$.

Suppose we multiply by $\exp(-\pi u^2) \exp(\pi u^2)$.

ur

$f(x) = \exp(-\pi x^2)\ F\{f(x)\} = F(u) = \exp(-\pi u^2)$.

Using method 1.

$$f(x) = e^{-\pi x^2} \Rightarrow f'(x) = -2\pi x\, e^{-\pi u^2} = -2\pi x\, f(x)$$

$$F\{f'(x)\} = F\{-2\pi x\, f(x)\}$$

$$2\pi j u \cdot F(u) = \frac{1}{j}\frac{dF(u)}{du} \Rightarrow \frac{dF(u)}{du} = -2\pi u \cdot du.$$

$$\ln(F(u)) = -\pi u^2 + c^{\nearrow 0} \Rightarrow F(u) = e^{-\pi u^2}$$

2. Using change of variable. FT. of: use

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$F\{a f(x)\} = a F(u)$$

$$F\{f(ax)\} = \frac{1}{|a|}F\left(\frac{u}{a}\right)$$

$$F\left\{\frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{-x^2}{2\sigma^2}\right)\right.$$

$$-\pi a^2 u^2 = -\frac{u^2}{2\sigma^2} \Rightarrow a = \sqrt{\frac{1}{2\pi\sigma^2}}$$

$$F\left\{\frac{1}{\sigma\sqrt{2\pi}} \cdot f(a)\right\} = \underbrace{\frac{1}{\sigma\sqrt{2\pi}}}_{a} \cdot \frac{1}{|a|}\, e^{-\pi\left(\frac{u}{a}\right)^2}$$

$$= \frac{1}{2\pi\sigma^2}\, e^{-2\pi^2\sigma^2 u^2}$$

$\Rightarrow$ Comment: It is inverse of each other $\rightarrow$ due to σ.

(*) Gaussian } important
(*) Laplacian }

EX4. 2D FT:

→ can be considered as a set of 1 D

$$F(u,v) = \iint_{x\,y} f(x,y) \exp[-2\pi i(ux + vy)]\,dx\,dy.$$

→ We can reduce this expression to a composition of two monodimensional transformations:

$$F(u,v) = \int_x \int_y f(x,y) \exp(-2\pi j u x) \exp(-2\pi j v y)\,dx\,dy.$$

$$= \int_x \underbrace{\int_y f(x,y) \exp(-2\pi j v y)\,dy}\ \exp(-2\pi j u x)\,dx$$

$$= \int_x F(x,v) \exp(-2\pi j u x)\,dx \quad = F(u)$$

EX 5. FT of a Laplacian:

$$\nabla^2 = \frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{dy^2}$$

$$f(x,y) \longrightarrow F(u,v)$$

$$F\left\{\frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2}\right\}$$

$$F(\nabla^2) = F\left(\frac{\delta^2 f}{\delta x^2}\right) + F\left(\frac{\delta^2 f}{\delta y^2}\right)$$

let $\frac{df}{dx} = g(x)$ and $\frac{df}{dy} = b(y)$

$$F\{g(x)\} = 2\pi j u F(u,v)$$

hence

$$\cancel{F(\nabla^2) = F\{g'(x)\} + F\{g'(y)\}}.$$

$$\cancel{= 2\pi j u}$$

Scanned with CamScanner

$$F\{g(x)\} = 2\pi j u\, F(u,v)$$

$$\text{hence } F(\nabla^2) = F\{g'(x)\} + F\{h'(y)\}$$

$$= 2\pi j u\, F\{g(x)\} + 2\pi j v\, F\{h(y)\}$$

$$= (2\pi j u)^2 F(u,v) + (2\pi j v)^2 F(u,v)$$

$$= -4\pi^2(u^2+v^2)\,F(u,v)$$

EX 6. DFT

$$F(k) = \sum_{0}^{3} f(n)\, e^{-2\pi j k \frac{n}{4}}$$

$$= 2\cdot e^{-2\pi j k \frac{0}{4}} + 3e^{-2\pi j k \frac{1}{4}} + 4e^{-2\pi j u \frac{2}{4}} + 4e^{-2\pi j u \frac{3}{4}}$$

$$= 2 + 3e^{-\pi j \frac{k}{2}} + 4e^{-\pi j k} + 4e^{-\frac{3\pi j k}{2}}$$

$$F(0) = 2 + 3 + 4 + 4 = 13$$

$$F(1) = 2 + 3e^{-\frac{\pi j}{2}} + 4e^{-\pi j} + 4e^{-\frac{3\pi j}{2}} = -2 + j$$

$$F(2) = 2 + 3e^{-\pi j} + 4e^{-2\pi j} + 4e^{-3\pi j} = -1$$

$$F(3) = 2 + 3e^{-\frac{3}{2}\pi j} + 4e^{-3\pi j} + 4e^{-\frac{8}{2}\pi j} = -2 - j$$

Q.

f(-1,-1)	f(-1,0)	f(-1,1)
f(0,-1)	f(0,0)	f(0,1)
f(1,-1)	f(1,0)	f(1,1)

$\longrightarrow$

	F(0,0)	

0	-1	0
-1	4	-1
0	-1	0

Laplacian is high pass filter. $\Rightarrow$ use fft2

$$f(x,y) * h(x,y) = F(u,v) \times \underset{\text{filter.}}{H(u,v)}$$

$$F(u,v) = \sum_{x}\sum_{y} f(x,y) \exp\left[-2\pi j \left(\frac{ux}{N} + \frac{vy}{N}\right)\right], \; N=3$$

$$= \sum_{x=-1}^{1}\sum_{y=-1}^{1} f(x,y) \exp\left[\frac{-2\pi j}{3}(ux+vy)\right]$$

$$F(u,v) = f(0,0)\exp\left[\frac{-2\pi j}{3}(0u+0v)\right] +$$
$$f(0,1)\exp\left[\frac{-2\pi j}{3}(0u+v)\right] +$$
$$f(1,0)\exp\left[\frac{-2\pi j}{3}(u+0v)\right] +$$
$$f(1,1)\exp\left[\frac{-2\pi j}{3}(u+v)\right] +$$
$$f(-1,0)\exp\left[\frac{-2\pi j}{3}(-u+0v)\right] +$$
$$f(-1,1)\exp\left[\frac{-2\pi j}{3}(-u+v)\right] + f(-1,-1)\exp\left[\frac{-2\pi j}{3}(-u-v)\right]$$
$$+ f(0,-1)\exp\left[\frac{-2\pi j}{3}(0u-v)\right] + f(1,-1)\exp\left[\frac{-2\pi j}{3}(u-v)\right].$$

$$F(u,v) = 4 - 1\exp\left[\frac{2\pi j v}{3}\right] - 1\exp\left[\frac{-2\pi j u}{3}\right] - 1\exp\left[\frac{+2\pi j u}{3}\right]$$
$$-1\exp\left[\frac{+2\pi j v}{3}\right].$$

$$F(0,0) = 4 - 1 - 1 - 1 - 1 = 0$$
$$F(0,1) = 4 - 1\exp\left[\frac{-2\pi j}{3}\right] - 1 - 1 - 1\exp\left[\frac{2\pi j}{3}\right] \dots \text{etc.}$$
$$F(1,0) =$$

Matlab.

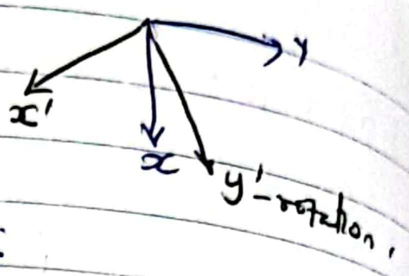
$$L = \begin{bmatrix} 0,-1,0 \\ -1,4,-1 \\ 0,-1,0 \end{bmatrix} \quad \cdot \; \text{fftshift(fft2(L))}$$

$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Ex8: Isotropy:

Suppose $x^* = x'\cos\theta - y'\sin\theta$

$y^* = x'\sin\theta + y'\cos\theta$.



$f \to$ image.

$$\underbrace{\frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2}}_{f * L} = \underbrace{\frac{\delta^2 f}{\delta x'^2} + \frac{\delta^2 f}{\delta y'^2}}_{f' * L}$$

$f' \to$ new image.

Show that ∇^2 is an isotropic operator. (inv.

$$g = \frac{\delta f}{\delta x'} = \frac{\delta f}{\delta x}\frac{\delta x}{\delta x'} + \frac{\delta f}{\delta y}\frac{\delta y}{\delta y'}$$

$$\frac{\delta f}{\delta y'} = \frac{\delta f}{\delta x}\frac{\delta x}{\delta y'} + \frac{\delta f}{\delta y}\frac{\delta y}{\delta y'}$$

Method 1

let $\cos\theta = c$

$\sin\theta = s$

first der.

$$\frac{\delta f}{\delta x'} = \frac{\delta f}{\delta x} c + \frac{\delta f}{\delta y} s$$

$$\frac{\delta f}{\delta y'} = -\frac{\delta f}{\delta x} s + \frac{\delta f}{\delta y} c$$

Second der.

$$\frac{\delta^2 f}{\delta x'^2} = \underbrace{\frac{\delta\left(\frac{\delta f}{\delta x}\right)}{\delta x'}}_{①} \cdot c + \underbrace{\frac{\delta\left(\frac{\delta f}{\delta y}\right)}{\delta x'}}_{②} \cdot s$$

① $\dfrac{\delta\left(\frac{\delta f}{\delta x}\right)}{\delta x'} =$

Method 2:

first der

$$\frac{\delta f}{\delta x'} = \frac{\delta f}{\delta x}\frac{\delta x}{\delta x'} + \frac{\delta f}{\delta y}\frac{\delta y}{\delta y'} = \frac{\delta f}{\delta x}\cos\theta + \frac{\delta f}{\delta y}\sin\theta \doteq g$$

$$\frac{\delta f}{\delta y'} = \frac{\delta f}{\delta x}\frac{\delta x}{\delta y'} + \frac{\delta f}{\delta y}\frac{\delta y}{\delta y'} = \frac{\delta f}{\delta x}(-\sin\theta) + \frac{\delta f}{\delta y}\cos\theta = h$$

second der

$$\frac{\delta g}{\delta x'} = \frac{\delta g}{\delta x}\frac{\delta x}{\delta x'} + \frac{\delta g}{\delta y}\frac{\delta y}{\delta x'} = \frac{\delta f^2}{\delta x^2}\cos\theta\cos\theta + \frac{\delta f^2}{\delta y^2}\sin\theta\sin\theta$$

$$\frac{\delta h}{\delta x'} = \frac{\delta h}{\delta x}\frac{\delta x}{\delta y'} + \frac{\delta h}{\delta y}\frac{\delta y}{\delta y'} = \frac{\delta f^2}{\delta x^2}(-\sin\theta)(-\sin\theta) + \frac{\delta f^2}{\delta y^2}\cos\theta\cos\theta$$

$$\Rightarrow \frac{\delta^2 f}{\delta x^2}\cos^2\theta + \frac{\delta^2 f}{\delta y^2}\sin^2\theta + \frac{\delta f^2}{\delta x^2}\sin^2\theta + \frac{\delta f^2}{\delta y^2}\cos^2\theta$$

$$= \frac{\delta f^2}{\delta x^2} + \frac{\delta f^2}{\delta y^2}$$

Ex 9. Average filter. → 8 pixels (excluding middle (all are 9))

1 Av. filter. $h = \frac{1}{8}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

with

$$H(u,v) = \frac{1}{8}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

to normalize

② use $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

2 If $h = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

filter

3x3 here N=3

4 - connecity.

	$f(x-1,y)$	·
$f(x,y-1)$	$f(x,y)$	$f(x,y+1)$
	$f(x+1,y)$	

→ at every pixel.

low pass → high value in origin' $e^{-2\pi j(\cdot}$
high pass → low value in middle

$\int f(t)\, h(x-t)\, dt$ → filter that we are moving with time t.

$$F\{f * h\} = F\{f(x-1,y) + f(x,y-1) + f(x,y+1) + f(x+1,y)\}.$$

$$F\{f(x,y)\} = F(u,v)$$ → use translation property (shifting & convolution)

max in middle → ~~F(u,v)~~ Recall $F\{f(x+a)\} = e^{\frac{-2\pi j a u}{N}} F(u)$

$$F(u,v) = e^{2\pi j \frac{u}{3}} F(u,v) + e^{2\pi j \frac{v}{3}} F(u,v) + e^{-2\pi j \frac{v}{3}} F(u,v) + e^{-2\pi j \frac{u}{3}} F(u,v)$$

$$= 2F(u,v)\left[\frac{e^{2\pi j \frac{u}{3}} + e^{-2\pi j \frac{u}{3}}}{2} + \frac{e^{2\pi j \frac{v}{3}} + e^{-2\pi j \frac{v}{3}}}{2}\right]$$

$$= 2F(u,v)\left[\cos\left(2\pi \frac{u}{3}\right) + \cos\left(2\pi \frac{v}{3}\right)\right]$$ max = (0,0) of (u,v) hence it is a LPF

Hence $H(u,v)$ has a maximum at $(u=0, v=0)$
h is a low pass filter.

$$L = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

EX10. → 0 in the middle
→ High pass filter

Ex 11 $\bar{f} = f * h = f(x-1,y) + f(x,y-1) + f(x,y+1) + f(x+1,y) + f(x,y)$.

$\nabla^2 = f * L = -f(x.$

Ex 11 : $\nabla^2 \simeq f - \bar{f}$

$\bar{f} = f * h = \frac{1}{5}\{f(x-1,y) + f(x,y-1) + f(x,y+1) + f(x+1,y)$
$+ f(x,y)$.

$\nabla^2 = f \times L = -f(x-1,y) - f(x,y-1) + 4f(x,y) - f(x,y+1)$
$-1 f(x+1,y)$

$f - \bar{f} = \frac{4}{5} f(x,y) - \frac{1}{5}\{f(x-1,y) + f(x,y-1) + f(x,y+1) +$
$+ f(x+1,y)\}$

$\nabla^2 = \frac{f - \bar{f}}{5} \Rightarrow \nabla^2 \simeq \frac{f - \bar{f}}{5}$

Filtering in Spacial Domain

✓ Linear filters – convolution.

✓ Non-linear filters

Properties of filters

- Symmetry: Center
- Separability
- Cascading

LPF

⊛ Average filtering $\xrightarrow{}$ rectangular. → reduces noise

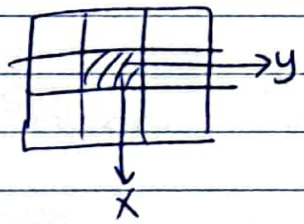
$$\underbrace{\frac{1}{9}}_{\text{Normalize}}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

→ Not good like Gaussian.

→ Cannot remove high frequencies coz of sinc f

More size of Av. filter → more blur

⊛ Gaussian filter



- replace (x,y) in G
- Choose the right value of σ
- Big value of σ & filter has to be big also & vice versa (pg 12)
- Normalize - divide by minimum value to have integral values in the filter

⊛ Binomial filters - Approximation of Gaussian - Preserve edges compared to average.

→ It is integral (normalized).

1D B.F. → Convolve several times same filter.

2D B.F. → use separability property of filters.

1 2 1 * $\begin{smallmatrix}1\\2\\1\end{smallmatrix}$

Sinc

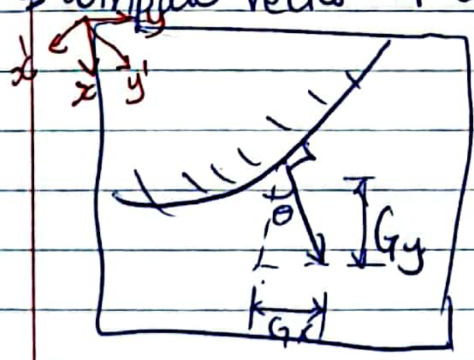
④ Nagao Filter
- Where the average value doesn't make sense like if the Variance is big. Choose possibility of low Variance i.e. average with low variance.

HPF.

* 1D
→ max 1st deri → point of transition.

Crosses zero at second derivative.

* 2D (Gradient) → first derivative
→ Compute vector $\binom{G_x}{G_y}$ at each pixel < Orientation → arctg. / Height (Length of vector) → Norm



$$\vec{G}_{(x,y)} \begin{cases} G_x = \frac{\delta f}{\delta x} = f * \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ G_y = \frac{\delta f}{\delta y} = f * [-1, 1] \end{cases}$$

Norm $|\vec{G}| = \sqrt{G_x^2 + G_y^2}$ > threshold — high — there is variation

→ symmetry on conv

$$\frac{\delta f}{\delta y} = (f(x, y+1) - f(x,y)) = f * [-1, 1]$$

− min	0	max
0	128	255
black	grey	white

$$\frac{\delta f}{\delta x} = f * \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

④ Robert Kernels — variant of Gradients
— We have less noise.

④ Prewitt Kernels → Average filter + first derivative.
Smoothing + derivation

④ Sobel

Hirsch Kernels

- choose the filter that gives us maximum value of Convolution & is considered as norm of gradient. and I will know the orientation in the filter of

1st order

① $G_x = f * \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$G_y = f * [-1, 1]$

$\vec{G} = \sqrt{G_x^2 + G_y^2} \rightarrow$ threshold (high value)

$\theta = \text{Arctg} \frac{G_y}{G_x}$

② $f * \vec{K_i} = G_i \rightarrow$ image ($K_1, K_2, \ldots$)

edge = Norm $\vec{G} = \max\{G_i\}$ & threshold. → highest max value.

zero crossings

orientation $\theta = i_{max}$.

Laplacian

Second derivative.

pg 32

1st image → Btwn positive & -ve value → zero crossings (white marks)

2nd image → Combine with smoothing filter (LPF Gaussian or average)

LOG → 2nd der. of Gaussian.

LOG(x, y) sum of second derivatives → mexican hat.

DOG → Subtracting two Gaussians we get LOG

2nd order

→ $f * L \rightarrow$ Zero crossings (but noisy).

$f * \underbrace{G_\sigma * L}_{\text{LOG}} \rightarrow$ Zero crossings

same; at end shape
$\sim$ same

$= f * G_{\sigma_1} - f * G_{\sigma_2} = f * \underbrace{(G_{\sigma_1} - G_{\sigma_2})}_{\text{DOG}} \rightarrow$ Zero crossings

→ easier

Non-linear Filter

→ Non-linear fnctn of the neighbours
- neighbourd value.
- Size.

(*) Median Filter. (m.F)
- median value of neighbourhood.
- remove impulse noise.
- We can't change relevant value of image unlike in average filter.

→ Increase size of (m.F.) → less noise.

Scanned with CamScanner

TD2: Filtering in Spatial Domain.

EX1: ① Calculate & name the foll. filters

$$\underbrace{\overset{f}{[1 \;\; 1]}}_{} * \overset{g}{[1 \;\; 1]} \qquad f*g = \sum_t f(t)\, g(x-t)$$

Average 1X2.

$f \qquad [1 \;\; 1] \qquad = [1 \;\; 2 \;\; 1]$

$g \quad [1 \;\; 1] \longrightarrow 1$

$[1 \;\; 1] \longrightarrow 2$

$[1 \;\; 1] \longrightarrow 1$

binomial 1X3

Low pass

Sum diff than 0 = low pass

Otherwise HPF.

② $\underset{Av.}{[1 \;\; 1]} * \underset{\frac{\delta f}{\delta y}}{[-1 \;\; 1]}$

$[1 \;\; 1] \qquad = [-1 \;\; 0 \;\; 1].$

$[1 \;\; -1] \longrightarrow -1$

$[1 \;\; -1] \longrightarrow 0$

$[1 \;\; -1] \longrightarrow 1$

Differential 1X3.

High pass

③ $\overset{\frac{\delta f}{\delta y}}{[1 \;\; -1]} * \overset{\frac{\delta f}{\delta y}}{[1 \;\; -1]}.$

$[1 \;\; -1] \qquad = [1 \;\; -2 \;\; 1]$

$[-1 \;\; 1] \longrightarrow 1$

$[-1 \;\; 1] \longrightarrow -2$

$[-1 \;\; 1] \longrightarrow 1$

2nd derivative $\left(\frac{\delta^2 f}{\delta y^2}\right)$

High pass.

$$\begin{array}{c} \\ 0 \\ -1 \\ \end{array}\begin{array}{c} f \\ x\ \ x\ \ x \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \\ x\ \ x\ \ x \end{array} * \begin{array}{c} g \\ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \end{array} + \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$$

4. $$\begin{array}{c} 1 \\ 0 \\ -1 \end{array}\begin{array}{c} x\ \ x\ \ x \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \\ x\ \ x\ \ x \end{array} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Av 1x3.

* ~~average 2x1~~
* $\frac{\delta f}{\delta x}$ 2x1

$\frac{\delta^2 f}{\delta y^2}$.

av 1x3 * av 2x1 * $\frac{\delta f}{\delta x} + \frac{\delta^2 f}{\delta y^2} \simeq \frac{\delta f}{\delta x} + \frac{\delta^2 f}{\delta y^2}$ → detect direction in both direction (x, y).

$-1\ 0\ 1 \rightarrow \begin{matrix} -1 \\ 0 \\ 1 \end{matrix}$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

* Inverse
* put centre overlap
* $(2m-1) =$ size

5. $$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 2 & 0 \\ 1 & 2 & 5 & 2 & 1 \\ 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Avg Avg = Binomial (low pass)

6. $$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \circledast \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Derivative Average Derivative (High pass).

7. $$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 & 1 & 1 \end{bmatrix}.$$

Av 1x2 $\circledast$ $\frac{df}{dy}$ av 1x3 high pass

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$