Solution to Assignment #2

#1

Here: $f(x) = e^{+x} + e^{-x}$

(a) Taylor expansion of $e^{\pm x} = 1 \pm x + \frac{x^2}{2!} \pm \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

$\therefore \boxed{f(x) = 2\left(1 + \frac{x^2}{2} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots\right)}$ 4

(b) Comparing to: $P_4(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$, we find: $\boxed{a_0 = 2;\ a_1 = 0;\ a_2 = 1;\ a_3 = 0;\ a_4 = \frac{1}{12}}$ 4

(c) $f(0.1) = e^{0.1} + e^{-0.1} \Rightarrow \boxed{f(0.1) = \cancel{2.10}\ 2.010008}$

and $P_4(0.1) = 2 + 1(0.1)^2 + \frac{1}{12}(0.1)^4 \Rightarrow \boxed{P_4(0.1) = 2.010008}$

(d) $\boxed{\%\ \text{Error} = 0\ \%}$ (within 7 sig fig). 4

#2

Here $x_0 = -1$; $x_1 = 0$ & $x_1 = 1 \Rightarrow$ So V is a 3×3 matrix

(a) $V = \begin{pmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \end{pmatrix} \Rightarrow \boxed{V = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}}$ 4

(b) $\det V = 1(0-0) - (-1)(1-0) + 1(1-0) \Rightarrow \boxed{\det V = 2}$ 4

(c) $V^{-1} = \frac{1}{\det V}(\text{Adj } V)^{T}$; using MAPLE to compute the inverse of V, we find.

$$\boxed{V^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \end{pmatrix}}$$ 4

(d) $\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = V^{-1} \begin{pmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \end{pmatrix}$

Here $f(x_0) = e^{-1} + e^{1} = 3.08616$

$f(x_1) = e^{0} + e^{-0} = 2$

& $f(x_2) = e^{+1} + e^{-1} = 3.08616$.

$$= \begin{pmatrix} 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 3.08616 \\ 2 \\ 3.08616 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1.08616 \end{pmatrix}$$

Comparing $\Rightarrow$ $\boxed{a_0 = 2 \ ; a_1 = 1 \text{ and } a_2 = 1.08616}$

Therefore: $\boxed{P_2(x) = 1.08616\, x^2 + 2}$

(e) $P_2(0.1) = (1.08616)(0.1)^2 + 2 \Rightarrow \boxed{P_2(0.1) = 2.01086.}$

& $f(0.1) = e^{0.1} + e^{-0.1} \Rightarrow \boxed{f(0.1) = \cancel{2.01000}\ 2.01001}$

(f) $\% \text{ Error} = \dfrac{|P_2(0.1) - f(0.1)|}{f(0.1)} \times 100\%$

$$= \frac{|2.01086 - 2.0100|}{2.01001} \times 100\%$$

$\Rightarrow \boxed{\% \text{ Error} \approx 0.04229\%}$