Assignment

Module-9

Problem 1

1. $P_2(x) = a_0 + a_1 x + a_2 x^2$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 4 & 16 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad b = \begin{pmatrix} 3 \\ -2 \\ 2 \\ 1 \end{pmatrix}$$

2. $$A^T A = \begin{pmatrix} 4 & 4 & 18 \\ 4 & 18 & 64 \\ 18 & 64 & 258 \end{pmatrix} \qquad A^T b = \begin{pmatrix} 4 \\ -9 \\ -29 \end{pmatrix}$$

3. $$(A^T A)^{-1} = \frac{1}{362} \begin{pmatrix} 137 & 30 & -17 \\ 30 & 177 & -46 \\ -17 & -46 & 14 \end{pmatrix}$$

$$(A^T A)^{-1}(A^T b) = \frac{1}{362} \begin{pmatrix} 771 \\ -139 \\ -60 \end{pmatrix}$$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \frac{1}{362} \begin{pmatrix} 771 \\ -139 \\ -60 \end{pmatrix} = \begin{pmatrix} 2.129 \\ -0.3839 \\ 0.1657 \end{pmatrix}$$

$$P_2(x) = 2.129 - 0.3839x - 0.1657x^2$$

Problem-2

1 $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 4 & 16 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $u_2 = \begin{pmatrix} 0 \\ 4 \\ -1 \\ 1 \end{pmatrix}$ $u_3 = \begin{pmatrix} 0 \\ 16 \\ 1 \\ 1 \end{pmatrix}$

2 $p_1 = u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $|p_1| = 2$

$q_1 = \frac{p_1}{|p_1|} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

$p_2 = u_2 - (u_2^T q_1) q_1$

$(u_2^T q_1) = \frac{1}{2}(0 \quad 4 \quad -1 \quad 1)\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 2$

$p_2 = \begin{pmatrix} 0 \\ 4 \\ -1 \\ 1 \end{pmatrix} - 2 \cdot \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix}$

$|p_2| = \sqrt{1+9+4} = \sqrt{14}$

$q_2 = \frac{p_2}{|p_2|} = \frac{1}{\sqrt{14}}\begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix}$

$$p_3 = u_3 - \left((u_3^T q_1) q_1 + (u_3^T q_2) q_2 \right)$$

$$(u_3^T q_1) = \frac{1}{2} \begin{pmatrix} 0 & 16 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \frac{18}{2} = 9$$

$$u_3^T q_2 = \frac{1}{\sqrt{14}} \begin{pmatrix} 0 & 16 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix} = \frac{46}{\sqrt{14}}$$

$$(u_3^T q_1) q_1 = \frac{9}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \qquad (u_3^T q_2) q_2 = \frac{46}{14} \begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix}$$

$$p_3 = \begin{pmatrix} 0 \\ 16 \\ 1 \\ 1 \end{pmatrix} - \frac{9}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \frac{23}{7} \begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} -17 \\ 23 \\ 43 \\ -49 \end{pmatrix}$$

$$q_3 = \frac{p_3}{|p_3|} \qquad |p_3| = 5.08$$

$$q_3 = \begin{pmatrix} -0.239 \\ 0.323 \\ 0.604 \\ -0.688 \end{pmatrix}$$

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$$R = \begin{pmatrix} u_1^T q_1 & u_2^T q_1 & u_3^T q_1 \\ 0 & u_2^T q_2 & u_3^T q_2 \\ 0 & 0 & u_3^T q_3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 9 \\ 0 & 3.74 & 12.3 \\ 0 & 0 & 5.08 \end{pmatrix}$$

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$$Rx = \begin{pmatrix} 2 & 2 & 9 \\ 0 & 3.74 & 12.3 \\ 0 & 0 & 5.08 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

$$= \begin{pmatrix} 2a_0 + 2a_1 + 9a_2 \\ 3.74 a_1 + 12.3 a_2 \\ 5.08 a_2 \end{pmatrix}$$

$$Q^T b = \begin{pmatrix} 2 \\ -3.475 \\ -0.843 \end{pmatrix} \quad \therefore$$

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$a_0 = 2.129$

$a_1 = -0.3833$

$a_2 = -0.1659$

$P_2(x) = 2.129 - 0.3833x - 0.1659x^2$