Module 10

Problem-1

$a=1,\ b=3,\ f(x)=\ln x$

1 $m=2.\quad h=\dfrac{b-a}{m}=\dfrac{3-1}{2}=1$

$x_0=1,\ x_1=1+1=2,\ x_2=2+1=3$

$$C_{1,2}=\frac{h}{2}\left[f(x_0)+2f(x_1)+f(x_2)\right]$$

$$=\frac{1}{2}\left[\ln 1+2\ln 2+\ln 3\right]=1.2424$$

1 $m=3,\quad h=\dfrac{3-1}{3}=\dfrac{2}{3}.$

$x_0=1,\ x_1=5/3,\ x_2=7/3,\ x_3=3$

$$C_{1,3}=\frac{2/3}{2}\left[\ln 1+2\ln 5/3+2\ln 7/3+\ln 3\right]=1.2716$$

3 $m=4,\quad h=\dfrac{3-1}{4}=\dfrac{1}{2}$

$x_0=1,\ x_1=3/2,\ x_2=2,\ x_3=5/2,\ x_4=3$

$$C_{1,4}=\frac{1/2}{2}\left[\ln 1+2\ln 3/2+2\ln 2+2\ln 5/2+\ln 3\right]$$

$$=1.2821$$

2 $\displaystyle\int_1^3 \ln x\,dx=\left[x\ln x-x\right]_1^3=3\ln 3-3+1=1.2958$

m	error	
2	0.053	~~0.0288~~
3	0.0242	~~0.0105~~
4	0.0137	

Problem·2

1. $h = \frac{3-1}{2} = 1$, $x_0 = 1, x_1 = 2, x_2 = 3$

2. $l_0 = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} = \frac{x-2}{1-2} \quad \frac{x-3}{1-3} = \frac{(x^2 - 5x + 6)}{2}$

$l_1 = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} = \frac{x-1}{2-1} \cdot \frac{x-3}{2-3} = \frac{x^2 - 4x + 3}{-1}$

$l_2 = \frac{x - x_1}{x_2 - x_1} \cdot \frac{x - x_0}{x_2 - x_0} = \frac{(x-1)(x-2)}{1 \cdot 2} = \frac{x^2 - 3x + 2}{2}$

3. $\sigma_0 = \int_1^3 l_0(x)\, dx = \frac{1}{3}$

$\sigma_1 = \int_1^3 l_1(x)\, dx = 4/3$

$\sigma_2 = \int_1^3 l_2(x)\, dx = \frac{1}{3}$

4. $I_2(f) = \sigma_0 f(x_0) + \sigma_1 f(x_1) + \sigma_2 f(x_2)$

$= \frac{1}{3} \ln 1 + \frac{4}{3} \ln 2 + \frac{1}{3} \ln 3$

$= 1.2904$

5. $I = 1.2958$

$\left| \frac{I - I_2}{I} \right| \times 100\% = \left| \frac{1.2958 - 1.2904}{1.2958} \right| \times 100\%$

$= 0.4167\%$

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