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section : 09

Assignment 4

Given data,

	x	$f(x)$	$f'(x)$
$x_0 =$	-1	0	1
$x_1 =$	0	1	0
$x_2 =$	1	0	1

$$\left[\begin{array}{l} \text{node} = 3 \\ \therefore n = (3-1) = 2 \end{array}\right]$$

we know,

$$P_{2n+1}(x) = f(x_0)(h_0(x)) + f(x_1)h_1(x) \dots f(x_n)\,h_n(x) + f'(x_0)\hat{h}_0(x)$$
$$+ f'(x_1)\,\hat{h}_1(x) \dots\dots f'(x_n)\,\hat{h}_n(x)$$

$$\therefore P_{2.2+1} = P_5(x) = h_0(x)\,f(x_0) + h_1(x)f(x_1) + h_2(x)\,f(x_2)$$
$$+ \hat{h}_0\,f'(x_0) + \hat{h}_1(x)\,f'(x_1) + \hat{h}_2(x_2)\,f'(x_2)$$
$$= h_0 \times 0 + h_1(x) \times 1 + h_2(x) \times 0 + \hat{h}_0 \times 1 +$$
$$\hat{h}_1(x) \times 0 + \hat{h}_2(x) \times 1$$
$$= h_1(x) + \hat{h}_0(x) + \hat{h}_2(x)$$

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Lagrange basis,

$$l_0 = \frac{(x-x_1)}{(x_0-x_1)} \frac{(x-x_2)}{(x_0-x_2)}$$

$$= \frac{x-0}{-1-0} \times \frac{x-1}{-1-1} = \frac{-x(x-1)}{-2}$$

$$l_0(x) = \frac{1}{2}(x^2-x)$$

$$\therefore l_0'(x) = \frac{2x-1}{2}$$

$$l_1 = \frac{(x-x_0)}{(x_1-x_0)} \frac{(x-x_2)}{(x_1-x_2)}$$

$$= \frac{x+1}{1} \times \frac{x-1}{-1} \Rightarrow -(x^2-1) \Rightarrow (1-x^2)$$

$$\therefore l_1'(x) = -2x$$

$$l_2 = \frac{(x-x_0)}{(x_2-x_0)} \frac{(x-x_1)}{(x_2-x_1)}$$

$$= \frac{x+1}{2} \times \frac{x}{1} \Rightarrow \frac{x^2+x}{2}$$

$$\therefore l_2'(x) = \frac{2x+1}{2}$$

$\underline{2}$

we know,

$\cancel{h_k(x) = \{1 - 2(x - x_k)\}(l_k'(x))^2}$

$$h_k(x) = \{1 - 2(x - x_k)\, l_k'(x_k)\}\{l_k(x)\}^2$$

$$\therefore h_0(x) = \{1 - 2(x - x_0) \times l_0'(x_0)\}\{l_0(x)\}^2$$

$$= \left\{1 - 2(x+1)\,\frac{2x_0 - 1}{2}\right\}\left\{\frac{1}{2}(x^2 - x)\right\}^2$$

$$= \{1 - (x+1)(2x_0 - 1)\}\,\frac{x^2(x-1)^2}{4}$$

$$\cancel{= \{1 - 2x^2 + x - 2x + 1\}\left(\frac{x^4 - 2x^3 + x^2}{4}\right)}$$

$$\cancel{= (2 - 2x^2 - x)\left(\frac{x^4 - 2x^3 + x^2}{4}\right)}$$

$\cancel{h_1(x) = \{1 - 2(x - x_1)\, l_1'(x}$

$$= \{1 + 3x + 3\}\left\{\frac{x^4 - 2x^3 + x^2}{4}\right\}$$

$[2x_0 - 1 = 2x - 1 - 1 = -3]$

$$= (4 + 3x)\left(\frac{x^4 - 2x^3 + x^2}{4}\right)$$

$$h_1(x) = \{1 - 2(x - x_1)\, l'_1(x_1)\}\, \{l_1(x)\}^2$$

$$= \{1 - 2(x-0) \times -2x_1\}\, (1-x^2)^2 \quad [x_1 = 0]$$

$$= \cancel{\{1 - 2x\}}\, (1 - 2x^2 + x^4)$$

$$h_2(x) = \{1 - 2(x - x_2)\, l'_2(x_2)\}\, \{l_2(x)\}^2$$

$$= \left\{1 - 2(x-1)\frac{2x_2+1}{2}\right\} \left\{\frac{x^2+x}{2}\right\}^2$$

$$= (4 - 3x)\left(\frac{x^4 + 2x^3 + x^2}{4}\right)$$

Now,

we know, $\hat{h}(x) = (x - x_k)\{l_k(x)\}^2$

$$\therefore \hat{h}_0(x) = (x - x_0)\{l_0(x)\}^2$$

$$= (x+1)\left(\frac{x^2-x}{2}\right)^2$$

$$= (x+1)\left(\frac{x^4 - 2x^3 + x^2}{4}\right)$$

$$\hat{h}_1(x) = (x - x_1)\{l_1(x)\}^2$$

$$= x \cdot (1-x^2)^2$$

$$= x - 2x^3 + x^5$$

$$\hat{h}_2(x) = (x - x_2)\{l_2(x)\}^2$$

$$= (x-1)\left\{\frac{x^2+x}{2}\right\}^2$$

$$= (x-1)\left(\frac{x^4+2x^3+x^2}{4}\right)$$

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From 2 we get,

$$h_1(x) = \cancel{(1-2x)}(1-2x^2+x^4)$$

$$\hat{h}_0(x) = (x+1)\left(\frac{x^4-2x^3+x^2}{4}\right)$$

$$\hat{h}_2(x) = (x-1)\left(\frac{x^4+2x^3+x^2}{4}\right)$$

So we get $P_5(x) = h_1(x) + \hat{h}_0(x) + \hat{h}_2(x)$

$$= \cancel{(1-2x)}(1-2x^2+x^4) + (x+1)\left(\frac{x^4-2x^3+x^2}{4}\right)$$

$$+ (x-1)\left(\frac{x^4+2x^3+x^2}{4}\right)$$

$$= \left(1 - 2x^2 + x^4\cancel{-2x+4x^3-2x^5}\right) + \left(\frac{x^5}{4} - \frac{x^4}{2} + \frac{x^3}{4} + \frac{x^4}{4}\right.$$

$$\left.- \frac{x^3}{2} + \frac{x^2}{4}\right) + \left(\frac{1}{4}x^2 + \frac{x^5}{4} + \frac{x^3}{4} + \frac{x^4}{2} - \frac{x^4}{4} - \frac{x^3}{2} - \frac{x^2}{4}\right)$$

$$= \frac{1}{2}x^5 + x^4 - \frac{1}{2}x^3 - 2x^2 + 1$$

(Ans)

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from 3 we get,

$$P_5(x) = -\frac{3x^5}{2} + x^4 + \frac{7}{2}x^3 - 2x^2 - 2x + 1$$

$$P_5(0.5) = -\frac{3\cdot(0.5)^5}{2} + (0.5)^4 + \frac{7}{2}(0.5)^3 - 2\cdot(0.5)^2 - 2\times 0.5 + 1$$

$$= -0.0468$$

(Ans)

4

from 3 we get,

$$P_5(x) = \frac{1}{2}x^5 + x^4 - \frac{1}{2}x^3 - 2x^2 + 1$$

$$\therefore P_5(0.5) = \frac{1}{2}(0.5)^5 + (0.5)^4 - \frac{1}{2}(0.5)^3 - 2(0.5)^2 + 1$$

$$= 0.5156$$

(Ans)