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section : 09

Assignment 5

Answer 1

1.

here given, $f(x) = 5e^{-2x}$, $h = 0.32$

we know, CD : $f'(x) = \dfrac{f(x+h) - f(x-h)}{2h}$

$$\therefore f(0.4) = \frac{f(0.4+0.32) - f(0.4-0.32)}{2 \times 0.32}$$

$$= \frac{f(0.72) - f(0.08)}{0.64}$$

$$= \frac{1.18463 - 4.26071}{0.64}$$

$$= -4.80637$$

2

Now, $h = 0.16$

$$\therefore f'(0.4) = \frac{f(x+h) - f(x-h)}{2 \cdot h}$$

$$= \frac{f(0.56) - f(0.24)}{0.32}$$

$$= \frac{1.63139 - 3.09391}{0.32}$$

$$= -4.57037$$

3

from 2 we get, $D(h) = -4.80637$

$D(h/2) = -4.57087$

we know, $D^{(1)}_{0.32} = \dfrac{2^2 \cdot D(h/2) - D(h)}{2^2 - 1}$

$$= \frac{(4 \times -4.57087) - (-4.80637)}{3}$$

$$= -4.49170$$

4

Given the exact value $f'(0.4) = -4.4933$

$\therefore$ Percentage error ; $\left| \frac{-4.4933 - D'_{0.32}}{-4.4933} \right| \times 100\%$

$= \left| \frac{-4.4933 + 4.4917}{-4.4933} \right| \times 100\%$

$= 0.03560\ \%$

Answer 2

1

we know, $D_{n}' = \frac{f(x_1+h) - f(x_1-h)}{2h}$

$$f(x_1+h) = f(x_1) + f'(x_1)h + \frac{f^2(x_1)h^2}{2!} + \frac{f^3(x_1)h^3}{3!} + \frac{f^4(x_1)h^4}{4!}$$
$$+ \frac{f^5(x_1)h^5}{5!} + \frac{f^6(x_1)h^6}{6!} + \frac{f^7(x_1)h^7}{7!} + O(h^8)$$

$$\therefore f(x_1-h) = f(x_1) - f'(x_1)h + \frac{f^2(x_1)h^2}{2!} - \frac{f^3(x_1)h^3}{3!} + \frac{f^4(x_1)h^4}{4!}$$
$$- \frac{f^5(x_1)h^5}{5!} + \frac{f^6(x_1)h^6}{6!} - \frac{f^7(x_1)h^7}{7!} + O(h^8)$$

$$\therefore f(x_1+h) - f(x_1-h) = 2f'(x_1)h + \frac{2f^3(x_1)h^3}{3!} + \frac{2f^5(x_1)h^5}{5!} + \frac{2f^7(x_1)h^7}{7!} + O(h^9)$$

$$\therefore \frac{f(x_1+h) - f(x_1-h)}{2h} = f'(x_1) + \frac{f^3(x_1)h^2}{3!} + \frac{f^5(x_1)h^4}{5!} + \frac{f^7(x_1)h^6}{7!} + O(h^8)$$

$$\therefore D'_h = f'(x_1) + \frac{f^3(x_1)h^2}{3!} + \frac{f^5(x_1)h^4}{5!} + \frac{f^7(x_1)h^6}{7!} + O(h^8)$$

$$\therefore D'_{\frac{h}{2}} = f'(x_1) + \frac{f^3(x_1)\left(\frac{h}{2}\right)^2}{3!} + \frac{f^5(x_1)\left(\frac{h}{2}\right)^4}{5!} + \frac{f^7(x_1)\left(\frac{h}{2}\right)^6}{7!} + O(h^8)$$

2

from 1 we get

$$D^1_{\frac{h}{2}} = f^1(x_1) + \frac{f^3(x_1)\left(\frac{h}{2}\right)^2}{3!} + \frac{f^5(x_1)\left(\frac{h}{2}\right)^4}{5!} + \frac{f^7(x_1)\left(\frac{h}{2}\right)^6}{7!} + O(8)$$

$$\therefore 2^4 D^1_{\frac{h}{2}} = 2^4 f^1(x_1) + \frac{2^2 f^3(x_1) h^2}{3!} + \frac{f^5(x_1) h^4}{5!}$$

$$+ \frac{1}{2^2}\frac{f^7(x_1) h^6}{7!} + O(8)$$

$$\therefore 2^4 D^1_{\frac{h}{2}} - D^1_h = (2^4-1) f^1(x_1) + (2^2-1)\frac{f^3(x_1)h^2}{3!}$$

$$+ \left(\frac{1}{2^2}-1\right)\frac{f^7(x_1)h^6}{7!} + O(8)$$

$$\therefore \frac{2^4 D^1_{\frac{h}{2}} - D^1_h}{2^4-1} = f^1(x) + \frac{(2^2-1)}{(2^4-1)\,3!} f^3(x_1)h^2$$

$$+ \frac{\left(\frac{1}{2^2}-1\right)}{(2^4-1)\,7!} f^7(x_1)h^6 + O(8)$$

Given $D^2_{(h)} = \dfrac{2^4 D'_{h/2} - D'_h}{2^4 - 1}$

$\therefore D^2_h = f'(x) + \dfrac{1}{30} f^3(x_1)h^2 - \dfrac{1}{20 \times 7!} f^7(x_1)h^6$

$+ O(8)$

(Ans)