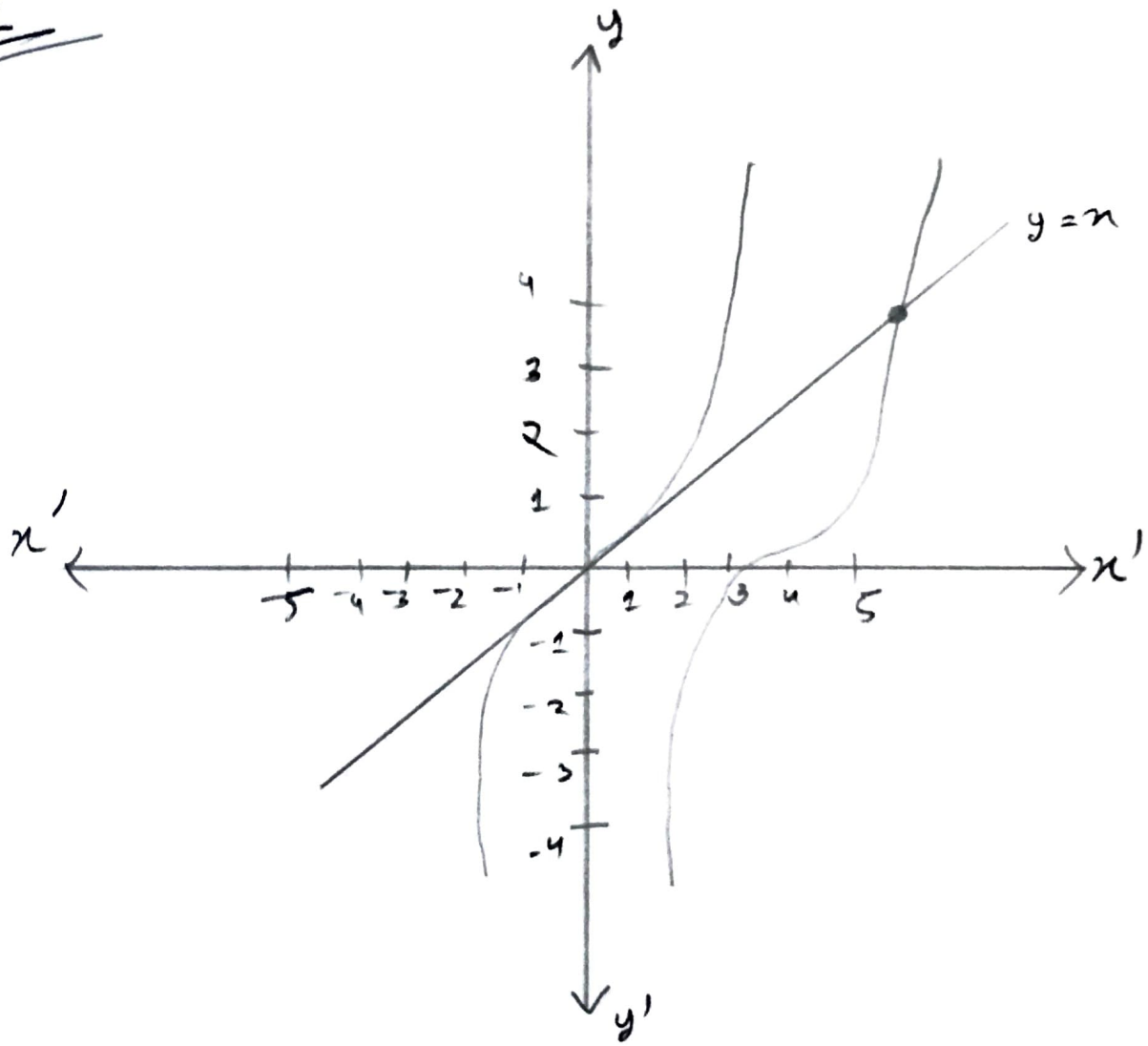
Name : Akib Zabed Ifti ID : 20101113

Section : 09

330 Assignment 6

Problem 1

1



the interval (4,5) contains the intersection of $y = x$ and $y = \tan x$

2. Let $f(x) = x - \tan x$

the interval we get from a is $[4, 5]$

$\therefore x_l = 4$

$x_u = 5$

$x_m = \frac{4+5}{2} = 4.5$

1st iteration,

$f(4) = 2.842$

$f(5) = ~~7.380~~ \; 8.380$

$f(4.5) = ~~-0.637~~ ~~+1.373~~ \; -0.137$

$\therefore f(x_l) \times f(x_m) = f(4) \times f(4.5) = -0.390 < 0$

$\therefore x_u = x_l$

$x_u = x_m = x_u = 4.5$

2nd iteration,

$x_m = \frac{4+4.5}{2} = 4.25$

$f(x_l) = f(4) = 2.842$

$f(x_u) = f(4.5) = ~~-0.637~~ \; 0.137$

$f(x_m) = f(4.25) = 2.243$

$\therefore f(x_l) \times f(x_m) = f(4) \times f(4.25) = 6.376 > 0$

$\therefore x_l = x_m = 4.25$

$x_u = 4.5$

3rd iteration

$f(x_l) = f(4.25) = 2.243$

$x_m = \frac{4.25+4.5}{2}$

$= 4.375$

$f(x_m) = f(4.375) = 1.524$

$\therefore f(x_l) \times f(x_m) = 2.43 \times 1.524 = 3.419 > 0$

$\therefore x_l = x_m = 4.375$

$x_h = 4.5$

4th iteration,

$f(x_l) = f(4.375) = 1.524$

$f(x_m) = f(4.4375) = 0.891$

$x_m = \frac{4.375+4.5}{2}$

$= 4.4375$

$\therefore f(x_l) \times f(x_m) = 1.524 \times 0.891$

$= 1.359 > 0$

$\therefore x_l = x_m = 4.4375$

$x_h = 4.5$

5th iteration

$\therefore x_l = f(4.4375) = 0.891$

$x_m = f(4.6875) = 0.445$

$x_m = \frac{4.5+4.4375}{2}$

$= 4.46875$

$\therefore x_l \times x_5 > 0$ [positive]

$\therefore x_l = x_m = 4.46875$

$x_u = 4.5$

6th iteration,

$\therefore x_l = f(4.46875) = 0.445$

$x_m = f(4.4843) = 0.174$

$x_m = \frac{4.5 + 4.46875}{2}$

$= 4.484375$

$\therefore x_l \times x_m > 0$

$\therefore x_l = x_m = 4.484375$

$x_u = 4.5$

7th iterations,

$x_m = 4.49.21$

$x_l = f(4.484375) = 0.175$

$x_m = f(4.4921) = 0.0245$

$\therefore x_l \times x_m > 0$

$\therefore x_l = x_5 = 4.4921$

$x_u = 4.5$

8th iteration

$x_m = \frac{4.49214.5}{2}$

$= 4.49605$

$x_l = f(4.4921) = 0.0245$

$x_m = f(4.49605) = -0.053$

$\therefore x_l \times x_5 < 0$

$\therefore x_l = 4.4921$

$x_U = x_m = 4.49605$

9th iteration

$x_m = \frac{4.49605 + 4.4921}{2}$

$= 4.494075$

$\therefore x_l = f(4.4921) = 0.0245$

$x_m = f(4.494075) = -0.013$

$\therefore x_l \times x_m < 0$

$\therefore x_l = x_l = 4.4921$

$x_U = x_m = 4.494075$

10th iteration,

$x_m = \frac{4.494075 + 4.4921}{2}$

$= 4.49308$

$\therefore x_l = f(4.4921) = 0.0245$

$x_m = f(4.49308) = 6.490 \times 10^{-3}$

$\therefore x_l \times x_m > 0$

∴ So $x_l = x_m = 4.49308$

and $x_u = 4.4934075$

Problem B

1 Given $f(x) = x - \tan x$

we have to construct a $g(x)$ for the $f(x)$ so that $g(x) \in I$ for $x \in I$

$\therefore \quad x - \tan x = 0$

$\Rightarrow x = \tan x$

$\Rightarrow \frac{1}{x} = \frac{1}{\tan x}$

$\Rightarrow x + \frac{1}{x} = \frac{1}{\tan x} + x$ [add x both sides]

$\Rightarrow x = \frac{1}{\tan x} + x - \frac{1}{x}$

$\Rightarrow g(x) = \frac{1}{\tan x} + x - \frac{1}{x}$

Now if we put the ~~th~~ value between $[4, 5)$ in the above $g(x)$ then we will have a value between $[4,5]$

for example

$$\frac{1}{\tan(4)} - \frac{1}{4} + 4 = 4.613$$

$$\frac{1}{\tan(5)} - \frac{1}{5} + 5 = 4.304$$

2

from the previous part we get

$$g(x) = \frac{1}{\tan(x)} - \frac{1}{x} + x$$

Now lets use fixed point iteration and lets assume $x = 4$

$$\therefore g(4) = \frac{1}{\tan(4)} - \frac{1}{4} + x$$

x_0	x_1	difference
$g(4) = \frac{1}{\tan(4)} - \frac{1}{4} + 4$	4.613	19.34228
$g(4.613)$	4.4956	2.55168
$g(4.4956)$	4.493411	0.05680
$g(4.49341)$	4.4934369	0.000032

As $0.000032 < 10^{-4}$ So Now we can stop our iteration

(Ans)