1. Use Newton's method to find all the roots of $2-x^2=\sin(x)$ accurate to six decimal places.

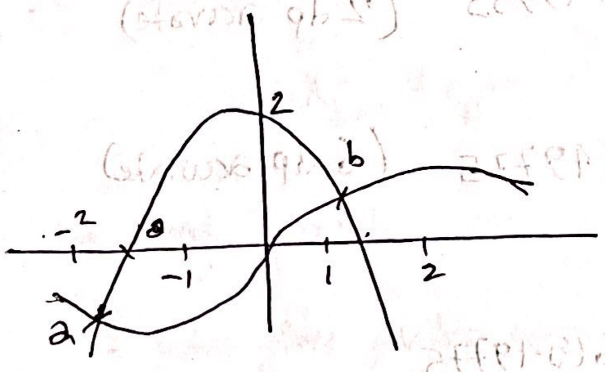
$$2-x^2=\sin x$$

$$\Rightarrow f(x)=0$$

$$\Rightarrow f(x)=2-x^2-\sin x \quad , f'(x)=-2x-\cos x$$

⊛ 2 solutions/roots as intersects at two points

⊛ $\sin x$ will not intersect with $2-x^2$ at more than 2 points since the quadratic equation is going down ~~& will not intersect with the trigonometric function~~




2 points of intersection,

Leftmost side,

let $x_0=-1.5$

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

$$x_1=-1.5-\frac{f(-1.5)}{f'(-1.5)}=-1.755181948$$

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}=-1.728754674 \text{ (1 dp accurate agree)}$$

$$x_3=x_2-\frac{f(x_2)}{f'(x_2)}=\cancel{-1.72}\ -1.728466353 \text{ (3 dp accurate)}$$

$$x_4=x_3-\frac{f(x_3)}{f'(x_3)}=-1.728466319 \text{ (7 dp accurate)}$$

left most root, $x \approx -1.728466319$

now,

Rightmost root,

let $x_0 = 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.062405571$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.061549933 \quad \text{(2 dp accurate)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.061549775 \quad \text{(6 dp accurate)}$$

rightmost root, $x \approx 1.061549775$

(2) (a)

(i) 2 initial guesses, $x_0, x_1$ such that $x_0 < x_1$ &

$$f(x_0)f(x_1) < 0$$

(ii) Compute $f(x_0)$ & $f(x_1)$

(iii) Compute,

$$x_2 = x_0 - \frac{f(x_0)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

(iv) Test for accuracy of $x_2$. If ~~$x_2$~~ $|f(x_i)| \geq$ Accuracy then $x_0 = x_1$, $x_1 = x_2$ & repeat step (iii) & (iv)

(v) If $|f(x_i)| \leq$ Accuracy then stop & the ~~rot~~ approximated root will be $x_i$.

(b) $f(x) = x^3 - x - 1$

$f(0) = -1$, $f(1) = -1$, $f(2) = 5$ ← selecting random values of x

Since there is a change in sign ~~for~~ between $f(1)$ & $f(2)$ so root must lie in between 1 & 2.

<u>Iteration 1</u>

$x_0 = 1, x_1 = 2$

$f(x_0) = -1$, $f(x_1) = 5$

$$\Rightarrow x_2 = x_0 - \frac{f(x_0)(x_1 - x_0)}{f(x_1) - f(x_0)} = 1.16667$$

$f(x_2) = -0.5787$

<u>Iteration 2</u>

~~$x_2 = 1.16667$~~, $x_1 = 2$

$x_2 = 1.16667$

$f(2) = +ve$, $f(1.16667) = -ve$

$$\Rightarrow x_3 = x_1 - \frac{f(x_1)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$x_3 = 1.25311$

$f(x_3) = -0.28536$

## Iteration 03

$x_2 = 1.16667$

$x_3 = 1.25311$

$f(1.16667) = -ve$

$f(1.25311) = -ve$

$$x_4 = x_2 - \frac{f(x_2)(x_3 - x_2)}{f(x_3) - f(x_2)}$$

$x_4 = 1.33721$

$f(x_4) = 0.05388$

## Iteration 04

$x_3 = 1.25311$

$x_4 = 1.33721$

$f(x_3) = -0.28536$

$f(x_4) = 0.05388$

$$\Rightarrow x_5 = x_3 - \frac{f(x_3)(x_4 - x_3)}{f(x_4) - f(x_3)}$$

$\Rightarrow x_5 = 1.32385$

$f(x_5) = -0.0037$

## Iteration 05

$x_4 = 1.33721$

$x_5 = 1.32385$

$f(x_4) = +ve$

$f(x_5) = -ve$

$$x_6 = x_4 - \frac{f(x_4)(x_5 - x_4)}{f(x_5) - f(x_4)}$$

$\Rightarrow x_6 = 1.32471$

$f(x_6) = -0.00004$

So, Approximated root $= \underline{1.32471}$