# Final Exam

Read the following instructions/rules before you start answering the questions:

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- **You MUST keep a Backup Data plan, in case your internet service is interrupted. NO EXCUSE.**
- **The exam time is 1 hour 15 minutes and an additional 15 minutes is for submission of answer script and uploading. So total is 1 hour 30 minutes. No more time time extension, and NO EMAIL SUBMISSION will be accepted.**
- Prepare a Title page containing a) Your Name, b) Your BRACU ID #, c) Your Theory class Section # and d) Date and e) Exam Name.
- Prepare the solution file in orderly fashion: first question first, then maintain the serial.
- Always start an answer to a question from a fresh page.
- Write legibly and neatly, FOLLOW the significant rule for all your calculations.
- YOU MUST WORK ALONE. INVOLVEMENT IN UNFAIR MEANS WILL BE REPORTED TO THE AUTHORITY.

**Question # 11**: A function $f(x) = x^3 - 7x^2 + 4x + 12$ has one root in the interval $I = [4.25, 8.95]$. In the following, we would like to find the root using Newton's method along with Aitken acceleration:

1. (1 Mark) Explain an advantage of using the Quasi-Newton Method over Newton's method.
2. (7 Marks) Starting from $x_0 = 7.23$ use Newton's iteration formula, up to four iterations, *i.e.,* $k = 0, 1, 2, \cdots, 4$, to find the approximate root of $f(x)$ by applying Aitken acceleration only once appropriately. Express your result up to five decimal places where necessary.
3. (2 Marks) Suppose the actual root is 6. Calculate the percentage error of your last iteration value of root. Express your result up to three decimal places.

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**Question # 23**: A linear system is described by the following equations:

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_1 + x_2 - 3x_3 &= 5 \\ 4x_1 - 7x_2 + x_3 &= -1 \end{aligned}$$

Solve the above linear system by answering the following:

1. (2 Marks) Does this system have any unique solution? Why or why not?
2. (3 Marks) Find the upper triangular matrix $U$.
3. (3 Marks) Solve the above linear system by Gaussian elimination method. Show your work.
4. (2 Marks) Suppose, you have constructed an Augmented matrix from a different linear system as given below,

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 0 & 4 & 1 \\ 0 & 9 & -6 & 8 \end{array}\right)$$

Explain why the Gaussian elimination method fails to solve this system? Also explain how we can overcome the problem to actually solve it (you do not have to solve this system)?

**Question # 36**: Consider a set of four data points: $f(0) = 0$, $f(2) = -1$, $f(-1) = 2$ and $f(1) = 2$. We now find the solution by *QR*-decomposition method using these four data points by answering the following:

1. (1.5 Marks) Write down the matrix $A$ and $b$. Also identify the linearly independent column vectors $u_1$, $u_2$ and $u_3$ from the matrix $A$.
2. (4.5 Marks) Using the Gram-Schmidt process construct the orthonormal column matrices (or vectors) $q_1$, $q_2$ and $q_3$ from the linearly independent column vectors $u_1$, $u_2$ and $u_3$ obtained in the previous part, and then write down the $Q$ matrix.
3. (2 Marks) Now calculate the matrix elements of $R$, and write down the matrix $R$.
4. (1 Mark) Compute $Rx$ and $Q^{\mathrm{T}}b$, where $x = (a_0 \;\; a_1 \;\; a_2)^{\mathrm{T}}$ is a column vector with $a_0$, $a_1$ and $a_2$ are the coefficients of the polynomial $p_2$.
5. (1 Mark) Using the above result, find the values of $a_0$, $a_1$ and $a_2$; and write the polynomial $p_2(x)$.

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**Question # 41**: Answer the following:

1. (2 Marks) Show that the upper bound error for numerical integration of a quadratic function is given by

$$\frac{f^{(3)}(\xi)}{3!}(x - x_0)(x - x_1)\,,$$

   where $\xi$ is a value within the limits of the integral that maximizes the error.

2. (3 Marks) The vertical distance covered by a rocket from $t = 8$ to $t = 30$ seconds is given by

$$x = \int_8^{30}\left(2000\ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t\right)dt\,.$$

   Use single segment trapezium rule to find the distance covered.

3. (3 Marks) Find the actual distance in the previous part, and also compute the actual error.
4. (2 Marks) How can the error be decreased?

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Final Exam Answer Script Submission:

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- Prepare a single .pdf file including (1) title page as described in the Final Exam Guidelines and (2) the orderly arranged answer scripts to the final exam questions.
- Rename the single .pdf file as "ID_FirstName_TheorySection" (for CSE students) or "ID_FirstName_205" (for MNS students. As for example: "98765432_Abu_9.pdf".
- The final exam exam solution will be accepted by the following Google Form and you will have ONLY ONE chance to submit with no possibility of correction after submission.
- Now, to submit your final exam answer script, open "Final Exam Answer Script Submission Form" (or Click here), and then fill-up the form, upload the solution file you prepared, and press the SUBMIT button.
- YOU ARE ALL DONE. GOOD JOB. Thanks.

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