Assignment 2

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Ques 1

1

Taylor series of

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$$

$$\therefore e^x + e^{-x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$$

$$= 2\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots\right)$$

2

$$P_4(x) = 2x^0 + 0 \cdot x + \frac{2}{2!}x^2 + 0 \cdot x^3 + \frac{2}{4!}x^4 + \dots$$

$\therefore$ $a_0 = 2$,

$a_1 = 0$

$a_2 = \frac{2}{2!} = 1$

$a_3 = 0$

$a_4 = \frac{2}{4!} = \frac{1}{12}$

3

$$f(0.1) = e^{0.1} + e^{-0.1} = 2.0100083$$

and $P_4(0.1) = 2 + \frac{(0.1)^2 \times 2}{2!} + \frac{(0.1)^4 \times 2}{4!}$

$$= 2 + (0.1)^2 + \frac{(0.1)^4}{12}$$

$= 2.0100083$

$$\underline{\underline{4}}$$

As $f(0.1) = = P_4(0.1)$

so the percentage error for interpreting f by P_4 at 0.1 is 0.

Ques no 2

1

Given, $f(x_0) = \cancel{e^{x}+e^{x}}\ e^{x} + e^{-x}$

Nodes given, $-1, 0, 1$

$\therefore f(x_0) = e^{-1} + e^{1}$

$= 3.08616$

$f(x_1) = e^{0} + e^{0}$

$= 1+1 = 2$

$f(x_2) = e^{1} + e^{-1}$

$= 3.08616$

we know,

$P_2(x) = a_0 + a_1 x + a_2 x^2$

$\therefore P_2(x_0) = a_0 + a_1(-1) + a_2(-1)^2 = 3.086$

$P_2(x_1) = \cancel{a}\ a_0 + a_1 \times 0 + a_2 0^2 = 2$

$P_2(x_2) = a_0 + a_1 \times 1 + a_2 \times 1^2 = 3.086$

$\therefore$ Vanderomonde matrix : $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

<u>2</u>

$$\det|V| = 1(0-0) - (-1)(1-0) + 1(1-0)$$
$$= 1+1$$
$$= 2 \quad \text{(Ans)}$$

<u>3</u>

$$\begin{vmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

for inverse,

$$\left|\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right|$$

if we perform gauss jordan elimination on the ~~b~~ Left matrix we get

$$= \left|\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 & -1 & 1/2 \end{array}\right|$$

$\therefore$ the inverse matrix

$$V^{-1} = \begin{vmatrix} 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \\ 1/2 & -1 & 1/2 \end{vmatrix}$$

<u>4</u>

coeffecients a_0, a_1, a_2

$$\therefore \begin{vmatrix} a_0 \\ a_1 \\ a_2 \end{vmatrix} = V^{-1} \begin{vmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \\ 1/2 & -1 & 1/2 \end{vmatrix} \begin{vmatrix} 3.0816 \\ 2 \\ 3.0861 \end{vmatrix}$$

$$= \begin{vmatrix} 2 \times 1 \\ -\frac{1}{2} \times 3.08 + \frac{1}{2} \times 3.08 \\ \frac{1}{2} \times 3.08 - 2 + \frac{1}{2} \times 3.08 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a_0 \\ a_1 \\ a_2 \end{vmatrix} = \begin{vmatrix} 2 \\ 0 \\ 1.086 \end{vmatrix}$$

$\therefore a_0 = 2$

$a_1 = 0$

$a_2 = 1.086$ (Ans)

5

we know, $P_2(x) = a_0 + a_1 x + a_2 x^2$

from 4 we get $a_0 = 2$

$a_1 = 0$

$a_2 = 1.086$

$\therefore P_2(x) = 2 + 1.0861\, x^2$

$\therefore P_2(0.1) = 2 + 1.0861\,(0.1)^2$

$= 2.01086$

$\therefore f(0.1) = e^{-0.1} + e^{0.1}$

$= 2.010008$

6

percentage error : $\left(\dfrac{2.010008 - 2.01086}{2.010008}\right) \times 100$

$= 0.0423\ \%$