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Section : 09

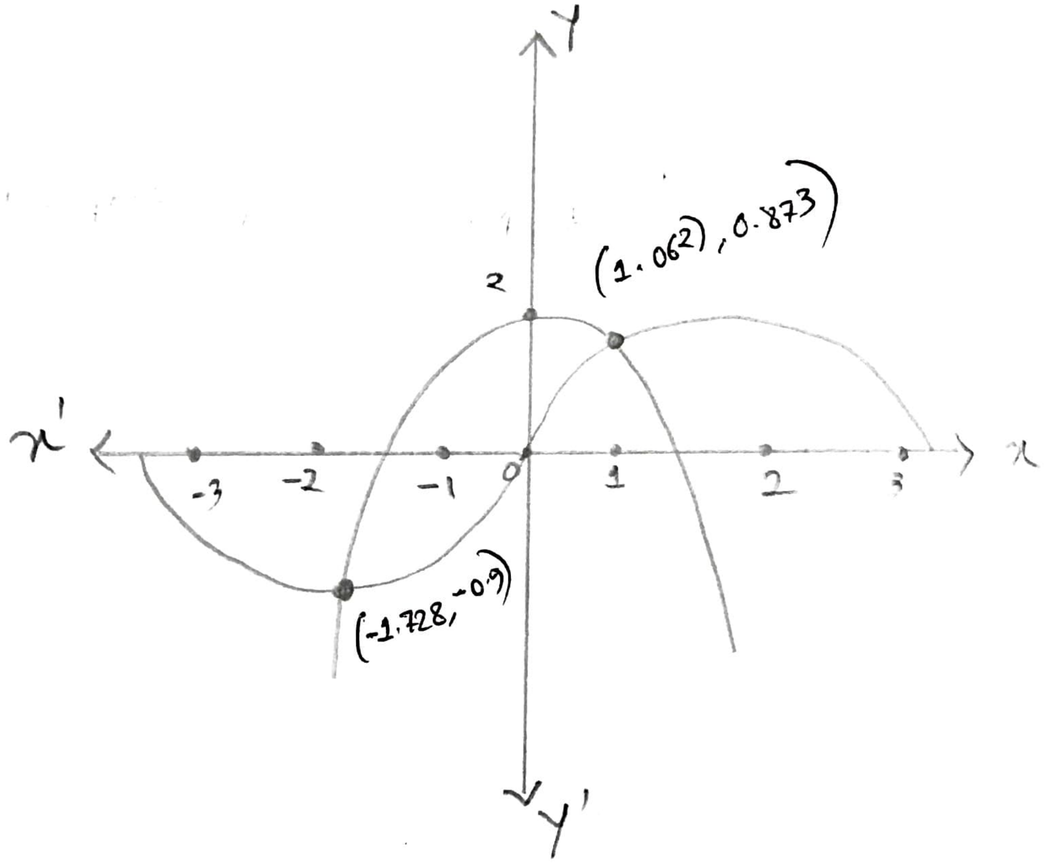
Assignment 7

Problem A

Given,

$$2-x^2 = \sin(x)$$

If we draw the graphs of $2-x^2$ and $\sin(x)$ we get



here we can ~~we~~ ~~the~~ see there are two roots of the above equation.

Now, for the leftroot assuming $x_0 = -1.5$

Lets perform Newtons method for the left root,

$$f(x) = 2 - x^2 - \sin(x)$$

$$f'(x) = -2x - \cos(x)$$

$$x_0 = -1.5$$

we know,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

n (iteration)	x_k	$f(x_k)$	$f'(x_k)$	x_{k+1}	
1	-1.5	0.747495	2.929263	-1.755182	$= x_1$
2	-1.755182	-0.097615	3.693706	-1.728755	$= x_2$
3	-1.728755	-0.001042	3.614812	$-1.72846$6	$= x_3$
4	-1.728466	0	3.61395	-1.728466	$= x_4$

Again,
For Righroot lets assume, $x_0 = 1$

Now Again performing Newtons Method for the Right Root,

$$f(x) = 2 - x^2 - \sin(x)$$
$$f'(x) = -2x - \cos x$$

$$x_0 = 1$$

and we know,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

n(Iterations)	x_0	$f(x_0)$	$f'(x_0)$	x_k
1	1	0.158529	-2.540302	1.062406
2	1.062406	-0.002235	-2.611583	1.06155
3	1.06155	0	-2.610619	1.06155
4				

Problem 2B

1

In Secant Method, to avoid the drawbacks of Newtons method we replace $f'(x)$ with a easily computable function.

The Algorith of secant method:

S-1 : we have to find the points x_0 and x_1 so that $x_0 < x_1$ & $f(x_0) \cdot f(x_1) < 0$

3-2 : the we have to find the next value using

$$x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}$$

3-3 : if $f(x_2) = 0$, then x_2 is an exact error

otherwise $x_0 = x_1$

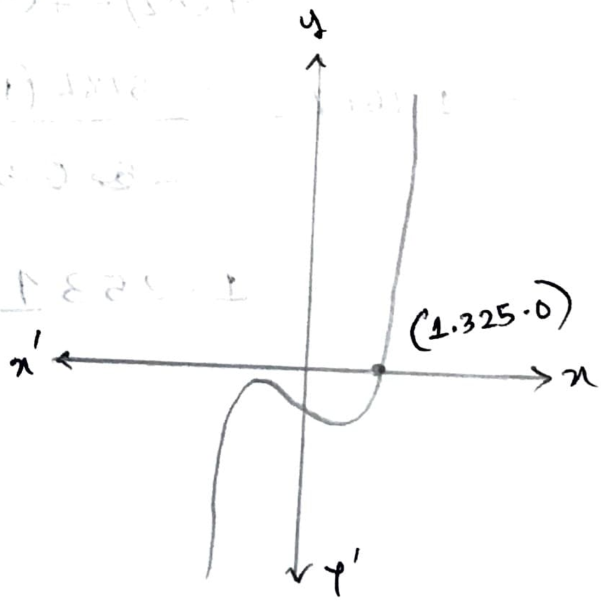
$x_1 = x_2$

and we have to repeat steps 2 and 3 until $f(x_i) = 0$

* here, the iteration starts with two initial values x_0 and x_1. So the first iterated point is x_2.

2

Given, $f(x) = x^3 - x - 1$



Lets assume,

$x_0 = 1$ and $x_1 = 2$

we know,

Secant method :

$$x_{k+1} = x_k - \frac{f(x_k)\,(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}$$

1st iteration

$$x_2 = x_1 - \frac{f(x_1)\,(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$f(x_0) = f(1) = (1)^3 - 1 - 1$
$= -1$

$f(x_1) = f(2) = (2)^3 - 2 - 1$
$= 5$

$$= 2 - \frac{5 \times 1}{5 - (-1)}$$

$$= 1.1667$$

2nd Iteration

$$x_3 = x_2 - \frac{f(x_2)\,(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$f(1.1667) = (1.1667)^3 - 1.1667 - 1$
$= -0.5787$

$$= 1.1667 - \frac{-0.5787\,(1.1667 - 2)}{-0.5787 - 5}$$

$$= 1.2531$$

$$x_4 = x_3 - \frac{f(x_3)(x_3 - x_2)}{f(x_3) - f(x_2)}$$

$f(x_3) = f(1.2531)$

$= -0.2854$

$$= 1.2531 - \frac{-0.2854(1.2531 - 1.1667)}{-0.2854 - (-0.5787)}$$

$$= 1.3372$$

4th iteration,

$$x_5 = x_4 - \frac{f(x_4)(x_4 - x_3)}{f(x_4) - f(x_3)}$$

$f(x_4) = f(1.3372)$

$= 0.0539$

$$= 1.3372 - \frac{0.0539(1.3372 - 1.2531)}{0.0539 - (-0.2854)}$$

$$= 1.3239$$

5th iteration,

$$x_6 = x_5 - \frac{f(x_5)(x_5 - x_4)}{f(x_5) - f(x_4)}$$

$f(x_5) = f(1.3239)$

$= -0.0037$

$$= 1.3299 - \frac{-0.0037(1.3299 - 1.3372)}{-0.0037 - 0.0539}$$

$= 1.3247$

$\therefore x_6 = \cancel{3.325}\ 1.325$

$\therefore f(x_6) = f(1.325) = (1.325)^3 - 1.325 - 1$

$\simeq 0$

$\therefore$ Approximate root of $x^3 - x - 1 = 0$ is 1.325