Midterm Exam

Read the following instructions/rules before you start answering the questions:

- **You MUST keep a Backup Data plan, in case your internet service is interrupted. NO EXCUSE.**
- **The exam time is 1 hour and an additional 15 minutes is for preparing the solution file and uploading. No more time time extension, and NO EMAIL SUBMISSION will be accepted.**
- Prepare a Title page containing a) Your Name, b) Your BRACU ID #, c) Your Theory class Section # and d) Date and e) Exam Name.
- Prepare the solution file in orderly fashion: first question first, then maintain the serial.
- Always start an answer to a question from a fresh page.
- Write legibly and neatly, FOLLOW the significant rule for all your calculations.
- YOU MUST WORK ALONE. INVOLVEMENT IN UNFAIR MEANS WILL BE REPORTED TO THE AUTHORITY.

Question # 13: Given $\beta = 2$, $m = 3$, $e_{min} = -1$ and $e_{max} = 0$. Now answer the following:

Question # 15: Given $p = 2$, $m = 3$, e_{min} = -1 and e_{max} = 3. Now answer the following:

1. [1 Mark] What will be the machine epsilon for $\pm(1.d_1d_2\cdots d_m)_\beta \beta^e$.
2. [4 Marks] Using your answer in the previous part, find out the values that each group represent for the sets of *e*,and plot them on a number line starting from 0.
3. [1 Mark] Calculate the machine epsilon, ϵ_M, for the problem.

Question # 23: Suppose you have a function $f(x) = \cos(x)$ and two nodes 0 and $\frac{\pi}{2}$.

1. [3 Marks] Find the interpolation polynomial of the given function and nodes using the Vandermonde matrix method.
2. [1.5 Marks] Roughly hand-draw a comparison graph between $f(x)$ and the interpolating polynomial.
3. [1.5 Marks] Compute the actual interpolation error at $x = 0$.

Question # 33: A rocket has been launched, and it's velocities at different times are collected. From these data, the acceleration of the rocket, $a(t)$ at $t = 16$ sec is calculated numerically by using different methods (using $h = 1$) as shown in the table below (up to six decimal places):

Difference method	Forward	Backward	Central
$a(t = 16)$	33.880085	32.898426	33.389255

Now, the velocity of a rocket as function of time obeys the equation below:

$$v(t) = 1900\ln\left(\frac{12 * 10^4}{12 * 10^4 - 2000t}\right) - 9.8t,$$

where v is in m/s and t is in seconds.

1. [4.5 Marks] Find the truncation errors for the acceleration at $t = 16$ sec for Forward, Backward and Central Difference methods.
2. [1.5 Marks] How do truncation error and rounding error behave when $h \to 0$.

Question # 45: Consider the following data set generated by a function, $f(x)$:

x	0.0	0.2	0.4
$f(x)$	1.03	1.23	1.53

1. [6 Marks] Find the interpolating polynomial for the above data of appropriate order using Newton's Divided/Difference method.
2. [1 Mark] Use the interpolating polynomial to find the approximate value of $f(0.37)$.
3. [4 Marks] Add a data point $f(0.8) = 1.83$ to the given data, and find the new interpolating polynomial that includes this new data point.

Question # 54: Consider the following data set:

x	-0.1	0	0.1
$f(x)$	0.97722	1	1.04390

for a function $f(x)$. For these data values answer the following, and write all your results up to five decimal places.

1. [2 Marks] Using the above data, compute the first derivative of the function numerically by using the central difference method.
2. [5 Marks] For in interval $[-0.1, 0.1]$, compute the error bound (truncation error) if the above data is generated by the function, $f(x) = e^{x/3} + x^2$.
3. [2 Marks] Also compute the actual error.
4. [2 Marks] Based on your error calculations in the previous two parts, explain why the result found for the numerical value of the first derivative in the first part of the question is acceptable or not.

MIDTERM EXAM SUBMISSION

Status

You have completed this assignment. Your final grade will be available when the assessments of your response are complete.

▸ Your Response due Nov 24, 2021 19:00 +06 (in 0 minutes) ✔ COMPLETE

Staff Grade NOT AVAILABLE

Waiting for a Staff Grade

Check back later to see if a course staff member has assessed your response. You will receive your grade after the assessment is complete.

Your Grade: Waiting for Assessments

You have completed your steps in the assignment, but some assessments still need to be done on your response. When the assessments of your response are complete, you will see feedback from everyone who assessed your response, and you will receive your final grade.

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