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Assignment 8

Question 1 :

$$x_1 - x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

Now, Let,

$$A = \begin{vmatrix} 1 & -1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{vmatrix}$$

$$\therefore \det A = 1(9-(-5)) - (-1)(12-(-3)) + 1(20-9)$$

$$= 14 + 15 + 11$$

$$\cancel{= 40} = 40 \neq 0$$

$\therefore$ This system has a unique solution.

Question 2

from Question 1 we get

$$\left|\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 4 & 3 & -1 & 6 \\ 3 & 5 & 3 & 4 \end{array}\right|$$

$$= \left|\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 7 & -5 & 2 \\ 3 & 5 & 3 & 4 \end{array}\right| \quad [R_2 - R_2 - 4\times R_1]$$

$$= \left|\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 7 & -5 & 2 \\ 0 & 8 & 0 & 1 \end{array}\right| \quad [R_3 = R_3 - 3\times R_1]$$

$$= \left|\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 7 & -5 & 2 \\ 0 & 0 & \frac{40}{7} & -\frac{9}{7} \end{array}\right| \quad [R_3 = R_3 \cdot \tfrac{8}{7} - R_2]$$

so from this we can get,

$$\frac{40}{7} x_3 = -\frac{9}{7} \quad \text{— (I)}$$

$$7x_2 - \cancel{x_3}\, 5x_3 = 2 \quad \text{— (II)}$$

$$x_1 - x_2 + x_3 = 1 \quad \text{— (III)}$$

from (I)

$$x_3 = -\frac{9}{7} \times \frac{7}{40}$$

$$= \frac{-9}{40}$$

from (II)

$$7x_2 - 5 \times \frac{-9}{40} = 2$$

$$7x_2 + \frac{9}{8} = 2$$

$$\Rightarrow x_2 = \frac{1}{8}$$

Now from (III)

$$x_1 - \frac{1}{8} + \frac{-9}{40} = 1$$

$$x_1 = 1 + \frac{1}{8} + \frac{9}{40}$$

$$x_1 = \frac{27}{20}$$

$$\therefore \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} \frac{27}{20} \\ \frac{1}{8} \\ -\frac{9}{40} \end{vmatrix} \quad \text{(Ans)}$$

Question 3

Given,

$$x_1 - x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

here the co-efficient matrix,

$$A^{(1)} = \begin{vmatrix} 1 & -1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{vmatrix}$$

$$\therefore F_1 = \begin{vmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -3 & 0 & 1 \end{vmatrix} \qquad \left[\begin{array}{l} R_2 = R_2 - \left(\frac{4}{1} \times R_1\right) \\ R_3 = R_3 - \left(\frac{3}{1} \times R_1\right) \end{array}\right]$$

$$\therefore A^2 = \cancel{A^{(1)}} \times F^1 \times A^1$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -3 & 0 & 1 \end{vmatrix} \times \begin{vmatrix} 1 & -1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -1 & 1 \\ 0 & 7 & -5 \\ 0 & 8 & 0 \end{vmatrix}$$

Now,

$$F^{(2)} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{8}{7} & 1 \end{vmatrix} \qquad \left[R_3 = R_3 - \left(\frac{8}{7} \times R_2\right)\right]$$

$$\therefore A^3 = U = F^2 \times A^2$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{8}{7} & 1 \end{vmatrix} \times \begin{vmatrix} 1 & -1 & 1 \\ 0 & 7 & -5 \\ 0 & 8 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -1 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & 40/7 \end{vmatrix} \quad \text{(Ans)}$$

$$\therefore L = \begin{vmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & \frac{8}{7} & 1 \end{vmatrix} \quad \text{(Ans)}$$

Question 4

Lets assume

a temporary variable = y

$$\therefore [L] \times \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{vmatrix} 1 \\ 6 \\ 4 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & \frac{8}{7} & 1 \end{vmatrix} \times \begin{vmatrix} y_1 \\ y_2 \\ y_3 \end{vmatrix} = \begin{vmatrix} 1 \\ 6 \\ 4 \end{vmatrix}$$

$\therefore y_1 = 1$ —— (I)

$4y_1 + y_2 = 6$ —— (II)

$3y_1 + \frac{8}{7} y_2 + y_3 = 4$ —— (III)

$\therefore$ from (II)

$4 \times 1 + y_2 = 6$

$y_2 = 2$

from (III)

$3 \times 1 + \frac{8}{7} \times 2 + y_3 = 4$

$y_3 = 4 - 3 - \left(\frac{8}{7} \times 2\right)$

$y_3 = -\frac{9}{7}$

$\therefore [y_1, y_2 \ y_3] = [1, 2, -\frac{9}{7}]$

$\therefore$ Now,

$$U \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & \frac{40}{7} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \\ -\frac{9}{7} \end{vmatrix}$$

$$x_1 - x_2 + x_3 = 1 \quad \text{——— (i)}$$

$$7x_2 - 5x_3 = 2 \quad \text{——— (ii)}$$

$$\frac{40}{7} x_3 = -\frac{9}{7} \quad \text{——— (iii)}$$

from (iii)

$$x_3 = -\frac{9}{7} \times \frac{7}{40}$$

$$= -\frac{9}{40} \quad \text{(Ans)}$$

from (ii)

$$7x_2 = 2 + 5 \times -\frac{9}{40}$$

$$7x_2 = \frac{7}{8}$$

$$\therefore x_2 = \frac{1}{8} \quad \text{(Ans)}$$

from, (i)

$$x_1 - \frac{1}{8} - \frac{9}{40} = 1$$

$$\therefore x_1 = \frac{27}{20} \quad \text{(Ans)}$$