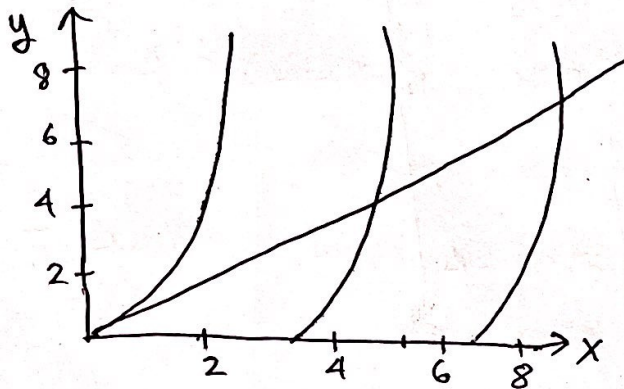## Question 01

(a) $y=x$, $y=\tan x$

(b)

first positive value of x. Since root lies between 4,5 so $I \in [4,5]$

$f(x) = x - \tan x$. [10 iterations]

k	a_k	m_k	b_k	$f(a_k)$	$f(m_k)$	$f(b_k)$	$x_* \in [a,b]$
0	4	4.5	5	2.84	-0.14	8.38	[4, 4.5]
1	4	4.25	4.5	2.84	2.24	-0.14	[4.25, 4.5]
2	4.25	4.375	4.5	2.24	1.52	0.89 -0.14	[4.375, 4.5]
3	4.375	4.4375	4.5	1.52	0.89	-0.14	[4.4375, 4.5]
4	4.4375	4.46875	4.5	0.89	0.45	-0.14	[4.46875, 4.5]
5	4.46875	4.484375	4.5	0.45	0.17	-0.14	[4.484375, 4.5]
6	4.484375	4.4921875	4.5	0.17	0.02	-0.14	[4.4921875, 4.5]
7	4.4921875	4.49609375	4.5	0.02	-0.05	-0.14	[4.4921875, 4.49609375]
8	4.4921875	4.4941406025	4.49609375	0.02	-0.01	-0.05	[4.4921875, 4.4941406025]
9	4.4921875	4.493164051	4.4941406025	0.02	4.95×10^{-3}	-0.01	[4.493164051, 4.4941406025]

X_* after 10 iterations = 4.493164051

$X = \tan x$ ~~$tan x + tan x$~~

$g(x) = \tan x \qquad x \in [4,5]$

(a) let, $x_0 = 4$

$g(4) = \tan 4 = 1.157$ (lies outside the range)

$g(x)$ is not valid since it falls outside the range.

Rearranging, $x = \tan x$

$$\Rightarrow \frac{1}{x} = \frac{1}{\tan x}$$

$$\Rightarrow \frac{1}{\tan x} - \frac{1}{x} = 0$$

$$\Rightarrow \frac{1}{\tan x} - \frac{1}{x} + x = x$$

$$\Rightarrow x = x + \frac{1}{\tan x} - \frac{1}{x}$$

$$\Rightarrow g(x) = x + \frac{1}{\tan x} - \frac{1}{x}$$

(b)

Now,

$x_0 = 4, \quad g_1(4) = 4.61369$ (falls within the range) $[4,5]$

$x_1 = 4.61369, \quad g_2(4.61369) = 4.49596$

$x_2 = 4.49596, \quad g_3(4.49596) = 4.49341$

$x_3 = 4.49341, \quad g_4(4.49341) = 4.49341$

now, $|g_4 - g_3| = |4.49341 - 4.49341| = 0 \times 10^{-4}$ so, accuracy within 10^{-4}.

[take higher decimal places for better precision]