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Assignment - 5

Sta - 201

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1

a) we know, $\sum f(x) = 1$

Again, $\sum f(x) = 3k \times 1 + 3k \times 3 + 3k \times 5 + k(7^2 + 0.5)$

$= \frac{153}{2} k$

$\therefore \frac{153}{2} k = 1 \Rightarrow k = \frac{2}{153}$ (Ans)

b)

x	$P(X=x)$
1	$\frac{2}{51}$
3	$\frac{2}{17}$
5	$\frac{10}{51}$
7	$\frac{11}{17}$

$\therefore P(3 < x \leq 7) = \frac{10}{51} + \frac{11}{17}$

$= \frac{43}{51}$ (Ans)

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c The exact value of $P(3<x<5) = 0$

d $E(x) = \left(1 \times \frac{2}{51}\right) + \left(3 \times \frac{2}{17}\right) + \left(5 \times \frac{10}{51}\right) + \left(7 \times \frac{11}{17}\right)$

$= \frac{301}{51} = 5.90$ (Ans)

e) for variance, $E(x^2) = \left(1 \times \frac{2}{51}\right) + \left(9 \times \frac{2}{17}\right) + \left(25 \times \frac{10}{51}\right)$

$+ \left(49 \times \frac{11}{17}\right)$

$= \frac{641}{17}$

$\therefore$ Variance : $E(x^2) - (E(x))^2$

$= \frac{641}{17} - \left(\frac{301}{51}\right)^2$

$= 2.87$ (Ans)

f $Var\left(-\frac{1}{3}x + 3\right) \Rightarrow \left(-\frac{1}{3}x\right)$

$= \left(-\frac{1}{3}\right)^2 \times Var(x) \Rightarrow \frac{1}{9} \times 2.87$

$= 0.3192$ (Ans)

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2

a $\int_0^4 \frac{1}{20} y \, dy + \int_4^6 \frac{1}{30}(10-y)\, dy$

$= \frac{1}{20}\left(\frac{y^2}{2}\right)_0^4 + \frac{1}{30}\left(10y - \frac{y^2}{2}\right)_4^6$

$= \frac{1}{20} \times \frac{16}{2} + \frac{1}{30} \times (42-32)$

$= 0.733$ (Ans)

b $\int (y>7) + \int (y<3) = \int_7^{10} \frac{1}{30}(10-y) + \int_0^3 \frac{1}{20} y \, dy$

~~$= \frac{1}{2} \times \frac{9}{2} + \frac{1}{30}$~~

$= \cancel{\frac{1}{30} \times 9} \; \frac{3}{20} + \frac{9}{40}$

$= 0.375$ (Ans)

c $E(y) = \int_0^4 y \frac{1}{20} y \, dy + \int_4^{10} y \frac{1}{30}(10-y)\, dy$

$= \frac{16}{15} + \frac{18}{5} \Rightarrow \frac{14}{3} \Rightarrow 4.667$ (Ans)

d $E(y^2) = \int_0^4 y^2 \frac{1}{20} y \, dy + \int_4^{10} y^2 \frac{1}{30}(10-y)\, dy$

$= 3.2 + 22.8 \Rightarrow 26$

$\therefore Var = E(y^2) - (E(y))^2 = 26 - (4.66)^2 = 4.22$

$\therefore SD(y) = \sqrt{Var(y)} = 2.0548$ (Ans)

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3

a) ~~toto~~ $P(A=B) = P(0,0) + P(1,1) + P(2,2) + P(3,3)$

$= 0.09 + 0.01 + 0.1 + 0.1$

$= 0.3$ (Ans)

b) $A+B=4$ so, $P(1,3) + (P(2,2) + P(3,1) + (4,0)$

$= 0.04 + 0.1 + 0.03 + 0.01$

$= 0.18$ (Ans)

Again, "at least four" $= P(A+B \geq 4)$

$= P(4,1) + P(2,3) + P(4,2) + P(3,2) + P(3,3) +$
$P(4,3) + P(A+B=4) = 0.15 + 0.07 +$

$0.05 + 0.01 + 0.1 + 0.06 + 0.18$

$= 0.62$ (Ans)

c)

A\B	0	1	2	3	
0	0.09	0.05	0.03	0	0.17
1	0.01	0.01	0.05	0.04	0.11
2	0.08	0.06	0.1	0.07	0.31
3	0	0.03	0.01	0.1	0.14
4	0.01	0.15	0.05	0.06	0.27
P(B)	0.19	0.3	0.24	0.27	1

$\therefore E(x) = 0 \times 0.19 + 1 \times 0.13 + 2 \times 0.24 + 3 \times 0.27$

$= 1.59$

d

$P(B=2 / A=3) = \dfrac{P(A=3 \cap B=2)}{P(A=3)} = \dfrac{0.01}{0.14} = 0.0714$

(Ans)

e

we have from chart: $P(0,0) = 0.09$

$P_A(0) = 0.17$ and $P_B(0) = 0.19$

$P_A(0) \times P_B(0) = 0.17 \times 0.19 = 0.032$

$\therefore 0.032 \neq 0.09$

$\therefore$ B and A are not independent.

4

a. we get, $p = \frac{3}{12} = \frac{1}{4}$

$\therefore$ probability of getting the first heart in 5th turn,

$\frac{1}{4}\left(1-\frac{1}{4}\right)^4 = 0.0791$ (Ans)

b. spade $= 2$, No spade $= 12-2 = 10$

$\therefore p = \frac{10}{12} = \frac{5}{6}$

$\therefore E(x) = \frac{1}{p} = \frac{1}{\frac{5}{6}} = \frac{6}{5} = 1.2$

$\simeq 2$ (Ans)

c. $p = \frac{7}{12}$

so the variance of the number of turns required to get one club is $\frac{1-p}{p^2} = \frac{1-\frac{7}{12}}{\left(\frac{7}{12}\right)^2}$

$= 1.2245$ (Ans)

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5

a $P(c) = \frac{6}{12}$

$n = 6, \ r = 4$

∴ probability of getting exactly 4 clubs after 6 turns

is $= {}^nC_r \, p^r (1-p)^{n-r} \Rightarrow {}^6C_4 \left(\frac{6}{12}\right)^4 \left(1 - \frac{6}{12}\right)^2$

$\Rightarrow 0.234$ (Ans)

b $P(c) = \frac{6}{12}$

Now the probability of picking more than 3 clubs, after 6 turns $= {}^6C_4 \left(\frac{6}{12}\right)^4 \left(1-\frac{6}{12}\right)^2 + {}^6C_5 \left(\frac{6}{12}\right)^5 \left(1-\frac{6}{12}\right) + {}^6C_6 \left(\frac{6}{12}\right)^6 \left(1-\frac{6}{12}\right)^0$

$= \frac{1}{64} + \frac{3}{32} + \frac{1}{64} \Rightarrow \frac{11}{32}$ (Ans)

c $P(h) = \frac{4}{12}, \ n = 60$

∴ Mean of hearts picked after 60 turns $= 60 \times \frac{4}{12}$

$= 20$ (Ans)

d $P(S) = \frac{2}{12}$

∴ standard deviation of number of spades picked after 36 turns $= \sqrt{36 \times \frac{2}{12} \times \left(1 - \frac{2}{12}\right)} = \sqrt{5} = 2.2361$ (Ans)

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6

a Mean facalities $\lambda_1 = 14$ [1 in a weak]

Per day $= \frac{14}{7} = 2$

$\therefore$ Mean facalities in a month $\lambda_4 = 30 \times 2$ [30 d = 1 month]

$= 60$ (Ans)

b $\lambda_1 = 14$, $\lambda_2 = 28$, $x = 40$

$\therefore$ Probability of 40 fatalities in the next two weeks $= \frac{e^{-28} \times (28)^{40}}{40!}$

$= 6.52 \times 10^{-3}$ (Ans)

c $\lambda_1 = 14$

$x = 1, 2, 3, 4, 5, 6, 7, 8, 9$

$$\therefore P(x \leq 9) = e^{-14} \left[\frac{14^0}{0!} + \frac{14}{1!} + \frac{14^2}{2!} + \frac{14^3}{3!} + \frac{14^4}{4!} + \frac{14^5}{5!} + \frac{14^6}{6!} + \frac{14^7}{7!} + \frac{14^8}{8!} + \frac{14^9}{9!} \right]$$

$= 0.109$ (Ans)

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$\underline{7}$

$\underline{a}$

(1) As mean = mode = median = 100

standard deviation = $\sqrt{36} = 6$

$\therefore X \sim N(\mu, \sigma^2) = N(100, 6^2)$

$\underline{\underline{i}}$

$$P(100 \le x \le 105) = P\left[\frac{100-\mu}{6} \le \frac{x-\mu}{6} \le \frac{105-\mu}{6}\right]$$

$$= \left[\frac{100-100}{6} \le z \le \frac{105-100}{6}\right]$$

$$= P[0 \le z \le 0.83]$$

$$= 0.2967$$

$$\therefore E(x) = 0.2967 \times 2000$$

$$\Rightarrow 593.4$$

$$\approx 593 \quad (\text{Ans})$$

$\underline{\underline{ii}}$

$$P(x > 110) = P\left[\frac{x-\mu}{6} > \frac{110-\mu}{6}\right] = P\left[\frac{x-\mu}{6} > \frac{110-100}{6}\right]$$

$$= P[z > 1.67]$$

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$= P[Z \geq 0] - P[0 \leq Z \leq \cancel{2.02}\ 1.67]$

$= 0.5 - 0.4525$

$= 0.0475$

$\therefore E(x) = 0.0475 \times 2000 \Rightarrow 95$ (Ans)

<u>iii</u>

$P(x<90) = P\left[\frac{x-\mu}{\sigma} < \frac{90-\mu}{\sigma}\right]$

$= \left[Z < \frac{90-100}{6}\right]$

$= [Z < 1.67]$

$= 1 - P[Z \geq 0] + P[\ Z \leq 90]$

$= 1 - 0.5 + 0.4525$

$= \cancel{0.475}\ 0.0475$

$\therefore E(x) = 0.0475 \times 2000$

$= 95$

(Ans)

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d iv

From (iii) $(x < 90) = 0.0475$

$(x > 107) = 0.5 - P\left(\frac{7}{6}\right)$

$= 0.5 - P(1.17)$

$= 0.5 - 0.3790 \Rightarrow 0.121$

$\therefore P(x < 90) + P(x > 107) = 0.0475 + 0.1210$

$= 0.1685$

$\therefore E(x) = 0.1685 \times 2000$

$= 337$ (Ans)

7 (b)

$P(x \leq x_{85}) = 0.8500$

$\Rightarrow P\left(\frac{x_{85} - 100}{6}\right) = 0.850$

$\Rightarrow \frac{x_{85} - 100}{6} = \phi(1.036)$

$= x_{85} = (1.036 \times 6) + 100$

$= 106.216$ (Ans)

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8

a

x = mintues elapsed

so, 30 customers arrived at average 60 mins

1 " " " " $\frac{60}{30}$ "

$\therefore$ 2 " " " " $\frac{60 \times 2}{30}$ "

$= 4$ minute

(Ans)

b

$\frac{1}{\lambda} = 2 \Rightarrow \lambda = \frac{1}{2} = 0.5$

$\therefore P(x>8) = 1 - P(x \leq 8)$

$\Rightarrow 1 - (1 - e^{-\lambda x})$

$= e^{-0.5 \times 8}$

$= 0.0183$ (Ans)

c

$P(x<2) = 1 - e^{-0.5 \times 2}$ $\left[\begin{array}{l} x = 2 \\ \lambda = 0.5 \end{array}\right]$

$= 0.63212$

(Ans)

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d

So, $P(x \le x) = 70\%$.

$$1 - e^{-0.5x} = 0.7$$

$$\Rightarrow x = \frac{\ln(1-0.70)}{-0.5}$$

$$\Rightarrow x = 2.407$$ (Ans).

e

From the Question, we found Mean = 2

Now for Median,

$$P(x \le u) = 0.50$$

$$\Rightarrow 1 - e^{-0.5u} = 0.5$$

$$\Rightarrow x = \frac{\ln(0.5)}{-0.5}$$

$$= 1.3863$$

$\therefore$ Median is less than Mean

$\therefore$ Mean $>$ Median

(Ans)