STA201 Assignment 4 Solution

1. In a simultaneous throw of a pair of fair 6-sided dice, find the probability of getting:

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

a. A sum of 8

For this event, E = { (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) }

So, the probability of getting 8 as the sum, P(E) = $\frac{5}{36}$

b. A doublet (two dice landing on the same value)

For this event, E = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }

So, the probability of getting a double, P(E) = $\frac{6}{36}$

c. A sum greater than 5

For this event, E = {(1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6), (4,2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6,4), (6,5), (6,6)}

So, the probability of getting a sum greater than 5, P(E) = $\frac{26}{36}$ = $\frac{13}{18}$

d. A sum less than 4 or greater than 8

For this event,
E = {(1, 1), (1, 2), (2, 1), (3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5,6), (6,3), (6, 4), (6, 5), (6, 6)}

So, the probability of getting a sum less than 4 or greater than 8, P(E) = $\frac{13}{36}$

e. An even number on the first die

For this event,
E = {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

So, the probability of getting an even number on first, P(E) = $\frac{18}{36}$ = $\frac{1}{2}$

f. An odd number on one and an even number on the other

For this event,
E = {(1,2), (1,4), (1,6), (2,1), (2, 3), (2, 5), (3,2), (3,4), (3,6), (4, 1), (4, 3), (4,5), (5, 2), (5, 4), (5, 6), (6, 1), (6, 3), (6, 5)}

So, the probability of getting an odd number on one and an even number on the other

P(E) = $\frac{18}{36}$

g. At least one 6

For this event,

E = {(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6,4), (6,5), (6,6)}

So, the probability of getting at least one 6, P(E) = $\frac{11}{36}$

h. At least one 6, if the two faces are different

For this event,

E = {(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6,4), (6,5)}

So, the probability of getting at least one 6, if the two faces are different, P(E) = $\frac{10}{36} = \frac{5}{18}$

2. A bag contains 30 balls numbered 1 through 30. Suppose drawing an even numbered ball is considered a 'Success'.

a. Two balls are drawn from the bag with replacement. Find the probability of getting:

i. Two successes

There are 15 even numbers from 1 to 30 and so the probability of selecting a ball having odd number in the first draw is, A = $\frac{15}{30}$.

Since two balls are drawn with replacement, for the second selection there are 15 even numbered balls. So the probability of selecting a ball having even number in the second draw is, B = $\frac{15}{30}$.

Here, the events A & B are independent.

Thus, the probability of getting two successes is,

P(E) = $\frac{15}{30} \times \frac{15}{30}$ = 0.25

ii. exactly one success

One even numbered ball can be selected either in the first draw or in the second draw. Thus, the probability of getting exactly one success is,

P(E) = ($\frac{15}{30} \times \frac{15}{30}$) + ($\frac{15}{30} \times \frac{15}{30}$) = 0.50

iii. at least one success

It can happen during the draw that two even numbered balls are selected.Also, it can occur that one even numbered ball can be selected either in the first draw or in the second draw. Thus, the probability of getting at least one success,

P(E) = ($\frac{15}{30} \times \frac{15}{30}$) + ($\frac{15}{30} \times \frac{15}{30}$) + ($\frac{15}{30} \times \frac{15}{30}$) = 0.75

iv. no successes

Odd, no even numbered can be selected during the drawn and so, the probability of getting no successes is,

P(E) = $\frac{15}{30} \times \frac{15}{30}$ = 0.25

[PTO for 2b]

b. Find the same four probabilities for the experiment where the two balls are drawn without replacement.

i) Two successes

There are 15 even numbers from 1 to 30 and so the probability of selecting a ball having odd number in the first draw is, A = $\frac{15}{30}$.

Since the selected ball earlier is drawn without replacement, for the second selection there are 15 even numbered balls. So the probability of selecting a ball having even number in the second draw is, B = $\frac{14}{29}$.

Thus, the probability of getting two successes,

P(E) = $\frac{15}{30} \times \frac{14}{29} = \frac{7}{29}$

ii) exactly one success

One even numbered ball can be selected either in the first draw or in the second draw. Thus, the probability of getting exactly one success is,

P(E) = $\left(\frac{15}{30} \times \frac{15}{29}\right) + \left(\frac{15}{30} \times \frac{15}{29}\right) = \frac{15}{29}$

i. at least one success

It is possible that two consecutive even numbered balls are selected. Also, it can occur that one even numbered ball can be selected either in the first draw or in the second draw. Thus, the probability of getting at least one success,

P(E) = $\left(\frac{15}{30} \times \frac{14}{29}\right) + \left(\frac{15}{30} \times \frac{15}{29}\right) + \left(\frac{15}{30} \times \frac{15}{29}\right) = \frac{22}{29}$

ii. no successes

Odd, no even numbered ball can be selected during the draw, so the probability of getting no successes is,

P(E) = $\frac{15}{30} \times \frac{14}{29} = \frac{7}{29}$

3. Suppose, you are rolling two regular six-sided dice and two four-sided dice together. Let's say, the sum of the numbers appearing on the two regular six-sided dice is 'A' and the sum of the numbers appearing on the two four-sided dice is 'B'. What is the probability that the product of A and B is 12?

Sample space of rolling a six-sided dice:

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Sample space of rolling a four-sided dice:

	1	2	3	4
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)

A: The sum of the numbers appearing on the two regular six-sided dice
B: The sum of the numbers appearing on the two four-sided dice

There can be four scenario where product of A and B will be 12
Scenario 1: When A = 2 and B = 6
Scenario 2: When A = 3 and B = 4
Scenario 3: When A = 4 and B = 3
Scenario 4: When A = 6 and B = 2

A = 2 will be in the following combinatio
(1, 1)
So, $P(A = 2) = \frac{1}{36}$
A = 3 will be in the following combinatio
(1, 2), (2, 1)
So, $P(A = 3) = \frac{2}{36}$
A = 4 will be in the following combinatio
(1, 3), (2, 2), (3, 1)
So, $P(A = 4) = \frac{3}{36}$
A = 6 will be in the following combinatio
(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)
So, $P(A = 6) = \frac{5}{36}$

B = 6 will be in the following combination
(2, 4), (3, 3), (4, 2)
So, $P(B = 6) = \frac{3}{16}$
B = 4 will be in the following combination
(1, 3), (2, 2), (3, 1)
So, $P(B = 4) = \frac{3}{16}$
B = 3 will be in the following combination
(1, 2), (2, 1)
So, $P(B = 3) = \frac{2}{16}$
B = 2 will be in the following combination
(1, 1)
So, $P(B = 2) = \frac{1}{16}$

Now we calculate the probability of different scenarios

Scenario 1: $P(A = 2)^* P(B = 6)$= $\frac{1}{36} * \frac{3}{16}$= $\frac{3}{576}$= $\frac{1}{192}$

Scenario 2: $P(A = 3)^* P(B = 4)$= $\frac{2}{36} * \frac{3}{16}$= $\frac{6}{576}$= $\frac{1}{96}$

Scenario 3: $P(A = 4)^* P(B = 3)$= $\frac{3}{36} * \frac{2}{16}$= $\frac{6}{576}$= $\frac{1}{96}$

Scenario 4: $P(A = 6)^* P(B = 2)$= $\frac{5}{36} * \frac{1}{16}$= $\frac{5}{576}$

The probability that the product of A and B is 12 will be,

$\frac{1}{192}$+ $\frac{1}{96}$+ $\frac{1}{96}$+ $\frac{5}{576}$= $\frac{5}{144}$

Alternatively:

Taking the sum values for the two types of dice,

Then the sample space for the sum of two six-sided dice will be (Possible values for A):

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

And the sample space for the sum of two four-sided dice will be (Possible values for B):

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Let E = {(A, B)}, where A*B = 12,

E = {(2, 6), (2, 6), (2, 6), (3, 4), (3, 4), (3, 4), (4, 3), (4, 3), (6, 2), (3, 4), (3, 4), (3, 4), (4, 3), (4, 3), (6, 2), (4, 3), (4, 3), (6, 2), (6, 2), (6, 2)}

So, $P(E) = \frac{20}{36 * 16}$= $\frac{20}{576} = \frac{5}{144}$

4. A secondary school is offering two extracurricular classes, one in Photography and the other in Swimming. These classes are open to all of the 250 students in the school. Suppose there are 48 students in the Photography class, 34 in the Swimming class, and 12 who are in both classes. If a student is randomly chosen, what is the probability that this student is not enrolled in any one of these classes?

Here, the probability that the student is enrolled in the Photography class is,
P(F) = $\frac{48}{250}$

And, the probability that the student is enrolled in the Swimming class is,
P(S) = $\frac{34}{250}$

Again, the probability that the student is enrolled in both classes is,
P(E ∩ S) = $\frac{12}{250}$

So, if an upper grade student is randomly chosen, the probability that this student is enrolled in any one of these classes,
P(E U S) = $\frac{48}{250} + \frac{34}{250} - \frac{12}{250}$

Thus, if an upper grade student is randomly chosen, the probability that this student is not enrolled in any one of these classes,
$P(E\ U\ S)^c$ = 1 – [$\frac{48}{250} + \frac{34}{250} - \frac{12}{250}$] = 0.72

5. Assume that the chances of a patient suffering from high blood pressure is 60%. It is also assumed that a course of meditation reduces the risk of high blood pressure by 45% and prescription of certain drugs reduces its chances by 55%. At a time, a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random does not suffer from high blood pressure. Find the probability that the patient chose a course of meditation?

Let us define the events,
A_1: Person is treated with meditation
A_2: Person is treated with drugs
B: Person has high blood pressure

We have to find the probability that the patient followed a course of meditation given that the patient selected at random does not suffer from high blood pressure.

$$P(A_1|B') = \frac{P(B'|A_1)*P(A_1)}{P(B')}$$

It is given that, meditation and drug has equal probabilities,

$$P(A_1) = 0.5,\ P(A_2) = 0.5$$

We know the chances of having high blood pressure without any treatment is 60%

Meditation reduces the risk by 45% so the risk becomes 55% of the original.
Therefore, the probability of having high blood pressure if treated with meditation,
$P(B|A_1) = 0.60 \times 0.55 = 0.33$

The drug reduces the risk by 55% so the risk becomes 45% of the original.
Therefore, the probability of having high blood pressure, if treated with drugs,
$P(B|A_2) = 0.60 \times 0.45 = 0.27$

Therefore,
$P(B) = P(B|A_1) * P\left(A_1\right) + P(B|A_2) * P\left(A_2\right) = (0.33 \times 0.5) + (0.27 \times 0.5) = 0.3$

Finally,
$P(A_1|B') = \frac{P(B'|A_1)*P(A_1)}{P(B')} = \frac{(1-P(B|A_1))*P(A_1)}{1-P(B)} = \frac{(1-0.33)*0.5}{1-0.3} = \frac{67}{140} = 0.4786$

6. Bag A contains 6 red and 7 black balls and Bag B contains 9 red and 6 black balls. One ball is transferred from Bag A to Bag B and then a ball is drawn from Bag B. The ball so drawn is found to be black in color. Find the probability that the transferred ball was red.

Let us define the events as follows:
E_1: ball transferred from Bag A to Bag B is red
E_2: ball drawn from Bag B is black

Probability of transferring a red ball from Bag A, $P(E_1) = \frac{6}{13}$

∴ Probability of transferring a black ball from Bag A, $P(E_1') = \frac{7}{13}$

When a red ball is transferred to Bag B, the probability of drawing a black ball is,

$P\left(E_2|E_1\right) = \frac{6}{16}$

∴ When a black ball is transferred to Bag B, the probability of drawing a black ball is,

$P\left(E_2|E_1'\right) = \frac{7}{16}$

Now, The probability of drawing a black ball from Bag B,

$P(E_2) = P\left(E_2|E_1\right).P\left(E_1\right) + P\left(E_2|E_1'\right).P\left(E_1'\right) = (\frac{6}{16} \times \frac{6}{13}) + (\frac{7}{16} \times \frac{7}{13}) = \frac{85}{208}$

We have to find, Probability of a black ball drawn given that a red ball was transferred,

$P\left(E_1|E_2\right) = \frac{P\left(E_2|E_1\right) \times P\left(E_1\right)}{P\left(E_2\right)} = \frac{36}{85} = 0.4235$

7. There are three coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the three coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

Let us define the events as follows:
E_1: two headed coin is chosen
E_2: fair coin is chosen
E_3: biased coin is chosen
H = Head is shown

Probability of choosing any of the coins will be the same. Therefore
$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Probability of getting head from the two headed coin, $P(H|E_1) = 1$

Probability of getting head from the fair coin, $P(H|E_2) = \frac{1}{2}$

Probability of getting head from the biased coin, $P(H|E_3) = \frac{3}{4}$

$P(H) = P(H|E_1) * P(E_1) + P(H|E_2) * P(E_2) + P(H|E_3) * P(E_3)$
$P(H) = (1 * \frac{1}{3}) + (\frac{1}{2} * \frac{1}{3}) + (\frac{3}{4} * \frac{1}{3}) = \frac{3}{4} = \frac{3}{4}$

Now, if a coin was flipped randomly and it showed head, then the probability of that coin being a two-headed coin will be,
$P(E_1|H) = \frac{P(H|E_1) \times P(E_1)}{P(H)} = \frac{1 * \frac{1}{3}}{\frac{3}{4}} = \frac{4}{9} = 0.44$