STAA 551 Case Study

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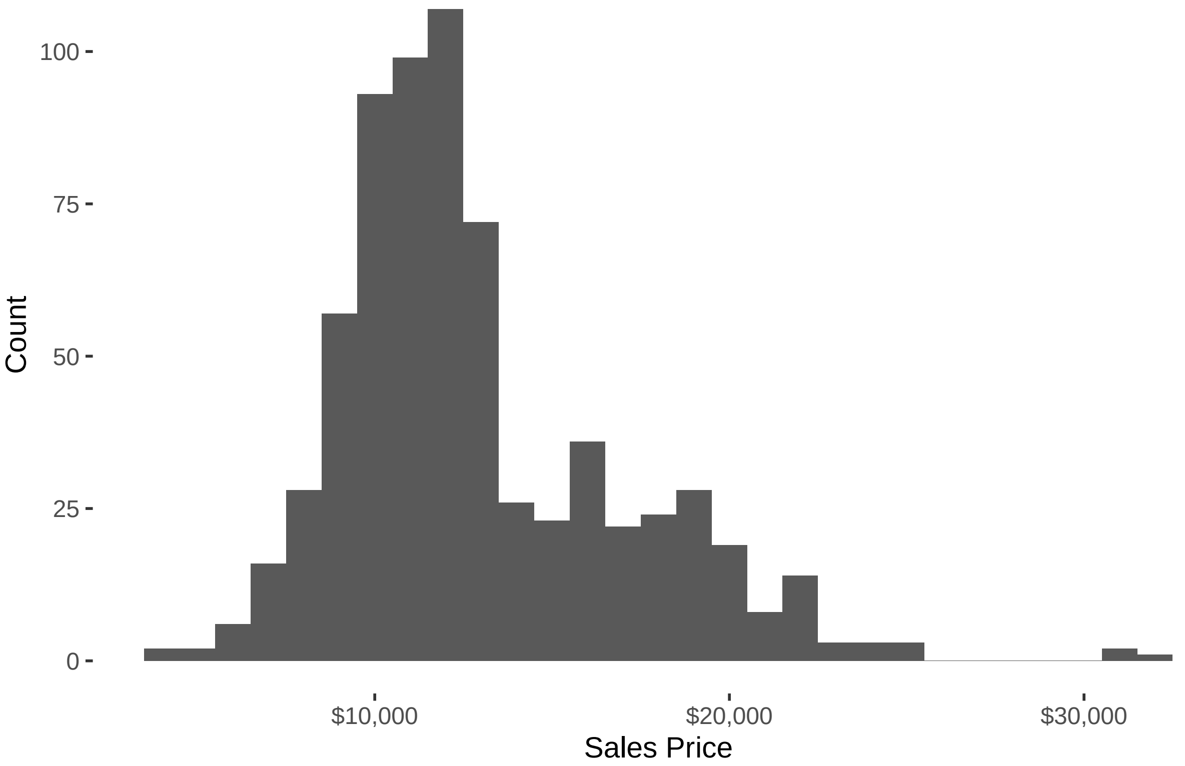
Introduction

Consumers shopping for new cars often have the option to trade in their current car for money towards their new one. Dealerships, in turn, sell the used cars at a slightly higher price. To do this effectively, the dealership must be able to accurately estimate the expected price that it can sell each used car for based only on characteristics of the car.



Distribution of Used Toyota Corolla Sales Prices (Figure 1)

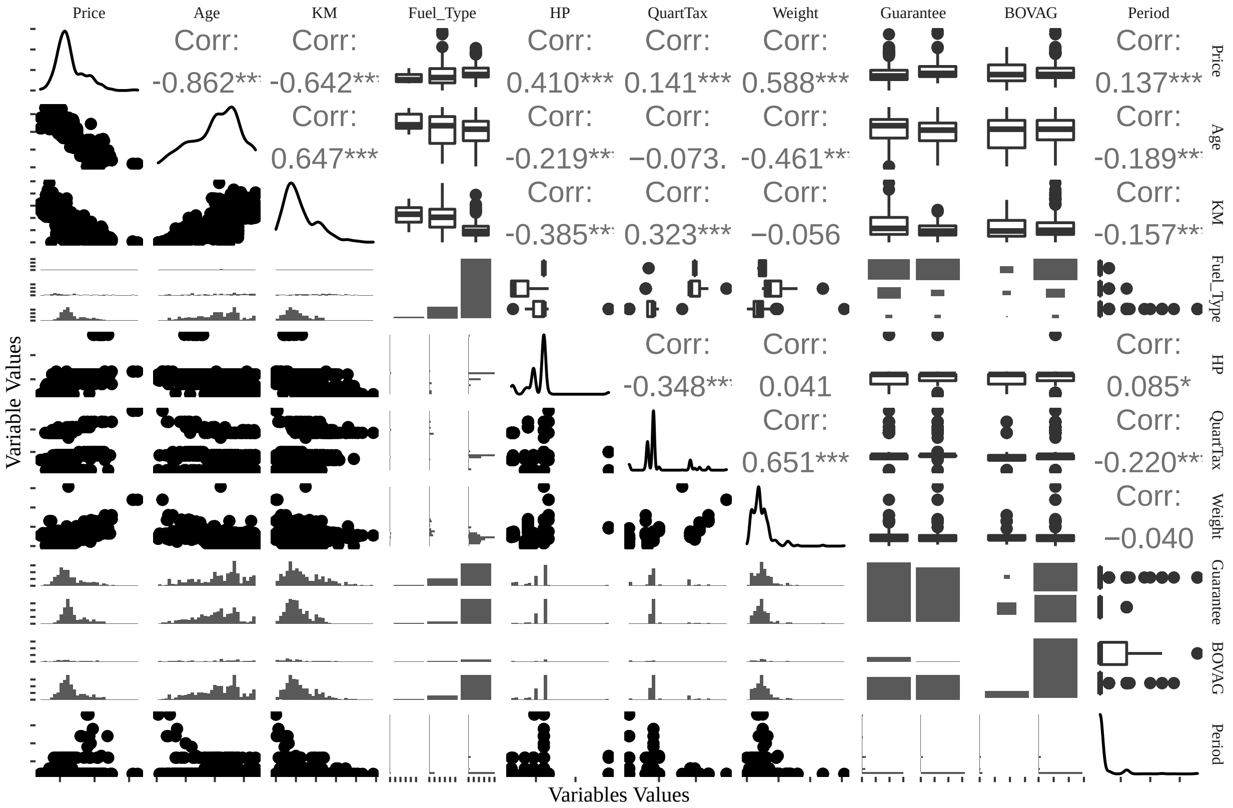
A Toyota dealership was interested in fitting a regression model to estimate `Price` (Figure 1) using 16 different predictor variables and a data set of 694 used Toyota Corolla sales. Numeric predictors included `Age`, `KM` (odometer reading), `HP` (horsepower), `CC` (cylinder volume), `QuartTax` (quarterly road tax), `Weight` and `Period` (guarantee period). Categorical variables included `Mfg_Month` (manufacturing month), `Mfg_Year` (manufacturing year from 1999-2004), `Fuel_type` (CNG, Diesel, or Petrol), `Metallic`, `Automatic`, `Doors` (3, 4, or 5), `Gears` (5 or 6), `Guarantee` (manufacturer's guarantee), and `BOVAG` (BOVAG guarantee). The `Cylinders` predictor was removed from the original data set because all recorded cars had 4 cylinders, and all categorical predictors were made into factors in `R`. No other cleaning was performed on the data.

Table 1: Summary Statistics

Variable	Mean	Std. Deviation	Minimum	Maximum
Age	41.31	15.77	1	68
BOVAG	1.90	0.30	1	2
Fuel_Type	2.80	0.44	1	3
Guarantee	1.48	0.50	1	2
HP	102.49	16.95	69	192
KM	60614.31	41309.23	1	243000
Period	4.03	3.81	3	36
Price	12928.45	4012.51	4350	32500
QuartTax	96.47	48.92	19	283
Weight	1090.51	62.13	1000	1615

A set of practical predictors of interest were chosen based on contextual knowledge and exploratory data visualizations to use for model fitting separately of any formal variable selection techniques. Intuitively, older cars (indicated by `Age` and `KM`) should be worth less and larger and more powerful cars (indicated by `Weight` and `HP`, respectively) should be worth more. It was assumed that cars that take different fuel types (`Fuel_Type`) are likely different kinds of cars and would likely vary in price in some way. Finally, `Guarantee`, `BOVAG`, and `Period` are financial-based incentives that should also intuitively lead to a higher car price. Summary statistics (Table 1) and plots (Figure 2) for these predictors of interest are shown below.

Paired Plots and Correlations (Figure 2)



Analysis

The relationships between many of the predictors and the response variable of interest, car price, was largely unknown. Aside from a selection of predictors deemed to likely be important based on contextual knowledge, it was not known which predictors should be included in a regression model. Therefore, an array of models were fit using various transformations, variable selection methods, and prior assumptions. All models were fit with the `stan_glm` function in the `rstanarm` package using a Bayesian framework that incorporates prior belief into into traditional regression by use of prior distributions. After fitting, all models were assessed for explanatory power, overfitting, and violations of model assumptions by comparing R^2 and R^2_{LOO} and by generating QQ-Normal plots for residuals and residual vs fitted values plots.

To begin, a full model (1) was fit using the default `stan_glm` priors ($N(0, 6.25)$) to serve as a comparative baseline for future models. The regression intercept was estimated to be $\beta_0 = -4,173.88$ and can be interpreted as the expected price (in dollars) of 3-door, manual, non-metallic, CNG-fueled car that was manufactured in January of 1999 with no manufacturer or BOVAG guarantee and a value of 0 for all numeric predictors included in the model. The regression coefficients, β_j, could be interpreted as the expected difference in expected car price associated with a one unit difference in the predictor (or associated with whether or not an observation assumes that specific level of a categorical predictor), with all other predictors remaining constant. These interpretations are unreasonable in a practical context and it makes no sense that the price of a car could be estimated to be a negative value. Therefore, another full model (2) was fit, also using the default `stan_glm` priors, but now with scaled numeric predictors and a log-transformation of car price. This was done to make the interpretation of coefficients more practical and comparable, and to restrict predictions of price to positive values. There may also be potential for modeling benefits from scaling. Then, regression coefficients for numeric predictors could be interpreted as the expected percent difference in expected car price associated with a one standard deviation increase in the value of the predictors, with all other predictors remaining constant.

The full models fitted utilized 32 predictors (16, not including dummy variables) for modeling. Using a large number of predictors can be damaging to a model due to redundancies in predictor information (multicollinearity) and the potential for overfitting. Several different variable selection methods were applied to the car prices data set to determine which predictors were most closely related with car price. First, a Bayesian variable selection method called a horseshoe prior was used to systematically exclude unimportant predictors from the model. A horseshoe prior places a large amount of prior mass near 0, forcing predictors to be highly related with the response to be included in the model. This model (3) determined 14 predictors (12, not including dummy variables) to be significant in predicting car price: `Age`, `Mfg_Month`, `Mfg_Year`, `KM`, `Fuel_Type`, `HP`, `Metallic`, `QuartTax`, `Weight`, `Guarantee`, `BOVAG`, and `Period`. A follow-up model (4) was fit using only these selected predictors, with only an inconsequential loss in R^2. Next, a model (5) was fit using a variable selection method called LASSO that penalizes a model for producing large coefficient estimates, forcing it to be selective with how it chooses estimates (equivalent to using a Laplace prior). This model determined 15 predictors (12, not including dummy variables) to be significant in predicting car price. These were the same predictors chosen by the horseshoe prior model previously fit. A follow-up model (6) was fit using only these selected predictors, again with only an inconsequential loss in R^2 (DO WE NEED THIS MODEL?? ISNT IT THE SAME AS 4??).

Thirdly, a more contextual approach was taken to variable selection and the predictors selected previously based on practical knowledge and data visualizations were used for modeling. These predictors were selected independently of the results of the horseshoe and LASSO selections, but aligned closely with these selections nonetheless. First, a model (7) was fit using these predictors and the `stan_glm` default priors. A follow-up model (8) was then fit using using weakly informative priors (Table 2) informed by prior knowledge and exploratory data visualizations. This model showed slightly less evidence of overfitting than the previous model but was otherwise similar. Finally, two models (9) & (10) were fit to explore potential interactions of interest. The first of these models included an interaction between `Age` and `KM`, and the second included an interaction between `Weight` and `HP`.

Table 2: Prior Parameters

Variable	Prior Mean	Prior Std. Deviation
Age	-100	625
BOVAG Guarantee	0	100
Fuel (Diesel)	1000	62500
Fuel (Petrol)	-1000	62500
Guarantee Period	1000	62500
HP	50	100
KM	-0.0625	0.125
Manufacturer Guarantee	1000	62500
Quarterly Tax	100	625
Weight	10	0.125

Results