Fitted Models

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```
set.seed(551)
```

Import Data

```
# read data from .csv file
data_raw <- read.csv('ToyotaCorollaData.csv')

data <- data_raw %>%
  # factor categorical predictors
  mutate_at(c('Metallic', 'Automatic', 'Doors', 'Cylinders', 'Gears',
              'Guarantee', 'BOVAG', 'Fuel_Type', 'Mfg_Month', 'Mfg_Year'),
            as.factor) %>%
  # remove singular predictor
  select(-Cylinders)

head(data)
```

```
##   Price Age Mfg_Month Mfg_Year    KM Fuel_Type HP Metallic Automatic   CC Doors
## 1 13500  23        10     2002 46986    Diesel 90        1         0 2000     3
## 2 13750  23        10     2002 72937    Diesel 90        1         0 2000     3
## 3 13950  24         9     2002 41711    Diesel 90        1         0 2000     3
## 4 14950  26         7     2002 48000    Diesel 90        0         0 2000     3
## 5 13750  30         3     2002 38500    Diesel 90        0         0 2000     3
## 6 12950  32         1     2002 61000    Diesel 90        0         0 2000     3
##   Gears QuartTax Weight Guarantee BOVAG Period
## 1     5      210   1165         0     1      3
## 2     5      210   1165         0     1      3
## 3     5      210   1165         1     1      3
## 4     5      210   1165         1     1      3
## 5     5      210   1170         1     1      3
## 6     5      210   1170         0     1      3
```

Scale data and log transform `Price`.

```
# log transform Price and scale predictors
data_scale <- cbind(log(data$Price),
                    (data %>%
                       select(-Price) %>%
                       mutate_if(is.numeric, scale))) %>%
```

```
  as.data.frame()
names(data_scale) <- names(data)
```

Diagnostic Function

```
resultsDF <- data.frame(model = 1:10,
                        predictors = rep(NA, 10),
                        R2 = rep(NA, 10),
                        LOO_R2 = rep(NA, 10),
                        LOO_CV = rep(NA, 10))

# function to assess model and display examination results
diagFit <- function(fit, modelNum, results, printPlots) {
  # print model fit
  print(fit,
        digits = 3,
        detail = FALSE)

  # extract number of predictors
  p <- length(fit$coefficients) - 1
  results$predictors[which(results$model == modelNum)] <- p

  # print model R^2 scores
  bayesR2 <- round(mean(bayes_R2(fit)), 3)
  results$R2[which(results$model == modelNum)] <- bayesR2
  print(paste('R^2: ', bayesR2))
  looR2 <- round(mean(loo_R2(fit)), 3)
  results$LOO_R2[which(results$model == modelNum)] <- looR2
  print(paste('LOO R^2: ', looR2))

  # print mode LOO CV score
  looFit <- loo(fit, k_threshold = 0.7)
  elpd <- round(looFit$estimates[1, 1], 2)
  # correction for log transformation of response
  if (elpd < 0) {
    looFit$pointwise[, 1] <- looFit$pointwise[, 1] + data_scale$Price
    elpd <- round(sum(looFit$pointwise[, 1]), 2)
  }
  results$LOO_CV[which(results$model == modelNum)] <- elpd
  print(paste('LOO ELPD: ', elpd))

  # generate QQ-Norm plot
  n <- length(fit$residuals)
  quants <- qnorm((1:n / n))

  plt_QQ <- ggplot(mapping = aes(x = quants,
                                 y = sort(scale(fit$residuals)))) +
    geom_point(alpha = .5) +
    geom_abline(intercept = 0,
                slope = 1,
                col = 'red',
```

```
                size = .2) +
    labs(title = 'QQ-Norm Plot',
         x = 'Theoretical Quantiles',
         y = 'Sorted Residuals')

  # generate residuals vs fitted values plot
  plt_res_fit <- ggplot(mapping = aes(x = fit$fitted.values,
                                      y = fit$residuals)) +
    geom_point(alpha = .5) +
    geom_hline(yintercept = 0,
               col = 'red',
               size = .2) +
    labs(title = 'Residuals vs Fitted Values Plot',
         x = 'Fitted Values',
         y = 'Residuals')

  # print plots if desired
  if (printPlots) {
    print(plt_QQ)
    print(plt_res_fit)
  }

  return(results)
}

# Bayesian simulation size
N <- 5000

# print model diagnostic plots?
printPlots <- FALSE
```

Model 1: Full, Unscaled, Default Priors

```
fit1 <- stan_glm(Price ~ .,
                 data = data,
                 refresh = 0,
                 iter = N)

resultsDF <- diagFit(fit1, 1, resultsDF, printPlots)
```

```
##                 Median    MAD_SD
## (Intercept)  -4173.885 23006.583
## Age            -84.341   336.838
## Mfg_Month2     205.219   384.232
## Mfg_Month3     348.234   702.700
## Mfg_Month4     146.004  1022.252
## Mfg_Month5     506.995  1357.477
## Mfg_Month6     211.164  1692.619
## Mfg_Month7     -85.699  2014.251
## Mfg_Month8      90.152  2359.808
## Mfg_Month9    -383.527  2704.662
```

```
## Mfg_Month10      -325.799  3022.337
## Mfg_Month11      -351.001  3380.437
## Mfg_Month12      -145.222  3683.105
## Mfg_Year2000     -185.908  4036.807
## Mfg_Year2001     -314.582  8079.371
## Mfg_Year2002     1617.607 12158.539
## Mfg_Year2003     2735.279 16201.595
## Mfg_Year2004     3915.591 20215.338
## KM                 -0.020     0.002
## Fuel_TypeDiesel  1075.313   414.333
## Fuel_TypePetrol  1688.934   438.442
## HP                 50.020     4.657
## Metallic1         179.532   109.084
## Automatic1        425.403   254.684
## CC                  0.075     0.092
## Doors4            -25.385   202.744
## Doors5             90.795   115.540
## Gears6             95.111   339.885
## QuartTax           13.535     2.314
## Weight             11.324     1.706
## Guarantee1        370.747   107.178
## BOVAG1            410.389   189.980
## Period             25.924    14.765
##
## Auxiliary parameter(s):
##       Median   MAD_SD
## sigma 1258.207   34.772
## [1] "R^2:  0.902"
## [1] "LOO R^2:  0.89"
## [1] "LOO ELPD:  563.82"
```

Model 2: Full, Scaled, Default Priors

```
fit2 <- stan_glm(Price ~ .,
                 data = data_scale,
                 refresh = 0,
                 iter = N)

resultsDF <- diagFit(fit2, 2, resultsDF, printPlots)
```

```
##                 Median MAD_SD
## (Intercept)      9.109  0.670
## Age             -0.086  0.396
## Mfg_Month2       0.024  0.030
## Mfg_Month3       0.038  0.053
## Mfg_Month4       0.018  0.078
## Mfg_Month5       0.041  0.102
## Mfg_Month6       0.027  0.127
## Mfg_Month7       0.006  0.152
## Mfg_Month8       0.024  0.177
## Mfg_Month9      -0.025  0.203
## Mfg_Month10     -0.009  0.228
```

```
## Mfg_Month11     -0.010  0.255
## Mfg_Month12      0.016  0.278
## Mfg_Year2000     0.025  0.303
## Mfg_Year2001     0.038  0.602
## Mfg_Year2002     0.181  0.906
## Mfg_Year2003     0.237  1.202
## Mfg_Year2004     0.235  1.506
## KM              -0.078  0.007
## Fuel_TypeDiesel  0.037  0.034
## Fuel_TypePetrol  0.162  0.034
## HP               0.051  0.006
## Metallic1        0.021  0.009
## Automatic1       0.034  0.021
## CC               0.004  0.004
## Doors4          -0.007  0.016
## Doors5           0.016  0.009
## Gears6           0.015  0.028
## QuartTax         0.076  0.009
## Weight           0.032  0.008
## Guarantee1       0.030  0.009
## BOVAG1           0.054  0.015
## Period           0.016  0.005
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 0.102  0.003
## [1] "R^2:  0.882"
## [1] "LOO R^2:  0.87"
## [1] "LOO ELPD:  568.85"
```

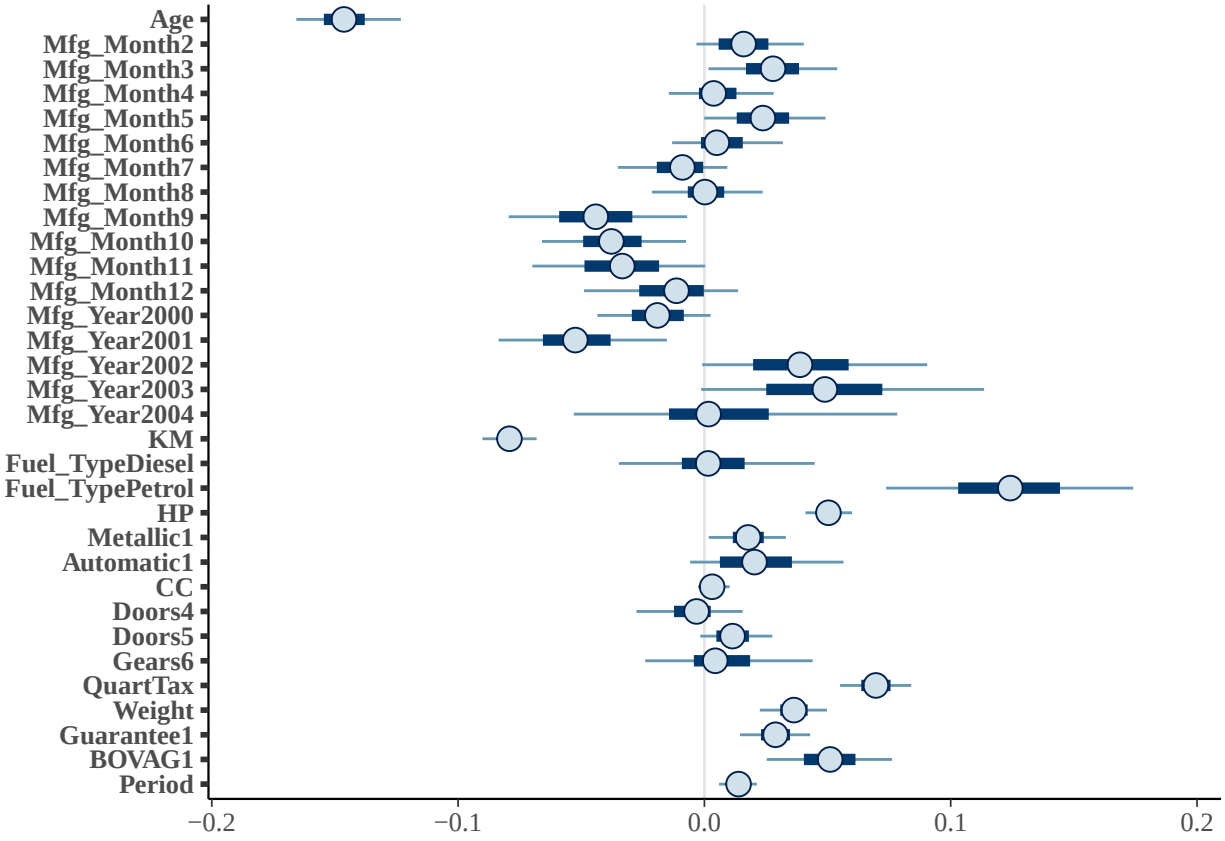
Model 3: Full, Scaled, Horseshoe Prior

```
p <- ncol(data) - 1
n <- nrow(data)

p0 <- 6

slab_scale <- sqrt(0.3 / p0) * sd(data_scale$Price)
global_scale <- (p0 / (p - p0)) / sqrt(n)

fit3 <- stan_glm(Price ~ .,
                 data = data_scale,
                 refresh = 0,
                 iter = N,
                 prior = hs(global_scale = global_scale,
                            slab_scale = slab_scale))

resultsDF <- diagFit(fit3, 3, resultsDF, printPlots)
```

```
##                 Median MAD_SD
## (Intercept)      9.251  0.036
## Age             -0.146  0.012
```

```
## Mfg_Month2       0.016  0.015
## Mfg_Month3       0.028  0.016
## Mfg_Month4       0.004  0.011
## Mfg_Month5       0.024  0.016
## Mfg_Month6       0.005  0.012
## Mfg_Month7      -0.009  0.013
## Mfg_Month8       0.000  0.011
## Mfg_Month9      -0.044  0.022
## Mfg_Month10     -0.038  0.018
## Mfg_Month11     -0.033  0.022
## Mfg_Month12     -0.011  0.018
## Mfg_Year2000    -0.019  0.016
## Mfg_Year2001    -0.052  0.020
## Mfg_Year2002     0.039  0.029
## Mfg_Year2003     0.049  0.035
## Mfg_Year2004     0.002  0.030
## KM              -0.079  0.007
## Fuel_TypeDiesel  0.002  0.019
## Fuel_TypePetrol  0.124  0.031
## HP               0.050  0.006
## Metallic1        0.018  0.009
## Automatic1       0.020  0.022
## CC               0.003  0.004
## Doors4          -0.003  0.011
## Doors5           0.011  0.010
## Gears6           0.004  0.016
## QuartTax         0.070  0.009
## Weight           0.036  0.008
## Guarantee1       0.029  0.009
## BOVAG1           0.051  0.015
## Period           0.014  0.005
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 0.102  0.003
## [1] "R^2:  0.88"
## [1] "LOO R^2:  0.87"
## [1] "LOO ELPD:  572.39"
```

```
as.data.frame(fit3) %>%
  select(-c('(Intercept)', 'sigma')) %>%
  mcmc_intervals()
```



Model 4: Horseshoe-Selected, Scaled, Default Priors

```
fit4 <- stan_glm(Price ~ Age + Mfg_Month + Mfg_Year + KM + Fuel_Type + HP +
                   Metallic + QuartTax + Weight + Guarantee + Period,
                 data = data_scale,
                 refresh = 0,
                 iter = N)

resultsDF <- diagFit(fit4, 4, resultsDF, printPlots)
```

```
##                 Median MAD_SD
## (Intercept)      9.174  0.655
## Age             -0.106  0.385
## Mfg_Month2       0.025  0.029
## Mfg_Month3       0.041  0.052
## Mfg_Month4       0.019  0.075
## Mfg_Month5       0.041  0.099
## Mfg_Month6       0.024  0.122
## Mfg_Month7       0.006  0.146
## Mfg_Month8       0.024  0.171
## Mfg_Month9      -0.030  0.197
## Mfg_Month10     -0.018  0.220
## Mfg_Month11     -0.018  0.243
## Mfg_Month12      0.009  0.269
## Mfg_Year2000     0.007  0.294
```

```
## Mfg_Year2001     0.009  0.586
## Mfg_Year2002     0.133  0.876
## Mfg_Year2003     0.172  1.171
## Mfg_Year2004     0.157  1.463
## KM              -0.076  0.007
## Fuel_TypeDiesel  0.027  0.034
## Fuel_TypePetrol  0.188  0.036
## HP               0.050  0.006
## Metallic1        0.021  0.009
## QuartTax         0.084  0.009
## Weight           0.038  0.008
## Guarantee1       0.035  0.009
## Period           0.011  0.004
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 0.103  0.003
## [1] "R^2:  0.88"
## [1] "LOO R^2:  0.869"
## [1] "LOO ELPD:  569.46"
```

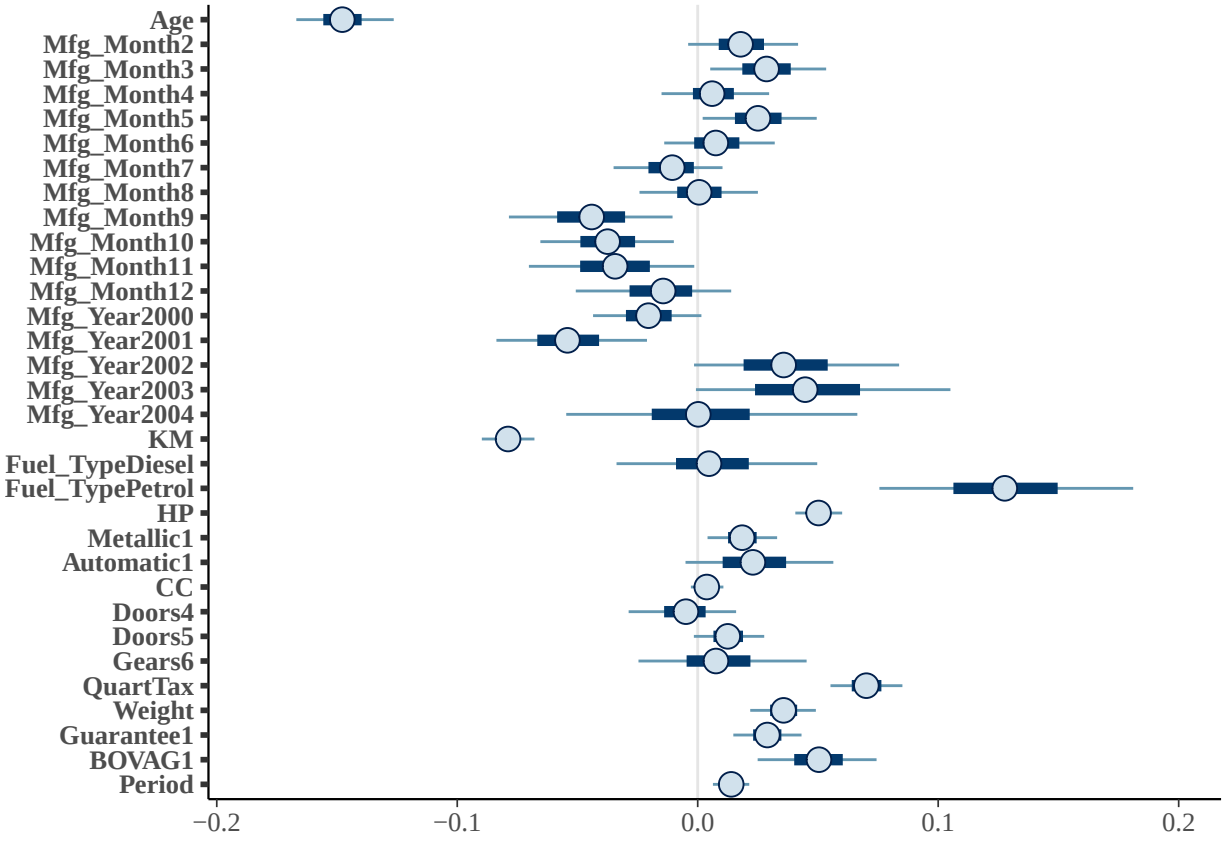
Model 5: Full, Scaled, LASSO Prior

```
fit5 <- stan_glm(Price ~ .,
                 data = data_scale,
                 refresh = 0,
                 iter = N,
                 prior = lasso())

resultsDF <- diagFit(fit5, 5, resultsDF, printPlots)
```

```
##                 Median MAD_SD
## (Intercept)      9.249  0.036
## Age             -0.148  0.012
## Mfg_Month2       0.018  0.014
## Mfg_Month3       0.029  0.015
## Mfg_Month4       0.006  0.013
## Mfg_Month5       0.025  0.014
## Mfg_Month6       0.008  0.014
## Mfg_Month7      -0.011  0.014
## Mfg_Month8       0.001  0.014
## Mfg_Month9      -0.044  0.021
## Mfg_Month10     -0.037  0.017
## Mfg_Month11     -0.034  0.021
## Mfg_Month12     -0.014  0.019
## Mfg_Year2000    -0.020  0.014
## Mfg_Year2001    -0.054  0.019
## Mfg_Year2002     0.036  0.026
## Mfg_Year2003     0.045  0.032
## Mfg_Year2004     0.000  0.030
## KM              -0.079  0.007
## Fuel_TypeDiesel  0.005  0.022
```

```
## Fuel_TypePetrol  0.128  0.032
## HP               0.050  0.006
## Metallic1        0.018  0.009
## Automatic1       0.023  0.019
## CC               0.004  0.004
## Doors4          -0.005  0.013
## Doors5           0.013  0.009
## Gears6           0.008  0.020
## QuartTax         0.070  0.009
## Weight           0.036  0.008
## Guarantee1       0.029  0.009
## BOVAG1           0.050  0.015
## Period           0.014  0.005
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 0.102  0.003
## [1] "R^2:  0.881"
## [1] "LOO R^2:  0.871"
## [1] "LOO ELPD:  572.17"
```

```
as.data.frame(fit5) %>%
  select(-c('(Intercept)', 'sigma')) %>%
  mcmc_intervals()
```



Model 6: LASSO-Selected, Scaled, Default Priors

```
fit6 <- stan_glm(Price ~ Age + Mfg_Month + KM + Fuel_Type + HP + QuartTax +
                   Weight + Guarantee + BOVAG + Period,
                 data = data_scale,
                 refresh = 0,
                 iter = N)

resultsDF <- diagFit(fit6, 6, resultsDF, printPlots)
```

```
##                  Median MAD_SD
## (Intercept)       9.215  0.036
## Age              -0.162  0.007
## Mfg_Month2        0.021  0.016
## Mfg_Month3        0.029  0.017
## Mfg_Month4        0.001  0.017
## Mfg_Month5        0.022  0.017
## Mfg_Month6       -0.001  0.018
## Mfg_Month7       -0.019  0.017
## Mfg_Month8        0.003  0.018
## Mfg_Month9       -0.049  0.022
## Mfg_Month10      -0.043  0.018
## Mfg_Month11      -0.043  0.022
## Mfg_Month12      -0.045  0.024
## KM               -0.069  0.007
## Fuel_TypeDiesel   0.018  0.034
## Fuel_TypePetrol   0.183  0.036
## HP                0.046  0.006
## QuartTax          0.072  0.009
## Weight            0.057  0.008
## Guarantee1        0.031  0.009
## BOVAG1            0.049  0.016
## Period            0.016  0.005
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 0.106  0.003
## [1] "R^2:  0.872"
## [1] "LOO R^2:  0.86"
## [1] "LOO ELPD:  546.19"
```

Model 7: Selective, Scaled, Default Priors

```
fit7 <- stan_glm(Price ~ Age + KM + Fuel_Type + HP + QuartTax + Weight +
                   Guarantee + BOVAG + Period,
                 data = data_scale,
                 refresh = 0,
                 iter = N)

resultsDF <- diagFit(fit7, 7, resultsDF, printPlots)
```

```
##                 Median MAD_SD
## (Intercept)      9.198  0.036
## Age             -0.155  0.007
## KM              -0.071  0.007
## Fuel_TypeDiesel  0.019  0.036
## Fuel_TypePetrol  0.195  0.037
## HP               0.047  0.006
## QuartTax         0.073  0.010
## Weight           0.061  0.008
## Guarantee1       0.028  0.009
## BOVAG1           0.053  0.016
## Period           0.017  0.005
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 0.108  0.003
## [1] "R^2:  0.867"
## [1] "LOO R^2:  0.857"
## [1] "LOO ELPD:  538.93"
```

Model 8: Selective, Unscaled, Weakly Informative Priors

```
priorMeans <- c(-100, -.0625, 1000, -1000, 50, 100, 10, 1000, 0, 1000)
priorVars <- c(625, .125, 62500, 62500, 100, 625, .125, 62500, 100, 62500)

fit8 <- stan_glm(Price ~ Age + KM + Fuel_Type + HP + QuartTax + Weight +
                   Guarantee + BOVAG + Period,
                 data = data,
                 refresh = 0,
                 iter = N,
                 prior = normal(priorMeans,
                                priorVars))

resultsDF <- diagFit(fit8, 8, resultsDF, printPlots)
```

```
##                  Median   MAD_SD
## (Intercept)     -870.040  803.751
## Age             -151.073    5.184
## KM                -0.016    0.002
## Fuel_TypeDiesel 1537.554  462.882
## Fuel_TypePetrol 2401.225  490.257
## HP                56.172    4.155
## QuartTax          18.417    2.468
## Weight            10.054    0.128
## Guarantee1       274.489  118.584
## BOVAG1            62.257   91.575
## Period            33.799   16.142
##
## Auxiliary parameter(s):
##       Median   MAD_SD
## sigma 1463.010   39.630
## [1] "R^2:  0.865"
```

```
## [1] "LOO R^2:  0.863"
## [1] "LOO ELPD:  488.67"
```

Model 9: Selective with 1 Interaction, Scaled, Default Priors

```
fit9 <- stan_glm(Price ~ Fuel_Type + HP + QuartTax + Weight +
                   Guarantee + BOVAG + Period + Age*KM,
                 data = data_scale,
                 refresh = 0,
                 iter = N)

resultsDF <- diagFit(fit9, 9, resultsDF, printPlots)
```

```
##                 Median MAD_SD
## (Intercept)      9.194  0.035
## Fuel_TypeDiesel  0.023  0.035
## Fuel_TypePetrol  0.195  0.036
## HP               0.048  0.006
## QuartTax         0.074  0.009
## Weight           0.059  0.008
## Guarantee1       0.029  0.009
## BOVAG1           0.053  0.015
## Period           0.015  0.005
## Age             -0.153  0.007
## KM              -0.075  0.007
## Age:KM           0.007  0.005
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 0.108  0.003
## [1] "R^2:  0.867"
## [1] "LOO R^2:  0.857"
## [1] "LOO ELPD:  539.57"
```

Model 10: Selective with 2 Interactions, Scaled, Default Priors

```
fit10 <- stan_glm(Price ~ Fuel_Type + HP + QuartTax + Weight +
                    Guarantee + BOVAG + Period + Age*KM + Weight*HP,
                  data = data_scale,
                  refresh = 0,
                  iter = N)

resultsDF <- diagFit(fit10, 10, resultsDF, printPlots)
```

```
##                 Median MAD_SD
## (Intercept)      9.189  0.036
## Fuel_TypeDiesel  0.030  0.036
## Fuel_TypePetrol  0.199  0.036
## HP               0.057  0.007
```

```
## QuartTax          0.074  0.009
## Weight            0.060  0.008
## Guarantee1        0.028  0.009
## BOVAG1            0.053  0.016
## Period            0.015  0.005
## Age              -0.152  0.007
## KM               -0.076  0.007
## Age:KM            0.008  0.005
## HP:Weight        -0.008  0.005
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 0.108  0.003
## [1] "R^2:  0.867"
## [1] "LOO R^2:  0.857"
## [1] "LOO ELPD:  539.63"
```

Results

```
resultsDF
```

```
##    model predictors    R2 LOO_R2 LOO_CV
## 1      1         32 0.902  0.890 563.82
## 2      2         32 0.882  0.870 568.85
## 3      3         32 0.880  0.870 572.39
## 4      4         26 0.880  0.869 569.46
## 5      5         32 0.881  0.871 572.17
## 6      6         21 0.872  0.860 546.19
## 7      7         10 0.867  0.857 538.93
## 8      8         10 0.865  0.863 488.67
## 9      9         11 0.867  0.857 539.57
## 10    10         12 0.867  0.857 539.63
```

```
loo1 <- loo(fit1, k_threshold = 0.7)
loo2 <- loo(fit2, k_threshold = 0.7)
loo3 <- loo(fit3, k_threshold = 0.7)
loo4 <- loo(fit4, k_threshold = 0.7)
loo5 <- loo(fit5, k_threshold = 0.7)
loo6 <- loo(fit6, k_threshold = 0.7)
loo7 <- loo(fit7, k_threshold = 0.7)
loo8 <- loo(fit8, k_threshold = 0.7)
loo9 <- loo(fit9, k_threshold = 0.7)
loo10 <- loo(fit10, k_threshold = 0.7)

loo1$pointwise[, 1] <- loo1$pointwise[, 1] + data_scale$Price
loo8$pointwise[, 1] <- loo8$pointwise[, 1] + data_scale$Price

loo_compare(loo1, loo2, loo3, loo4, loo5, loo6, loo7, loo8, loo9, loo10)
```

```
##       elpd_diff se_diff
## fit3    0.0       0.0
```

```
## fit5   -0.8       1.1
## fit4   -3.3       5.2
## fit2   -4.4       3.5
## fit6  -26.5      14.9
## fit9  -32.5      15.3
## fit10 -34.4      16.3
## fit7  -35.0      18.5
## fit1  -13.1      29.2
## fit8  -84.6      23.9
```