IN-STK5000 Project 2

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1 Introduction

In the following we will explain and discuss our procedure for development of a *vaccine* strategy against the Coronavirus disease (covid). We use simulated data containing the features age, gender, income and 128 genes. As outcome the data includes covid status (positive or negative) and 10 symptoms of relevant comorbidities.

We specify a policy that decides whether to give one of three vaccines or no vaccine. The data simulation is non-deterministic with respect to outcomes thus necessitating a stochastic predictive model of the outcomes. In order to optimize the policy we define a utility function that the policy aims to maximize. We also consider privacy issues and fairness of our vaccine policy. Finally, we experiment with changes to our policy with the goal of improvement and we also test the robustness of the policy using a new simulated data set.

The following report is structured as follows. In the subsequent preliminary sections, we introduce notations and define policies as well as the utility function considered. In Section 2, we perform a privacy analysis and estimate the privacy-utility trade-off in Section 2.4. Section 3 addresses the fairness of a given vaccination strategy. Here, we consider the notion of balanced decisions as our fairness framework. We then measure the potential imbalance of a decision rule and also measure the variation in outcomes. In Section 4, we estimate the historical policy and provide our own improved policy. We compare both policies on the historical data as well as simulated data.

1.1 Preliminaries

Let $\mathcal{X}$ denote some feature space and let $\mathbf{x} \in \mathcal{X}^N$ denote the features of a population of N individuals, enumerated by $[N] = \{1, \dots, N\}$. Let $\mathcal{Y}$ denote some outcome space and let $\mathbf{y} \in \mathcal{Y}^N$ be the outcomes of individuals $[N]$. We will consider the case where $\mathbf{y} \in \mathbb{R}^{N \times d}$, where d is the number of relevant outcomes. This means that each individual i has some outcome $\mathbf{y}_i \in \mathbb{R}^d$. For example, $\mathbf{y}_{i,j}$ could correspond to whether individual i is covid-positive. Then, a decision-maker will assign this event (individual i is covid-positive) a scalar utility. The formally introduce the utility of a specific decision rule, we will first have to define policies and policy spaces.

1.2 Policies

Let $\mathcal{A} = \{0, 1\}^3$ denote the action space and let $\mathbf{a} \in \mathcal{A}$ so that $\mathbf{a}_i = 1$ if vaccine i is planned to be given under action $\mathbf{a}$. Let Π be the space of policies $\pi : \mathcal{X} \to \Delta(\mathcal{A})$, i.e. a mapping from feature vectors $\mathbf{x}$ to distributions over $\mathcal{A}$. We then write $\pi(\mathbf{a} \mid \mathbf{x})$ to denote the probability of action $\mathbf{a}$ given features $\mathbf{x}$. Since the number of vaccinations available at a given time step may be restricted, we distribute the vaccine only with probability $p \in [0, 1]$. In this way, in expectation we distribute only Np vaccines.

1.3 Predictive Model

Note that since we do not know the future outcomes $\mathbf{y}$ given a decision $\mathbf{a}$ and population $\mathbf{x}$, we must predict the outcomes using some model $\mathbb{P}(\mathbf{y} \mid \mathbf{x}, \mathbf{a})$. We will use the same predictive model as in Project 1.

1.4 The Utility Function

For an action a, a feature vector x and an outcome $\mathbf{y}$, we define the utility by some function

$$u : X \times \mathcal{A} \times Y \to \mathbb{R}.$$

According to our utility, for each individual, the utility (initialized to 0) is decreased by 2 if the person gets a vaccine to account for the cost of vaccines. If a person was already covid-positive, recovered or vaccinated, the utility decreases by 50 if we choose to vaccinate, since it will not be beneficial to vaccinate them. Added symptoms also reduce the utility - 100 for death, 10 for covid and 2 for headache for covid-positive and 1 for headache for covid-negative people (since side-effects are not desirable but are still tolerable). In particular, we thus trade-off the cost of vaccinations, the risk of deaths, increased infection rates as well as symptoms and side-effects.

We optimize the policy according to the utility. Ideally we would optimise the actual utility

$$u(\pi, \mathbf{y}) = \mathbb{E}[u(\mathbf{a}, \mathbf{y}) \mid \pi] = \sum_{\mathbf{a}} \pi(\mathbf{a} \mid \mathbf{x}) u(\mathbf{a}, \mathbf{y}).$$

However, since we do not know the outcome $\mathbf{y}$ given action $\mathbf{a}$ and population $\mathbf{x}$, we could only maximise the *expected utility* under model $\mathbb{P}(\mathbf{y} \mid \mathbf{x}, \mathbf{y})$ defined as

$$U(\pi) = \mathbb{E}\big[u(\mathbf{a}, \mathbf{y}) \mid \mathbf{x}, \pi\big] = \sum_{\mathbf{a} \in \mathcal{A}} \pi(\mathbf{a} \mid \mathbf{x}) \sum_{\mathbf{y} \in \mathcal{Y}} \mathbb{P}(\mathbf{y} \mid \mathbf{x}, \mathbf{a}) u(\mathbf{a}, \mathbf{y}).$$

2 Privacy Analysis

In this section, we will address privacy concerns raised by the existence of the database and our forthcoming analysis.

2.1 Protecting the Privacy of Patients - The Database

While the database would typically be in the possession of a selected set of people, one could still be concerned of the privacy of each individual as the database contains sensitive information. So far, our goal has been to e.g. estimate the efficacy of vaccinations or predict the effect of specific treatments. To perform these estimations successfully we in fact do not need *precise* knowledge of any single individual's features, but rather an average result over the whole population. In order to *protect each individual's privacy* – also against the authors of this project – we aim to use an ε-Differential Privacy (DP) mechanism. For example we could use the *Randomised Response Mechanism* in the following manner: for the feature that corresponds to whether an individual has been tested covid-positive, we will toss a coin and set the answer to covid-positive[1] if the coin shows tails and otherwise do not change it. We thus simulate the Randomised Response Mechanism retroactively. The Randomised Response Mechanism then guarantees ε-DP.

2.2 Differentially Private Policies

One can reasonably expect that a decision rule for distributing vaccinations across the population has to be made public. Thus, we have to guarantee that the data of individuals that receive a vaccination is not being leaked (through deployment of this policy). We will protect the data of the people by using the *Laplace Mechanism* that guarantees ε-DP and thus bounds the maximal amount of data leaked about any single individual.

2.3 A Private Decision Making Mechanism

We implement the *Laplace Mechanism* by adding noise to the policy function π in the following manner

$$F(\mathbf{x}) = \pi(\mathbf{x}) + \mathit{Laplace}\left(0, \frac{s}{\varepsilon}\right).$$

The resulting output $F(x)$ will satisfies ε-DP. Here, s is the sensitivity of the policy and the noise is sampled from a Laplace distribution with center 0 and scale s/ε. In our case, the sensitivity is $s = 1$ because we allow only one action, with probability in the interval $[0, 1]$. We clip the resulting value to be between 0 and 1 to get a valid probabilities.

2.4 Estimating the Privacy-Utility Trade-Off

It is interesting to consider how much utility a decision-maker will loose by ensuring the privacy of each individual's data. To this end, we plot the utility of differentially private policies using different epsilon values. The results are presented in Figure 1. We observe that there is a clear trade-off between the guarantee of privacy (increasing ε) and the utility of the decision-maker.

[1] If the patient was tested covid-positive, we don't change anything

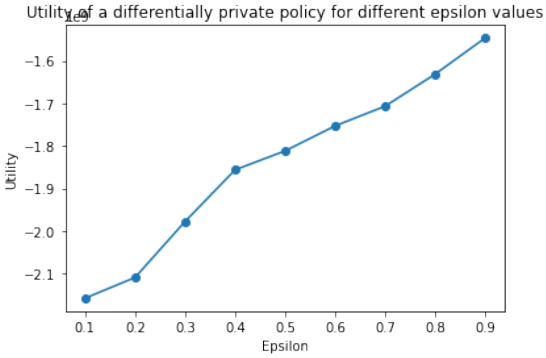


Figure 1: Privacy-Utility Trade-Off. We observe there is a cost associated with guaranteeing the (differentially) privacy of the policy.

3 Fairness of Policy

In this section, we explore the fairness of our policy with respect to the sensitive features gender, age and income. Firstly, we look for bias of the sensitive features in the training data. Then, we measured the fairness of our policy.

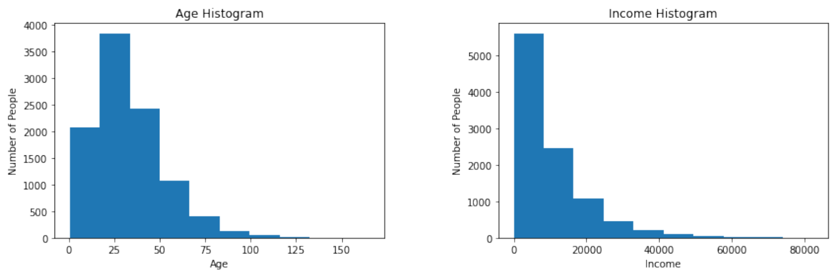
3.1 Definition of fairness

We focus on fairness in form of balance. For balanced decisions, a policy must make decisions independent from sensitive attributes, here denoted $\mathbf{z}$. More precisely, the probability of an action $\mathbf{a}_i$ concerning individual $\mathbf{x}_i$ must be independent from the sensitive attribute $\mathbf{z}_i$:

$$\mathbb{P}^\pi(\mathbf{a}_i \mid \mathbf{x}_i, \mathbf{z}_i) = \mathbb{P}^\pi(\mathbf{a}_i \mid \mathbf{x}_i).$$

3.2 Sensitive features in the training data

Bias in representation of sensitive features in the training data can affect the fairness of the policy as the optimization of the policy can be weighted more in favour of particular subgroups of the features. We therefore examined the frequency of the sensitive feature in the training data. Gender was approximately equal between the sexes, however, age and income differed substantively (Figure 4).

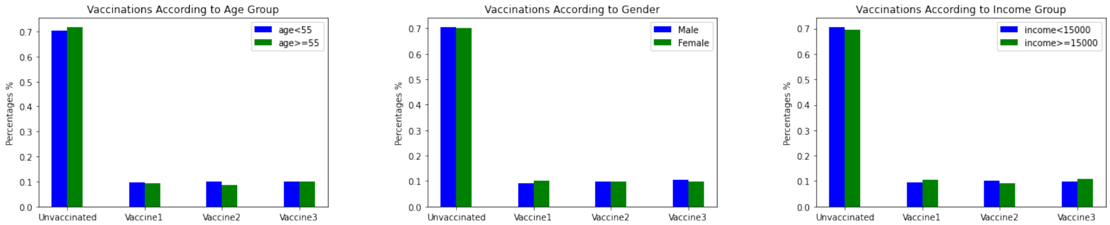


(a) Age Historgram (b) Income Histogram

Figure 2: Age and gender distribution in training data

3.2.1 Bias in the policy used for data collection

In this section, we (briefly) address the question of whether the policy used for data collection includes bias. To this end, we provide the following three histograms that show that the policy used for the study collected data proportionally to population sizes and thus in a balanced way.



(a) Age Groups (b) Gender Groups (c) Income Groups

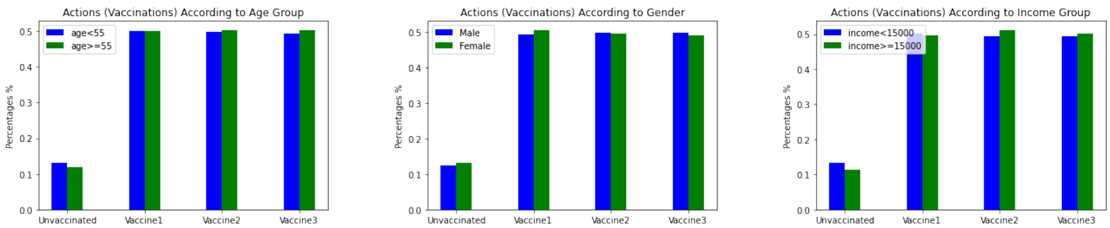
Figure 3: Proportions of unvaccinated and vaccinated individuals according to age, gender and income from the data collection. We observe balanced data elicitation among age, sex and income groups.

3.3 Measuring fairness of our policy

Note that we are currently using a simplistic policy that only allocates vaccine number three. In a next step (next deadline), we will derive an improved policy. These analyses will then be performed for the updated policy again.

3.3.1 Balance of our policy

In a first step, we are interested in whether our policy allocates resources, that is, vaccines, fairly. Here, we say that a fair allocation corresponds to a balanced allocation, i.e. an allocation proportional to the group size. We observe that our current policy indeed allocates vaccines proportional to the group sizes (see Figure 4).



(a) Age Groups (b) Gender Groups (c) Income Groups

Figure 4: Decisions made by our policy. We see that our policy makes balanced decisions among demographic groups. Note that for the sake of simplicity our current policy only allocates vaccine no. 3. We will extend the policy to allocate all three vaccines in a next step.

3.3.2 Measuring variation in outcomes

It is also interesting to look at the variation of outcomes for demographic groups. For instance, as for the case of the Astra Zeneca vaccine one may identify a higher risk of strong side-effects for female patients. To this end, we compare the observed symptoms of unvaccinated and vaccinated individuals for different demographic groups in Figures 5-7.

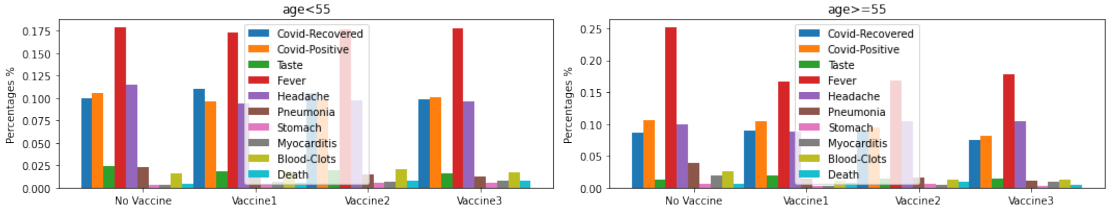


Figure 5: The likelihood of outcomes (symptoms) for two different age groups.

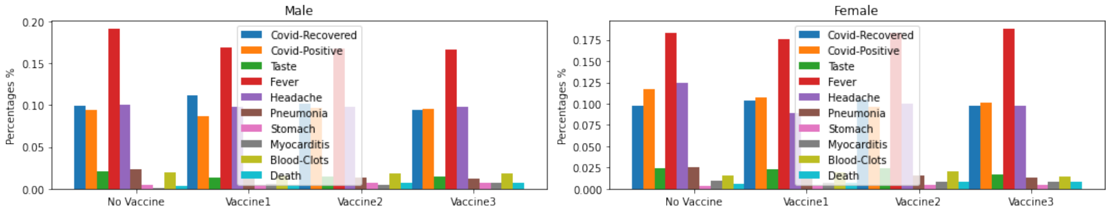


Figure 6: The likelihood of outcomes (symptoms) for male and female patients.

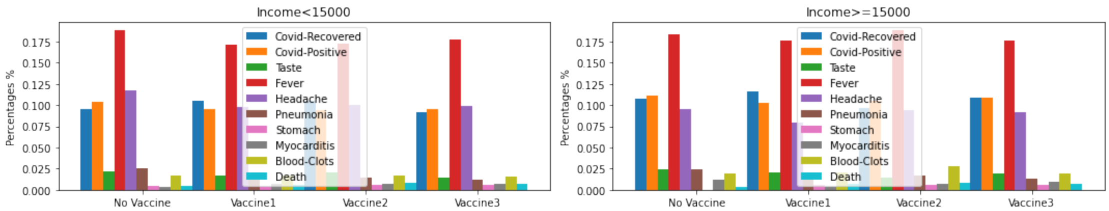


Figure 7: The likelihood of outcomes (symptoms) for two different income groups.

4 Experiment Design

4.1 Estimating the Historical Policy

We have access to the historical policy, as we have access to the code. However, we can also estimate it by, for instance, running multi-class logistic regression. Here, we assume for the historical data set that before the historical policy was deployed no individual was vaccinated. However, our estimation of the historical policy is apparently bugged, since it gives us a policy that never vaccinates. As a result, it achieves zero utility (according to our definition of utility). In some sense, the estimated historical policy thus 'hacks' the utility / 'cheats'. (Again, this is most likely due to a bug in our code.)

4.2 An Improved Policy

We use the predictive model $\mathbb{P}(\mathbf{y} \mid \mathbf{x}, \mathbf{a})$ (same model used as in Project 1) to obtain an improved policy, i.e. a policy that achieves higher expected utility than the historical policy. The improved policy is computed by evaluating the utility of vaccinating (with vaccine 1 or 2 or 3) or not vaccinating an individual and then choosing the utility maximising action for this individual. For instance, for individual i we compute the utility of vaccinating i with vaccine 1, 2 or 3 and the utility of not vaccinating i at all and then choose the action that yields highest utility, e.g. vaccinating i with vaccine 1. Note that the utility of an action is computed using the predictive model as we do not know the true utility of any action.

4.3 Expected Utility on Historical Data

In Figure 8, we compare the expected utility of the actual historical policy (taken from the code), our estimated historical policy, and the improved policy on the *historical data*. We average over 10 runs, where a run corresponds to sampling actions from the respective policy. As a result, we also obtain error bounds on the calculated utility - here provided as the standard deviation of the computed utilities. We observe very tight error bounds, which means we obtained reliable results. In general, we see that the improved policy is in fact performing worse than the historical policies. We suspect that is due to the predictive model making incorrect predictions.

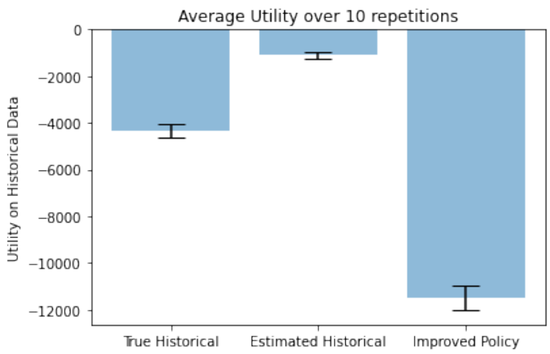


Figure 8: Expected utility of the historical and improved policy on simulated data including error bounds.

4.4 Expected Utility on Simulated Data

Similarly to before, we compute the expected utility of historical policies (true and estimated) and the improved policy on *simulated data*. Again, we provide error bounds by repeatedly sampling from the policies and computing the utility of the action. We observe that there is fairly low variation in the computed utility over the runs. We also observe on the simulated data that the (arguably) improved policy performs worse than the historical policies.

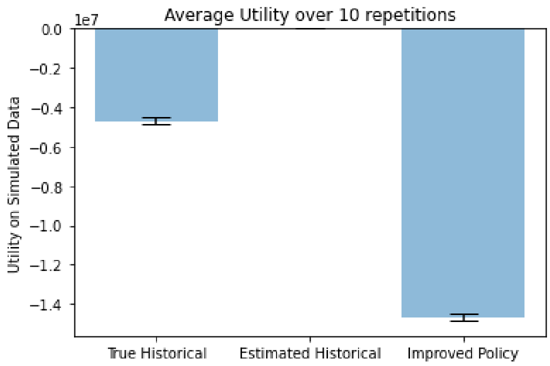


Figure 9: Expected utility of the historical and improved policy on simulated data including error bounds.

5 Code

The code is available under: colab link.