Observations from a numeric simulation of a small solar system

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Knowledge of remote solar system is most often achieved through interpretation of indirect observations, which is associated with uncertainties. To confirm if a particular model is appropriate a simulation of the model and observations of the simulation can be valuable. I have here simulated solar system composed of one star and 3 planets, where the planets mass was in the order 1:1000 to 1:1000 000 of the star mass. The simulation is based on Newton's gravitational force between the planets and the star, omitting the forces between the planets. The inclinations angle of all the stars to line of observation sight was 90°. The star had maximum radial velocity of 8.343 m/s and peculiar velocity of – 4.562 m/s. A simplified model of relative flux from the star is used to characterize the eclipse of the planets. During eclipse the fluxes was reduced in the order 10^{-5}- 10^{-3}.

To evaluate the significance of disturbances of the data Gaussian distributed noise was added. Adding noise with a standard deviation of one fifth of the maximum radial velocity did not influence the estimates the velocity, however information about the orbit of the less massive planets was masked. Addition of noise to the relative flux measurements was directedly proportional to information about eclipse as the eclipse basically caused a spike like change in the flux data, which could not be distinguished from noise spikes.

Introduction

Astrophysicists have developed a battery of complex method to gain information and knowledge of other solar systems. However, these methods are often relatively indirect and they also depend on intricate measuring equipment [1]. Both of these can lead to uncertainties about the truthiness of the interpretation of data. An attractive approach to this problem is to build models of solar systems of interest and compare observations of the models with real data.

Here, I have build a model of a solar system with one star and three relatively large planets using numerical simulations based on classical mechanics. I have included the forces between the planets and the star but omitted the forces between the planets. The operative force is Newtons universal law of gravitation.

Two of the more successful methods in astrophysical research in discovery of extrasolar planets is the radial velocity method and the transit method [1, 2]. The radial velocity method uses the doppler effect of the emitted light from the star to measure the velocity of the star relative to the point of observation. This is called the radial velocity as it is radial to the observation point (se fig. 1). Changes in the radial velocity of the star confirms the presence of a gravitational force by a nearby planet. I have here calculated the radial velocity of the model. The transit method measure changes in flux as planets eclipse the star, which means that the planet is in orbit position between the star and the observation point. I have calculated the change in relative flux of the star as the planet move around the star by implementing a simple model of flux reduction during the planets eclipse. Finally, I have tested the addition of Gaussian noise in order to acquire an impression of the significance of disturbances in the observational data.

Method

Model

The solar system is composed of one star and three planets. Se table 1 for information about masses, radii and initial velocity and position parameters. The line of sight, which is the line between the observation point and the mass center of the solar system, is along the x axis (fig 1). For all the planetary orbits the inclination angel of the orbits is set to 90°. The inclination angel is defined as the angel between the line of sight and the normal vector of the orbital plane (fig 1). The model can therefore be described solely by an x-y coordinate system.

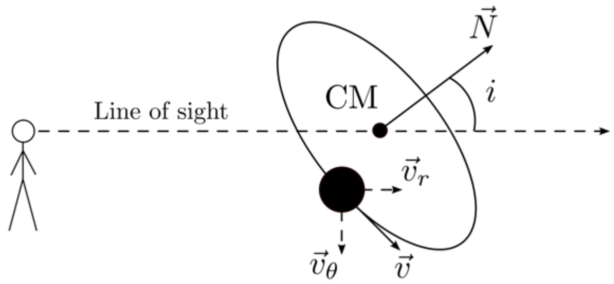

Figure 1. Illustration of the geometry of data recording from a solar system. The large dark circle represents a star orbiting the center of mass, CM, of its solar system. The star has a velocity *v*. From earthly observations the trajectory can be decomposed into a radial component, v_r pointing directly away from us and an angular component, v_θ pointing normal to the line of sight. v_r can be recorded through the Doppler effect. The position of the normal vector of the orbit plan N, and the inclination angle i, is shown. Copied with permission from Frode Hansen.

Table 1. Overview of the masses, radii, initial position and velocities of the planet and the star in the solar system. Astronomical units (AU) is the distance between the earth and the sun.

Object	Mass (earth masses)	x-position (AU)	y-position (AU)	x-velocity (m/year)	y-velocity (m/year)	Radius (km)
Planet 0	3.18	1.943	0.0	0.0	30053	9261
Planet 1	0.620	0.260	2.649	-25799	3605	6610
Planet 2	814	-16.89	5.594	-3667	-9465	84986

	(sun masses)					
Star	1.99	0	0	0	0	$1.137 \cdot 10^6$

The forces driving the motion of the objects is Newton's law of universal gravitation given by

$$\boldsymbol{F} = -\frac{G\, m_1\, m_2}{r^2}\boldsymbol{e}_r,$$

where $\boldsymbol{F}$ is the force, G is the gravitational constant, m_1 and m_2 is the mass of the two objects attracting each other, r is the distance between the objects and $\boldsymbol{e}_r$ is the unit vector from object one to object two. As the forces of interest here is the attractive forces on object one while the unit vector points from the object two to object one a minus sign is included in the equation.

Newtons second law of motion ($F = ma$) is used to transform the forces into movement. This leads to a second order differential equation which is solved numerically be the Euler-Cromer method. I have previously described this approach thoroughly [3]. Briefly, the equation is first converted into two coupled first-order differential equations. Secondly, the equations are converted into numeric relation where the infinitesimal is substituted for a real value Δt by assuming linear acceleration between the time steps, giving

$$\boldsymbol{v}(t+1) = \boldsymbol{v}(t) - \frac{G\, M\, m}{r^2}\boldsymbol{e}_r \Delta t$$

$$\boldsymbol{r}(t+1) = \boldsymbol{r}(t) + \boldsymbol{v}(t+1)\Delta t,$$

where t is time, v is the velocity and r is the position vectors.

Radial velocity

The velocity along the line of sight is called the radial velocity (fig. 1). In the model presented here the line of sight is long the x-axis, thus, the x-component of the velocity is the radial velocity.

Relative flux

The flux from the star is constant besides during eclipse. Because of the relatively short distance between the planets of the solar system and the star compared to the distance to the observation point the relative reduction in flux during eclipse is proportional to area of the star disc minus the area of the eclipsing planet disc. Thus, the relative flux becomes

$$flux = \frac{A_{\text{star}} - A_{\text{planet}}}{A_{\text{star}}},$$

where $A_{\text{star}}$is the area of the star disc and A_{planet} is the area of the planet disc enveloped by the star disc. When A_{planet} is outside the star disc the relative flux becomes one. When the A_{planet} is fully enveloped by the star disc the relative flux becomes

$$flux = \frac{R_{star}^2 - R_{planet}^2}{R_{star}^2} = 1 - \frac{R_{planet}^2}{R_{star}^2}, \quad (1)$$

since area is πR^2, where R is the radius of the star and the planets. For the situation in-between these two extremities I have made a simplification. I have assumed that because of the large star compared to the planet, the segment of planet disc in front of the star is defined by a straight line through the planet area. Furthermore, I have estimated the area of the planet in front of the star to be half the area of an ellipse with semi major axis equal to the planet radius and semi minor axis equal to the length of overlap. This gives

$$A_{\text{planet}} = \pi \frac{R_{planet}(R_{planet} + R_{star} - d)}{2},$$

where *d* is distance between the center of the star and the center of the planet. At half eclips this gives correct values, but at other position it will underestimate the reductions in flux. The equation of relative flux based on these assumptions becomes

$$flux = 1 - \frac{0.5 \cdot R_{planet}(R_{planet} + R_{star} - d)}{R_{star}^2}. \qquad (2)$$

Addition of noise

In order to understand the impact of noise on data of the model, Gaussian distributed noise with the mean value at zero and equal standard deviation at all time points is added.

Results

A graphical representation of the orbits in the solar system over a time period of 300 years in figure 2. The outermost planets used approximately 56 years on one orbit. The orbits are slightly moved with time which suggests a drift of the solar system. The orbit of the star follows almost the movement of the position of a point on a rolling wheel, which suggest that the solar system drifts the length of trajectory of one star orbit. The time period of full orbit of the star is close to 100 years

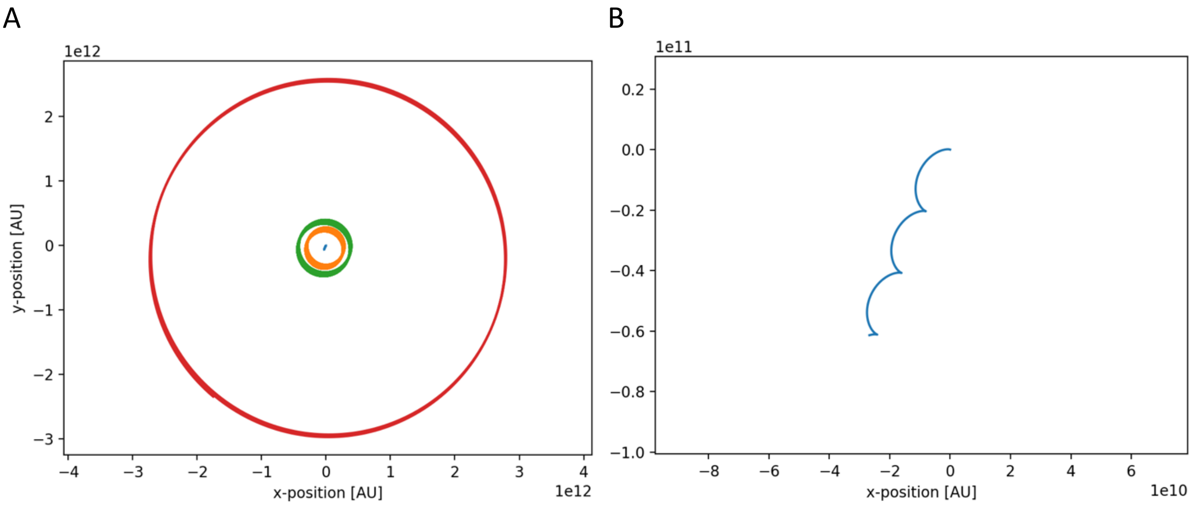

Figure 2. A) The orbits of the three planets and the star. A drift with time is reviled. B) The orbits of the star indicate the orientation and velocity of the drift. The simulation is done over a time period of 300 years with timesteps of 100 hours.

Radial velocity

The radial velocity has max value of 8.343 m/s. The change in velocity over time appeared to be composed of a combination of two sine functions (fig. 3). The most prominent sine had time period of approximately 56 years which is in accordance with the orbit time of the heaviest of the three planets. In addition, there was a more frequent sine with period of around two years, which is the in accordance with orbit time of next heaviest planet.

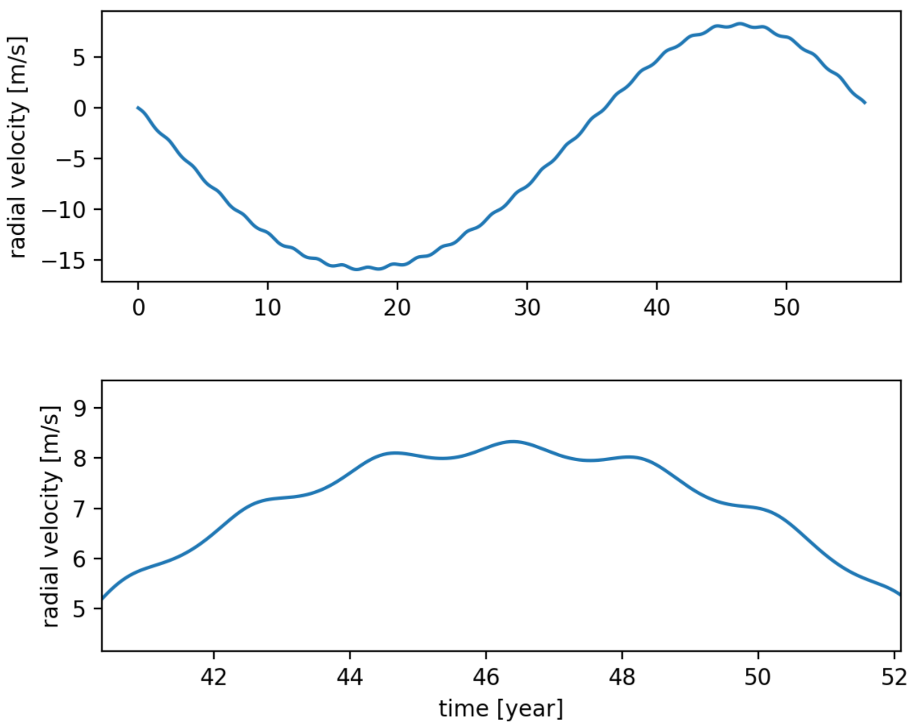

Figure 3. The radial velocity of the star. Visual inspection the clearly shows a sine like curve with period around 56 years (upper panel). Closer inspection revealed another sine with period of approximately 2 years (lower panel).

Flux and eclipse

Figure 4 presents the relative flux data of the model. The simulation was done with a time resolution of 0.001 years. Notably, all the reduction in flux had a step shape. This is most likely because the time resolution of the simulation is not small enough to pick up the period from start of the eclipse to full eclipse or equivalently at the end of the eclipse. I also tested with time resolution of 10^{-5} years with identical result. The decrease in relative flux at each eclipse is in the order 10^{-5}- 10^{-3} this is in accordance with the relatively large star disc compared to the planets.

The reduction in the relative flux was highest for the outermost planet and smallest for the intermediate planet. This correlated well with the area of the disc of the planets. The time length of the individual eclipses was longest for the outermost planet and progressively shorter for the planets closer to the star (data not shown). This is in accordance with the time it takes for the different planets to pass the star. Furthermore, the innermost planet showed the highest frequency of eclipse and the outermost the lowest frequency. This correspond well with the differences in orbit frequency of the planets.

The time period is near regular, but not fully regular. This is most likely because the star is orbiting with a different frequency of the planet causing a small change in the period over time. Of importance, the eclipse frequency of the planets is twice that actual eclipse as the equations simulating the eclipse also include when the planets pass that star on the opposite side of the observations point (equation 1 and 2).

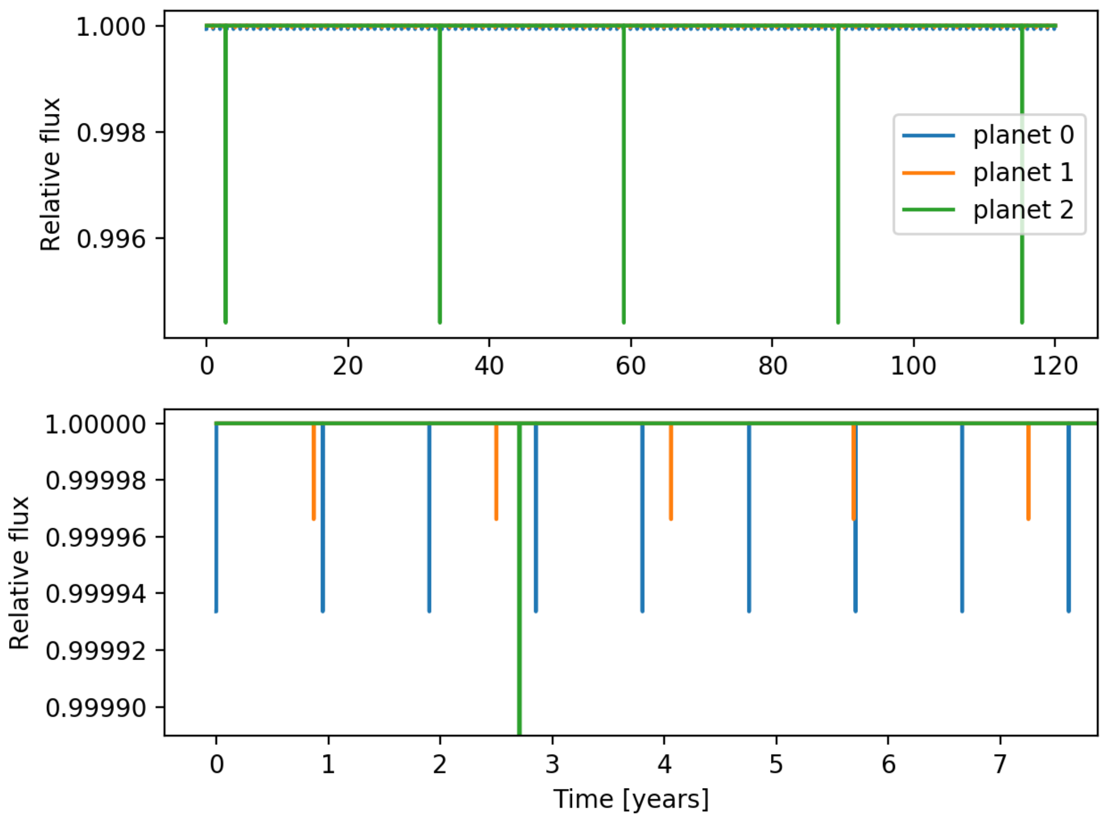

Fig 4. The horizontal lines in the flux curves represent the planets eclipsing the star. In the upper panel the dominant flux change is caused by the outermost planet due to its

substantially larger disc area then the other planets. The lower panel shoves the eclipse of the two innermost planets. The eclipse frequency shows a near regular time pattern.

Noise analysis

Addition of noise with a standard deviating of 20% of the maximum radial velocity did not mask the data's sinusoidal pattern caused by gravitational force of the most massive planet (fig. 5). However, with respect to the next largest planet it was not possible by visual inspection to distinguish an underling sine function.

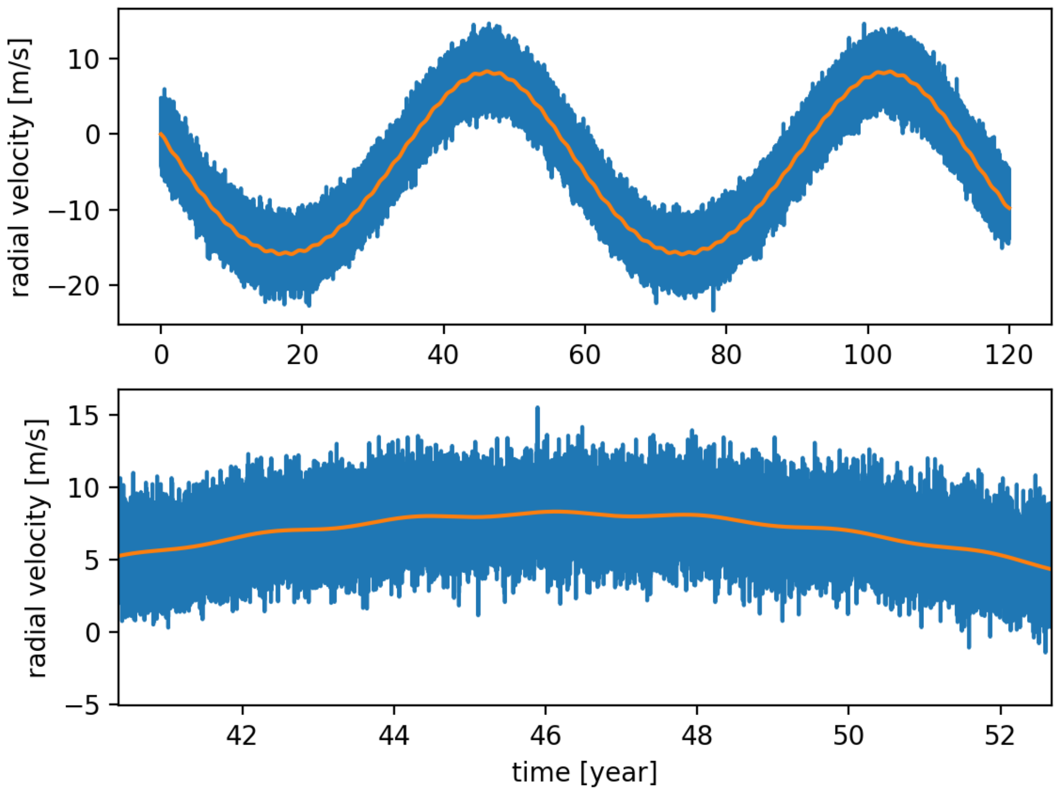

Figure 5. The radial velocity supplemented with normal distributed noise. The orange curve is the radial velocity without noise. In the upper panel the change in velocity over time due to the most massive star is easily recognized. The lower panel shows the difficulties of render the time pattern caused by the next most massive planet.

Addition of noise with a standard deviating of 0.2 to the relative noise data completely masked all the eclipse of the planets (fig. 6). As the eclipse of the planet with the largest disc only lower the flux by 0.005 this is as anticipated. E.g. lowering the noise to 0.001 uncovers the eclipse of the largest planet.

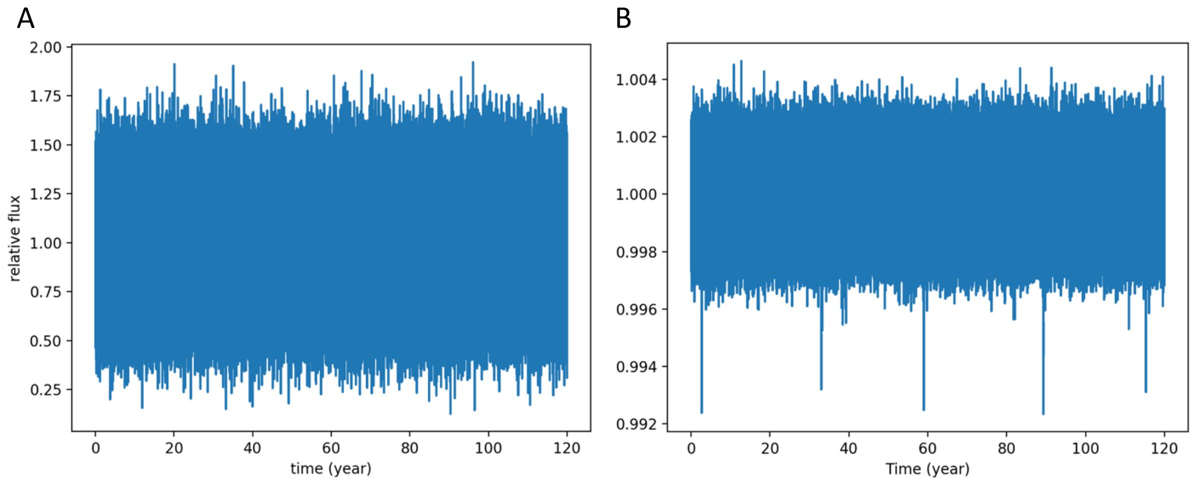

Figure 6. Addition of noise to the flux data. A) a noise level with standard deviation of 0.2 completely masks all information about eclipsing planets. B) Decreasing the noise to standard deviation of 0.001 makes it possible to observe the eclipse of the planet with the largest disc area.

Discussion and conclusion

I have here simulated a solar system using only forces between the star and the planet. Omitting the forces between the planets is a most likely of little significance since the star has a mass of 1000 times the largest planet. The radial velocity reassembles sine functions, which is the expected from the near circular movement of star around its mass center. Indeed, the data obtained from the simulation reassembles data from analysis of similar data obtain from a real solar system [2]. The solar system is drifting. This is because the initial conditions of the simulation promote a velocity of the mass center of the solar system (called peculiar velocity) [see 1 for more thorough explanation]. The model has potential for analyzing the significance of noise in the recording of real data. Here, I have only described this qualitatively, but this could be expanded to analysis using algorithms as e.g the least square methods [2]. A flaw in the simulation of the flux change during eclipse case doble count of eclipses. This can be repaired by introducing a condition of velocity sign since the radial velocity change sign around time of eclipse. In conclusion the model appears mostly as anticipated.

Reference

[1] Hansen, F. K., 2020, (2020) AST2000 Lecture Notes; part 1C
[2] Kielland, A., 2020, AST2000 Solution to Exercise 1C.4
[3] Kielland, A., 2020, AST2000 Solution to Exercise 1B.7