Discovery of extrasolar planets by examining the light emission from their respective stars

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Characterizing objects within far remote solar system is beyond the reach of telescopic methods. However, examining properties of the light emitted from the star can give information of orbiting planets. I have here examined variation in the wavelength of the H-alpha spectral line of five stars to determine existence of orbiting planets and further characterize their properties. Changes in wavelength caused by the doppler effect was used to estimate radial velocity of the stars. Potential eclipsing planets was determined by change in flux. These data were further used to estimate the mass of the planets and to create models of the planet's radial velocities. The model was obtained by fitting a sine function to the data using the least squared method. I discovered orbiting planets around four of the stars. Three showed eclipse, which enabled estimates of the mass and radial velocity. The masses were between 5.5 and 20.3 earth masses and the radial velocity were between 8.4 and 34.2 m/s. In the non-eclipsing stars only a lower bound of the mass and velocity cloud be estimated. I also examined the eclipse data with the goal of revealing the planets radius, however, the time resolution was too low. Hence, the density of the planets could not be estimated.

Introduction

The vast amount of astrophysical research suggests high prevalence of stars with orbiting planets [Hansen]. There are two main strategies for discovering planets. Direct imaging by telescoping and indirect methods based on changes in the light pattern of stars orbited by planets. Telescopic imaging is limited by their angular resolutions, which means that distance between objects, as a star and a planet, and the distance between observation point and objects determines the detection potential (Hansen 2020). These are both highly relevant parameters in the universe were extreme distances is a prominent factor. Thus, telescoping is limited to nearby solar systems.

To gain information from more remote solar systems indirect methods have been fruitful. Here, I have used two of these methods, the radial velocity method and the transit method to explore five solar system with the goal of finding orbital planets and further characterize the planets velocity, mass, radius and density. To surmount the noise in the data I have generated a simple model of the radial velocity of the star through a least square fit to a sine function. The estimation methods I have used assume that the source of the physical data is only from one star and one planet; a so called two-body problem (Hansen 2020). The examined data is provided by numerous observation stations around the earth and contains flux and wavelengths over time (Hansen 2020). The data also contains the mass of the stars.

An attractive question is if there exist life external to earth. The life type we know is dependent on liquid water. Thus, as the data provided by these methods, as density of the planets, can be used to confirm the potential presence of liquid water on other planets, they can help in reveling this question.

Methods

The radial velocity method

In this method, first the star velocity relative to an observation point on the earth is determined. Thereafter, the velocity of the center of mass of the stars solar system is determined by averaging over time. The latter is called the peculiar velocity. Radial velocity has various definitions. Here, it is defined as the velocity component oriented in radial direction from the observer along the line of sight subtracted the peculiar velocity (Fig 1).

In a solar system containing a planet that have a relatively large mass compared to the star the center of mass of the solar system will be localized distal to the star. The star will orbit this center of mass. The center of mass has a near constant peculiar velocity with respect to the center of mass of our solar system (Hansen 2000). If there exist a large enough planet in a solar system, the radial velocity of the star will change relative to us, as it orbits the mass center, in an extent that is observable from the earth. If there is no planet in the solar system or if the masses of the planets are small compared to the star the center of mass will be in or close to the center of the star. Consequently, such a radial velocity change will not exist or not be observable.

There are two challenges with the method. Firstly, the earth as an observation point is not in the center of our solar system causing a relative change in velocity of the observation point too. However, this velocity is well characterized and is adjusted for in the data set provided (Hansen 2020). Secondly, a more vital challenge is the orbital angel of the star relative to the direction of observation (fig 1). The orbit of the star forms a plane. The angel of the normal vector to this plane and the line of sight is called the inclination. If the inclination is 90°, the radial velocity measured halfway between the stars nearest and most distance point relative to us will represent the radial velocity of the star. However, if the inclination angle is 0°no change in radial velocity can be observed.

The doppler effect can be used to measure the radial velocity. The doppler effect is a phenomenon occurring when two objects are moving relatively to each other. In this situation, light emitted from one object will be observed with a different wavelength at the other object (Hansen 2020). The observed wavelengths will be shortened if the objects are closing in on each other and prolonged with increasing distance. This relation between wavelengths and velocities is given by the doppler formula

$$v = \frac{\lambda - \lambda_0}{\lambda_0} c, \qquad (1)$$

where *v* is the relative velocity between the two objects, λ_0 is emitted wavelength and λ is observed wavelength (Hansen 2020). λ_0 is here the H-α spectral line with a wavelength of

656.28 nm. This spectral line is caused by absorption of light by hydrogen gas in the atmosphere of the star (Hansen 2020). As c I have used the speed of light in vacuum.

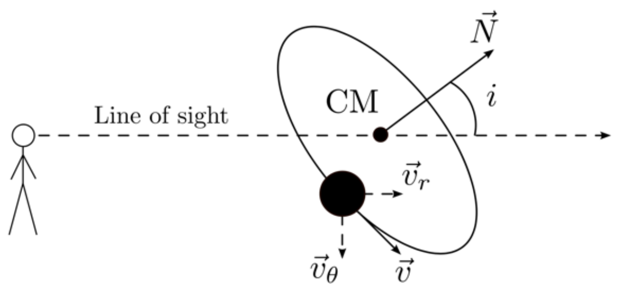


Figure 1. Illustration of the geometry of data recording from a solar system. The large dark circle represents a star orbiting the center of mass, CM, of its solar system. The star has a velocity *v*. From earthly observations the trajectory can be decomposed into a radial component, v_r pointing directly away from us and an angular component, v_θ pointing normal to the line of sight. v_r can be recorded through the Doppler effect. The position of the normal vector of the orbit plan N, and the inclination angle i, is shown. See method for explanation of these terms. Copied with permission from Frode Hansen.

The transit method

In this method the amount of flux from the star is recorded over time. If a planet during orbit passes between the star and the point of observation the observed flux may be reduced. From the flux change during the transit, the radius of the planet can be estimated (fig. 2). The method depends on knowledge about the radial velocity of the planets orbit v_p. This can in a two-body situation be achieved from the masses of the star, m_* and the mass of the planet m_p and the radial velocity of the star with this equation

$$v_p = v_{*r}\frac{m_*}{m_p} \tag{2}$$

(Hansen 2020). v_{*r} is the radial velocity of the star. The equation is derived from the Newtonian solution to the two-body problem which assume only gravitational forces between the two objects. The mass of the star is often known from spectroscopic measurements (Hansen 2020). The mass of the planets, m_p can be estimated with

$$m_p \sin i = \frac{m_*^{2/3} v_{*r} P^{1/3}}{(2\pi G)^{1/3}} \tag{3}$$

(Hansen 2020). Where m_* is the mass of the star v_{*r} is radial velocity of the star, *P* is the period of the system, *i* is the inclination angle and *G* is the gravitational constant. In this equation the planets orbit is assumed to be circular. Furthermore, as the inclination angle is not directly measurable this equation will only give an acceptable mass estimate of eclipsing planets. Eclipse imply an inclination of near 90°. In case of other angles only a lower bound of the mass can be estimated. Thus, by taking the time it takes from the start of the eclipse to the planets disc is just fully enveloped by the star disk and multiply by the velocity of the planet with respects to star, the following equation reveals the radius R

$$2R = (v_{*r} + v_p)(t_1 - t_0) \tag{4}$$

(Hansen 2020). Where the symbols for the velocities, *v* is as in equation 3 and the t stands for time points that is described in figure 2.

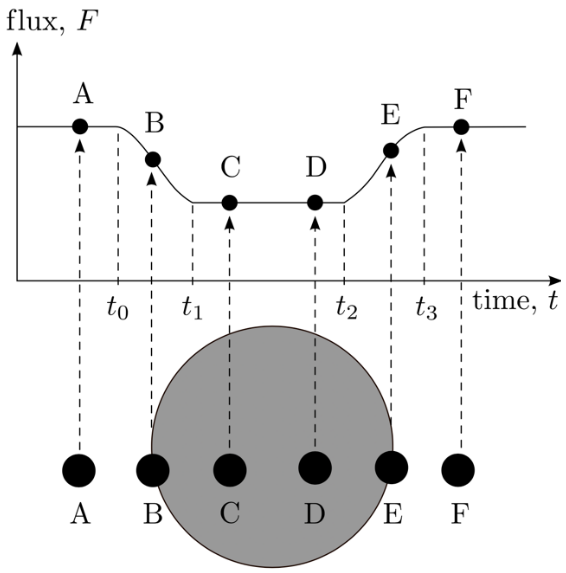


Fig. 2. Schematic representation of the time development of a planet eclipsing a star. The capital letters denote the position of the planet at different timepoints, the graph shows the change in flux. The eclipse starts at t_0 and ends at t_3. At t_1 the disc of the planet is just fully in front of the star disc and at t_2 the planet disk is beginning to leave the star disk. Copied with permission from Frode Hansen.

From the radius the volume can be calculated, and in combination with the mass the density of the planets can be estimated with

$$\rho_p = \frac{3m_p}{4\pi {R_p}^3} \tag{5}$$

(Hansen 2020). Where ρ_p is the density of the planet, $m_p$is the mass of the planet and $R_p$is the radius of the planet.

Mathematical model of the radial velocity

If the inclination angle of the star orbit is near 90 degree and circular orbit is assumed the radial velocity follows a sine function (Hansen 2020) as

$$v_{*r}^{\mathrm{model}} = v_{*r} \cos\left(\frac{2\pi}{P}(t - t_0)\right), \tag{6}$$

where v_{*r}^{model} is the radial velocity of the model, v_{*r} is the redial velocity of the star, *P* is the period, t_0 is time of max velocity and t is time.

To estimate the parameters for the model I used the least square method to make a best fit of the sine function to the data (Hansen 2020). In this method the quadratic sum of the distance between the data points and sine function is calculated for numerous combinations of the parameters. The parameters that leads to the lowest value is selected. Thus, the lowest value, Δ of the expression

$$\Delta(P, v_{*r}, t_0) = \sum_{i=1}^{N}\left(v_i - v_{*r} cos\frac{2\pi}{P}(t_i - t_0)\right)^2 \tag{7}$$

will be the best fit. The symbols are the same as in equation 6 with the addition of the indexing *i* that represents timepoints in the test range of the parameters as described below. The least square sum assumes normal distributed noise that is constant over time. Existence of normal noise is controlled by a qq-plot. Standard deviation of the noise is compared at various time points.

The method needs a test ranges for the three parameters. This are provided by a computerized algorithm that first select timepoints where the velocity curve changes signs, meaning goes from positive to negative values. Next the algorithm selects two zero crossings where the curve is positive in between. The distance between these two points is used as an estimate of half a period, *P*. The timepoints where the velocity is at its max value between the two points is used as an estimate of t0. The test ranges used in the method are *P* +/- 20% and t_0 +/- 20%. As test range of the radial velocity the max value minus 150% of the noise level too minus 50% of the noise level is used. The necessary number of values, *N* in the range was determine by visual comparison of the data with the model fit.

Results

Peculiar velocity

The peculiar velocity of the five star is calculated by first using equation 1 and then averaging. The respective data is presented in table 1. Besides star 3 all of the stars are coming closer to our solar system.

Table 1. Peculiar velocity of the five stars

Star	$V_{peculiar}$ (km/s)
Star 0	- 422.1
Star 1	- 259.6
Star 2	- 362.0
Star 3	176.4
Star 4	- 44.6

Qualitative evaluation of the solar systems

In order to build a first impression of the solar systems of the five stars I visually inspected a graphical representation the respective radial velocities of the stars around their centers of masses, and further the changes in flux over time (fig. 3). The star 0, 1, 2 and 4 show clear periodic changes in the velocity with curve chapes resembling sine functions. This suggest that the center of mass of their solar systems is remote to the star, which further suggest the presence of other objects in their solar systems. As the curves appear to be sinusoidal it is suggestive of a system composed of two objects: one star and one planet (a two-body system). The planet also needs to be of a mass relatively big compare to the star. I cannot observe any changes in the velocity of star 3, thus potential other object in its solar system likely have less mass than object of the other star. Velocities in range substantially lower than the recording noise, long or very short orbiting times would prevent detection by pure visual inspection. The noise in the systems seems of approximately same size for all measurements and similar along the recording periods.

Regarding the flux measurements star 0, 1 and 4 had clear dip in the flux level at two time points with distance in-between similar to the periods of the velocity curves (big fig). The time of the dip is at the location where the stars are in the orbit furthest away from us (zero radial velocity that in the next moment is negative). These strongly suggest that planets are eclipsing the stars during the orbit. For the stars not showing this dip in flux there are to my knowledge four potential explanations. Either the star has no orbiting planets, the time resolution of the measurement is to low, planets are not eclipsing, or the planets disc is to small compared to the disc of the star to absorb enough light that we can extract out of the noise of the data.

Flux data can be used to estimate radius of a planet by the transit method. This method needs time information about the start of the eclipse and when full disc eclipse. A closer look at the flux data of star 0, 1 and 4 show only one data point representing the eclipse time. Figure 4 shows the dip in the flux graph of star 4. Data from the other eclipsing star are not shown. In conclusion, the resolution limits the use of this method.

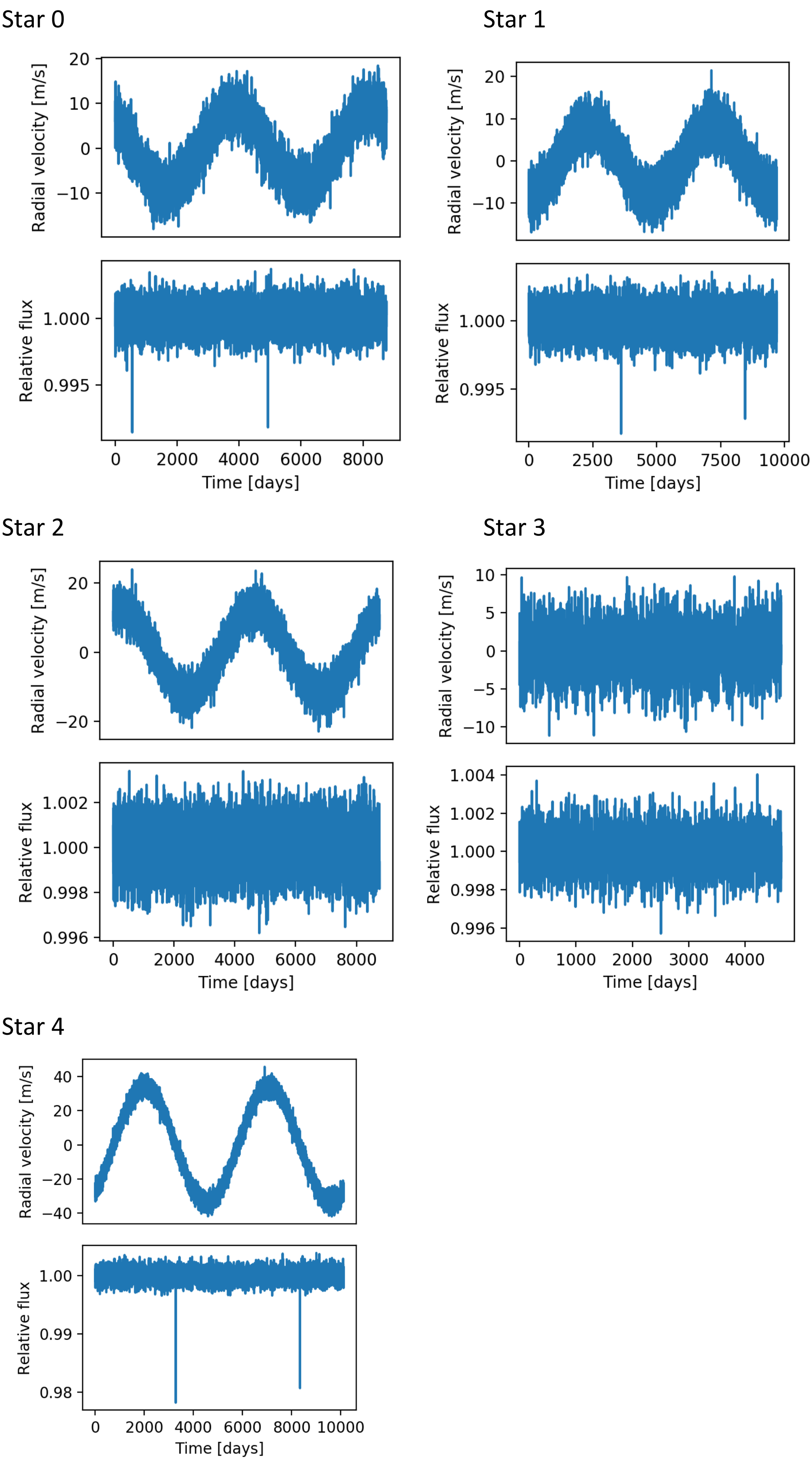


Fig 3. Graphs of the calculated radial velocity and measured relative flux of the five stars.

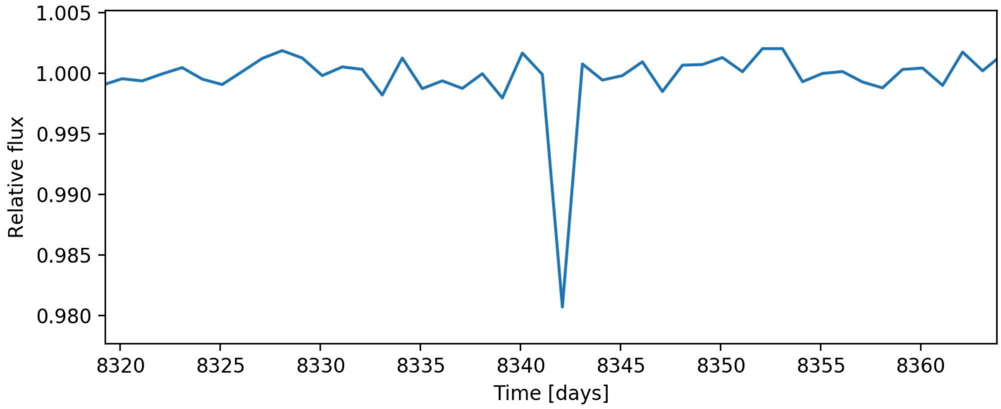


Fig 4. Timewise closer look at the relative flux from star 4. Only one timepoint detects the eclipse.

The mass of an orbiting planet can be estimated by equation 3. However, this require information about the inclination angle, which is not easily available. Nevertheless, a system with an eclipsing planet suggest and inclination angle of near 90°. A lesser angle will lead to an overestimate of the mass. Thus, using a visual estimation of the radial velocity and the periods in combination with knowledge of star mass, we can estimate a lower bound of the planet mass and in the eclipsing planet the estimate might be close to the real mass. See table 2 for estimated values and calculation of lower mass limits for the planets.

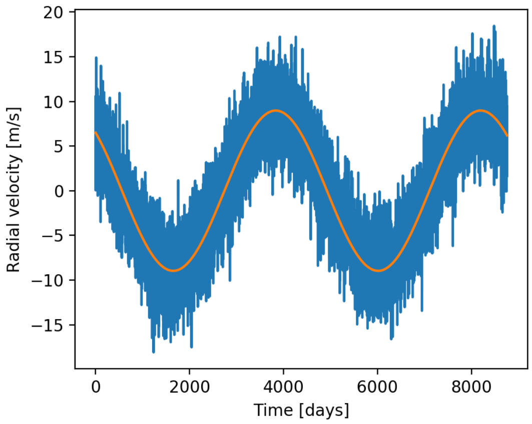
Tabell 2: Estimation of radial velocity V_{*r}, period, T and mass of the planets by visual inspection of velocity and relative flux curves of their respective stars. The masses of the planets orbiting star 0, 1 and 4 are estimates close to exact masses of the planet, while for star 2 the estimate is for a lower bound of mass.

Star nr.	Star mass [sun masses]	V_{*r} [m/s]	P [days]	Planet mass [earth masses]
0	1.63	8.2	4400	6.6
1	1.15	7.5	4700	4.9
2	0.84	13	2100	5.2
4	0.97	33	5000	19.6

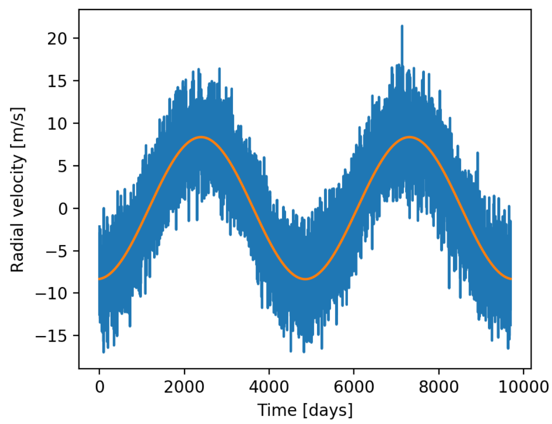
Curve fitting of star velocity

The least square method is used to fit a sine function to the velocity data of star 0, 1, 2, 4. The resulting parameters of the sine function is presented in table 3. Figure 5 display combined data of the recorded velocities and the models. Correctness of the models was evaluated by visual inspection. The curve of the sine functions was localized to the center of the noise in the data. The least square fit was not noticeable different for test ranges composed of 30 to 100 values. Bellow 30 the curves of some of the stars where a little out of phase. Mass of planets based on curve fitting meddles are presented in table 3.

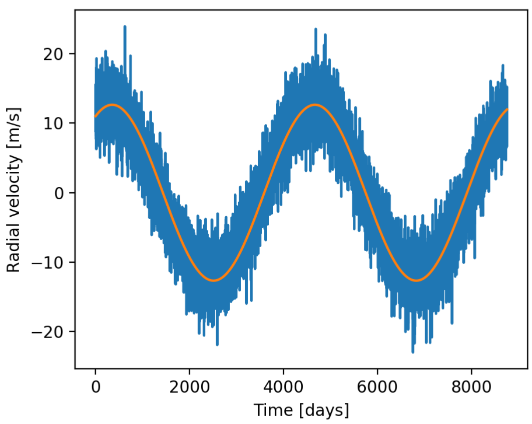
Star 0



Star 1



Star 2



Star 4

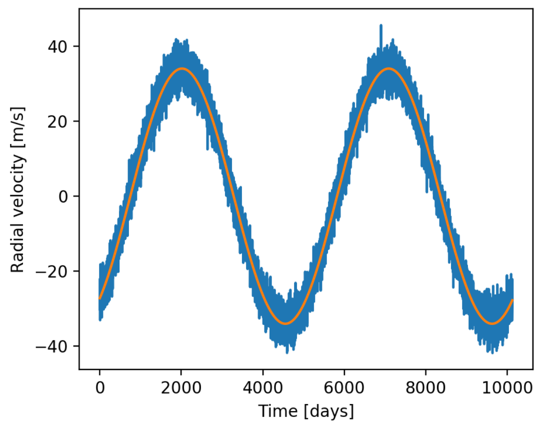


Fig. 5. The radial velocity based on Doppler analysis of spectral data overlayed be a sine function that is adapted to the data by the least square method (orange curve).

Table 3. Parameter of the best fit models and estimate of planet masses based on the model. The masses of the planets orbiting star 0, 1 and 4 are estimates close to exact masses of the planet, while for star 2 the estimate is for a lower bound of mass.

Star nr.	t_0 [days]	V_{*r} [m/s]	P [days]	Planet mass [earth masses]
0	3834	8.97	4363	7.16
1	2396	8.36	4914	5.52
2	4669	12.65	4316	6.49
4	2017	34.08	5073	20.3

This least square method assumes normal distribution of the noise over time (Hansen 2020). Linearity in a qq-plot of the noise data confirm that the noise is normal distributed (fig. 6). Only data from star 0 is shown as the noise of the other stars show near identical result.

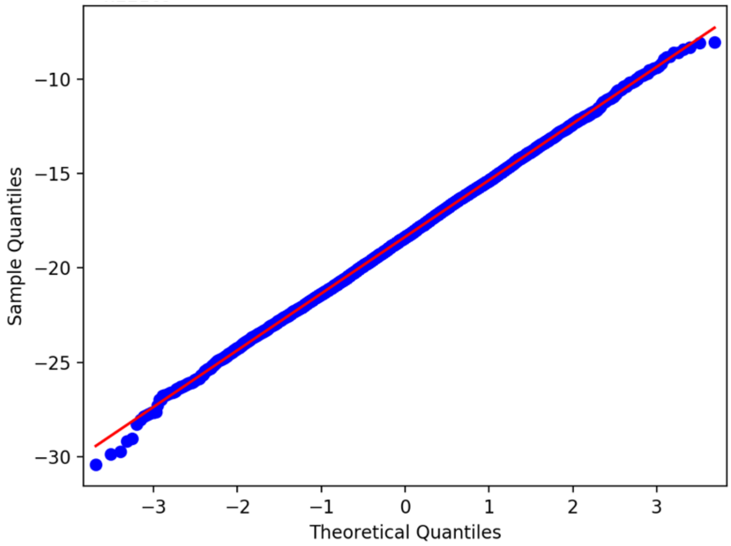


Fig 6. qq-plot of noise of star 0. The correlation of theoretical and sample quantiles is near linear supporting normal distribution of the noise in the data.

Discussion

I have here detected and estimated the radial velocity, period and mass of three planets in a remote solar system. Additionally, I have estimated a lower bound mass of a fourth planet. The masses are in the order of 5 to 20 times larger than the earth, while the stars are 0.8 to 1.6 of the sun. This is important for the sensitivity of the method as a large planet relative to the star will create a larger Doppler effect and this is probably the reason I observe clear sinusoidal changes in the radial velocity. To more thoroughly evaluation of the correctness of the masses the Newtonian variant of Kepler's 3 law could be used to estimate the distance to the center of mass for the planet.

I observed only one data point in the flux data suggesting eclipse. This is somewhat sparse and could be the result of some sort of error in the recording. However, the data points were separated by the time-period of the signal and was also localized at the time where the stars are furthers apart from us. This confirms the correctness of the eclipse data.

The masses based on visual estimates was in the worst case separated from the models bases approach by 8%. This relatively low number is not too surprising as the noise where stable and it was not too difficult to find point in the center of the noise.

Reference

Hansen, F. K., 2020, Forelesningsnotat 1C i kurset AST2000