**Numeric simulation of planetary orbits in a solar system**

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I have here simulated the orbit of eight planet in a one-star solar system by numerical simulations. I assume a two-body problem between each planet and the star ignoring forces between the planets. I have evaluated the numerical methods forward Euler, Euler-Cromer and Runge-Kutta four with respect to trajectory of the planets and energy stability. In addition, I have simulated a solar system with two stars and one planet including forces between all three objects. I found that RK4 is superior to forward Euler and Euler-Cromer and Euler-Cromer to be far superior to forward Euler. The eight planets of the on star solar system showed stable elliptical trajectories. The planet of the two-star solar system on the other hand shoved a peculiar trajectory and the planet appeared to be thrown out of the solar system with time.

**Introduction**

Of fundamental importance in understanding the astrophysical world is the description of the position and movement of celestial objects in relation to each other. Central to this is the orbiting behavior. Here I have examined the orbits of planets within two modeled solar systems using numerical approaches. One solar system is composed of one star and eight planets, while the other contains two stars and one planet. To characterize the orbits of the eight planets in a one-star solar system I only included gravitational forces from the star on the planets, omitting all forces from the planets. This reduces the problem to a classical two-body problem [1]. I used numerical simulation to solve the problem. However, the two body problem have an analytical solutions which describes the trajectory of the orbits as elliptical [1]. In the second solar system I include all forces between the stars and the planet. The system is not much more complex than the above described system, but nevertheless with no analytical solution [1]. Interestingly, about half of the know stars are binary stars [1].

In order to describe motions of astronomical objects, numerical approaches are of high relevance as the complexity of such systems is most often out of scop for an analytically approach. There are numerous numerical methods that are suitable for computer-based algorithms. I have evaluated here evaluated the forward Euler, Euler-Cromer, Runge-Cutta four methods as they are extensively in use and fast and easy to implement. Conservation of energy and deviation between trajectories of successive orbits is used to compare the and evaluate the stabilities of the various method.

The 2-body problem is a problem where you have two objects floating freely in space with given initial velocities and positions. The problem asks how the trajectory of object 2 looks like from the viewpoint of object 1. The analytic solution of the problem is

r=p 1+ecosf

where r is the distance between the objects, f is the angle from the perihelion in the orbit to object 2 (see Hansen,2019). Depending on the initial conditions, this solution gives an elliptical orbit, a parabolic trajectory, or a hyperbolic trajectory. In practice, a parabolic tra- jectory will never happen since it is only a borderline between an elliptic orbit and a hyperbolic trajectory.

The mass of the star was 1.99 sun masses. The masses of the planets is presented in table 1 together with their initial position and velocity.

Precise knowledge of orbits to nearby planets is necessary information for potential space voyages. Furthermore, planetary orbits will likely influence the environment of the planet and thus is of interest to one of the greatest questions to humankind: do their exits biological life on other planets. Surely, emphasizing the relevance of these types of studies.

Table 1. Overview of the masses and initial position and velocities of the planet in the solar system. Astronomical units (AU) is the distance between the earth and the sun.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Planet | Mass  (earth masses) | x-position  (AU) | y-position  (AU) | x-velocity  (m/year) | y-velocity  (m/year) |
| 0 | 3.18 | 1.943 | 0.0 | 0.0 | 30053 |
| 1 | 0.620 | 0.260 | 2.649 | -25799 | 3605 |
| 2 | 814 | -16.89 | 5.594 | -3667 | -9465 |
| 3 | 0.287 | -5.025 | -3.487 | 9871 | -13523 |
| 4 | 0.00179 | -12.518 | 1.465 | -1576 | -11721 |
| 5 | 0.136 | -7.246 | 1.671 | -3542 | -15219 |
| 6 | 0.0306 | 2.786 | -2.665 | 14460 | 15299 |
| 7 | 0.0171 | 1.194 | 0.499 | -15467 | 33936 |

**Method**

*Simulation of the one-star solar system*

In the one star-system I have modeled the planets trajectory based on the forces between the respective planet and the star. I have neglected interplanetary forces and external forces of the solar system. To simplify the problem further I placed the star in the origo and only characterized the orbit of the planets. Since the mass of the star is in order 1000 times the planet the movement of the star caused by gravitational pull from the planet is smalI. To simulate the force on the planets, I used classical mechanics which for this system follows Newton’s law of universal gravitation given by

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where is the force, *G* is the gravitational constant, *M* is the mass of the star, *m* is the mass of the planet, *r* is the distance between the star and the planet and ***e***r is the unit vector from the star to the planet As the forces of interest here is the attractive forces on object one while the unit vector points from the object two to object one a minus sign is included in the equation.

*Simulation of the two-star system*

I have approached the binary-star-one-planet system as a Newtonian three-body problem, which means I have used the same gravitational interaction (equation XX) as for the one-star system as described above but now with interactions between the three objects. Thus, the force acting on each object is now a sum of the forces from the two other objects.

Numerical integration methods

To translate force on the planets into description of motion I have used Newton’s second law of motion. Newton second law can be written as *F = ma,* which leads to *a = F/m*. *a* is the acceleration, which is the derivative of the velocity *v*. *v* is the derivative of the position *r*. Thus, this leads to a second ordered differential equation. To implement a numerical solution of the equation it is first written as two coupled first-order differential equations

The second step can be understood through the definition of the derivative. Her presented for *a*

This lim is converted to a numeric relation by assuming linear acceleration when the time steps are very small. Based on this assumption the infinitesimal *dt* is substituted for a real value

Solving for gives the equation

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Numerical implementation of change in position over time can be derived equivalently as for velocity. However, by changing *a* for *v* and *v* for *r* in the derivation above. This leads to

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Thus, for each time step firstly the velocity is updated based on the acceleration that is derived from the forces on the object. Thereafter the velocity is used to update the position. This method is caller forward Euler [1]. An extension of this method is the Euler-Cromer which use the velocity at the end of the step instead of to update the position. The Euler-Cromer is shown to perform better in energy conserved systems, which is relevant for the simulations here [2]. The following set of equations describes the total algorithm of one timestep for Euler-Cromer,

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The forward Euler use the slope at the beginning of the given step to calculate the next value, while the Euler-Cromer used the velocity slope at the end of the step. An even more refined method uses a combination of the beginning step, the final step, but also two intermediate steps to estimate the slope. This method is called the fourth order Runge-Kutta and it is regard as significantly more accurate than the two other methods [2 MAT-INF]. In principle the forward Euler is used four times and a slope value k is calculated at four locations. First a half step with the slope k1 calculated at the beginning of the step. Second another half step from the beginning, but now with the slope calculated at the end of the previous half step k2. The slope is also calculated at the end of this half step giving k3. Finally, a full step from the beginning with slope k3. The last slope k4 is calculated at the end of this final step. The actuall slope used to move on step uses the forward Euler with the weighted average of these four slopes

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where *k* is either *a* of *v* depending of whether it is the next velocity or position that is determined, respectably. It can be regarded as a combination of forward Euler, Euler-Cromer and evaluations at the midpoint of the timestep and is among the most popular numerical methods of differential equations [MAT-INF]

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Additionally, immobilizing/stationary and placing the star in the center of the system that can be solved analytically with the following solution (ref Hansen):

r = a(1-e^2)/(1-e cos (f))

the exentressity nedd to be e < 1 else the solutions which is of relevance for non elipitc situations. The sixe of e is dependent of the enrgy of the system and…

Thus I have studied each planet and the star isolated from the other planets. In order to do this, I neglect gravitational force between the planets. I also place the center of gravity in the star center . Furthermore, the external gravitational forces to the solar system is also neglected.

*Numerical integrations of the two-star system*

I have approached the binary-star-one-planet system as a Newtonian three-body problem, which means I have used all the gravitational interaction between the three objects. I have used the same numerical approach as described above besides the force acting on each object is now from the two other objects instead of one.

**Results**

To compare the three numeric methods, I ran the different algorithms with the same timestep of 526 hours for just enough time for the outermost planet to orbit one time (fig XX). The orbits, as known from the analytical solution of the two-body problem, are not supposed to deviate over time. From the plot of the trajectories it is clear that this is not the case for the forward Euler. The Euler-Cromer is substantially better, however a closer inspection reveal multiple trajectories of the innermost planets. The Rungo-Kutta 4 showed clear improvement over Euler-Cromer, but the innermost planet showed multiple trajectories.

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Fig. XX. Trajectories of the eight planets for 60 days with three different numerical methods.

As the planet orbit conserves energy, I compared the change in energy over time (Fig. XX)

To accomplish a more quantitative comparison the was calculated

Euler: 1.71662664e+14 2.39237723e+13 1.09756822e+15 5.43320277e+12

5.92159788e+09 1.56795875e+12 9.38599214e+11 8.44592730e+11

**Discussion**

Euler Cromer is known to be more stable than forward Euler (ref: Malte…) and RK4 even better. This is in accordance with my observations.

The mass of the star is in the order of 1000 times larger than the total mass of the planets indicating substantially less gravitational force form the planets compared to the stars.

* Star in origo and only forses from the star.

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Only put on gravitational force?

Dv/dt = lim v(t+dt) – v(t)/ dt

hat can be akcelarationChanning lim dt for delta\_t and put back into N2L gives

F/m = v(t+dt) – v(t)/ dt

v(t+dt) = v(t) +F/m\*dt

dr/dt = v = r(t+dt)-r(t)/dt

r(t+dt) = r(t) + v \* dt

Put togheter gives the following alorythm form change ion position over time by a Newtonina force.

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A variant of this is to use the velocity at t+dt instead of the one at t. This methos is called Euler cromer and is proven to be more stable in cyclic movement ().

Finnaly there is method called RK4.

Numerical integrations:

Euler Cromer

Hvor mye skal jeg si?

Aktuelt å sammenligne med andre metoder?

Combining the gravitational force and the numerical method based on the second law give the following equation for acceleration on the planets:

N2L med alle former

a = F/m

NG: - G Mm/r^2

From () we se that the acceleration on each planet is:

a = -G x M x **r** / r^3